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Power System Frequency Estimation Based on Simulated Annealing: A Variable Frequency Model

S.A. Soliman*, R.A. Alammari[1], A.H. Mantaway[2] and M.E. El-Hawary[3]

This paper presents a new technique for measuring power system frequency, rate of change of frequency and the voltage amplitude and phase angle using the Simulated Annealing (SA) based optimization algorithm. The algorithm uses the digitized samples of the voltage signal at the relay location and minimizes a cost function of the sum of the absolute error between the actual estimated signal samples. The proposed algorithm does not need any filter or model for the system frequency before and during the estimation process. The effects of the number of samples, sampling frequency and harmonics contamination on the estimated parameters are tested and discussed in the paper.

INTRODUCTION

In power system protection, power system voltage amplitude and local frequency are very important for frequency relaying purposes, for the Automatic Voltage Regulator (AVR) function and the operating of Uninterruptible Power Supplies (UPS). The widespread use of power electronics devices in power generation, transmission, distribution and utilization is responsible for corrupting voltage signal waveforms with noise and/or harmonics. Harmonics cause operational problems in power systems, such as signal interference and malfunction of relays, particularly in solid-state and microprocessor controlled apparatus used to estimate frequency and its rate of change. As a result, there is a need to find a fast and accurate algorithm for measuring system frequency and voltage signal amplitude in such an environment and for such applications [1,2].

Many digital techniques have been developed over the past two decades to measure frequency and voltage amplitude. The required data window size varies between algorithms, but a feature common to most techniques is the assumption that the voltage signal is free of noise and/or harmonics. If the voltage signal were corrupted by harmonics, a few periods would be required for this measurement [3-7].

The rate of change of frequency is an important factor in power system control, especially during system disturbance. It gives an indication to the decision maker whether to increase the generated power or shed some of the system load [8-10].

The orthogonal FIR digital filter is applied in [11-13] with a least error square algorithm for measuring the operating frequency of a power system. This algorithm has beneficial features, including fixed sampling rate, fixed data window size and easy implementation. Discrete Fourier transform with Poney's estimation are applied in [14] for measuring system frequency with a variable data window to filter out the noise and harmonics associated with the signal.

Static estimation algorithms have been applied for the last three decades to estimate system frequency and voltage phasor amplitude. The Least Error Squares (LES) algorithm and the Least Absolute Value algorithm (LAV) technique are used to estimate system frequency and voltage phasor amplitude from digitized samples of the voltage signal of one of the phases. Each one of these algorithms has its own figure of merit and is only suitable for the system it works with. Also, all the available algorithms are either tested off-line or online and they produce, in most cases, good estimates for the purposes for which they were designed [14,15].

*. *Corresponding Author, Department of Electrical Engineering, University of Qatar, P.O. Box 2713, Doha, Qatar.*

1. *Department of Electrical Engineering, University of Qatar, P.O. Box 2713, Doha, Qatar.*
2. *Department of Electrical Engineering, King Fahad University for Petroleum and Minerals (KFUPM), Dhahran, Saudi Arabia.*
3. *Department of Electrical and Computer Engineering, Dalhousie University, Halifax, Nova Scotia, Canada.*

The complex Kalman filter and $\alpha\beta$-transformation for measuring system frequency are presented in [16]. A nonlinear state-space formulation is obtained and the nonlinear equations are solved using the extended Kalman filter. The solution is obtained under the assumption that the three-phase voltage has a constant frequency during the data window size.

An algorithm for frequency estimation, based on demodulation of two signals and the $\alpha\beta$-transformation, is presented in [17]. The algorithm in this reference demodulates the complex phasor resulting from the transformation using a complex signal with a known frequency. The resulting demodulated signal does not contain the double frequency signal, as does the old demodulation, described in the literature.

This paper deals with measuring power system frequency, rate of change of frequency, voltage amplitude and phase angle using the simulated annealing based optimization algorithm. The algorithm uses samples of the voltage signal at the relay location and minimizes a nonlinear cost function of the sum of the absolute error between the actual and estimated signal samples. The proposed algorithm does not need any filter or model for the system frequency before and during the estimation process. Effects of the number of samples, sampling frequency and harmonics contamination on the estimated parameters are tested and discussed in the paper.

