



A novel memristive chaotic system with hidden attractors and a line of equilibria

Zhen Wang ^{a,b}, Sridevi Sriram ^c, Huaigu Tian ^b, Karthikeyan Rajagopal ^{d, e, *}

a. School of Mathematics and Computer Science, Yan'an University, Yan'an, China.

b. School of Computer Science, Xi'an Key Laboratory of Human-Machine Integration and Control Technology for Intelligent Rehabilitation, Xijing University, Xi'an, China.

c. Centre for Computational Biology, Chennai Institute of Technology, Tamil Nadu, India.

d. Center for Research, Easwari Engineering College, Chennai, India.

e. Center for Research, SRM TRP Engineering College, Trichy, India.

* Corresponding author: karthikeyan.rajagopal@citchennai.net (K. Rajagopal)

Received 13 November 2022; Received in revised form 13 February 2023; Accepted 15 August 2023

Keywords

Memristor;
Chaos;
Hidden attractor;
Line of equilibria;
Dissipative system.

Abstract

This paper introduces a newly designed four-dimensional memristive chaotic system. The novel oscillator is chaotic as the system's dynamics have one positive Lyapunov Exponent (LE). Also, based on the results of the equilibrium points analysis, it is shown that the oscillator has a line of equilibria, so the attractors of this system are hidden. Moreover, the study of energy dissipation of this system, power spectrum, and Poincaré sections are conducted to investigate the system's dynamics. The complex features of this system are investigated with the aid of bifurcation diagrams, LE spectra, approximate entropy, and basins of attraction.

1. Introduction

Chaotic behavior is one of the characteristics of complex and interactive systems [1]. Chaos theory is a branch of mathematics and physics investigating the behavior of dynamical systems whose temporal dynamics, despite not being unstable, are susceptible to initial conditions, making predicting their future impossible [2]. Dynamical systems with these properties are called chaotic systems. Many systems in various fields show this behavior, and in recent decades, much attention has been focused on studying and investigating chaotic systems and signals [3]. Chaos can be considered a nonlinear and deterministic process that is not random but seems random in its time series [4]. The main reason for the output fluctuations, which seem unpredictable in the time series, is related to the internal mechanisms of the nonlinear data generator system and should not be misperceived as external shocks to the system, such as noise and changes in parameters [5].

Memristive elements, known as memory elements, including memristor, memcapacitor, and meminductor, can

be easily implemented, and their behavior can be simulated with electronic circuits [6]. A memory resistor, or memristor, is an element with a function similar to the resistor, which is known as the fourth element of electrical circuits and is made in nanometer dimensions [7]. The amount of this resistance depends on the amount of voltage and voltage polarity [8]. The hysteretic current-voltage curve in the memristor allows this element to act as a non-volatile resistive memory and remember information until a voltage with a different value and polarity is applied to it, even a year later [9]. In 1971, Chua proved the basis of this technology with mathematical and physical formulas [10]; but it took 37 years for its first technology to be practically registered [11]. Memristors can replace many transistors in some circuits and occupy less space [12]. A memristor has a voltage-dependent resistance, unlike an electrical resistor with a constant value [13]. The memristor material should be a reversible resistor with voltage change [14]. Ideal memristors are symmetrical, but the actual memristor's current-voltage characteristic is

To cite this article:

Z. Wang, S. Sriram, H. Tian, K. Rajagopal "A novel memristive chaotic system with hidden attractors and a line of equilibria", *Scientia Iranica* (2025) 32(17): 7307. <https://doi.org/10.24200/sci.2023.61440.7307>

usually skewed [15]. Memristive components are nonlinear and can exhibit chaotic behavior in electronic circuits [16]. In recent years, many researchers have worked on designing and implementing chaotic oscillators based on memory elements [17-19]. Considering that the memristor has many industrial and exciting applications, the application of the memristor in a chaotic system is investigated.

From the computational standpoint, it is natural to propose the classification of attractors based on the basin of attraction: The attractor is called a hidden attractor whose basin of attraction does not collide with the small neighborhoods of the equilibrium points; otherwise, it is called a self-excited attractor [20]. The self-excited attractor can be localized numerically by the standard computational methods [21,22]. On the other hand, the numerical calculation of hidden attractors requires the development of unique analytical-numerical methods [23,24]. For example, hidden chaotic attractors are attractors in systems with no equilibrium point [25], a stable equilibrium point [26], or a line of equilibrium points [25,27].

A novel memristive chaotic system with hidden attractors is proposed in this paper. The rest of the article is divided into three more sections. The next part is focused on introducing the newly designed system and examining its properties. The third part is devoted to the outcomes of further numerical simulations of this system to look at it in more depth and discover more exciting behaviors. In the end, the paper is concluded in the last section.

2. The proposed memristive system

The mathematical description of the newly designed system is given by System (1):

$$\begin{aligned}\dot{x} &= z, \\ \dot{y} &= w, \\ \dot{z} &= -2.1x - 0.1z - w^2 + 0.11xz + 0.5zw, \\ \dot{w} &= z - w + kwY(y),\end{aligned}\quad (1)$$

where $Y(y) = 1 + 0.24y^2 - 0.0016y^4$ is the memristor function introduced by Pham et al. in 2014 [28], and k is the system parameter. This memristor function's complete analysis and characteristics were conducted before, so this

part could be skipped. The rest of the paper is dedicated to studying the properties of the proposed system.

2.1. Equilibrium points and stability analysis

For many years, the equilibrium points were used in finding chaotic attractors [29]. Chaotic attractors were found in dynamical systems with unstable equilibrium points (saddle points). However, later, some dynamical systems did not have an equilibrium point at all [30] or had a stable equilibrium point [31] and a chaotic attractor. As the first step in studying a dynamical system, calculating its equilibrium points and investigating their stability is essential. The equilibrium points of dynamical systems can be obtained by setting all velocity variables to zero. In other words, all the right-hand side expressions should be set to zero in the differential equations. By applying this procedure and solving the equations, the equilibrium points of System (1) are as:

$$\{(x, y, z, w) | x = z = w = 0, y \in \mathbb{R}\},$$

which indicates that System (1) has a line of equilibria located on the y -axis. Thus, the attractors of this system belong to the group of hidden attractors. It is worth investigating whether this line of equilibria consists of stable or unstable points. The Jacobian matrix of System (1) should be formed to examine this issue. The Eigenvalues of this system at the equilibrium points indicate their stability. These Eigenvalues could be obtained by solving

$$|\lambda I - J|_{Equilibrium} = 0,$$

leading to the characteristic equation.

