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A Convergent Genetic Algorithm for Pipe Network Optimization

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A highly convergent Genetic Algorithm (GA) for pipe network optimization is presented in this paper. An artificial genotype passing mechanism, an alternative penalty cost calculation method, an iterative setting of the penalty parameters prior to the GA search and, more importantly, a new selection operator, are introduced in the proposed GA. The genotype passing mechanism leads to a monotonically decreasing convergence curve of the GA search and, therefore, paves the way for introducing a logical convergence criterion for genetic algorithms. The use of an alternative penalty cost calculation leads to a better distribution of the fitness function in the search space, compared to conventional methods and, therefore, helps the GA to locate useful genes. Penalty parameters used for the calculation of the penalty cost are determined prior to a GA search, via use of a mathematical programming method, eliminating the possibility of choosing too low or high parameter values. Finally, a new selection operator is designed in an attempt to simulate the process of natural mating more closely, leading to an improvement in the optimality and convergence characteristics of the method. The efficiency of the proposed GA is shown by applying the method to the optimal design of three well-known benchmark networks, namely two-loop, Hanoi and New York networks. The method produces results comparable to the best results presented in the literature with much less computational effort.

INTRODUCTION

The problem of network optimization requires the determination of pipe sizes from a set of commercially available diameters ensuring a feasible least-cost solution. Various methods, with different degrees of success, have been devised by different researchers to solve this problem. These methods can be grouped into three classes: Enumeration, mathematical programming and random search methods. Enumeration methods, capable of finding a global optimum solution to a pipe network design problem, are very costly and cannot be used for the optimization of real-world networks [1,2]. On the other hand, mathematical programming methods are very efficient from a computational point of view, but, are often trapped in saddle points in their search for the global optimum of the pipe network design problem. The computational efficiency of mathematical programming methods is, of course, limited to continuous solutions, which are not favored from an engineering point of view [3-9]. Random search methods have shown to logically balance between computational efficiency and the capability of approaching a global optimum. Among the random search methods, the GA has gained more popularity for pipe network optimization in recent years. The early research was primarily concentrated on developing a methodology for applying GA to pipe network optimization problems using simple genetic algorithms [10-14]. More recent investigations on the application of GA to pipe network optimization have focused on the development of new genetic algorithms to yield less costly solutions than already existing algorithms. These improvements are mostly achieved via modifications of the simple genetic algorithm or by introducing new operators and features to the basic algorithms [15-20]. This paper presents a new genetic algorithm with improved convergence characteristics compared to simple and even recently improved genetic algorithms. The improvements are achieved via four modifications made to the simple and basic genetic algorithm, as follows: (1) Introduction

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of an artificial genotype passing mechanism; (2) Predetermination of the penalty parameters; (3) Use of an alternative method of penalty cost calculation and, more importantly, (4) Introduction of a new selecting operator. The proposed algorithm is shown to outperform the existing algorithm from a convergence characteristics point of view, while producing results comparable to the cheapest solutions available in the literature.

PROBLEM FORMULATION

The optimal design of a pipe network with a prespecified layout in its standard form can be described as:

$$\min C = \sum_{l=1}^{m} L_l C_l, \tag{1}$$

subject to:

1. Hydraulic constraints:

$$\sum_{l\in k} q_l = Q_k \qquad k = 1, 2, \cdots, n,$$

$$\sum_{l\in p} J_l = 0 \qquad p = 1, 2, \cdots, P,$$

$$q_l = Kch_l d_l^{\alpha} (J_l/L_l)^{\beta}, \tag{2}$$

   with $\alpha = 2.63, \beta = 0.54$ and $K = 0.06393160$ for q in cm/hr and d in inches.

