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Performance Analysis of Time-Hopping Ultra-Wideband Systems in Multipath Fading Channels (Uncoded and Coded Schemes)

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In this paper, the performances of both uncoded and coded multiple access TH-UWB systems, introduced in [1-3] in multipath Rayleigh fading channels, are evaluated. The receiver is a selective diversity combining receiver, known as SRake. Based on a Gaussian distribution assumption for the multiple access interference at the output of the SRake receiver and by using a virtual branch technique, as introduced in [4,5], the bit error rates for uncoded and coded schemes are derived. The performance analysis shows that the effective order of diversity achieved by the coded scheme is the product of the number of branches of the SRake receiver and the Hamming distance of the code applied. Furthermore, the results indicate that the coded scheme significantly outperforms the uncoded scheme without requiring any extra bandwidth, further than what is required by the uncoded system.

INTRODUCTION

TH-UWB systems are spread spectrum systems that do not use sinusoidal carriers to raise the signal to higher frequencies. In this system, which was first introduced in [1,2], data is transmitted using a time-hopping baseband signal composed of subnanosecond pulses. Due to using subnanosecond pulses, the power spectrum of the system has an ultra-wide bandwidth, about several gigahertz, and a very low density, well below the thermal noise floor. As a result, the TH-UWB systems do not much interfere with narrow bandwidth systems operating in the same band. The capability of the system to highly resolve the multipaths with differential delays, to the order of one nanosecond or less, and its ability to penetrate materials, make the system viable for high-quality, fully mobile, short-range indoor radio communication [2]. For more details on TH-UWB systems see [1-3,6-8].

In [3], a practical, low-rate coding scheme is applied to the TH-UWB system, which does not require any extra bandwidth further than what is required by spread spectrum modulation. The error correcting code used for demonstrating the coded system performance was a super orthogonal code [9], with rate of $1/N_s$, where N_s denotes the number of pulses transmitted for each bit in the system. The system performance analysis in [3], which was for an AWGN channel, indicates that the coded scheme outperforms the uncoded scheme significantly or, more importantly, at a given bit error rate, the coding scheme increases the number of MA (multiple access) users by a factor that is logarithmic in N_s, as defined above.

In this paper, the performance of the system is evaluated for both uncoded and coded schemes in a multipath Rayliegh fading channel. Recently, there has been a large amount of attention paid to multiuser detectors for the TH-UWB-CDMA systems, for instance see [10-13]. However, in this work, a conventional single user receiver is considered, which is not optimal in our multiuser applications. Since the TH-UWB can resolve the closely spaced multipath components, it can efficiently combine them to provide high order diversity. In [2], two kinds of Rake receivers, namely ARake (All Rake) and SRake (Selective Rake), are introduced. The ARake receiver combines all the resolved multipath components. This receiver has a high complexity, due to the high number of resolvable multipath components. SRake has been introduced to reduce the complexity of the ARake.

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This receiver combines only L paths with the largest signal to noise ratio among all available resolvable paths.

In this paper, for performance evaluation, a SRake receiver with Maximum Ratio Combining (MRC) is considered. Also, the multiple access interference at the output of the receiver is assumed to have a Gaussian distribution. Since the ordered branch variables of the SRake receiver are statistically dependent, by using the virtual branch technique [4,5], the L dependent variables can be transformed into a new set of independent virtual variables. Then, the bit error probability in a Rayleigh fading channel can be derived with an arbitrary Power Delay Profile (PDP) and, also, as a special case, with a uniform PDP.

The authors' analysis indicates that the order of diversity achieved by the coded scheme is equal to the product of the number of branches in the SRake receiver and the Hamming distance of the code applied. Furthermore, as expected, it indicates that, just as in AWGN channels, the coded scheme significantly outperforms the uncoded scheme.

This paper is organized as follows. First, uncoded and coded TH-UWB systems are described. Then, the performance of the uncoded system is derived and the performance of the coded system is evaluated. After that, the numerical results are presented and, finally the paper is concluded.

SYSTEM DESCRIPTION

Transmitted Signal

In this system, the duration of each bit is divided into N_s frames, each with duration of T_f, so, the bit rate is equal to $R_s = 1/N_s T_f$. In each frame, one pulse, with a duration of less than 1 ns, is transmitted. The modulation is BPPM (Pulse Position Modulation), in which the pulses corresponding to bit 1 are sent δ seconds later than the pulses corresponding to bit 0. Location of the pulses in each frame is determined by a user dedicated pseudorandom sequence. The transmitted waveform of user k is:

$$S_{\mathrm{tr}}^k(t) = \sum_j w_{\mathrm{tr}}(t - jT_f - c_j^k T_c - \delta d_j^k), \tag{1}$$

where the index, j, indicates the frame number, $w_{\mathrm{tr}}(t)$ represents the transmitted monocycle pulse and $\{c_j^k\}$ is the dedicated pseudorandom sequence of user k, which can take on integer values between 0 to $N_h - 1$. T_c is the chip duration and satisfies $N_h T_c \leq T_f$ and $\{d_j^k\}$ is the binary sequence of the transmitted symbols corresponding to user k. For the uncoded systems, the sequence is N_s repetitions of the transmitted data sequence, i.e., if the transmitted binary data sequence is $\{D_i^k\}$, then:

$$d_j^k = D_i^k \quad \text{for} \quad i = \left[\frac{j}{N_s}\right]. \tag{2}$$

Thus, one can consider an uncoded TH-UWB system as a coded system, in which a simple repetition block code with rate $1/N_s$ is used. To improve the system performance, in [3], the authors proposed the application of a near optimal low-rate convolutional code, called a super-orthogonal code [9], instead of the above simple repetition code. A super-orthogonal code with constraint length K, has a rate of $1/2^{K-2}$. Since the rate of $1/N_s$ is required, the constraint length of the code must be equal to $2 + \log_2 N_s$. Now, for a coded scheme, d_j^k, in Equation 1, is the jth coded symbol corresponding to the current input bit of the code. That is, the location of each pulse in each frame is determined by the user dedicated pseudorandom sequence, along with the code symbol corresponding to that frame.

