

Numerical investigation of the heat transfer and peristaltic flow through an asymmetric channel having variable viscosity and electric conductivity

Jamil Abbas Haider ^{a,*}, Sana Gul ^a, Sohail Nadeem ^{b,c}

a. Abdus Salam School of Mathematical Sciences, Government College University, Lahore, Pakistan.

b. Department of Mathematics, Quaid-i-Azam University, Islamabad, Pakistan.

c. Department of Mathematics, Wenzhou University, Wenzhou, P.R. China.

* Corresponding author: jamil@sms.edu.pk (J.A. Haider)

Received 12 May 2022; Received in revised form 06 July 2023; Accepted 09 October 2023

Keywords

Nanofluids;
 Heat transfer;
 Shooting Method;
 Peristaltic flow;
 Variable viscosity;
 Variable electric conductivity.

Abstract

The peristaltic flow of nanofluids is a topic of growing interest in fluid dynamics. This study investigates the effect of temperature-dependent viscosity and electric conductivity on the peristaltic flow of nanofluids. The mathematical model of the peristaltic flow is developed using the governing equations of continuity, momentum, and energy for a Newtonian fluid. Large wavelength and small Reynolds number (Re) assumptions are used to study peristaltic flow to simplify the equations of continuity, momentum, and energy. In this article, the nanofluids are assumed to be electrically conducting and temperature dependent, and the effects of Hartman number (Ha) and Eckert number (Ec) is studied. The resulting equations are solved using the Shooting Method. The results show that the temperature-dependent viscosity and electric conductivity significantly affect the peristaltic flow of nanofluids. The flow rate and pressure gradient decrease with increasing viscosity and conductivity while the temperature and heat transfer rate increase. Moreover, the nanofluid concentration and particle size significantly impact the flow characteristics. In conclusion, this study comprehensively analyses the peristaltic flow of nanofluids with temperature-dependent viscosity and electric conductivity. The results can be useful for understanding the behavior of nanofluids in various applications, such as drug delivery systems, microfluidics, and thermal management.

1. Introduction

The word "nanofluid" was first used by Choi and Eastman [1]. Nanofluid is a colloidal suspension containing nanoparticles with sizes ranging from 1 to 100 nanometers dispersed in a base fluid, such as water, oil, or ethylene glycol [2,3]. The nanoparticles can be made of various materials, including metals, metal oxides, and carbon-based materials. Nanofluids exhibit unique properties due to the nanoparticles' high surface area to volume ratio, which can enhance the base fluid's thermal conductivity, electrical conductivity, and viscosity [4]. The study of nanofluids has gained considerable attention in recent years due to their potential applications in various fields such as energy, biomedical, and industrial sectors [5]. Nanofluids have been studied as a promising candidate for heat transfer applications in the energy sector, as they exhibit higher thermal conductivity than conventional fluids [6]. In the

biomedical field, nanofluids have been studied as potential drug-delivery vehicles due to their ability to penetrate biological barriers [7].

The peristaltic flow of nanofluids is a field of fluid mechanics that combines the study of peristaltic motion, which is a type of fluid flow induced by periodic contraction and relaxation of a tube or channel, with the unique properties of nanofluids, which are colloidal suspensions containing nanoparticles dispersed in a base fluid [8]. The study of the peristaltic flow of nanofluids has gained considerable attention in recent years due to its potential applications in various fields, such as biomedical, industrial, and energy sectors [9]. The unique properties of nanofluids, such as high thermal conductivity and enhanced viscosity, can affect the behavior of peristaltic

To cite this article:

J.A. Haider, S. Gul and S. Nadeem "Numerical investigation of the heat transfer and peristaltic flow through an asymmetric channel having variable viscosity and electric conductivity", *Scientia Iranica* (2025) 32(13): 6783. <https://doi.org/10.24200/sci.2023.60404.6783>

flow. Additionally, the peristaltic motion can induce particle migration and concentration gradients in the fluid, leading to potential applications in drug delivery, microfluidics, and heat transfer [10].

Studying the peristaltic flow of nanofluids requires a multidisciplinary approach that combines fluid mechanics, nanoscience, and engineering. Various analytical and numerical techniques have been used to study the peristaltic flow of nanofluids, including perturbation methods, numerical simulations, and experimental measurements [11,12]. Understanding the peristaltic flow of nanofluids is of great importance for designing and optimizing microfluidic devices and heat transfer systems. The potential applications of peristaltic flow of nanofluids in the biomedical field, such as drug delivery systems and targeted therapy, have also generated significant interest among researchers [13,14]. The study of the peristaltic flow of nanofluids is a rapidly growing field that presents numerous opportunities for research and development, with potential applications in various areas, including energy, industrial, and biomedical sectors [15-18]. The peristaltic flow of nanofluids with heat transfer is a field of fluid mechanics that combines the study of peristaltic motion with the unique properties of nanofluids, which are colloidal suspensions containing nanoparticles dispersed in a base fluid, and heat transfer [19]. The study of the peristaltic flow of nanofluids with heat transfer has gained considerable attention in recent years due to its potential applications in various fields, such as energy, biomedical, and industrial sectors [20]. Peristaltic motion induced by periodic contraction and relaxation of a tube or channel can enhance heat transfer due to the mixing of the fluid and the generation of vortices. Additionally, the unique properties of nanofluids, such as high thermal conductivity, can further enhance heat transfer in peristaltic flow. Studying the peristaltic flow of nanofluids with heat transfer requires a multidisciplinary approach that combines fluid mechanics, nanoscience, and heat transfer. Various analytical and numerical techniques have been used to study the peristaltic flow of nanofluids with heat transfer, including perturbation methods, numerical simulations, and experimental measurements. Understanding the peristaltic flow of nanofluids with heat transfer is of great importance for the design and optimization of microfluidic devices and heat transfer systems, as well as for the development of new materials for heat transfer applications [21-23].