2. Head and flow constraints:

$$H_k \geq H_{\min} \qquad k = 1, 2, \cdots, n,$$

$$q_l \geq q_{\min} \qquad l = 1, 2, \cdots, m. \tag{3}$$

3. Pipe size availability constraints:

$$d_l \in [d_{id}], \quad l = 1, 2, \cdots, m, \ id = 1, 2, \cdots, nd, \tag{4}$$

where:

L_l	length of the lth pipe,
C_l	per unit cost of the lth pipe,
d_l	diameter of the lth pipe,
$[d_{id}]$	set of commercially available diameters,
q_l	flow in the lth pipe,
J_l	head loss in the lth pipe,
H_k	nodal head at the kth node,
$H_{\min}$	minimum allowable hydraulic head,
$q_{\min}$	minimum allowable pipe flow,
$d_{\min}$	minimum allowable pipe diameter,
$d_{\max}$	maximum allowable pipe diameter,
n, p, m	total number of nodes, loops and links in the network, respectively.

The first set of constraints describes the flow continuity at nodes, head loss balance in loops and the Hazen-Williams equation. The second set refers to the minimum nodal head and pipe flow requirements while the last constraint requires that the optimal pipe diameters should be chosen from a set of commercially available diameters. Equation 1 describes the total cost of the pipes in the network.

Here, as in all GA searches, the second set of constraints is included in the objective function via the use of an exterior penalty method, resulting in the following penalized problem:

$$\min C_p = \sum_{l=1}^{m} C_l L_l + \sum_{l=1}^{m} \alpha_l (q_l - q_{\min})^2 + \sum_{k=1}^{n} \alpha_k (H_k - H_{\min})^2, \tag{5}$$

where α_l and α_k are the pipe flow and nodal head penalty parameters, respectively, with large values when corresponding constraints are violated and zero values otherwise.

A SIMPLE GA FORMULATION

The following steps are taken in a simple GA search for the optimal design of the pipe networks:

1. Encoding the design variables. The genetic algorithm requires that any trial solution of the design problem be represented by a coded string of finite length, similar to the structure of a chromosome of a genetic code. This is usually achieved by defining a selected mapping between the possible values of the design variables and a set of coded sub-strings with a required number of binary bits. For example, a four-bit sub-string can be coded to represent any of the 16 commercially available pipe diameters. Here, a binary coding is used to represent the possible values of the pipe diameters;
2. Generation of an initial population. The GA randomly generates an initial population, of size N, of coded strings representing some trial solutions to the pipe network design problem;
3. Computation of network cost. Each of the N members of the population is considered in turn and decoded to the corresponding pipe networks. The cost of each trial solution of the current population is then calculated;
4. Hydraulic analysis of the network. A steady-state analysis is carried out for each network of the current population to find the pressure and velocity constraint violations. In this work, the hydraulic

constraints are satisfied via the use of an element-by-element simulation program, which explicitly solves the set of hydraulic constraints for nodal heads [21];

5. Computation of the total penalized cost. The penalty cost of the networks in the population is computed if the trial design does not satisfy the pressure and velocity constraints. The total penalized cost is considered as the sum of the network and penalty cost;
6. Computation of the fitnesses. The fitness of a trial design is taken as some function of the total network cost. Investigators use different forms of the fitness functions [15,22]. Here, the deficit of the total cost from a big number and the sum of the maximum and minimum total network cost of the current generation, is used as the fitness of each network;
7. Generation of a new population. The GA generates the members of the new generation by a roulette wheel selection scheme. In this scheme, the probability of a string i, p_i, to be selected for the next generation, is given by:

$$p_i = \frac{f_i}{\sum_{i=1}^{N} f_i}. \tag{6}$$

8. The crossover operation. Two off-springs are formed via partial exchange of bits between two selected parents by using a crossover operator. Crossover occurs with some specified probability of crossover, p_c, for each pair of parents selected in the previous step;
9. Mutation. A bit-wise mutation, with some specified probability of mutation, p_m, is carried out for each of the strings that have undergone crossover. The bit-wise mutation changes the value of the selected bit to the opposite value (i.e., 0 to 1 or 1 to 0);
10. Production of successive generations. The three operators described above produce a new generation of pipe network trial designs. This procedure is repeated to create successive generations. Typically, a GA will evaluate between 100 and 1000 generations, depending on the problem size.