Received Signal in Multipath Channel

The channel, the transmitter and the receiver antennas alter the shape of the transmitted pulse, $w_{\mathrm{tr}}(t)$, and convert it to $w_{\mathrm{rec}}(t)$ at the output of the receiver antenna [2]. An uncorrelated scattering multipath Rayliegh fading channel is assumed, which, for each user, k, has the following impulse response:

$$h_k(t) = \sum_{l=1}^{n_k} \alpha_l^k(t)\delta(t - \tau_l^k(t)), \tag{3}$$

where $\alpha_l^k(t)$ is the attenuation of the lth path, which has Rayleigh distribution, $\tau_l^k(t)$ is the delay of the lth path and n_k denotes the number of resolvable paths of user k. Therefore, in this channel, for each transmitted pulse in each frame, several pulses with different attenuation and delays, each corresponding to one resolvable path, are received. Thus, the received signal of the kth user at the receiver antenna output is:

$$\begin{aligned} S_{\mathrm{rec}}^k(t) &= S_{\mathrm{tr}}^k(t)^* h_k(t) \\ &= \sum_j \sum_{l=1}^{n_k} \alpha_{jl}^k w_{\mathrm{rec}}(t - jT_f - c_j^k T_c - \delta d_j^k - \tau_{jl}^k), \end{aligned} \tag{4}$$

where $w_{\mathrm{rec}}(t)$ is the received signal with the duration T_w and $\alpha_{jl}^k \equiv \alpha_l^k(jT_f)$ and $\tau_{jl}^k \equiv \tau_l^k(jT_f)$ are the amplitude and delay of the lth received path, due to the jth transmitted pulse, respectively. It is assumed that the channel is slowly fading, so that the channel parameters remain constant during several bit intervals. Since the multiple access capability of the uncoded and coded schemes in multipath fading channels are to be compared, for simplicity of computations and evaluations, in the following, the channel parameters estimation errors are ignored and the availability of perfect channel state information at the receiver is assumed. The power controlled channel, i.e., the received power of all users is considered to be equal:

$$\sum_{l=1}^{n_k} \left(\alpha_{jl}^{k}\right)^2 = 1, \qquad k = 1, \cdots, N_u,$$

where N_u is the number of active users.

The total received signal in the multiuser channel is:

$$r(t) = \sum_{k=1}^{N_u} S_{\mathrm{rec}}^{k}(t) + n(t), \tag{5}$$

where $n(t)$ is the zero mean Additive White Gaussian Noise (AWGN), with two-sided spectral density equal to $N_0/2$.

Receiver Structure for Multipath Channel

Uncoded Scheme

Without any loss of generality, suppose that the desired user is user 1. The SRake receiver with MRC is used. It is assumed that the receiver is completely synchronized with the desired transmitter (user 1), i.e., the receiver knows the amplitudes and delays of the L strongest paths and the pseudorandom sequence of user 1, i.e., $\{c_j^1\}$. Then, the SRake receiver decides, based on the following rule:

"decide that $D_0^1 = 0$" $\Leftrightarrow$

$$u = \sum_{j=0}^{N_s-1} \underbrace{\int_{t_1}^{t_2} r(t)\left(\sum_{l=1}^{L} \alpha_j^1(l) v(t - jT_f - c_j^1 T_c - \tau_j^1(l))\right) dt}_{u_j} > 0, \tag{6}$$

where u indicates the SRake output, u_j is the pulse SRake output, L is the number of branches in the SRake receiver and $\alpha_j^1(l)$ and $\tau_j^1(l)$ are the amplitude and delay of the lth strongest (selected) path of the jth transmitted pulse, respectively. Note that the quantity, $\alpha_j^1(l)$, is the ordered α_{jl}^1, i.e., $\alpha_j^1(1) > \alpha_j^1(2) > \cdots > \alpha_j^1(n_1)$. $[t_1, t_2]$ is the interval that includes selected L paths. $v(t)$ is the template signal and it is defined by $v(t) = w_{\mathrm{rec}}(t) - w_{\mathrm{rec}}(t - \delta)$. Since $w_{\mathrm{rec}}(t)$ has the duration T_w, the template signal, $v(t)$, has a duration of $T_w + \delta$.

In fading channels, the successive frames do not usually experience independent fading. By using an interleaver in the transmitter, the frames belonging to the same bit are reordered, in order not to be transmitted successively. Rather, they are transmitted at large enough intervals, such that the pulses belonging to successive frames fade independently and, in the receiver, a deinterleaver after the demodulator puts them back into the original order.

Coded Scheme

As mentioned previously, for a super orthogonal code with required rate $1/N_s$, the code constraint length must be equal to $K = \log_2 N_s + 2$. In this case, each code symbol modulates one pulse and, as a result, the coded scheme requires the same bandwidth as the uncoded scheme. Decoding is performed using a soft input Viterbi algorithm with 2^{K-1} states and each path metric is evaluated by using the quantity u_j in Relation 6, which is called pulse SRake output. For further information on super-orthogonal codes see [9].

PERFORMANCE EVALUATION (UNCODED SCHEME)

In this section, the bit error rate is evaluated as a function of the number of users. It is assumed that:

- The elements $\{c_j^k\}$ for $k = 1, 2, \cdots, N_u$ and all j, are i.i.d. random variables with uniform distribution on $[0, N_h - 1]$;
- The users transmit their data independently;
- Multiuser interference at the output of SRake has a Gaussian distribution. This assumption is reasonable, since, for a large number of users and a large value of L, the central limit theorem can be applied;
- The received waveform ($w_{\mathrm{rec}}(t)$) satisfies the relation $\int_{-\infty}^{+\infty} w_{\mathrm{rec}}(t)dt = 0$ [2,3].

By these assumptions, first, the Signal to Interference Ratio (SIR) is evaluated at the output of the receiver. Then, as the modulation is BPPM, the bit error rate will be equal to $P_e = E(Q(\sqrt{SIR}))$, where the expectation is taken on fading parameters.

SIR Evaluation

First, it is assumed that there is no interference between successive frames. That is, all multipath components received from each transmitted monocycle pulse are located in the same frame. If the desired transmitter (user 1) transmits a zero bit, the received signal, due to the jth transmitted pulse of user 1 at the receiver antenna output, will be:

$$r_j^1(t) = \sum_{i=1}^{n_1} \alpha_{ji}^1 w_{\mathrm{rec}}(t - jT_f - c_j^1 T_c - \tau_{ji}^1). \tag{7}$$

The pulse SRake output in Relation 6, due to user $1(u_j^1)$, equals:

$$u_j^1 = \int_{jT_f}^{(j+1)T_f} \left(\sum_{i=1}^{n_1} \alpha_{ji}^1 w_{\text{rec}}(t - jT_f - c_j^1 T_c - \tau_{ji}^1) \right) \left(\sum_{l=1}^{L} \alpha_j^1(l) v(t - jT_f - c_j^1 T_c - \tau_j^1(l)) \right) dt$$

$$= \sum_{l=1}^{L} \alpha_j^1(l) \sum_{i=1}^{n_1} \alpha_{ji}^1 \int_0^{T_f} w_{\text{rec}}(t - c_j^1 T_c - \tau_{ji}^1) v(t - c_j^1 T_c - \tau_j^1(l)) dt. \quad (8)$$

