

Three-dimensional stretched boundary layer flow of Casson nanofluid in rotating frame with bio-convection phenomenon

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3D flow;
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Nanofluid;
Optimal Homotopy Asymptotic Method (OHAM);
Rotating frame;
Stretching sheet;
Buongiorno model;
Bio-convection.

Abstract

This paper describes the three-dimensional Casson nanofluid's rotating flow with bio-convection phenomenon containing microorganisms, thermal radiation, and magnetic effects. This research has real-world applications in a variety of processes, including oceanography, crystal growth, computer storage devices, lubrication, and rotating machinery. The present model incorporates the Buongiorno nanofluid model, which describes the Brownian and thermophoresis motion of nanoparticles. The boundary layer approximation and non-Newtonian three-dimensional Casson fluid model are encompassed in the modelling of the nonlinear partial differential system. Finding an analytical clarification to the 3D Casson nanofluid flow past a rotating frame involving bio-convection phenomenon nonlinear differential equation is the goal of this study. Graphical results are obtained using the Optimal Homotopy Asymptotic Method (OHAM). The effects of different parameters are examined with respect to velocities, thermal field, magnetic field, nanoparticle concentration, and microorganism field. The temperature distribution is reduced with a greater Prandtl number. The temperature and solute field of a species increase with an upsurge in the estimation of the thermophoresis parameter. The microorganism field decreases when the Peclet parameter, bio-convection Lewis number, and microorganism difference number increase.

1. Introduction

Nanofluids are used in engineering branches, like mechanical engineering, chemical engineering, power engineering, agricultural engineering, refrigerators, aero planes, and vehicles. The use of nanofluid has Enhanced Oil Recovery (EOR) in saltwater environments. Choi and Eastman [1] have presentation the original study that popularized the idea of nanofluids. Nayak et al. [2] report nanofluid convective flow involving chemical reaction across an exponentially extending sheet. Shehzad et al. [3] explored the Casson nanofluid models. Sajid and Ali [4] provide a critical study of recent advancements in the use of nanofluid in heat transfer devices. Chakraborty and Panigrahi [5] explored the stability of nanofluid. The use of convective boundary conditions, Nadeem et al. [6] examined Casson nanofluid flow across a linear stretched sheet within a three-dimensional Magneto-

hydrodynamics (MHD) boundary layer. Convective boundary conditions were employed by Sulochana et al. [7] to investigate 3D Casson nanofluid flow on a stretched sheet. Heat transfer with slip, convective boundary conditions, and MHD stagnating point flow were employed by Ibrahim and Makinde [8] to investigate the Casson nanofluid flow across a stretched sheet. Santoshi et al. [9] analyzed Casson-Carreau nanofluids numerically in three dimensions. Kumaran and Sandeep [10] examined Casson fluid flow while taking into account Brownian motion. Archana et al. [11] tested Casson nanofluid is used as a lubricant. The Shaheen et al. [12] was studied Dufour-sort effect with Casson nanofluid in three-dimensional containing dust particles and changing characteristics in a permeable medium. An excellent case study for researching the Casson nanofluid instabilities of

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material across a stretched sheet, as investigated. Under thermal radiation, Jamshed et al. [13] carried out an excellent case study with an unstable Casson nanofluid assessment across a stretched sheet.

Many industrial applications involved in rotating frame systems. Whether a vortex formed or a little motion was created, the issue of spinning fluids was inevitably brought up. The current experiment concentrated on how rotating frame systems affected the flow. Rotating frame flow is frequently used in the broad range of industries, including the design of turbines, cheerleaders, processing of food, engine, gearing, brake parts, disc cleaners, medical devices, viscometers, distinctive rotating geometry for internal air conditioning, compressors, and cyclone separators. Wang [14] sought to comprehend rotational flow over-stretching geometries at first and finally acquired the required results using the perturbation approach. Rotating flow in three-dimension with suction over a sheet that is diminishing exponentially explored by Rosali et al. [15]. Radha et al. [16] analyzed Casson nanofluid motion in three dimensions using a stretched sheet. For MHD radiative Casson nanofluid flow using a moving disc saturated in a porous medium. Noranuar et al. [17] has studied dual diffusing in non-coaxial rotating frame. Ibrahim and Fawzy [18] conducted semi-numerical computations to examine the rotating Casson nanofluid flow with Arrhenius energy effects and velocity slip possessions under convective conditions. Raju and Sandeep [19] have investigated the ferrous nanoparticles were suspended in a Casson nanofluid in a rotating frame above a rotating cone. Khan et al. [20] study utilized Buongiorno's model to carry out a computational assessment of nanofluid heat and flow transmission across a rotating frame. Analytical methodologies for the 3D investigation of condensing nanofluid layers on an angled rotating frame were discussed by Hatami et al. [21]. A rotating flow's Casson nanofluid's reaction to nonlinear thermal radiation inspected by Archana et al. [22]. Nanofluids in an orbiting frame were the subject of a Darcy-Forchheimer flow deliberate by Hayat et al. [23]. Rashid et al. [24] examine how Darcy-Forchheimer operates within a porous material in order to evaluate the velocity field in rotating frame examinations. Saeed et al. [25] finite element analysis used the thermal flux modal to examine how the MHD affects a Casson nanofluid's rotating flow. Carbon nanotubes in an MHD Casson nanofluid move non-coaxially past a rotating frame with the porosity effects addressed by Noranuar et al. [26]. In three dimensions, Aziz et al. [27] scrutinized the numerical modelling of a spinning nanofluid with entropy production. Aziz et al. [28] looked at the entropy analysis of the Powell-Eyring hybrid nanofluid while taking viscous dissipation and linear thermal radiation into consideration. To investigate the theoretical framework of Cattaneo-Christov thermal flux consequences on Williamson hybrid nanofluids based on engine oil, Jamshed et al. [29] conducted a thermal case study. Jamshed and Aziz [30]

