Robust Regression-Based Ratio-Type Estimators in Simple, Ranked Set, and Median Ranked Set Sampling: Applications in Sustainable Agriculture

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Abstract: Recently, researchers have developed robust ratio-type estimators for estimating the population mean under simple random sampling. These estimators exploit robust regression techniques such as Least Absolute Deviation (LAD), Huber-M, Huber-MM, Tukey-M, Least Trimmed Squares (LTS), and Least Median of Squares (LMS). The present study aims to introduce a novel and efficient class of robust regression-based ratio-type estimators for estimating the finite population mean under simple random sampling (SRS), ranked set sampling (RSS), and median ranked set sampling (MRSS) schemes. The proposed estimators exhibit superior reliability and efficiency compared to competing methods across all three sampling designs. The mathematical properties of the proposed estimators are analytically investigated, including derivations of bias and mean squared error. Furthermore, Monte Carlo simulation studies are conducted using two real agricultural field populations to assess the empirical performance of the proposed estimators. The numerical results clearly demonstrate the effectiveness and improved efficiency of the proposed estimators relative to existing estimators.

Key Words: Auxiliary information; mean squared error; median ranked set sampling; ranked set sampling; robust regression methods; simple random sampling.

Classification: 62D05 & 62P30

1. Introduction

Massive literature is available for estimating finite population mean/total, coefficient of variation, median, variance and proportion in sample surveys e.g. Irfan et al. [1], Javed et al. [2], Koc [3], Cekim [4], Irfan et al. [5], Shahzad et al. [6, 7], Ahmad et al. [8], Hussain et al. [9], Pandey et al. [10] and many more. The efficiency of ratio, product and regression estimators are suspicious in the presence of outliers. To handle this situation, Zaman and Bulut [11], Bulut and Zaman [12], Zaman et al. [13], Cetin and Koyuncu [14] and Babatunde and Oladugba [15] used robust regression methods for the estimation of population mean under simple random sampling (SRS). Robust regression methods, such as Least Absolute Deviation (LAD), Huber-M, Huber-MM,

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Tukey-M, Least Trimmed Squares (LTS), and Least Median of Squares (LMS), are designed to reduce the influence of extreme values or outliers in the data. These methods offer a more reliable alternative to traditional regression techniques by providing estimates that are less sensitive to outliers, thereby mitigating the negative effects of extreme values on the analysis.

Ranked set sampling (RSS) initially conceptualized by McIntyre [16], is an alternative sampling scheme to SRS. It is used under the following conditions: (i) when the incorporated measurement of the study construct is too costly and time-intensive (ii) to boost the efficiency of the estimator of the population mean (iii) in case of infinite population's (iv) if all the sample data is not available. The dilemma of calculating population mean under RSS has been widely addressed by several authors such as Al-Nasser [17], Al-Omari et al. [18], Koyuncu [19], Mehta and Mandowara [20], Zamanzada and Al-Omari [21], Koyuncu and Al-Omari [22], Sabry and Almetwally [23], Mahdizadeh and Zamanzade [24, 25], Mohammadkhani [26], Taconeli [27]. Both theoretically and numerically, all the above mentioned authors proved that their suggested estimators are more efficient as compared to SRS. Classical estimator(s) are highly sensitive to the outliers which reduces the efficiency of the estimators under RSS. This is the eminent disadvantage of existing estimators under RSS.

We go as follows to obtain a ranked set sample from a population. First from a population, an initial simple random sampling units are chosen and ranked according to the attributes of interest. The smallest unit in this ranking is included as the first item in the ranked set sampling, and the attribute of interest for the unit is formally measured. The first observation is known as the first judgment order statistic and it is denoted by square bracket $\left(X_{[1]}\right)$ around the value instead of normal round bracket since in simple random sampling, the smallest attribute observation among the units may or may not exist, despite our ranking judging it to be the smallest. The second smallest unit is included as the second ranked set sampling unit, and it represent the second judgment order statistic. Similarly, m^{th} units selected and ranked which is the m^{th} judgment order statistic $\left(X_{[m]}\right)$. The remaining observations (other than those in our initial simple random sampling) are not used to draw inferences about the population. The complete process results in the m measured observation $\left(X_{[1]}, X_{[2]}, X_{[3]}, \ldots, X_{[m]}\right)$ is referred to be a cycle. The number of observations m in each SRS is known as set size. Therefore, to complete a solo ranked set cycle, we must require to use a m^2 units from population to rank independent SRS of size m in each subset. To get the mr ranked set samples, repeat the whole process m times.

Median ranked set sampling (MRSS) was suggested by Muttlak [28]. The goal of MRSS is to collect the data that are intended to encompass the population's whole range of values and, thus, it is more demonstrative of it by using only median values, under the slight errors in ranking, then the same amount of observation gained through SRS. Many authors have worked on MRSS e.g., Al-Odat and Al-Selah [29] and Koyuncu and Al-Omari [22]. MRSS comprises of picking m SRS each of size m and calculating the median of every set. First of all, select the m^2 simple random samples from population, and ranked every set of samples with respect to personal judgment or any other cost-effective method. After that, if the m is odd then select the $\left(\frac{m+1}{2}\right)^{th}$ set of observations for the median ranked set sampling. If the sample size m is even then first choose $\frac{m}{2}$ values from $\left(\frac{m}{2}\right)^{th}$ set and next $\frac{m}{2}$ values choose from the $\left(\left(\frac{m}{2}\right)+1\right)^{th}$ set of observation. The MRSS consist of total m sampling units, but these units are more likely to each other and give minimum variation. That is why median ranked set sampling is more efficient as compare to simple random sampling.

The novelty of the current study lies in its proposal of an efficient class of robust regression-based ratio-type estimators for estimating the population mean under SRS, RSS, and MRSS, incorporating robust regression methods. The innovation behind this work is:

- Robust regression methods are employed in SRS, RSS, and MRSS schemes to estimate the population mean while effectively handling outliers in the data.
- Non-conventional measures such as quartile deviation, interquartile deviation, median, and the Hodge-Lehmann estimator are introduced for the first time as auxiliary information in both RSS and MRSS schemes, expanding the available tools for estimation in these contexts.
- The proposed generalized estimator is versatile and can generate various robust estimators, depending on the auxiliary information available.
- To evaluate the effectiveness of the proposed generalized estimator, a robustness study is conducted under the SRS, RSS, and MRSS sampling schemes, specifically in the presence of outliers, demonstrating the estimator's potential in real-world scenarios with outliers.

Remark 1.1 Robust regression methods can be calculated as:

$$LAD = min\sum_{i=1}^{n}\left|\varepsilon_i\right|$$

$$Huber - M = min\sum_{i=1}^{n}\rho\left(\varepsilon_i\right)$$

$$Huber - MM = \frac{1}{n}min\sum_{i=1}^{n}\rho\left(\frac{\varepsilon_i\left(\beta\right)}{s_n}\right)$$

$$Tukey - M = \rho(y) = \begin{cases} \frac{1}{6}\left(1-\left(1-\left(\frac{y}{k}\right)^2\right)^3\right) & |y| \le k \\ \frac{1}{6} & |y| > k \end{cases}$$

$$LTS = \min \sum_{i=1}^{z} \left(y_i - \hat{\beta}_o - \hat{\beta}_1 x_i \right)^2 \ where\ z = \frac{n}{2} + 1$$

$$LMS = \min Median\left(\varepsilon_i^2\right)$$

Remark 1.2 Non-conventional measures related to auxiliary variable are the following:

Quartile deviation: Q_D $\quad = \dfrac{Q_3 - Q_1}{2}$

Inter quartile range: Q_R $\quad = Q_3 - Q_1$

Hodge Lehman: H_L $\quad = Median\left(\dfrac{X_j + X_k}{2}\right), 1 \le j \le N; 1 \le k \le N$

Median: M_x $\quad = \left(\dfrac{n+1}{2}\right)^{th}$

First quartile: Q_1 $\quad = \left(\dfrac{n}{4} + 1\right)^{th}$

Third quartile: Q_3 $\quad = \left(\dfrac{3n}{4} + 1\right)^{th}$

2. Existing/Modified Estimators

Zaman and Bulut [11] suggested ratio estimators for the population mean using robust regression methods under simple random sampling.