By a change of variable and noting that $w(t)$ is zero out of the interval $[0, T_w]$, u_j^1 will be equal to:

$$u_j^1 = \sum_{l=1}^{L} \alpha_j^1(l) \sum_{i=1}^{n_1} \alpha_{ji}^1 \int_0^{T_w} w_{\text{rec}}(t) v(t - (\tau_j^1(l) - \tau_{ji}^1)) dt. \quad (9)$$

Since the resolvable multipath components are considered, then:

$$|\tau_{jl}^1 - \tau_{jp}^1| \geq T_w \quad \text{for} \quad l \neq p, \quad (10)$$

so, only for $\tau_{ji}^1 = \tau_j^1(l)$, the integral in Equation 9 is not zero. Consequently, u_j^1 is equal to:

$$u_j^1 = \sum_{l=1}^{L} \left(\alpha_j^1(l)\right)^2 \int_0^{T_w} w_{\text{rec}}(t) v(t) dt = m \sum_{l=1}^{L} \left(\alpha_j^1(l)\right)^2, \quad (11)$$

where:

$$m = \int_0^{T_w} w_{\text{rec}}(t) v(t) dt, \quad (12)$$

and, according to Relation 6, the SRake output, due to user 1, will be:

$$u_{\text{self}} = m \sum_{j=0}^{N_s - 1} \sum_{l=1}^{L} \left(\alpha_j^1(l)\right)^2. \quad (13)$$

To study the effect of multiuser interference at the SRake output, first, the mean and variance of one interfering user component is evaluated at the output of the receiver and, then, as the interfering users are independent, the mean and variance of total interference at the receiver output is obtained.

Suppose that user $k (k \neq 1)$ is the interfering user. The received signal of user k at the j' frame interval is:

$$r_{j'}^k(t) = \sum_{i=1}^{n_k} \alpha_{j'i}^k w_{\text{rec}}(t - j'T_f - c_{j'}^k T_c - \delta d_{j'}^k - \tau_{j'i}^k), \quad (14)$$

so, the jth pulse SRake output, due to this interfering frame, is:

$$u_j^k = \int_{jT_f}^{(j+1)T_f} \left(\sum_{i=1}^{n_k} \alpha_{j'i}^k w_{\text{rec}}(t - j'T_f - c_{j'}^k T_c - \delta d_{j'}^k - \tau_{j'i}^k) \right) \left(\sum_{l=1}^{L} \alpha_j^1(l) v(t - jT_f - c_j^1 T_c - \tau_j^1(l)) \right) dt$$

$$= \sum_{l=1}^{L} \alpha_j^1(l) \sum_{i=1}^{n_k} \alpha_{j'i}^k \int_0^{T_w+\delta} w_{\text{rec}}\Big(t - (j' - j)T_f + (c_{j'}^k - c_j^1)T_c + \delta d_{j'}^k + (\tau_{j'i}^k - \tau_j^1(l))\Big) v(t) dt. \quad (15)$$

The delay in the above equation can be written as the following:

$$(j' - j)T_f + (c_{j'}^k - c_j^1)T_c + \delta d_{j'}^k + (\tau_{j'i}^k - \tau_j^1(l)) = m_{li}^k T_f + \tau_{li}^k,$$

$$\tau_{li}^k \sim U\left(-\frac{T_f}{2}, \frac{T_f}{2}\right), \quad (16)$$

τ_{li}^k is well assumed to have uniform distribution at interval $(-T_f/2, T_f/2)$. Therefore:

$$u_j^k = \sum_{l=1}^{L} \alpha_j^1(l) \sum_{i=1}^{n_x} \alpha_{j'i}^k \int_0^{T_w+\delta} w_{\text{rec}}(t - m_{li}^k T_f - \tau_{li}^k) v(t) dt. \quad (17)$$

Thus, in order for the received j'th pulse of user k to have a component at the jth pulse SRake output, one must have $m_{li}^k = 0$. So, u_j^k, as defined in Equation 17, is a function of τ_{li}^k, with the above uniform distribution. Since it is assumed that the received power of user k is equal to one, i.e., $\sum_{i=1}^{n_k} \left(\alpha_{ji}^k\right)^2 = 1$, it can be easily shown that u_j^k has zero mean and variance equal to the following:

$$\sigma_{u_j^k}^2 = \sigma_a^2 \sum_{l=1}^{L} \left(\alpha_j^1(l)\right)^2, \quad (18)$$

where:

$$\sigma_a^2 = T_f^{-1} \int_{-\infty}^{\infty} \left(\int_0^{T_w+\delta} v(t) w_{\text{rec}}(t - \tau) dt \right)^2 d\tau. \quad (19)$$

Since the interference at each pulse of the SRake output is independent from those at the others, the variance of total interference at the SRake output, due to interfering user k, will be:

$$\sigma^2_{u^k} = \sigma^2_a \sum_{j=0}^{N_s \;\; 1} \sum_{l=1}^{L} \;\; \alpha^1_j(l))^2 . \tag{20}$$

Then, since the interferences of different users are independent, the variance of the multiuser interference at the SRake output is:

$$\sigma^2_{\mathrm{MA}} = (N_u \quad 1)\sigma^2_a \sum_{j=0}^{N_s \;\; 1} \sum_{l=1}^{L} \;\; \alpha^1_j(l))^2 . \tag{21}$$

It is easily verified that the noise at the SRake output (n_0) has zero mean and variance equal to the following:

$$\mathrm{Var}(n_0) = \frac{N_0}{2} \sum_{j=0}^{N_s \;\; 1} \sum_{l=1}^{L} \;\; \alpha^1_j(l))^2 \int_0^{T_w+\delta} v^2(t)dt. \tag{22}$$

So, from Equations 13, 21 and 22, the SIR is obtained as the following:

$$\mathrm{SIR} = \frac{(u_{\mathrm{self}})^2}{\sigma^2_{\mathrm{MA}} + \mathrm{Var}(n_0)}$$

$$= \frac{m^2}{(N_u \quad 1)\sigma^2_a + \frac{N_0}{2} \int\limits_0^{T_w+\delta} v^2(t)dt} \sum_{j=0}^{N_s \;\; 1} \sum_{l=1}^{L} \;\; \alpha^1_j(l))^2 . \tag{23}$$

Since the system is multiple access interference limited and, also, the maximum achievable multiple access capability is under consideration, for the rest of paper, the effect of Gaussian noise, i.e., $N_0 = 0$ is neglected.

