

Three-dimensional elasticity stress analysis of composite single-lap adhesive joints using the multi-term extended Kantorovich method

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Abstract

In the present study, three-dimensional distributions of interfacial stresses in adhesively bonded single-lap joints with composite adherends are predicted using the generalized multi-term extended Kantorovich method. Sets of coupled governing equilibrium equations are derived based on the three-dimensional elasticity theory using the principle of minimum total potential energy. Using the state-space approach, the governing equilibrium equations with partial derivatives are converted into sets of ordinary differential equations, which are solved iteratively in three dimensions. The method proposed in this study represents a new analytical framework for three-dimensional interlaminar stress analysis of thin and thick as well as narrow and wide adhesively bonded composite joints with common geometries. Interlaminar stress distributions in composite joints with general layup under various in-plane and out-of-plane loading conditions, including uniaxial tensile and shear loadings as well as bending and torsional moments were predicted. Results in comparison with the full layerwise theory and finite element simulation indicate that the proposed method can accurately estimate the stress distribution, particularly near the edges of the bonding region with high stress concentrations due to edge effects.

1. Introduction

The application of adhesive joints has significantly increased across various industries, including aerospace, automotive, construction, and marine sectors. This is primarily attributed to their high strength, low weight, good corrosion resistance, and ease of maintenance compared to traditional mechanical connections [1-3]. Although adhesive joints offer an improved stress distribution compared to other mechanical joints, the interlayer stresses rise sharply near the edges - a phenomenon known as the edge effect - making these regions more susceptible to failure [4, 5]. In addition to appropriate surface preparation and selection of suitable adhesive materials, proper geometrical design of the adherends and the bonding layer can considerably lower the stress concentrations and improve the joint strength [6-8].

Systematic modeling and optimal design of an adhesive joint requires accurate prediction of the interlayer stress distributions in the bonding region, particularly near the edges. To date no experimental technique is available to measure these stresses, and this highlights the need to use accurate analytical and numerical methods [9].

Numerical methods have been widely used in the study of adhesively bonded composite joints under different loading conditions, focusing mainly on design of joint geometry, the adhesive layer thickness, and the composite layup [10-12]. Numerical simulations offer several advantages over analytical methods, including more flexibility in modeling complex geometries and material models [13, 14], along with improved ease of use. However, analytical methods provide higher accuracy, particularly near the edges, where the stress behavior is singular [15-17]. Recently, analytical frameworks assisted by machine learning have been proposed to improve stress prediction in asymmetric adhesive joints by accounting for complex eccentric loading effects and geometric asymmetries. These approaches significantly enhance prediction accuracy while still relying on rigorous analytical formulations as their physical foundation, demonstrating that high-quality analytical solutions remain essential even in emerging data-driven methodologies [18, 19].

Analytical methods can precisely estimate the interlaminar stresses in an adhesive joint; however, their application is limited to simple geometries. Early analytical methods for modeling adhesive joints had significant drawbacks [20], as they, in particular, have oversimplified the adhesive joint as a bar [21]. Unlike advanced theories developed in recent years, the application of early analytical approaches was mostly limited to specific simple geometric configurations [20], not applicable to composite and unbalanced joints (i.e., adhesive joints with adherends of different thickness and/or material properties) [20]. An example is the use of equivalent single-layer (ESL) theories, such as Euler-Bernoulli and Timoshenko beam theories [22-24]. A primary drawback is the inherent two-dimensional assumptions made in these theories. Moreover, they are unable to accurately capture the continuity of interlaminar stress distribution through the laminate thickness, considering the fact that the displacements and their derivatives (i.e., the strains) are continuous functions. Recently, researchers have introduced a new analytical method based on the Timoshenko beam theory, called the matrix method, which enables the analysis of various symmetric and asymmetric joint geometries solely through the assembly of the stiffness matrix and load vector [25]. Despite its simplifying assumptions, this method accounts for the coupling between all loading conditions.

To account for nonlinear variations of shear deformation in modeling adhesive joints, which are not captured by the first-order shear deformation theory (FSDT) [26-28], higher-order shear deformation theories (HSDTs) have been employed [29]. Although the incorporation of nonlinear through-thickness displacements in these theories has significantly improved the accuracy of results, the HSDTs still lack sufficient accuracy in resolving continuous interlaminar stresses.

Several researchers have expanded the use of sandwich-type theories by developing the classical analytical methods [20]. In these theories, the overlap region is explicitly modeled as two adherends and an adhesive layer, analyzed through the FSDT. In a latest effort, an extended displacement-based approach using polynomial functions was developed to more accurately model the deformation behavior of a thick adhesive layer, particularly through its thickness [30].

Zigzag-based methods, which were developed based on linear variation of displacements through the thickness and discontinuities at layer interfaces, have partially addressed the limitations of the ESL theories [31]. In a refined zigzag theory the displacement field was formulated so that it allows for variations in through-thickness displacements, enabling more precise modeling of abrupt stepwise changes (step gradients) [32]. In a recent study, the use of peridynamic differential operator in solving the equilibrium equations of the refined zigzag theory showed better results, particularly at the edges [33].

To enhance the accuracy of stress distributions in adhesive joints, some researchers have employed the full layerwise theories (FLWTs) [34, 35]. In this approach, each physical layer is divided into a number of mathematical sublayers, and the displacement components are continuous throughout the thickness. However, the derivatives of displacements with respect to the thickness coordinate may be discontinuous at certain points, and consequently, the transverse stresses can vary continuously along the thickness direction [34]. Despite an improved accuracy in prediction of the interlaminar stresses in adhesive joints, particularly those with thick adherends, the FLWTs also have some limitations. A key drawback is the substantial escalation in the computational demand, particularly in adhesive joints with very thin adherends or a large number of physical layers through the thickness direction.

The use of elasticity theories can overcome most of these challenges. Two- and three-dimensional (3-D) elasticity theories have been used for stress analysis of adhesive joints [36, 37]. Unlike other methods, these approaches do not rely on simplifying assumptions and provide significantly more accurate predictions for the interlaminar stresses. In summary, to obtain highly accurate results for the interfacial stress distributions in adhesive joints, the 3-D elasticity and layerwise theories are the most appropriate choices [38].

Recent research has increasingly shifted from merely evaluating joint strength toward accurately predicting damage initiation, delamination evolution, and fracture processes in composite bonded structures [39]. High-fidelity numerical studies have demonstrated that reliable prediction of mixed-mode delamination strongly depends on accurate characterization of local interlaminar stress fields. Meanwhile, machine-learning techniques have emerged as promising tools for rapid stress and failure prediction; however, their training and calibration continue to rely on accurate physics-based analytical and numerical benchmark solutions [40, 41]. These developments further emphasize the continuing importance of rigorous analytical approaches for predicting three-dimensional interlaminar stress fields [42].

In the framework of 3-D elasticity theory, the present study employs the multi-term extended Kantorovich method (MTEKM) to accurately predict the interlayer stress distributions in adhesively bonded composite joints. In brief, by geometrically dividing a single-lap joint (SLJ) into three connected regions (i.e., a bonding mid-region and two adjacent side-regions) and using the displacement field of the MTEKM, the governing equations of equilibrium is formulated based on 3-D elasticity theory with no common simplifying assumptions proposed in beam or plate theories. For this, the principle of minimum total potential energy is employed to derive a system of fully coupled second-order ordinary differential equations (ODEs), which generally cannot be solved in analytical form. Hence, using the Kantorovich method, these equations are subsequently transformed into systems of coupled first-order ODEs using state-space variables, which can then be solved iteratively within an analytical framework. Finally, the stress distributions are predicted upon solving the equations for various in-plane and out-of-plane loading conditions in both symmetric and asymmetric composite layups.

In the Kantorovich approach, a function of three independent spatial variables is approximated as the product of three separate functions, each depending on a single spatial coordinate. Previous studies have also demonstrated that such a separated representation, or separation-of-variables form, can provide highly accurate approximations when a multi-term formulation is used. Accordingly, the present study employs the multi-term Kantorovich method to improve the accuracy of the solution. Furthermore, it has been shown in earlier works that achieving accurate results typically requires multiple iterations of the solution procedure. Therefore, in the present research, the extended multi-term Kantorovich method is adopted to ensure convergence and improve solution accuracy.

This method offers significant advantages compared to previous approaches. It is well-suited for interlaminar stress analysis of wide adhesive joints with common geometries, which are subjected to any combination of three-dimensional mechanical loading. Moreover, the proposed method takes advantage of high speed and accuracy in solving problems, avoiding extensive and complex computations in the solution procedure.

2. Mathematical formulation

The proposed adhesive joint comprises two composite laminates bonded together by a thin layer of adhesive to form an SLJ with a total thickness of h , as illustrated in Figure 1. The thicknesses of the upper and lower adherends and the adhesive layer are denoted

by t_1 , t_2 , and t_a , respectively (Figure 1a). The joint length L , shown in Figure 1b, is divided into three connected regions: left (L_1), middle (L_a), and right (L_2), and the joint width is denoted by b . The left end of the joint is fixed, while its right end may experience tension, bending, shear, or torsion. The coordinate system of the joint is positioned at its mid-length and mid-thickness, as shown in Figure 1. Furthermore, a local coordinate system $(\bar{x}, \bar{y}, \bar{z})$ is defined, whose origin is placed at the mid-thickness of the lower adherend.

In what follows, the MTEKM is used within the framework of 3-D elasticity theory to derive the governing equilibrium equations for the left region of the SLJ, which is denoted by I in Figure 1b. Similar equations are derived for the other two regions. By applying proper boundary and loading conditions at each end of the joint and satisfying appropriate continuity conditions between the adjacent regions, a set of fully coupled second-order ODEs for the whole joint is obtained.

The proposed displacement field of the MTEKM is defined as follows:

$$\begin{aligned} u(x, y, z) &= \sum_{i=1}^n u_i(x) \bar{u}_i(y) \tilde{u}_i(z) \\ v(x, y, z) &= \sum_{i=1}^n v_i(x) \bar{v}_i(y) \tilde{v}_i(z) \quad i = 1, 2, \dots, n \\ w(x, y, z) &= \sum_{i=1}^n w_i(x) \bar{w}_i(y) \tilde{w}_i(z) \end{aligned} \quad (1)$$

where $u(x, y, z)$, $v(x, y, z)$, and $w(x, y, z)$ represent the displacements of a point at coordinates (x, y, z) in the x , y , and z directions, respectively. The displacement in each direction is assumed to be expressed as the product of functions corresponding to each respective direction. The parameter n denotes the number of terms considered in the summation. Notably, the Kantorovich method is referred to as the single-term extended Kantorovich method for $n = 1$, and as the multi-term extended Kantorovich method for $n \geq 2$. Based on the displacement field defined in Eq. (1), the strain-displacement relations can be expressed as follows:

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} = \sum_{i=1}^n u_i' \bar{u}_i \tilde{u}_i \\ \varepsilon_y &= \frac{\partial v}{\partial y} = \sum_{i=1}^n v_i \bar{v}_i' \tilde{v}_i \\ \varepsilon_z &= \frac{\partial w}{\partial z} = \sum_{i=1}^n w_i \bar{w}_i \tilde{w}_i' \\ \gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \sum_{i=1}^n (v_i \bar{v}_i \tilde{v}_i' + w_i \bar{w}_i' \tilde{w}_i) \\ \gamma_{xz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \sum_{i=1}^n (u_i \bar{u}_i \tilde{u}_i' + w_i \bar{w}_i \tilde{w}_i') \\ \gamma_{xy} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \sum_{i=1}^n (u_i \bar{u}_i' \tilde{u}_i + v_i \bar{v}_i \tilde{v}_i') \end{aligned} \quad i = 1, 2, \dots, n \quad (2)$$

where a prime symbol denotes an ordinary derivative with respect to the appropriate variable x , y , or z .