explored the effects of variable thermal conductivity, numerous nanoparticle shapes, and thermal radiation on the Fe_3O_4 /methanol Powell-Eyring nanofluid and Cu within solar thermal accumulators using a comparative entropy-based methodology. Jamshed et al. [31] scrutinized the fluid flow over an absorbent shrinkable sheet with Ohmic heat resistance utilizing numerical heat and solvent transfer simulation. Sajid et al. [32] introduced this study, which looked at the influence of tetra nanoparticles that are hybrids on binary fluids with ethanol biofuels serving as the base fluid and the catalytic interaction, Crosser model, and novel Hamilton. Kai et al. [33] conducted a scrutiny on the stimulus of applied restricted magnetic fields on the Ag-TiO₂ hybrid fluid flow within an enclosure that is induced by numerous magnetic sources. Hanif et al. [34] conducted a numerical Crank-Nicolson methodology investigation on the hybridity of an aluminum alloy nanofluid that was flowing through a stretchy horizontal plate and had a thermal resistive effect. Kai et al. [35] inspected the temperature and created vortices in a hybridized nanofluid that was flowing in an implicit solution in two different directions. According to Akolade and Tijani [36], there are nonlinear radiation effects when it comes to how different transport parameters affect the magnetized 3D flow of the Casson nanofluid across a Riga plate.

Stretching sheet usually refers to a theoretical model that is used to investigate how a nanofluid flows and transfers heat along a stretching surface. A stretched sheet's orientation was the focus of the heat transfer and stagnation-point flows in a Casson fluid were discovered by Mustafa et al. [37]. The idea for a 3D Casson nanofluid that is flow across a porous sheet that is linearly expanding was created by Nadeem et al. [38]. Using a linear stretched sheet, Mahanta and Shaw [39] discussed the Casson fluid flow. Oyelakin et al. [40] expressed an unsteady Casson nanofluid that flows onto a sheet that was stretched with conditions including heat radiation and convection. Shah et al. [41] investigates thermophoresis's impact on a 3D non-Newtonian nanofluid with continuous current, related to the Hall effects in a Brownian motion rotating frame. Research on the effects of diffusing on the hydrodynamics Anwar et al. [42] looks at how the Casson nanofluid boundary zone moves across a stretched surface. Considering viscous dissipation, a viscous Casson fluid can be injected and sucked via a stretched surface by Hussanan et al. [43]. Melting affects radiation-induced Magnetohydrodynamic Casson nanofluid motion via a stretched sheet via a porous media, as demonstrated by Mabood and Das [44]. Ullah et al. [45] investigated a Williamson-dusty nano-liquid flow across a stretched plate using MHD mixed convection. Using Cattaneo-Christov theory, Khan et al. [46] surveyed the stratification consequence brought on by an exponentially stretched sheet in a rotating frame beneath the inspiration of

a magnetic field as well as heat transfer analysis of Casson fluid flow. 3D Casson nanofluid flow was inspected by Lone et al. [47] on a bidirectional linear stretchy sheet that was home to gyrotactic microorganisms. Shah et al. [48] examine the Casson ferrofluid on a stretched sheet utilizing the efficient thermal conductivity model. With the use of an MHD Casson nanofluid, Gangaiah et al. [49] examined how heat radiation impacts mixed convection flow across an exponentially increasing sheet. Using the Keller box as a Buongiorno model, Rafique et al. [50] examined a three-dimensional Casson nanofluid study on an inclined surface that combined Brownian and thermophoretic dispersion. Ragupathi et al. [51] have studied 3D Casson nanofluid flow with exponential thermal effects and Arrhenius activation energy on a stretched sheet. Puneeth et al. [52] have premeditated a 3D hybrid Casson nanofluid flow that crosses a non-linear stretching surface. According to Al-Mamun et al. [53], a permeable stretching sheet was utilized to replicate periodic MHD Casson nanofluid flow. Gangadhar et al. [54] investigation of the Casson flows through a stretched sheet. In 2006, R. Buongiorno expanded the conventional single-phase fluid model and proposed the Buongiorno model to account for the inclusion of nanoparticle in a base fluid. Garoosi et al. [55] research suggests that the Buongiorno model might mimic the nanofluid's natural convection in heat exchangers. Buongiorno nanofluid concept impacts on optical heat and mass transmission have been demonstrated by Turkiymazoglu [56]. 3D Magnetohydrodynamics (MHD) viscous flow with thermal radiation and dissipation of viscous fluid is examine by Akbar and Sohail [57]. Li et al. [58] explored the under-slip circumstances effects of buoyancy and dissipation of viscous on a 3D Magnetohydrodynamic viscous hybrids nano fluid (MgO-TiO₂). Nazir et al. [59] investigated the use of mesh-free studies and the Finite Element (FE) approach in a non-Fourier way to investigate uses for changeable thermal features in Carreau materials including ion hall and slide forces towards cone. A monophasic-magnetized nanofluid with Ohmic heating and a thermal hop was studied for its entropy and heating case specification by Zhang et al. [60] using finite element techniques. To investigate second-grade nanofluid flow's radiative heat exchange via a porous flat surface, Jamshed et al. [61] employed a one-phase mathematical model. To analyze the flow of an Ag-Cu/EO hybrid Williamson nanofluid across a stretched surface with an outline factor, Jamshed et al. [62] performed single-phase research. The Casson nanofluid generated by an unstable stretching surface was premeditated by Ahmad et al. [63] using features of chemical reaction and heat radiation in Magnetohydrodynamic time-dependent three-dimensional simulations. A second-class fluid with varying thermal conductivity was studied by Haider et al. [64] in terms of the transfer of energy dependent on time Cattaneo-Christov double

