Evaluation of Solid Waste Disposal Strategies in Chennai Using Complex q-Rung Orthopair Fuzzy Yager Aggregation Techniques

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Abstract

This study presents an innovative Multi-Criteria Decision-Making (MCDM) approach for evaluating municipal solid waste disposal strategies in Chennai. The city faces a serious waste problem, producing 6,084 tonnes of waste each day. The methodology tackles this issue by combining Complex q-Rung Orthopair Fuzzy Sets (Cq-ROFS) with Yager aggregation operators. We propose four new operators, Cq-ROFYPWA, Cq-ROFYPWG, Cq-ROFYPOWA, and Cq-ROFYPOWG. These operators leverage a 2-D representation of uncertainty (amplitude and phase) and weighted criteria to provide more accurate discrimination between waste management options than traditional techniques such as TOPSIS. The framework effectively evaluates trade-offs between economic feasibility, environmental impact, and technical performance. Applied to landfill management, bio-mining, and waste-to-energy conversion, the model demonstrates high consistency in rankings and sensitivity to performance variations. Fundamental mathematical properties—idempotency, boundedness, and monotonicity—are validated. This advanced, scalable framework serves as a highly robust, comprehensive, data-driven instrument for fostering sustainable waste management policy-making in rapidly urbanizing emerging economies.

Highlights

- Presents a Cq-ROFS-Yager framework for ranking solid waste disposal strategies.
- Proposes four new Cq-ROFS aggregation operators based on Yager operator.
- Two dimensional amplitude-phase fuzzy modelling improves accuracy of decision-making.
- Validates the framework with a real world case study on waste system of Chennai.

1. Introduction

Global population growth increased municipal waste generation to 2.24 billion tonnes in 2020 (0.79 kg per capita daily), with projections suggesting a 73% surge to 3.88 billion tonnes by mid-century [1]. Ecological stability requires innovative technologies, regulatory frameworks, and citizen engagement [2]. Primary resource reclamation pathways include material recovery and energy conversion [3], alongside carbon mitigation to reduce greenhouse gas emissions [4]. These challenges are critical for developing nations like India. Chennai's 8 million residents across 1189 square km generate 6084 TPD, equivalent to 0.76 kg per person daily (Figure 1). This rate exceeds other major Indian cities, highlighting an urgent waste management crisis [5]. Full abbreviations are provided in Appendix Table A.1.

1.1. Status and Environmental Implications of Waste Disposal in Chennai

Waste management is a major problem in Chennai. Source segregation is only 20% and 80% of daily waste is dumped into the landfill at Kodungaiyur (2,600 TPD) and Perungudi (2,400 TPD) [6]. Despite efforts like bio-mining for land reclamation, bio-methanation at Koyambedu Wholesale Market and bio-CNG plants, these problems have not been mitigated [6–8]. The system is also plagued by operational inefficiencies and underutilization of micro-composting centres. Such disposal practices are causing serious degradation of the environment. The groundwater in Perungudi is contaminated with unsafe levels of copper and mercury due to the infiltration of industrial and medical waste [9]. Also, open burning of non-recyclables causes dangerous air pollution. Incineration and biogas are other options with poor pollution controls and are serious threats to public health [10].

Addressing these crises requires holistic strategies emphasizing strict segregation, advanced recycling, and public participation to improve SWDS efficiency and sustainability (Figure 2).

1.2. Justification for Complex q-Rung Orthopair Fuzzy Information

Effective SWDS need frameworks that can handle complex uncertainty and interdependent criteria. Though q-ROFS [11] is an approach to deal with uncertainty, its one-dimensional membership (MG) and non-membership (NMG) representation is restrictive. Extensions such as Complex Fuzzy Sets (CFS) [12], Complex Intuitionistic Fuzzy Sets (CIFS) [13], and Complex Pythagorean Fuzzy Sets (CPyFS) [14] include amplitude and phase but are still insufficient for complex waste-disposal evaluations. In this study, the Complex q-Rung Orthopair Fuzzy Sets (Cq-ROFS) [15] are used to combine MG and NMG in the two-dimensional amplitude (magnitude) and phase (direction). In the Chennai context, the amplitude indicates the method's efficacy (e.g., sustainability and cost), and the phase indicates the trade-offs between conflicting priorities.

1.3. Role and Benefits of Yager-Based Aggregation in Chennai's Solid Waste Disposal Strategy

Yager Aggregation Operators (AOs) [11] help in optimizing the SWDS in Chennai in an effective way since they offer computationally efficient ways to compromise between economic considerations and environment. Yager-based methods provide a more robust alternative, while traditional frameworks such as Einstein [16], Frank [17], Aczel-Alsina [18], and Schweizer-Sklar [19] often suffer from computational overhead or inconsistent performance in different waste settings [20]. In particular, Yager-Ordered Aos [21] ensure an objective ranking, while Yager Prioritized AOs [22, 23] allow to consider critical criteria such as sustainability and public health in the decision-making process. This finding, known as Yager's Prioritized Ordered approach, provides a two-tier system that combines prioritization and orderliness for the decision-maker to address critical issues within the urban setting in a strategic evaluative context.

1.4. Literature Review

Applications of the Cq-ROFS AOs, such as Einstein, Dombi, and ExpoLogarithmic operators, have been used in green supply chain and innovation in urban areas [16, 24, 25]. The use of Yager operators [26] and the power weighted forms of Yager operators is flexible in the application in the energy and medical industry [27, 28] while evaluating infrastructure [29]. The recent focus is on the prioritized AOs for handling the hierarchy of criteria [30] and evaluating sustainable suppliers or linguistic fuzzy problems [31, 32] . Additionally, Yager's weighted operators enhance consensus-based model reliability [33, 34], representing a strategic shift from traditional aggregation toward robust, hybrid methodologies.

1.5. Research Gap, Objectives, and Significance

There is no study on the inclusion of the ordered and prioritized AOs as per Yager into the Cq-ROFS paradigm. This paper attempts to fill this research void by devising four new operators (Cq-ROFYPWA, Cq-ROFYPWG, Cq-ROFYPOWA, and Cq-ROFYPOWG). Some of the major contributions made are:

- Theoretical validation through the proof of properties like idempotency, boundedness, and monotonicity.
- A sound methodology that accounts for the challenges in sustainable economic and environmental development.
- Improved performance in terms of ranking stability and sensitivity.
- Application in decision-making regarding urban sanitation strategies.

1.6. Structure of the Study

In Section 1, an overview of the SWDS, its impacts in Chennai and a research gap section are provided. Section 2 describes the Cq-ROFS model, the Yager operators and the MCDM methodology. The framework is then adapted for the waste initiatives in Chennai (Section 3). Numerical results, model sensitivity and comparisons are provided in Section 4. In section 5, important findings and implications are summarized. The data in the form of a mathematical proof are provided in the Supplementary Data.

2. Materials and Methods

The present section outlines the core theoretical foundations and introduces a set of four original aggregation operators, namely the Cq-ROFYPWA, Cq-ROFYPWG, Cq-ROFYPOWA, and Cq-ROFYPOWG models.

2.1. Theoretical Definition

In this section, the score function, Yager's t-operators, Prioritized Average (PRA) AO, and AO for Cq-ROF numbers (Cq-ROFN) will be presented.

Definition 2.1. [15] The Cq-ROFS $\mathcal{C}$ over a universe X is defined as $\mathcal{C}=\left\{x,\sigma(x)e^{i2\pi\eta(x)},\tau(x)e^{i2\pi\gamma(x)}:x\in X\right\}$, (1)

where $\sigma(x)e^{i2\pi\eta(x)}$ and $\tau(x)e^{i2\pi\gamma(x)}$ denote the MG and NMG of $x\in X$, respectively. In this context, $\sigma(x)$ and $\tau(x)$ represent the amplitude of the MG, while $\eta(x)$ and $\gamma(x)$ represent the phase of the NMG. All these components belong to the interval [0,1] and satisfy the below conditions:

$$\sigma(x)^q + \tau(x)^q \le 1 \text{ and } \eta(x)^q + \gamma(x)^q \le 1, \tag{2}$$

where $q \ge 1$. Note that if $\eta(x)=0, \gamma(x)=0,$ then Eq. (1) is called q-ROFS [11]. If $q = 2$ in Eq. (2), then Eq. (1) is called CPyFS [14]. A Cq-ROFN is expressed as $\mathcal{C}=\langle(\sigma,\eta),(\tau,\gamma)\rangle$ or $\mathcal{C}_i=\langle(\sigma_i,\eta_i),(\tau_i,\gamma_i)\rangle$ where i is a positive integer.

Definition 2.2. [35] For any Cq-ROFN ($q \ge 1$) denoted by $\mathcal{C}=\langle(\sigma,\eta),(\tau,\gamma)\rangle$, the score function is calculated as:

$$\mathbb{S}(\mathcal{C}) = \frac{1}{2} + \frac{1}{4}\left(\sigma^q + \eta^q - \tau^q - \gamma^q\right) \in [0,1]. \tag{3}$$

The relationship between the scores of two Cq-ROFNs, $\mathcal{C}_1$ and $\mathcal{C}_2$, is defined as follows: (i) If $\mathbb{S}(\mathcal{C}_1) < \mathbb{S}(\mathcal{C}_2)$, then $\mathcal{C}_1 < \mathcal{C}_2$ (ii) If $\mathbb{S}(\mathcal{C}_1) > \mathbb{S}(\mathcal{C}_2)$, then $\mathcal{C}_1 > \mathcal{C}_2$ (iii) If $\mathbb{S}(\mathcal{C}_1) = \mathbb{S}(\mathcal{C}_2)$, then $\mathcal{C}_1 = \mathcal{C}_2$.

Definition 2.3. [11] For any two $x, y \in [0,1]$, and $\beta \in (0,\infty)$, Yager's t-norm (YTN) and Yager's t-conorm (YTCN) are defined as follows: $\text{YTN}(x,y) = \min\left\{1, \left(x^\beta + y^\beta\right)^{1/\beta}\right\}$ and $\text{YTCN}(x,y) = 1 - \min\left\{1, \left((1-x)^\beta + (1-y)^\beta\right)^{1/\beta}\right\}$.

Definition 2.4. [21] For a set of criteria $\mathcal{C}_i$ where $i = 1,2,\dots,m,$ the PRA AO is defined as: $\text{PRA}(\mathcal{C}_1,\mathcal{C}_2,\dots,\mathcal{C}_m) = \oplus_{i=1}^m \delta_i \mathcal{C}_i$, where $\delta_i = \dfrac{\lambda_i}{\sum_{k=1}^m \lambda_k}$ with $\lambda_1 = 1$ and $\lambda_i = \prod_{k=1}^{i-1} \mathbb{S}(\mathcal{C}_k)$ for $i = 1,2,\dots,m.$

Definition 2.5. [36] For a set of criteria $\mathcal{C}_i$ where $i = 1,2,\dots,m,$ the Ordered Weighted Average (OWA) operator with an associated weight vector $\omega = [\omega_1, \omega_2, \dots, \omega_m]$, where $\omega_i > 0$ and $\sum_{i=1}^m \omega_i = 1$, is defined as: $\text{OWA}(\mathcal{C}_1,\mathcal{C}_2,\dots,\mathcal{C}_m) = \oplus_{i=1}^m \omega_i \mathcal{C}_{(i)}$, where (1), (2),…,(m) represents a permutation of $(i = 1,2,\dots,m)$ such that $\mathcal{C}_{(i-1)} \ge \mathcal{C}_{(i)}$ for $i = 1,2,\dots,m.$

Definition 2.6. [37] The Yager operations $\oplus$ and $\otimes$ between two Cq-ROFNs, $\mathcal{C}_1=\langle(\sigma_1,\eta_1),(\tau_1,\gamma_1)\rangle$ and $\mathcal{C}_2=\langle(\sigma_2,\eta_2),(\tau_2,\gamma_2)\rangle$, with $q \ge 1$ and $\beta, \delta > 0,$ are defined as follows:

(i)Addition:

$$\mathcal{C}_1 \oplus \mathcal{C}_2 = \left\langle \begin{array}{l} \left(\left[1-\min\left\{1,\left(\left(1-\sigma_1^q\right)^\beta + \left(1-\sigma_2^q\right)^\beta\right)^{1/\beta}\right\}\right]^{1/q}, \left[1-\min\left\{1,\left(\left(1-\eta_1^q\right)^\beta + \left(1-\eta_2^q\right)^\beta\right)^{1/\beta}\right\}\right]^{1/q}\right), \\ \left(\left[\min\left\{1,\left(\left(\tau_1^q\right)^\beta + \left(\tau_2^q\right)^\beta\right)^{1/\beta}\right\}\right]^{1/q}, \left[\min\left\{1,\left(\left(\gamma_1^q\right)^\beta + \left(\gamma_2^q\right)^\beta\right)^{1/\beta}\right\}\right]^{1/q}\right) \end{array} \right\rangle. \tag{4}$$

(ii) Multiplication:

$$\mathcal{C}_1 \otimes \mathcal{C}_2 = \left\langle \begin{array}{l} \left(\left[\min\left\{1,\left(\left(\sigma_1^q\right)^\beta + \left(\sigma_2^q\right)^\beta\right)^{1/\beta}\right\}\right]^{1/q}, \left[\min\left\{1,\left(\left(\eta_1^q\right)^\beta + \left(\eta_2^q\right)^\beta\right)^{1/\beta}\right\}\right]^{1/q}\right), \\ \left(\left[1-\min\left\{1,\left(\left(1-\tau_1^q\right)^\beta + \left(1-\tau_2^q\right)^\beta\right)^{1/\beta}\right\}\right]^{1/q}, \left[1-\min\left\{1,\left(\left(1-\gamma_1^q\right)^\beta + \left(1-\gamma_2^q\right)^\beta\right)^{1/\beta}\right\}\right]^{1/q}\right) \end{array} \right\rangle.$$

(iii) Scalar Multiplication:

$$\delta\,\mathcal{C}_1 = \left\langle \begin{array}{l} \left(\left[1-\min\left\{1,\left(\delta\left(1-\sigma_1^q\right)^\beta\right)^{1/\beta}\right\}\right]^{1/q}, \left[1-\min\left\{1,\left(\delta\left(1-\eta_1^q\right)^\beta\right)^{1/\beta}\right\}\right]^{1/q}\right), \\ \left(\left[\min\left\{1,\left(\delta\left(\tau_1^q\right)^\beta\right)^{1/\beta}\right\}\right]^{1/q}, \left[\min\left\{1,\left(\delta\left(\gamma_1^q\right)^\beta\right)^{1/\beta}\right\}\right]^{1/q}\right) \end{array} \right\rangle. \tag{5}$$

(iv) Scalar Power:

$$\mathcal{C}_1^{\delta} = \left\langle \begin{array}{l} \left(\left[\min\left\{1,\left(\delta\left(\sigma_1^q\right)^\beta\right)^{1/\beta}\right\}\right]^{1/q}, \left[\min\left\{1,\left(\delta\left(\eta_1^q\right)^\beta\right)^{1/\beta}\right\}\right]^{1/q}\right), \\ \left(\left[1-\min\left\{1,\left(\delta\left(1-\tau_1^q\right)^\beta\right)^{1/\beta}\right\}\right]^{1/q}, \left[1-\min\left\{1,\left(\delta\left(1-\gamma_1^q\right)^\beta\right)^{1/\beta}\right\}\right]^{1/q}\right) \end{array} \right\rangle.$$

2.2. Complex q-rung Orthopair Fuzzy Yager Prioritized Weighted Average Aggregation Operator

This portion of the study introduces the Cq-ROFYPWA aggregation tool and provides a detailed analysis of its essential mathematical characteristics.

Definition 2.7. Let $\mathcal{C}_i = \langle (\sigma_i, \eta_i), (\tau_i, \gamma_i) \rangle$ $(i = 1, 2, ..., m)$ denote the Cq-ROFNs with associated weights $\omega = [\omega_1, \omega_2, ..., \omega_m]$, where $\omega_i > 0$ and $\sum_{i=1}^{m} \omega_i = 1$. The Cq-ROFYPWA AO is expressed as:

Cq-ROFYPWA$(\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_m) = \oplus_{i=1}^{m} \delta_i \mathcal{C}_i$,

where $$\delta_i = \frac{\omega_i \lambda_i}{\sum_{k=1}^{m} \omega_k \lambda_k}, i = 1, 2, ..., m, \tag{6}$$

$$\lambda_1 = 1, \tag{7}$$

$$\lambda_i = \prod_{k=1}^{i-1} \mathbb{S}(\mathcal{C}_k), i = 1, 2, ..., m. \tag{8}$$

Unless stated otherwise, δ_i is kept constant through out the paper.

From our results, we can see that using the Cq-ROFYPWA operator to aggregate the values of $\mathcal{C}_i$ for $i = 1, 2, ..., m$, always yields a value in the Cq-ROFN range.

Theorem 2.1. Let $\mathcal{C}_i = \langle (\sigma_i, \eta_i), (\tau_i, \gamma_i) \rangle$ $(i = 1, 2, ..., m)$ represent the Cq-ROFNs with associated weights $\omega = [\omega_1, \omega_2, ..., \omega_m]$, where $\omega_i > 0$ and $\sum_{i=1}^{m} \omega_i = 1$. The Cq-ROFYPWA operator can be derived from Cq-ROFN and is computed as follows:

Cq-ROFYPWA$(\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_m) = \oplus_{i=1}^{m} \delta_i \mathcal{C}_i$

$$= \left\langle \begin{array}{l} \left(\left[1 - \min\left\{ 1, \left(\sum_{i=1}^{m} \delta_i \left(1 - \sigma_i^{\,q}\right)^{\beta} \right)^{1/\beta} \right\} \right]^{1/q}, \left[1 - \min\left\{ 1, \left(\sum_{i=1}^{m} \delta_i \left(1 - \eta_i^{\,q}\right)^{\beta} \right)^{1/\beta} \right\} \right]^{1/q} \right), \\ \left(\left[\min\left\{ 1, \left(\sum_{i=1}^{m} \delta_i \left(\tau_i^{\,q}\right)^{\beta} \right)^{1/\beta} \right\} \right]^{1/q}, \left[\min\left\{ 1, \left(\sum_{i=1}^{m} \delta_i \left(\gamma_i^{\,q}\right)^{\beta} \right)^{1/\beta} \right\} \right]^{1/q} \right) \end{array} \right\rangle. \tag{9}$$

Remark: The mathematical proof of Theorem 2.1 is available in the Supplementary material, Proof of Theorem 2.1.

Example 2.1 offers a rigorous numerical illustration of Theorem 2.1 and hence the mathematical soundness of the prioritized aggregation approach is in the Supplementary material, Example 2.1.

Theorem 2.2. (Idempotency Property) If $\mathcal{C}_i = \langle (\sigma_i, \eta_i), (\tau_i, \gamma_i) \rangle$ $(i = 1, 2, ..., m)$ and $\mathcal{C}_i = \mathcal{C}$ where $\mathcal{C} = \langle (\sigma, \eta), (\tau, \gamma) \rangle$, then the aggregation operator Cq-ROFYPWA, when applied to $\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_m$, results in $\mathcal{C}$.

Theorem 2.3. (Boundedness Property) Let $\mathcal{C}_i = \langle (\sigma_i, \eta_i), (\tau_i, \gamma_i) \rangle$ $(i = 1, 2, ..., m)$ denote the Cq-ROFNs.

Define $\mathcal{C}^- = \left\langle \left(\min_i \{\sigma_i\}, \min_i \{\eta_i\} \right), \left(\max_i \{\tau_i\}, \max_i \{\gamma_i\} \right) \right\rangle$,

$\mathcal{C}^+ = \left\langle \left(\max_i \{\sigma_i\}, \max_i \{\eta_i\} \right), \left(\min_i \{\tau_i\}, \min_i \{\gamma_i\} \right) \right\rangle$.

The following inequality is satisfied: $\mathcal{C}^- \leq$ Cq-ROFYPWA$(\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_m) \leq \mathcal{C}^+$.

Theorem 2.4. (Monotonicity Property) Let $\mathcal{C}_i = \langle (\sigma_i, \eta_i), (\tau_i, \gamma_i) \rangle$ $(i = 1, 2, ..., m)$ and $\mathcal{C}_i' = \langle (\sigma_i', \eta_i'), (\tau_i', \gamma_i') \rangle$ $(i = 1, 2, ..., m)$ be the Cq-ROFNs. If $\mathcal{C}_i \leq \mathcal{C}_i'$ for all $i = 1, 2, ..., m$, then Cq-ROFYPWA$(\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_m) \leq$ Cq-ROFYPWA$(\mathcal{C}_1', \mathcal{C}_2', ..., \mathcal{C}_m')$.

Note: The full mathematical proofs for Theorems 2.2–2.4 are provided in the Supplementary material, Proof of Theorems 2.2-2.4.

2.3. Complex q-rung Orthopair Fuzzy Yager Prioritized Weighted Geometric Aggregation Operator

The current segment outlines the Cq-ROFYPWG aggregation model and evaluates its core mathematical traits.

Definition 2.8. Let $\mathcal{C}_i = \langle (\sigma_i, \eta_i), (\tau_i, \gamma_i) \rangle$ $(i = 1, 2, ..., m)$ represent the Cq-ROFNs with associated $\omega = [\omega_1, \omega_2, ..., \omega_m]$, where $\omega_i > 0$ and $\sum_{i=1}^{m} \omega_i = 1$. The Cq-ROFYPWG AO is defined as

$$\text{Cq-ROFYPWG}(\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_m) = \otimes_{i=1}^{m} \mathcal{C}_i^{\delta_i},$$

where δ_i is defined as in Eq. (6).

Theorem 2.5. Let $\mathcal{C}_i = \langle (\sigma_i, \eta_i), (\tau_i, \gamma_i) \rangle \; (i = 1, 2, ..., m)$ represent the Cq-ROFNs with associated $\omega = [\omega_1, \omega_2, ..., \omega_m]$, where $\omega_i > 0$ and $\sum_{i=1}^{m} \omega_i = 1$. The Cq-ROFYPWG operator can be derived from Cq-ROFN and is computed as follows:

Cq-ROFYPWG$(\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_m) = \otimes_{i=1}^{m} \mathcal{C}_i^{\delta_i}$

$$= \left\langle \begin{array}{l} \left(\left[\min\left\{ 1, \left(\sum_{i=1}^{m} \delta_i \left(\sigma_i^{q}\right)^{\beta} \right)^{1/\beta} \right\} \right]^{1/q}, \left[\min\left\{ 1, \left(\sum_{i=1}^{m} \delta_i \left(\eta_i^{q}\right)^{\beta} \right)^{1/\beta} \right\} \right]^{1/q} \right), \\ \left(\left[1 - \min\left\{ 1, \left(\sum_{i=1}^{m} \delta_i \left(1 - \tau_i^{q}\right)^{\beta} \right)^{1/\beta} \right\} \right]^{1/q}, \left[1 - \min\left\{ 1, \left(\sum_{i=1}^{m} \delta_i \left(1 - \gamma_i^{q}\right)^{\beta} \right)^{1/\beta} \right\} \right]^{1/q} \right) \end{array} \right\rangle. \tag{10}$$

Proof: The proof is similar to Theorem 2.1.

The analytical framework used to evaluate the Cq-ROFYPWA operator in Section 2.2 also applies to the Cq-ROFYPWG operator with respect to its idempotency, boundedness, and monotonicity.

2.4. Complex q-rung Orthopair Fuzzy Yager Prioritized Ordered Weighted Average Aggregation Operator

The theoretical formulation of the Cq-ROFYPOWA operator is detailed in this segment, followed by a comprehensive assessment of its core mathematical behaviors, such as its bounded, idempotent, and monotonic nature.