The potential applications of the peristaltic flow of nanofluids with heat transfer in the energy sector, such as in designing cooling systems for electronic devices and developing thermal energy storage systems, have generated significant interest among researchers [24]. In the biomedical field, the peristaltic flow of nanofluids with heat transfer can potentially be used in hyperthermia treatment for cancer therapy [25]. The study of the peristaltic flow of nanofluids with heat transfer is a rapidly growing field that presents numerous opportunities for research and development, with potential applications in various areas, including energy, biomedical, and industrial sectors [26]. The peristaltic flow of nanofluids with heat transfer having temperature-dependent viscosity and electric conductivity is an advanced field of fluid mechanics that combines the study of peristaltic motion with the unique properties of nanofluids and their temperature-dependent viscosity and electric conductivity. The analysis of the peristaltic flow of

nanofluids with heat transfer and variable properties has gained considerable attention in recent years due to its potential applications in various fields, such as energy, biomedical, and industrial sectors [27-29]. The properties of nanofluids, such as high thermal conductivity and enhanced viscosity, can affect the behavior of peristaltic flow [30]. Additionally, the peristaltic motion can induce particle migration and concentration gradients in the fluid, leading to potential applications in drug delivery, microfluidics, and heat transfer [31]. However, the properties of nanofluids can vary significantly with temperature, which can affect the behavior of peristaltic flow and heat transfer.

Studying the peristaltic flow of nanofluids with heat transfer having temperature-dependent viscosity and electric conductivity requires a multidisciplinary approach that combines fluid mechanics, nanoscience, heat transfer, and electromagnetics. Various analytical and numerical techniques have been used to study the peristaltic flow of nanofluids with variable properties, including perturbation methods, numerical simulations, and experimental measurements [32,33]. Understanding the peristaltic flow of nanofluids with heat transfer and variable properties is of great importance for the design and optimization of microfluidic devices and heat transfer systems [34]. The potential applications of peristaltic flow of nanofluids with variable properties in the energy sector, such as in designing cooling systems for electronic devices and developing thermal energy storage systems, have generated significant interest among researchers. In the biomedical field, the peristaltic flow of nanofluids with variable properties can potentially be used in hyperthermia treatment for cancer therapy and drug delivery systems [35].

Studying the peristaltic flow of nanofluids with heat transfer having temperature-dependent viscosity and electric conductivity is an emerging and challenging field that presents numerous opportunities for research and development, with potential applications in various areas, including energy, biomedical, and industrial sectors. The temperature-dependent viscosity and electric conductivity of nanofluids can significantly affect the behavior of peristaltic flow and heat transfer. The following are some of the results that have been observed in the study of the peristaltic flow of nanofluids with variable properties:

1. Temperature-dependent viscosity: The viscosity of nanofluids can increase significantly with increasing temperature due to Brownian motion and particle aggregation. This can affect the peristaltic motion and the generation of vortices, leading to changes in heat transfer characteristics. Furthermore, an increase in viscosity can also result in higher pressure drop, which can affect the pumping efficiency of the system.
2. Temperature-dependent electric conductivity: The electric conductivity of nanofluids can also vary with temperature due to changes in the mobility of ions and electrons. The peristaltic motion can induce electrical charges and create electric fields, leading to potential applications in electro kinetics and electroosmotic flow. The electric conductivity can also affect the Joule heating and the overall heat transfer characteristics of the system.
3. Particle migration and concentration gradients: The peristaltic motion can induce particle migration and con-

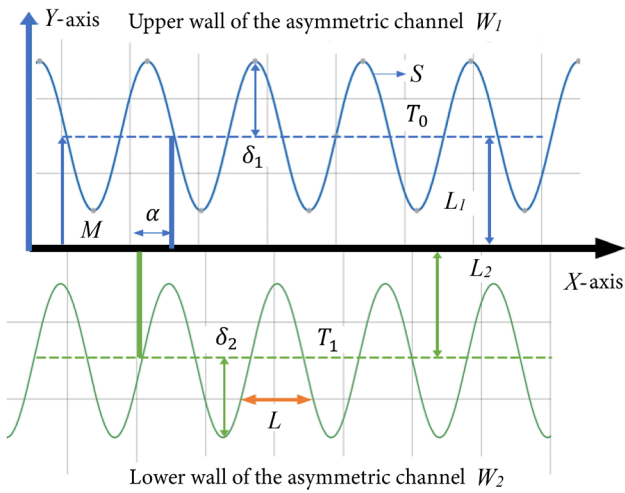


Figure 1. The problem's geometrical characteristics.

centration gradients in the fluid due to the combined effects of fluid flow and electric fields. The migration and concentration of nanoparticles can affect the thermal conductivity and heat transfer characteristics of the system.

4. Non-Newtonian behavior: Nanofluids can exhibit non-Newtonian behavior due to the presence of nanoparticles and their interaction with the base fluid. The non-Newtonian behavior can affect the peristaltic motion and the overall heat transfer characteristics of the system.

The temperature-dependent viscosity and electric conductivity of nanofluids can significantly affect the behavior of peristaltic flow and heat transfer. The study of the peristaltic flow of nanofluids with variable properties is an emerging field that presents numerous opportunities for research and development, with potential applications in various areas, including energy, biomedical, and industrial sectors.

2. Problem formulation

Flow of an incompressible nanofluid in a $l_1 + l_2$ width asymmetric channel. The flow is caused by the propagation of sinusoidal waves over the channel's non-conducting and flexible walls.

The mathematical equations for the channel walls, as illustrated in Figure 1, are given by [36]:

$$Y = W_1(X, t) = l_1 + \delta_1 \cos\left[\frac{2\pi}{\lambda}(X - st)\right],$$

$$Y = W_2(X, t) = -l_1 - \delta_2 \cos\left[\frac{2\pi}{\lambda}(X - st) + \alpha\right]. \quad (1)$$

In the case of rectangular coordinates, the length of the channel and the normal to its walls are used, while time coordinate is represented by t . The amplitudes of the upper and lower walls of the channel are denoted by the letters l_1 and l_2 , respectively. These amplitudes may be used to calculate the Reynolds number (Re), which is a dimensionless parameter that describes the ratio of inertial forces to viscous forces in the fluid. The Re is an important parameter in fluid mechanics, as it can determine the nature of the fluid flow behavior, such as laminar or turbulent flow. Overall, the choice of coordinate system and the parameters

used to describe fluid flow behavior depend on the specific problem being analyzed and the physical system being studied, while the phase difference is denoted by the letters $\alpha \in [0, \pi]$ and the wavelength is denoted by the letter λ . Furthermore, the amplitudes δ_1, δ_2 , the width of the channel $l_1 + l_2$ and the phase difference meets the following constraint: amplitudes δ_1, δ_2 , width of the channel $l_1 + l_2$. In addition, the magnetic field is present.