A HIGHLY CONVERGENT GENETIC ALGORITHM

The highly convergent GA formulation is achieved by introducing the following features into the simple and basic GA: (1) Alternative penalty cost calculation; (2) Pre-determination of penalty parameters; (3) Best genotype passing mechanism; and (4) New selection operator.

Penalty Cost Calculation

It is a common practice to use the maximum constraint violation for calculating the penalty cost [15,17,19,23]. In this method, the GA could not distinguish between two different designs with the same maximum constraint violation but with a different number of constraint violations. Here, the penalty cost is calculated according to Equation 5, in which all the constraint violations are used for the penalty cost calculation. This method ensures that non-proper networks would have more penalty costs and, therefore, leads to a better distribution of the fitness function in the search space, compared to the conventional method, helping the GA to locate useful genes.

Pre-Determination of the Penalty Parameters

The values of the penalty parameters used in the GA formulation of the pipe network optimizations are usually specified by the user [15,19,23]. This method of specifying the penalty parameters requires a trial-and-error procedure to get the proper range for the parameter values before the final runs. The proper setting of the penalty parameters is very important in the GA solution to the pipe network optimization problems, as a low value of the penalty parameters could lead to a constraint-violating solution, while a high value of the parameters would result in rejecting some of the constraint-violating solutions with useful genes from the evolution process [22]. Here, the proper setting of the penalty parameters is achieved before starting the genetic search by use of a mathematical programming method to solve the problem for it's continuous solution. For this, an iterative setting of the penalty parameter is chosen, whereby the minimization of the penalized cost function in Equation 5 is substituted with a series of minimization problems with different values of penalty parameter values. The procedure starts with a value of unity for penalty parameters and the design problem is solved. The solution is then checked for constraints violation, upon which the next design problem is defined by an increased value in the penalty parameters, by an order of magnitude, if corresponding constraints are not satisfied. This procedure is continued until all the constraints are satisfied, i.e., all the penalized terms are equal to zero. The computational effort required for the setting of the penalty parameters is negligible compared to the one required by the genetic search.

Best Genotype Passing Mechanism

One of the recognized problems of the GA is the uncertainty about the termination of the search, since

Table 1. GA parameter values.

GA Parameter	Test No.		
	1	**2**	**3**
Population size, N	100,200	250,500	250,500
Probability of crossover	1	1	1
Probability of mutation	0.5	0.5	0.5
Crossover operator	One point	One point	One point
Mutation operator	One bit	One bit	One bit
Pressure penalty parameter (automatically determined prior to GA search)	10^8	10^8	10^{10}

the best solution of the generation could oscillate during the evolution process. A too loose convergence criterion could, therefore, lead to an early termination, while a too stringent termination criterion would increase the computational cost. Here, the fittest string of the current generation is directly copied into a randomly chosen position of the next generation. This mechanism has two advantages. First, useful information in the fittest string is directly passed to the next generation as a member, in addition to the information carried by its off-springs. This ensures that this information is never lost during the evolutionary process. Second, the oscillatory nature of the best solution of the generations is eliminated, paving the way for the definition of a logical convergence criterion. In this work, the GA search is assumed to have converged, if the best solution of the generations remains constant for a specified number of generations.