$$\hat{\bar{Y}}_{zb1} = \frac{\bar{y} + b_{rob(zb)}\left(\bar{X} - \bar{x}\right)}{\bar{x}} \bar{X} \qquad (1)$$

$$\hat{\bar{Y}}_{zb2} = \frac{\bar{y} + b_{rob(zb)}\left(\bar{X} - \bar{x}\right)}{\bar{x} + C_x} \left(\bar{X} + C_x\right) \qquad (2)$$

$$\hat{\bar{Y}}_{zb3} = \frac{\bar{y} + b_{rob(zb)}\left(\bar{X} - \bar{x}\right)}{\bar{x} + \beta_{2(x)}} \left(\bar{X} + \beta_{2(x)}\right) \qquad (3)$$

$$\hat{\bar{Y}}_{zb4} = \frac{\bar{y} + b_{rob(zb)}\left(\bar{X} - \bar{x}\right)}{\bar{x}\beta_{2(x)} + C_x} \left(\bar{X}\beta_{2(x)} + C_x\right) \qquad (4)$$

$$\hat{\bar{Y}}_{zb5} = \frac{\bar{y} + b_{rob(zb)}\left(\bar{X} - \bar{x}\right)}{\bar{x}C_x + \beta_{2(x)}} \left(\bar{X}C_x + \beta_{2(x)}\right) \qquad (5)$$

where C_x and $\beta_{2(x)}$ are the population coefficient of variation and coefficient of the kurtosis, respectively, of the auxiliary variate; $\bar{y}$ and $\bar{x}$ are the sample mean of the study and auxiliary variable, respectively; $\bar{X}$ is the known population mean; and $b_{rob(zb)}$ is the regression coefficient, zb means the methods of LAD, Huber-M, Huber-MM, Tukey-M, LTS and LMS, respectively.
We adopted the estimators proposed by Zaman and Bulut [11] and implemented their modified versions under RSS and MRSS schemes.

$$\hat{\bar{Y}}_{zb1} = \frac{\bar{y}_{(j)} + b_{rob(zb)}\left(\bar{X} - \bar{x}_{(j)}\right)}{\bar{x}_{(j)}}\bar{X} \tag{6}$$

$$\hat{\bar{Y}}_{zb2} = \frac{\bar{y}_{(j)} + b_{rob(zb)}\left(\bar{X} - \bar{x}_{(j)}\right)}{\bar{x}_{(j)} + C_x}\left(\bar{X} + C_x\right) \tag{7}$$

$$\hat{\bar{Y}}_{zb3} = \frac{\bar{y}_{(j)} + b_{rob(zb)}\left(\bar{X} - \bar{x}_{(j)}\right)}{\bar{x}_{(j)} + \beta_{2(x)}}\left(\bar{X} + \beta_{2(x)}\right) \tag{8}$$

$$\hat{\bar{Y}}_{zb4} = \frac{\bar{y}_{(j)} + b_{rob(zb)}\left(\bar{X} - \bar{x}_{(j)}\right)}{\bar{x}_{(j)}\beta_{2(x)} + C_x}\left(\bar{X}\beta_{2(x)} + C_x\right) \tag{9}$$

$$\hat{\bar{Y}}_{zb5} = \frac{\bar{y}_{(j)} + b_{rob(zb)}\left(\bar{X} - \bar{x}_{(j)}\right)}{\bar{x}_{(j)}C_x + \beta_{2(x)}}\left(\bar{X}C_x + \beta_{2(x)}\right) \tag{10}$$

where *j* represents SRS, RSS and MRSS. Mean squared error of the estimators under SRS, RSS and MRSS up to first order of approximation are:

$$MSE\left(\bar{Y}_{zbi}\right) \cong \gamma\left[R_{KCi}^2 S_{X(j)}^2 + 2b_{rob(zb)}R_{KCi}S_{(X)j}^2 + b_{rob(zb)}^2 S_{X(j)}^2 - 2R_{KCi}S_{YX(j)} - 2b_{rob(zb)}S_{YX(j)} + S_{Y(j)}^2\right] \tag{11}$$

where $R_{KC1} = R = \frac{\bar{Y}}{\bar{X}}$, $R_{KC2} = \frac{\bar{Y}}{\bar{X} + C_x}$, $R_{KC3} = \frac{\bar{Y}}{\bar{X} + \beta_{2(x)}}$, $R_{KC4} = \frac{\bar{Y}\beta_{2(x)}}{\bar{X}\beta_{2(x)} + C_x}$,

$R_{KC5} = \frac{\bar{Y}C_x}{\bar{X}C_x + \beta_{2(x)}}$ and $\gamma = \left(\frac{1}{n} - \frac{1}{N}\right)$.

3. Suggested estimators based on robust regression methods

A significant limitation of many existing estimators, including those proposed by Kadilar et al. [30], Al-Omari et al. [18], Koyuncu [19], and Irfan et al. [1], is their inefficiency in the presence of extreme observations or outliers, which can severely distort the accuracy of population mean estimation. Although Irfan et al. [1] introduced ratio-type estimators based on the coefficient of correlation under the SRS scheme, their performance remains sensitive to extreme values. Motivated by this shortcoming, we propose a new class of robust regression based estimators for estimating the population mean under SRS, RSS, and MRSS schemes. By incorporating robust regression techniques along with non-conventional auxiliary measures, the proposed estimators aim to improve the accuracy and stability of population mean estimation in the presence of outliers. Additionally, non-conventional measures related to the auxiliary variable, which are outlined in Remark 1.2, are also used to further improve the robustness of the estimators. The class of estimators proposed by Irfan et al. [1] is summarized below:

$$\bar{Y}_{IR1} = t\left[\bar{y}\left(\frac{\bar{X}\rho_{XY}+R}{\bar{x}\rho_{XY}+R}\right)^{\frac{\bar{X}\rho_{XY}}{\bar{X}\rho_{XY}+R}} + b\left(\bar{X}-\bar{x}\right)\right] \tag{12}$$

$$\bar{Y}_{IR2} = t\left[\bar{y}\left(\frac{\bar{X}\rho_{XY}+RC_X}{\bar{x}\rho_{XY}+RC_X}\right)^{\frac{\bar{X}\rho_{XY}}{\bar{X}\rho_{XY}+RC_X}} + b\left(\bar{X}-\bar{x}\right)\right] \tag{13}$$

$$\bar{Y}_{IR3} = t\left[\bar{y}\left(\frac{\bar{X}\rho_{XY}+R\beta_{2(X)}}{\bar{x}\rho_{XY}+R\beta_{2(X)}}\right)^{\frac{\bar{X}\rho_{XY}}{\bar{X}\rho_{XY}+R\beta_{2(X)}}} + b\left(\bar{X}-\bar{x}\right)\right] \tag{14}$$

$$\bar{Y}_{IR4} = t\left[\bar{y}\left(\frac{\bar{X}\rho_{XY}+RC_X\beta_{2(X)}}{\bar{x}\rho_{XY}+RC_X\beta_{2(X)}}\right)^{\frac{\bar{X}\rho_{XY}}{\bar{X}\rho_{XY}+RC_X\beta_{2(X)}}} + b\left(\bar{X}-\bar{x}\right)\right] \tag{15}$$

$$\bar{Y}_{IR5} = t\left[\bar{y}\left(\frac{\bar{X}+R\rho_{XY}}{\bar{x}+R\rho_{XY}}\right)^{\frac{\bar{X}}{\bar{X}+R\rho_{XY}}} + b\left(\bar{X}-\bar{x}\right)\right] \tag{16}$$

$$\bar{Y}_{IR6} = t\left[\bar{y}\left(\frac{\bar{X}+R\rho_{XY}C_X}{\bar{x}+R\rho_{XY}C_X}\right)^{\frac{\bar{X}}{\bar{X}+R\rho_{XY}C_X}} + b\left(\bar{X}-\bar{x}\right)\right] \tag{17}$$

$$\bar{Y}_{IR7} = t\left[\bar{y}\left(\frac{\bar{X}+R\rho_{XY}\beta_{2(X)}}{\bar{x}+R\rho_{XY}\beta_{2(X)}}\right)^{\frac{\bar{X}}{\bar{X}+R\rho_{XY}\beta_{2(X)}}} + b\left(\bar{X}-\bar{x}\right)\right] \tag{18}$$

$$\bar{Y}_{IR8} = t\left[\bar{y}\left(\frac{\bar{X}+R\rho_{XY}C_X\beta_{2(X)}}{\bar{x}+R\rho_{XY}C_X\beta_{2(X)}}\right)^{\frac{\bar{X}}{\bar{X}+R\rho_{XY}C_X\beta_{2(X)}}} + b\left(\bar{X}-\bar{x}\right)\right] \tag{19}$$

where "t" represents the unknown constant, $\bar{X}$ and $\bar{Y}$ are the population means of the auxiliary and study variables, ρ_{XY} is the correlation coefficient between auxiliary and study variables and "b" is the known regression coefficient.