$$\sqrt{\delta_1 + \delta_2 + 2\delta_1\delta_2 \cos(\alpha)} \leq (l_1 + l_2). \quad (2)$$

It is assumed that constant wall temperature of the upper wall denoted T_0 and the constant temperature of the lower wall satisfies the following the condition $T_0 \leq T_1$. When a magnetic field is present, it can exert a force on the charged particles in the fluid, which in turn can affect the fluid flow behavior. To model the effects of a magnetic field in a fluid, the magnetic field strength is typically described using the magnetic field intensity, denoted by the symbol $M = [0; M; 0]$. Temperature-dependent viscosity and electric conductivity are two crucial properties that are at the heart of many scientific research studies. Viscosity is a measure of a fluid's resistance to flow, while electric conductivity is a measure of a material's ability to conduct electrical current. Both of these properties are highly dependent on temperature, which can have a significant impact on the behavior of materials and fluids. In many industrial and scientific applications, understanding the temperature-dependent behavior of viscosity and electric conductivity is essential. For example, in the petroleum industry, knowledge of viscosity is crucial for predicting the flow behavior of crude oil through pipelines. Similarly, in the field of materials science, understanding the electric conductivity of materials at different temperatures is important for developing new technologies such as solar cells and batteries. Research into the temperature-dependent behavior of viscosity and electric conductivity is ongoing, and new insights into these properties are continually being discovered. As scientists continue to deepen our understanding of these properties, we are able to better predict and control the behavior of materials and fluids, leading to advances in a wide range of industries and fields.

2.1. Mathematical modeling

The peristaltic flow of nanofluids with heat transfer can be mathematically modeled using the Navier-Stokes equations for fluid flow and the energy equation for heat transfer. The governing equations are supplemented by the continuity equation and the Maxwell equations for the electric field.

The continuity equation is presented in [37]:

$$\frac{\partial \rho}{\partial t} + \Delta(\rho u) = 0, \quad (3)$$

where ρ is the density, t is time, u is the velocity vector, and Δ is the gradient operator.

The Navier-Stokes equations are [38]:

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\left(\frac{1}{\rho}\right) \Delta p + \mu \Delta^2 u + f, \quad (4)$$

where p is the pressure, μ is the dynamic viscosity, and f is the body force per unit mass. For peristaltic flow, f is modeled as the wave velocity and amplitude.

The energy equation for heat transfer is given by [39]:

$$\rho c_p \left(\frac{\partial T}{\partial t} + u \cdot \Delta T \right) = \Delta \cdot (k \Delta T) + Q, \quad (5)$$

where c_p is the specific heat, T is the temperature, k is the thermal conductivity, and Q is the heat generation or absorption term.

For nanofluids, the viscosity and electric conductivity can be modeled as temperature-dependent functions. The viscosity can be modeled by the Vogel-Fulcher-Tamman (VFT) Eq. (6) [40]:

$$\mu(T) = \mu_0 \exp \left[\frac{\Delta E}{k_b T - T_0} \right], \quad (6)$$

where μ_0 is the viscosity at a reference temperature T_0 , ΔE is the activation energy, k_b is the Boltzmann constant, and T is the temperature.

Following [41], the electrical conductivity is modeled using the Arrhenius Eq. (7):

$$\sigma(T) = \sigma_0 \exp \left[-\frac{E_a}{k_b} \left(\frac{1}{T} - \frac{1}{T_0} \right) \right], \quad (7)$$

where σ_0 is the electric conductivity at a reference temperature T_0 , E is the activation energy, and T is the temperature.

The Maxwell equations for the electric field are given in [42]:

$$\Delta \cdot E = \rho_e / \epsilon_0, \quad \Delta \times E = -\frac{\partial B}{\partial t}, \quad (8)$$

where E is the electric field, ρ_e is the charge density, ϵ_0 is the electric constant, and B is the magnetic field.

Governing equations in cartesian form is written as which is modeled on the basis of temperature depend.

The governing equations for continuity, momentum, and energy are given below [43].

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (9)$$

$$\begin{aligned} \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = & \\ & - \frac{\partial p}{\partial x} + 2 \left[\frac{\partial \mu(T)}{\partial T} \left(\frac{\partial T}{\partial x} \frac{\partial u}{\partial x} \right) + \mu(T) \frac{\partial^2 u}{\partial x^2} \right] \\ & + \frac{\partial}{\partial y} \left[\mu(T) \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] - \sigma(T) M_0^2 u, \end{aligned} \quad (10)$$

$$\begin{aligned} \rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = & \\ & - \frac{\partial p}{\partial y} + 2 \left[\frac{\partial \mu(T)}{\partial T} \left(\frac{\partial T}{\partial y} \frac{\partial u}{\partial y} \right) + \mu(T) \frac{\partial^2 u}{\partial y^2} \right] \\ & + \frac{\partial}{\partial x} \left[\mu(T) \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right], \end{aligned} \quad (11)$$

$$(\rho s)_{nf} \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_{nf} \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] + \sigma(T) M_0^2$$

$$+ \mu(T) \left[\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + 2 \left(\frac{\partial u}{\partial x} \right)^2 + 2 \left(\frac{\partial u}{\partial y} \right)^2 \right]. \quad (12)$$

The following equations are used to express the frames that relate unsteady and steady flow phenomena, which are referred to as the fixed frame and the moving frame, respectively:

$$\begin{aligned} X &= \bar{X} + st, & Y &= \bar{Y} + st, & U(X, Y, t) &= \bar{U}(\bar{X}, \bar{Y}) + s, \\ V(X, Y, t) &= \bar{V}(\bar{X}, \bar{Y}), & P(X, Y, t) &= \bar{P}(\bar{X}, \bar{Y}), \\ T(X, Y, t) &= \bar{T}(\bar{X}, \bar{Y}). \end{aligned} \quad (13)$$