Selection Operator

The genetic algorithm owes its credit to the claim of simulating the real-world evolutionary process engineered by nature. Three basic GA operators, such as selection (mating), crossover and mutation operators used in the reproduction stage of any genetic search, are designed to mimic nature as closely as possible. If this is the case, then, the proportionate selection scheme used in the simple GA is not simulating nature. This method often leads to the dominance of a few good solutions and, hence, premature convergence, which is not seen in nature. What really happens in the natural mating process is twofold. First, the genotypes will have a tendency to look for a fitter mate or mates. This means that genotypes of very low and very high fitnesses are unlikely to mate. Second, the search for the mate is often limited to a small community rather than the whole population. The size of this community is, of course, larger for a big size population and vice-versa. In an effort to closely simulate this mating procedure, a new selection operator is devised and used in this work. A small community of random size between 2 and $N^{0.5}$ is randomly generated out of the current population and, then, the two fittest members are chosen to mate. This operator is believed to be the most responsible for the efficiency of the proposed GA.

Here, only the results of the improved GA, considering all the modifications, will be presented and the effect of individual modifications will not be discussed. The parameters used for the improved GA are shown in Table 1. In all the examples, the GA search is terminated if the cost of the best generations remains constant for 50 subsequent generations.

TEST PROBLEMS

The first problem to be considered is a two-loop network with 8 pipes, 7 nodes and one reservoir shown in Figure 1 [23]. All the pipes are 1000 m long and the Hazen-Williams coefficient is assumed to be 130 for all the pipes. The minimum nodal head requirement for all demand nodes is 30 m. There are 14 commercially available pipe diameters, as listed in Table 2. Figure 2 shows the best generation cost against the number of network analyses required for two different numbers of population in each generation. The solution of 420,000 units is obtained at the expense of 3,400

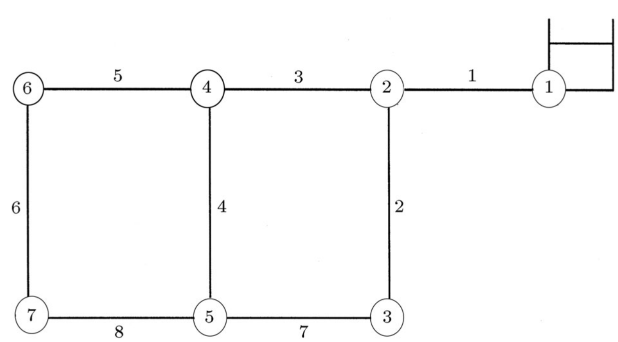

Figure 1. Two-loop network.

Table 2. Cost data for the two-loop network.

Diameter (inch)	1	2	3	4	6	8	10	12	14	16	18	20	22
Cost (units/m)	2	5	8	11	16	23	32	50	60	90	130	170	300

Table 3. Optimal pipe diameters along with some of the available discrete results for two-loop network.

Pipe	**Present Work**		**Abebe and Solomatine [23]**	**Wu et al. [25]**	**Savic and Walters [17]***	
	P100	**P200**		**FMGA**	**GA2**	**GA1**
1	18	18	18	18	20	18
2	14	10	14	10	10	10
3	14	16	14	16	16	16
4	1	4	1	4	1	4
5	14	16	14	16	14	16
6	6	10	1	10	10	10
7	14	10	14	10	10	10
8	10	1	12	1	1	1
Cost (units)	420000	419000	424000	419000	420000	419000

* These solutions are obtained using different numerical conversion constant for the head loss equations.

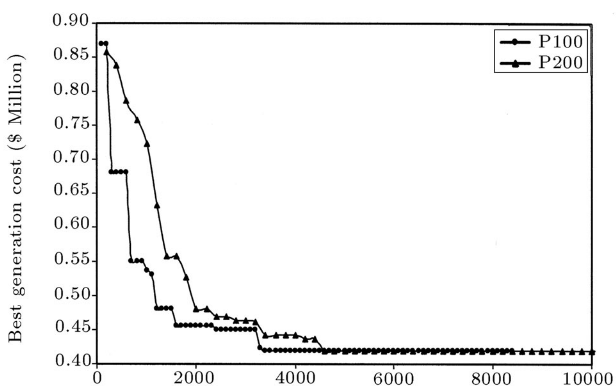

Figure 2. Best of generation cost for two-loop network.