Minimum mean squared error of Irfan et al. [1] estimators is given in Eq. (20):

$$MSE\left(\bar{Y}_{IRi}\right) \cong \left[\bar{Y}^2 - \frac{C_{1i}^2}{2C_{2i}}\right] \tag{20}$$

where $i = 1, 2, \ldots, 8$

$$C_{1i} = \bar{Y}^2 + \frac{\gamma}{2}\left[\left(\varnothing_i^4 + \varnothing_i^3\right)R^2 S_X^2 - 2\varnothing_i^2 R S_{YX}\right]$$

$$C_{2i} = \bar{Y}^2 + \gamma\left[S_Y^2 + \left\{2\varnothing_i^4 R^2 + b^2 + \varnothing_i^3 R^2 + 2\varnothing_i^2 bR\right\} S_X^2 - \left\{4\varnothing_i^2 R + 2b\right\} S_{YX}\right]$$

$$\varnothing_1 = \frac{\bar{X}\rho_{YX}}{\bar{X}\rho_{YX} + R}, \varnothing_2 = \frac{\bar{X}\rho_{YX}}{\bar{X}\rho_{YX} + RC_X}, \varnothing_3 = \frac{\bar{X}\rho_{YX}}{\bar{X}\rho_{YX} + R\beta_{2(X)}}, \varnothing_4 = \frac{\bar{X}\rho_{YX}}{\bar{X}\rho_{YX} + RC_X\beta_{2(X)}},$$

$$\varnothing_5 = \frac{\bar{X}}{\bar{X} + R\rho_{YX}}, \varnothing_6 = \frac{\bar{X}}{\bar{X} + R\rho_{YX}C_X}, \varnothing_7 = \frac{\bar{X}}{\bar{X} + R\rho_{YX}\beta_{2(X)}}, \varnothing_8 = \frac{\bar{X}}{\bar{X} + R\rho_{YX}C_X\beta_{2(X)}}.$$

We generalized the form of the estimator introduced by Irfan et al. [1] for estimating the finite population mean $\bar{Y}$ under SRS, RSS, and MRSS.

$$\bar{Y}_{(j)k} = w\left[\bar{y}_{(j)}\left(\frac{\bar{X}k_1 + k_2}{\bar{x}_{(j)}k_1 + k_2}\right)^{\delta_k} + b_{rob(zb)}\left(\bar{X} - \bar{x}_{(j)}\right)\right] \tag{21}$$

where "w" is the weight to be chosen such that the MSE becomes minimum, $\delta_k = \frac{\bar{X}k_1}{\bar{X}k_1 + k_2}$, $k = 1, 2, \ldots, 11$, $\left(k_1 \text{ \& } k_2\right)$ are the known functions of any known population parameters of auxiliary variable, $b_{rob(zb)}$ are the regression coefficients obtained from LAD, Huber-M, Huber-MM, Tukey-M, LTS, and LMS regression methods.

Following relative errors are used to get bias, MSE and minimum MSE of $\bar{Y}_{(j)k}$:

$$e_{o(j)} = \frac{\bar{y}_{(j)} - \bar{Y}}{\bar{Y}} \; and \; e_{1(j)} = \frac{\bar{x}_{(j)} - \bar{X}}{\bar{X}} \text{ such that } E\left(e_{o(j)}\right) = E\left(e_{1(j)}\right) = 0$$

$$E\left(e_{o(j)}^2\right)=\frac{Var\left(\bar{y}_{(j)}\right)}{\bar{Y}^2}=\frac{S_{Y(j)}^2}{\bar{Y}^2}, E\left(e_{1(j)}^2\right)=\frac{Var\left(\bar{x}_{(j)}\right)}{\bar{X}^2}=\frac{S_{X(j)}^2}{\bar{X}^2} \; and$$

$$E\left(e_{o(j)}e_{1(j)}\right)=\frac{Cov\left(\bar{y}_{(j)},\bar{x}_{(j)}\right)}{\bar{X}\bar{Y}}=\frac{S_{YX(j)}}{\overline{YX}}.$$

where

$$S_{Y(SRS)}^2=\left(N-1\right)^{-1}\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)^2, \qquad S_{X(SRS)}^2=\left(N-1\right)^{-1}\sum_{i=1}^{N}\left(x_i-\bar{X}\right)^2$$

$$S_{YX(SRS)}=\left(N-1\right)^{-1}\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)\left(x_i-\bar{X}\right)$$

$$S_{Y(RSS)}^2=\left[\frac{S_Y^2}{m}-\frac{1}{m^2}\sum_{i=1}^{m}\left(\mu_{Y(i)}-\bar{Y}\right)^2\right], \; S_{X(RSS)}^2=\left[\frac{S_X^2}{m}-\frac{1}{m^2}\sum_{i=1}^{m}\left(\mu_{X(i)}-\bar{X}\right)^2\right]$$

$$S_{YX(RSS)}=\left[\frac{S_{XY}}{m}-\frac{1}{m^2}\sum_{i=1}^{m}\left(\mu_{Y(i)}-\bar{Y}\right)\left(\mu_{X(i)}-\bar{X}\right)\right]$$

$$S_{Y(MRSS)}^2=\frac{1}{n}S_{Y\left[\frac{(n+1)}{2}\right]}^2, \; S_{X(MRSS)}^2=\frac{1}{n}S_{X\left[\frac{(n+1)}{2}\right]}^2, \qquad S_{YX(MRSS)}=\frac{1}{n}S_{YX\left[\frac{(n+1)}{2}\right]}$$

Expressing suggested class of estimators $\bar{Y}_{(j)k}$ in terms of relative error terms as

$$\bar{Y}_{(j)k}=w\left[\left(\bar{Y}+\bar{Y}e_{o(j)}\right)\left\{1-\delta_k^2 e_{1(j)}+\frac{1}{2}\delta_k^3\left(\delta_k+1\right)e_{1(j)}^2\right\}-\bar{X}b_{rob(zb)}e_{1(j)}\right] \tag{22}$$

Subtract $\bar{Y}$ from Eq. (22) and expanding up to first degree of approximation

$$\bar{Y}_{(j)k}-\bar{Y}=\left[\begin{array}{c} w\bar{Y}+w\bar{Y}e_{o(j)}-w\bar{Y}\delta_k^2 e_{1(j)}-w\bar{X}b_{rob(zb)}e_{1(j)}+\frac{1}{2}w\bar{Y}\left(\delta_k^4+\delta_k^3\right)e_{1(j)}^2- \\ w\bar{Y}\delta_k^2 e_{0(j)}e_{1(j)}-\bar{Y}\end{array}\right]$$

The bias, mean squared error and minimum mean squared error of the estimator $\bar{Y}_{(j)k}$ under SRS, RSS, and MRSS using a first-order approximation, are given as follows:

$$\text{Bias}\left(\bar{Y}_{(j)k}\right)=E\left(\bar{Y}_{(j)k}-\bar{Y}\right)\cong\bar{Y}\left(w-1\right)+\bar{Y}w\left[\frac{\left(\delta_k^4+\delta_k^3\right)}{2}\frac{S_{X(j)}^2}{\bar{X}^2}-\delta_k^2\frac{S_{YX(j)}}{\overline{YX}}\right]$$

$$\text{MSE}\left(\bar{Y}_{(j)k}\right)=E\left(\bar{Y}_{(j)k}-\bar{Y}\right)^2\cong\left(w\bar{Y}-\bar{Y}\right)^2+w^2a_1-wa_2$$

$$MSE_{min}\left(\bar{Y}_{(j)k}\right)\cong\left[\bar{Y}^2-\frac{w_1^2}{2w_2}\right]$$

where $w_1=2\bar{Y}^2+a_2\ and\ w_2=2\bar{Y}^2+2a_1$

$$a_1=S_{Y(j)}^2+\left\{2R^2\delta_k^4+b_{rob(zb)}^2+R^2\delta_k^3+2Rb_{rob(zb)}\delta_k^2\right\}S_{X(j)}^2-\left\{4R\delta_k^2+2b_{rob(zb)}\right\}S_{YX(j)}$$

$$a_2=\left(\delta_k^4+\delta_k^3\right)R^2S_{X(j)}^2-2R\delta_k^2S_{YX(j)}$$

Remark 3.1. Robust regression methods provide efficient estimates in the presence of unusual observations in the data. It is not advisable to remove unusual observations from the data.