The equilibrium equations are derived using the principle of minimum total potential energy. That is [38],

$$\delta \Pi = \delta U + \delta V = 0 \quad (3)$$

where Π represents the total potential energy, U is the internal strain energy, and V stands for the potential of the work done by external loads. The external loads may include any combination of tension, bending, shear, or torsion, which is applied to the right end of the SLJ (see Figure 1c). For example, the variation of V , δV , can be obtained in case of uniaxial tension as follows:

$$\delta V_{Axial} = - \int_0^b \int_{-t_2/2}^{t_2/2} \bar{\sigma}_x \delta u(\bar{x}, \bar{y}, \bar{z}) \big|_{x=L/2} d\bar{z} d\bar{y} \quad (4)$$

The variation of the internal strain energy, resulting from internal stresses, is also expressed as [43]:

$$\delta U = \iiint_V \sigma_{ij} \delta \varepsilon_{ij} dV \quad (5)$$

By substituting the strain-displacement relations from Eq. (2) into Eq. (5), and back-substitution of the result into Eq. (3) along with Eq. (4), the equilibrium equations along the length for region I are obtained as follows:

$$\begin{aligned}
\delta u_i : \frac{dN_x^i}{dx} - N_{xy1}^i - Q_{x1}^i &= 0 \\
\delta v_i : \frac{dN_{xy2}^i}{dx} - N_y^i - Q_{y1}^i &= 0 \quad i=1,2,\dots,n \\
\delta w_i : \frac{dQ_{x2}^i}{dx} - Q_{y2}^i - N_z^i &= 0
\end{aligned} \tag{6}$$

in which, the stress resultants are defined as:

$$\begin{aligned}
(N_x^i, N_y^i, N_z^i, N_{xy1}^i, N_{xy2}^i) &= \int_{-h/2}^b \int_{-h/2}^{h/2} (\sigma_x \bar{u}_i \bar{u}_i', \sigma_y \bar{v}_i \bar{v}_i', \sigma_z \bar{w}_i \bar{w}_i', \sigma_{xy} \bar{u}_i \bar{v}_i', \sigma_{xy} \bar{v}_i \bar{v}_i') dz dy \\
(Q_{y1}^i, Q_{y2}^i, Q_{x1}^i, Q_{x2}^i) &= \int_{-h/2}^b \int_{-h/2}^{h/2} (\sigma_{yz} \bar{v}_i \bar{v}_i', \sigma_{yz} \bar{w}_i \bar{w}_i', \sigma_{xz} \bar{u}_i \bar{u}_i', \sigma_{xz} \bar{w}_i \bar{w}_i') dz dy
\end{aligned} \tag{7}$$

Considering the adherends generally as the composite laminates, the 3-D linear constitutive relations for the k -th layer can be written as [43]:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{Bmatrix}^{(k)} = \begin{bmatrix} \bar{C}_{11} & \bar{C}_{12} & \bar{C}_{13} & 0 & 0 & \bar{C}_{16} \\ \bar{C}_{12} & \bar{C}_{22} & \bar{C}_{23} & 0 & 0 & \bar{C}_{26} \\ \bar{C}_{13} & \bar{C}_{23} & \bar{C}_{33} & 0 & 0 & \bar{C}_{36} \\ 0 & 0 & 0 & \bar{C}_{44} & \bar{C}_{45} & 0 \\ 0 & 0 & 0 & \bar{C}_{45} & \bar{C}_{55} & 0 \\ \bar{C}_{16} & \bar{C}_{26} & \bar{C}_{36} & 0 & 0 & \bar{C}_{66} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix}^{(k)} \tag{8}$$

where $[\bar{C}]^k$ represents the transformed stiffness matrix for the k -th layer. Substituting the strain-displacement relations from Eq. (2) into Eq. (8) and substituting the subsequent results into Eq. (7) yields the stress resultants in terms of the displacement terms. Upon substitution of these results into Eq. (6), the governing equilibrium equations in x -direction are obtained as follows:

$$\begin{aligned}
\sum_{j=1}^n [T_{11,11}^{ij} u_j'' + (T_{12,11}^{ij} - T_{21,11}^{ij}) u_j' - (T_{22,11}^{ij} + T_{11,22}^{ij}) u_j + T_{13,13}^{ij} v_j'' + (T_{14,13}^{ij} - T_{23,13}^{ij}) v_j' \\
- (T_{24,13}^{ij} + T_{13,24}^{ij}) v_j + (T_{15,16}^{ij} - T_{15,25}^{ij}) w_j' - (T_{25,16}^{ij} + T_{16,25}^{ij}) w_j] &= 0 \\
\sum_{j=1}^n [T_{31,31}^{ij} u_j'' + (T_{32,31}^{ij} - T_{41,31}^{ij}) u_j' - (T_{42,31}^{ij} + T_{31,42}^{ij}) u_j + T_{33,33}^{ij} v_j'' + (T_{34,33}^{ij} - T_{43,33}^{ij}) v_j' \\
- (T_{44,33}^{ij} + T_{33,44}^{ij}) v_j + (T_{35,36}^{ij} - T_{35,45}^{ij}) w_j' - (T_{45,36}^{ij} + T_{36,45}^{ij}) w_j] &= 0 \\
\sum_{j=1}^n [(T_{51,52}^{ij} - T_{51,61}^{ij}) u_j' - (T_{61,52}^{ij} + T_{52,61}^{ij}) u_j + (T_{53,54}^{ij} - T_{53,63}^{ij}) v_j' - (T_{63,54}^{ij} + T_{54,63}^{ij}) v_j \\
+ T_{55,55}^{ij} w_j'' + (T_{56,55}^{ij} - T_{65,55}^{ij}) w_j' - (T_{66,55}^{ij} + T_{55,66}^{ij}) w_j] &= 0
\end{aligned} \tag{9}$$

where $T_{pq,rs}^{ij}$ are constant coefficients defined in the Appendix (see Eq. (A.1)). It is important to recall that Eq. (9) represents the governing equation for each one of three connected regions depicted in Figure 1, and modeling the entire problem requires solving Eq. (9) for all three regions simultaneously by applying respective boundary and continuity conditions.

In general, the definition of boundary conditions in x -direction requires that either the displacement functions, say the “primary variables”, or the stress resultant components as functions of x , say the “secondary variables”, to be specified as below:

$$\begin{aligned}
\text{Primary variables:} \quad & u_i, v_i, w_i \quad i=1,2,\dots,n \\
\text{Secondary variables:} \quad & N_x^i, N_{xy2}^i, Q_{x2}^i
\end{aligned} \tag{10}$$

As illustrated in Figure 1b, the adhesive joint is assumed to be clamped at its left end, implying that $u_i(x)$, $v_i(x)$, and $w_i(x)$ are zero for $i = 1, 2, \dots, n$ in region I at $x = -L/2$. Moreover, the continuity and equilibrium conditions between the right end of region I and the left end of region II should be satisfied [44]. That is,

Displacement continuity conditions:

$$u_i^{(I)} \Big|_{x=-L_0/2} = u_i^{(II)} \Big|_{x=-L_0/2}, \quad v_i^{(I)} \Big|_{x=-L_0/2} = v_i^{(II)} \Big|_{x=-L_0/2}, \quad w_i^{(I)} \Big|_{x=-L_0/2} = w_i^{(II)} \Big|_{x=-L_0/2} \tag{11}$$

Stress equilibrium conditions:

$$N_x^{i(I)} \Big|_{x=-L_0/2} = N_x^{i(II)} \Big|_{x=-L_0/2}, \quad N_{xy}^{i(I)} \Big|_{x=-L_0/2} = N_{xy}^{i(II)} \Big|_{x=-L_0/2}, \quad Q_{x2}^{i(I)} \Big|_{x=-L_0/2} = Q_{x2}^{i(II)} \Big|_{x=-L_0/2} \quad (12)$$

Similarly, the continuity and equilibrium conditions are maintained between the right end of region *II* and the left end of region *III*. Finally, at the right end of the joint (i.e., at $x = L/2$), the loading conditions for different scenarios (see Figure 1c) can be written for $i = 1, 2, \dots, n$ as:

$$\text{Tension:} \quad N_x^i = P_x^i, \quad N_{xy2}^i = Q_{x2}^i = 0 \quad (13a)$$

$$\text{Shear:} \quad N_x^i = N_{xy2}^i = 0, \quad Q_{x2}^i = P_{xz}^i \quad (13b)$$

$$\text{Bending:} \quad N_x^i = F_{Bending}^i, \quad N_{xy2}^i = Q_{x2}^i = 0 \quad (13c)$$

$$\text{Torsion:} \quad N_x^i = 0, \quad N_{xy2}^i = F_{Torsion}^i, \quad Q_{x2}^i = 0 \quad (13d)$$

In these equations, the tensile, shear, bending, and torsion loads are respectively defined as:

$$P_x^i = \int_0^b \int_{-t_2/2}^{t_2/2} \bar{\sigma}_x \bar{u}_i \bar{u}_i \, d\bar{z} d\bar{y} \quad (14)$$

$$P_{xz}^i = \int_0^b \int_{-t_2/2}^{t_2/2} \bar{\sigma}_{xz} \bar{w}_i \bar{w}_i \, d\bar{z} d\bar{y} \quad (15)$$

$$F_{Bending}^i = \int_0^b \int_{-t_2/2}^{t_2/2} \bar{\sigma}_1 \bar{u}_i \bar{u}_i \, \bar{z} \, d\bar{z} d\bar{y} \quad (16)$$

$$F_{Torsion}^i = \int_0^b \int_{-t_2/2}^{t_2/2} \bar{\sigma}_{12} \bar{v}_i \bar{v}_i \, \bar{z} \, d\bar{z} d\bar{y} \quad (17)$$

In Eqs. (14) and (15), the terms $\bar{\sigma}_x$ and $\bar{\sigma}_{xy}$ represent the uniform tensile and shear stresses applied to the right end of region *III*, respectively. Eq. (16) calculates the force resulting from a bending moment, where $\bar{\sigma}_x = \bar{\sigma}_1 \bar{z}$ represents the linear variation of the axial stress through the thickness at the same boundary. Here, $\bar{\sigma}_1$ is the maximum axial stress applied to the top and bottom surfaces of this boundary to form the bending moment. Eq. (17) also calculates the torsional moment, where $\bar{\sigma}_{xy} = \bar{\sigma}_{12} \bar{z}$ represents the linear variation of the shear stress through the thickness at the same boundary. Here, $\bar{\sigma}_{12}$ is the maximum shear stress applied at this boundary to form the torsional moment. By applying the normal and shear stresses that vary linearly through the thickness at the right end of region *III*, the bending and torsional moments can be expressed as follows:

$$M_0 = \int_0^b \int_{-t_2/2}^{t_2/2} \bar{\sigma}_x \, \bar{z} \, d\bar{z} d\bar{y} \quad (18)$$

$$T_0 = \int_0^b \int_{-t_2/2}^{t_2/2} \bar{\sigma}_{xy} \, \bar{z} \, d\bar{z} d\bar{y} \quad (19)$$

Without loss of generality and for the sake of brevity, the remaining equations are presented exclusively for the case of tensile loading, and the other scenarios can be obtained likewise.