evaluations, while the best ever solution of 419,000 units is reached after 4,600 evaluations. This compares favorably with $\sim 250,000$ evaluations required by the method of Savic and Walters [17], $\sim 53,000$ evaluations required by the method of Cuncha and Sousa [24], 9,201 evaluations required by the fast messy genetic algorithm of Boulos et al. [19] and 7,467 evaluations required by the fast messy genetic algorithm of Wu et al. [25] to get the least cost solution of 419,000 units. Table 3 compares the results produced by the presented method to some of the random search results available in the literature [17,19,23,24]. These solutions are obtained in less than 1 minute on a 366 HZ PC.

The second test problem is that of the Hanoi network shown in Figure 3, with 34 pipes, 31 demand nodes and one reservoir [23]. The minimum nodal head requirement at all demand nodes is 30 m. Table 4 shows the diameters of commercially available pipes and their cost calculated, based on the analytical cost function 1.1 $D^{1.5}$ [9]. This is a difficult problem amongst the three benchmark examples in the literature and has been considered by only a few of the GA researchers [17,23,25]. Figure 4 shows the best generation cost against the number of network analyses required for two different numbers of population in each generation. The solution of \$6.31 million is obtained at the expense of 13,000 evaluations, while the cheaper solution of \$6.14 million is reached after 23,000 evaluations. This compares favorably with about 1,000,000 evaluations required by the method of Savic and Walters [17] to obtain the corresponding solutions of \$6.19 million and \$6.07 million and 113600 evaluations by the fast messy genetic algorithm of Wu et. al [25] to produce solutions of \$6.182 million and \$6.056 million. Table 5 compares the results produced by the proposed method with some of the available GA search solutions [17,23]. It should be noted that these solutions are obtained with 8 and 15 minutes of CPU time on a 366Hz PC.

The third test problem concerns the rehabilitation of the New York City water supply network with 21 pipes, 20 demand nodes and one reservoir, as shown in Figure 5 [15]. The commercially available pipe diameters and their respective costs are listed in Table 6, while the pipe and nodal data of the existing network are shown in Table 7. Figure 6 shows the best generation cost against the number of network analyses

Table 4. Cost data for the Hanoi network.

Diameter (inch)	12	16	20	24	30	40
Cost (units/m)	45.73	70.40	98.39	129.33	180.75	278.28

Table 5. Optimal pipe diameters and nodal heads obtained by different methods for Hanoi network.

Pipe	**Present Work**		**Abebe and Solomatine [23]**	**Savic and walters [17]***		**Wu et al. [25]***	
	P250	**P500**		**GA1**	**GA2**	**fmGA1**	**fmGA2**
1	40	40	40	40	40	40	40
2	40	40	40	40	40	40	40
3	40	40	40	40	40	40	40
4	40	40	40	40	40	40	40
5	40	40	30	40	40	40	40
6	40	40	40	40	40	40	40
7	40	40	40	40	40	40	40
8	30	40	30	40	40	40	40
9	40	40	30	40	30	40	40
10	24	24	30	30	30	30	30
11	30	30	30	24	30	24	24
12	30	24	30	24	24	24	24
13	16	16	16	20	16	16	20
14	16	16	24	16	16	12	16
15	12	12	30	12	12	12	12
16	12	12	30	12	16	12	12
17	20	20	30	16	20	20	16
18	24	20	40	20	24	24	20
19	20	30	40	20	24	24	20
20	40	40	40	40	40	40	40
21	20	20	20	20	20	20	20
22	12	12	20	12	12	12	12
23	40	40	30	40	40	40	40
24	30	30	16	30	30	30	30
25	30	30	20	30	30	30	30
26	30	20	12	20	20	24	20
27	12	12	24	12	12	12	12
28	12	12	20	12	12	12	12
29	16	16	24	16	16	16	16
30	12	12	30	16	16	16	12
31	12	12	30	12	12	12	12
32	20	30	30	12	12	16	16
33	20	16	30	16	16	16	16
34	24	24	12	20	20	24	24
Cost ($M)	6.31	6.14	7.0	6.07	6.19	6.182	6.056

* These solutions are obtained using different numerical conversion constant for the head loss equations.