diffusion. Employing the spectral relaxation technique, Rasool et al. [65] have studied the bio-convection and chemical reactions for an elevated influenced Cross nanofluid over a persuaded cylinder. The Casson nano-fluid stream over a plate was premeditated by Farooq et al. [66] using a variety of techniques, such as bioconvection, exponential heat sources, and motile micro-organisms. Within MHD, bio-convection Kumaraswamy Naidu et al. [67] planned the Casson nanofluid, which is made up of motile microorganisms using convective boundary conditions. The study's findings have significant ramifications for biological and technical systems, including the petroleum industry, bio-microsystems, designing bio-cells, biomedicine, and other fields which microorganisms and nanoparticles are crucial to improving energy conservation efficiency. Tawade et al. [68] looked into the issue of continual laminar flow of a nanofluid across a barrier layer utilizing Cassona heat transfer via a linearly increasing sheet. Variable heat conductivity and a Lorentz force were employed by Botmart et al. [69] in their mathematical formulation of the cross model to study the nanoscale thermal transmission phenomena of fluids. The current study advances the numerical treatment relating a mix of convective inclination magnetized Cross nanofluid with changing heat conductivity across a moving permeable surface, based on variable viscosity, as studied by Darvesh et al. [70]. Al-Kouz and Owhaib [71] conducted a numerical study utilizing a three-dimensional Casson's nanofluid that was flowing via a rotating frame and subjected to viscous warming and calculated heat flux.

The Optimal Homotopy Asymptotic Method (OHAM) is used for bio-convective Casson model to produce graphical findings. The impacts of various parameters are investigated in relation to temperature field, nanoparticle concentration, microbiological field, and velocities. The temperature distribution is lessened with a greater Prandtl number. As the thermophoresis parameter is estimated more, the specie's solute zone and heat grow. The microorganism's field shrinkages with increasing ethics of the bio-convection Lewis number, Peclet parameter, and microorganism's difference value.

2. Problem formulation

We examine the extended non-Newtonian Casson nanofluid's three-dimensional laminar flow. The fluid space is regarded as being at $z \geq 0$. A constant rate of rotation ω about the z -axis is experienced by the incompressible Casson nanofluid. Figure 1 depicts a schematic illustration of the problem being studied.

The present problem under investigation governing equations is defined below, see [71].

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (1)$$

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - 2\omega v \right) = \mu \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial z^2} - \sigma B_0^2 u, \quad (2)$$

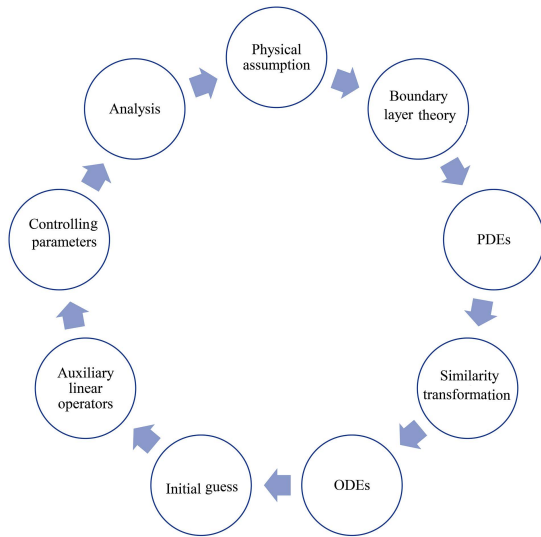


Figure 2. Pictorial presentation of used scheme.

$$\text{Re}_y^{0.5} S f_y = \left(1 + \frac{1}{\beta}\right) g''(0), \quad (17)$$

$$\text{Re}_x^{-0.5} N u_x = -\theta'(0), \quad (18)$$

$$\text{Re}_x^{-0.5} S h_x = -\varphi'(0). \quad (19)$$

where $\text{Re}_x = \frac{u_w x}{\nu}$ is Reynolds number

3. Methodology

The nonlinear Boundary Value Problem (BVP), Eqs. (9) to (13) are analytically solved, and graphical results are generated utilizing the Homotopy Analytical Method (HAM) BVPPh2.0 as explained through Figure 2. These are following the initial estimates and auxiliary linear operators:

$$f_0(\eta) = 1 - \frac{1}{\tilde{\exp}(\eta)} + S, \quad g_0(\eta) = \alpha \left(1 - \frac{1}{\tilde{\exp}(\eta)}\right),$$