The equilibrium point is stable if the real part of every Eigenvalue is negative. In contrast, the equilibrium point is unstable if just one of them is positive. A zero eigenvalue without a positive one requires in-depth investigation. Suppose the largest Eigenvalue of the Jacobian matrix is zero or has a zero real part. In that case, the equilibrium point is non-hyperbolic, and its stability cannot be determined from the sign of the Eigenvalues of the Jacobian matrix. Because the Jacobian matrix of this system has a zero column at the equilibrium points, it is singular and thus has a zero Eigenvalue. This procedure is shown in Eqs. (2) to (5), step by step, as shown in Box I.

$$J = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2.1 + 0.11z & 0 & -0.1 + 0.11x + 0.5w & -2w + 0.5z \\ 0 & kw(0.48y - 0.0064y^3) & 1 & -1 + k(1 + 0.24y^2 - 0.0016y^4) \end{bmatrix}, \quad (2)$$

$$J|_{Equilibrium} = J|_{x=z=w=0} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2.1 & 0 & -0.1 & 0 \\ 0 & 0 & 1 & -1 + k(1 + 0.24y^2 - 0.0016y^4) \end{bmatrix}, \quad (3)$$

$$|\lambda I - J|_{Equilibrium} = 0 \rightarrow \lambda(0.0016\lambda^2 + 0.00016\lambda + 0.00336)(0.00016ky^4 - 0.024ky^2 - 0.1k + 0.1\lambda + 0.1) = 0 \rightarrow \quad (4)$$

$$\lambda_1 = 0, \lambda_2 = -0.05 + \frac{\sqrt{839}i}{20}, \lambda_3 = -0.05 - \frac{\sqrt{839}i}{20}, \lambda_4 = -0.0016y^4 + 0.24ky^2 + k - 1. \quad (5)$$

According to the Eigenvalues achieved in Eq. (5), System (1) has a zero, a pair of complex conjugate Eigenvalues with a negative real part, and one more eigenvalue whose value depends on the state variable y and the system parameter k . To analyze λ_4 in detail, the extrema of the fourth-order function are obtained. If the maximum values of λ_4 are negative, then the negativity of λ_4 for all values of y is guaranteed. To find the extrema of λ_4 , the derivative of it with respect to y should be computed. Setting the derivative expression equal to zero and substituting the resulting answers into λ_4 , gives the extremum points that could be minimum or maximum. The extremums of λ_4 are obtained in Eqs. (6) and (7). λ_4 has three extremums, but the one corresponding to zero is a local minimum, so we are not interested in it.

$$\frac{d\lambda_4}{dy} = -0.0064y^3 + 0.48ky = y(-0.0064y^2 + 0.48k) = 0 \rightarrow$$

$$y_1 = 0, y_{2,3} = \pm 5\sqrt{3k}, \quad (6)$$

$$\max \lambda_4 = \lambda_4|_{y=\pm 5\sqrt{3k}} = -0.0016 \times 5625k^2$$

$$+ 0.24 \times 75k^2 + k - 1 = 9k^2 + k - 1. \quad (7)$$

According to Eq. (7), the maximum value of λ_4 is a quadratic function of k . When the value of k is between the two roots of Eq. (7), the sign of the resulting value is negative (since the sign of the coefficient of k^2 is positive). Solving this equation leads to an interval such that, if k falls in it, the maximum value of λ_4 is negative. However, because of the zero Eigenvalue, the stability of the equilibria needs more investigations carried out in future parts. The interval of k to have a negative value is found in Eq. (8):

$$9k^2 + k - 1 < 0 \rightarrow \frac{-1 - \sqrt{1+36}}{18} < k < \frac{-1 + \sqrt{1+36}}{18}$$

$$\rightarrow -0.393 < k < 0.282. \quad (8)$$

Assume $k = 0.0001$, which is in the interval. To better show this result, the variation of λ_4 concerning y is shown in Figure 1. It is perceived from Figure 1 that for all values of y , λ_4 is negative.

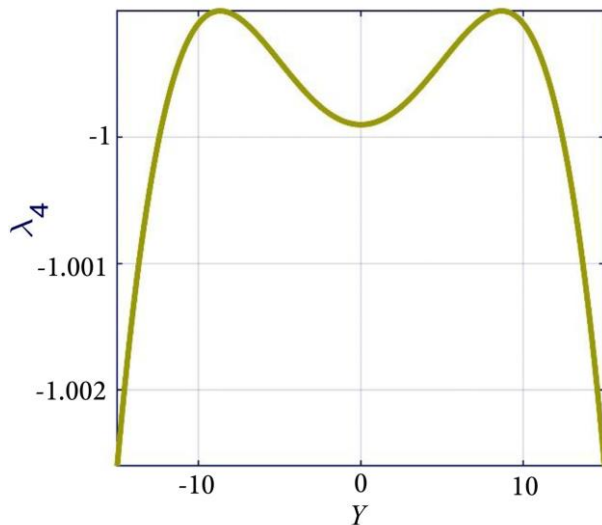


Figure 1. The variation of λ_4 concerning y with $k = 0.0001$. If k is selected from the desired interval, the maximum value of λ_4 is negative.

2.2. Time series and state space

Chaos is a bounded, irregular behavior of a system. In chaotic behavior, the attractor is bounded and convergent but not periodic. The time series and the attractor of System (1) with $k=0.0001$ projected on the X - Y , Y - Z , and Z - W planes after removing the transient parts are depicted in Figure 2(a)-(d), respectively. By observing Figure 2, System (1)'s chaotic behavior in time series and state space can be recognized. The Lyapunov Exponents (LEs) of this chaotic attractor are $LE_1 = 0.1428, LE_2 = LE_3 = 0, LE_4 = -3.7978$. Since, with $k = 0.0001$, System (1) has one positive LE, it has chaotic behavior.

2.3. Energy dissipating

A simple analysis of any dynamical system determines how the system treats the energy. This feature can be determined by the sign of the system's divergence, the Jacobian matrix's trace [32]. If the divergence is negative, the system is dissipative; otherwise, the system is classified as conservative if the divergence is zero. If a dynamical system is bounded and has a positive LE, then the system is chaotic. Chaotic behavior can be observed in conservative and dissipative systems [33]. If the sum of the absolute values of the negative exponents is greater than the sum of the positive exponents, or in other words, if the attraction is greater than the repulsion, the chaotic system is dissipative. However, suppose the sum of the absolute values of the negative exponents is equal to the sum of the positive exponents. In that case, the sum of its LEs becomes zero, and the chaotic system is conservative (chaotic sea). Sometimes the divergence computation does not lead to a number, but it depends on the system's state variables. In this case, its average (DC value) determines the divergence. After this substitution, the divergence is transformed into a number that indicates whether the chaotic system is conservative or dissipative. The divergence of System (1) is calculated in Eq. (9):

$$\nabla = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} = -0.1 + 0.11x + 0.5w - 1$$