Taking similar steps, one can derive the other set of ODEs in y -direction. If the functions $u_i(x)$, $v_i(x)$, $w_i(x)$, $\tilde{u}_i(z)$, $\tilde{v}_i(z)$, and $\tilde{w}_i(z)$ are assumed to be known, the equilibrium equations in y -direction can be written as follows:

$$\begin{aligned} \delta \bar{u}_i : \frac{d\bar{N}_{xy1}^i}{dy} - \bar{N}_x^i - \bar{Q}_{x1}^i &= -\bar{P}_x^i \\ \delta \bar{v}_i : \frac{d\bar{N}_y^i}{dy} - \bar{N}_{xy2}^i - \bar{Q}_{y1}^i &= 0 \\ \delta \bar{w}_i : \frac{d\bar{Q}_{y2}^i}{dy} - \bar{Q}_{x2}^i - \bar{N}_z^i &= 0 \end{aligned} \quad i = 1, 2, \dots, n \quad (20)$$

where the stress resultants are defined as:

$$\begin{aligned} (\bar{N}_x^i, \bar{N}_y^i, \bar{N}_z^i, \bar{N}_{xy1}^i, \bar{N}_{xy2}^i) &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} (\sigma_x u_i' \tilde{u}_i, \sigma_y v_i' \tilde{v}_i, \sigma_z w_i' \tilde{w}_i, \sigma_{xy} u_i' \tilde{u}_i, \sigma_{xy} v_i' \tilde{v}_i) dz dx \\ (\bar{Q}_{y1}^i, \bar{Q}_{y2}^i, \bar{Q}_{x1}^i, \bar{Q}_{x2}^i) &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} (\sigma_{yz} v_i' \tilde{v}_i, \sigma_{yz} w_i' \tilde{w}_i, \sigma_{xz} u_i' \tilde{u}_i, \sigma_{xz} w_i' \tilde{w}_i) dz dx \end{aligned} \quad (21)$$

Similarly, the primary and secondary variables in this direction are defined as below:

$$\text{Primary variables:} \quad \bar{u}_i, \bar{v}_i, \bar{w}_i \quad i = 1, 2, \dots, n \quad (22)$$

Secondary variables: $\bar{N}_{xy1}^i, \bar{N}_y^i, \bar{Q}_{y2}^i$

The equilibrium equations in y -direction can be written in terms of the displacements by substituting the generalized stress resultants from Eq. (21) into Eq. (20). This leads to a system of ODEs with constant coefficients. That is,

$$\begin{aligned} \sum_{j=1}^n & \left[\bar{T}_{11,11}^{ij} \bar{u}_j'' + (\bar{T}_{12,11}^{ij} - \bar{T}_{21,11}^{ij}) \bar{u}_j' - (\bar{T}_{22,11}^{ij} + \bar{T}_{11,22}^{ij}) \bar{u}_j + \bar{T}_{13,13}^{ij} \bar{v}_j'' + (\bar{T}_{14,13}^{ij} - \bar{T}_{23,13}^{ij}) \bar{v}_j' \right. \\ & \left. - (\bar{T}_{24,13}^{ij} + \bar{T}_{13,24}^{ij}) \bar{v}_j + (\bar{T}_{15,16}^{ij} - \bar{T}_{15,25}^{ij}) \bar{w}_j' - (\bar{T}_{25,16}^{ij} + \bar{T}_{16,25}^{ij}) \bar{w}_j \right] = -\bar{P}_x^i \\ \sum_{j=1}^n & \left[\bar{T}_{31,31}^{ij} \bar{u}_j'' + (\bar{T}_{32,31}^{ij} - \bar{T}_{41,31}^{ij}) \bar{u}_j' - (\bar{T}_{42,31}^{ij} + \bar{T}_{31,42}^{ij}) \bar{u}_j + \bar{T}_{33,33}^{ij} \bar{v}_j'' + (\bar{T}_{34,33}^{ij} - \bar{T}_{43,33}^{ij}) \bar{v}_j' \right. \\ & \left. - (\bar{T}_{44,33}^{ij} + \bar{T}_{33,44}^{ij}) \bar{v}_j + (\bar{T}_{35,36}^{ij} - \bar{T}_{35,45}^{ij}) \bar{w}_j' - (\bar{T}_{45,36}^{ij} + \bar{T}_{36,45}^{ij}) \bar{w}_j \right] = 0 \\ \sum_{j=1}^n & \left[(\bar{T}_{51,52}^{ij} - \bar{T}_{51,61}^{ij}) \bar{u}_j' - (\bar{T}_{61,52}^{ij} + \bar{T}_{52,61}^{ij}) \bar{u}_j + (\bar{T}_{53,54}^{ij} - \bar{T}_{53,63}^{ij}) \bar{v}_j' - (\bar{T}_{63,54}^{ij} + \bar{T}_{54,63}^{ij}) \bar{v}_j \right. \\ & \left. + \bar{T}_{55,55}^{ij} \bar{w}_j'' + (\bar{T}_{56,55}^{ij} - \bar{T}_{65,55}^{ij}) \bar{w}_j' - (\bar{T}_{66,55}^{ij} + \bar{T}_{55,66}^{ij}) \bar{w}_j \right] = 0 \end{aligned} \quad (23)$$

where the inhomogeneous term on the right-hand side of Eq. (23), resulting from the axial loading applied at the right edge of the joint, is given by:

$$\bar{P}_x^i = \int_{-t_2/2}^{t_2/2} \bar{\sigma}_x \tilde{u}_i \big|_{x=L/2} d\bar{z} \quad (24)$$

In Eq. (23), the constant coefficients $\bar{T}_{pq,rs}^{ij}$ are given in the Appendix (see Eq. (A.2)). Since the front and back surfaces of the joint (i.e., at $y = 0$ and $y = b$) are free, the boundary conditions in y -direction for all loading cases are given by:

$$\bar{N}_{xy1}^i = \bar{N}_y^i = \bar{Q}_{y2}^i = 0 \quad i = 1, 2, \dots, n \quad (25)$$

Next, assuming that the functions $u_i(x)$, $v_i(x)$, $w_i(x)$, $\bar{u}_i(y)$, $\bar{v}_i(y)$, and $\bar{w}_i(y)$ are known, the equilibrium equations in z -direction for each layer can be expressed as follows:

$$\begin{aligned} \delta \tilde{u}_i : \frac{d\tilde{Q}_{x1}^i}{dz} - \tilde{N}_x^i - \tilde{N}_{xy1}^i &= -\tilde{P}_x^i \\ \delta \tilde{v}_i : \frac{d\tilde{Q}_{y1}^i}{dz} - \tilde{N}_y^i - \tilde{N}_{xy2}^i &= 0 \quad i = 1, 2, \dots, n \\ \delta \tilde{w}_i : \frac{d\tilde{N}_z^i}{dz} - \tilde{Q}_{x2}^i - \tilde{Q}_{y2}^i &= 0 \end{aligned} \quad (26)$$

where

$$\begin{aligned} (\tilde{N}_x^i, \tilde{N}_y^i, \tilde{N}_z^i, \tilde{N}_{xy1}^i, \tilde{N}_{xy2}^i) &= \int_{-L/2}^{L/2} \int_0^b (\sigma_x u_i' \bar{u}, \sigma_y v_i' \bar{v}, \sigma_z w_i' \bar{w}, \sigma_{xy} u_i' \bar{u}', \sigma_{xy} v_i' \bar{v}') dy dx \\ (\tilde{N}_x^i, \tilde{N}_y^i, \tilde{N}_z^i, \tilde{N}_{xy1}^i, \tilde{N}_{xy2}^i) &= \int_{-L/2}^{L/2} \int_0^b (\sigma_x u_i' \bar{u}, \sigma_y v_i' \bar{v}, \sigma_z w_i' \bar{w}, \sigma_{xy} u_i' \bar{u}', \sigma_{xy} v_i' \bar{v}') dy dx \end{aligned} \quad (27)$$

The primary and secondary variables in the thickness direction are defined as follows:

$$\begin{aligned} \text{Primary variables: } & \tilde{u}_i, \tilde{v}_i, \tilde{w}_i \\ \text{Secondary variables: } & \tilde{N}_{xy1}^i, \tilde{N}_y^i, \tilde{Q}_{y2}^i \quad i = 1, 2, \dots, n \end{aligned} \quad (28)$$

Using Eqs. (2), (8), and (27), the stress resultants can be expressed in terms of displacement functions. By substituting the obtained results into Eq. (26), the governing equilibrium equations can be written as:

$$\begin{aligned} \sum_{j=1}^n & \left[\tilde{T}_{11,11}^{ij} \tilde{u}_j'' - (\tilde{T}_{22,11}^{ij} + \tilde{T}_{21,12}^{ij} + \tilde{T}_{12,21}^{ij} + \tilde{T}_{11,22}^{ij}) \tilde{u}_j + \tilde{T}_{13,13}^{ij} \tilde{v}_j'' \right. \\ & \left. - (\tilde{T}_{23,14}^{ij} + \tilde{T}_{24,13}^{ij} + \tilde{T}_{13,24}^{ij} + \tilde{T}_{14,23}^{ij}) \tilde{v}_j + (\tilde{T}_{15,16}^{ij} + \tilde{T}_{16,15}^{ij} - \tilde{T}_{15,25}^{ij} - \tilde{T}_{25,15}^{ij}) \tilde{w}_j' \right] = -\tilde{P}_x^i \\ \sum_{j=1}^n & \left[\tilde{T}_{31,31}^{ij} \tilde{u}_j'' - (\tilde{T}_{32,32}^{ij} + \tilde{T}_{31,42}^{ij} + \tilde{T}_{42,31}^{ij} + \tilde{T}_{41,32}^{ij}) \tilde{u}_j + \tilde{T}_{33,33}^{ij} \tilde{v}_j'' \right. \\ & \left. - (\tilde{T}_{33,44}^{ij} + \tilde{T}_{34,43}^{ij} + \tilde{T}_{43,34}^{ij} + \tilde{T}_{44,33}^{ij}) \tilde{v}_j + (\tilde{T}_{35,36}^{ij} + \tilde{T}_{36,35}^{ij} - \tilde{T}_{35,45}^{ij} - \tilde{T}_{45,35}^{ij}) \tilde{w}_j' \right] = 0 \end{aligned} \quad (29)$$

$$\sum_{j=1}^n \left[\left(\tilde{T}_{52,51}^{ij} + \tilde{T}_{51,52}^{ij} - \tilde{T}_{61,51}^{ij} - \tilde{T}_{51,61}^{ij} \right) \tilde{u}'_j + \left(\tilde{T}_{53,54}^{ij} + \tilde{T}_{54,53}^{ij} - \tilde{T}_{63,53}^{ij} - \tilde{T}_{53,63}^{ij} \right) \tilde{v}'_j \right. \\ \left. + \tilde{T}_{55,55}^{ij} \tilde{w}_j'' - \left(\tilde{T}_{65,56}^{ij} + \tilde{T}_{66,55}^{ij} + \tilde{T}_{55,66}^{ij} + \tilde{T}_{56,65}^{ij} \right) \tilde{w}_j \right] = 0$$

where $\tilde{T}_{pq,rs}^{ij}$ are constant coefficients which are defined in the Appendix (see Eq. (A.3)), and $\tilde{P}_x^i$ resulting from tensile loading is defined as:

$$\tilde{P}_x^i = \int_0^b \bar{\sigma}_x \bar{u}_i u_i|_{x=L/2} d\bar{y} \quad (30)$$