Remark 3.2. It's imperative to elucidate that the parameters manifest itself in othe form of optimal weights and the minimum MSEs are generally unpredictable. Nonetheless, the investigated parameters can be determined quite accurately from the preliminary data sets or performed repated surveys in different rounds based on sampling in over several occasions. Various authors have discussed the applicability of parameters use of prior information on parameters at the estimation stage. See Irfan et al. [31] for details.

Remark 3.3. Table 1 presents some members of $\hat{\hat{\bar{Y}}}_{(j)k}$ based on regression methods and non-conventinoal measures.

4. Numerical Illustration

Estimators of the population mean are essential in sustainable agriculture as they provide reliable summaries of key agricultural indicators at regional and national levels. Accurate estimation of variables such as crop yield, fertilizer use, irrigation demand, farm income, and agricultural loan supports evidence-based decision-making. Agricultural data are often influenced by heterogeneity, measurement errors, and extreme values due to climatic and market variability. Robust population mean estimators reduce the impact of such irregularities. This results in more stable and credible estimates. These estimates are crucial for sustainable resource management and policy evaluation. In this section, we assess the performance of the proposed class of robust regression-based ratio-type estimators under SRS, RSS, and MRSS using two real-life datasets related to the field of agriculture that include outliers. The required parameters for both populations were computed using the R software. All necessary calculations and statistical analyses were carried out in R to ensure accuracy and consistency. The computed results, including the estimated parameters and related summary statistics, are presented in Tables 2 and 3 for clarity and comparison.

Population I [Agriculture Field] is sourced from Mehta and Mandowara [20]. This dataset contains 50 observations of agricultural loans outstanding from all functional banks in distinct states of the USA in 1997. The auxiliary variable "X" represents non-real estate loans in thousands of dollars, while the study variable "Y" corresponds to real estate loans in thousands of dollars.

Population II [Agriculture Field] is from Kadilar and Cingi [32], with a dataset comprising 94 observations. In this case, the study variable is the amount of apple production, and the auxiliary variable is the number of apple trees.

Figure 1 and Figure 2 illustrate the scatter plots for Population I and Population II, respectively. From the plots, it is evident that both datasets contain some potential outliers.
We calculated the mean squared error (MSE) for the modified and proposed estimators under SRS, RSS and MRSS. The superiority of these estimators is evaluated through a Monte Carlo simulation study conducted using R software. For this analysis, two real-life populations were used, with different sample sizes. The MSE expressions for all the existing/modified and proposed estimators are provided in detail in section 2. The empirical findings are then summarized in Tables 4-15. Several key insights can be drawn from these tables, as outlined below:

- ✓ From Tables 4 and 5, it is observed that the modified estimators i.e. $\bar{Y}_{zb}$ under SRS have a lower MSE compared to the traditional regression estimator i.e. $\bar{Y}_{lr}$. The incorporation of robust measures in the form of regression estimators has contributed to a reduction in the MSE, highlighting the effectiveness of robust methods in improving estimation accuracy.
- ✓ An examination of Tables 6 and 7 indicates that the modified estimators under RSS exhibit greater efficiency compared to those based on SRS.
- ✓ A detailed examination of the results presented in Tables 8 and 9 reveals that the modified estimators developed under the MRSS scheme consistently demonstrate superior efficiency compared to their RSS based estimators.
- ✓ From Tables 10 and 11, it is evident that the proposed estimators i.e. $\bar{Y}_{(j)k}$ under SRS outperform the $\bar{Y}_{lr}$ and $\bar{Y}_{zb}$ estimators in terms of MSE. This improvement can be attributed to the use of robust measures in the regression estimators, which play a crucial role in reducing estimation error and improving accuracy.
- ✓ A closer examination of the columns in $\bar{Y}_{(j)k}$ under SRS reveals that the LMS based estimator achieves the lowest minimum MSE among all the proposed estimators for Population I.
- ✓ Tables 12 and 13 show that the proposed estimators $\bar{Y}_{(j)k}$ under ranked set sampling are more efficient compared to the SRS scheme. Furthermore, the LMS based estimator provides the lowest MSE among all the proposed estimators for Population I,

demonstrating its superior performance in terms of efficiency under the ranked set sampling scheme.

- ✓ From Tables 14 and 15, it is evident that the proposed estimators $\bar{Y}_{(j)k}$ under median ranked set sampling are more efficient than those under SRS and RSS schemes.
- ✓ From Tables 4-15, it is observed that the MSE of all the proposed estimators under SRS, RSS, and MRSS decreases as the sample sizes increase. This suggests that larger sample sizes lead to more accurate estimates, reducing the variability in the estimator's performance across the different sampling schemes.

5. Concluding Remarks

Robust regression methods are highly effective when the data is non-normal and contains outliers. In this study, we proposed efficient estimators for estimating the population mean under simple random sampling (SRS), ranked set sampling (RSS), and median ranked set sampling (MRSS) schemes, using robust regression methods. Theoretical results, including bias and mean squared error (MSE), were presented in Section 2. The proposed estimators demonstrate superior efficiency compared to existing methods under all three sampling schemes. Additionally, a Monte Carlo simulation study was conducted in R software, integrating two real-life populations, to assess the performance of the proposed class of estimators. Although the findings are promising, the present study has some limitations that can be explored and refined in future work. Despite the encouraging results, this study has some limitations. The proposed estimators were evaluated under specific non-normal distributions and contamination settings, which may limit their generalizability to other data structures. The analysis was confined to SRS, RSS, and MRSS schemes, and their performance under other sampling designs was not explored. Additionally, the ranking in RSS and MRSS was assumed to be accurate, whereas ranking errors may affect estimator efficiency in practice.

Future studies may focus on extending the proposed framework to develop new estimators for alternative ranked set sampling designs, such as stratified ranked set sampling, double ranked set sampling, or partially ranked set sampling. By incorporating robust regression techniques, these estimators could better accommodate non-normality, outliers, and ranking errors commonly encountered in practical applications.

6. References

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Table 1. Some members of $\bar{Y}_{(j)k}$

k_1	k_2	k	Robust ratio-type estimators $\bar{Y}_{(j)k} = w\left[\bar{y}_{(j)}\left(\dfrac{\bar{X}k_1 + k_2}{\bar{x}_{(j)}k_1 + k_2}\right)^{\delta_k} + b_{rob(zb)}\left(\bar{X} - \bar{x}_{(j)}\right)\right]$
1	$\beta_{2(X)}$	1	$\bar{Y}_{(j)1} = w\left[\bar{y}_{(j)}\left(\dfrac{\bar{X} + \beta_{2(X)}}{\bar{x}_{(j)} + \beta_{2(X)}}\right)^{\delta_1} + b_{rob(zb)}\left(\bar{X} - \bar{x}_{(j)}\right)\right]$
1	C_X	2	$\bar{Y}_{(j)2} = w\left[\bar{y}_{(j)}\left(\dfrac{\bar{X} + C_X}{\bar{x}_{(j)} + C_X}\right)^{\delta_2} + b_{rob(zb)}\left(\bar{X} - \bar{x}_{(j)}\right)\right]$
1	Q_2	3	$\bar{Y}_{(j)3} = w\left[\bar{y}_{(j)}\left(\dfrac{\bar{X} + Q_2}{\bar{x}_{(j)} + Q_2}\right)^{\delta_3} + b_{rob(zb)}\left(\bar{X} - \bar{x}_{(j)}\right)\right]$
1	Q_1	4	$\bar{Y}_{(j)4} = w\left[\bar{y}_{(j)}\left(\dfrac{\bar{X} + Q_1}{\bar{x}_{(j)} + Q_1}\right)^{\delta_4} + b_{rob(zb)}\left(\bar{X} - \bar{x}_{(j)}\right)\right]$
1	Q_3	5	$\bar{Y}_{(j)5} = w\left[\bar{y}_{(j)}\left(\dfrac{\bar{X} + Q_3}{\bar{x}_{(j)} + Q_3}\right)^{\delta_5} + b_{rob(zb)}\left(\bar{X} - \bar{x}_{(j)}\right)\right]$
1	QD	6	$\bar{Y}_{(j)6} = w\left[\bar{y}_{(j)}\left(\dfrac{\bar{X} + QD}{\bar{x}_{(j)} + QD}\right)^{\delta_6} + b_{rob(zb)}\left(\bar{X} - \bar{x}_{(j)}\right)\right]$