$$+ k(1 + 0.24y^2 - 0.0016y^4) = -1.1 + k + 0.11x + 0.5w$$

$$+ 0.24ky^2 - 0.0016ky^4 \text{ State Dependent}$$

$$\nabla = -1.1 + k + 0.11DC(x) + 0.5DC(w)$$

$$+ 0.24kDC(y^2) - 0.0016kDC(y^4) \quad DC(w) = 0$$

$$\nabla = -1.1 + k + 0.11DC(x) + 0.24kDC(y^2)$$

$$- 0.0016kDC(y^4). \quad (9)$$

The average value of all velocity variables must be zero to avoid resulting in unbounded behavior. In other words, the DC value of all right-hand side expressions in the differential equations describing the system's bounded dynamics is zero. Thus, regarding System (1), the DC value of the z and w state variables is zero. However, the other expressions in Eq. (9) should be obtained by simulating the system for a long time and calculating the DC value of x , y^2 , and y^4 , and replacing them in Eq. (9). Finally, the divergence of the system is achieved numerically. Eq. (9) also depends on the parameter k .

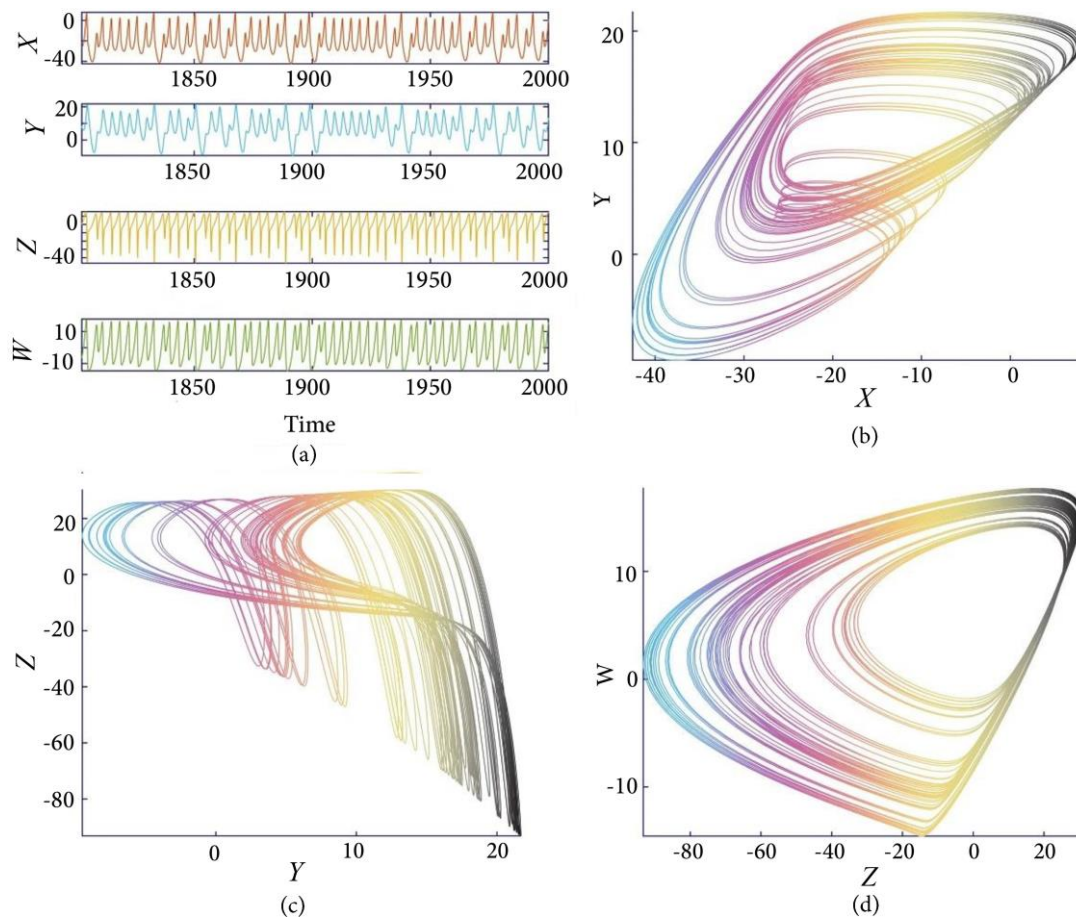


Figure 2. Time series and projection of attractors of System (1) with $k = 0.0001$ and $(x_0, y_0, z_0, w_0) = (-28, 0, 0, 0)$: (a) Time series, (b) projection of attractor on X-Y, (c) Y-Z, (d) Z-W planes. System (1) can exhibit chaotic time series and bounded but non-periodic cycles (strange attractors). It should be noted that different colors in the attractors do not have any special meaning, and such a colormap is used for better illustration.

Here, this parameter is fixed at 0.0001, and the numerical divergence values of System (1) are acquired over time. The result shown in Figure 3 means that as time passes, the value of the divergence of System (1) becomes negative, so System (1) is dissipative.

2.4. Power spectrum

Periodic signals have peaks in a fundamental component and harmonics in frequency space [34]. Also, the peak of quasiperiodic signals is located in linear combinations of two or more related frequencies [34]. Chaotic dynamics have wideband components in the spectrum [35]. This feature is used as a criterion to identify a dynamic as chaos [34,35]. The power spectrum of System (1) with $k = 0.0001$ is demonstrated in Figure 4. Since System (1) is four-dimensional, the power spectrum is calculated along four distinct variables and plotted in a different color. When a system is in its chaotic regime, the power spectrum of all system variables is broadband. Usually, the significant frequency interval in which the power spectrum components are prominent and cannot be neglected is similar for all system variables. However, in some unique systems like the well-known Rössler system, the significant power spectrum interval of the system variables is not the same and can be distributed in different ranges [36]. According to Figure 4, the power spectrum of System (1) with this parameter is wideband and does not exhibit any peaks in specific frequencies. This spectrum is consistent with the results obtained in Figure 3 and confirms the chaoticity of this newly designed system.

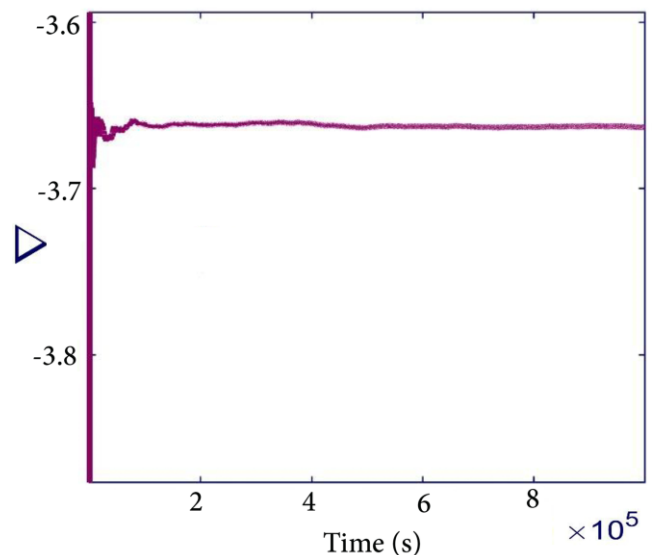


Figure 3. Numerical calculation of System (1)'s divergence over time with $k = 0.0001$ and $(x_0, y_0, z_0, w_0) = (-28, 0, 0, 0)$. The divergence results in a negative value, meaning System (1) is dissipative.