$$\theta_0(\eta) = \frac{1}{\tilde{\exp}(\eta)}, \quad \varphi_0(\eta) = \frac{1}{\tilde{\exp}(\eta)}, \quad \chi_0(\eta) = \frac{1}{\tilde{\exp}(\eta)}, \quad (20)$$

$$G_f = f''' - f', \quad G_g = g''' - g', \quad G_\theta = \theta'' - \theta',$$

$$G_\varphi = \varphi'' - \varphi', \quad G_\chi = \chi'' - \chi'. \quad (21)$$

by:

$$G_f [F_1 + F_2 e^\eta + F_3 e^{-\eta}] = 0, \quad G_g [F_4 + F_5 e^\eta + F_6 e^{-\eta}] = 0,$$

$$G_\theta [F_7 e^\eta + F_8 e^{-\eta}] = 0, \quad G_\varphi [F_9 e^\eta + F_{10} e^{-\eta}] = 0,$$

$$G_\chi [F_{11} e^\eta + F_{12} e^{-\eta}] = 0. \quad (22)$$

4. Results and discussion

We study the rotational flow of a three-dimensional Casson nanofluid with heat radiation, magnetic effects, and a bio-convection phenomenon including microorganisms. This study's analytical solution is shown in Figures 3–19. Figure 3 shows the when β levels are raised, the axial velocity is decreased, then magnitude of axial velocity is decreased.

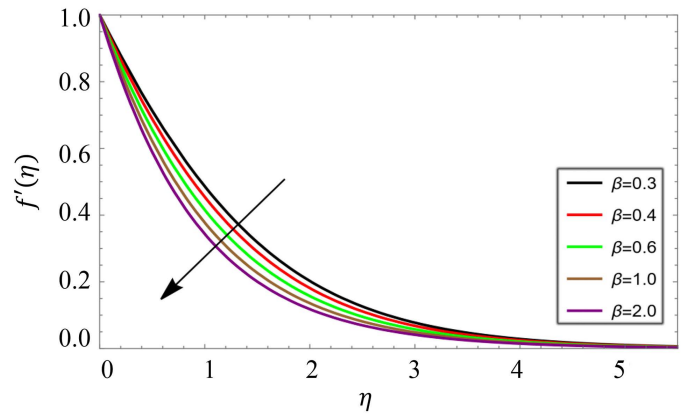


Figure 3. β impact on $f'(\eta)$ for the values $S=0.8$; $Ec=0.2$; $\alpha=0.8$; $M=0.0$; $Pr=1.0$; $Nt=0.2$; $Ro=0.1$; $Lb=0.5$; $Le=0.2$; $Nb=0.1$; $Pe=1.0$; $\delta_1=0.0$.

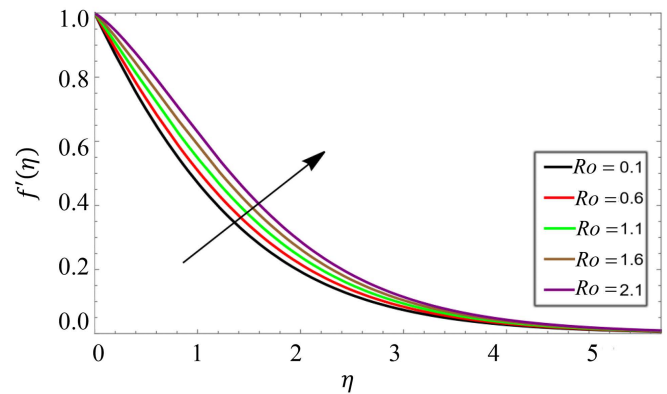


Figure 4. Ro impact on $f'(\eta)$ for the values $S=0.8$; $\alpha=0.8$; $M=0.0$; $\beta=1.0$; $Ec=0.2$; $Lb=0.5$; $Le=0.2$; $Nt=0.2$; $Pr=1.0$; $Nb=0.1$; $Pe=1.0$; $\delta_1=0.0$.

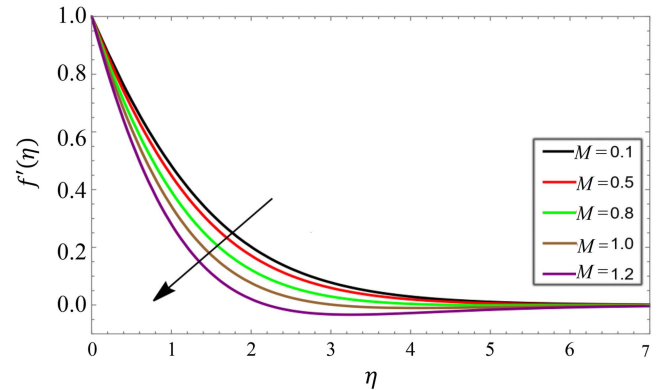


Figure 5. M impact on $f'(\eta)$ for the values $S=0.8$; $\alpha=0.8$; $Ro=0.1$; $Ec=0.2$; $\beta=1.0$; $Nb=0.1$; $Lb=0.5$; $Le=0.2$; $Pr=1.0$; $Nt=0.2$; $Pe=1.0$; $\delta_1=0.0$.

Figure 4 shows the higher rotation factor values denote a lower axial velocity rate, greater values of Ro therefore result in a decrease in the axial velocity's magnitude. As the M rises, the axial velocity profile shrinkages, as seen in Figure 5. Both the particle nanofluid and carrier-based fluid have their velocity reduced by the magnetic field. Figure 6 shows how the transverse velocities $g'(\eta)$ increases in magnitude as β rises. Figure 7 shows the magnitude for the transverse velocities $g'(\eta)$ growing as Ro increases.