Definition 2.9. Let $\mathcal{C}_i = \langle (\sigma_i, \eta_i), (\tau_i, \gamma_i) \rangle \; (i = 1, 2, ..., m)$ represent the Cq-ROFNs with associated weights $\omega = [\omega_1, \omega_2, ..., \omega_m]$, where $\omega_i > 0$ and $\sum_{i=1}^{m} \omega_i = 1$. The Cq-ROFYPOWA is defined as

$$\text{Cq-ROFYPOWA}(\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_m) = \oplus_{i=1}^{m} \delta_{(i)} \mathcal{C}_{(i)},$$

where $((1), (2), ..., (m))$ denotes a permutation of $(i = 1, 2, ..., m)$ such that $\mathcal{C}_{(i-1)} \geq \mathcal{C}_{(i)}$ for $i = 1, 2, ..., m$. The priority ordered weighting coefficients, $\delta_{(i)}$ are calculated as:

$$\delta_{(i)} = \frac{\omega_i \, \lambda_{(i)}}{\sum_{k=1}^{m} \omega_k \, \lambda_{(k)}}, i = 1, 2, ..., m, \tag{11}$$

$\lambda_{(1)} = 1, \;\; \lambda_{(i)} = \prod_{k=1}^{i-1} \mathbb{S}\left(\mathcal{C}_{(k)}\right), \; i = 1, 2, \ldots, m.$

The $\delta_{(i)}$ parameters are kept uniform for all $i = 1, 2, ..., m$ throughout this work.

Theorem 2.6. Consider a collection of Cq-ROFNs denoted as $\mathcal{C}_i = \langle (\sigma_i, \eta_i), (\tau_i, \gamma_i) \rangle \; (i = 1, 2, ..., m)$ These are individual instances of Cq-ROFNs. When we aggregate these using the Cq-ROFYPOWA operator, the result remains a Cq-ROFN. Therefore, Cq-ROFYPOWA$(\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_m) = \oplus_{i=1}^{m} \delta_{(i)} \mathcal{C}_{(i)}$

$$= \left\langle \begin{array}{l} \left(\left[1 - \min\left\{ 1, \left(\sum_{i=1}^{m} \delta_{(i)} \left(1 - \sigma_{(i)}^{q}\right)^{\beta} \right)^{1/\beta} \right\} \right]^{1/q}, \left[1 - \min\left\{ 1, \left(\sum_{i=1}^{m} \delta_{(i)} \left(1 - \eta_{(i)}^{q}\right)^{\beta} \right)^{1/\beta} \right\} \right]^{1/q} \right), \\ \left(\left[\min\left\{ 1, \left(\sum_{i=1}^{m} \delta_{(i)} \left(\tau_{(i)}^{q}\right)^{\beta} \right)^{1/\beta} \right\} \right]^{1/q}, \left[\min\left\{ 1, \left(\sum_{i=1}^{m} \delta_{(i)} \left(\gamma_{(i)}^{q}\right)^{\beta} \right)^{1/\beta} \right\} \right]^{1/q} \right) \end{array} \right\rangle. \tag{12}$$

Proof: The proof is conducted similarly to Theorem 2.1.

2.5. Complex q-rung Orthopair Fuzzy Yager Prioritized Ordered Weighted Geometric Aggregation Operator

The present segment introduces the Cq-ROFYPOWG aggregation model and performs a detailed validation of its primary mathematical attributes, which consist of its monotonic, bounded, and idempotent traits.

Definition 2.10. Let $\mathcal{C}_i = \langle (\sigma_i, \eta_i), (\tau_i, \gamma_i) \rangle \; (i = 1, 2, ..., m)$ represent the Cq-ROFNs with associated weights $\omega = [\omega_1, \omega_2, ..., \omega_m]$, where $\omega_i > 0$ and $\sum_{i=1}^{m} \omega_i = 1$. The Cq-ROFYPOWG AO is defined as

$$\text{Cq-ROFYPOWG}(\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_m) = \otimes_{i=1}^{m} \mathcal{C}_{(i)}^{\delta_{(i)}},$$

Where $((1),(2),\ldots,(m))$ denotes a permutation of $(i=1,2,\ldots,m)$ such that $\mathcal{C}_{(i-1)} \geq \mathcal{C}_{(i)}$ for $i=1,2,\ldots,m$, where $\delta_{(i)}$ is defined as in Eq. (11).

Theorem 2.7. Consider a collection of Cq-ROFNs denoted as $C_i = \langle (\sigma_i, \eta_i), (\tau_i, \gamma_i) \rangle (i=1,2,\ldots,m)$ These are individual instances of Cq-ROFNs. The Cq-ROFYPOWG operator can be derived from Cq-ROFN and is computed as follows: Therefore, Cq-ROFYPOWG $(\mathcal{C}_1,\mathcal{C}_2,\ldots,\mathcal{C}_m) = \otimes_{i=1}^{m} \mathcal{C}_{(i)}^{\delta_{(i)}}$

$$= \left\langle \begin{array}{l} \left(\left[\min\left\{1, \left(\sum_{i=1}^{m} \delta_{(i)} \left(\sigma_{(i)}^{q}\right)^{\beta} \right)^{1/\beta} \right\} \right]^{1/q}, \left[\min\left\{1, \left(\sum_{i=1}^{m} \delta_{(i)} \left(\eta_{(i)}^{q}\right)^{\beta} \right)^{1/\beta} \right\} \right]^{1/q} \right), \\ \left(1 - \left[\min\left\{1, \left(\sum_{i=1}^{m} \delta_{(i)} \left(1-\tau_{(i)}^{q}\right)^{\beta} \right)^{1/\beta} \right\} \right]^{1/q}, \left[1 - \min\left\{1, \left(\sum_{i=1}^{m} \delta_{(i)} \left(1-\gamma_{(i)}^{q}\right)^{\beta} \right)^{1/\beta} \right\} \right]^{1/q} \right) \end{array} \right\rangle. \tag{13}$$

Proof: The proof follows a similar approach to Theorem 2.6.

2.6. Multi-Criteria Decision-Making Framework to evaluate

This section details a Cq-ROFN-based MCDM framework. Alternatives are evaluated against criteria and aggregated into a final ranking, as illustrated in Figure 3:

Step 1: Define Alternatives: Define the alternatives as $\mathcal{A}_1,\mathcal{A}_2,\ldots,\mathcal{A}_n$.

Step 2: Identify Criteria: Specify the criteria as $\mathcal{E}_1,\mathcal{E}_2,\ldots,\mathcal{E}_m$.

Step 3: Criteria Weighting: Assign a specific value ω_i to each criterion $\mathcal{E}_i$, ensuring $\omega_i > 0$ for $i=1,2,\ldots,m$ and $\sum_{i=1}^{m} \omega_i = 1$.

Step 4: Evaluate Alternatives Using Cq-ROFNs: Evaluate each criterion $\mathcal{E}_i$ $(i=1,2,\ldots,m)$ in two dimensional way (amplitude and phase) against each alternative $\mathcal{A}_j$ $(j=1,2,\ldots,n)$ to create Cq-ROFNs. Represent the resulting Cq-ROFN as $\mathcal{C}_{ij} = \langle (\sigma_{ij},\eta_{ij}), (\tau_{ij},\gamma_{ij}) \rangle \; \forall\, i, j$.

Step 5: Normalize the Cq-ROFNs: Evaluate the Normalize Cq-ROFN $\mathcal{C}_{ij} = \langle (\sigma_{ij},\eta_{ij}), (\tau_{ij},\gamma_{ij}) \rangle \; \forall\, i, j$ from Step 4 using the following rule: $\mathcal{C}_{ij} = \begin{cases} \langle (\sigma_{ij},\eta_{ij}), (\tau_{ij},\gamma_{ij}) \rangle \text{ if } \mathcal{C}_j \text{ is the optimistic criterion} \\ \langle (\tau_{ij},\gamma_{ij}), (\sigma_{ij},\eta_{ij}) \rangle \text{ if } \mathcal{C}_j \text{ is the pessimistic criterion} \end{cases}$.

Step 6: Aggregate Cq-ROFNs: For each alternative $\mathcal{A}_j$ $(j=1,2,\ldots,n)$, aggregate the Cq-ROFNs $\mathcal{C}_{ij}$ $(i=1,2,\ldots,m)$ by any one of the following proposed AOs: Cq-ROFYPWA (Eq. (9)), Cq ROFYPWG (Eq. (10)), Cq-ROFYPOWA (Eq. (12)) or Cq-ROFYPOWG (Eq. (13)).

Step 7: Alternative Ranking and Score Determination: This final procedure involves utilizing Eq. (3) to calculate the numerical scores for the data consolidated in Step 5. Based on these resulting values, the relative hierarchy of the alternatives $\mathcal{A}_j$ $(j=1,2,\ldots,n)$ is established.

3. Case Study on Chennai's Waste Disposal Strategies

Addressing Chennai's urbanization-driven waste crisis, this study ranks disposal strategies using a Yager-Cq-ROFS framework to inform municipal planning across ecological, financial, and societal dimensions.

3.1. Landfills at Perungudi and Kodungaiyur Dump Yards

The Kodungaiyur (269 acres) and Perungudi (200 acres) dumps are among the largest solid waste dumps in Chennai. Around 80% of the city's daily garbage load of 4,500-7,600 tonnes is dumped here. Each of these facilities faces daily inflows of refuse ranging from 2,400 to 2,800 metric tonnes [38]. Consequently, the historical buildup of garbage has reached staggering proportions, with 30.60 lakh cubic meters at the Perungudi site and 64 lakh cubic meters at Kodungaiyur [38]. A local health survey found that 30.8% of people residing near Kodungaiyur have respiratory problems due to emissions from the site [39]. The Kodungaiyur landfill faces a serious issue of frequent fires (at least twice a year) due to methane's flammability (as illustrated in Figure 4) [39]. Disposal of medical and industrial waste at Perungudi has led to high concentrations of mercury and copper in groundwater, with multi-colored drinking water [9]. When Japanese university clinical findings revealed high levels of dioxin in breast milk samples from residents of Perungudi [9], they identified a public health crisis. Furthermore, unlined trenches at the Perungudi site also allow the seepage of effluents, greatly increasing the possibility of groundwater pollution [40] as shown in Figure 5.

3.2. Bio-Methanation Plant to utilize organic waste from Koyambedu Wholesale Market Complex (KWMC)

The KWMC has a dedicated bio-methanation plant that generates power from 30 metric tonnes of perishable organic waste per day [7]. The facility is developed on 1.16 acres of land, is invested with Rs. 5 crore, produces 2,000 kWh of power, and successfully reduces greenhouse gas emissions by 80% [7]. This initiative has the double advantage of saving up to Rs. 2 million per annum in waste transport costs and generating high-quality organic manure for agriculture [7]. KWMC is also a pioneer in using banana waste to produce biodegradable products, reinforcing its commitment to urban sustainability [41]. It can be seen from Figures 6 and 7 that the large amount of waste in the market would otherwise overload the landfills in Chennai. Therefore, the KWMC is an important waste-to-energy model, and needs to be continuously innovated and tightly controlled to maintain its environmental impact [41].

3.3. Bio-Mining Initiatives at Kodungaiyur and Perungudi Landfills

In March 2024, the Greater Chennai Corporation (GCC) initiated a bio-mining project at the 352-acre Kodungaiyur site to extract 6.6 million cubic meters of legacy waste [42]. After their success in Perungudi, where they recovered 96 acres, Zigma Global Environ Solutions has already converted 1.5 million tonnes of waste into industrial fuel [42]. This initiative also contributes to Chennai's target of 100% waste segregation by 2030 and addresses long-term environmental risks [42]. A planned 2,000 tonne-per-day incinerator at Kodungaiyur has raised local concerns about air quality, although 84,000 tonnes have been processed already [43]. To address this, the GCC has proposed strict emission controls and state-of-the-art ash-handling protocols for the new plant [43]. These efforts represent a paradigm shift in reclaiming urban land and resolving systemic waste challenges in the metropolitan area. Figures 8 and 9 show the visual results and progress of these bio-mining operations [42, 44].