1	IQR	7	$\bar{Y}_{(j)7} = w\left[\bar{y}_{(j)}\left(\dfrac{\bar{X}+IQR}{\bar{x}_{(j)}+IQR}\right)^{\delta_7} + b_{rob(zb)}\left(\bar{X}-\bar{x}_{(j)}\right)\right]$
1	HL	8	$\bar{Y}_{(j)8} = w\left[\bar{y}_{(j)}\left(\dfrac{\bar{X}+HL}{\bar{x}_{(j)}+HL}\right)^{\delta_8} + b_{rob(zb)}\left(\bar{X}-\bar{x}_{(j)}\right)\right]$
$\beta_{2(X)}$	C_X	9	$\bar{Y}_{(j)9} = w\left[\bar{y}_{(j)}\left(\dfrac{\bar{X}\beta_{2(X)}+C_X}{\bar{x}_{(j)}\beta_{2(X)}+C_X}\right)^{\delta_9} + b_{rob(zb)}\left(\bar{X}-\bar{x}_{(j)}\right)\right]$
C_X	$\beta_{2(X)}$	10	$\bar{Y}_{(j)10} = w\left[\bar{y}_{(j)}\left(\dfrac{\bar{X}C_X+\beta_{2(X)}}{\bar{x}_{(j)}C_X+\beta_{2(X)}}\right)^{\delta_{10}} + b_{rob(zb)}\left(\bar{X}-\bar{x}_{(j)}\right)\right]$
Q_1	Q_3	11	$\bar{Y}_{(j)11} = w\left[\bar{y}_{(j)}\left(\dfrac{\bar{X}Q_1+Q_3}{\bar{x}_{(j)}Q_1+Q_3}\right)^{\delta_{11}} + b_{rob(zb)}\left(\bar{X}-\bar{x}_{(j)}\right)\right]$

where

$$\delta_1 = \frac{\bar{X}}{\bar{X}+\beta_{2(X)}}, \delta_2 = \frac{\bar{X}}{\bar{X}+C_X}, \delta_3 = \frac{\bar{X}}{\bar{X}+Q_2}, \delta_4 = \frac{\bar{X}}{\bar{X}+Q_1}, \delta_5 = \frac{\bar{X}}{\bar{X}+Q_3}, \delta_6 = \frac{\bar{X}}{\bar{X}+QD},$$

$$\delta_7 = \frac{\bar{X}}{\bar{X}+IQR}, \delta_8 = \frac{\bar{X}}{\bar{X}+HL}, \delta_9 = \frac{\bar{X}\beta_{2(X)}}{\bar{X}\beta_{2(X)}+C_{(X)}}, \delta_{10} = \frac{\bar{X}C_{(X)}}{\bar{X}C_{(X)}+\beta_{2(X)}} \text{ and } \delta_{11} = \frac{\bar{X}Q_1}{\bar{X}Q_1+Q_3}.$$

Table 2. Characteristics of Population I.

$\bar{Y}=555.43$,	$\bar{X}=878.16$,	$S_Y^2=342021.5$,	$S_X^2=1176526$,	$C_Y^2=1.108$,
$C_X^2=1.525$,	$\rho_{YX}=0.803$,	$\beta_{2(X)}=1.921$,	$R=0.632$,	$Q_1=63.450$,
$Q_2=452.517$,	$Q_3=1177.151$,	$IQR=1113.7$,	$QD=556.850$,	$HL=634.438$.

Table 3. Characteristics of Population II.

$\bar{Y} = 9384.3$, $\bar{X} = 72409.9$, $S_Y^2 = 894457433$, $S_X^2 = 25842911895$, $C_Y^2 = 10.156$,
$C_X^2 = 4.928$, $\rho_{YX} = 0.901$, $\beta_{2(X)} = 26.136$, $R = 0.129$, $Q_1 = 6325$,
$Q_2 = 18305$, $Q_3 = 55650$, $IQR = 49325$, $QD = 24662.5$, $HL = 27061$.

Table 4. MSE of modified estimators $\bar{Y}_{zb}$ under SRS for Population I

Estimator	Robust (b_k)							
	m	$\bar{Y}_{lr}$	LAD	Huber-M	Huber-MM	Tukey-M	LTS	LMS
$\bar{Y}_{zb1}$	4	136090	122949	128998	108853	131142	108955	106378
	5	106505	96221	100955	85189	102633	85269	83252
	6	86782	78402	82260	69413	83627	69478	67835
$\bar{Y}_{zb2}$	4	135786	122665	128704	108590	130845	108692	106119
	5	106267	95998	100725	84984	102401	85063	83050
	6	86588	78221	82072	69246	83437	69311	67670
$\bar{Y}_{zb3}$	4	135617	122507	128542	108445	130681	108546	105976
	5	106135	95875	100598	84870	102272	84949	82938
	6	86480	78120	81968	69153	83333	69218	67579
$\bar{Y}_{zb4}$	4	135931	122801	128845	108716	130988	108818	106243
	5	106381	96105	100835	85082	102512	85162	83147
	6	86681	78308	82162	69326	83528	69391	67749
$\bar{Y}_{zb5}$	4	135707	122591	128628	108522	130769	108624	106052
	5	106205	95941	100666	84930	102341	85010	82997
	6	86538	78174	82024	69202	83388	69267	67628

Table 5. MSE of modifed estimators $\bar{Y}_{zb}$ under SRS for Population II

Estimator	Robust (b_k)							
	m	$\bar{Y}_{lr}$	LAD	Huber-M	Huber-MM	Tukey-M	LTS	LMS
$\bar{Y}_{zb1}$	4	144144154	72782597	84750515	43526666	51287837	43609158	48112749
	5	114034042	57579121	67047074	39434429	40574378	35917467	39315863
	6	93960634	47443471	55244780	31372938	33432071	30123007	33451273
$\bar{Y}_{zb2}$	4	144137783	72779032	84746345	43525535	51285760	45609888	48113602
	5	114029002	57576301	67043775	39433534	40572735	34918044	37316539
	6	93956481	47441147	55242062	33372200	33430718	29123482	31451830
	4	144069192	72740654	84701458	48513359	51263409	44617757	49122810

$\bar{Y}_{zb3}$	5	113974738	57545940	67008265	38423902	40555053	34924270	37323823
	6	93911769	47416130	55212803	31364264	33416148	29128612	33457831
$\bar{Y}_{zb4}$	4	144143910	72782461	84750355	43526623	51287757	46609186	47112781
	5	114033849	57579013	67046948	35434395	40574315	39917489	37315889
	6	93960475	47443382	55244676	31372910	33432020	31123025	34451294
$\bar{Y}_{zb5}$	4	144110379	72763698	84728411	43520668	51276828	47613029	45117278
	5	114007322	57564170	67029587	36429684	40565669	33920529	37319447
	6	93938617	47431151	55230371	30369028	33424896	29125530	34454226

Table 6. MSE of modified estimators $\bar{Y}_{zb}$ under RSS for Population I

Estimator	Robust (b_k)							
	m	$\bar{Y}_{lr}$	LAD	Huber-M	Huber-MM	Tukey-M	LTS	LMS
$\bar{Y}_{zb1}$	4	135290	122133	128188	108025	130335	108127	105550
	5	105800	95524	100254	84504	101930	84583	82569
	6	85450	77168	80980	68287	82332	68351	66728
$\bar{Y}_{zb2}$	4	134985	121847	127894	107762	130038	107864	105291
	5	105562	95301	100024	84298	101698	84378	82367
	6	85258	76989	80795	68122	82144	68186	66565
$\bar{Y}_{zb3}$	4	134817	121690	127731	107617	129873	107718	105147
	5	105431	95178	99897	84184	101570	84264	82255
	6	85152	76889	80692	68030	82041	68094	66475
$\bar{Y}_{zb4}$	4	135131	121984	128035	107888	130180	107990	105415
	5	105676	95408	100134	84397	101809	84476	82464
	6	85350	77075	80884	68201	82234	68265	66643
$\bar{Y}_{zb5}$	4	134906	121774	127818	107694	129961	107796	105224
	5	105501	95243	99964	84245	101638	84324	82315
	6	85209	76942	80747	68079	82096	68143	66523