Table 6. Pipe cost data for New-York network.

Diameter (inch)	0	36	48	60	72	84	96	108
Cost ($/ft)	0	93.5	134.0	176.0	221.0	267.0	316.0	365.0
Diameter (inch)	120	132	144	156	168	180	192	204
Cost ($/ft)	417.0	469.0	522.0	577.0	632.0	689.0	746.0	804.0

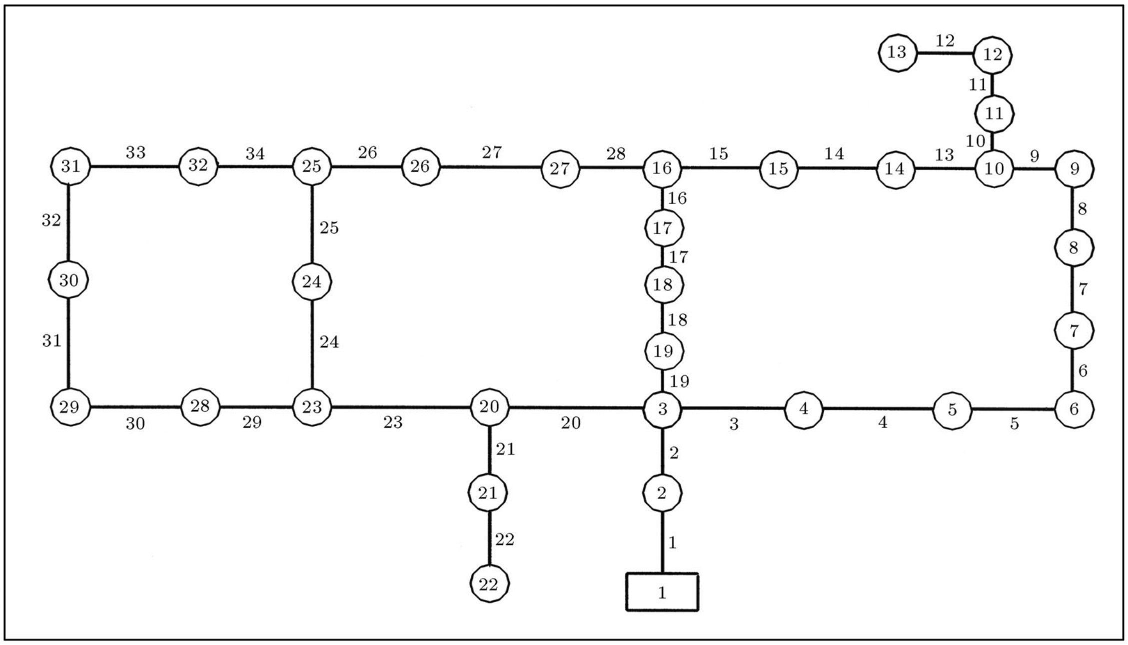

Figure 3. Hanoi network.

Table 7. Pipe and nodal data for New York tunnel network.