3.4. Handling Construction and Demolition (C&D) Refuse at Kodungaiyur and Perungudi

Chennai is an urban sprawl and the amount of C&D waste generated is huge and needs to be processed and disposed of properly and efficiently. The two plants at Kodungaiyur and Perungudi have the capacity to recycle up to 800 tonnes per day, which will reduce the need to send C&D waste to landfills, as C&D waste accounts for around 30% of the total recycled material [45]. These initiatives have helped to improve material recovery, but the majority of waste that is processed has been disposed of in landfill rather than being utilised as a sustainable building material [6, 46]. The GCC has created guidelines for proper disposal, such as the compulsory registration of private transporters and fees charged for generating bulk waste [47]. In addition, rigorous fines have been imposed on illegal dumping (which continues to be a problem), up to Rs. 25,000. Also, strict penalties have been imposed to control illegal dumping, which remains a problem, with fines of Rs. 3,000 to 5,000 per ton [47]. Persistent improper disposal in public spaces and water bodies—especially during monsoon desilting—necessitates improved policies to boost recycling and reuse rates sustainably.

3.5. Bio-Compressed Natural Gas Plants in Chetpet and Madhavaram

Bio-CNG production in Chennai is expanding from Chetpet and Madhavaram to Sholinganallur, Koyambedu, and Manali (Rs 93.3 million per plant) [48]. These facilities convert 100 tonnes of food waste into 4,000 kg of fuel, providing a 30% cost saving over LPG while capturing methane and producing agricultural manure. Some innovations in this process include the reuse of the ash produced by the incineration process for sustainable building purposes in Manali [49]. For cost savings in transportation and more efficient waste collection, there is an effort being made to decentralize the operation. Continuous infrastructure investments will be crucial to reducing customer costs and maximizing the environmental gains. Furthermore, the variability in performance across these decentralized plants provides a robust dataset for the proposed MCDM framework analysis.

4. Numerical Illustration

This section applies the proposed MCDM framework to rank Chennai's waste management options. Sensitivity analysis and benchmarking validate the efficiency and stability of the proposed AOs.

4.1. Application of MCDM

This section applies the Section 2.6 methodology to rank Chennai's sanitation protocols. To validate the framework, we analyze the five refuse-handling alternatives identified in the case study.

Step 1: Define Alternatives: Five alternatives—comprising conventional (landfill, bio-mining) and innovative (bio-methanation, C&D recycling, bio-CNG) methods—were selected to provide a balanced framework for evaluating Chennai's SWDS, as detailed in Section 3:

$\mathcal{A}_1$: Landfills at Perungudi and Kodungaiyur dump yards,

$\mathcal{A}_2$: Bio-Methanation Plant to utilize organic waste from Koyambedu Wholesale Market Complex,

$\mathcal{A}_3$: Bio-Mining Initiatives at Kodungaiyur and Perungudi Landfills,

$\mathcal{A}_4$: Handling C&D Refuse at Kodungaiyur and Perungudi,

$\mathcal{A}_5$: Bio-CNG Plant operational at Chetpet and Madhavaram.

Step 2: Define Criteria: The criteria $\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{E}_4,$ and $\mathcal{E}_5,$ for evaluating the alternatives outlined in Step 1 are Waste Reduction Efficiency, Cost-Effectiveness, Environmental Sustainability, Public Health Concerns, and Infrastructure Constraints, respectively. Detailed descriptions and the significance of these criteria are presented in Table 1.

Step 3: Determination of Criteria Weighting: Relative importance is allocated to each criterion ε_i for $i = 1,2,3,4,5,$ by assigning weights ω_1 = 0.30, ω_2 = 0.20, ω_3 = 0.25, ω_4 = 0.15 and ω_5 = 0.10, ensuring they satisfy the normalization condition $\sum_{i=1}^{5} \omega_i = 1.$. These weight distributions are organized in detail within Table 2.

Step 4: Alternative Assessment via Cq-ROFNs: The various alternatives $\mathcal{A}_j$, $j = (1,2,3,4,5)$ are appraised against the specified criteria ε_i, $i = (1,2,3,4,5)$ using Cq-ROFN values ($\mathcal{C}_{ij} = \langle (\sigma_{ij}, \eta_{ij}), (\tau_{ij}, \gamma_{ij}) \rangle$.), with the full evaluative matrix reported in Table 3. A five-member expert panel (environmental engineering, waste management, urban planning) evaluated Chennai's strategies against Table 3 criteria via a two-stage Delphi process to ensure consensus and assessment reliability.

Step 5: Normalize the Cq-ROFNs: A normalization procedure was performed on the Cq-ROFN data to standardize the assessment framework. Because ε_4 and ε_5 are categorized as cost-type (pessimistic) factors, while the remaining criteria are treated as benefit-type (optimistic), and was utilized to compute the normalized Cq-ROFN values ($q = 3$). The outputs of this transformation are summarized in Table 4.

Step 6: Aggregate Cq-ROFNs: The synthesis of the Cq-ROFNs listed in Table 4 is conducted via the Cq-ROFYPWA model (Eq. 9) or the Cq-ROFYPWG method (Eq. 10), assuming parameters $q = 3$ and $\beta = 5$. The specific application of the Cq-ROFYPWA operator is detailed in the following section. Focusing on alternative $\mathcal{A}_1$, the Cq-ROFN values relative to criteria $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4,$ and ε_5 are taken from Table 4 as follows:
$\mathcal{C}_{11}$ = ⟨(0.7,0.3), (0.8,0.6)⟩, $\mathcal{C}_{21}$ = ⟨(0.6,0.4), (0.2,0.8)⟩, $\mathcal{C}_{31}$ = ⟨(0.6,0.5), (0.6,0.5)⟩, $\mathcal{C}_{41}$ = ⟨(0.8,0.2), (0.6,0.9)⟩, $\mathcal{C}_{51}$ = ⟨(0.1,0.4), (0.5,0.7)⟩.
Applying the score function in Eq. (3), we have $\mathbb{S}(\mathcal{C}_{11}) = 0.4105$, $\mathbb{S}(\mathcal{C}_{21}) = 0.44$, $\mathbb{S}(\mathcal{C}_{31}) = 0.5$, $\mathbb{S}(\mathcal{C}_{41}) = 0.3938$, and $\mathbb{S}(\mathcal{C}_{51}) = 0.3993$. Based on Eqs. (7)-(8), we get $\lambda_1 = 1$, $\lambda_2 = \mathbb{S}(\mathcal{C}_{11}) = 0.4105$, $\lambda_3 = \mathbb{S}(\mathcal{C}_{11})\,\mathbb{S}(\mathcal{C}_{21}) = 0.1806$, $\lambda_4 = \mathbb{S}(\mathcal{C}_{11})\,\mathbb{S}(\mathcal{C}_{21})\,\mathbb{S}(\mathcal{C}_{31}) = 0.0903$ and $\lambda_5 = \mathbb{S}(\mathcal{C}_{11})\,\mathbb{S}(\mathcal{C}_{21})\,\mathbb{S}(\mathcal{C}_{31})\,\mathbb{S}(\mathcal{C}_{41}) = 0.0356$. Therefore, $\sum_{k=1}^{5} \omega_k \lambda_k = 0.44436$, and from Eq. (6), we have δ_1 = 0.6751321, δ_2 = 0.1847612, δ_3 = 0.1016186, δ_4 = 0.0304856 and δ_5 = 0.0080025.
We calculate the aggregated Cq-ROFN as:

$$\text{Cq-ROFYPWA}(\mathcal{C}_{11}, \mathcal{C}_{21}, \mathcal{C}_{31}, \mathcal{C}_{41}, \mathcal{C}_{51}) = \oplus_{i=1}^{5} \delta_{i1}\, \mathcal{C}_{i1} = \langle (\sigma, \eta), (\tau, \gamma) \rangle,$$

where σ is calculated using Eq. (9) as follows:

$$\sigma = \left[1 - \min\left\{1, \left(\sum_{i=1}^{5} \delta_{i1}\left(1 - \sigma_{i1}{}^{q}\right)^{\beta}\right)^{1/\beta}\right\}\right]^{1/q}$$

$$= \left[1 - \min\left\{1, \left(0.68\left(1-0.7^3\right)^5 + 0.18\left(1-0.6^3\right)^5 + 0.10\left(1-0.6^3\right)^5 + 0.03\left(1-0.8^3\right)^5 + 0.01\left(1-0.1^3\right)^5\right)^{1/5}\right\}\right]^{1/3}$$

$$= 0.6644076.$$

Similarly, we can calculate $\eta, \tau,$ and γ using Eq. (9). The final aggregated Cq-ROFN is:

$$\text{Cq-ROFYPWA}(\mathcal{C}_{11}, \mathcal{C}_{21}, \mathcal{C}_{31}, \mathcal{C}_{41}, \mathcal{C}_{51}) = \langle (0.6644076, 0.3464486), (0.7794564, 0.7491419) \rangle. \tag{14}$$

A parallel computational approach is utilized for the other alternatives $\mathcal{A}_j$, $j = 2,3,4,5,$ based on the criteria $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4,$ and ε_5 provided in Table 4. Similarly, the Cq-ROFYPWG operator serves to synthesize the Cq-ROFN data, with the consolidated findings presented in Table 5.

On the other hand, the Cq-ROFYPOWA operator is used to aggregate the Cq-ROFNs $\mathcal{C}_{11}, \mathcal{C}_{21}, \mathcal{C}_{31}, \mathcal{C}_{41},$ and $\mathcal{C}_{51}$. Subsequently, to utilize the ordered weighted average, it is necessary first to calculate the scores of these Cq-ROFNs, as described by Eq. (3). $\mathbb{S}(\mathcal{C}_{11}) = 0.4105$, $\mathbb{S}(\mathcal{C}_{21}) = 0.44$, $\mathbb{S}(\mathcal{C}_{31}) = 0.5$, $\mathbb{S}(\mathcal{C}_{41}) = 0.3938$, and $\mathbb{S}(\mathcal{C}_{51}) = 0.3993$. Based on these values, the ranking is established as

$$\mathbb{S}(\mathcal{C}_{31}) > \mathbb{S}(\mathcal{C}_{21}) > \mathbb{S}(\mathcal{C}_{11}) > \mathbb{S}(\mathcal{C}_{51}) > \mathbb{S}(\mathcal{C}_{41}).$$

Using the Cq-ROFYPOWA operator, we get

$$\text{Cq-ROFYPOWA}(\mathcal{C}_{11}, \mathcal{C}_{21}, \mathcal{C}_{31}, \mathcal{C}_{41}, \mathcal{C}_{51}) = \oplus_{i=1}^{5} \delta_{(i1)}\, \mathcal{C}_{(i1)}$$

$$= \delta_{(11)}\, \mathcal{C}_{31} \oplus \delta_{(21)}\, \mathcal{C}_{21} \oplus \delta_{(31)}\, \mathcal{C}_{11} \oplus \delta_{(41)}\, \mathcal{C}_{51} \oplus \delta_{(51)}\, \mathcal{C}_{41}$$

$$=\langle(\sigma,\eta),(\tau,\gamma)\rangle.$$

Using Eq. (12), we compute the aggregated value for σ as follows:

$$\sigma=\left[1-\min\left\{1,\left(\sum_{i=1}^{5}\delta_{(i1)}\left(1-\sigma_{(i1)}{}^{q}\right)^{\beta}\right)^{1/\beta}\right\}\right]^{1/q}$$

$$=\left[1-\min\{1,(0.64(1-0.6^3)^5+0.21(1-0.6^3)^5+0.12(1-0.7^3)^5+0.03(1-0.1^3)^5+0.01(1-0.8^3)^5)^{\frac{1}{5}}\}\right]^{\frac{1}{3}}$$

$$=0.6011016.$$

Similarly, we can calculate $\eta,\tau,$ and γ using Eq. (12). The final aggregated Cq-ROFN is:

Cq-ROFYPOWA$\left(\mathcal{C}_{11},\mathcal{C}_{21},\mathcal{C}_{31},\mathcal{C}_{41},\mathcal{C}_{51}\right)$ = ⟨(0.6011016, 0.4560438), (0.6964797, 0.7317474)⟩.