Table 7. MSE of modified estimators $\bar{Y}_{zb}$ under RSS for Population II

Estimator	Robust (b_k)							
	m	$\bar{Y}_{lr}$	LAD	Huber-M	Huber-MM	Tukey-M	LTS	LMS
$\bar{Y}_{zb1}$	4	140333668	72640822	83996448	44860383	52236792	43001694	43476187
	5	104736148	52900390	61494776	32596862	37761986	32523011	32995193
	6	80335746	41385875	47762712	26906255	30400103	27931954	28373961
$\bar{Y}_{zb2}$	4	140327626	72637439	83992492	44859306	52234820	43002381	43476992
	5	104731480	52897842	61491771	32596151	37760561	32523706	32995983
	6	80332204	41383994	47760472	26905812	30399102	27932613	28374692
	4	140262574	72601021	83949905	44847720	52213591	43009792	43485671

$\bar{Y}_{zb3}$	5	104681219	52870404	61459416	32588507	37745230	32531204	33004491
	6	80294078	41363736	47736365	26901052	30388323	27939707	28382578
$\bar{Y}_{zb4}$	4	140333437	72640692	83996297	44860342	52236717	43001720	43476218
	5	104735970	52900293	61494661	32596835	37761931	32523037	32995223
	6	80335610	41385803	47762626	26906238	30400065	27931980	28373989
$\bar{Y}_{zb5}$	4	140301636	72622888	83975476	44854675	52226336	43005339	43480457
	5	104711399	52886878	61478843	32593095	37754434	32526699	32999380
	6	80316972	41375899	47750840	26903909	30394794	27935445	28377841

Table 8. MSE of modified estimators $\bar{Y}_{zb}$ under MRSS for Population I

Estimator	Robust (b_k)							
	m	$\bar{Y}_{lr}$	LAD	Huber-M	Huber-MM	Tukey-M	LTS	LMS
$\bar{Y}_{zb1}$	4	47250	43644	45308	39734	45897	39762	39042
	5	55990	51766	53717	47173	54406	47207	46359
	6	28372	26309	27261	24063	27598	24079	23665
$\bar{Y}_{zb2}$	4	47168	43565	45228	39661	45815	39689	38970
	5	55893	51674	53622	47087	54310	47120	46274
	6	28324	26263	27215	24021	27551	24037	23623
$\bar{Y}_{zb3}$	4	47122	43522	45183	39620	45770	39648	38930
	5	55839	51623	53570	47039	54258	47073	46227
	6	28298	26239	27190	23997	27526	24014	23600
$\bar{Y}_{zb4}$	4	47207	43603	45266	39696	45854	39724	39005
	5	55940	51718	53668	47128	54356	47162	46315
	6	28347	26285	27237	24041	27574	24057	23643
$\bar{Y}_{zb5}$	4	47146	43545	45207	39642	45794	39670	38951
	5	55868	51650	53598	47065	54286	47098	46252
	6	28312	26252	27203	24010	27539	24026	23612

Table 9. MSE of modified estimators $\bar{Y}_{zb}$ under MRSS for Population II

Estimator	Robust (b_k)							
	m	$\bar{Y}_{lr}$	LAD	Huber-M	Huber-MM	Tukey-M	LTS	LMS
$\bar{Y}_{zb1}$	4	80929254	46738041	53314400	24652012	32394093	12500463	11835758
	5	21742056	12223027	14040701	6200829	8293100	2994123	2823289
	6	15508155	8869358	10143015	4612154	6099867	2296118	2170555
$\bar{Y}_{zb2}$	4	80926548	46735986	53312205	24650521	32392383	12499403	11834728
	5	21741298	12222461	14040093	6200429	8292635	2993850	2823025
	6	15507628	8868961	10142590	4611869	6099538	2295918	2170360
	4	80897416	46713860	53288570	24634470	32373973	12487999	11823634

$\bar{Y}_{zb3}$	5	21733129	12216359	14033546	6196121	8287631	2990913	2820180
	6	15501957	8864679	10138009	4608791	6095993	2293762	2168266
$\bar{Y}_{zb4}$	4	80929150	46737963	53314316	24651955	32394028	12500422	11835719
	5	21742027	12223006	14040677	6200814	8293082	2994112	2823279
	6	15508135	8869343	10142999	4612143	6099854	2296111	2170547
$\bar{Y}_{zb5}$	4	80914909	46727146	53302762	24644108	32385028	12494847	11830295
	5	21738034	12220023	14037477	6198708	8290636	2992676	2821888
	6	15505363	8867250	10140760	4610639	6098121	2295057	2169523

Table 10. MSE of proposed estimators $\bar{Y}_{(j)k}$ under SRS for Population I

Estimator	Robust (b_k)							
	m	$\bar{Y}_{lr}$	LAD	Huber-M	Huber-MM	Tukey-M	LTS	LMS
$\bar{Y}_1$	4	83247	77019	79928	69944	80942	69997	68658
	5	73083	67423	70063	61042	70984	61089	59887
	6	65086	59909	62320	54106	63163	54149	53060
$\bar{Y}_2$	4	83315	77087	79996	70012	81010	70065	68725
	5	73160	67499	70139	61117	71060	61164	59961
	6	65166	59988	62399	54182	63242	54225	53135
$\bar{Y}_3$	4	46600	42024	44099	37474	44848	37505	36727
	5	38149	34297	36040	30490	36671	30516	29868
	6	32292	28969	30472	25700	31016	25722	25167
$\bar{Y}_4$	4	76925	70697	73597	63715	74612	63767	62458
	5	66298	60720	63313	54517	64222	54563	53406
	6	58237	53197	55536	47627	56358	47668	46632
$\bar{Y}_5$	4	32104	30203	30999	28975	31315	28980	28864
	5	26001	24431	25087	23419	25348	23423	23327
	6	21847	20511	21069	19650	21291	19654	19573
$\bar{Y}_6$	4	42423	38384	40199	34530	40861	34556	33920
	5	34582	31205	32720	28001	33274	28023	27496
	6	29187	26288	27587	23548	28063	23567	23117
$\bar{Y}_7$	4	32583	30536	31401	29135	31740	29141	28991
	5	26390	24700	25413	23545	25694	23551	23427
	6	22175	20736	21343	19755	21582	19759	19655
$\bar{Y}_8$	4	40032	36361	37999	32975	38602	32997	32456
	5	32564	29507	30869	26700	31371	26718	26271
	6	27444	24826	25992	22431	26422	22447	22066
$\bar{Y}_9$	4	83373	77146	80055	70071	81068	70123	68784
	5	73226	67565	70204	61181	71126	61228	60025
	6	65234	60055	62466	54248	63310	54291	53200
	4	83284	77055	79965	69981	80978	70033	68694

$\bar{Y}_{10}$	5	73125	67464	70103	61082	71025	61129	59927
	6	65129	59951	62362	54147	63205	54190	53100
$\bar{Y}_{11}$	4	81585	75345	78257	68281	79273	68333	67000
	5	71237	65588	68219	59242	69139	59289	58096
	6	63185	58035	60430	52284	61269	52327	51250

Table 11. MSE of proposed estimators $\bar{Y}_{(j)k}$ under SRS for Population II