The non-dimensional parameters are provided as follows:

$$\begin{aligned} \xi &= \frac{\bar{X}}{\lambda}, & \nu &= \frac{V}{\tau s}, & w_2 &= \frac{w_2}{L_1}, & L &= \frac{L_2}{L_1}, \\ \gamma^* &= \gamma(\bar{T}_1 - \bar{T}_0), & \text{Pr} &= \frac{\mu_0 (S_p)_b}{k_b}, & \mu_{nf}(\theta) &= \frac{\mu_{nf}(\bar{T})}{\mu_0}, \\ \eta &= \frac{Y}{L_1}, & \tau &= \frac{L}{\lambda}, & \varepsilon_1 &= \frac{\delta_1}{L_1}, & p &= \frac{p L_1^2}{\lambda s \mu_0}, \\ \theta &= \frac{T - T_0}{T_1 - T_0}, & Ec &= \frac{s^2}{(S_p)_b (\bar{T}_1 - \bar{T}_0)}, \\ u &= \frac{\bar{U}}{s}, & w_1 &= \frac{W_1}{L_1}, & \varepsilon_2 &= \frac{\delta_2}{L_1}, & Ha^2 &= \frac{\sigma_0 M_0^2 L_1^2}{\mu_0}, \\ \text{Re} &= \frac{\rho s L_1}{\mu_0}. \end{aligned} \quad (14)$$

The following dimensionless equations are obtained by apply the assumptions of the long wavelength and low Re then also apply the stream function on the obtained governing equations such as:

$$u = \frac{\partial \psi}{\partial \eta}, \quad v = -\frac{\partial \psi}{\partial \xi}, \quad (15)$$

Also substituting the values of the $\mu_{nf}(\theta)$, $\sigma_{nf}(\theta)$ and κ_{nf} in the obtained governing equations such as:

$$\frac{\partial p}{\partial \xi} = A_1 \frac{\partial}{\partial \eta} \left((1 - \beta^* \theta) \frac{\partial^2 \psi}{\partial \eta^2} \right) - (1 + \gamma^* \theta) A_2 Ha^2 \left(\frac{\partial \psi}{\partial \eta} + 1 \right), \quad (16)$$

$$\frac{\partial p}{\partial \eta} = 0, \quad (17)$$

$$A_3 \left(\frac{\partial^2 \theta}{\partial \eta^2} \right) + A_1 \text{Pr} Ec (1 - \beta^* \theta) \left(\frac{\partial^2 \psi}{\partial \eta^2} \right) + \text{Pr} Ec Ha^2 A_2 (1 + \theta \gamma^*) = 0. \quad (18)$$

The values of the A_1 , A_2 , and A_3 are as follows:

$$\begin{aligned} A_1 &= \frac{1}{(1 - \chi)^{2.5}}, \\ A_2 &= \frac{\sigma_s + 2\sigma_0(1 + \gamma^* \theta) + 2\chi(\sigma_s - \sigma_0(1 + \gamma^* \theta))}{\sigma_s + 2\sigma_0(1 + \gamma^* \theta) - \chi(\sigma_s - \sigma_0(1 + \gamma^* \theta))}, \\ A_3 &= \frac{k_s + 2k_b + 2\chi(k_s - k_b)}{k_s + 2k_f - \chi(k_s - k_b)}. \end{aligned}$$

Eqs. (16) and (17) are simplified by removing the pressure gradient.

$$Ec = 0.1, \beta^* = 0.1, \gamma^* = 0.2, \epsilon_1 = 0.5, \epsilon_2 = 0.6,$$

$$L = 1, x = 0.3, K^* = 1.5, \alpha = \frac{\pi}{4}$$

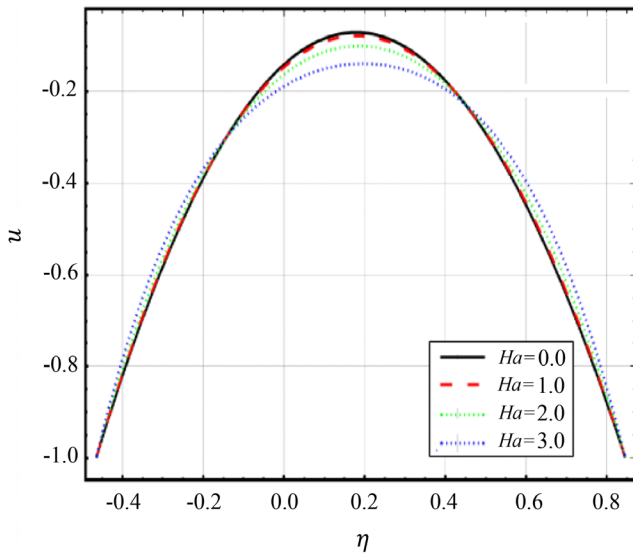


Figure 2. Variation in velocity $u(\xi, \eta)$ with Ha .

$$A_1 \left[1 - \beta^* \theta \frac{\partial^3 \psi}{\partial \eta^3} - \beta^* \frac{\partial^2 \psi}{\partial \eta^2} \frac{\partial^2 \theta}{\partial \eta^2} - 2\beta^* \frac{\partial^3 \psi}{\partial \eta^3} \right]$$

$$- A_2 Ha^2 \gamma^* \frac{\partial \theta}{\partial \eta} \left(\frac{\partial \psi}{\partial \eta} + 1 \right) - (1 + \gamma^* \theta) A_2 Ha^2$$

$$\left[(1 + \gamma^* \theta) A_2 Ha^2 \left(\frac{\partial \psi}{\partial \eta} + 1 \right) \right] = 0. \quad (19)$$

The dimensionless boundary conditions are given by:

$$\psi = -\frac{F}{2}, \quad \frac{\partial \psi}{\partial \eta} = 0, \quad \theta = 1 \quad \text{at } \eta = \omega_2,$$

$$\psi = -\frac{F}{2}, \quad \frac{\partial \psi}{\partial \eta} = 0, \quad \theta = 0 \quad \text{at } \eta = \omega_1, \quad (20)$$

$F = K^* - L$, where F and K^* are the dimensionless mean flow rates in the fixed and wave frames, respectively.

2.2. Solution methodology

The numerical approach, implemented in Maple 2022, employs the shooting technique to solve the simplified Eqs. (16)–(19) subject to the boundary conditions specified by Eq. (20).