A comparable computational procedure is executed for alternatives $\mathcal{A}_j,\ j=2,3,4,5,$ via the Cq-ROFYPOWA method, utilizing the criteria defined in Table 4. Furthermore, the Cq-ROFYPOWG operator is employed for the fusion of these fuzzy values, with the final integrated results compiled in Table 6.

Step 7: Alternative Ranking and Score Determination: To determine the numerical performance of the aggregated Cq-ROFNs, Eq. (3) is employed to derive the score values. For instance, the specific Cq-ROFN result from Eq. (14): $\mathcal{C}$ = ⟨(0.6644076,0.3464486), (0.7794564,0.7491419)⟩ yields a score of 0.3602221 upon evaluation. Using this identical methodology, scores for the remaining entries in Tables 5 and 6 were computed across all four operators (Cq-ROFYPWA, Cq-ROFYPWG, Cq-ROFYPOWA, and Cq-ROFYPOWG) with the parameter q set to 3. The finalized score outcomes and the subsequent hierarchical rankings are documented in Table 7.

4.2. Operator Robustness: Impact of β Variability under Invariant q

The sensitivity analysis calculates the presentation of the proposed AOs, namely Cq-ROFYPWA, Cq-ROFYPWG, Cq-ROFYPOWA, and Cq-ROFYPOWG, by studying the effect of varying β with q fixed. The assessment is oriented toward ranking alternatives $\mathcal{A}_1$ through $\mathcal{A}_5$ based on the score values obtained from these AOs. As we see how the rankings change with different values (5, 30, 150, and 300, with $q=3$ fixed), we can gain insight into the robustness and adaptability of the operators. As shown in Table 8, the alternative $\mathcal{A}_3$ is always the best-ranked choice and is also the least preferred choice. Moreover, increasing the value of contributes to the stability of the AOs. The geometric AOs (Cq-ROFYPWG and Cq-ROFYPOWG) are always more optimistic in their evaluations than the averaging AOs (Cq-ROFYPWA and Cq-ROFYPOWA). The pattern is shown in Figure 10.

4.3. Operator Stability: Impact of q Variability under Invariant β

Table 9 presents a robustness evaluation of the developed operators, illustrating how fluctuations in the q parameter affect performance while β is held at a fixed value of 5. The table showcases the score values obtained for five alternatives ($\mathcal{A}_1$ to $\mathcal{A}_5$) using these AOs across different q values. The primary focus is to observe how the ranking of these alternatives changes as it q varies. The results indicate that as q increases, the score values converge towards 0.5, leading to a diminishing distinction between the alternatives, and at q =100, all alternatives are evaluated equally. Notably, the ranking $\mathcal{A}_3>\mathcal{A}_5>\mathcal{A}_2>\mathcal{A}_4>\mathcal{A}_1$ is maintained for lower values of q; this stability of the AOs is seen when q is less than 30. Figure11 depicts the stabilization and convergence of scores as q increase with β fixed.

4.4. Comparative Analysis

4.4.1. Comparison with Javeed et al. [28] and Wu et al. [37]

The results obtainable in Table 10 show a significant advantage over the Yager AOs introduced by Javeed et al. [28]. Javeed's models identify the top-ranked alternative but also assign low or negative scores to some alternatives, limiting their discriminatory power. In contrast, our proposed operators produce consistently higher positive scores, leading to better differentiation and more accurate decision-making. Our AOs exhibit competitive performance and higher stability than the Cq-ROFPYA operator reported by Wu et al. [37]. As shown in Table 11, both frameworks generate the same rankings $\mathcal{A}_1>\mathcal{A}_4>\mathcal{A}_2>\mathcal{A}_5>\mathcal{A}_3$. The achieved consistency further strengthens the reliability of our proposed methods as dependable alternatives that preserve decision integrity and provide increased computational flexibility. As shown in Figures 12 and 13, the proposed operators are better at distinguishing between alternatives, leading to improved decision-making accuracy and confirming the robustness of our approach.

4.4.2. Comparison with Other Models

The comparisons in detail with Frank Aos [17], Power Aos [20], and Aczel-Alsina family [50, 51] are presented in Table 11–12. In all the above-mentioned cases, the proposed AOs demonstrate larger ranges of scores and better discriminative power, especially in the context of the uncertainty of the waste disposal system in Chennai, where the current models fail to provide non-zero or consistent score distribution. The advantage of the proposed operators is evident from Figures 13 and 14 through their better ranking of the alternatives.

4.5. Advantages and Disadvantages of the Proposed Framework

Inclusion of amplitude-phase degrees of belonging helps the Cq-ROFS framework [17] in evaluation of SWDS of Chennai city and outperforms 1D frameworks such as q-ROFS in handling uncertainties. However, despite being generalized and implementable (Table A.2), the framework does not allow for interval, hesitant, soft or rough set representations of information.

5. Conclusion

The current research paper provides a highly efficient approach to assess urban waste management through the application of Yager AOs along with the Cq-ROFS model. Four newly suggested operators provide a solution to dealing with ambiguities in two-dimensional space as well as handling the criteria preferences hierarchically, which is not achievable by traditional decision making procedures. The mathematical modeling proved the robustness of the models as well as their discrimination capacity especially when it comes to the consideration of economic and ecological issues of the case study – Chennai city.

5.1. Study Limitations and Future Directions

Interval valued or fuzzy sets are not included in the existing model and could be included for better handling of the ambiguity associated with the expert judgment. Moreover, the developed model has been found to be successful in terms of Chennai and its use is restricted to a limited geographical area only. The future work on this topic should be based on these ideas and should cover hybrid fuzzy environments as well as geographic information systems.

5.2. Implications of the Study

The above results give rise to an effective approach that can help local government bodies in achieving synergy between economic effectiveness and public health and environmental objectives. Through giving preference to recycling and energy recovery projects, the model becomes an evidence-based instrument for developing sustainable waste disposal policies. In addition to refuse management, the above Cq-ROFS-Yager approach can be used as a decision-making instrument in urban infrastructure construction.

GenAI

ChatGPT and AI Studio (OpenAI) were used to edit the manuscript for readability. The authors assure the originality of their intellectual contribution, analysis of data and conclusions.

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Authors' Credit

Petchimuthu Subramanian: Conceptualization; Supervision; Methodology; Investigation; Validation; Writing – original draft.
Felix Nes Mabel M. (Corresponding Author): Conceptualization; Methodology; Software; Data curation; Visualization; Writing – review & editing; Project administration.
Jeya Sutha M: Resources; Data curation; Validation; Writing – review & editing.
Karaaslan Faruk: Formal analysis; Visualization; Writing – review & editing.

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Conflicts of interest

The authors have no conflicts of interest to declare.

Supplementary Material

Supplementary data related to this article, including [Proof of Theorems 2.1-2.4], can be found in the online version of this manuscript.

Appendix A

Appendix A.1

Table A.1. Abbreviation Reference List.

Abbreviation	Full form
AOs	Aggregation Operators
C&D	Construction and Demolition
CFS	Complex Fuzzy Sets
Bio-CNG	Bio-Compressed Natural Gas
Cq-ROFN	Complex q-Rung Orthopair Fuzzy Number
CIFS	Complex Intuitionistic Fuzzy Sets
CPyFS	Complex Pythagorean Fuzzy Sets
Cq-ROFS	Complex q-Rung Orthopair Fuzzy Sets
Cq-ROFYPOWA	Complex q-Rung Orthopair Fuzzy Yager Prioritized Ordered Weighted Average
Cq-ROFYPOWG	Complexq-Rung Orthopair Fuzzy Yager Prioritized Ordered Weighted Geometric
Cq-ROFYPWA	Complex q-Rung Orthopair Fuzzy Yager Prioritized Weighted Average
Cq-ROFYPWG	Complex q-Rung Orthopair Fuzzy Yager Prioritized Weighted Geometric
GCC	Greater Chennai Corporation
IFS	Intuitionistic Fuzzy Sets
KWMC	Koyambedu Wholesale Market Complex
MCDM	Multi-Criteria Decision-Making
MG	Membership Grade
NMG	Non-membership Grade
OWA	Ordered Weighted Average
PRA	Prioritized Average
q-ROFS	q-Rung Orthopair Fuzzy Sets
SWDS	Solid Waste Disposal Strategies
TPD	Tonnes of Waste per Day
YTCN	Yager's t-conorm
YTN	Yager's t-norm

Appendix A.2

Table A.2. Extending Fuzzy Set Frameworks to the Cq-ROFS Environment.

Fuzzy Information	Fuzzy Number	Cq-ROFN	Condition ($q \geq 1$)
q-ROFS [11]	(σ,τ)	$\langle(\sigma,0),(\tau,0)\rangle$	$0 \leq \sigma^q + \tau^q \leq 1$
CFS [12]	(σ,η)	$\langle(\sigma,\eta),(0,0)\rangle$	$0 \leq \sigma,\eta \leq 1$
CIFS [13]	$\langle(\sigma,\eta),(\tau,\gamma)\rangle$	$\langle(\sigma,\eta),(\tau,\gamma)\rangle$	$0 \leq \sigma+\tau, \eta+\gamma \leq 1$
CPyFS [14]	$\langle(\sigma,\eta),(\tau,\gamma)\rangle$	$\langle(\sigma,\eta),(\tau,\gamma)\rangle$	$0 \leq \sigma^2+\tau^2,\ \eta^2+\gamma^2 \leq 1$
Cq-ROFS [17]	$\langle(\sigma,\eta),(\tau,\gamma)\rangle$	$\langle(\sigma,\eta),(\tau,\gamma)\rangle$	$0 \leq \sigma^q+\tau^q,\ \eta^q+\gamma^q \leq 1$

The supplementary data is available at:
https://scientiairanica.sharif.edu/jufile?ar_sfile=232382

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Figure Captions

1. Total Waste Generation and Waste per Person in Major Indian Cities
2. Chennai's Solid Waste Disposal Strategies
3. Flowchart of the Multi-Criteria Decision-Making Framework
4. The Kodungaiyur dump yard in Chennai experiences fires at least twice a year
5. Unlined trenches pose a groundwater risk at the Perungudi dumpsite
6. The KWMC generates 100 tonnes of waste daily, with 30 tonnes fit for power generation
7. The unused waste from the KWMC is sent to landfills in Perungudi dump yard
8. Evaluating the Impact of Bio-mining on Environmental Recovery at Kodungaiyur
9. Bio-mining at Perungudi landfill processed 30.62 lakhs cubic meters of waste, reclaiming 96 acres
10. Sensitivity evaluation of the proposed AOs with a fixed q = 3 and varying β
11. Sensitivity evaluation of the proposed AOs with a fixed β = 5 and varying q
12. Comparison of Yager and Frank AOs with proposed AOs
13. Comparison of Power AOs with proposed AOs
14. Comparison of Aczel-Alsina and its Power AOs with proposed AOs

Table Captions

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2. Criteria, Weight, and Justification Based on
3. Expert Knowledge Expert Evaluation in Cq-ROFN (q = 3)
4. Evaluation of Normalized Cq-ROFN (q = 3)
5. Aggregated Cq-ROFNs using the proposed AOs (q = 3 and $\beta = 5$)
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10. Comparison of Yager and Frank operators in the Cq-ROFS Environment
11. Comparison with Existing Cq-ROFS Power Operators
12. Comparison of Aczel-Alsina Power Operators in the Cq-ROFS Environment