Since the top and bottom surfaces of the joint are free, the boundary conditions at these surfaces are defined as follows:

$$\tilde{Q}_{x1}^i = \tilde{Q}_{y1}^i = \tilde{N}_z^i = 0 \quad i = 1, 2, \dots, n \quad (31)$$

The continuity and equilibrium conditions at the interface of layers of each region must be satisfied. The displacement continuity conditions and the stress equilibrium conditions at the interface of layers k and $k+1$ ($k=1, 2, \dots, N-1$) can be written as [44]:

$$\tilde{u}_i^{(k)} \Big|_{z=z_{k+1}} = \tilde{u}_i^{(k+1)} \Big|_{z=z_{k+1}}, \quad \tilde{v}_i^{(k)} \Big|_{z=z_{k+1}} = \tilde{v}_i^{(k+1)} \Big|_{z=z_{k+1}}, \quad \tilde{w}_i^{(k)} \Big|_{z=z_{k+1}} = \tilde{w}_i^{(k+1)} \Big|_{z=z_{k+1}} \quad (32)$$

and

$$\tilde{Q}_{x1}^{i(k)} \Big|_{z=z_{k+1}} = \tilde{Q}_{x1}^{i(k+1)} \Big|_{z=z_{k+1}}, \quad \tilde{Q}_y^{i(k)} \Big|_{z=z_{k+1}} = \tilde{Q}_y^{i(k+1)} \Big|_{z=z_{k+1}}, \quad \tilde{N}_z^{i(k)} \Big|_{z=z_{k+1}} = \tilde{N}_z^{i(k+1)} \Big|_{z=z_{k+1}} \quad (33)$$

where z_k is the coordinate of the interface in z -direction, and N is the total number of layers of each region.

3. Solution procedure

In this study, the 3-D Kantorovich method converts a system of partial differential equations into three sets of ODEs corresponding to x , y , and z directions (see Eqs. (9), (23), and (29)). Briefly, by assuming the displacement functions in two directions, the problem is simplified to one set of ODEs in the third direction. The problem is solved in an arbitrary direction by using the state-space approach, and the resulting displacement functions are used as the assumed functions for the subsequent iterations. This procedure is repeated iteratively for each direction until convergence is achieved.

To solve the equilibrium equation in x -direction, say Eq. (9), the displacement functions in y - and z -directions (i.e., $\bar{u}_i(y)$, $\bar{v}_i(y)$, $\bar{w}_i(y)$, $\tilde{u}_i(z)$, $\tilde{v}_i(z)$, and $\tilde{w}_i(z)$) are assumed. These functions allow for the calculation of the constants $T_{pq,rs}^{ij}$, as outlined in the Appendix. To solve the Eq. (9), the following state-space variables are defined:

$$\begin{aligned} \{X_1(x)\} &= \{u(x)\}, \quad \{X_2(x)\} = \{X_1'\} = \{u'(x)\} \\ \{X_3(x)\} &= \{v(x)\}, \quad \{X_4(x)\} = \{X_3'\} = \{v'(x)\} \\ \{X_5(x)\} &= \{w(x)\}, \quad \{X_6(x)\} = \{X_5'\} = \{w'(x)\} \end{aligned} \quad (34)$$

where, for example we have:

$$\{u'\}^T = [u'_1, u'_2, \dots, u'_n] \quad (35)$$

The set of state-space variables considered in Eq. (34) for the displacement functions in the longitudinal direction is included in vector $\{X\}$. Substitution of Eq. (34) into Eq. (9) results in a system of $6n$ coupled first-order ODEs with constant coefficients, which may be presented as:

$$\{X'\} = [A]\{X\} + \{F\} \quad (36)$$

in which, $\{F\}$ represents the force vector, which is zero for the system of equations in x -direction. Since all the coefficients $T_{pq,rs}^{ij}$ given in the Appendix are constant, the coefficients of the matrix $[A]$ are also constant. Therefore, the general solutions of Eq. (36) are given by [45]:

$$\{X\} = [U]_x [Q(x)] \{K\}_x - [A]^{-1} \{F\} \quad (37)$$

where

$$[Q(x)] = \text{diag}(\exp(\lambda_1 x), \exp(\lambda_2 x), \dots, \exp(\lambda_{6n} x)) \quad (38)$$

In Eqs. (37) and (38), $[U]$ and λ_m ($m = 1, 2, \dots, 6n$) are, the matrix of eigenvectors and the eigenvalues of $[A]$, respectively, which may include complex values. $\{K\}$ contains the unknown constants to be determined by imposing the boundary conditions at the ends of the joint and the displacement continuity and equilibrium conditions between adjacent regions, as given in Eqs. (11) and (12).

To solve the equations in y -direction, say Eq. (23), a similar iterative procedure is applied. Instead of guessing new functions, the displacement functions obtained from the first round of solutions are used for the x -direction, while the same guessed functions from the first round are used for the z -direction. By substituting the derived displacement functions, the constants $\tilde{T}_{pq,rs}^{ij}$ and the force terms are calculated.

A similar process is carried out for solving the equations in z -direction, say Eq. (29), with the key difference that there is no need to use any guessed functions. Subsequently, by substituting the previously obtained functions into the equilibrium equations in x -direction and solving them, the second stage of the solution procedure is completed. This iterative procedure continues for the coupled equations in Eqs. (9), (23), and (29) until convergence is achieved. It is important to note that the initial guess for the displacement functions does not need to satisfy all the boundary conditions.

4. Results and discussion

4.1 Validation of the model; comparison with analytical and finite element results

The proposed problem for modelling validation is an SLJ with the configuration depicted in Figure 1a. The joint has the lengths $L_1 = L_2 = L_a = 15$ mm, with composite adherends consisting of four layers. The thickness of each adherend is $t_1 = t_2 = 1.92$ mm, and the adhesive layer thickness is $t_a = 0.48$ mm. The total thickness h is 4.32 mm, and it is assumed that the joint width b is equal to its thickness. The adherends made up of glass/epoxy are bonded together using Epoxy AY103 as the adhesive. The material properties are listed in Table 1.

The proposed SLJ is subjected to a uniform axial tension of $\bar{\sigma}_x = 1$ MPa at one end and fixed at the other end (Figure 1). The lower and upper adherends are composed of four layers arranged as $(0/\pm 45/90)$ and $(90/\mp 45/0)$ respectively. As shown in Figure 2, the shear and peel stresses along the bonding region, say the normalized coordinate x/L_a , are computed at the mid-thickness of the adhesive layer (i.e., $z=0$ and $y=b/2$) and compared to those obtained by the FLWT reported by Yousefsani and Tahani [34] as well as the numerical results obtained by finite element simulations using Abaqus [46]. The finite element simulation is performed in three dimensions, and the joint geometry is meshed using C3D8R elements. Considering that the stress exhibits singular behavior at the edges of the bonding region [47], mesh refinement is conducted near the common interfaces between the layers and between the regions. A mesh sensitivity analysis is also performed to ensure the independence of results from the element size [48].

As shown in Figure 2, the MTEKM provides results that are more consistent with the finite element analysis, offering good agreement with both the layerwise method and FEM results. The present results are more accurate in comparison with the FLWT considering that the governing equations solved here are obtained using the 3-D elasticity theory with no simplifying assumption.

4.2 Effect of increasing n and number of iterations

The SLJ analyzed in Section 4.1 is reconsidered here. Figures 3a and 3b illustrates the effect of increasing the number of unknown functions, say n in Eq. (4), on the distribution of shear and peel stresses, respectively, at $z=0$ and $y=b/2$ (i.e., along the green line in Figure 3) after achieving convergence. It is observed that for $n=1$, the Kantorovich method is unable to accurately capture the stress distribution, particularly in the vicinity of the joint edges. However, increasing the number of independent terms enhances the accuracy of the method, leading to more precise predictions of stress distributions, particularly near the edges [49, 50]. As Figures 3a and 3b show, the results for $n=3$ and $n=4$ are nearly identical, indicating that the convergence has been achieved. In the following sections, all results are obtained, unless otherwise specified, using four solution terms in the series (i.e., $n=4$ in Eq. (4)).

The effect of iteration cycles on the convergence of results is also examined in Figures 3c and 3d in terms of the shear and peel stress distributions. As it can be seen, irregular and non-converging results are observed for the initial iterations, where lower maximum stress at the edges of the bonding region are observed. However, by the sixth and seventh iterations, the results converge showing an improved accuracy.

4.3 Interlaminar stress distribution in an SLJ made of composite adherends

In this section, the shear and peel stress distributions in an SLJ with composite adherends are analyzed. The upper and lower adherends are composed of four composite layers with general stacking sequences of $(\mp 45/90/0)$ and $(-45/90/0/45)$, respectively.

The joint geometry is identical to the configuration described in previous section, as shown in Figure 1a, and the stress distributions resulting from various loading conditions are presented and analyzed. Additionally, the effect of adhesive layer thickness on the shear and peel stress distributions are investigated, with particular attention to the stress variation through the thickness of the adhesive layer.

4.3.1 Interlayer stress distributions under different loadings

The prescribed adhesive joint is analyzed under tensile, shear, bending, and torsional loads (see Figure 1c). The applied loads include a uniaxial tension of $\bar{\sigma}_x = 1$ MPa, a bending moment of $\bar{M} = 12$ N.m, a shear load of $\bar{\sigma}_{xz} = 1$ MPa, and a torsional moment of $\bar{T} = 8$ N.m. Figure 4 illustrates the distribution of shear and peel stresses at the mid-surface of the adhesive layer. Additionally, the corresponding deformations of the adhesive joint under different loading conditions are depicted. The results were obtained by considering four independent terms (i.e., $n=4$ in Eq. (4)) in the equilibrium equations with 7 to 8 iterations. On average, each of the problems addressed in this paper required about two to five minutes of computation on a standard computer, demonstrating a substantially higher computational efficiency compared to conventional numerical methods.

The results indicate a significant stress concentration at the edges of the bonding region, which reflects the non-uniform nature of load transfer between the adherends. Depending on the loading condition, either the left or right edge experiences higher stress levels, indicating a more localized transmission of load.

Under the uniaxial tension, the stress concentration is much higher at the left edge of the SLJ, which can be attributed to the larger bending moment, as previously discussed in Ref. [34]. From Figure 4a, it can be inferred that the distribution of shear stress under tensile loading remains almost unchanged except near the left edge, whereas the peel stress varies more significantly along the bonding region. The presence of a severe stress concentration at the left edge makes this area prone to failure, which should be taken into account in the joint design. The deformed shape of the joint after loading exhibits upward bending, which results from the asymmetric force distribution inherently observed in the SLJs; under tensile loading, this asymmetry induces a bending moment that causes the adherends to peel apart at the edges [51].

As depicted in Figure 4b, the results indicate elevated interlayer stresses near the edges of the bonding region under shear loading, although they almost vanish away from the edges. Because of the joint configuration, the stress is more concentrated near the left edge. A key point observed in the stress distribution diagrams in this case is the change in the sign of the stresses from the left edge to the right one. This behavior is of great importance in joint design considerations, as it indicates the transition between tensile and compressive loading regions across the adhesive layer [52]. The tensile regions are more prone to interface separation and adhesive failure, whereas the compressive zones may lead to localized crushing or buckling of the adhesive material.