A VARIABLE FREQUENCY MODEL, LINEAR FREQUENCY VARIATION

In the following, it is assumed that the frequency of the voltage signals has a linear variation as:

$$f = f_0 + bt, \tag{1}$$

where f_0 is the nominal frequency 50 or 60 Hz and b is the rate of change of frequency, measured by Hz/s. Then:

$$\omega(t) = 2\pi f = 2\pi f_0 + 2\pi bt. \tag{2}$$

The angle of the voltage signal, in this case, is given by:

$$\theta(t) = \int \omega(t)dt = (2\pi f_0 + \pi bt)t + \phi. \tag{3}$$

The voltage signal can be written as:

$$v(t) = \sqrt{2}V \sin\theta(t) = \sqrt{2}V \sin[(2\pi f_0 + \pi bt)t + \phi]. \tag{4}$$

Equation 4 can be written at any sample k; $k = 1, 2 \ldots m$, where m is the total number of samples in the data window size, as:

$$v(t)=\sqrt{2}V \sin\lfloor(2\pi f_0 k\Delta T+\pi bk^2(\Delta T)^2)+\phi\rfloor+\xi(k), \tag{5}$$

where:

V	the rms. of the signal amplitude,
ΔT	the sampling time $= 1/F_s$; F_s is the sampling frequency,
k	the sampling step; $k = 1, \cdots, m$,
ϕ	the voltage phase angle,
$\xi(k)$	the noise terms, which may contain harmonics.

Equation 5 describes the voltage signal for a time-varient frequency. If $b = 0$, Equation 4 becomes a voltage signal with a constant frequency [18].

Problem Formulation

Given m samples of the voltage signal at the relay location, these samples may, or may not, be contaminated with harmonics and/or noise. It is required to estimate signal parameters, voltage amplitude, nominal frequency f_0, rate of change of frequency b and phase angle ϕ, so that the sum of the absolute value of the error is minimum. This can be expressed mathematically as:

$$J = \sum_{k=1}^{m} \Big| v(k\Delta T) - \sqrt{2}V \sin\Big[(2\pi f_0 k\Delta T + \pi bk^2(\Delta T)^2) + \phi\Big]\Big|. \tag{6}$$

The techniques used earlier tried to employ some sorts of approximation for this cost function, like the Taylor's series expansion, to make this cost function linear in the parameters and the linear programming based simplex method is used to solve the resulting problem [1,2]. This may produce accurate estimates if the power system frequency variation is small and close to the nominal value. But, if the frequency variation is too large, the estimates will be poor.

PROPOSED SA ALGORITHM

SA is a Monte Carlo technique for finding solutions for optimization problems [18-24]. In applying the SAA to solve optimization problems, the basic idea is to choose a feasible solution at random and, then, get a neighbor to this solution. A move to this neighbor is performed if it has either a better (lower) objective value or a higher objective function value, if exp $(-\Delta E/Cp) \geq R(0,1)$, where ΔE is the increase in objective value if one moves to the neighbor, Cp is a control parameter representing the temperature and $R(0,1)$ is a random number between 0 and 1. The algorithm starts with a high value of Cp, accepting solutions of higher objective function, which makes a diversion of the search. The effect of decreasing Cp during the algorithm is that the probability of accepting an increase in the objective function value is decreased during the search, which intensifies the search around the local minima to find the best solution.

The proposed algorithm is aimed to find the best (optimal) estimate for signal amplitude, frequency and phase angle of the power system, having a constant frequency during data windows. To estimate optimal parameters, problem is formulated as a nonlinear optimization problem in continuous variables.

The objective criterion (function) (Equation 3), J, is chosen to minimize the sum of the absolute value of the error between the sampling signal and the estimated signal at all sampling time periods. Implementation details of the SAA are given in the following section.

The major steps of the algorithm are summarized as follows:

Step 0: Set iteration counter ITR = 0. Set the initial temperature of the cooling schedule that results in a high probability of accepting new solutions. Initialize step size vector, $\mathrm{G}_{\mathrm{stepo}}(i)$ for all values of variables $G(i)$, $i = 1, 2, 3$;

Step 1: Find, randomly, initial values for the estimated parameters and set it as the current and best solution;

Step 2: Determine the error for the current estimated parameters;

Step 3: Generate randomly a new estimate (new trial solution); as a neighbor to the current solution;

Step 4: Calculate the performance index at the new estimate;

Step 5: Perform the SAA acceptance test; to accept or reject the trial solution (see SAA Test);

Step 6: Check for equilibrium at this temperature (see Equilibrium Test). If equilibrium is reached, go to Step 7, else, go to Step 3;

Step 7: If the pre-specified maximum number of iterations is reached, then stop, else, go to Step 8;

Step 8: If the step size vector values, for all variables (G_{step}) are less than a prespecified value, then stop, else, go to Step 9;

Step 9: Update the step size vector values (see Step Size Vector Adjustment). Decrease the temperature according to the polynomial time cooling schedule (see Cooling Schedule). Go to Step 3.