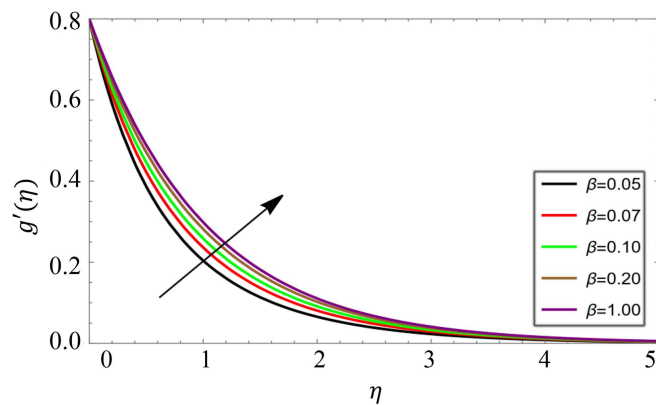


Figure 6. β impact on $g'(\eta)$ for the values $S=0.8$; $Ec=0.2$; $M=0.0$; $Nb=0.1$; $Nt=0.2$; $Pe=1.0$; $\delta_1=0.0$; $\alpha=0.8$; $Pr=1.0$; $Ro=0.1$; $Lb=0.5$; $Le=0.2$.

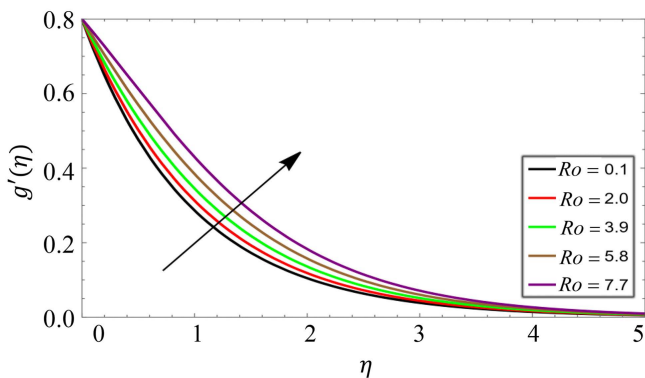


Figure 7. Ro impact on $g'(\eta)$ for the values $S=0.8$; $Ec=0.2$; $M=0.0$; $Lb=0.5$; $Nt=0.2$; $Le=0.2$; $\alpha=0.8$; $\beta=1.0$; $Pr=1.0$; $Nb=0.1$; $Pe=1.0$; $\delta_1=0.0$.

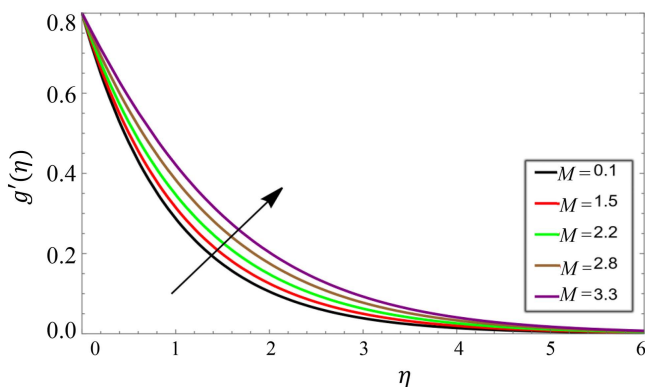


Figure 8. M impact on $g'(\eta)$ for the values $S=0.8$; $\alpha=0.8$; $Ec=0.2$; $\beta=1.0$; $Nt=0.2$; $Lb=0.5$; $Le=0.2$; $Pr=1.0$; $Ro=0.1$; $Nb=0.1$; $Pe=1.0$; $\delta_1=0.0$.

The longitudinal velocity profile rises as the M upsurges, as seen in Figure 8. The velocity of the particle nanofluid and the carrier-based fluid both increase in the existence of a magnetic field. As the Prandtl number surges, Figure 9 shows how the ambient temperature profile decreases and the nanofluid's thermal conductivity shrinkages. Figure 10 shows the stimulus of the Brownian parameter that is physically linked to the expected velocity of collisions and nanoparticles. Figure 11 illustrates the temperature profile rises with higher Nt values and the nanofluid produces more heat.

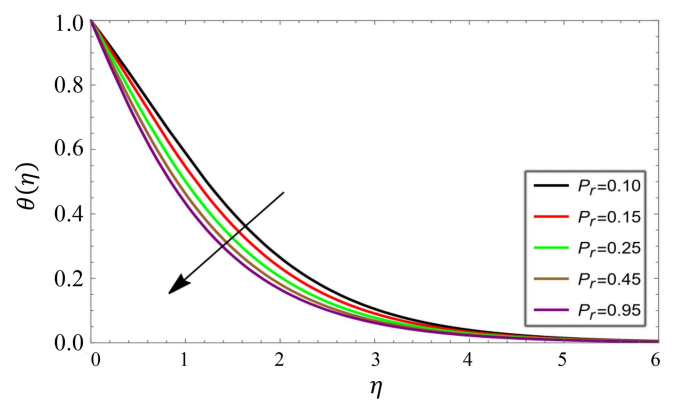


Figure 9. Pr impact on $\theta(\eta)$ for the values $S=0.8$; $Nb=0.1$; $Ro=0.1$; $Ec=0.2$; $Lb=0.5$; $Le=0.2$; $M=0.0$; $\alpha=0.8$; $\beta=1.0$; $Nt=0.2$; $Pe=1.0$; $\delta_1=0.0$.

