Scientia Iranica (2025) **32**(16): 5536

Sharif University of Technology
Scientia Iranica
Transactions on Industrial Engineering
https://scientiairanica.sharif.edu

A new artificial neural network approach for time series analysis

Najmeh Neshat [a,*], Hashem Mahlooji [b], Murat Kaya [c]

a. Department of Industrial Engineering, Meybod University, Yazd, Iran.
b. Department of Industrial Engineering, Sharif University of Technology, Tehran, Iran.
c. Program of Industrial Engineering, Sabanci University, Turkey.

* Corresponding author: neshat@meybod.ac.ir (N. Neshat)

Received 4 April 2021; Received in revised form 12 February 2022; Accepted 9 May 2022

Keywords

Time series;
Natural gas price;
Artificial Neural Networks (ANNs);
Fuzzy logic.

Abstract

Time series analysis and accurate forecasting of energy prices are critical for both policymakers and market participants. In the practical analysis of price time series, the coefficients play vital roles; however, their accurate estimation is a challenging issue, as they are affected by external factors. This study proposes a new modeling approach for Artificial Neural Networks (ANNs) models based on fuzzy logic. For this purpose, we reformulated an ANN model as a fuzzy Non-Linear Regression (NLR) model to capture the advantages of both fuzzy regression and ANN methodologies. This clear-box model can be applied not only to uncertain, ambiguous, and complex environments, but it is also capable of modeling nonlinear patterns. To illustrate the capability of the proposed approach, we report a case study of Liquefied Natural Gas (LNG) prices in Japan's market (as one of the world's largest natural gas importers). The results support that the performance of the proposed approach is acceptable; moreover, it can deal with uncertain and complex environments as a clear-box model.

1. Introduction

Analysis and forecasting of energy prices are at the heart of energy management, which enables policymakers to manage many frequent tactical decisions such as balancing energy demand and risk management. Price fluctuations influence the distribution and flow of various resources in the energy market and have substantial economic leverage; however, it is difficult to forecast price time series accurately due to their complicated and uncertain nature [1]. Accordingly, energy price analysis and forecasting has been the center of attention from both academic and practical points of view.

Different time series forecasting approaches, from statistical to computational intelligence, have been applied in the past [2]. Based on the number of techniques, the related literature is typically divided into two main categories: Stand-alone and hybrid methods, consisting of one and more than one technique, respectively. Based on the type of the underlying technique, stand-alone methods can be divided into three classes, namely Statistical, Causal and Computational intelligence methods. Statistical methods model the dynamic relationship between lagged values of determinants and the forecasted price based on historical data. Examples include Autoregressive (AR) and Double Seasonal Holt-Winter (DSHW) models [3], AR with Xogenous inputs (ARX) models [4], Threshold ARX (TARX) models [5], Generalized AR Conditional Heteroscedasticity (GARCH) based models [6-9], AR Integrated Moving Average (ARIMA) models [10,11], semi/nonparametric models [12], Seasonal AR Integrated Moving Average (SARIMA) [13], Transfer Function (TF) models [6], and grey models [14-16]. Certain hybrid versions of the mentioned methods are also suggested, e.g., wavelet-based models [14,17].

Causal methods focus on formulating the dynamic relationships between causal variables (determinants) and the forecasted energy price. These models may utilize the

To cite this article:
N. Neshat, H. Mahlooji, M. Kaya "A new artificial neural network approach for time series analysis", *Scientia Iranica* (2025) **32**(16): 5536. https://doi.org/10.24200/sci.2022.58046.5536

least-squares method [10] to determine the forecasted energy price in terms of its determinants and lagged data [18]. Different causal methods, e.g., Linear Regression (LR) [19], Non-Linear Regression (NLR) [20-23] and Logistic or Logit Regression (LoR) [24] have been widely utilized in the literature.

Energy price forecasting, similar to many real-life problems, has a nonlinear nature. Statistical and causal models, on the other hand, are usually linear in nature [25]; hence, they may not perform well with financial data, considering some features such as leptokurtosis, volatility clustering, and leverage effects. In addition, the nonlinear behavior of energy prices might become too complicated to predict [25]. However, statistical and causal models offer the advantage of having a clear internal logic; this is why they are referred to as "white-box" methods (the type of models for which one can clearly explain how they behave, how they produce predictions, and what the influencing variables are).