Now, the previous assumption is relaxed and it is assumed that there is multipath interference from one frame to the successive frames. That is, the multipath components received from each transmitted pulse last up to M frames, i.e., in each frame, there are $M \quad 1$ interfering signals from the previous frames of the same user. As the location of pulses in each frame are determined by a pseudorandom sequence with i.i.d. elements, it can well be assumed that these $M \quad 1$ interfering signals from $M \quad 1$ previous frames are independent and they can be considered as interfering signals from $M \quad 1$ virtual users. So, the current problem can be converted to the previous problem with the number of interfering users increased from $N_u \quad 1$ to $MN_u \quad 1$. By the assumption that all of these virtual interfering signals have the same powers, an upper bound on σ^2_{MA} is obtained as the following:

$$\sigma^2_{\mathrm{MA}} < \sigma^2_a(MN_u \quad 1) \sum_{j=0}^{N_s \;\; 1} \sum_{l=1}^{L} \;\; \alpha^1_j(l))^2 . \tag{24}$$

Then, the SIR is lower bounded by the following:

$$\mathrm{SIR} > \frac{m^2}{\sigma^2_a(MN_u \quad 1)} \sum_{j=0}^{N_s \;\; 1} \sum_{l=1}^{L} \;\; \alpha^1_j(l))^2$$

$$= \sum_{j=0}^{N_s \;\; 1} \sum_{l=1}^{L} a \;\; \alpha^1_j(l))^2 , \tag{25}$$

where:

$$a = \frac{m^2}{\sigma^2_a(MN_u \quad 1)}. \tag{26}$$

Error Probability Evaluation

Let γ_{jl} be the lower bound of SIR of the lth diversity branch of the jth transmitted pulse of desired user (user 1), defined by $\gamma_{jl} = a\left(\alpha^1_{jl}\right)^2$, where a is given in Equation 26. Since α^1_{jl} has been assumed to have Rayleigh distribution, so, γ_{jl} will have an exponential distribution:

$$f_{\gamma_{jl}}(x) = \frac{1}{_{jl}} \exp(\quad \frac{x}{_{jl}}), \qquad x \geq 0, \tag{27}$$

where $_{jl} = E(\gamma_{jl})$. In the rest of the article, it is assumed that $_{jl} = \; _l$, for all j, which can be based on the channel stationarity. $\gamma = \sum_{j=0}^{N_s \;\; 1} \sum_{l=1}^{L} \gamma_j(l)$ is defined, where $\gamma_j(l) = a \;\; \alpha^1_j(l))^2$ is the ordered γ_{jl}, i.e., $\gamma_j(1) > \gamma_j(2) > \cdots > \gamma_j(L)$. So, the upper bound on the bit error rate equals:

$$p_u(e) = E(Q(\sqrt{\gamma})) = E\left(\frac{1}{\pi}\int\limits_0^{\frac{\pi}{2}} \exp(\quad \frac{\gamma}{2\sin^2\theta})d\theta\right). \tag{28}$$

In the above equation, the geometric interpretation of $Q(x)$ has been used.

Although the variables $\{\gamma_j(l)\}_{j=0}^{N_s \;\; 1}$ are independent for a fixed l, the ordered variables, $\{\gamma_j(l)\}_{l=1}^{L}$, for a fixed j, are not independent. By using the virtual branch technique [4,5], these dependent variables can be transformed to a new set of independent variables and, then, the bit error rate can be easily obtained. To this end, the same approach introduced in [4,5] is followed. It has been shown in [4] that the dependent variables, $\{\gamma_j(l)\}_{l=1}^{n_1}$, can be written as a linear combination of another set of variables, $\{V_{jn}\}_{n=1}^{n_1}$, as the following:

$$\gamma_j(l) = \sum_{n=l}^{n_1} \left(\frac{1}{n}\sum_{m=1}^{n} \frac{1}{_m}\right)^{\;1} \frac{1}{n} V_{jn}, \quad 0 < V_{jn} < \infty, \tag{29}$$

where:

$$\left(\sum_{m=1}^{l}\frac{1}{\quad_{m}}\right)^{\quad 1} V_{jl} = \gamma_j(l) \quad \gamma_j(l+1),$$

$$l = 1, 2, 3, \cdots, n_1, \quad \gamma_j(n_1+1) = 0. \tag{30}$$

Then, one has the following:

$$\gamma = \sum_{j=0}^{N_s \;\; 1}\sum_{l=1}^{L}\gamma_j(l) = \sum_{j=0}^{N_s \;\; 1}\sum_{n=1}^{n_1} b_n V_{jn}, \tag{31}$$

where:

$$b_n = \begin{cases} \left(\frac{1}{n}\sum_{m=1}^{n}\frac{1}{\quad_{m}}\right)^{\quad 1} & 1 \le n \le L \\ \left(\frac{1}{n}\sum_{m=1}^{n}\frac{1}{\quad_{m}}\right)^{\quad 1} \frac{L}{n} & L < n \le n_1 \end{cases}. \tag{32}$$

As a result, Equation 28 can be written as the following:

$$P_u(e) = E\left(Q\left(\sqrt{\sum_{j=0}^{N_s \;\; 1}\sum_{n=1}^{n_1} b_n V_{jn}}\right)\right). \tag{33}$$

To compute the expectation, let S_{n_1} be the set of all permutations of integers $\{1, \cdots, n_1\}$ and σ denote the particular function $\sigma : (1, 2, \cdots, n_1) \to (\sigma_1, \sigma_2, \cdots, \sigma_{n_1})$, which permutes the integers $\{1, \cdots, n_1\}$, then, it has been shown [4] that $\{V_{jn}|\sigma\}_{n=1}^{n_1}$ are independent exponential random variables, with the following probability density function:

$$f_{V_{jn}|\sigma}(v|\sigma) = \begin{cases} \frac{1}{\tilde{\quad}_n}\exp(\quad \frac{v}{\tilde{\quad}_n}) & 0 < v < \infty \\ 0 & \text{otherwise} \end{cases}, \tag{34}$$

where:

$$\tilde{\quad}_n = \left(\frac{1}{n}\sum_{m=1}^{n}\frac{1}{\quad_{m}}\right)\left(\frac{1}{n}\sum_{k=1}^{n}\frac{1}{\quad_{\sigma_k}}\right)^{\quad 1}. \tag{35}$$