2.5. Poincaré section

Another well-known tool to investigate dynamical systems is the Poincaré section. As a result of intersecting a plane or hyperplane with the attractor in the N -dimensional state space, the Poincaré section is obtained with a lower dimension.

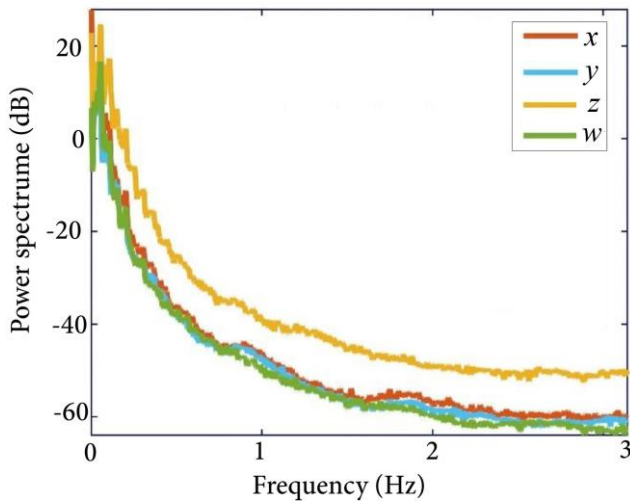


Figure 4. Power spectrum of System (1) with $k = 0.0001$ and $(x_0, y_0, z_0, w_0) = (-28, 0, 0, 0)$. System (1) is a chaotic system due to the wideband spectrum.

Dimension reduction by this method sometimes enables visualizing the attractors that could not be visualized in the N -dimensional state space. Regarding the fact that System (1) is a four-dimensional system and the most visualizable dimension is three, the Poincaré section of the attractors of System (1) on the $X=0$, $Y=0$, $Z=0$, and $W=0$ hyperplanes with $k = 0.0001$ are presented in Figure 5 (a) and (d), respectively. The gray diagram depicts the three-dimensional attractors, and the colorful dots demonstrate the system trajectory's intersection with the previously mentioned hyperplanes. In plotting the intersection points, the transition from the negative side of the desired hyperplane to the positive side is considered. The pattern of the dots shows that the attractor of System (1) with the selected parameter is chaotic.

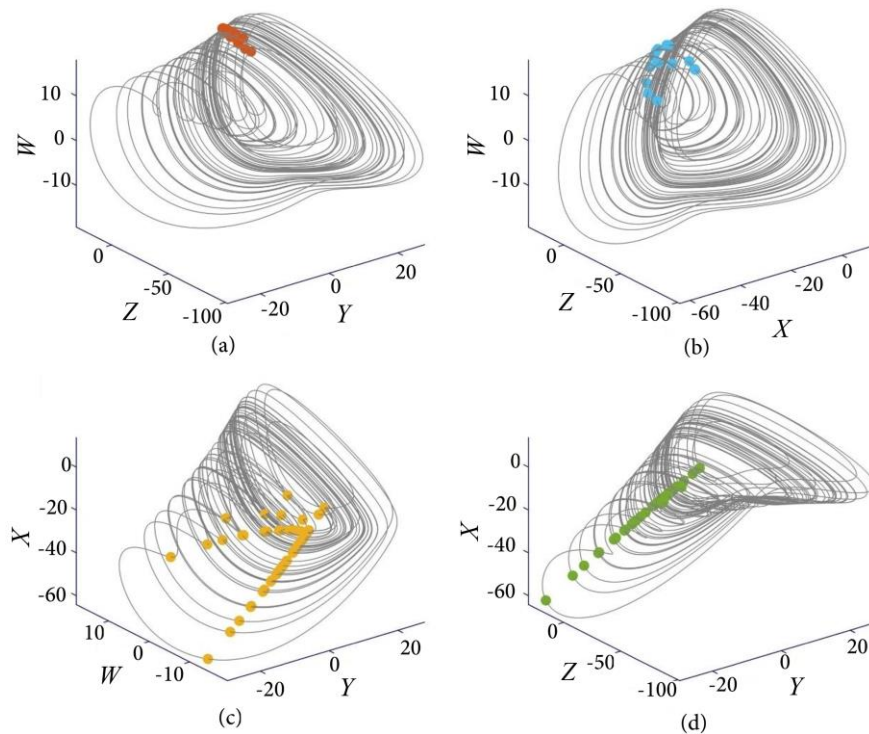


Figure 5. Poincaré sections and attractors of System (1) with $k = 0.0001$ and $(x_0, y_0, z_0, w_0) = (-28, 0, 0, 0)$ on: (a) $X=0$; (b) $Y=0$; (c) $Z=0$; and (d) $W=0$ hyperplanes. The gray diagram and colored dots correspond to the system trajectory and the Poincaré section.

3. Dynamical analysis of the proposed system

In this section, a further and more in-depth analysis of System (1) with varying system parameter k and the initial conditions is carried out.

3.1. The effect of parameter k on the system dynamics

In dynamical equations, the solutions of the equations are strongly dependent on the value of the parameters. Studying the equations' behavior is possible by changing the parameters' values. The parameter whose effect of changing is examined is called the control parameter (parameter k in System (1)). The bifurcation diagram of System (1), by changing k using the final solution of the equations in the long term ($y_{\max}$) and with constant initial conditions, is plotted in Figure 6(a). Periodic and chaotic behavior and periodic windows in the middle of chaos are observed in System (1). By decreasing parameter k , the period-doubling route to chaos is followed. If the bifurcation parameter changes, the geometrical structure of the phase space also changes; then a critical point occurs. These critical points are called bifurcation points; for instance, in $k = 0.0210$, $k = 0.0105$, and $k = 0.0083$ in Figure 6(a).

Moreover, another applicable analytical criterion that can describe chaos is employed. This criterion is called the LE. The LEs describe how quickly a tiny gap between two initially close states grows. There are various methods of determining LEs. In this work, the Wolf method with a runtime of 10000 seconds is applied [37,38]. The LE spectra of System (1) by varying parameter k are shown in Figure 6(b). System (1) always has a negative and relatively large LE (LE_4) that assures the system's boundedness. Also, this system always has a zero LE. A system can be considered chaotic in the case of one positive LE.