**** Figures***

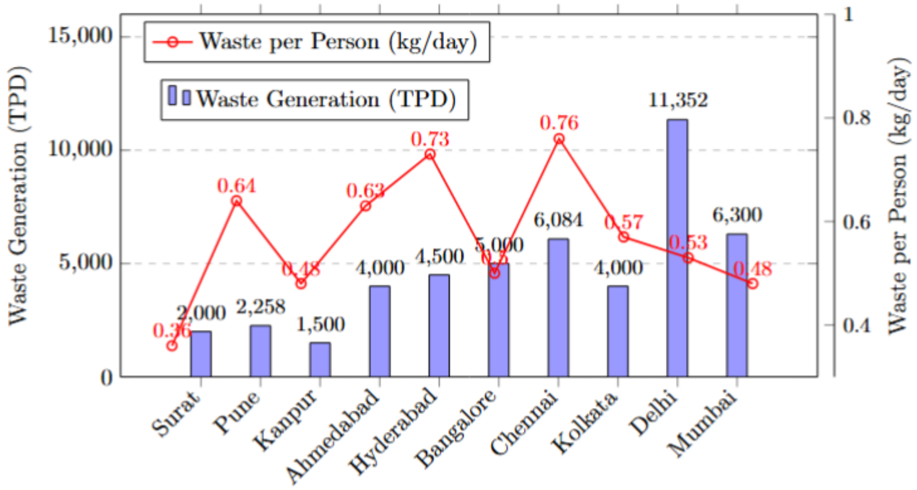

Figure 1. Total Waste Generation and Waste per Person in Major Indian Cities [5]

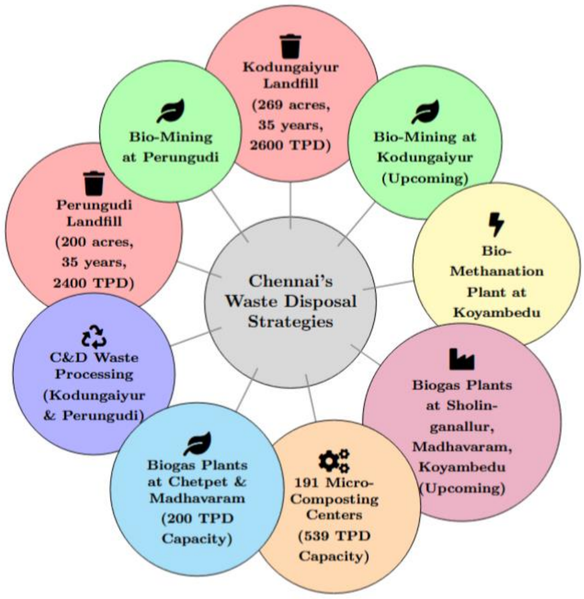

Figure 2. Chennai's Solid Waste Disposal Strategies

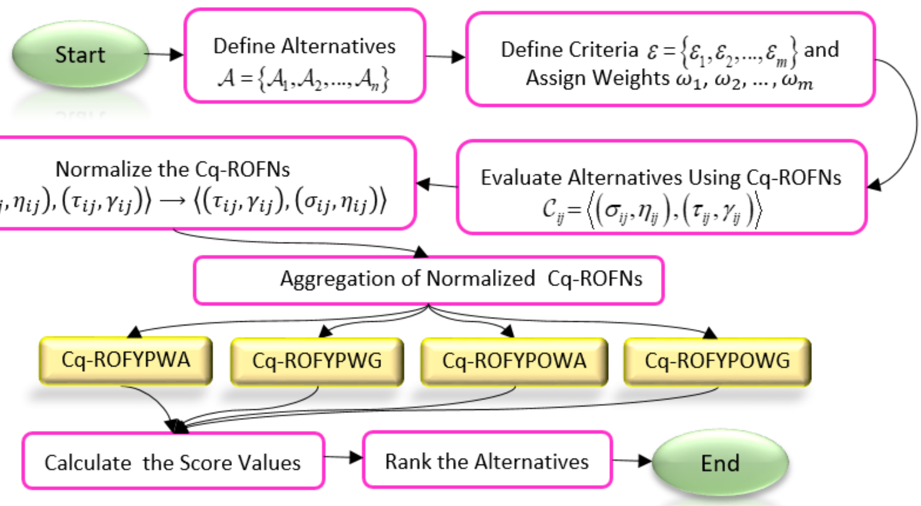

Figure 3. Flowchart of the Multi-Criteria Decision-Making Framework

Figure 4. The Kodungaiyur dump yard in Chennai experiences fires at least twice a year [39]

Figure 5. Unlined trenches pose a groundwater risk at the Perungudi dumpsite [40]

Figure 6. The KWMC generates 100 tonnes of waste daily, with 30 tonnes fit for power generation [8]

Figure 7. The unused waste from the KWMC is sent to landfills in Perungudi dump yard [8]

Figure 8. Evaluating the Impact of Bio-mining on Environmental Recovery at Kodungaiyur [42]

Figure 9. Bio-mining at Perungudi landfill processed 30.62 lakhs cubic meters of waste, reclaiming 96 acres [44]

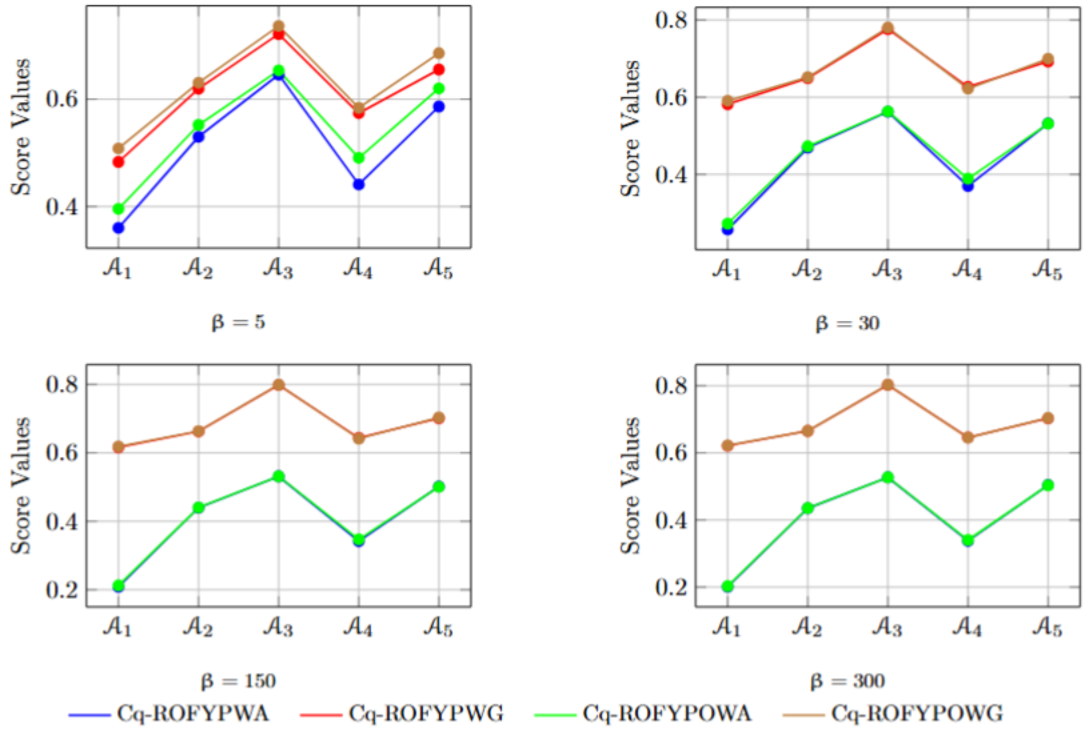

Figure 10. Sensitivity evaluation of the proposed AOs with a fixed q = 3 and varying β

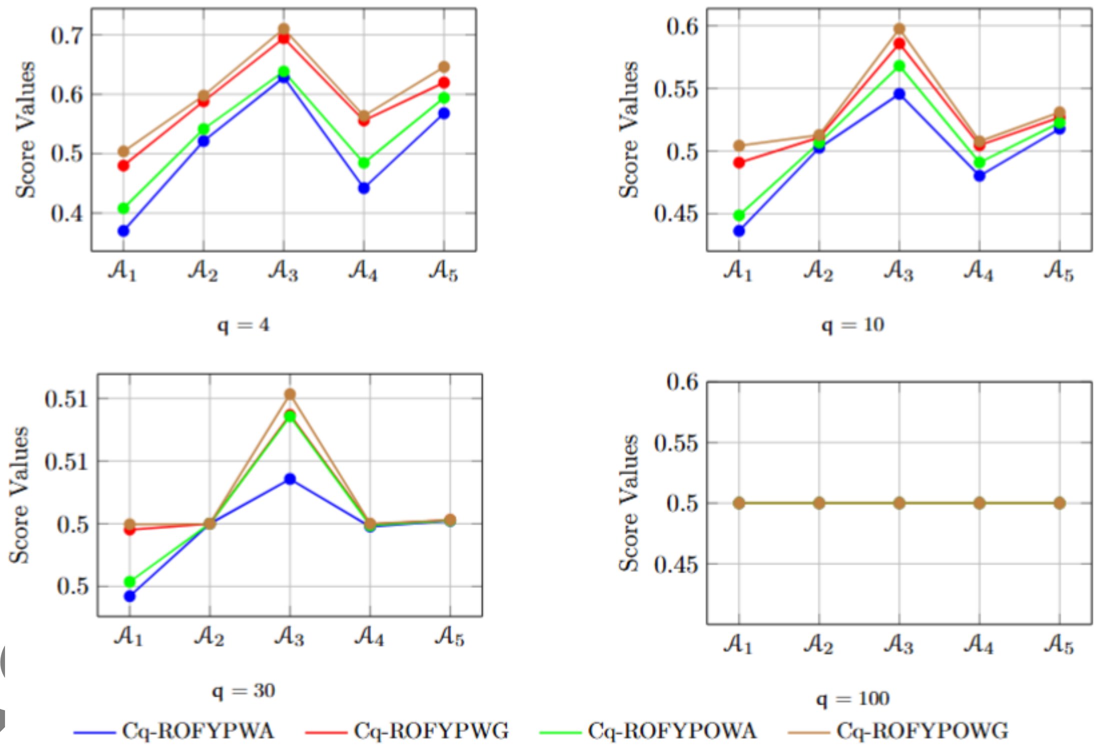

Figure 11. Sensitivity evaluation of the proposed AOs with a fixed β = 5 and varying q