DETAILS OF THE SAA

SAA Test

The implementation steps of the SAA test, as applied to each iteration in the algorithm, are described as follows [19,24]:

Step 1: At the same calculated temperature, c_p^k, apply the following acceptance test for the new trial solution;

Step 2: Acceptance test: If $E_j \leq E_i$, or, if $\exp[(\mathrm{E}_i - E_j)/C_p] \geq R(0,1)$, then, accept the trial solution, set $X_i = X_j$ and $E_i = E_j$. Otherwise, reject the trial solution, where X_i, X_j, E_i, E_j are the SAA current solution, the trial solution and their corresponding cost, respectively;

Step 3: Go to the next step in the algorithm.

Equilibrium Test

The sequence of trial solutions generated in the SAA at a fixed temperature is stopped as soon as thermodynamic equilibrium, detected by some adequate condition, is reached. Then, the temperature and step vectors are suitably adjusted [18,24].

The test of equilibrium is done as follows: If the NTRACP (T) < $N1$*n and NTR (T) < $N2$*n, then, continue at the same temperature, otherwise, end the temperature stage, where NTRACP (T), NTR (T) are the number of trials accepted and attempted at temperature T, respectively, n is the number of variables in the problem and $N1$ and $N2$ are end temperature stage parameters.

Step Size Vector Adjustment

In this work, the step vector is updated jointly with the cooling schedule temperature, according to the acceptance rate of the attempted moves at the previous temperature stage [18,24]. All the components are updated simultaneously. The following steps explain how the step vector is adjusted mathematically:

Step 1: Calculate $P(i)$ = NTRACP(i)/NTR(i), i = 1...N, where NTRACP (i) is the number of trials accepted, then, variable i is changed. NTR (i) is the number of trials attempted by changing variable i;

Step 2: If $P(i)$ >PMAX, then, STEP(i) = STEP(i)* STEPMAX; if $P(i)$ <PMIN, then STEP(i) = STEP(i)*STEPMIN, where PMAX, PMIN, STEPMAX and STEPMIN are parameters taken in this implementation as 0.05, 0.5, 0.8 and 1.2, respectively [19,24].

Cooling Schedule

A finite-time implementation of the SAA can be realized by generating homogenous Markov chains of finite length for a finite sequence of descending values of the control parameter. To achieve this, one must specify a set of parameters that governs the convergence of the algorithm. These parameters form a cooling schedule. The parameters of the cooling schedules are: An initial value of the control parameter decrement function for decreasing the control parameter; a final value of the control parameter specified by the stopping criterion; and a finite length of each homogenous Markov chain. In this work, a polynomial-time cooling schedules is used, in which the temperature is decreased based on the statistics of the trial solutions acceptance or rejection during the search.

TESTING THE ALGORITHM WITH SIMULATED DATA

In this section, the proposed algorithm is tested using simulated examples. Two tests are performed. In the first test, the signal is assumed to be a noise free signal and the effects of number of samples and sampling frequency on the estimated parameters are studied. The voltage signal waveform is given as:

$$v(t) = \sqrt{2}\sin(2\pi 50t + 0.2\pi t^2 + 30°).$$

This signal is sampled using a sampling frequency of 1000 Hz and 200 samples are used to estimate the signal parameters. It has been found that the proposed algorithm estimates the signal parameters accurately. These estimates are:

$$V = 1.0 \text{ p.u}, \quad f_0 = 50.0, \quad b = 0.10, \quad \phi = 30.0°.$$

Effects of Number of Samples

The effects of a number of samples on the estimated parameters are studied in this section, where the sampling frequency is kept constant at 1000 Hz and the number of samples changes from 50 to 250. Table 1 provides the results obtained for the test.

Examining Table 1 reveals the following remarks:

- For a number of samples greater than 50, the SAA produces an accurate estimate for the signal parameters;
- At a number of samples equal to 50, an inaccurate estimate for the rate of frequency change and phase angle is obtained, while an accurate estimate for the voltage amplitude and nominal frequency is produced;
- For an integer number of cycles, an accurate estimate for the parameters is obtained.