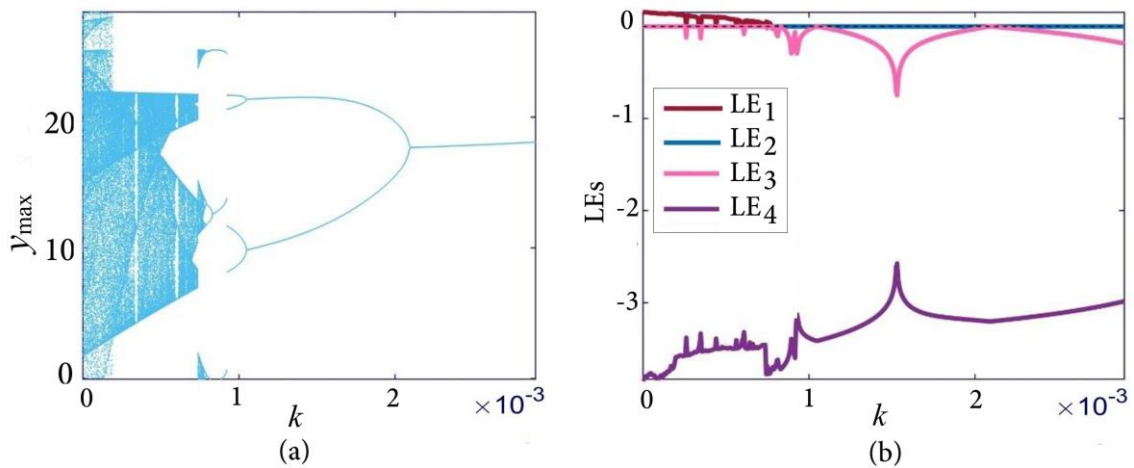


Figure 6. (a) Bifurcation diagram ($y_{\max}$) and (b) Lyapunov Exponents (LEs) spectra of System (1) while changing the parameter k with $(x_0, y_0, z_0, w_0) = (-28, 0, 0, 0)$. Periodic solutions approach chaotic solutions along the period-halving route.

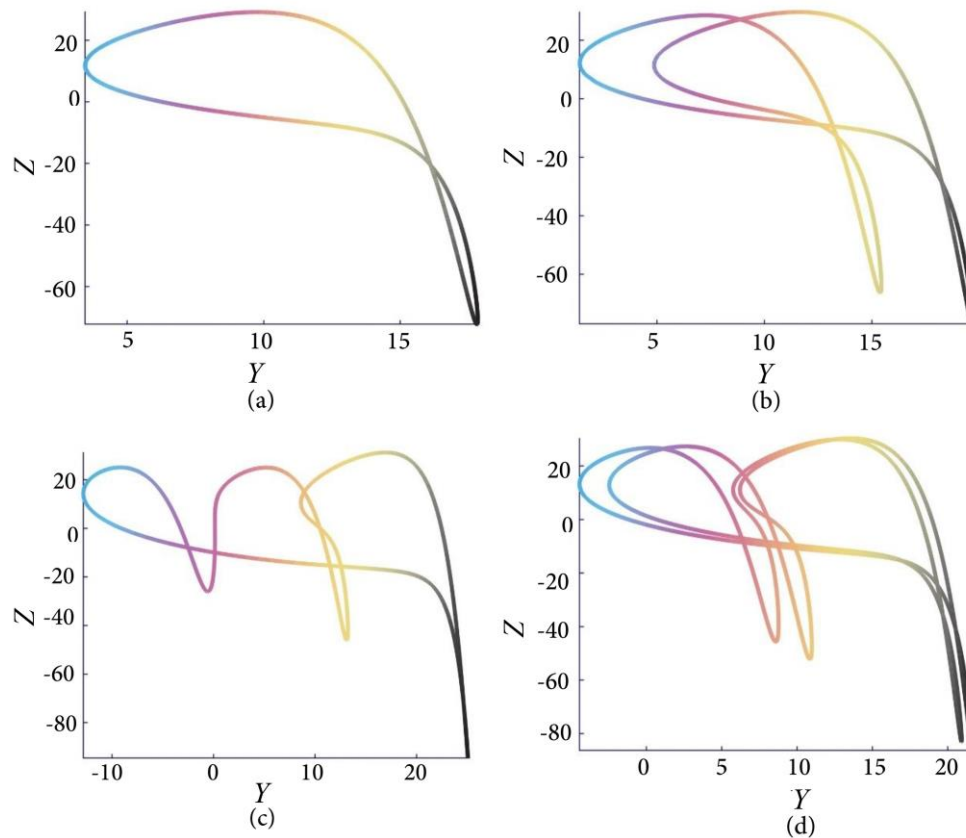


Figure 7. The projection of limit cycles of System (1) on the $Y-Z$ plane with $(x_0, y_0, z_0, w_0) = (-28, 0, 0, 0)$ and (a) $k = 0.0025$, (b) $k = 0.002$, (c) $k = 0.0009$, and (d) $k = 0.001$. System (1) can exhibit limit cycles with various periods.

Previously, a strange attractor in System (1) was confirmed. According to this system's bifurcation diagram, it can also exhibit periodic behaviors. By choosing the system parameter as $k=0.0025$, $k=0.002$, $k=0.0009$, and $k=0.001$, period one, two, three, and four attractors were achieved, respectively. The projection of these periodic attractors on the $Y-Z$ plane is illustrated in Figure 7.

One of the time analysis methods for measuring the complexity of signals is to use approximate entropy. Approximate entropy is a statistical measure that quantifies order in a time series. The disorder of the signals in the chaotic dynamics increases, and its amount equals the

approximate entropy. Approximate entropy is a parameter used to characterize the irregularity of the signal. This entropy's growth shows the signal's irregularity and complexity during parameter change. To measure this entropy, $N-m+1$ delayed subsets of the signal are extracted and named $Y_{1,N-m+1}$, where m is the embedding dimension, and N is the number of samples in the signal. In the next step, each Y_k is selected. The number of others Y_i s like it is calculated as N_i in Eq. (10). The similarity criterion is that the difference between attributes of the two subsets must be less than R , the similarity radius, and 1 is the indicator function. Then, as in Eq. (11), Φ_m is calculated based on the

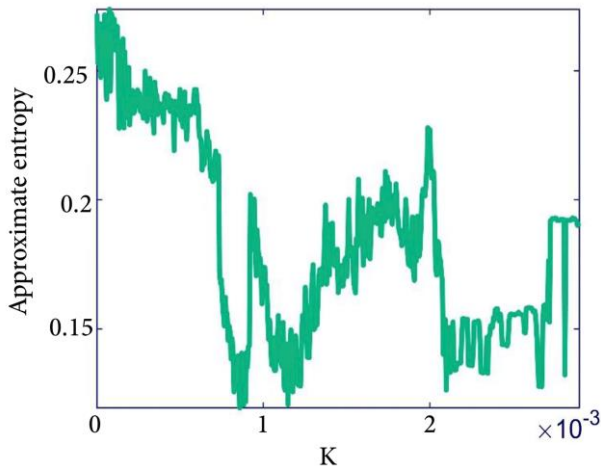


Figure 8. Approximate entropy of System (1) while changing the parameter k with $(x_0, y_0, z_0, w_0) = (-28, 0, 0, 0)$. Approximate entropy in chaotic zones is more than that of periodic ones.