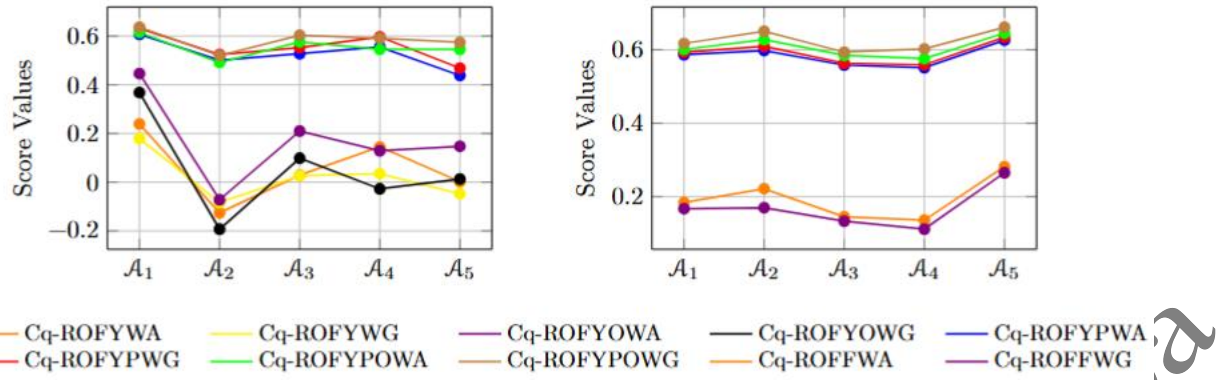

Figure 12. Comparison of Yager and Frank AOs with proposed AOs

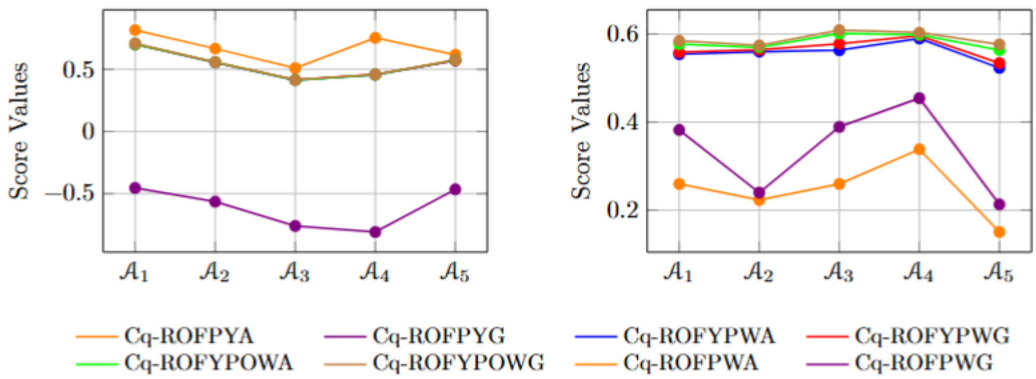

Figure 13. Comparison of Power AOs with proposed AOs

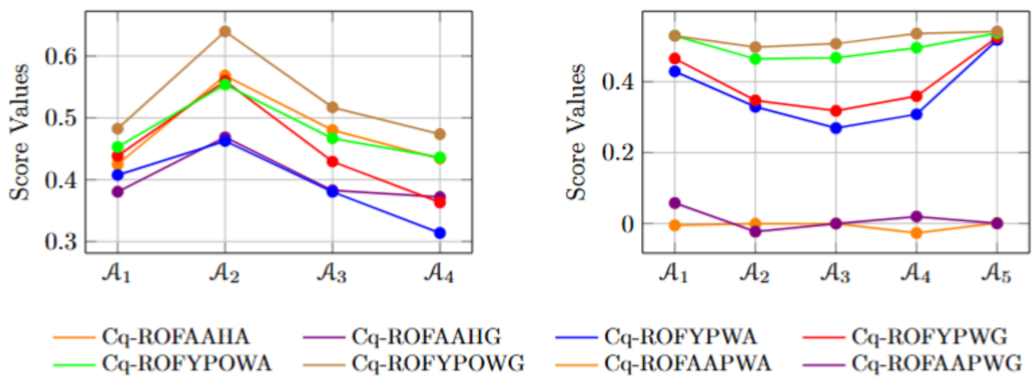

Figure 14. Comparison of Aczel-Alsina and its Power AOs with proposed AOs

**** Tables***

Table 1. Evaluation Criteria to Prioritize Waste Management Strategies

Criteria	Description
Waste Reduction Efficiency	Measures how effectively the strategy reduces waste volume, optimizes resources, and diverts waste from landfills [52].
Cost-Effectiveness	An analysis of economic viability, taking into account the balance between costs of implementing the strategy and its advantages such as saving landfill space or earning money by recycling [53].
Environmental Sustainability	A measurement of the ability of the approach to cut down emissions, pollution, and save natural resources [54].
Public Health Concerns	An evaluation of the possible risks to human health due to waste management and disposal [55].
Infrastructure Constraints	An assessment of the existing facilities' potential to support the process [56].

Table 2. Criteria, Weight, and Justification Based on Expert Knowledge

Criteria	Weight	Justification
Waste Reduction Efficiency	0.30	Volume reduction and efficient use of resources are very important aspects of managing waste.
Cost-Effectiveness	0.20	Helps to make the strategy economically viable by balancing cost and benefit in the form of recycling income and reduced landfills.
Environmental Sustainability	0.25	Aids in cutting down emission and pollutants, along with saving natural resources.
Public Health Concerns	0.15	Addresses potential health risks associated with improper waste disposal.
Infrastructure Constraints	0.10	Considers the feasibility of implementing the strategy given existing infrastructure limitations.

Table 3. Expert Evaluation in Cq-ROFN ($q=3$)

$\varepsilon / \mathcal{A}$	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$	$\mathcal{A}_5$
ε_1	⟨(0.7,0.3), (0.8,0.6)⟩	⟨(0.5,0.7), (0.2,0.4)⟩	⟨(0.8,0.7), (0.3,0.5)⟩	⟨(0.6,0.4), (0.5,0.3)⟩	⟨(0.8,0.6), (0.7,0.3)⟩
ε_2	⟨(0.6,0.4), (0.2,0.8)⟩	⟨(0.7,0.6), (0.4,0.7)⟩	⟨(0.9,0.7), (0.3,0.4)⟩	⟨(0.7,0.5), (0.4,0.8)⟩	⟨(0.7,0.7), (0.3,0.2)⟩
ε_3	⟨(0.6,0.5), (0.6,0.5)⟩	⟨(0.7,0.7), (0.5,0.2)⟩	⟨(0.9,0.6), (0.5,0.2)⟩	⟨(0.6,0.7), (0.5,0.4)⟩	⟨(0.8,0.7), (0.3,0.3)⟩
ε_4	⟨(0.6,0.9), (0.8,0.2)⟩	⟨(0.3,0.4), (0.4,0.5)⟩	⟨(0.2,0.3), (0.6,0.8)⟩	⟨(0.7,0.5), (0.5,0.4)⟩	⟨(0.3,0.2), (0.6,0.7)⟩
ε_5	⟨(0.5,0.7), (0.1,0.4)⟩	⟨(0.4,0.3), (0.6,0.5)⟩	⟨(0.1,0.2), (0.8,0.5)⟩	⟨(0.4,0.7), (0.6,0.5)⟩	⟨(0.3,0.2), (0.7,0.5)⟩

Table 4. Evaluation of Normalized Cq-ROFN ($q=3$)

$\varepsilon / \mathcal{A}$	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$	$\mathcal{A}_5$
ε_1	⟨(0.7,0.3), (0.8,0.6)⟩	⟨(0.5,0.7), (0.2,0.4)⟩	⟨(0.8,0.7), (0.3,0.5)⟩	⟨(0.6,0.4), (0.5,0.3)⟩	⟨(0.8,0.6), (0.7,0.3)⟩
ε_2	⟨(0.6,0.4), (0.2,0.8)⟩	⟨(0.7,0.6), (0.4,0.7)⟩	⟨(0.9,0.7), (0.3,0.4)⟩	⟨(0.7,0.5), (0.4,0.8)⟩	⟨(0.7,0.7), (0.3,0.2)⟩
ε_3	⟨(0.6,0.5), (0.6,0.5)⟩	⟨(0.7,0.7), (0.5,0.2)⟩	⟨(0.9,0.6), (0.5,0.2)⟩	⟨(0.6,0.7), (0.5,0.4)⟩	⟨(0.8,0.7), (0.3,0.3)⟩
ε_4	⟨(0.8,0.2), (0.6,0.9)⟩	⟨(0.4,0.5), (0.3,0.4)⟩	⟨(0.6,0.8), (0.2,0.3)⟩	⟨(0.5,0.4), (0.7,0.5)⟩	⟨(0.6,0.7), (0.3,0.2)⟩
ε_5	⟨(0.1,0.4), (0.5,0.7)⟩	⟨(0.6,0.5), (0.4,0.3)⟩	⟨(0.8,0.5), (0.1,0.2)⟩	⟨(0.6,0.5), (0.4,0.7)⟩	⟨(0.7,0.5), (0.3,0.2)⟩

Table 5. Aggregated Cq-ROFNs using the proposed AOs ($q=3$ and $\beta=5$)

Alternative	Cq-ROFYPWA	Cq-ROFYPWG
$\mathcal{A}_1$	⟨(0.6644076,0.3464486), (0.7794564,0.7491419)⟩	⟨(0.6965928,0.4312445), (0.6288467,0.6190343)⟩
$\mathcal{A}_2$	⟨(0.5622207,0.6557639), (0.4419677,0.6330135)⟩	⟨(0.6557175,0.6851226), (0.3314746,0.4473096)⟩
$\mathcal{A}_3$	⟨(0.7810900,0.6723399), (0.4487777,0.4759721)⟩	⟨0.8589097,0.7130309), (0.3434647,0.4143600)⟩
$\mathcal{A}_4$	⟨(0.6136407,0.4612724), (0.5729945,0.7221699)⟩	⟨(0.6441561,0.6103189), (0.4867411,0.4371396)⟩
$\mathcal{A}_5$	⟨(0.7452134,0.6365598), (0.6703760,0.2927731)⟩	⟨(0.7830817,0.6683107), (0.5148356,0.2759284)⟩

Table 6. Aggregated Cq-ROFNs using the proposed ordered AOs (q =3 and $\beta=5$)

Alternative	Cq-ROFYPOWA	Cq-ROFYPOWG
$\mathcal{A}_1$	⟨(0.6011016,0.4560438), (0.6964797,0.7317474)⟩	⟨(0.6393794,0.4855397), (0.5457715,0.5650635)⟩
$\mathcal{A}_2$	⟨(0.6223703,0.6470183), (0.4796979,0.5782651)⟩	⟨(0.6768685,0.6873701), (0.4267204,0.3258578)⟩
$\mathcal{A}_3$	⟨(0.7933401,0.6692597), (0.4529621,0.4513666)⟩	⟨(0.8808271,0.7115328), (0.3506572,0.3803071)⟩
$\mathcal{A}_4$	⟨(0.6022461,0.5790819), (0.5371098,0.6657223)⟩	⟨(0.6130690,0.6742652), (0.4843198,0.4446880)⟩
$\mathcal{A}_5$	⟨(0.7122131,0.6775722), (0.5526178,0.2870370)⟩	⟨(0.7694007,0.6952742), (0.3170547,0.2600703)⟩

Table 7. Alternative Ranking and Score Determination

Operator	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$	$\mathcal{A}_5$	Ranking
Cq-ROFYPWA	0.3602221	0.5299312	0.6455634	0.4411139	0.5863558	$\mathcal{A}_3 > \mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_1$
Cq-ROFYPWG	0.4830807	0.6194016	0.7211233	0.5739427	0.6553061	$\mathcal{A}_3 > \mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_1$
Cq-ROFYPOWA	0.3955924	0.5520463	0.6535480	0.4906584	0.6199833	$\mathcal{A}_3 > \mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_1$
Cq-ROFYPOWG	0.5082144	0.6306432	0.7363768	0.5838568	0.6855265	$\mathcal{A}_3 > \mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_1$

Table 8. Analysis of the proposed AOs' sensitivity to changes in β values and $q=3$

q	β	AOs	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$	$\mathcal{A}_5$	Ranking
	5	Cq-ROFYPWA	0.3602221	0.5299312	0.6455634	0.4411139	0.5863558	$\mathcal{A}_3 > \mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_1$
		Cq-ROFYPWG	0.4830807	0.6194016	0.7211233	0.5739427	0.6553061	
		Cq-ROFYPOWA	0.3955924	0.5520463	0.6535480	0.4906584	0.6199833	
		Cq-ROFYPOWG	0.5082144	0.6306432	0.7363768	0.5838568	0.6855265	

β	q	AOs	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$	$\mathcal{A}_5$	Ranking
3	30	Cq-ROFYPWA	0.2572441	0.4689919	0.5622704	0.3696941	0.5319151	$\mathcal{A}_3 > \mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_1$
		Cq-ROFYPWG	0.5815273	0.6492278	0.7762930	0.6265369	0.6922599	
		Cq-ROFYPOWA	0.2722389	0.4725482	0.5631662	0.3887790	0.5313737	
		Cq-ROFYPOWG	0.5906412	0.6517765	0.7803607	0.6221189	0.6995441	
	150	Cq-ROFYPWA	0.2088512	0.4396417	0.5312064	0.3417232	0.5016038	$\mathcal{A}_3 > \mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_1$
		Cq-ROFYPWG	0.6164975	0.6626459	0.7982783	0.6435726	0.7012792	
		Cq-ROFYPOWA	0.2124414	0.4404149	0.5314345	0.3469671	0.5007781	
		Cq-ROFYPOWG	0.6185994	0.6635673	0.7990074	0.6417017	0.7032281	
	300	Cq-ROFYPWA	0.2010787	0.4349664	0.5269984	0.3376297	0.5039411	$\mathcal{A}_3 > \mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_1$
		Cq-ROFYPWG	0.6212291	0.6650563	0.8026869	0.6461543	0.7030462	
		Cq-ROFYPOWA	0.2027511	0.4353596	0.5271131	0.3402739	0.5035165	
		Cq-ROFYPOWG	0.6222795	0.6655124	0.8030554	0.6452084	0.7040691	