So, Equation 33 can be written as follows:

$$P_u(e) = E_\sigma(P_u(e|\sigma)) = \sum_{\sigma \in S_{n_1}} \Pr(\sigma) P_u(e|\sigma), \tag{36}$$

where $\Pr(\sigma)$ has the following distribution [4]:

$$\Pr(\sigma) = \prod_{k=1}^{n_1}\frac{1}{\quad_{\sigma_k}}\left(\sum_{m=1}^{k}\frac{1}{\quad_{\sigma_m}}\right)^{\quad 1}, \tag{37}$$

and:

$$P_u(e|\sigma) = E_{\{V_{jn}|\sigma\}}\left(Q(\sqrt{\gamma})\right)$$

$$= E_{\{V_{jn}|\sigma\}}\left(\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\exp\left(\quad\frac{1}{2\sin^2\theta}\sum_{j=0}^{N_s \;\; 1}\sum_{n=1}^{n_1} b_n V_{jn}\right)d\theta|\sigma\right)$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} E_{\{V_{jn}|\sigma\}}\left(\exp\left(\quad\frac{1}{2\sin^2\theta}\sum_{j=0}^{N_s \;\; 1}\sum_{n=1}^{n_1} b_n V_{jn}\right)|\sigma\right)d\theta$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\prod_{j=0}^{N_s \;\; 1}\prod_{n=1}^{n_1} E_{V_{jn}|\sigma}\left(\exp\left(\quad\frac{1}{2\sin^2\theta} b_n V_{jn}\right)|\sigma\right)d\theta$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\prod_{j=0}^{N_s \;\; 1}\prod_{n=1}^{n_1}\Psi_{V_{jn}|\sigma}\left(\quad\frac{1}{2\sin^2\theta} b_n\right)d\theta. \tag{38}$$

$\Psi_{V_{jn}|\sigma}$ is the moment generating function of $V_{jn}|\sigma$ and is equal to:

$$\Psi_{V_{jn}|\sigma}(s) = \frac{1}{1 \quad s\,\tilde{\quad}_n}. \tag{39}$$

By substituting Equation 39 in Equation 38, the following relation is obtained:

$$P_u(e|\sigma) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(\prod_{n=1}^{n_1}\frac{1}{\frac{b_n}{2\sin^2\theta}\tilde{\quad}_n + 1}\right)^{N_s} d\theta$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(\prod_{n=1}^{n_1}\frac{\sin^2\theta}{\frac{b_n}{2}\tilde{\quad}_n + \sin^2\theta}\right)^{N_s} d\theta, \tag{40}$$

where, from Equations 32 and 35, one obtains the following:

$$b_n\tilde{\quad}_n = \begin{cases} \left(\frac{1}{n}\sum_{m=1}^{n}\frac{1}{\quad_{\sigma_m}}\right)^{\quad 1} & 1 \le n \le L \\ \left(\frac{1}{n}\sum_{m=1}^{n}\frac{1}{\quad_{\sigma_m}}\right)^{\quad 1} \frac{L}{n} & L \le n \le n_1 \end{cases}. \tag{41}$$

If it is assumed that all multipath components have equal average power (uniform PDP), i.e., $\quad_l = \quad$ for $1 \le l \le n_1$, then, $\{V_{jn}\}_{n=1}^{n_1}$ will be i.i.d. exponential random variables [5] and the above error probability simplifies to the following:

$$P_u(e) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(\left(\frac{\sin^2\theta}{\frac{\quad}{2} + \sin^2\theta}\right)^{L}\right.$$

$$\left.\prod_{n=L+1}^{n_1}\left(\frac{\sin^2\theta}{\frac{L}{2n} + \sin^2\theta}\right)\right)^{N_s} d\theta. \tag{42}$$

PERFORMANCE EVALUATION (CODED SCHEME)

In this section, the performance of the coded TH-UWB system, using super orthogonal codes, is derived. Since, for a convolutional code, only the upper and lower bounds on the bit error rate, using an ML (Maximum Likelihood) decoder (through a Viterbi algorithm), are available, for the coded scheme, only the upper and lower bounds on the bit error rate can be computed. For this purpose, the path generating function of the code is required. This function for a super-orthogonal code is given as follows [9]:

$$T(X,Y) = \frac{XW^{K+2}(1-W)}{1-W(1+X(1+W^{K-3}-2W^{K-2}))}, \tag{43}$$

where $W = Y^{2^{K-3}}$. Expanding the above expression, one obtains a polynomial in X and Y. The coefficients and powers of X and Y in each term of the polynomial indicate the number of paths and output-input path weights, respectively. The free distance of the code is obtained from the first term of the expansion as $d_f = 2^{K-3}(K+2) = \frac{N_s}{2}(\log_2 N_s + 4)$.

Suppose that a zero bit sequence is transmitted and the receiver selects an incorrect path with Hamming weight d instead of the zero sequence path. Let $P(d)$ denote the probability of this error event. Then, an upper bound on the bit error rate, using a union bound is obtained as follows:

$$P_b \le \sum_{d=d_f}^{\infty} \beta_d P(d), \tag{44}$$

where β_d is the number of incorrectly decoded bits corresponding to all error events with the weight of d.

A lower bound on the probability of bit error is obtained by considering only the first term of the path generating function. The result is as follows:

$$P_b \ge P(d_f). \tag{45}$$

To evaluate $P(d)$, suppose that $CM(0)$ and $CM(d)$ are the metrics of the zero sequence path and the path with weight d, respectively. Since the coded bits in the correct and incorrect paths are identical, except in the d positions, then:

$$p(d|\{\gamma_j(l)\}) = P(CM(0)-CM(d)<0) = P\left(\sum_{j=1}^{d} u_j < 0\right)$$
$$= Q\left(\sqrt{\sum_{j=1}^{d}\sum_{l=1}^{L}\gamma_j(l)}\right), \tag{46}$$

where $\{u_j\}_{j=1}^{d}$ are the pulse SRake outputs corresponding to the coded symbol positions at which the two paths differ. Using the virtual branch variables, as in Equation 29, the following is obtained:

$$P(d|\{V_{jn}\}) = Q\left(\sqrt{\sum_{j=1}^{d}\sum_{n=1}^{n_1} b_n V_{jn}}\right). \tag{47}$$

If one applies the Chernoff bound, $(Q(\sqrt{x}) \le \frac{1}{2}e^{-\frac{x}{2}})$, to Equation 47, $P(d|V_{jn})$ is upper bounded as follows:

$$P(d|\{V_{jn}\}) \le \frac{1}{2}\exp\left(-\frac{1}{2}\sum_{j=1}^{d}\sum_{n=1}^{n_1} b_n V_{jn}\right). \tag{48}$$