Effects of Sampling Frequency

The effects of sampling frequency on the estimated parameters are studied in this section, where the number of samples is kept constant at 200 and the sampling frequency changes from 250 Hz and 1500 Hz. Table 2 shows the results obtained for this test. Examining this table, it is noted that:

- The proposed algorithm, at the specified number of samples and sampling frequency, produces very accurate estimates for the signal parameters;
- For this test, a number of samples equalling 200 and a sampling frequency of 750 Hz are recommended to produce accurate estimates.

Effects of Harmonics

Today, due to the widespread use of power electronics devices in power system operation and control, the voltage waveforms are polluted with all kind of harmonics. In this test, it is assumed that the signal is contaminated with the third and fifth harmonics. Also, 200 samples with a sampling frequency equal to 2000 Hz are used for the following results:

$$V = 1.00 \text{ (p.u)}, \quad f_0 = 50.0 \text{ Hz},$$

$$b = 0.10198, \quad \phi = 30.0°.$$

Here, it is assumed that the harmonics frequencies are an integral number of the nominal frequency, which is assumed to be 50 Hz. Examining these results, one

Table 1. Effects of number of samples, sampling frequency = 1000 Hz.

m	# of Cycles	V (p.u)	$f_0 = a$ (Hz)	b (Hz/sec)	ϕ
50	2.5	1.0	50.0	0.06818	29.996
100	5	1.0	50.0	0.099771	30.0
150	7.5	1.0	50.0	0.10132	30.0
200	10	1.0	50.0	0.10037	30.0
250	12.5	1.0	50.0	0.099781	29.999

Table 2. Effects of number of sampling frequency, $m = 200$ samples.

Sampling Frequency	# of Cycles	V (p.u)	$f_0 = a$ (Hz)	b (Hz/sec)	ϕ
250	40	1.0	50.0	0.09996	30.0
500	20	1.0	50.0	0.09996	30.0
750	40/3	1.0	50.0	0.100	30.0
1000	10	1.0	50.0	0.10037	30.0
1250	8	1.0	50.0	0.10016	30.0
1500	20/3	1.0	50.0	0.10019	30.0

notices that the SAA produces very accurate estimates for the signal parameters from a harmonics polluted signal. Figure 1 shows the simulated and estimated waveforms, together with the error.

It can be noticed, from the figure, that the SAA produces the same signal exactly, since the error in all samples is almost zero.

EXPONENTIAL DECAYING FREQUENCY

Another test is conducted in this section, where it is assumed that the frequency of the voltage signal has the form of:

$$f = f_0 + be^{-ct},$$

where f_0, b and c are the parameters to be estimated. The voltage signal equation, in this case, becomes:

$$v(t) = \sqrt{2}V \sin(2\pi f_0 t - \frac{2\pi b}{c}e^{-ct} + \phi_0). \tag{7}$$

This type of variable frequency could be obtained at transient operation in power systems. The cost function to be minimized, based on least absolute error

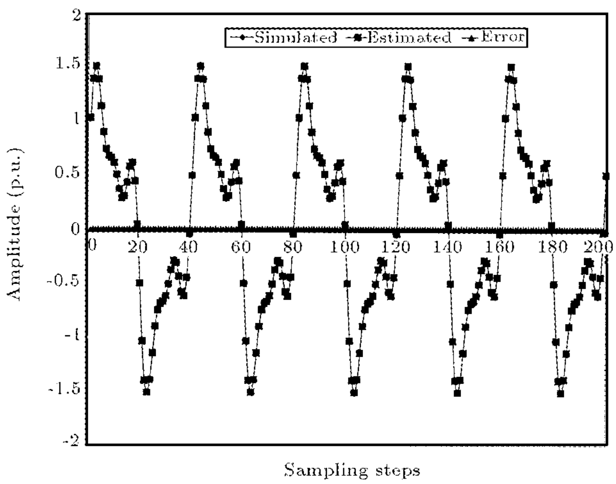

Figure 1. Simulated and estimated signal with the error.

in this case, is given by:

$$J = \sum_{i=1}^{m} \left| v(t_i) - \sqrt{2}V \sin(2\pi f_0 t_i - \frac{2\pi b}{c}e^{-ct_i} + \phi_O) \right|. \tag{8}$$

In this test, it is assumed that $f_0 = 50.0, b = 0.1, c = -10$ and $\phi = 30°$. The signal is sampled at 1000 Hz and 200 samples are used. The results obtained for this simulation are:

$$V = 1.0, \quad f_0 = 50.0, \quad b = 0.0996,$$

$$c = 10.18, \quad \phi = 29.92.$$

The error in the estimated value of b equals 0.4 percent while the estimated value of c equals 1.8 percent. These are acceptable for a highly non-linear estimation.