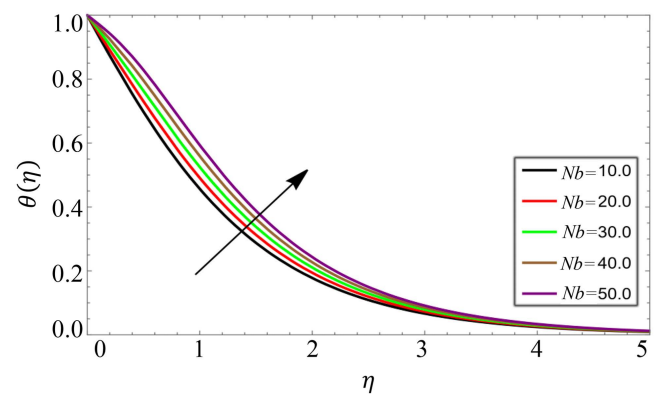


Figure 10. Nb impact on $\theta(\eta)$ for the values $S=0.8$; $\alpha=0.8$; $Le=0.2$; $Ec=0.2$; $Pr=1.0$; $Nt=0.2$; $M=0.0$; $\beta=1.0$; $Ro=0.1$; $Lb=0.5$; $Pe=1.0$; $\delta_1=0.0$.

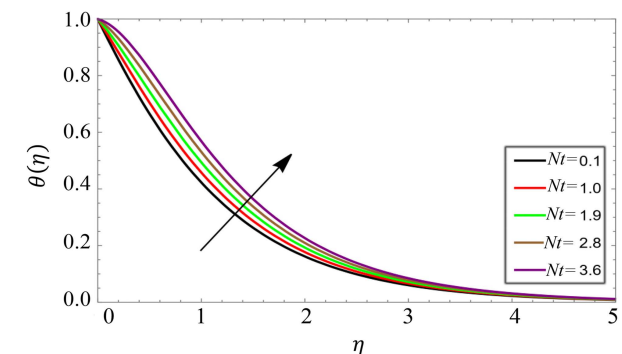


Figure 11. Nt impact on $\theta(\eta)$ for the values $S=0.8$; $Le=0.2$; $M=0.0$; $Ec=0.2$; $Pr=1.0$; $Pe=1.0$; $\delta_1=0.0$; $Nb=0.1$; $\alpha=0.8$; $\beta=1.0$; $Ro=0.1$; $Lb=0.5$.

The increasing values of Ec in Figure 12 demonstrate the layer structure's tendency to rise with temperature. The rising Casson fluid parameter values and the layer structure's propensity to contract with temperature are depicted in Figure 13. As can be seen in Figure 14, the concentration profile rises as the thermophoresis parameter is raised. A rise in the thermophoresis parameter grades in a growth in the concentration boundary layer thickness. As can be shown in Figure 15, the concentration of nanoparticles increases with an upturn in the Brownian parameter Nb .

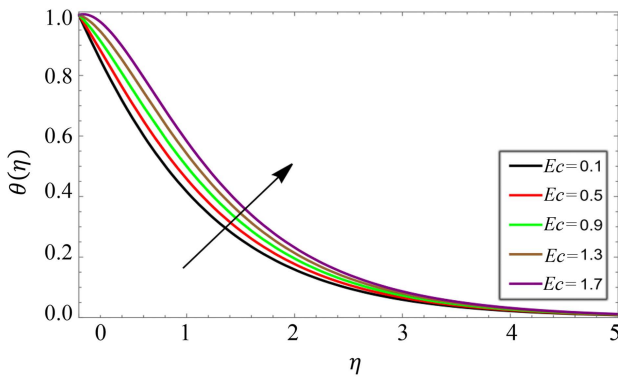


Figure 12. Ec impact on $\theta(\eta)$ for the values $S=0.8$; $\alpha=0.8$; $\beta=1.0$; $Le=0.2$; $M=0.0$; $Pr=1.0$; $Nb=0.1$; $Nt=0.2$; $Ro=0.1$; $Lb=0.5$; $Pe=1.0$; $\delta_1=0.0$.

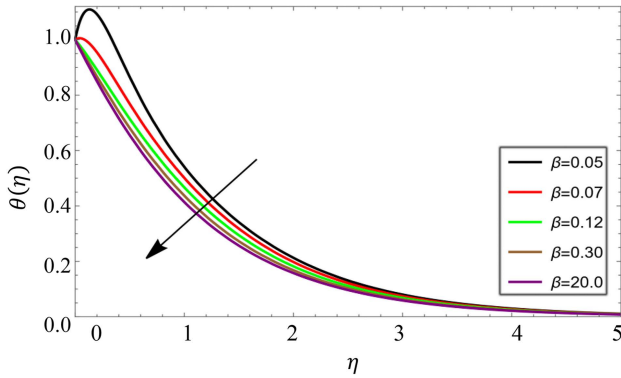


Figure 13. β impact on $\theta(\eta)$ for the values $S=0.8$; $Ec=0.2$; $M=0.0$; $Pr=1.0$; $Nb=0.1$; $Nt=0.2$; $\alpha=0.8$; $Ro=0.1$; $Lb=0.5$; $Le=0.2$; $Pe=1.0$; $\delta_1=0.0$.