Estimator	Robust (b_k)							
	m	$\bar{Y}_{lr}$	LAD	Huber-M	Huber-MM	Tukey-M	LTS	LMS
$\bar{Y}_1$	4	70000553	56577227	41495784	41856561	42671729	42803037	47606347
	5	63199378	48050489	33585216	33904377	34630008	34747480	39160040
	6	57457459	41656587	28183955	28465996	29109663	29214185	33205918
$\bar{Y}_2$	4	70001107	56586288	41475210	41833997	42677664	42809281	47617559
	5	63204959	48064652	33569352	33886757	34637868	34755651	39174107
	6	57466680	41673108	28171091	28451587	29117927	29222747	33220654
$\bar{Y}_3$	4	63436154	45482282	57106761	57932997	45277252	45105628	42053304
	5	54172438	36558971	47435010	48278887	36382420	36235010	33669020
	6	47089954	30545727	40448716	41258179	30392661	30265038	28069269
$\bar{Y}_4$	4	68898935	53096221	47504689	48255217	41723185	41743552	44335464
	5	60912568	43738791	38496855	39178482	33458442	33475559	35689567
	6	54382842	37103254	32321826	32931719	27918928	27933579	29846081
$\bar{Y}_5$	4	46345734	40882036	63516228	64100836	53465921	53260484	47491294
	5	38223451	33214384	55942372	56605284	45182923	44974833	39309062
	6	32475073	27942600	49868166	50563644	39042823	38841692	33478432
$\bar{Y}_6$	4	59869334	43085602	59935015	60697044	47934134	47726970	43061244
	5	50462860	34563673	50532653	51347255	38792258	38606016	34543096
	6	43468072	28846752	43536867	44343036	32541499	32375959	28829093
$\bar{Y}_7$	4	48389921	40939820	63410058	64013489	53061957	52852276	47045887
	5	39950669	33125445	55527425	56210859	44518176	44308126	38679071
	6	33960316	27793508	49260338	49974185	38262513	38061452	32788102
$\bar{Y}_8$	4	58533232	42502346	60685394	623959	48802626	48589646	43562529
	5	49145221	34094545	51427775	52227320	39631226	39436580	34995208
	6	42222734	28454846	44472105	45270893	33315525	33140780	29230314
$\bar{Y}_9$	4	70001155	56587095	41473374	41831984	42678194	42809839	47618559
	5	63205456	48065915	33567937	33885185	34638571	34756381	39175362
	6	57467503	41674582	28169944	28450302	29118666	29223512	33221970
$\bar{Y}_{10}$	4	70000890	56582674	41483424	41843007	42675294	42806787	47613084
	5	63202733	48058998	33575685	33893792	34634727	34752386	39168489
	6	57463000	41666511	28176226	28457339	29114624	29219324	33214768
	4	70000958	56583800	41480866	41840201	42676032	42807564	47614478

$\bar{Y}_{11}$	5	63203427	48060759	33573712	33891601	34635705	34753402	39170238
	6	57464146	41668565	28174627	28455548	29115652	29220390	33216600

Table 12. MSE of proposed estimators $\bar{Y}_{(j)k}$ under RSS for Population I

Estimator	Robust (b_k)							
	m	$\bar{Y}_{lr}$	LAD	Huber-M	Huber-MM	Tukey-M	LTS	LMS
$\bar{Y}_1$	4	65501	61511	63370	57020	64020	57054	56207
	5	54238	50993	52503	47356	53032	47383	46699
	6	46045	43344	44600	40322	45040	40344	39777
$\bar{Y}_2$	4	65561	61571	63430	57078	64080	57111	56264
	5	54295	51048	52559	47409	53088	47436	46752
	6	46097	43394	44651	40370	45091	40393	39824
$\bar{Y}_3$	4	40872	37901	39264	34771	39749	34794	34231
	5	33153	30794	31878	28292	32263	28310	27857
	6	27895	25948	26843	23869	27161	23884	23507
$\bar{Y}_4$	4	60386	56497	58305	52156	58939	52188	51375
	5	49563	46434	47888	42953	48398	42979	42328
	6	41878	39290	40492	36415	40914	36436	35898
$\bar{Y}_5$	4	31007	29062	29936	27196	30255	27209	26897
	5	25182	23561	24294	21956	24559	21967	21691
	6	21215	19826	20457	18419	20685	18428	18183
$\bar{Y}_6$	4	38261	35505	36765	32645	37216	32665	32157
	5	31046	28846	29854	26540	30213	26557	26143
	6	26135	24311	25148	22382	25446	22396	22047
$\bar{Y}_7$	4	31401	29401	30301	27468	30629	27481	27157
	5	25503	23844	24595	22192	24867	22203	21919
	6	21487	20069	20714	18628	20946	18638	18385
$\bar{Y}_8$	4	36720	34106	35298	31423	35726	31442	30969
	5	29804	27708	28667	25529	29009	25545	25157
	6	25098	23353	24153	21520	24438	21533	21203
$\bar{Y}_9$	4	65613	61622	63481	57128	64131	57161	56314
	5	54343	51096	52607	47456	53136	47482	46798
	6	46141	43437	44694	40412	45135	40434	39866
$\bar{Y}_{10}$	4	65533	61543	63402	57051	64052	57084	56238
	5	54268	51023	52533	47384	53062	47411	46727
	6	46073	43370	44627	40348	45067	40370	39802
$\bar{Y}_{11}$	4	64066	60099	61946	55644	62592	55677	54839
	5	52902	49686	51182	46089	51705	46115	45440
	6	51714	47801	49609	43539	50247	43570	42783

Table 13. MSE of proposed estimators $\bar{Y}_{(j)k}$ under RSS for Population II

Estimator	Robust (b_k)							
	m	$\bar{Y}_{lr}$	LAD	Huber-M	Huber-MM	Tukey-M	LTS	LMS
$\bar{Y}_1$	4	67046650	53167343	56622954	40735397	44810202	44721181	40312030
	5	50441092	34522678	32280138	32842496	27708105	27701852	28803805
	6	49837436	32230672	25575609	26109201	22402839	22436681	24761150
$\bar{Y}_2$	4	67050102	53178658	56632692	40740255	44821276	44732194	40292514
	5	50459299	34541872	32259934	32821327	27706921	27701057	28813253
	6	49855413	32253481	25557602	26089877	22407586	22441842	24776449
$\bar{Y}_3$	4	59143965	42860789	54495939	55270069	43766430	43611496	40686931
	5	37276853	27712244	45082644	45809698	34645422	34471644	30261591
	6	35818270	22453573	39459928	40290687	27946931	27766403	23667255
$\bar{Y}_4$	4	65269536	49464914	45857749	46543200	40211304	40212345	41992972
	5	45471178	30377337	37618651	38336379	29262342	29166397	27705222
	6	44782188	26954044	30850901	31619783	22764062	22693509	22225029
$\bar{Y}_5$	4	43247879	39359410	60891088	61478251	51122083	50927977	45540983
	5	27455404	30525453	51723622	52310390	42410045	42229857	37185123
	6	24024605	25002796	48311293	48979026	37759838	37557704	31966636
$\bar{Y}_6$	4	55576066	41000213	57134477	57865821	46080351	45894486	41653209
	5	34212737	27910217	47440498	48135974	37026260	36841664	32095259
	6	32249665	22218499	42424790	43224625	30717528	30517762	25583765
$\bar{Y}_7$	4	45036517	39397948	60712131	61316310	50708577	50511874	45126961
	5	28197234	30106894	51354853	51956113	41851850	41669276	36594764
	6	24932647	24455507	47743887	48430242	36960429	36755741	31147368
$\bar{Y}_8$	4	54287203	40558458	57855577	58569771	46836841	46645089	42092850
	5	33272947	28096223	48120270	48803719	37775814	37589550	32728613
	6	31130553	22320290	43304143	44090634	31630642	31427238	26300722
$\bar{Y}_9$	4	67050409	53179667	40290772	40649753	40635551	40740689	44822264
	5	50460924	34543587	32258131	32819437	27706817	27700988	28814099
	6	49857017	32255518	25555995	26088153	22408012	22442305	24777817
$\bar{Y}_{10}$	4	67048727	53174142	40300306	40660109	40633334	40738313	44816854
	5	50452028	34534203	32268003	32829781	27707388	27701369	28809475
	6	49848234	32244369	25564791	26097592	22405684	22439774	24770334
$\bar{Y}_{11}$	4	67049155	53175549	40297880	40657473	40633898	40738917	44818231
	5	50454292	34536591	32265490	32827149	27707242	27701271	28810650
	6	49850470	32247206	25562552	26095189	22406275	22440417	24772237

Table 14. MSE of proposed estimators $\bar{Y}_{(j)k}$ under MRSS for Population I

	Robust (b_k)