Actual Recorded Data

The proposed algorithm is tested on actual recorded data generated from EMTP, due to a fault in a power system. The frequency is, first, assumed to be a linear time-variant and, second, with exponential decaying. The results for the linear time-variant is: $V = 0.976$ (p.u), $f_0 = 49.8$ Hz, $b = 0.370$ Hz/s and $\phi = 90.94°$. Figure 2 compares the actual and the estimated signal waveforms.

Examining this curve carefully reveals that:

- A large error in the estimated wave is produced in the first quarter of the cycle, since the frequency is constant to the nominal value of 50 Hz and the model assumes a linear variation in this part of the data window size;
- During the fault, the error reaches a small value until the end of the data window size. It means that the model for the frequency is adequate for this part of the data window size.

In the second test, an exponential decay model is assumed for the frequency, which leads to the following results:

$$V = 0.976 \text{ (p.u)}, \quad f_0 = 49.93 \text{ Hz}, \quad b = 0.03,$$

$$c = 14.26, \quad \phi = 90.73.$$

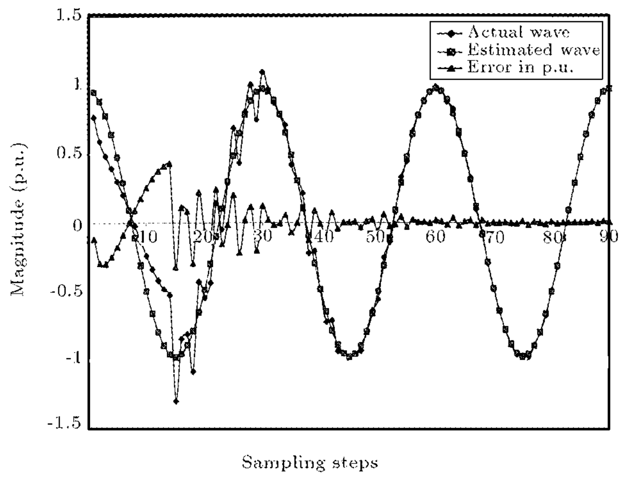

Figure 2. Actual and estimated waveform with the error for the actual recorded data.

By examining Figure 3, the same conclusions as of Figure 2 are reached, except in the decaying model. The decaying term goes to zero very quickly, since the coefficient, c, is relatively large. However, for the two frequency models used, the frequency is a time-varient, thus, it needs a dynamic estimation algorithm to track the frequency variation at each instant.

CONCLUSIONS

In this paper, the simulated annealing algorithm (SAA) is used to estimate the frequency of a power system, where a time-variant frequency model is assumed for the voltage signal. The proposed algorithm uses a digitized sample of the voltage waveform at the relay location and is tested using simulated and actual data. The algorithm is able to predict frequency and rate of frequency change from a highly nonlinear function and does not need any approximations. It has been shown that the proposed algorithm is a little sensitive to the number of samples used in the estimation, but the sampling frequency should satisfy the sampling theory and the data window size must be an integer number of cycles.

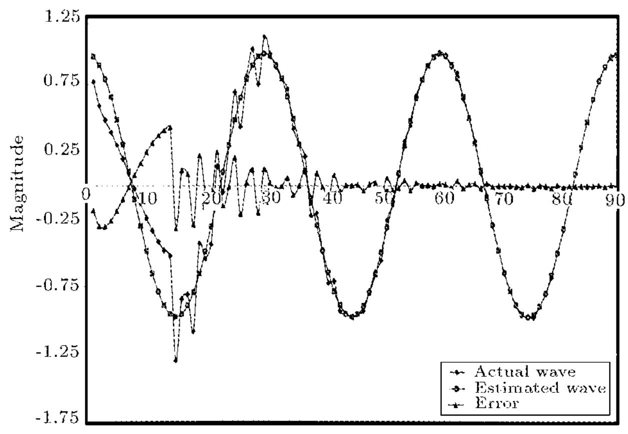

Figure 3. Actual and estimated waveform with the error for the second test where $c = 14.26$.

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