According to Figure 4c, it can be observed that the external bending moment can cause sign reversal of the interlayer stresses from positive to negative and vice versa, as also depicted in Ref. [27]. These changes in the stress state can lead to failure and delamination of the adhesive joint [34]. Under bending moment, the pronounced stress concentrations increase the likelihood of failure at the right edge of the bonding region.

A different trend in the stress distribution plots shown in Figure 4 is observed for the shear stress across the bonding region of the SLJ under torsional moment (see Figure 4d). Unlike other loading conditions, the interlayer shear stress under torsion never reaches a constant value away from the edges. However, the peel stress distribution under torsion closely resembles the peel stress distribution under bending, with the right edge experiencing higher stress concentration. Disregarding the slight stress variation at the edges, the distribution of peel stress under torsional loading can be considered approximately symmetric with respect to the center of the bonding region.

4.3.2 Stress variations across the adhesive layer thickness

This section focuses on the shear and peel stress variations through the thickness of the adhesive layer in an SLJ subjected to uniaxial tension. The results including the interlayer stress distributions are plotted on three parallel paths: the upper and lower bonding interfaces and the mid-surface of the adhesive layer, as shown in Figure 5. The upper interface is between the adhesive layer and the upper adherend, while the lower interface is between the adhesive layer and the lower adherend.

As indicated in Figure 5, the distributions of shear and peel stresses differ along the joint length, with pronounced variations near the left edge of the bonding region due to edge effects. Away from the edge, the shear stress remains nearly constant and tends towards zero, whereas the peel stress continues to vary along the bonding region. The sharp stress concentration observed at the left edge makes this area prone to failure. Therefore, consideration of such edge effects is essential in the design and evaluation of adhesive joints. Although the peel stress values do not differ significantly across the different levels of the joint, the shear stress exhibits a decreasing trend from the lower to the upper interface; in other words, its value at the upper interface is lower than that at the mid-surface and the lower interface. This indicates that the largest portion of the load between the adherends is transferred through the lower interface, making this region highly susceptible to debonding and delamination.

4.3.3 Effect of adhesive thickness on stress distribution

One of the key factors in design of an appropriate adhesive joint is the thickness of the adhesive layer, which may affect the effective bonding strength [53]. In this section, the effect of adhesive thickness on the interlayer stress distributions along the mid-surface of the adhesive layer of an SLJ under uniaxial tension is investigated. Two thicknesses of $t_a=0.48$ mm and $t_a=0.96$ mm are considered for the thin and thick adhesive layers, respectively. The layup configuration, the applied load, and all geometric parameters other than adhesive thickness are kept identical to those used in the earlier analysis.

Figure 6 shows that as the adhesive layer thickness increases, the distributions of shear and peel stresses undergo noticeable changes. In both SLJs with thin and thick adhesive layers, the stress concentration is observed at the edges of the bonding region, as also shown in Refs. [54, 55]. By increasing the adhesive thickness, the shear stress magnitude at the left edge is significantly reduced, which is in agreement with previous studies [34, 37, 56]. It is noteworthy that the peel stress at the left edge of the bonding region has also increased significantly, while its magnitude at the right edge remains relatively unchanged. In other words, when the adhesive layer is thick, the load transfer between the upper and lower adherends is more evenly distributed along the bonding length, as confirmed in Ref. [57]. The experimental findings reveal that an overly thick adhesive layer can negatively impact the joint performance and reduce its strength [58].

4.4 Interlaminar stress distribution in a wide SLJ: a 3-D perspective

In this section, the interlaminar stress distribution in a wide adhesive joint is examined. The SLJ shown in Figure 1 is considered assuming $b=5h$. Other geometrical details and material properties remain the same as previously described. The 3-D stress distributions here are predicted at the mid-surface of the adhesive layer ($z=0$) along the joint width (i.e., in y -direction), at several longitudinal positions as depicted in Figure 7. The horizontal axis shows the normalized coordinate with respect to the total joint width b .

Similar to the results obtained in previous section, it can be observed that the stress distributions near the end points of the bonding region indicate significant stress elevation, which is more pronounced in peel stress plots. Furthermore, Figure 7 shows the stress concentration and uneven stress distribution along the free edges in width direction. The reduction in peel stress with increasing distance from the joint edge occurs more rapidly than the reduction in shear stress. The concentration of peel stress at the edges of adhesive joints with composite adherends leads to delamination and joint failure [59, 60].

The 3-D contour plots of the interlaminar stresses at the mid-surface of the adhesive layer ($z=0$) are also presented in Figure 8, showing a detailed view of the 3-D stress distributions, particularly near the edges. It is noteworthy that the two-dimensional theoretical models are inadequate for capturing the stress distributions across the width of wide structural joints.

Figure 8 indicates that the interlaminar stresses vanish within the adhesive layer except at small regions near the edges of the bonding region, where the stresses are singular. The pronounced stress concentration at the edges of the bonding region, where the joint is more prone to failure, emphasizes the necessity for development of methods for highly accurate 3-D stress analysis of adhesive joints.

As illustrated in Figure 8, 3-D distributions of shear and peel stresses reveal that the shear stress component σ_{yz} displays a more pronounced non-uniformity at the left edge of the bonding region relative to the other components. As previously discussed in Section 4.3.1, the deformed configuration of the joint presented in Figure 8d clearly demonstrates the bending deformation induced by the axial tension [51].

5. Conclusions

In this study, the multi-term extended Kantorovich method was employed to analyze the interfacial stresses in single-lap adhesive joints with composite adherends of general layup under various in-plane and out-of-plane loading conditions. In addition, the stress variations through the joint thickness and the effect of the adhesive layer thickness on the distribution of interlayer stresses were examined. The proposed method, which is developed based on three-dimensional elasticity theory, can effectively model both thin and thick as well as narrow and wide adhesive joints. To validate the proposed method, the resulting stress distributions were compared to those obtained by the full layerwise theory and finite element simulation. These comparisons demonstrated the accuracy of the MTEKM in predicting the interfacial stresses, particularly the stress concentrations near the edges of bonding region.

Results of modeling and stress analysis of single-lap joints under different loads indicated the flexibility of the proposed method in modeling different boundary and loading conditions for common joint geometries. In practice, an adhesive joint may experience combined loading. The three-dimensional analytical method presented in this study is capable of providing accurate predictions of the interlaminar stress distributions under combined loadings. Moreover, the results showed that the proposed method offers highly accurate stress predictions with very good convergence speed. The three-dimensional estimation of the interlaminar stress responses in a single-lap joint, especially near the edges of bonding region, is of great importance for an accurate joint failure analysis. The present method, which uses the MTEKM in the framework of three-dimensional elasticity theory, can provides an effective analytical approach for precise modeling of common adhesive joints.

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Conflict of interest

The authors declare that they have no conflict of interest concerning the contents of this article.

Appendix

The constant coefficients appearing in governing equations of the x direction (i.e., Eq. (9)) are defined as:

$$\begin{aligned}
 T_{11,11}^{ij} &= \int_0^b \int_{-h/2}^{h/2} \bar{C}_{11}^{(k)} \bar{u}_i \bar{u}_j \tilde{u}_i \tilde{u}_j \, dz dy, & T_{11,22}^{ij} &= \int_0^b \int_{-h/2}^{h/2} \bar{C}_{55}^{(k)} \bar{u}_i \bar{u}_j \tilde{u}_i' \tilde{u}_j' \, dz dy \\
 T_{12,11}^{ij} &= \int_0^b \int_{-h/2}^{h/2} \bar{C}_{16}^{(k)} \bar{u}_i \bar{u}_j \tilde{u}_i \tilde{u}_j \, dz dy, & T_{13,13}^{ij} &= \int_0^b \int_{-h/2}^{h/2} \bar{C}_{16}^{(k)} \bar{u}_i \bar{v}_j \tilde{u}_i \tilde{v}_j \, dz dy \\
 T_{13,24}^{ij} &= \int_0^b \int_{-h/2}^{h/2} \bar{C}_{45}^{(k)} \bar{u}_i \bar{v}_j \tilde{u}_i' \tilde{v}_j' \, dz dy, & T_{14,13}^{ij} &= \int_0^b \int_{-h/2}^{h/2} \bar{C}_{12}^{(k)} \bar{u}_i \bar{v}_j \tilde{u}_i \tilde{v}_j \, dz dy \\
 T_{15,16}^{ij} &= \int_0^b \int_{-h/2}^{h/2} \bar{C}_{13}^{(k)} \bar{u}_i \bar{w}_j \tilde{u}_i \tilde{w}_j' \, dz dy, & T_{15,25}^{ij} &= \int_0^b \int_{-h/2}^{h/2} \bar{C}_{55}^{(k)} \bar{u}_i \bar{w}_j \tilde{u}_i' \tilde{w}_j' \, dz dy \\
 T_{16,25}^{ij} &= \int_0^b \int_{-h/2}^{h/2} \bar{C}_{45}^{(k)} \bar{u}_i \bar{w}_j \tilde{u}_i' \tilde{w}_j' \, dz dy, & T_{21,11}^{ij} &= \int_0^b \int_{-h/2}^{h/2} \bar{C}_{16}^{(k)} \bar{u}_i' \bar{u}_j \tilde{u}_i \tilde{u}_j \, dz dy \\
 T_{22,11}^{ij} &= \int_0^b \int_{-h/2}^{h/2} \bar{C}_{66}^{(k)} \bar{u}_i' \bar{u}_j' \tilde{u}_i \tilde{u}_j \, dz dy, & T_{23,13}^{ij} &= \int_0^b \int_{-h/2}^{h/2} \bar{C}_{66}^{(k)} \bar{u}_i' \bar{v}_j \tilde{u}_i \tilde{v}_j \, dz dy
 \end{aligned} \tag{A.1}$$