3. Results and discussions

The impacts of different physical factors on heat transfer coefficient, dimensionless velocity $u(\xi, \eta)$, temperature $\theta(\xi, \eta)$, and axial pressure gradient $\frac{dp}{d\xi}$ were investigated and described in this section. Figures 2–13 show the influence of the Hartman number (Ha) on the velocity, temperature and axial pressure gradient respectively, in order to better understand the phenomenon. Each of these graphs represents a nanofluid of water ($-\text{Fe}_3\text{O}_4$). The Ha is a dimensionless parameter used to characterize the behavior of a conducting fluid in the presence of a magnetic field. It is defined as the

$$Ec = 0.1, \beta^* = 0.1, \gamma^* = 0.2, \epsilon_1 = 0.5, \epsilon_2 = 0.6,$$

$$L = 1, K^* = 1.5, \alpha = \frac{\pi}{4}$$

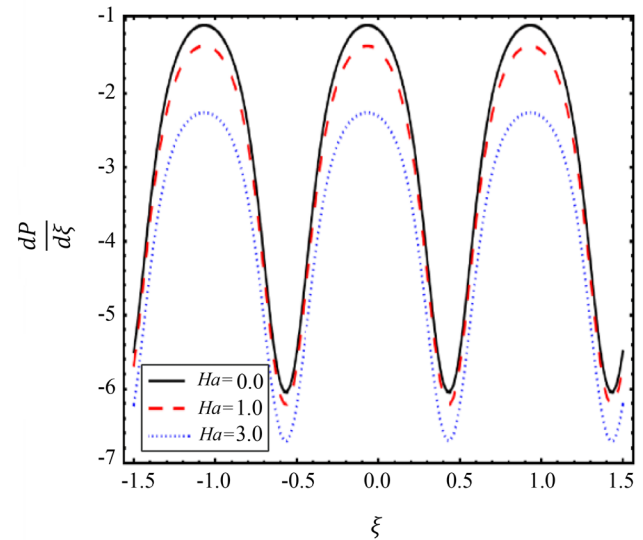


Figure 3. Variation in pressure gradient $\frac{dp}{d\xi}$ with Ha .

ratio of the magnetic field strength (B) to the velocity of the fluid $u(\xi, \eta)$ times the square root of the fluid's electrical conductivity (σ) and density (ρ):

$$Ha = \frac{B}{u(\xi, \eta)} * \sqrt{\rho^* \sigma}.$$

The relationship between velocity and Ha in Figure 2 can be analyzed by considering the effects of a magnetic field on the fluid flow. The magnetic field can cause the fluid to become more viscous and resistive, which in turn can affect the velocity of the fluid. In general, for a fixed magnetic field strength and electrical conductivity, as the velocity of the fluid increases, the Ha decreases. This is because the denominator of the Ha equation, $u(\xi, \eta) * \sqrt{\rho^* \sigma}$, increases as the velocity increases, leading to a smaller Ha value. Conversely, for a fixed velocity and electrical conductivity, as the magnetic field strength increases, the Ha increases. This is because the numerator of the Ha equation, B , increases as the magnetic field strength increases, leading to a larger Ha value. The relationship between velocity and Ha can have significant implications for the behavior of conducting fluids in magnetic fields. For example, at high Ha s, the magnetic field can dominate the fluid flow and suppress turbulence, while at low Ha s, the fluid flow can become turbulent and chaotic. Overall, the relationship between velocity and Ha is complex and depends on the specific conditions of the fluid flow and magnetic field. However, understanding this relationship can help in predicting and controlling the behavior of conducting fluids in the presence of magnetic fields. The pressure gradient, in Figure 3 on the other hand, is a measure of how the pressure of a fluid change with respect to distance. In general, a positive pressure gradient means that the pressure increases in the direction of flow, while a negative pressure gradient means that the pressure decreases in the direction of flow.

$$Ec = 0.1, \beta^* = 0.1, \gamma^* = 0.2, \epsilon_1 = 0.5, \epsilon_2 = 0.6,$$

$$L = 1, x = 0.3, K^* = 1.5, \alpha = \frac{\pi}{4}$$

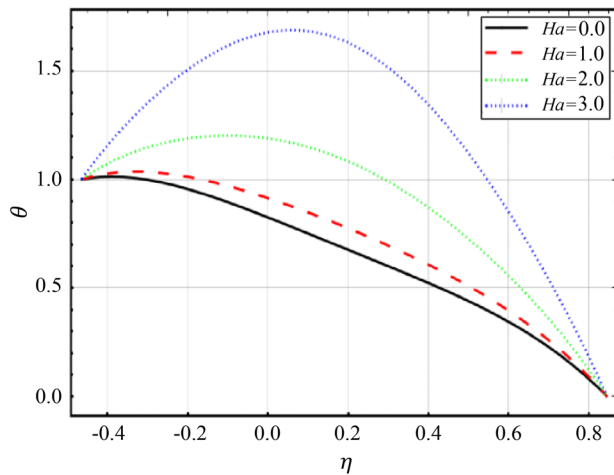


Figure 4. Variation in temperature $\theta(\xi, \eta)$ with Ha .

The relationship between the pressure gradient $\frac{dp}{d\xi}$ and Ha can be analyzed by considering the effects of a magnetic field on the fluid flow. The magnetic field can cause the fluid to become more viscous and resistive, which can in turn affect the pressure gradient of the fluid. In general, for a fixed magnetic field strength and electrical conductivity, as the pressure gradient increases, the Ha decreases. This is because the denominator of the Ha equation, $u(\xi, \eta) \sqrt{\rho^* \sigma}$ increases as the pressure gradient increases, leading to a smaller Ha value. Conversely, for a fixed pressure gradient and electrical conductivity, as the magnetic field strength increases, the Ha increases. This is because the numerator of the Ha equation, B , increases as the magnetic field strength increases, leading to a larger Ha value. The relationship between pressure gradient and Ha can have significant implications for the behavior of conducting fluids in magnetic fields. For example, at high Ha s, the magnetic field can dominate the fluid flow and suppress turbulence, while at low Ha s, the fluid flow can become turbulent and chaotic. Similarly, changes in the pressure gradient can affect the behavior of the fluid, including its stability and the formation of vortices and eddies. The trapping phenomenon is a well-known phenomenon that occurs in Magneto Hydro Dynamics (MHD), where a magnetic field is applied to a conducting fluid. The Ha , which is a dimensionless parameter, is used to characterize the strength of the magnetic field. In this discussion, we will examine the trapping phenomenon for various values of the Ha . The trapping phenomenon occurs when the magnetic field is strong enough to slow down the fluid flow in the direction perpendicular to the magnetic field lines, but not strong enough to completely stop it. This results in a build-up of fluid in Figure 4 in the direction perpendicular to the magnetic field, leading to the trapping phenomenon. The strength of the magnetic field is characterized by the Ha , which is given by:

$$Ha = B_0 d \sqrt{(\sigma / \rho \mu)},$$

$$Ec = 0.1, Ha = 1.5, \gamma^* = 0.1, \epsilon_1 = 0.5, \epsilon_2 = 0.6,$$

$$L = 1, x = 0.3, K^* = 1.5, \alpha = \frac{\pi}{4}$$

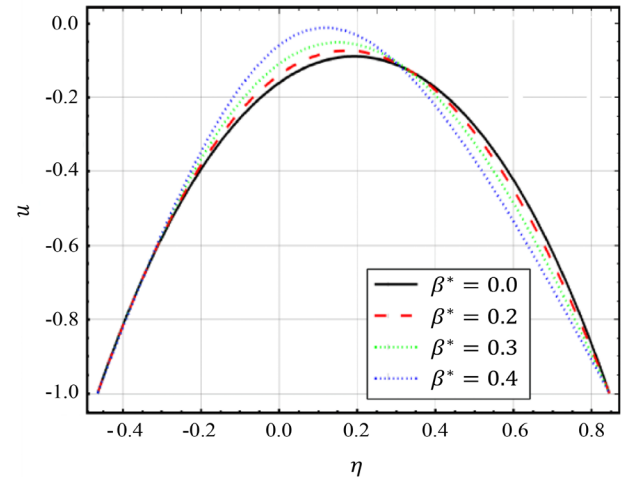


Figure 5. Variation in velocity $u(\xi, \eta)$ with variable viscosity.

where B_0 is the strength of the magnetic field, d is the characteristic length scale of the system, σ is the electrical conductivity of the fluid, ρ is the density of the fluid, and μ is the magnetic permeability of the fluid. When the Ha is small, the magnetic field is not strong enough to cause trapping, and the fluid flows freely. As the Ha increases, the magnetic field becomes stronger, and the fluid flow is slowed down. Eventually, a critical Ha is reached, at which point the trapping phenomenon occurs. Beyond this critical Ha , the fluid is trapped and does not flow freely anymore. The critical Ha depends on the specific system being studied, as well as the parameters of the system such as the fluid conductivity and density. However, in general, the critical Ha increases with increasing fluid conductivity and decreasing density. In conclusion, the trapping phenomenon is a well-known phenomenon that occurs in magnetohydrodynamics when the magnetic field is strong enough to slow down the fluid flow in the direction perpendicular to the magnetic field lines, but not strong enough to completely stop it. The strength of the magnetic field is characterized by the Ha , and the critical Ha at which the trapping phenomenon occurs depends on the specific system being studied, as well as the parameters of the system such as the fluid conductivity and density. In general, the Ha is used to analyze the effect of magnetic fields on the behavior of conducting fluids. The Ha varies with temperature because the electrical conductivity of the fluid depends on temperature as shown in the Figure 5. As the temperature $\theta(\xi, \eta)$ increases, the electrical conductivity generally increases as well. If we consider a system with a constant magnetic field strength and length scale, we can study how the Ha changes with temperature $\theta(\xi, \eta)$ by looking at the other parameters in the equation. As the electrical conductivity σ increases with temperature, the Ha will increase as well, since it appears in the denominator of the equation. On the other hand, the fluid density ρ and kinematic viscosity ν generally decrease with increasing temperature $\theta(\xi, \eta)$, which would tend to decrease the Ha .

$$Ec = 0.1, Ha = 0.5, \gamma^* = 0.2, \epsilon_1 = 0.5, \epsilon_2 = 0.6,$$

$$L = 1, K^* = 1.5, \alpha = \frac{\pi}{4}$$

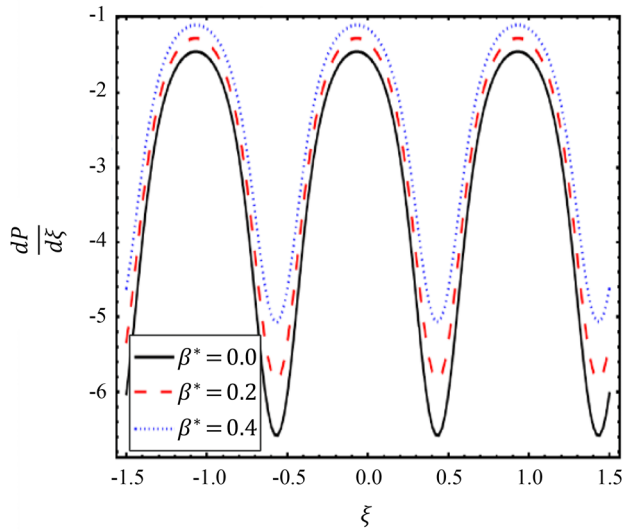


Figure 6. Variation in pressure gradient $\frac{dp}{d\xi}$ with variable viscosity.

$$Ec = 0.1, Ha = 1.5, \gamma^* = 0.1, \epsilon_1 = 0.5, \epsilon_2 = 0.6,$$

$$L = 1, x = 0.3, K^* = 1.5, \alpha = \frac{\pi}{4}$$

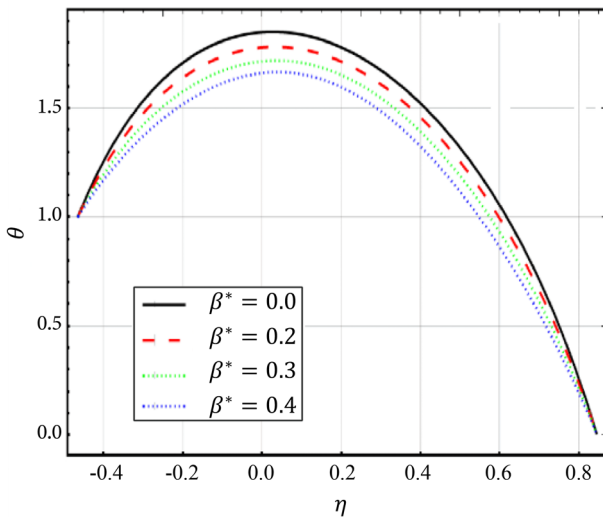


Figure 7. Variation in temperature $\theta(\xi, \eta)$ with variable viscosity.

Therefore, the net effect of temperature $\theta(\xi, \eta)$ on the Ha would depend on the relative magnitudes of these factors. Without more specific information about the system in question, it is difficult to say more about how the Ha might vary with temperature $\theta(\xi, \eta)$.