Computational intelligence methods such as Artificial Neural Network (ANN) and Support Vector Machine (SVM) are widely used in this research area due to their ability to handle the hidden features of data as well as their nonlinear modeling capability (e.g., [18], [26-28]). ANNs are examples of flexible regression approaches but they are different from the standard methods in some aspects such as: (1) They do not require a prior assumption of the model form in the model building process; (2) They provide better solutions for modeling complex non-linear relationships compared to the parametric models; and (3) They have high robustness when the scope of the appraisal is widened to include aspects such as outliers, non-linearity and other kinds of dependence among data. However, ANNs are not capable of handling the problems caused by uncertain situations, and they are identified as "black box" techniques. Uncertain conditions often arise due to the rapid development of new technologies or due to imprecise and inadequate data in energy pricing.

Fuzzy regression models are suitable to address such uncertainties; however, these models do not map the function with nonlinear behavior. A number of studies have used fuzzy time series to forecast energy prices in Australia and Singapore electricity markets [29], and Ontario and New England markets [30]; however, recent studies indicate that hybrid models of ANN and fuzzy systems outperform standalone fuzzy regression models [25,31-32]. These models of ANNs and fuzzy systems correspond to a fuzzy model of Takagi-Sugeno, wherein the weights of the neural network model are similar to the parameters of the fuzzy system, and hence, they behave as black box models. On the other hand, in clear-box models in which the coefficients are identified, in addition to the magnitude of the correlation between a certain response variable and the determinants, one can also infer the effects of each response variable.

In light of the above-mentioned considerations, we propose a reformulated ANN model as a nonlinear fuzzy time series capable of dealing with uncertain situations and nonlinear functions. The proposed model brings together the advantages of fuzzy Linear Regression (LR) and ANN models, while mitigating the shortcomings of these two models. As we illustrate with a case study on the Japanese Liquefied Natural Gas (LNG) market price, the flexibility of the model allows it to be applied to uncertain, ambiguous, or complex environments.

The remainder of this paper is organized as follows: In the remaining subsections of Section 1, the techniques embedded in the proposed approach are explained. Section 2 introduces the proposed modeling approach and Section 3 is devoted to experiments. Conclusions and final remarks are presented in Section 4.

1.1. Regression modelling

Generally, let x_{mn} and y_m represent the *n*th independent variable and the dependent variable, respectively, in the *m*th observation, β_i be the parameter associated with the *i*th independent variable (*i*=1,2,.., *n*), and ε_m be the error term associated with the *m*th observation. The classical regression model can be stated as Eq. (1):

$$y_m = \beta_0 + \beta_1 x_{m1} + ... + \beta_n x_{mn} + \varepsilon_m, \qquad m = 1,2,...,k. \tag{1}$$

The regression parameter β_i must usually be estimated from sample data. Let β_i be estimated under uncertainty in the adequacy of independent variables or sample data, and $X_0 = 1$, then β_i falls into the category of fuzzy regression analysis as Eq. (2):

$$\tilde{Y} = \tilde{\beta}_0 + \tilde{\beta}_1 X_1 + \tilde{\beta}_2 X_2 + ... + \tilde{\beta}_n X_n = \sum_{i=0}^{n} \tilde{\beta}_i X_i = X'\tilde{\beta}, \tag{2}$$

where $\tilde{Y}$ is the fuzzy output,

$$X' = [X_0, X_1, X_2, ..., X_n]^T,$$

is the real-valued input vector, and $\tilde{\beta} = \{\tilde{\beta}_0, \tilde{\beta}_1, ..., \tilde{\beta}_n\}$, is a set of fuzzy numbers which we want to determine according to some criteria of goodness of fit. In the form of triangular fuzzy numbers, for $\tilde{\beta}_i$, we have Eq. (3) as follows:

$$\mu_{\tilde{\beta}_i} = \begin{cases} 1 - \dfrac{|\alpha_i - \beta_i|}{c_i} & \text{if} \quad \alpha_i - c_i \le \beta_i \le \alpha_i + c_i \\ 0 & \text{otherwise;} \end{cases} \tag{3}$$

where $\mu_{\tilde{\beta}_i}$ is the membership function; the membership function is generally developed according to [33]; however, here i= {1, 2, … , n} of $\tilde{\beta}_i$, α_i is the center of the fuzzy number, and c_i is the width or spread around the center of the fuzzy number. According to the extension principle, the membership function of the fuzzy numbers $\tilde{y}_m = x_m'\tilde{\beta}$ can be indicated using pyramidal fuzzy parameter β as Eq. (4) [34]:

$$\mu_{\tilde{Y}} = \begin{cases} 1 - \dfrac{|y_m - x_m.\alpha|}{c'|x_m|} & \text{for} \quad x_m \ne 0 \\ 1 & \text{for} \quad x_m = 0; \quad y_m = 0 \\ 0 & \text{for} \quad x_m = 0; \quad y_m \ne 0 \end{cases} \tag{4}$$

where α and c denote vectors of model values and spreads, all model parameters, respectively. The problem of determining $\tilde{\beta}_i$ is formulated as a Linear Programming