$$\begin{aligned}
T_{24,13}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{26}^{(k)} \bar{u}_i' \bar{v}_j' \tilde{u}_i \tilde{v}_j \, dz dy, & T_{25,16}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{36}^{(k)} \bar{u}_i' \bar{w}_j' \tilde{u}_i \tilde{w}_j \, dz dy \\
T_{31,31}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{16}^{(k)} \bar{v}_i \bar{u}_j \tilde{v}_i \tilde{u}_j \, dz dy, & T_{31,42}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{45}^{(k)} \bar{v}_i \bar{u}_j \tilde{v}_i' \tilde{u}_j' \, dz dy \\
T_{32,31}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{66}^{(k)} \bar{v}_i \bar{u}_j' \tilde{v}_i \tilde{u}_j \, dz dy, & T_{33,33}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{66}^{(k)} \bar{v}_i \bar{v}_j' \tilde{v}_i \tilde{v}_j \, dz dy \\
T_{33,44}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{44}^{(k)} \bar{v}_i \bar{v}_j' \tilde{v}_i' \tilde{v}_j' \, dz dy, & T_{34,33}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{26}^{(k)} \bar{v}_i \bar{v}_j' \tilde{v}_i \tilde{v}_j \, dz dy \\
T_{35,36}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{36}^{(k)} \bar{v}_i \bar{w}_j' \tilde{v}_i \tilde{w}_j' \, dz dy, & T_{35,45}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{45}^{(k)} \bar{v}_i \bar{w}_j' \tilde{v}_i \tilde{w}_j' \, dz dy \\
T_{36,45}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{44}^{(k)} \bar{v}_i \bar{w}_j' \tilde{v}_i' \tilde{w}_j' \, dz dy, & T_{41,31}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{12}^{(k)} \bar{v}_i \bar{u}_j \tilde{v}_i \tilde{u}_j \, dz dy \\
T_{42,31}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{26}^{(k)} \bar{v}_i \bar{u}_j' \tilde{v}_i \tilde{u}_j \, dz dy, & T_{43,33}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{26}^{(k)} \bar{v}_i \bar{v}_j' \tilde{v}_i \tilde{v}_j \, dz dy \\
T_{44,33}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{22}^{(k)} \bar{v}_i \bar{v}_j' \tilde{v}_i \tilde{v}_j \, dz dy, & T_{45,36}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{23}^{(k)} \bar{v}_i \bar{w}_j' \tilde{v}_i \tilde{w}_j' \, dz dy \\
T_{51,52}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{55}^{(k)} \bar{w}_i \bar{u}_j' \tilde{w}_i \tilde{u}_j' \, dz dy, & T_{51,61}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{13}^{(k)} \bar{w}_i \bar{u}_j' \tilde{w}_i \tilde{u}_j' \, dz dy \\
T_{52,61}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{36}^{(k)} \bar{w}_i \bar{u}_j' \tilde{w}_i' \tilde{u}_j' \, dz dy, & T_{53,54}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{45}^{(k)} \bar{w}_i \bar{v}_j' \tilde{w}_i \tilde{v}_j' \, dz dy \\
T_{53,63}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{36}^{(k)} \bar{w}_i \bar{v}_j' \tilde{w}_i' \tilde{v}_j' \, dz dy, & T_{54,63}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{23}^{(k)} \bar{w}_i \bar{v}_j' \tilde{w}_i' \tilde{v}_j' \, dz dy \\
T_{55,55}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{55}^{(k)} \bar{w}_i \bar{w}_j' \tilde{w}_i \tilde{w}_j \, dz dy, & T_{55,66}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{33}^{(k)} \bar{w}_i \bar{w}_j' \tilde{w}_i' \tilde{w}_j' \, dz dy \\
T_{56,55}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{45}^{(k)} \bar{w}_i \bar{w}_j' \tilde{w}_i \tilde{w}_j \, dz dy, & T_{61,52}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{45}^{(k)} \bar{w}_i \bar{u}_j' \tilde{w}_i \tilde{u}_j' \, dz dy \\
T_{63,54}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{44}^{(k)} \bar{w}_i \bar{v}_j' \tilde{w}_i' \tilde{v}_j' \, dz dy, & T_{65,55}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{45}^{(k)} \bar{w}_i \bar{w}_j' \tilde{w}_i \tilde{w}_j \, dz dy \\
T_{66,55}^{ij} &= \int_{-h/2}^b \int_{-h/2}^{h/2} \bar{C}_{44}^{(k)} \bar{w}_i \bar{w}_j' \tilde{w}_i' \tilde{w}_j' \, dz dy
\end{aligned}$$

The constant coefficients present in the governing equations for the y direction (i.e., Eq. (23)) are expressed as:

$$\begin{aligned}
\bar{T}_{11,11}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{66}^{(k)} u_i u_j \tilde{u}_i \tilde{u}_j \, dz dx, & \bar{T}_{11,22}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{55}^{(k)} u_i u_j \tilde{u}_i' \tilde{u}_j' \, dz dx \\
\bar{T}_{12,11}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{16}^{(k)} u_i u_j \tilde{u}_i \tilde{u}_j \, dz dx, & \bar{T}_{13,13}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{26}^{(k)} u_i v_j \tilde{u}_i \tilde{v}_j \, dz dx \\
\bar{T}_{13,24}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{45}^{(k)} u_i v_j \tilde{u}_i' \tilde{v}_j' \, dz dx, & \bar{T}_{14,13}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{66}^{(k)} u_i v_j \tilde{u}_i \tilde{v}_j \, dz dx \\
\bar{T}_{15,16}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{36}^{(k)} u_i w_j \tilde{u}_i \tilde{w}_j' \, dz dx, & \bar{T}_{15,25}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{45}^{(k)} u_i w_j \tilde{u}_i' \tilde{w}_j' \, dz dx \\
\bar{T}_{16,25}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{55}^{(k)} u_i w_j \tilde{u}_i' \tilde{w}_j' \, dz dx, & \bar{T}_{21,11}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{16}^{(k)} u_i' u_j \tilde{u}_i \tilde{u}_j \, dz dx \\
\bar{T}_{22,11}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{11}^{(k)} u_i' u_j \tilde{u}_i \tilde{u}_j \, dz dx, & \bar{T}_{23,13}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{12}^{(k)} u_i' v_j \tilde{u}_i \tilde{v}_j \, dz dx
\end{aligned}$$

(A.2)

$$\begin{aligned}
\bar{T}_{24,13}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{16}^{(k)} u_i' v_j' \tilde{u}_i' \tilde{v}_j' dz dx, & \bar{T}_{25,16}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{13}^{(k)} u_i' w_j' \tilde{u}_i' \tilde{w}_j' dz dx \\
\bar{T}_{31,31}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{26}^{(k)} v_i' u_j' \tilde{v}_i' \tilde{u}_j' dz dx, & \bar{T}_{31,42}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{45}^{(k)} v_i' u_j' \tilde{v}_i' \tilde{u}_j' dz dx \\
\bar{T}_{32,31}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{12}^{(k)} v_i' u_j' \tilde{v}_i' \tilde{u}_j' dz dx, & \bar{T}_{33,33}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{22}^{(k)} v_i' v_j' \tilde{v}_i' \tilde{v}_j' dz dx \\
\bar{T}_{33,44}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{44}^{(k)} v_i' u_j' \tilde{v}_i' \tilde{v}_j' dz dx, & \bar{T}_{34,33}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{26}^{(k)} v_i' v_j' \tilde{v}_i' \tilde{v}_j' dz dx \\
\bar{T}_{35,36}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{36}^{(k)} v_i' u_j' \tilde{v}_i' \tilde{w}_j' dz dx, & \bar{T}_{35,45}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{44}^{(k)} v_i' u_j' \tilde{v}_i' \tilde{w}_j' dz dx \\
\bar{T}_{36,45}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{45}^{(k)} v_i' w_j' \tilde{v}_i' \tilde{w}_j' dz dx, & \bar{T}_{41,31}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{66}^{(k)} v_i' u_j' \tilde{v}_i' \tilde{u}_j' dz dx \\
\bar{T}_{42,31}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{16}^{(k)} v_i' u_j' \tilde{v}_i' \tilde{u}_j' dz dx, & \bar{T}_{43,33}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{26}^{(k)} v_i' v_j' \tilde{v}_i' \tilde{v}_j' dz dx \\
\bar{T}_{44,33}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{66}^{(k)} v_i' v_j' \tilde{v}_i' \tilde{v}_j' dz dx, & \bar{T}_{45,36}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{36}^{(k)} v_i' u_j' \tilde{v}_i' \tilde{w}_j' dz dx \\
\bar{T}_{51,52}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{45}^{(k)} w_i' u_j' \tilde{w}_i' \tilde{u}_j' dz dx, & \bar{T}_{51,61}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{36}^{(k)} w_i' u_j' \tilde{w}_i' \tilde{u}_j' dz dx \\
\bar{T}_{52,61}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{13}^{(k)} w_i' u_j' \tilde{w}_i' \tilde{u}_j' dz dx, & \bar{T}_{53,54}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{44}^{(k)} w_i' u_j' \tilde{w}_i' \tilde{v}_j' dz dx \\
\bar{T}_{53,63}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{23}^{(k)} w_i' v_j' \tilde{w}_i' \tilde{v}_j' dz dx, & \bar{T}_{54,63}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{36}^{(k)} w_i' v_j' \tilde{w}_i' \tilde{v}_j' dz dx \\
\bar{T}_{55,55}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{44}^{(k)} w_i' w_j' \tilde{w}_i' \tilde{w}_j' dz dx, & \bar{T}_{55,66}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{33}^{(k)} w_i' w_j' \tilde{w}_i' \tilde{w}_j' dz dx \\
\bar{T}_{56,55}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{45}^{(k)} w_i' w_j' \tilde{w}_i' \tilde{w}_j' dz dx, & \bar{T}_{61,52}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{55}^{(k)} w_i' u_j' \tilde{w}_i' \tilde{u}_j' dz dx \\
\bar{T}_{63,54}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{45}^{(k)} w_i' v_j' \tilde{w}_i' \tilde{v}_j' dz dx, & \bar{T}_{65,55}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{45}^{(k)} w_i' w_j' \tilde{w}_i' \tilde{w}_j' dz dx \\
\bar{T}_{66,55}^{ij} &= \int_{-L/2}^{L/2} \int_{-h/2}^{h/2} \bar{C}_{55}^{(k)} w_i' w_j' \tilde{w}_i' \tilde{w}_j' dz dx
\end{aligned}$$

Finally, the constant coefficients appearing in governing equations of the z direction (i.e., Eq. (29)) are defined as:

$$\begin{aligned}
\tilde{T}_{11,11}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{55}^{(k)} u_i' u_j' \bar{u}_i' \bar{u}_j' dy dx, & \tilde{T}_{11,22}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{66}^{(k)} u_i' u_j' \bar{u}_i' \bar{u}_j' dy dx \\
\tilde{T}_{12,21}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{16}^{(k)} u_i' u_j' \bar{u}_i' \bar{u}_j' dy dx, & \tilde{T}_{13,13}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{45}^{(k)} u_i' v_j' \bar{u}_i' \bar{v}_j' dy dx \\
\tilde{T}_{13,24}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{26}^{(k)} u_i' v_j' \bar{u}_i' \bar{v}_j' dy dx, & \tilde{T}_{14,23}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{66}^{(k)} u_i' v_j' \bar{u}_i' \bar{v}_j' dy dx \\
\tilde{T}_{15,16}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{45}^{(k)} u_i' w_j' \bar{u}_i' \bar{w}_j' dy dx, & \tilde{T}_{15,25}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{36}^{(k)} u_i' w_j' \bar{u}_i' \bar{w}_j' dy dx \\
\tilde{T}_{16,15}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{55}^{(k)} u_i' w_j' \bar{u}_i' \bar{w}_j' dy dx, & \tilde{T}_{21,12}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{16}^{(k)} u_i' u_j' \bar{u}_i' \bar{u}_j' dy dx \\
\tilde{T}_{22,11}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{11}^{(k)} u_i' u_j' \bar{u}_i' \bar{u}_j' dy dx, & \tilde{T}_{23,14}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{12}^{(k)} u_i' v_j' \bar{u}_i' \bar{v}_j' dy dx
\end{aligned}$$