N_i s found in the previous step, and in the end, the approximate entropy is obtained by Eq. (12) [39-41].

$$N_i = \sum_{i=1, i \neq k}^{N-m+1} 1(\|Y_i - Y_k\|_{\infty} < R), \quad (10)$$

$$\Phi_m = \frac{\sum_{i=1}^{N-m+1} \log N_i}{N-m+1}, \quad (11)$$

$$\text{Approximate Entropy} = \Phi_m - \Phi_{m+1}. \quad (12)$$

The approximate entropy of the time series produced by System (1), while varying the parameter k , is calculated and shown in Figure 8. Comparing this result with the bifurcation diagram, in the region of chaotic behavior, the quantity of approximate entropy is larger than in regions with periodic behavior. This trend is because the chaotic signal is more irregular and complex than the ordered periodic ones.

3.2. The effect of initial conditions on system dynamics

As the last part of the dynamical analysis of System (1), the basins of attraction of this system with two different initial conditions are depicted in Figure 9. The results are obtained

in x_0 - y_0 , y_0 - z_0 , and z_0 - w_0 planes with $k=0.0001$ in parts (a)-(c) of Figure 9, respectively. A 300×300 grid of initial conditions is used to obtain this figure, and the two other initial conditions are constant. The light orange, red, and white regions correspond to chaotic, fixed points, and unbounded solutions. A spectrum of other colors illustrates the periodic behavior; as the number of periods rises, the time series tend to be complex, and the color of these points changes. The black points are the projection of the line of equilibria of System (1). Regarding the equilibrium point stability analysis conducted in part 2.1, the line of equilibria of System (1) with $k=0.0001$ is stable, and all red zones converge to it. This system has a broad region of initial conditions that leads to chaotic behavior, so it is flexible in choosing the initial conditions.

4. Discussion and conclusion

This paper studied a chaotic system that took advantage of the characteristics of a previously introduced memristor. The system was not only able to bring up a strange attractor, but its attractor was also hidden. According to the nature of chaotic signals, it is necessary to investigate the state and behavior of these systems with more analytical methods. To verify the possibility of this behavior, numerical simulations were used to check the system's behavior. Chaos can be recognized from the bifurcation diagrams and the largest LE. Some criteria to prove the presence of chaos in this system were investigated: the system's dissipation, time series, a strange attractor, a positive Lyapunov Exponent (LE), Poincaré mapping, and a broad frequency spectrum. All these criteria showed the existence of chaos in this system, and they have been used to distinguish periodic and chaotic behaviors from each other.

The system had a dissipative chaotic attractor and a line of equilibrium points. Its line of equilibria was located on the y -axis. The system had a zero, two complex conjugates (with negative real parts), and a real Eigenvalue whose value depends on both the parameter k and the y value of the equilibria. Because of the appearance of a zero Eigenvalue, the stability of the equilibria cannot be obtained quickly and needs extensive investigations like basins of attraction. Besides, since the divergence of the system was state-

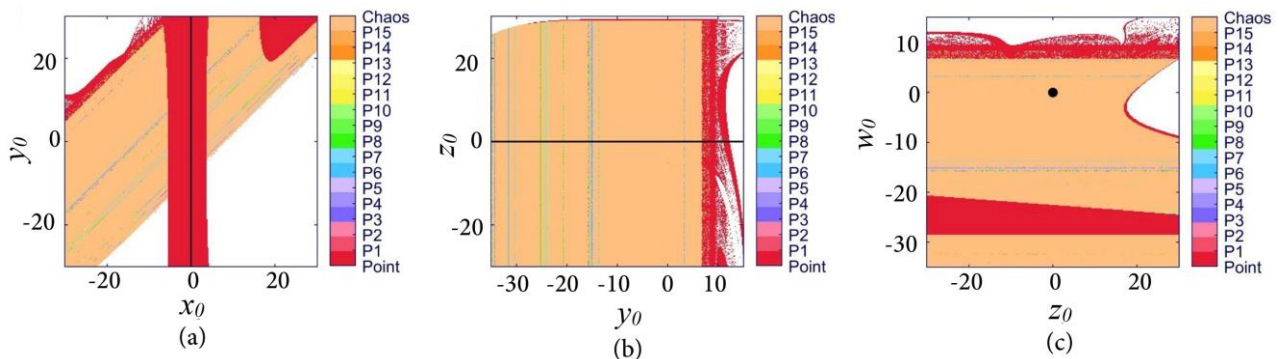


Figure 9. Basins of attraction of System (1) with $k = 0.0001$ in a 300×300 grid of initial conditions in (a) $(x_0, y_0, z_0, w_0) = (x_0, y_0, 0, 0)$, (b) $(x_0, y_0, z_0, w_0) = (-28, y_0, z_0, 0)$, and (c) $(x_0, y_0, z_0, w_0) = (-28, 0, z_0, w_0)$. The red, light orange, and white regions correspond to the fixed point, chaotic, and unbounded solutions. The black points are the projection of System (1) equilibrium points in each plane. This system also has many initial conditions that lead to chaos. P1, P2, ..., and P15 are periodic oscillations with different period numbers.

dependent, and the average value of the state-dependent terms could not be algebraically calculated; the average values were computed numerically. After a sufficient run time, the divergence of the system converged to a negative value. This proved that the chaotic trajectories were strange attractors.

The claimed properties of this system were evaluated with the help of powerful dynamical analysis tools like Poincaré sections, power spectrum, bifurcation diagram, LE spectrum, the basin of attraction, and approximate entropy. The inverse route of period doubling and periodic windows were observed in the bifurcation diagram. Attraction basins verified the coexistence of fixed points, periodic, and chaotic attractors. Moreover, the basin of attraction confirmed that the previously calculated line of equilibria is stable. Due to the outcomes of these examinations and the fact that there has been a promising development in implementing memristors, the proposed system can have potential real-world applications.

Acknowledgments

The authors would like to thank the editor and the reviewers for their valuable comments and constructive suggestions, which have significantly improved the quality of this manuscript.

Data availability statement

All data that support the findings of this study are included within the article.