So, the upper bound on $P(d)$ is as follows:

$$P(d) \le \frac{1}{2}E\left(\exp\left(-\frac{1}{2}\sum_{j=1}^{d}\sum_{n=1}^{n_1} b_n V_{jn}\right)\right)$$
$$= \frac{1}{2}E_\sigma\left(\prod_{j=1}^{d}\prod_{n=1}^{n_1} E_{V_{jn}|\sigma}\left(\exp\left(-\frac{1}{2}b_n V_{jn}\right)|\sigma\right)\right)$$
$$= \frac{1}{2}E_\sigma\left(\prod_{j=1}^{d}\prod_{n=1}^{n_1}\Psi_{V_{jn}|\sigma}\left(-\frac{1}{2}b_n\right)\right)$$
$$= \frac{1}{2}E_\sigma\left(\left(\prod_{n=1}^{n_1}\frac{1}{\frac{b_n}{2}\tilde{\gamma}_n+1}\right)^{d}\right)$$
$$= \frac{1}{2}\sum_{\sigma\in S_{n_1}}\Pr(\sigma)\left(\prod_{n=1}^{n_1}\frac{1}{1+\frac{1}{2}b_n\tilde{\gamma}_n}\right)^{d}, \tag{49}$$

where the second equality is obtained by applying Equation 39. From Relations 44 and 49, an upper bound on the bit error rate is given as follows:

$$p_b \le \sum_{d=d_f}^{\infty}\beta_d P(d)$$
$$\le \frac{1}{2}\sum_{\sigma\in S_{n_1}}\Pr(\sigma)\sum_{d=d_f}^{\infty}\beta_d\left(\prod_{n=1}^{n_1}\frac{1}{1+\frac{1}{2}b_n\tilde{\gamma}_n}\right)^{d}$$
$$= \frac{1}{2}\sum_{\sigma\in S_{n_1}}\Pr(\sigma)\frac{\partial T(X,Y)}{\partial X}|_{X=1}, \tag{50}$$

where:

$$\frac{\partial T(X,Y)}{\partial X}|_{X=1} = \frac{W^{K+2}}{(1-2W)^2}\left(\frac{1-W}{1-W^{K-2}}\right)^2, \tag{51}$$

in which $W = Y^{2^K \;\; 3}$ and $Y = \prod_{n=1}^{n_1} \frac{1}{1+\frac{1}{2}b_n \tilde{\;}_n}$. The lower bound is as follows:

$$P_b \geq P(d_f) = \sum_{\sigma \in S_{n_1}} \Pr(\sigma) P(d_f|\sigma), \tag{52}$$

where:

$$P(d_f|\sigma) = E_{\{V_{jn}|\sigma\}}\left(Q\left(\sqrt{\sum_{j=1}^{d_f}\sum_{n=1}^{n_1} b_n V_{jn}}\right)\right)$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(\prod_{n=1}^{n_1}\frac{\sin^2\theta}{\frac{b_n}{2}\tilde{\;}_n + \sin^2\theta}\right)^{d_f} d\theta. \tag{53}$$

The second equality is obtained using a geometric interpretation of $Q(x)$ in Equation 28.

For uniform PDP, in which $\;_l = \;$ for $1 \leq l \leq n_1$, a tighter bound can be obtained. In this case, by applying the inequality $Q\left(\sqrt{x+y}\right) \leq Q(\sqrt{x})e^{\;\frac{y}{2}}$ to Equation 47 and considering Equation 32, $P(d|\{V_{jn}\})$ is upper bounded by the following:

$$P\left(d|\{V_{jn}\}\right) = Q\left(\sqrt{\sum_{j=1}^{d}\left(\sum_{n=1}^{L}\;V_{jn} + \sum_{n=L+1}^{n_1}\;\frac{L}{n}V_{jn}\right)}\right)$$

$$\leq Q\left(\sqrt{\sum_{j=1}^{d}\sum_{n=1}^{L}\;V_{jn}}\right)\exp\left(\;\frac{1}{2}\sum_{j=1}^{d}\sum_{n=L+1}^{n_1}\;\frac{L}{n}V_{jn}\right). \tag{54}$$

Since $\{V_{jn}\}$ are independent random variables, $P(d)$ is upper bounded by the following:

$$P(d) = E\left(P(d|\{V_{jn}\})\right)$$

$$\leq E\left(Q\left(\sqrt{\sum_{j=1}^{d}\sum_{n=1}^{L}\;V_{jn}}\right)\right)$$

$$.E\left(\exp\left(\;\frac{1}{2}\sum_{j=1}^{d}\sum_{n=L+1}^{n_1}\;\frac{L}{n}V_{jn}\right)\right). \tag{55}$$

To evaluate an upper bound for the first term of Relation 55, the following is introduced:

$$\lambda = \sum_{j=1}^{d}\sum_{n=1}^{L}\;V_{jn}. \tag{56}$$

Since λ is a linear combination of Ld independent exponential random variables, it has a chi-square distribution with Ld degrees of freedom, as follows:

$$f(\lambda) = \frac{1}{\;(Ld)\;\;^{Ld}}\lambda^{Ld\;\;1}\exp(\;\;\frac{\lambda}{\;})U(\lambda). \tag{57}$$

So, one has the following:

$$E(Q(\sqrt{\lambda})) = \int_0^{\infty} Q(\sqrt{\lambda})f(\lambda)d\lambda. \tag{58}$$

By using the inequality $Q(\sqrt{x}) \leq \frac{1}{\sqrt{2\pi x}}e^{\;\frac{x}{2}}$ [14] and the following:

$$\int_0^{\infty} u^n e^{\;\alpha u}du = \frac{\;(n+1)}{\alpha^{n+1}}, \qquad \alpha > 0, \qquad n > \;\;1, \tag{59}$$

an upper bound on Equation 58 is obtained as follows:

$$E\left(Q\left(\sqrt{\sum_{j=1}^{d}\sum_{n=1}^{L}\;V_{jn}}\right)\right) \leq \frac{1}{\sqrt{2\pi}}$$

$$.\frac{\;(Ld\;\;\frac{1}{2})}{\;(Ld)}.\sqrt{\frac{1}{2}+\frac{1}{\;}}.\left(\left(1+\frac{\;}{2}\right)^{\;L}\right)^{d}. \tag{60}$$