(A.3)

$$\begin{aligned}
\tilde{T}_{24,13}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{16}^{(k)} u_i' v_j' \bar{u}_i \bar{v}_j \, dy dx, & \tilde{T}_{25,15}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{13}^{(k)} u_i' w_j' \bar{u}_i \bar{w}_j \, dy dx \\
\tilde{T}_{31,31}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{45}^{(k)} v_i u_j' \bar{v}_i \bar{u}_j \, dy dx, & \tilde{T}_{31,42}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{26}^{(k)} v_i u_j' \bar{v}_i \bar{u}_j' \, dy dx \\
\tilde{T}_{32,41}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{12}^{(k)} v_i u_j' \bar{v}_i' \bar{u}_j \, dy dx, & \tilde{T}_{33,33}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{44}^{(k)} v_i v_j' \bar{v}_i \bar{v}_j \, dy dx \\
\tilde{T}_{33,44}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{22}^{(k)} v_i v_j' \bar{v}_i' \bar{v}_j \, dy dx, & \tilde{T}_{34,43}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{26}^{(k)} v_i v_j' \bar{v}_i' \bar{v}_j \, dy dx \\
\tilde{T}_{35,36}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{44}^{(k)} v_i w_j' \bar{v}_i \bar{w}_j \, dy dx, & \tilde{T}_{35,45}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{23}^{(k)} v_i w_j' \bar{v}_i' \bar{w}_j \, dy dx \\
\tilde{T}_{36,35}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{45}^{(k)} v_i w_j' \bar{v}_i \bar{w}_j \, dy dx, & \tilde{T}_{41,32}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{66}^{(k)} v_i' u_j \bar{v}_i' \bar{u}_j \, dy dx \\
\tilde{T}_{42,31}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{16}^{(k)} v_i' u_j \bar{v}_i' \bar{u}_j \, dy dx, & \tilde{T}_{43,34}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{26}^{(k)} v_i' v_j \bar{v}_i' \bar{v}_j \, dy dx \\
\tilde{T}_{44,33}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{66}^{(k)} v_i' v_j \bar{v}_i' \bar{v}_j \, dy dx, & \tilde{T}_{45,35}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{36}^{(k)} v_i' w_j \bar{v}_i' \bar{w}_j \, dy dx \\
\tilde{T}_{51,52}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{36}^{(k)} w_i u_j' \bar{w}_i \bar{u}_j \, dy dx, & \tilde{T}_{51,61}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{45}^{(k)} w_i u_j' \bar{w}_i' \bar{u}_j \, dy dx \\
\tilde{T}_{52,51}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{13}^{(k)} w_i u_j' \bar{w}_i \bar{u}_j \, dy dx, & \tilde{T}_{53,54}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{23}^{(k)} w_i v_j' \bar{w}_i' \bar{v}_j \, dy dx \\
\tilde{T}_{53,63}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{44}^{(k)} w_i v_j' \bar{w}_i' \bar{v}_j \, dy dx, & \tilde{T}_{54,53}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{36}^{(k)} w_i v_j' \bar{w}_i' \bar{v}_j \, dy dx \\
\tilde{T}_{55,55}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{33}^{(k)} w_i w_j' \bar{w}_i \bar{w}_j \, dy dx, & \tilde{T}_{55,66}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{44}^{(k)} w_i w_j' \bar{w}_i' \bar{w}_j \, dy dx \\
\tilde{T}_{56,65}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{45}^{(k)} w_i w_j' \bar{w}_i' \bar{w}_j \, dy dx, & \tilde{T}_{61,51}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{55}^{(k)} w_i' u_j \bar{w}_i' \bar{u}_j \, dy dx \\
\tilde{T}_{63,53}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{45}^{(k)} w_i' v_j \bar{w}_i' \bar{v}_j \, dy dx, & \tilde{T}_{65,56}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{45}^{(k)} w_i' w_j \bar{w}_i' \bar{w}_j \, dy dx \\
\tilde{T}_{66,55}^{ij} &= \int_{-L/2}^{L/2} \int_0^b \bar{C}_{55}^{(k)} w_i' w_j \bar{w}_i' \bar{w}_j \, dy dx
\end{aligned}$$

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Figure Captions

Figure 1. Geometry, coordinate systems, and boundary and loading conditions of the proposed SLJ, (a, b) detailed views of the joint and (c) boundary loads applied at the right end of the joint.

Figure 2. Interlaminar stress distributions along the adhesive mid-surface ($z=0$) of the SLJ under uniaxial tension of $\bar{\sigma}_x = 1$ MPa, (a) shear stress σ_{xz} and (b) peel stress σ_{zz} (results obtained by finite element simulation and the FLWT [34] are also depicted for comparison).

Figure 3. Effect of increasing (a, b) the number of solution terms in the series and (c, d) the number of iterations on the shear and peel stress distributions at $z=0$, $y=b/2$ in the SLJ under uniaxial tension.

Figure 4. Interlaminar stress distributions at mid-surface of adhesive layer ($y=b/2$, $z=0$) and deformed shape of the composite SLJ with general layup under (a) uniaxial tension, (b) shear load, (c) bending moment, and (d) torsional moment.

Figure 5. Interlaminar (a) shear and (b) peel stress distributions at $y=b/2$ along different paths in the adhesive layer of the composite SLJ with a general layup under uniaxial tension.

Figure 6. Interfacial (a) shear and (b) peel stress distributions at the mid-surface of the adhesive layer ($y=b/2$, $z=0$) of the SLJ with thin and thick adhesive layers under uniaxial tension.

Figure 7. Interlaminar (a) shear and (b) peel stress distributions along the adhesive width (in y -direction) at the mid-surface of the adhesive layer ($z=0$) across several longitudinal positions of a wide SLJ under uniaxial tension of $\bar{\sigma}_x = 12$ MPa.

Figure 8. Contour plots of the interlaminar (a,b) shear stresses, σ_{xz} and σ_{yz} , and (c) peel stress at the mid-surface of the adhesive layer and (d) deformed shape of the wide SLJ under uniaxial tension of $\bar{\sigma}_x = 12$ MPa.

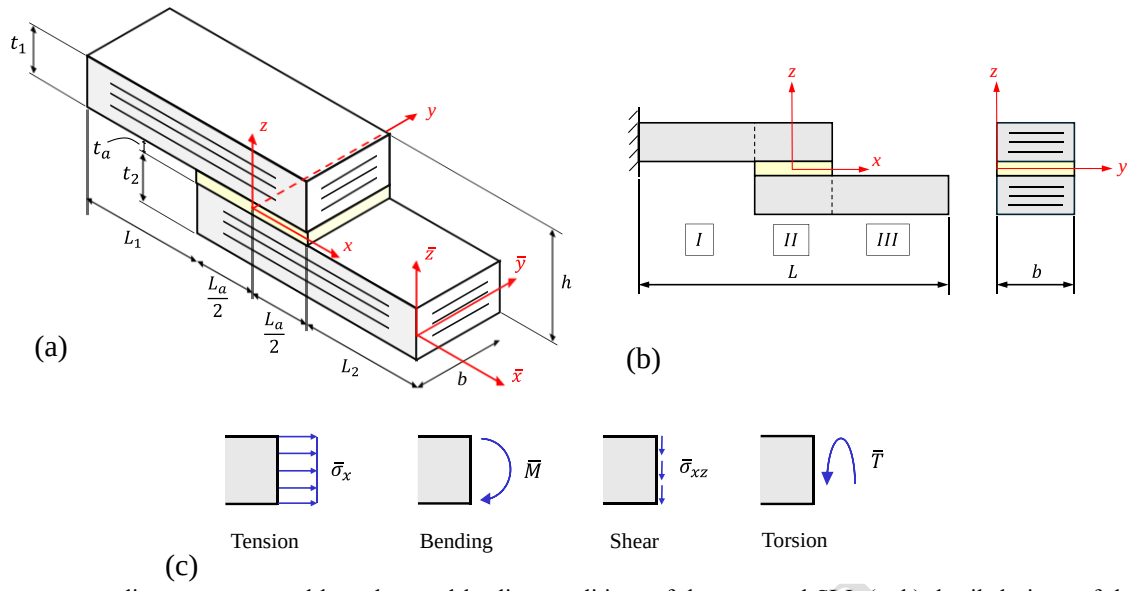


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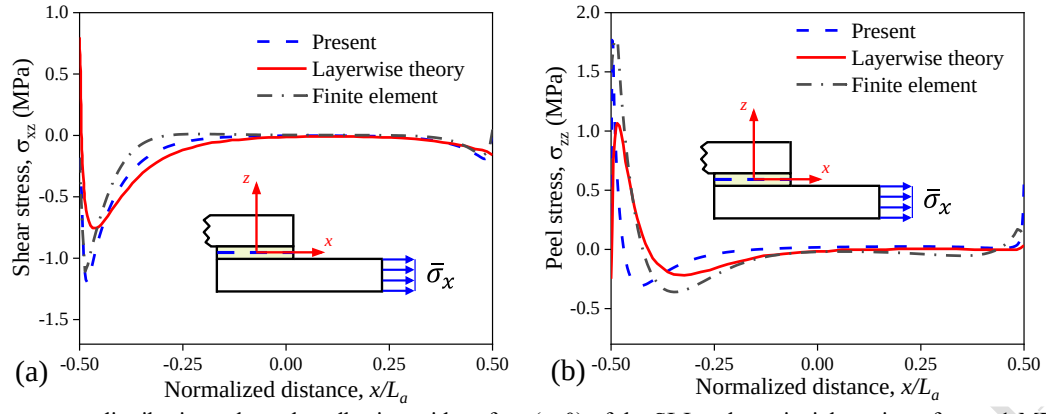


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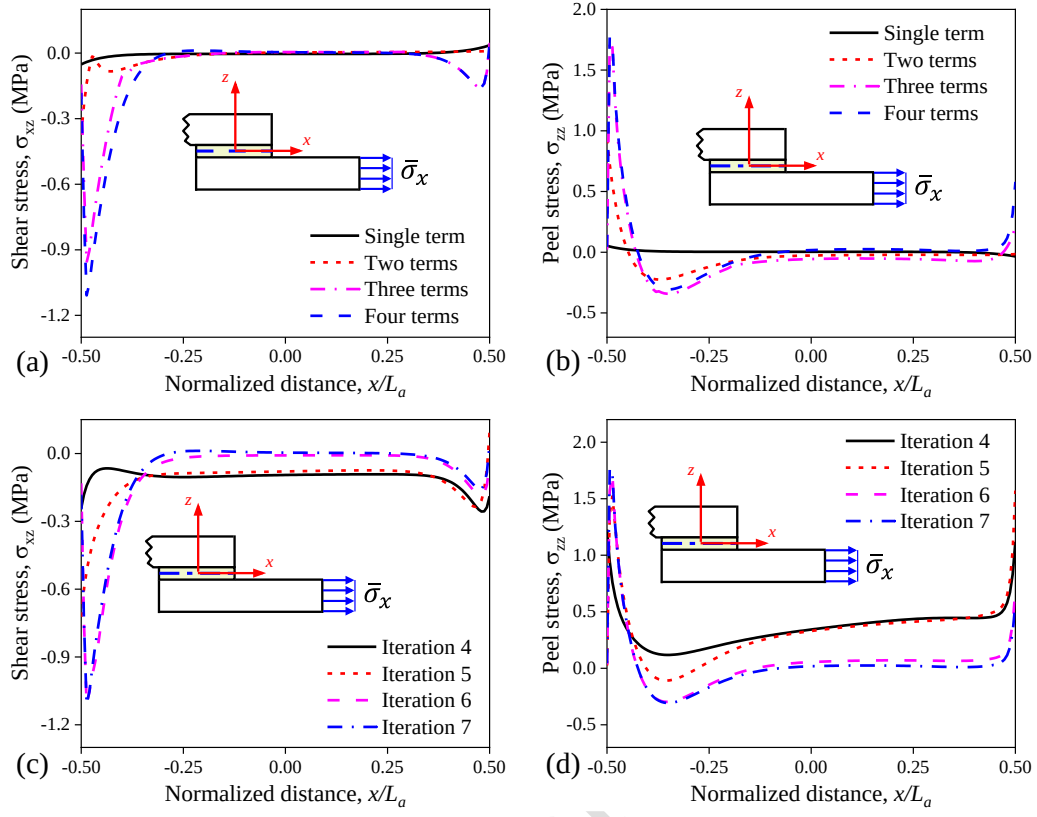