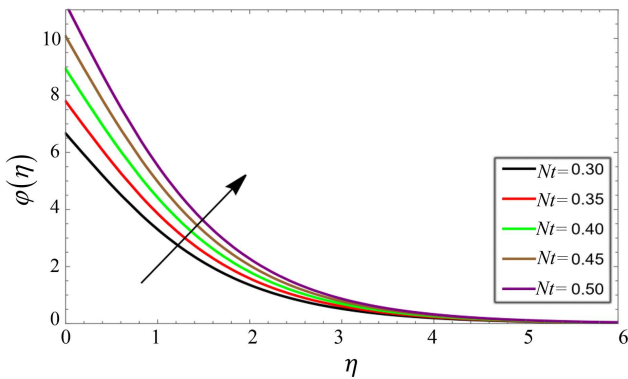


Figure 14. Nt impact on $\phi(\eta)$ for the values $S=0.8$; $\alpha=0.8$; $Ro=0.1$; $Lb=0.5$; $Le=0.2$; $M=0.0$; $Ec=0.2$; $\beta=1.0$; $Pr=1.0$; $Nb=0.1$; $Pe=1.0$; $\delta_1=0.0$.

Figure 16 depicts the reduced concentration of nanoparticles layer structure induced by lower mass diffusivity due to higher Le values. Figure 17 illustrates both factors have a markedly reducing influence on $\chi(\eta)$ due to the reduction in diffusion of the motile density of the bacterium caused by Lb . Figure 18 shows the rise in the value of the Peclet number Pe induces a rise in the diffusivity of the microbe, and therefore the microorganism concentration function $\chi(\eta)$ decreases. Figure 19 shows the accumulative value of the microorganism concentration difference number δ_1 reduces the

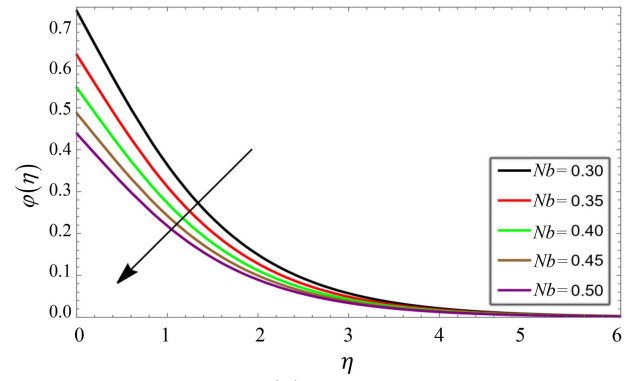


Figure 15. Nb impact on $\phi(\eta)$ for the values $S=0.8$; $\alpha=0.8$; $Ro=0.1$; $Lb=0.5$; $Le=0.2$; $M=0.0$; $Ec=0.2$; $\beta=1.0$; $Pr=1.0$; $Nt=0.2$; $Pe=1.0$; $\delta_1=0.0$.

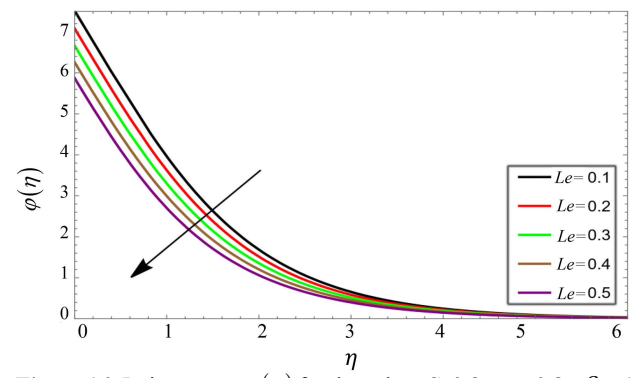


Figure 16. Le impact on $\phi(\eta)$ for the values $S=0.8$; $\alpha=0.8$; $\beta=1.0$; $Pr=1$; $Nt=0.2$; $Pe=1.0$; $Nb=0.1$; $Ec=0.2$; $Ro=0.1$; $Lb=0.5$; $M=0.0$; $\delta_1=0.0$.

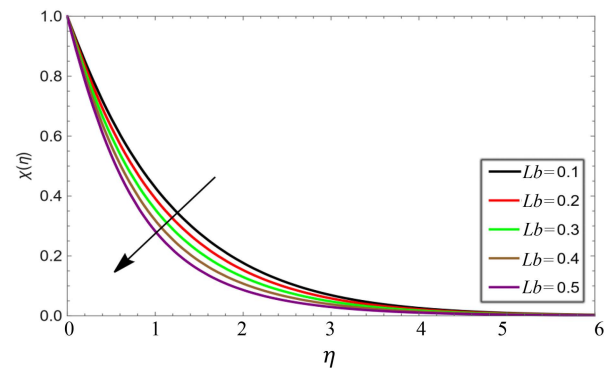


Figure 17. Lb impact on $\chi(\eta)$ for the values $S=0.8$; $\alpha=0.8$; $Pr=1.0$; $Nt=0.2$; $Pe=1.0$; $Nb=0.1$; $\delta_1=0.0$; $Ec=0.2$; $Ro=0.1$; $Le=0.2$; $M=0.0$; $\beta=1.0$.