Using the same approach taken in deriving Equation 38, the second term of Relation 55 is simplified as follows:

$$E\left(\exp\left(\;\frac{1}{2}\sum_{j=1}^{d}\sum_{n=L+1}^{n_1}\;\frac{L}{n}V_{jn}\right)\right)$$

$$= \left(\prod_{n=L+1}^{n_1}\frac{1}{1+\;\frac{L}{2n}}\right)^{d}. \tag{61}$$

By substituting Relation 60 and Equation 61 in Relation 55, Equation 44 simplifies to the following:

$$P_b \leq \sum_{d=d_f}^{\infty}\beta_d P(d) = \frac{1}{\sqrt{2\pi}}.\sqrt{\frac{1}{2}+\frac{1}{\;}}$$

$$.\sum_{d=d_f}^{\infty}\beta_d\frac{\;(Ld\;\;\frac{1}{2})}{\;(Ld)}\left(\frac{\prod_{n=L+1}^{n_1}\frac{1}{1+\;\frac{L}{2n}}}{\left(1+\frac{\;}{2}\right)^{L}}\right)^{d}. \tag{62}$$

As $\;(Ld\;\;\frac{1}{2})/\;(Ld)$ is a monotonically decreasing sequence on d, the right hand of the above equation is upper bounded by the following:

$$P_b \leq \frac{1}{\sqrt{2\pi}}.\frac{\;(Ld_f\;\;\frac{1}{2})}{\;(Ld_f)}.\sqrt{\frac{1}{2}+\frac{1}{\;}}.\frac{\partial T(X,Y)}{\partial X}|_{X=1}, \tag{63}$$

where $Y = \frac{\prod_{n=L+1}^{n_1}\frac{1}{1+\;\frac{L}{2n}}}{\left(1+\frac{\;}{2}\right)^{L}}$. The lower bound in Relation 52, in this case (uniform PDP), also simplifies to the following:

$$P_b \geq P(d_f) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(\left(\frac{\sin^2\theta}{\frac{}{2}+\sin^2\theta}\right)^L \prod_{n=L+1}^{n_1}\left(\frac{\sin^2\theta}{\frac{L}{2n}+\sin^2\theta}\right)\right)^{d_f} d\theta. \quad (64)$$

From the above equation, it can be realized that in the coded scheme, the effective order of diversity is equal to the product of L (the number of branches in the SRake receiver) and the Hamming distance of the code (d_f).

NUMERICAL EXAMPLES

To evaluate the performance of the TH-UWB system in multipath fading channels and, also, to demonstrate the advantages of using coding in the system, in this section, some numerical results are presented, based on the analytical derivations presented in the previous sections. As in [2], the received signal is modeled as $w_{\text{rec}}(t+\frac{T_w}{2}) = \left[1 \quad 4\pi\left(\frac{t}{\tau_m}\right)^2\right]\exp\left(\quad 2\pi\left(\frac{t}{\tau_m}\right)\right)$ where $\tau_m = 0.2877$ ns and $T_w = 0.7$ ns (Figure 1). $\delta = 0.156$ ns, $T_f = 100$ ns, $\frac{m^2}{\sigma_a^2} \approx 504$ and $M = 3$ are also set. The number of resolvable paths for user $1(n_1)$ is assumed to be 50. A uniform PDP is considered, in which all multipath components have the same powers. The bit error rate of the uncoded scheme is obtained from Equation 42 and the upper and lower bounds on the bit error rate for the coded scheme are obtained from Relations 63 and 64, respectively.

Figure 2 presents the plots of bit error rate versus the number of users in an uncoded system, in both nonfading and Rayleigh fading channels, for various values of L (the number of branches in the SRake receiver), at the input bit rate $R_s = 1/T_f N_s = 1.25$ Mbps ($N_s = 8$). As expected, by increasing L, the performance improves, however, at high values of L, this improvement is negligible. It must be noted that the Gaussian distribution assumption for the MAI component, based on using the Central Limit Theorem (C.L.T.) is accurate at only a moderate to high number of users.

Figures 3 and 4 show the plots of the bit error rate versus the number of diversity branches of the fading channel in the receiver (L), in uncoded and coded systems, for various values of N_s, where $N_s = 2$ corresponds to $R_s = 5$ Mbs, $N_s = 32$ corresponds to $R_s = 0.3125$ Mbs and vice versa, respectively. The number of users is assumed to be 100. From these figures, it can be realized that the coded system outperforms the uncoded system substantially, despite requiring the same bandwidth as the uncoded scheme. For example, it is seen that for $N_s = 8$ and $L = 30$, the

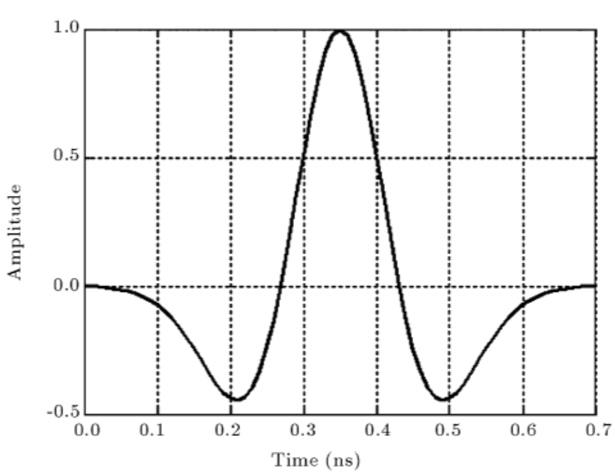

Figure 1. The monocycle received waveform.

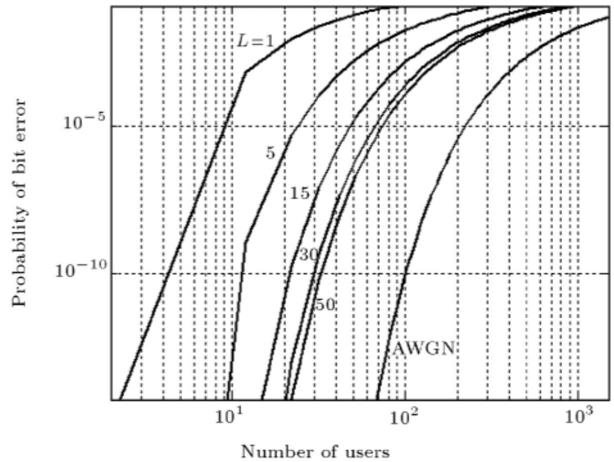

Figure 2. The bit error rate versus the number of users in fading and nonfading channels at input bit rate 1.25 Mbps ($N_s = 8$).