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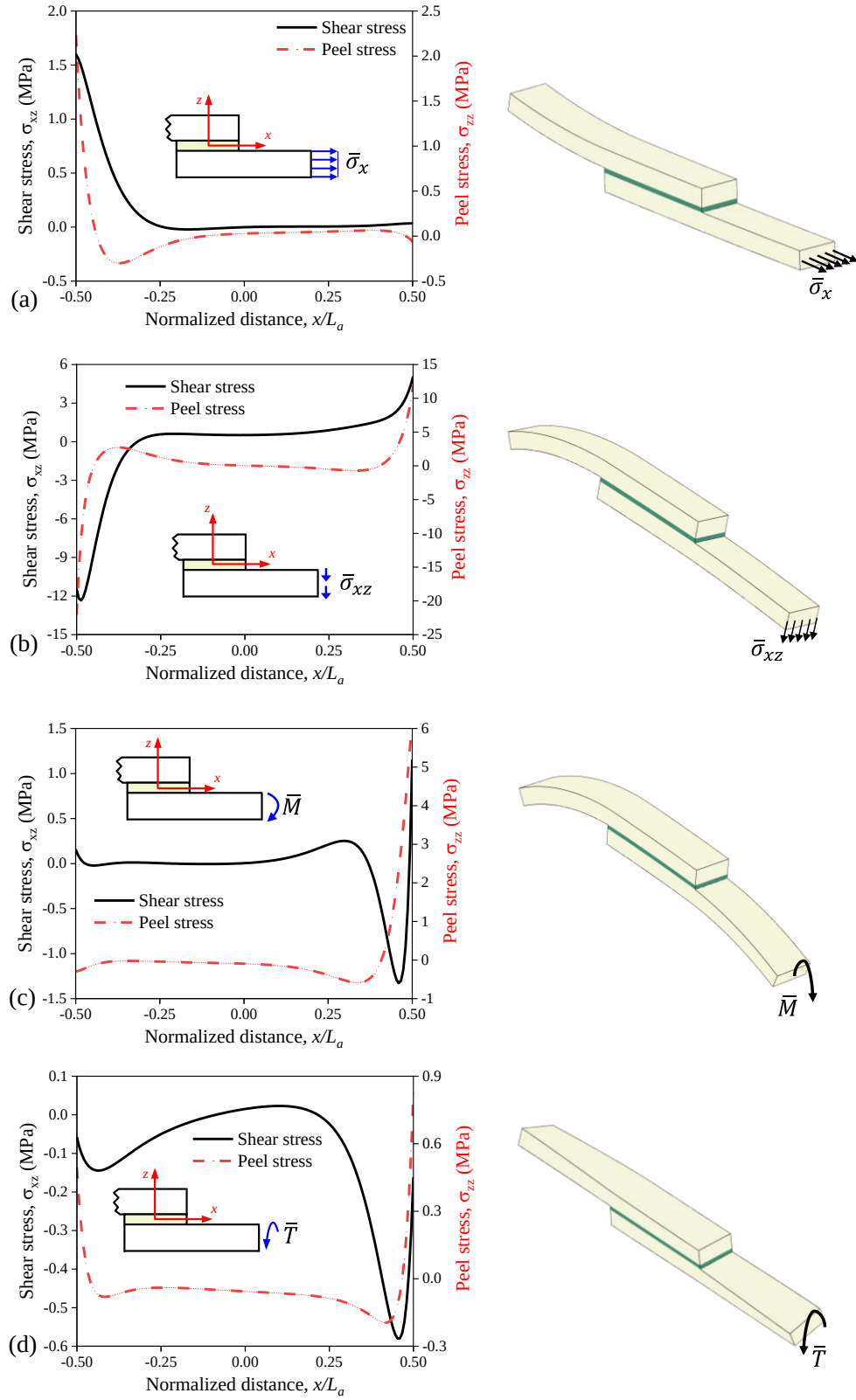


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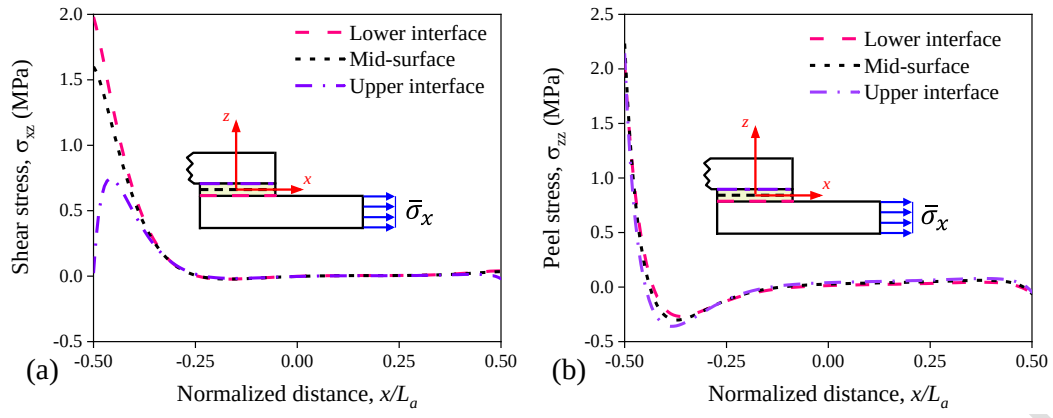


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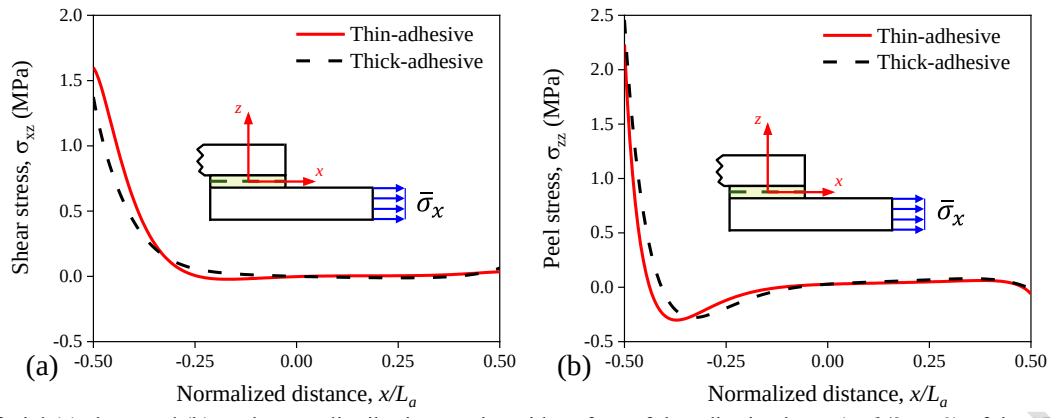


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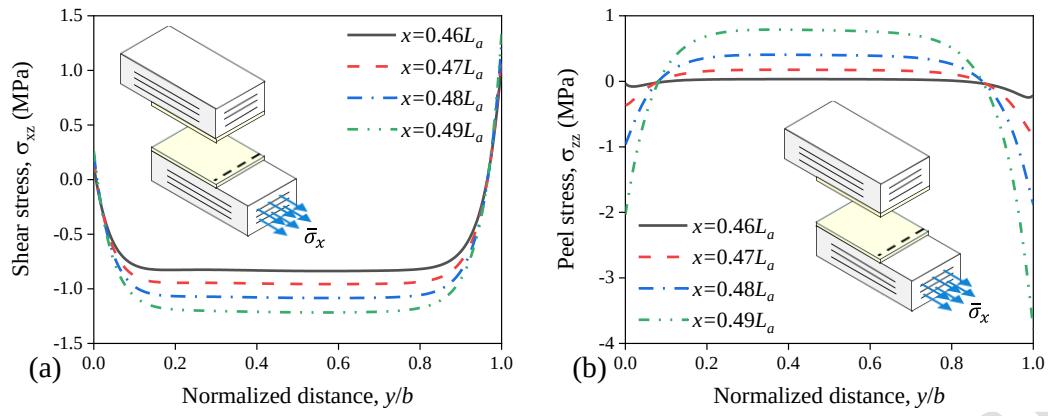


Figure 7. Interlaminar (a) shear and (b) peel stress distributions along the adhesive width (in y -direction) at the mid-surface of the adhesive layer ($z=0$) across several longitudinal positions of a wide SLJ under uniaxial tension of $\bar{\sigma}_x = 12$ MPa.

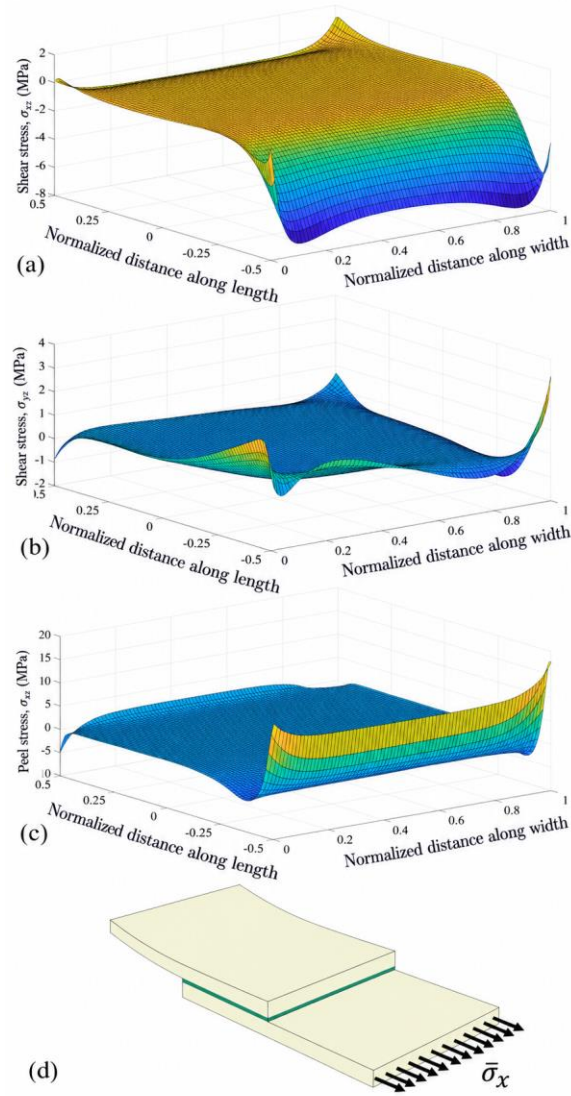


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Table Caption

Table 1. Material properties of the adherends and adhesive [61].

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Table 1. Material properties of the adherends and adhesive [61].

Material properties	Adherends	Adhesive
	Glass/epoxy	Epoxy AY103
Young's modulus (GPa)	$E_1 = 36.8, E_2 = E_3 = 8.27$	$E = 2.80$
Shear modulus (GPa)	$G_{12} = G_{13} = 4.14, G_{23} = 3$	$G = E / 2(1+\nu)$
Poisson's ratio	$\nu_{12} = \nu_{13} = 0.26, \nu_{23} = 0.38$	$\nu = 0.4$