Estimator	m	$\bar{Y}_{lr}$	LAD	Huber-M	Huber-MM	Tukey-M	LTS	LMS
$\bar{Y}_1$	4	33249	25091	24689	22128	25239	22147	21018
	5	25198	19458	18176	16596	18147	16146	16097
	6	19175	14568	15789	13148	15189	13500	13489
$\bar{Y}_2$	4	31129	30014	29145	34002	31166	29028	28163
	5	26147	29156	25569	30015	28189	24589	22183
	6	20026	19149	19123	22486	24132	20147	19258
$\bar{Y}_3$	4	37414	34083	35611	30579	36155	30604	29975
	5	25806	23810	24723	21728	25049	21742	21370
	6	22454	20596	21448	18643	21751	18657	18306
$\bar{Y}_4$	4	36089	34033	34014	32122	30037	32128	31012
	5	23139	25144	21089	19188	22369	27123	25239
	6	19963	18169	18796	17796	19156	20589	21177
$\bar{Y}_5$	4	26248	24093	25060	22047	25413	22060	21722
	5	19088	17832	18395	16637	18601	16645	16447
	6	16110	14883	15436	13685	15637	13694	13490
$\bar{Y}_6$	4	34446	31357	32769	28160	33274	28183	27616
	5	23973	22141	22976	20257	23276	20270	19938
	6	20739	19022	19807	17238	20087	17251	16933
$\bar{Y}_7$	4	26689	24470	25467	22345	25831	22360	22006
	5	19345	18051	18632	16812	18844	16820	16613
	6	16357	15099	15667	13863	15873	13871	13661
$\bar{Y}_8$	4	32696	29769	31103	26775	31581	26796	26270
	5	22909	21183	21968	19426	22251	19439	19131
	6	19740	18113	18855	16438	19121	16450	16153
$\bar{Y}_9$	4	36259	34489	34147	32258	30239	32135	31189
	5	23147	25236	21289	19741	22245	27879	25367
	6	19963	18147	18364	17387	19147	20693	21125
$\bar{Y}_{10}$	4	26589	24169	25159	22193	25896	22258	21148
	5	19144	17333	18487	16879	18358	16796	16456
	6	16589	14589	15369	13337	15347	13148	13896
$\bar{Y}_{11}$	4	34356	31766	32125	28896	33144	28359	27125
	5	23589	22558	22148	20145	23789	20225	19478
	6	20877	19012	19239	17789	20963	17089	16896

Table 15. MSE of proposed estimators $\bar{Y}_{(j)k}$ under MRSS for Population II

Estimator	Robust (b_k)							
	m	$\bar{Y}_{lr}$	LAD	Huber-M	Huber-MM	Tukey-M	LTS	LMS
$\bar{Y}_1$	4	14135216	7874877	1645041	1547157	3661554	3708729	5178950
	5	6263979	3596505	862982	811402	1806344	1827074	2460948

	6	2296544	1459878	533438	513268	873681	880771	1093102
$\bar{Y}_2$	4	14144036	7882398	1648136	1550085	3666875	3714087	5185280
	5	6268151	3599852	864684	813050	1808795	1829538	2463778
	6	2297896	1460976	534115	513932	874541	881634	1094060
$\bar{Y}_3$	4	9254989	4065203	753859	757016	1370430	1393467	2214870
	5	4101730	1947831	190033	169233	694978	707848	1124081
	6	1606335	914255	224482	211379	462075	467282	626688
$\bar{Y}_4$	4	12063077	6166770	1059169	1002078	2522441	2560288	3782866
	5	5313918	2849519	512537	473651	1276255	1293784	1839357
	6	1991201	1214617	387063	369855	684338	690641	880865
$\bar{Y}_5$	4	5359803	1714536	1524750	1621073	761991	759635	873870
	5	2492589	879931	69735	77482	159711	164990	368799
	6	1092531	534584	60557	53831	205124	208617	319726
$\bar{Y}_6$	4	8203148	3354306	805489	832657	1074945	1091738	1741427
	5	3662801	1639134	115955	102340	517214	528200	891928
	6	1467234	808663	173121	161627	387282	392058	539328
$\bar{Y}_7$	4	5740817	1905053	1369825	1456483	754397	754828	947829
	5	2649435	975122	60445	65105	195685	201799	429075
	6	1143365	570724	73233	65816	227892	231571	347935
$\bar{Y}_8$	4	7864873	3136088	843970	879030	998098	1012794	1604514
	5	3522781	1543046	98009	86769	464874	475234	821461
	6	1422784	775311	157681	146717	364110	368744	512005
$\bar{Y}_9$	4	14144824	7883070	1648413	1550347	3667351	3714566	5185845
	5	6268524	3600151	864836	813197	1809014	1829759	2464031
	6	2298017	1461074	534176	513992	874618	881712	1094146
$\bar{Y}_{10}$	4	14140513	7879393	1646899	1548915	3664749	3711946	5182751
	5	6266484	3598514	864004	812391	1807816	1828553	2462647
	6	2297356	1460537	533844	513667	874197	881289	1093677
$\bar{Y}_{11}$	4	14141610	7880328	1647284	1549279	3665411	3712612	5183538
	5	6267003	3598931	864215	812596	1808120	1828860	2462999
	6	2297524	1460674	533929	513749	874304	881397	1093797

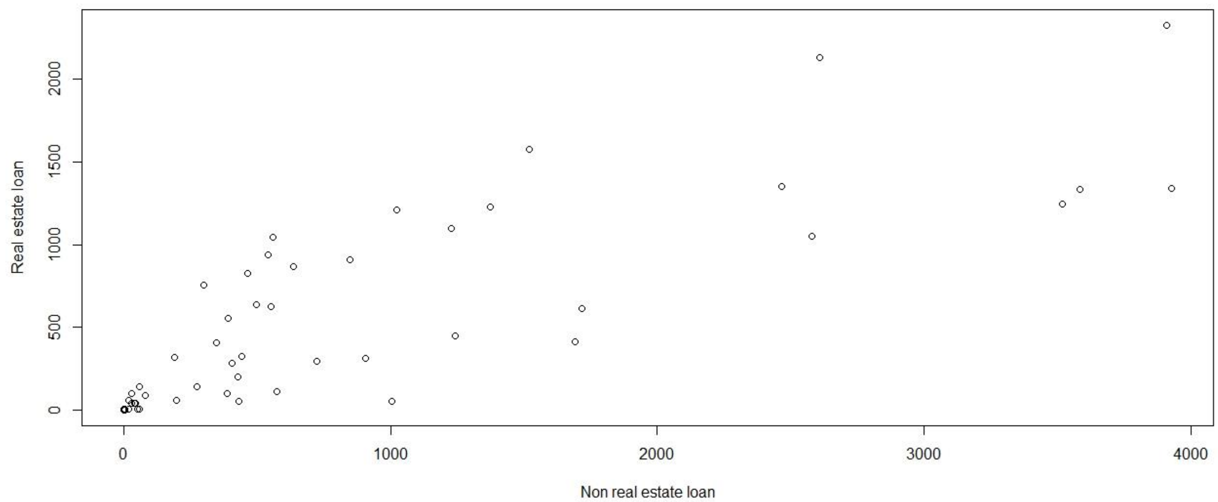

Figure.1. Scatter plot of non-real estate loan and real estate loan

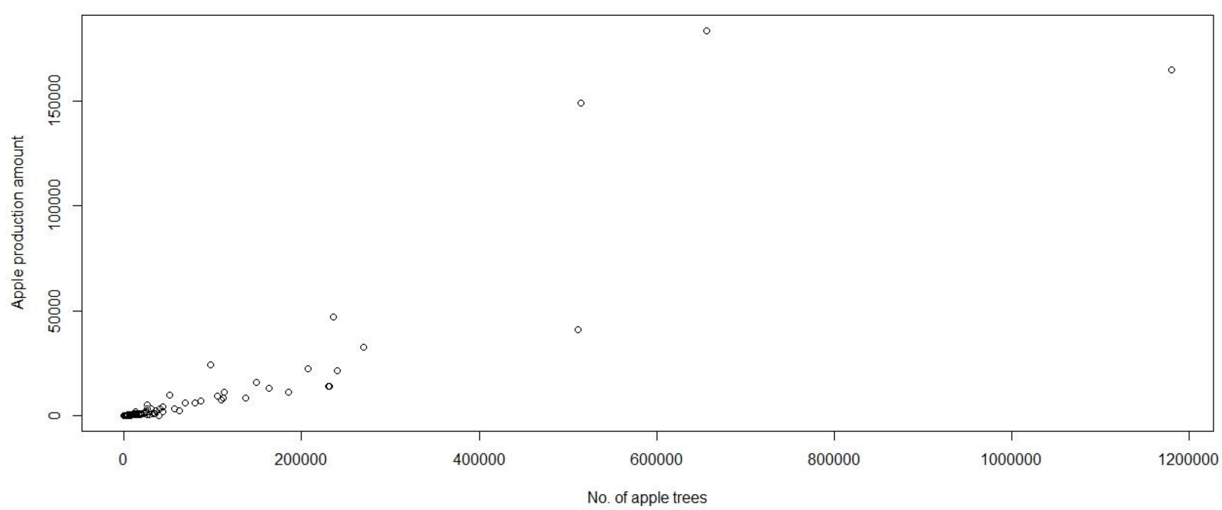

Figure.2. Scatter plot of number of apple trees and apple production amount