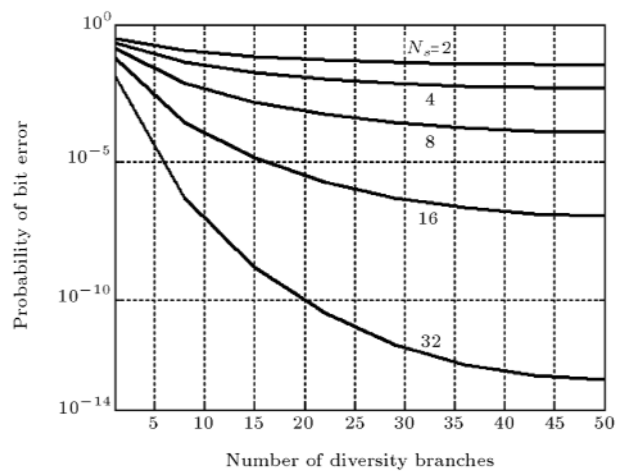

Figure 3. The bit error rate versus the number of diversity branches in an uncoded system for $N_u = 100$.

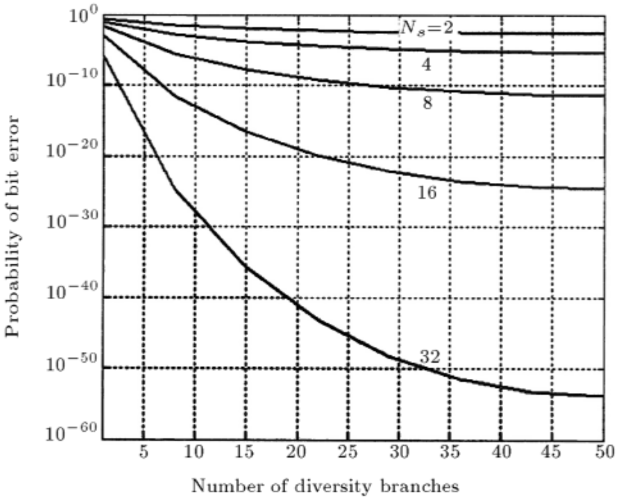

Figure 4. The lower bound on the bit error rate versus the number of diversity branches in a coded system for $N_u = 100$.

bit error rate of the uncoded scheme is approximately 10 ^{4}, while, for the coded scheme, it is about 10 ^{10}, which shows a superior improvement.

Figure 5 represents the plots of bit error rate versus the number of users in nonfading and multipath fading channels for $L = 32$ and at an input rate of 5 Mbps. In this figure, the upper and lower bounds of the bit error rate of the coded system in a fading channel are also plotted. It is observed that the two bounds are very close when the number of users is less than 30, but are separated when the number of users exceeds 30.

Figures 6 and 7 show the plots of the bit error rate versus the number of users for $L = 32$ and at input rates of 1.25 Mbps and 78.12 Kbps, respectively. It can be realized that there is a great improvement in the performance of the coded system compared to the uncoded system.

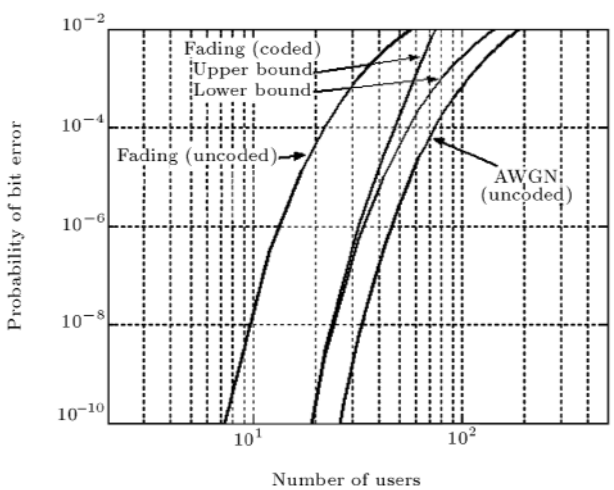

Figure 5. The bit error rate versus the number of users at input bit rate 5 Mbps ($N_s = 2$) and $L = 32$.

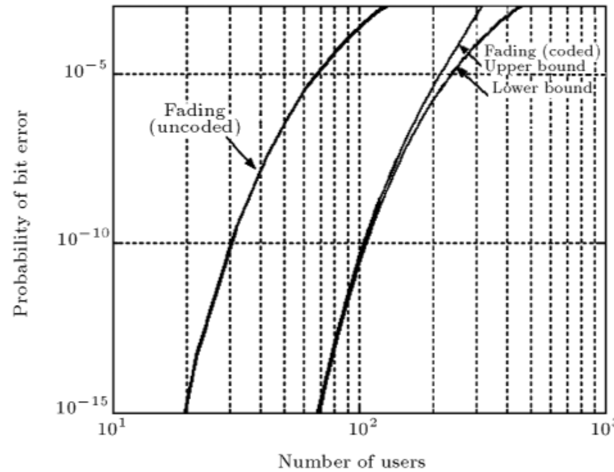

Figure 6. The bit error rate versus the number of users at input bit rate 1.25 Mbps ($N_s = 8$) and $L = 32$.

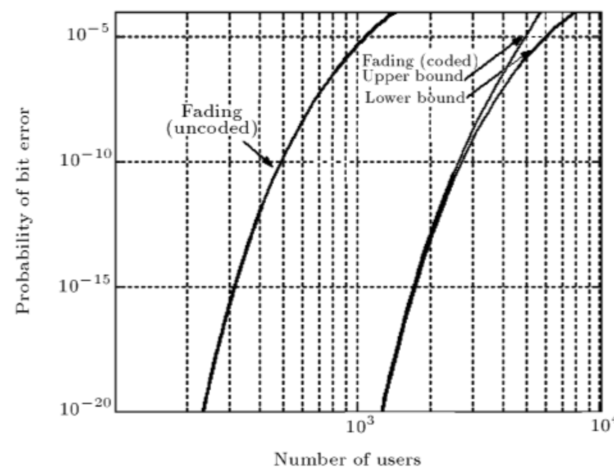

Figure 7. The bit error rate versus the number of users at input bit rate 78.12 Kbps ($N_s = 128$) and $L = 32$.

CONCLUSION

In this paper, the performances of both uncoded and coded TH-UWB systems in a multipath Rayleigh fading channel are evaluated, using the SRake receiver with MRC. It was assumed that the interference at the output of the pulse SRake receiver has Gaussian distribution. Then, by using a virtual branch technique and transforming the dependent ordered path variables to virtual independent variables, the bit error rates were derived for both uncoded and coded schemes. The performance analysis indicates that the effective order of diversity achieved by the coded scheme is the product of the number of branches in the SRake receiver (L) and the Hamming distance of the code. Moreover, it also shows that the coded scheme substantially outperforms the uncoded scheme, without requiring any extra bandwidth compared to the uncoded scheme.

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