Table 9. Analysis of the proposed AOs' sensitivity to changes in q values and β =5

β	q	AOs	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$	$\mathcal{A}_5$	Ranking
5	4	Cq-ROFYPWA	0.3699791	0.5212080	0.6285182	0.4420407	0.5679620	$\mathcal{A}_3 > \mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_1$
		Cq-ROFYPWG	0.4798117	0.5880679	0.6943884	0.5557730	0.6195353	
		Cq-ROFYPOWA	0.4079847	0.5415418	0.6384297	0.4844555	0.5939706	
		Cq-ROFYPOWG	0.5036519	0.5976720	0.7101394	0.5636446	0.6459586	
	10	Cq-ROFYPWA	0.4363264	0.5026450	0.5455217	0.4802202	0.5177697	$\mathcal{A}_3 > \mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_1$
		Cq-ROFYPWG	0.4907858	0.5108789	0.5857425	0.5046124	0.5269314	
		Cq-ROFYPOWA	0.4487246	0.5066140	0.5680304	0.4908806	0.5226731	
		Cq-ROFYPOWG	0.5042970	0.5128275	0.5975081	0.5077275	0.5309875	
	30	Cq-ROFYPWA	0.4942116	0.5000020	0.5035667	0.4997735	0.5002282	$\mathcal{A}_3 > \mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_1$
		Cq-ROFYPWG	0.4995317	0.5000087	0.5086759	0.4999442	0.5002928	
		Cq-ROFYPOWA	0.4953665	0.5000054	0.5085612	0.4998712	0.5002549	
		Cq-ROFYPOWG	0.4999520	0.5000104	0.5103353	0.5000026	0.5003017	
	100	Cq-ROFYPWA	0.5	0.5	0.5	0.5	0.5	$\mathcal{A}_3 = \mathcal{A}_5 = \mathcal{A}_2 = \mathcal{A}_4 = \mathcal{A}_1$
		Cq-ROFYPWG	0.5	0.5	0.5	0.5	0.5	
		Cq-ROFYPOWA	0.5	0.5	0.5	0.5	0.5	
		Cq-ROFYPOWG	0.5	0.5	0.5	0.5	0.5	

Table 10. Comparison of Yager and Frank operators in the Cq-ROFS Environment

Problems	AOs	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$	$\mathcal{A}_5$	Ranking
Javeed et al. [28]	Cq-ROFYWA [28]	**0.2395**	-0.1256	0.0290	0.1451	0.0024	$\mathcal{A}_1 > \mathcal{A}_4 > \mathcal{A}_3 > \mathcal{A}_5 > \mathcal{A}_2$
	Cq-ROFYWG [28]	**0.1795**	-0.0829	0.0268	0.0351	-0.0470	$\mathcal{A}_1 > \mathcal{A}_4 > \mathcal{A}_3 > \mathcal{A}_5 > \mathcal{A}_2$
	Cq-ROFYOWA [28]	**0.4461**	-0.0719	0.2103	0.1292	0.1476	$\mathcal{A}_1 > \mathcal{A}_3 > \mathcal{A}_5 > \mathcal{A}_4 > \mathcal{A}_2$
	Cq-ROFYOWG [28]	**0.3683**	-0.1925	0.0986	-0.0274	0.0132	$\mathcal{A}_1 > \mathcal{A}_3 > \mathcal{A}_5 > \mathcal{A}_4 > \mathcal{A}_2$
	Cq-ROFYPWA	**0.6072**	0.5010	0.5274	0.5555	0.4384	$\mathcal{A}_1 > \mathcal{A}_4 > \mathcal{A}_3 > \mathcal{A}_2 > \mathcal{A}_5$
	Cq-ROFYPWG	**0.6324**	0.5237	0.5532	0.5969	0.4685	$\mathcal{A}_1 > \mathcal{A}_4 > \mathcal{A}_3 > \mathcal{A}_2 > \mathcal{A}_5$
	Cq-ROFYPOWA	**0.6151**	0.4916	0.5762	0.5455	0.5456	$\mathcal{A}_1 > \mathcal{A}_3 > \mathcal{A}_5 > \mathcal{A}_4 > \mathcal{A}_2$
	Cq-ROFYPOWG	**0.6367**	0.5202	0.6035	0.5911	0.5742	$\mathcal{A}_1 > \mathcal{A}_3 > \mathcal{A}_4 > \mathcal{A}_5 > \mathcal{A}_2$
Du et al.[17]	Cq-ROFFWA[17]	0.1839	0.2217	0.1452	0.1358	**0.2816**	$\mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_1 > \mathcal{A}_3 > \mathcal{A}_4$
	Cq-ROFFWG[17]	0.1672	0.1698	0.1331	0.1115	**0.2651**	$\mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_1 > \mathcal{A}_3 > \mathcal{A}_4$
	Cq-ROFYPWA	0.5869	0.5981	0.5583	0.5513	**0.6262**	$\mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_1 > \mathcal{A}_3 > \mathcal{A}_4$
	Cq-ROFYPWG	0.5933	0.6097	0.5633	0.5592	**0.6341**	$\mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_1 > \mathcal{A}_3 > \mathcal{A}_4$
	Cq-ROFYPOWA	0.6010	0.6276	0.5850	0.5759	**0.6455**	$\mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_1 > \mathcal{A}_3 > \mathcal{A}_4$
	Cq-ROFYPOWG	0.6174	0.6506	0.5938	0.6028	**0.6618**	$\mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_1 > \mathcal{A}_4 > \mathcal{A}_3$

Table 11. Comparison with Existing Cq-ROFS Power Operators

Problems	AOs	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$	$\mathcal{A}_5$	Ranking
Wu et al [37]	Cq-ROFPYA [37]	**0.8160**	0.6663	0.5095	0.7527	0.6169	$\mathcal{A}_1 > \mathcal{A}_4 > \mathcal{A}_2 > \mathcal{A}_5 > \mathcal{A}_3$
	Cq-ROFPYG [37]	**-0.4539**	-0.5649	-0.7596	-0.8077	-0.4656	$\mathcal{A}_1 > \mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_3 > \mathcal{A}_4$
	Cq-ROFYPWA	**0.7017**	0.5558	0.4140	0.4567	0.5718	$\mathcal{A}_1 > \mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_3$
	Cq-ROFYPWG	**0.7023**	0.5562	0.4144	0.4573	0.5721	$\mathcal{A}_1 > \mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_3$
	Cq-ROFYPOWA	**0.7017**	0.5583	0.4140	0.4567	0.5750	$\mathcal{A}_1 > \mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_3$
	Cq-ROFYPOWG	**0.7077**	0.5587	0.4144	0.4573	0.5753	$\mathcal{A}_1 > \mathcal{A}_5 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_3$
Garg et al [20]	Cq-ROFPWA [20]	0.2595	0.2233	0.2597	**0.3382**	0.1506	$\mathcal{A}_4 > \mathcal{A}_3 > \mathcal{A}_1 > \mathcal{A}_2 > \mathcal{A}_5$
	Cq-ROFPWG [20]	0.3821	0.2399	0.3892	**0.4544**	0.2132	$\mathcal{A}_4 > \mathcal{A}_3 > \mathcal{A}_1 > \mathcal{A}_2 > \mathcal{A}_5$
	Cq-ROFYPWA	0.5545	0.5599	0.5633	**0.5899**	0.5230	$\mathcal{A}_4 > \mathcal{A}_3 > \mathcal{A}_2 > \mathcal{A}_1 > \mathcal{A}_5$
	Cq-ROFYPWG	0.5587	0.5643	0.5780	**0.5962**	0.5338	$\mathcal{A}_4 > \mathcal{A}_3 > \mathcal{A}_2 > \mathcal{A}_1 > \mathcal{A}_5$
	Cq-ROFYPOWA	0.5771	0.5699	**0.6013**	0.5980	0.5636	$\mathcal{A}_3 > \mathcal{A}_4 > \mathcal{A}_1 > \mathcal{A}_2 > \mathcal{A}_5$
	Cq-ROFYPOWG	0.5844	0.5741	**0.6092**	0.6033	0.5764	$\mathcal{A}_3 > \mathcal{A}_4 > \mathcal{A}_1 > \mathcal{A}_2 > \mathcal{A}_5$

Table 12. Comparison of Aczel-Alsina Power Operators in the Cq-ROFS Environment

Problems	AOs	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$	$\mathcal{A}_5$	Ranking
Ali et al. [50]	Cq-ROFAAHA [50]	0.4248	**0.5684**	0.4804	0.4341	…	$\mathcal{A}_2 > \mathcal{A}_3 > \mathcal{A}_4 > \mathcal{A}_1$
	Cq-ROFAAHG [50]	0.3805	**0.4686**	0.3926	0.3719	…	$\mathcal{A}_2 > \mathcal{A}_3 > \mathcal{A}_1 > \mathcal{A}_4$
	Cq-ROFYPWA	0.4077	**0.4626**	0.3804	0.3139	…	$\mathcal{A}_2 > \mathcal{A}_1 > \mathcal{A}_3 > \mathcal{A}_4$
	Cq-ROFYPWG	0.4384	**0.5598**	0.4294	0.3634	…	$\mathcal{A}_2 > \mathcal{A}_1 > \mathcal{A}_3 > \mathcal{A}_4$
	Cq-ROFYPOWA	0.4530	**0.5539**	0.4669	0.4362	…	$\mathcal{A}_2 > \mathcal{A}_3 > \mathcal{A}_1 > \mathcal{A}_4$
	Cq-ROFYPOWG	0.4826	**0.6399**	0.5170	0.473	…	$\mathcal{A}_2 > \mathcal{A}_3 > \mathcal{A}_1 > \mathcal{A}_4$
Khan et al. [51]	Cq-ROFAAPWA [51]	-0.0052	-0.0009	-0.0003	-0.0267	**0.0016**	$\mathcal{A}_5 > \mathcal{A}_3 > \mathcal{A}_2 > \mathcal{A}_1 > \mathcal{A}_4$
	Cq-ROFAAPWG [51]	**0.0576**	-0.0230	-0.0002	0.0194	0.0002	$\mathcal{A}_1 > \mathcal{A}_4 > \mathcal{A}_5 > \mathcal{A}_3 > \mathcal{A}_2$
	Cq-ROFYPWA	0.4291	0.3292	0.2691	0.3082	**0.5177**	$\mathcal{A}_5 > \mathcal{A}_1 > \mathcal{A}_2 > \mathcal{A}_4 > \mathcal{A}_3$
	Cq-ROFYPWG	0.4653	0.3475	0.3177	0.3593	**0.5251**	$\mathcal{A}_5 > \mathcal{A}_1 > \mathcal{A}_4 > \mathcal{A}_2 > \mathcal{A}_3$
	Cq-ROFYPOWA	0.5298	0.4642	0.4676	0.4959	**0.5367**	$\mathcal{A}_5 > \mathcal{A}_1 > \mathcal{A}_4 > \mathcal{A}_3 > \mathcal{A}_2$
	Cq-ROFYPOWG	**0.5668**	0.4976	0.5074	0.5358	0.5420	$\mathcal{A}_1 > \mathcal{A}_5 > \mathcal{A}_4 > \mathcal{A}_3 > \mathcal{A}_2$

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