Figure 6 depicts the effect of the parameter velocity $u(\xi, \eta)$. The behavior on the wall surfaces is diametrically opposed. The velocity profile rises as the distance between the higher wall and the ground increases. With an increase in the variable viscosity parameter Figure 7, the amount of the pressure gradient reduces in magnitude. Fluid is trapped in the variable viscosity case as compared to the constant velocity case. Impact of variable viscosity parameter on the dimensionless temperature $\theta(\xi, \eta)$ is captured in Figure 8. Temperature of the fluid $\theta(\xi, \eta)$ decreases by increasing the variable viscosity parameter.

$$Ec = 0.1, Ha = 1.5, \beta^* = 0.1, \epsilon_1 = 0.5, \epsilon_2 = 0.6,$$

$$L = 1, x = 0.3, K^* = 1.5, \alpha = \frac{\pi}{4}$$

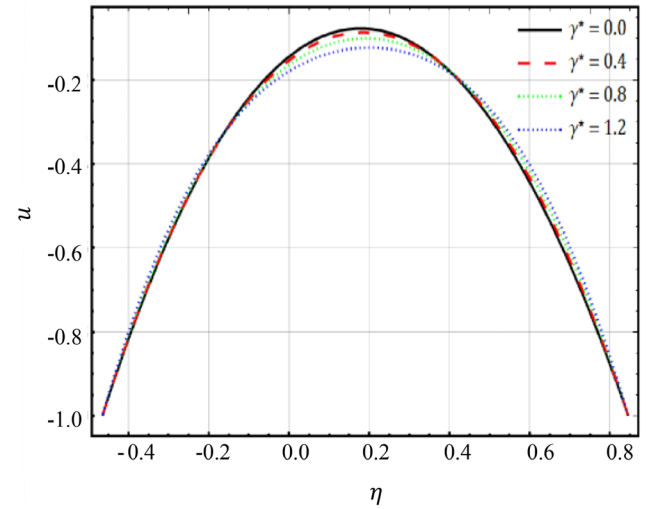


Figure 8. Variation in velocity $u(\xi, \eta)$ with variable electric conductivity.

$$Ec = 0.1, Ha = 0.5, \beta^* = 0.3, \epsilon_1 = 0.5, \epsilon_2 = 0.6,$$

$$L = 1, K^* = 1.5, \alpha = \frac{\pi}{4}$$

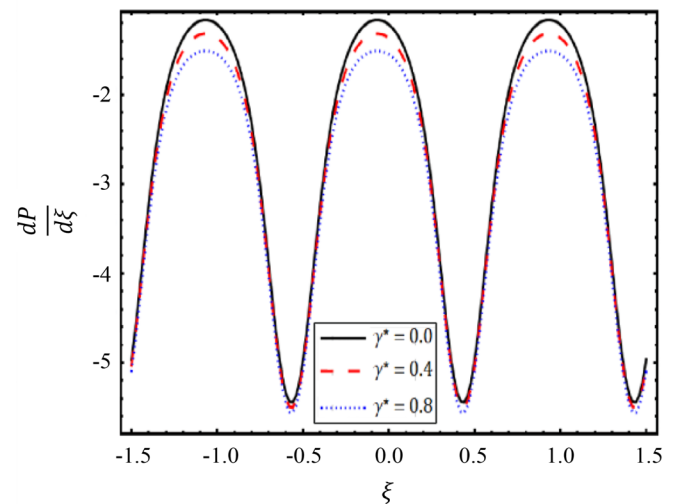


Figure 9. Variation in pressure gradient $\frac{dp}{d\xi}$ with variable electric conductivity.

Figure 9 is presented to see the variation in the velocity $u(\xi, \eta)$ with variable electric conductivity. Increase in electric conductivity results in velocity decrease in the vicinity center of channel. Because electrical conductivity enters in the momentum equation as a function of the Lorentz force, it may have the same effect as the Ha .

Figure 10 show that effect of variable electric conductivity parameter on magnitude of the pressure gradient $\frac{dp}{d\xi}$ is also similar to Ha , that is, it decreases by increasing electric conductivity. Size of trapped bolus also decreases by increasing γ^* .

Figure 11 is plotted to see the impact of electric conductivity on temperature. The graph exhibits decrease in temperature with the increase in γ^* .

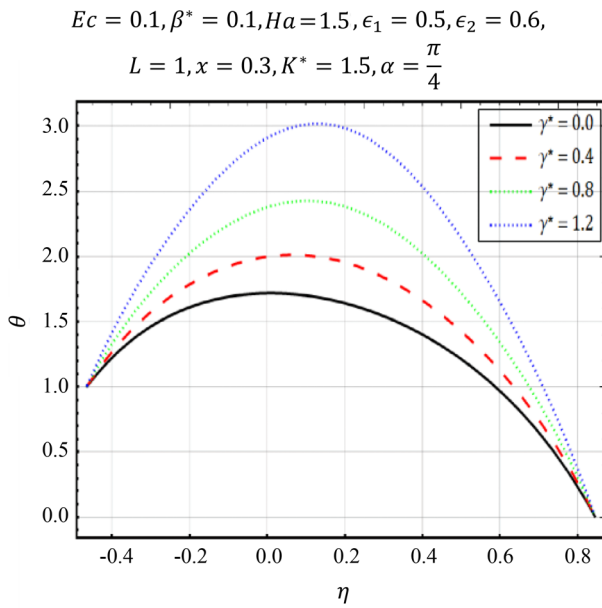


Figure 10. Variation in temperature $\theta(\xi, \eta)$ for electric conductivity.

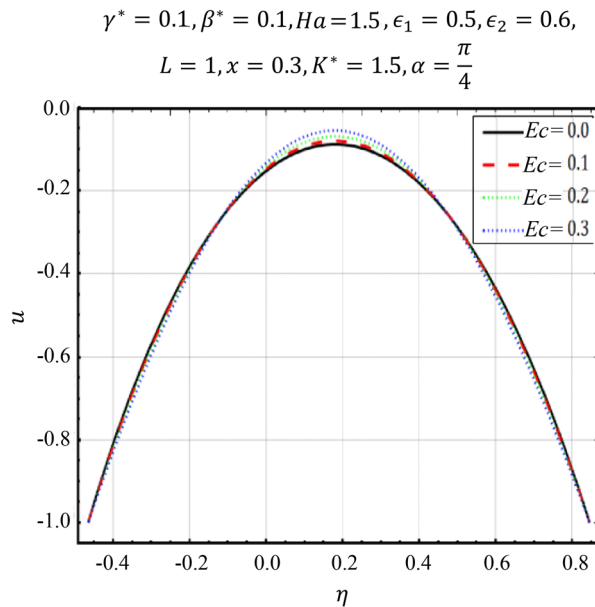


Figure 11. Variation in velocity $u(\xi, \eta)$ for Ec .