Pipe Data					Nodal Data		
Pipe	**Start Node**	**End Node**	**Length (ft)**	**Existing Diameter (inch)**	**Node**	**Demand (Cft/s)**	**Min Total Head (ft)**
1	1	2	11600	180	1	Reservoir	300
2	2	3	19800	180	2	92.4	255
3	3	4	7300	180	3	92.4	255
4	4	5	8300	180	4	88.2	255
5	5	6	8600	180	5	88.2	255
6	6	7	19100	180	6	88.2	255
7	7	8	9600	132	7	88.2	255
8	8	9	12500	132	8	88.2	255
9	9	10	9600	180	9	170	255
10	11	9	11200	204	10	1	255
11	12	11	14500	204	11	170	255
12	13	12	12200	204	12	117.1	255
13	14	13	24100	204	13	117.1	255
14	15	14	21100	204	14	92.4	255
15	1	15	15500	204	15	92.4	255
16	10	17	26400	72	16	170	260
17	12	18	31200	72	17	57.5	272.8
18	18	19	24000	60	18	117.1	255
19	11	20	14400	60	19	117.1	255
20	20	16	38400	60	20	170	255
21	9	16	26400	72			

Table 8. Optimal pipe diameters and nodal heads obtained by different methods for New York network.

Pipe	**Present Work**		**Dandy et al. [15]**	**Boulos et al. [19]***		**Murphy et al. [13]**	**Savic and Walters [17]***		**Wu et al. [25]***	
	P250	**P500**		**GA1**	**GA2**		**GA1**	**GA2**	**fmGA1**	**fmGA2**
1	96	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0
7	84	0	0	124	108	0	108	0	124	108
8	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0
15	0	108	120	0	0	120	0	144	0	0
16	96	96	84	96	96	84	96	84	96	96
17	120	96	96	96	96	96	96	96	96	96
18	72	84	84	84	84	84	84	84	84	84
19	72	72	72	72	72	72	72	72	72	72
20	0	0	0	0	0	0	0	0	0	0
21	72	72	72	72	72	72	72	72	72	72
Cost ($M)	41.9	39.28	38.80	37.83	37.13	38.80	37.13	40.42	37.83	37.13

* These solutions are obtained using different numerical conversion constant for the head loss equations.

required for two different numbers of population in each generation. The solution of $41.9 million is obtained at the expense of 10,500 evaluations, while the less costly solution of $39.28 million is reached after 20,500 evaluations. This compares favorably with $\sim 200,000$ evaluations required by the method of Murphy et al. [13] to get the solution of $38.80, $\sim 46,000$ evaluations required by Lippai et al. [26] to get their solution of $37.83, $\sim 1,000,000$ evaluations required by the method of Savic and Walters [17] to get the solutions of $40.42 and $37.13 and, finally, 37,186 evaluations required by the fast messy genetic algorithm of Boulos et al. [19] and Wu et al. [25] to get the solutions of $37.83 million and $37.13 million. The solution to this problem is shown in Table 8, along with some of the available GA solutions. These solutions are obtained with 154 and 300 seconds of CPU time on the same machine used previously.

It should be noted that a criterion of 'no solution change for 50 consecutive generations' is used as a convergence criterion for all the examples presented.

CONCLUDING REMARKS

A highly convergent Genetic Algorithm (GA) for pipe network optimization was presented in this paper. An artificial genotype passing mechanism, an alternative penalty cost calculation method, an iterative setting of the penalty parameters prior to the GA search and, more importantly, a new selecting operator were

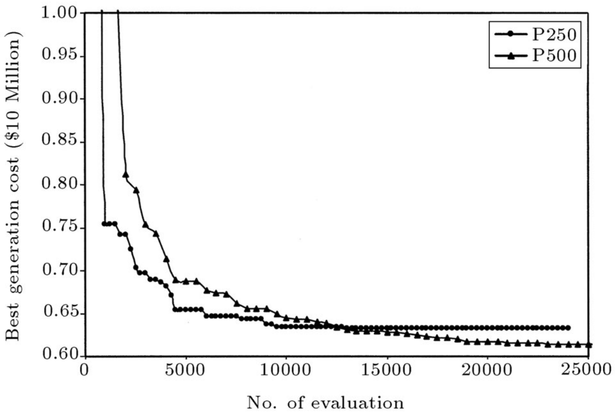

Figure 4. Best of generation cost for Hanoi network.