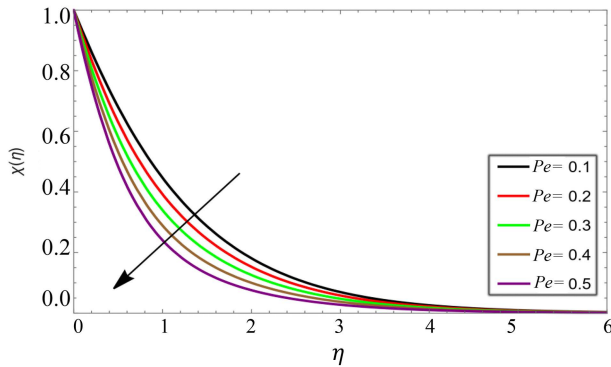
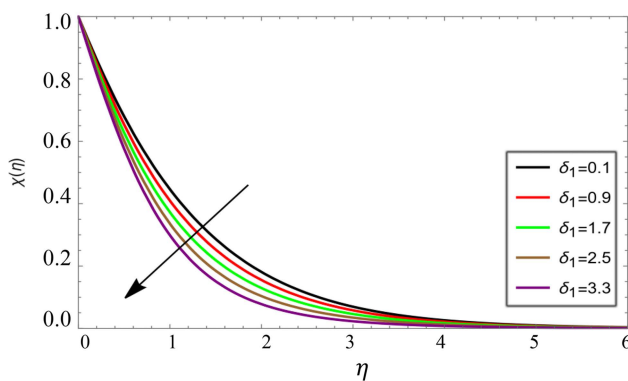
microorganism concentration function $\chi(\eta)$. Increasing the values of Casson fluid parameters β marks in a drop in the Nusselt numbers, Sherwood number, and skin friction, as Table 1 illustrates.

4. Conclusions

The rotational flow of a 3D Casson nanofluid comprising microorganisms, heat radiation, and magnetic influences is studied. The following summarizes the main findings of the current research:

Table 1. Numerical Outcomes $Nb=0.1$ and $Nt=0.05$.

β	$Re_x^{0.5} Sf_x$	$Re_x^{0.5} Sf_y$	$Re_x^{-0.5} Nu_x$	$Re_x^{-0.5} Sh_x$
0.2	-4.319850835	-1.945788288	1.000000000	0.500000000
0.4	-3.563695142	-1.605193443	1.000000000	0.500000000
0.6	-3.325422742	-1.497868523	1.000000000	0.500000000
0.8	-3.239888144	-1.459341217	1.000000000	0.500000000

**Figure 18.** Pe impact on $\chi(\eta)$ for the values $S=0.8$; $\alpha=0.8$; $Ec=0.2$; $Ro=0.1$; $M=0.0$; $Le=0.2$; $Nb=0.1$; $\beta=1.0$; $Pr=1.0$; $Nt=0.2$; $Lb=0.5$; $\delta_1=0.0$.**Figure 19.** δ_1 impact on $\chi(\eta)$ for the values $S=0.8$; $Ro=0.1$; $Le=0.2$; $M=0.0$; $Pr=1.0$; $Nt=0.2$; $Lb=0.5$; $\alpha=0.8$; $Ec=0.2$; $\beta=1.0$; $Nb=0.1$; $Pe=1.0$.

- A similar behavior is observed when increasing or decreasing the magnetic field or the caisson fluid profile;
- The increase in heat and solute thickness of layers is caused by the thermo-diffusion caused by nanoparticles;
- When both Nb and Nt are adjusted to greater levels, the heat transfer rate at the frame achieves its maximum. This suggests that raising the Nusselt number values centrals to an increased heat transfer rate at the frame;
- The concentration of nanoparticles decreases as the microbe concentration and Lewis value both increase;
- Increasing the changes of the Lewis number of bio-convection Lb , Peclet number Pe , and the microorganisms variance number δ_1 results in a decrease of the motile micro-organisms density field $\chi(\eta)$.

Nomenclature

β	Dimensionless casson fluid factor
C_w	Plate nanoparticles concentration
C_p	Specific heat
D_B	Brownian diffusion factor
D_T	Thermophoresis diffusion factor
Ec	Eckert number
k	Heat conductivity
Le	Lewis coefficient
Nb	Factor of Brownian motion
Nt	Factor of Thermophoresis
Pr	Prandtl coefficient
Re	Reynolds coefficient
Ro	Rotation parameter
T_w	Temperature of fluid
T_∞	Temperature of ambient
φ	Dimensionless nanoparticle concentration
ρC_p	Casson nanofluid heat capacity
$(\rho C_p)_p$	Nanoparticles heat capacity
A	Thermal diffusivity ($\frac{m^2}{s}$)
Np	Nanoparticles
Pe	Peclet number
M	Magnetic parameter
W_c	Maximum cell swimming speed
Dm	Microorganism coefficient
Lb	Concentration of microorganisms
B_0	Magnetic field
δ_1	microorganisms difference number
χ	density field of motile microorganisms

Authors contribution statement

First author: Resources; Software;

Second author: Data curation; Formal analysis; Investigation; Methodology; Writing - review and editing;

Third author: Data curation; Formal analysis; Investigation; Methodology; Writing - review and editing;

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