Figures 12 and 13 are displayed to demonstrate the effect of Eckert number (Ec) on the velocity $u(\xi, \eta)$, axial pressure gradient $\frac{dp}{d\xi}$ and the temperature $\theta(\xi, \eta)$, respectively. Ec appears in the energy equation when we incorporate the viscous dissipation effects. Due to viscous dissipation, kinetic energy is converted to heat energy in the flow process and increasing the Ec increases the heat energy. Therefore, both velocity $u(\xi, \eta)$ and temperature $\theta(\xi, \eta)$ of the fluid enhances by increasing the Ec . Magnitude of the axial pressure gradient $\frac{dp}{d\xi}$ decreases by increasing Ec .

4. Conclusion

This paper investigates the peristaltic transport of nanofluid via an asymmetric channel under the influence of a magnetic

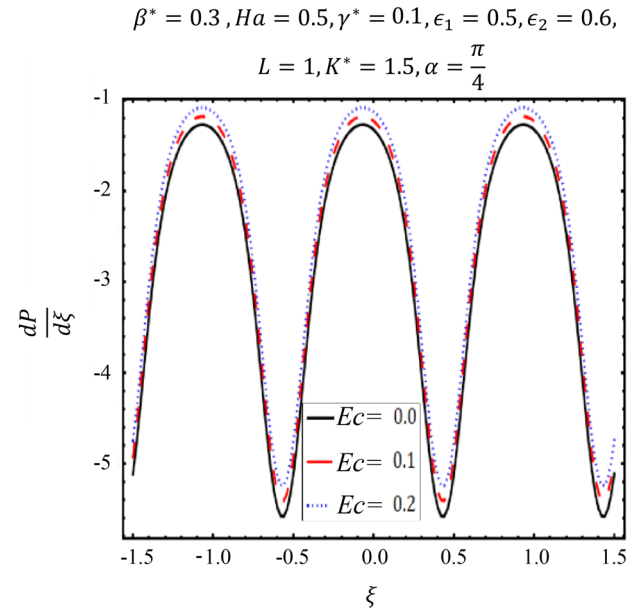


Figure 12. Variation in pressure gradient $\frac{dp}{d\xi}$ for Ec .

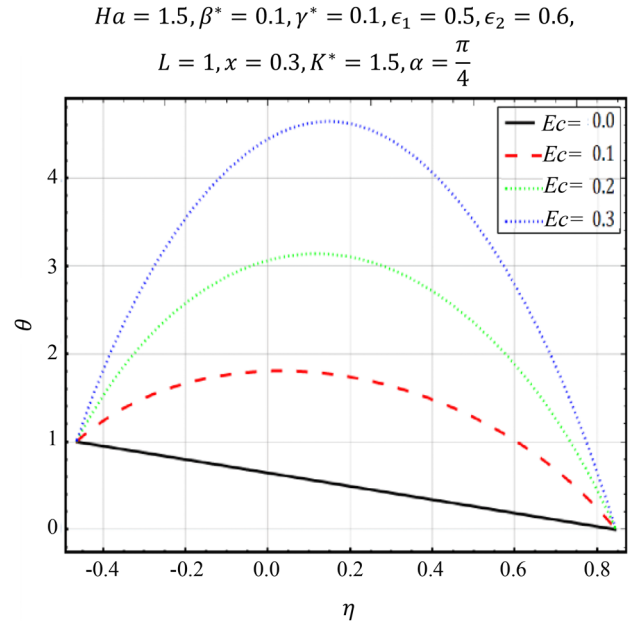


Figure13. Variation in temperature $\theta(\xi, \eta)$ for Ec .

field in which the viscosity and electric conductivity of the fluid vary. The effects of the many physical factors under consideration are analyzed, and their behavior is expanded via the use of graphs and tables. The following are the most significant findings:

- When nanoparticles are introduced to a base fluid, the rate of heat transmission rises significantly;
- The addition of 2 percent nanoparticles to the base fluid results in the greatest increase in heat transfer rate in the case of blood-Au nanofluid, which is 5.82 percent higher than the base fluid;
- The fluid's velocity falls as the Ha and the variable electric conductivity parameter increase;

- Increases in the Ha and the variable electric conductivity parameter both enhance the rate of heat transmission but increases in the variable viscosity parameter reduce the rate of heat transfer;
- When the variable viscosity parameter is increased, the magnitude of the pressure gradient increases, but the reverse tendency is seen when the variable electric conductivity parameter is increased;
- The size of the trapped bolus diminishes when the intensity of the magnetic field is increased.

Acknowledgment

The authors acknowledge their respective universities for their support.

Authors contribution statement

All three authors contributed to the conceptualization, methodology, solutions, and writing; however, the major work was performed by the corresponding and lead author.

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Biographies

Jamil Abbas Haider is a PhD scholar at the Abdus Salam School of Mathematical Sciences (AS-SMS), Government College University, Lahore, Pakistan. His research interests include applied mathematics, computational fluid dynamics, and dynamical systems, with a current focus on stochastic differential equations. He earned his Master's degree from COMSATS University Islamabad, Pakistan, under the supervision of Professor Saleem Asghar. He joined the Fluid Mechanics Lab at the Department of Mathematics, Quaid-i-Azam University, Islamabad, Pakistan, under the supervision of Professor Sohail Nadeem (Sitara-e-Imtiaz).

Sana Gul is a PhD scholar at the Abdus Salam School of Mathematical Sciences (AS-SMS), Government College University, Lahore, Pakistan. Her research interests in applied mathematics, and she has published several papers in international journals. She completed her Master's degree under the supervision of Professor Shamsul Qamar from COMSATS University Islamabad, Pakistan.

Sohail Nadeem is a Professor of Mathematics at Quaid-i-Azam University, Islamabad, Pakistan. His contribution to the field of applied mathematics is recognized at a national and international level. He has produced a good number of PhD and Master's students. He is also the editor of several prestigious international journals.