Funding

This paper is supported by the Doctoral Research Project of Yan'an University (YAU202512551), the Scientific Research Program Funded by the Education Department of Shaanxi Provincial Government (25JK0737), and the Innovation and Entrepreneurship Training Program Projects for Undergraduates of Yan'an University (D2025116).

Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Authors contribution statement

First author: Conceptualization; Investigation; Software; Writing - Original draft.

Second author: Data Curation; Methodology; Supervision; Writing - Original draft.

Third author: Formal analysis; Project administration; Validation; Writing - Review & Editing.

Fourth author: Funding acquisition; Resources; Visualization; Writing - Review & Editing.

References

- Nazarimehr, F., Jafari, S., Chen, G., et al. "A tribute to J.C. Sprott", *International Journal of Bifurcation and Chaos*, **27**(14), 1750225 (2017).
<https://doi.org/10.1142/S021812741750225X>
- Wei, Z., Zhang, W., Moroz, I., et al. "Codimension one and two bifurcations in Cattaneo-Christov heat flux model", *Discrete and Continuous Dynamical Systems-B*, **26**(10), pp. 5305-5319 (2021).
<https://doi.org/10.3934/dcdsb.2020344>
- Kant, N.A., Dar, M.R., Khanday, F.A., et al. "Ultra-low-voltage integrable electronic realization of integer-and fractional-order Liao's chaotic delayed neuron model", *Circuits, Systems, and Signal Processing*, **36**, pp. 4844-4868 (2017).
<https://doi.org/10.1007/s00034-017-0615-5>
- Wei, Z., Zhu, B., and Escalante-González, R. "Existence of periodic orbits and chaos in a class of three-dimensional piecewise linear systems with two virtual stable node-foci", *Nonlinear Analysis: Hybrid Systems*, **43**, 101114 (2021).
<https://doi.org/10.1016/j.nahs.2021.101114>
- Wang, F., Liu, T., Kuznetsov, N.V., et al. "Jacobi stability analysis and the onset of chaos in a two-degree-of-freedom mechanical system", *International Journal of Bifurcation and Chaos*, **31**(5), 2150075 (2021).
<https://doi.org/10.1142/S0218127421500759>
- Itoh, M. and Chua, L.O. "Memristor oscillators", *International Journal of Bifurcation and Chaos*, **18**(11), pp. 3183-3206 (2008).
<https://doi.org/10.1142/S0218127408022354>
- Adhikari, S.P., Sah, M.P., Kim, H., et al. "Three fingerprints of memristor", *IEEE Transactions on Circuits and Systems I: Regular Papers*, **60**(11), pp. 3008-3021 (2013).
<https://doi.org/10.1109/TCSI.2013.2256171>
- Xu, Q., Cheng, S., Ju, Z., et al. "Asymmetric coexisting bifurcations and multi-stability in an asymmetric memristive diode-bridge-based jerk circuit", *Chinese Journal of Physics*, **70**(1), pp. 69-81 (2021).
<https://doi.org/10.1016/j.cjph.2020.11.007>
- Majzoub, S., Elwakil, A., Psychalinos, C., et al. "On the mechanism of creating pinched hysteresis loops using a commercial memristor device", *AEU-International Journal of Electronics and Communications*, **111**, 152923 (2019).
<https://doi.org/10.1016/j.aecue.2019.152923>
- Chua, L. "Memristor-the missing circuit element", *IEEE Transactions on Circuit Theory*, **18**(5), pp. 507-519 (1971).
<https://doi.org/10.1109/TCT.1971.1083337>
- Strukov, D.B., Snider, G.S., Stewart, D.R., et al. "The missing memristor found", *Nature*, **443**, pp. 80-83 (2008).
<https://doi.org/10.1038/nature06932>
- He, S., Zhan, D., Wang, H., et al. "Discrete memristor and discrete memristive systems", *Entropy*, **24**(6), 786 (2022).
<https://doi.org/10.3390/e24060786>