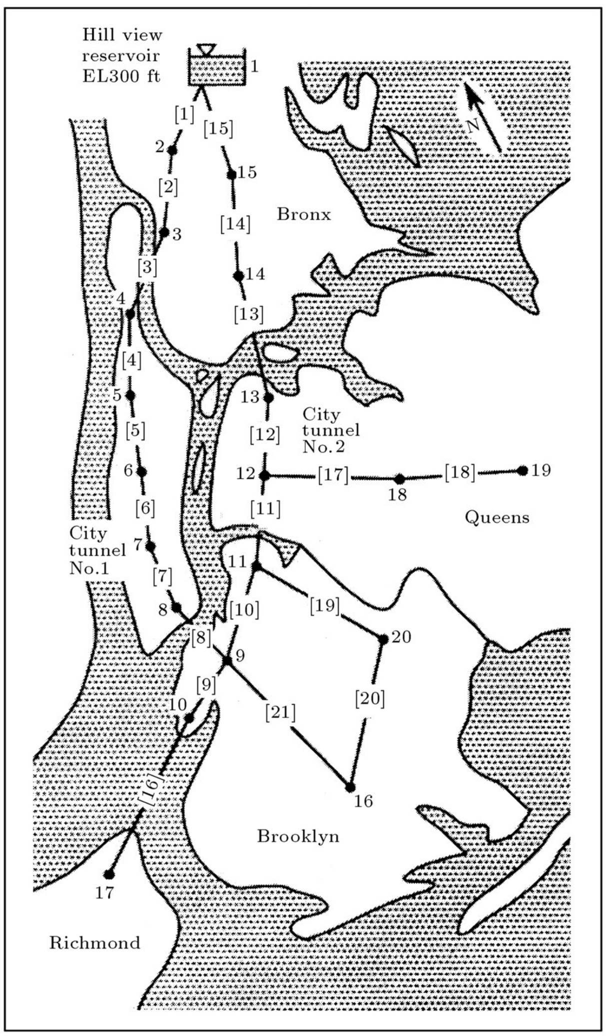

Figure 5. New York tunnel network.

introduced in the proposed GA. The genotype passing mechanism leads to a monotonically decreasing convergence curve and, therefore, paves the way for introducing a logical convergence criterion for genetic algorithms. The use of an alternative penalty cost leads

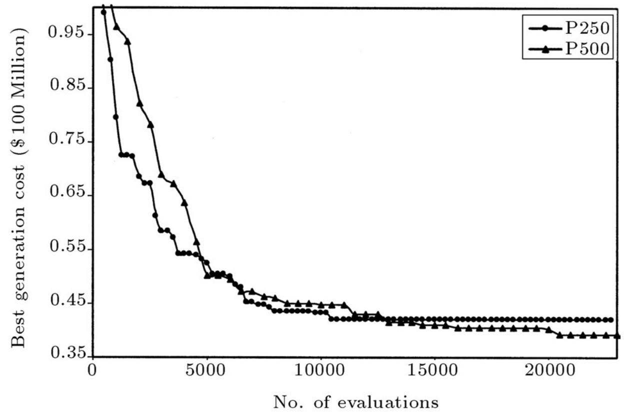

Figure 6. Best of generation cost for New-York network.

to a better distribution of the fitness function compared to the conventional method and, therefore, helps the GA to locate useful genes. Penalty parameters used for the calculation of the penalty cost were determined prior to the GA search, via use of a mathematical programming method, eliminating the possibility of choosing too low or high values of the parameters. Finally, a new selecting operator was designed in an attempt to better simulate the process of natural mating, leading to an improvement in the accuracy and convergence characteristics of the method. The efficiency of the proposed GA was shown by applying the method to the optimal design of three well-known benchmark networks, namely two-loop, Hanoi and New York networks. The method produced results comparable to the results presented in the literature with much less computational effort.

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