13. Xu, Q., Liu, T., Feng, C.-T., et al., "Continuous non-autonomous memristive Rulkov model with extreme multistability", *Chinese Physics B*, **30**, 128702 (2021).
<https://doi.org/10.1088/1674-1056/ac2f30>
14. Xu, Q., Ju, Z., Ding, S., et al. "Electromagnetic induction effects on electrical activity within a memristive Wilson neuron model", *Cognitive Neurodynamics*, **16**, pp. 1221-1231 (2022).
<https://doi.org/10.1007/s11571-021-09764-0>
15. Xu, Q., Lin, Y., Bao, B., et al. "Multiple attractors in a non-ideal active voltage-controlled memristor based Chua's circuit", *Chaos, Solitons and Fractals*, **83**, pp. 186-200 (2016).
<https://doi.org/10.1016/j.chaos.2015.12.007>
16. Lin, H., Wang, C., Xu, C., et al. "A memristive synapse control method to generate diversified multi-structure chaotic attractors", *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, **42**(3), pp. 942-955 (2023).
<https://doi.org/10.1109/TCAD.2022.3186516>
17. Lin, H., Wang, C., Sun, Y., et al. "Generating n -scroll chaotic attractors from a memristor-based magnetized Hopfield neural network", *IEEE Transactions on Circuits and Systems II: Express Briefs*, **70**(1), pp. 311-315 (2023).
<https://doi.org/10.1109/TCSII.2022.3212394>
18. Lin, H., Wang, C., Cui, L., et al. "Hyperchaotic memristive ring neural network and application in medical image encryption", *Nonlinear Dynamics*, **110**, pp. 841-855 (2022).
<https://doi.org/10.1007/s11071-022-07630-0>
19. Ding, S., Wang, N., Bao, H., et al. "Memristor synapse-coupled piecewise-linear simplified Hopfield neural network: Dynamics analysis and circuit implementation", *Chaos, Solitons and Fractals*, **166**, 112899 (2023).
<https://doi.org/10.1016/j.chaos.2022.112899>
20. Leonov, G., Kuznetsov, N., and Vagitsev, V. "Hidden attractor in smooth Chua systems", *Physica D: Nonlinear Phenomena*, **241**(18), pp. 1482-1486 (2012).
<https://doi.org/10.1016/j.physd.2012.05.016>
21. Bao, B., Hu, F., Chen, M., et al. "Self-excited and hidden attractors found simultaneously in a modified Chua's circuit", *International Journal of Bifurcation and Chaos*, **25**(5), 15500753 (2015).
<https://doi.org/10.1142/S0218127415500753>
22. Chen, M., Li, M., Yu, Q., et al. "Dynamics of self-excited attractors and hidden attractors in generalized memristor-based Chua's circuit", *Nonlinear Dynamics*, **81**, pp. 215-226 (2015).
<https://doi.org/10.1007/s11071-015-1983-7>
23. Leonov, G.A., Kuznetsov, N.V., and Mokaev, T.N. "Homoclinic orbits, and self-excited and hidden attractors in a Lorenz-like system describing convective fluid motion", *The European Physical Journal Special Topics*, **224**, pp. 1421-1458 (2015).
<https://doi.org/10.1140/epjst/e2015-02470-3>
24. Leonov, G.A., Kuznetsov, N.V., and Mokaev, T.N. "Hidden attractor and homoclinic orbit in Lorenz-like system describing convective fluid motion in rotating cavity", *Communications in Nonlinear Science and Numerical Simulation*, **28**(1-3), pp. 166-174 (2015).
<https://doi.org/10.1016/j.cnsns.2015.04.007>
25. Wang, Z., Wei, Z., Sun, K., et al. "Chaotic flows with special equilibria", *The European Physical Journal Special Topics*, **229**(6), pp. 905-919 (2020).
<https://doi.org/10.1140/epjst/e2020-900239-2>
26. Wang, X. and Chen, G. "A chaotic system with only one stable equilibrium", *Communications in Nonlinear Science and Numerical Simulation*, **17**(3), pp. 1264-1272 (2012).
<https://doi.org/10.1016/j.cnsns.2011.07.017>
27. Wang, X. and Chen, G. "Constructing a chaotic system with any number of equilibria", *Nonlinear Dynamics*, **71**, pp. 429-436 (2013).
<https://doi.org/10.1007/s11071-012-0669-7>
28. Pham, V.-T., Volos, C., and Gambuzza, L.V. "A memristive hyperchaotic system without equilibrium", *The Scientific World Journal*, **2014**, 368986 (2014).
<https://doi.org/10.1155/2014/368986>
29. Marwan, N., Romano, M.C., Thiel, M., et al. "Recurrence plots for the analysis of complex systems", *Physics Reports*, **438**(5-6), pp. 237-329 (2007).
<https://doi.org/10.1016/j.physrep.2006.11.001>
30. Wei, Z. "Dynamical behaviors of a chaotic system with no equilibria", *Physics Letters A*, **376**(2), pp. 102-108 (2011). <https://doi.org/10.1016/j.physleta.2011.10.040>
31. Wei, Z. and Yang, Q. "Dynamical analysis of a new autonomous 3-D chaotic system only with stable equilibria", *Nonlinear Analysis: Real World Applications*, **12**(1), pp. 106-118 (2011).
<https://doi.org/10.1016/j.nonrwa.2010.05.038>
32. Hilborn, R.C., *Chaos and Nonlinear Dynamics: An Introduction for Scientists and Engineers*, Oxford University Press (2000).
<https://doi.org/10.1093/acprof:oso/9780198507239.001.0001>
33. Leng, X., Du, B., Gu, S., et al. "Novel dynamical behaviors in fractional-order conservative hyperchaotic system and DSP implementation", *Nonlinear Dynamics*, **109**(2), pp. 1167-1186 (2022).
<https://doi.org/10.1007/s11071-022-07498-0>
34. Dumont, R.S. and Brumer, P. "Characteristics of power spectra for regular and chaotic systems", *The Journal of Chemical Physics*, **88**, pp. 1481-1496 (1988).
<https://doi.org/10.1063/1.454126>

35. Antonsen Jr, T.M., Fan, Z., Ott, E., et al. "The role of chaotic orbits in the determination of power spectra of passive scalars", *Physics of Fluids*, **8**, pp. 3094-3104 (1996).
<https://doi.org/10.1063/1.869083>
36. Zandi-Mehran, N., Nazarimehr, F., Rajagopal, K., et al. "FFT bifurcation: A tool for spectrum analyzing of dynamical systems", *Applied Mathematics and Computation*, **422**, 126986 (2022).
<https://doi.org/10.1016/j.amc.2022.126986>
37. Wolf, A., Swift, J.B., Swinney, H.L., et al. "Determining Lyapunov exponents from a time series", *Physica D: Nonlinear Phenomena*, **16**(3), pp. 285-317 (1985).
[https://doi.org/10.1016/0167-2789\(85\)90011-9](https://doi.org/10.1016/0167-2789(85)90011-9)
38. Kantz, H. "A robust method to estimate the maximal Lyapunov exponent of a time series", *Physics Letters A*, **185**(1), pp. 77-87 (1994).
[https://doi.org/10.1016/0375-9601\(94\)90991-1](https://doi.org/10.1016/0375-9601(94)90991-1)
39. Pincus, S.M. "Approximate entropy as a measure of system complexity", *Proceedings of the National Academy of Sciences*, **88**(6), pp. 2297-2301 (1991).
<https://doi.org/10.1073/pnas.88.6.2297>
40. Pincus, S.M. and Huang, W.-M. "Approximate entropy: Statistical properties and applications", *Communications in Statistics-Theory and Methods*, **21**(11), pp. 3061-3077 (1992).
<https://doi.org/10.1080/03610929208830963>
41. Pincus, S. "Approximate entropy (ApEn) as a complexity measure", *Chaos: An Interdisciplinary Journal of Nonlinear Science*, **5**(1), pp. 110-117 (1995).
<https://doi.org/10.1063/1.166092>

Biographies

Zhen Wang received his BSc and MSc degrees in 2004 and 2008 from Shaanxi University of Science and Technology, China, and his PhD degree in 2022 from Xi'an University of Technology, China. He is a Professor at the School of Mathematics and Computer Science at Yan'an University. His research interests include ordinary differential equations and dynamical systems, nonlinear control system theory, etc.

Sridevi Sriram is a mathematician who completed her BSc degree (1992) and MSc degree (1994), specializing in Mathematics, and graduated from Madurai Kamaraj University. She also added another degree of Qualification in Bachelor of Education (BEd.), graduated in 1995 from Madurai Kamaraj University. She is affiliated with the Center for Nonlinear Systems, Chennai Institute of Technology, Chennai. Her research interests are applied and computational mathematics, nonlinear dynamics, fractional-order nonlinear systems and control, stability analysis of nonlinear systems, and numerical methods for solving nonlinear equations.

Huaigu Tian received his BSc and MSc degrees in 2018 and 2021, respectively, from Xijing University, China. He is an Assistant at the Xi'an Key Laboratory of Human-Machine Integration and Control Technology for Intelligent Rehabilitation at Xijing University. His research interests include chaos dynamics, nonlinear control system theories, etc.

Karthikeyan Rajagopal is a globally renowned researcher and academic leader, currently serving as the Director of Research at SRM Group of Institutions, Chennai Ramapuram and Trichy campuses. His research areas span chaos, nonlinear dynamics, complex networks, artificial intelligence, embedded systems, robotics, signal processing, communication systems, edge computing, and neuroscience.