(LP) model (Eq. (5)). To the best of our knowledge, this is the first LP formulation developed for estimating pyramidal fuzzy parameters [34]:

$$\text{Minimize} \quad S=\sum_{m=1}^{k} c'\left|x_m\right|$$

$$\text{Subject to} \begin{cases} x'_m \cdot \alpha+(1-h) c'\left|x_m\right| \geq y_m \\ x'_m \cdot \alpha-(1-h) c'\left|x_m\right| \leq y_m \\ c \geq 0 \end{cases} \tag{5}$$

where the objective function minimizes the total vagueness (*S*), defined as the sum of the individual spreads of the model's fuzzy parameters. Here, *k* denotes the number of observations, and *h* represents the threshold (or α -cut) of the membership function of *Y*. Fuzzy thresholds support multi-valued logic and can be implemented using mathematical functions that produce sigmoidal curves. This approach enables the definition of decision points along a continuum rather than at fixed boundaries. Further details on the basic operations and functions of fuzzy logic can be found in [35].

1.2. Neural Network (NN) models

NN models are a class of flexible nonlinear models that can find patterns adaptively from data. Through processing experimental data, these systems transfer the knowledge or rules hidden in data to the network's structure. They can predict a phenomenon's future by taking into account its history.

The relationship between the output y_m and the inputs $x_{m1}, x_{m2}, \ldots x_{mn}$ in a neural network model has the following mathematical representation as Eq. (6):

$$\begin{aligned} y_m &= f(W_0+\sum_{q=1}^{Q} w_{i,q} . g(W_q+\sum_{i=1}^{n} w_{i,q} . x_{mi})) \\ &= f(W_0+\sum_{q=1}^{Q} w_{i,q} . X_{mq})=f(\sum_{q=1}^{Q} W_q . X_{mq}), \end{aligned} \tag{6}$$

where $w_{i,q}(i=1,2, \ldots ., n ; q=1,2,3 ., \ldots, Q)$,

and $W_q(q=0,1,2, \ldots ., Q)$,

are model parameters called connection weights; *n* is the number of input nodes;

$X_{mq}=g(W_q+\sum_{i=1}^{n} w_{i,q} . x_{mi})$;

Q is the number of hidden neurons used as a logistic function. Also, transfer functions are used to model the behavior of intended functions and can be chosen based on LR or trial and error. In this sense, the neural network is equivalent to a nonlinear multiple regression model. In practice, a network consisting of one hidden layer that has a small number of hidden nodes generally works well in out-of-sample forecasting [35]. This may be due to the overfitting effect typically found in the neural network modeling process. An overfitted model has a good fit to the sample used in the model building process but has poor ability to generalize to data outside the sample [35].

To improve the performance of ANNs, certain data mining approaches are often applied in the network training process, namely, dynamic, multiple, prune, exhaustive prune data-mining methods, as well as networks with Radial Basis Function Network (RBFN) [36].

Dynamic method: In this method, first an initial topology for the network is created, and then during the training process this topology is modified by adding or deleting hidden parts of the network. This method is very suitable when there are nonlinear relationships between variables. Under these conditions, the dynamic model has a great deal of flexibility in regression applications (for prediction) to explain nonlinear behaviors of the response variable. In addition, the performance of this model is evaluated in situations where the observations are time series.

Multiple method: This method first creates several different topologies of possible scenarios (although their number depends on the data selected for training) and puts each of them in parallel in the training process. After the training phase, the model with the least squared errors is chosen as the final model.

Pruning method: The main idea of this type of network is that the efficiency of the network can be increased by eliminating some connections between neurons (weights). These networks consider a selection criterion, which is usually "the inverse of the network error" on a portion of the training data. If this value is increased, it indicates the correct progress of the network; otherwise, the network is growing to no avail and the algorithm stops. On the other hand, after adding new network neurons using this selection criterion, one of the weights is removed if this value is increased, which indicates that the decision is correct, and otherwise, the weight is returned. This method usually takes a long execution time, but improves the model by removing redundant variables and results in increased model generalization.

Exhaustive pruning data-mining method: Similar to the pruning method, the process begins with a very large network, and then the weakest units are removed from the input and hidden layers during the training process.

2. Formulating the proposed model

In this section, we explain how we reformulate an ANN model as a fuzzy NLR model that is capable of dealing with uncertain situations and nonlinear functions. This proposed model brings together the advantages of both fuzzy regression and ANN models while their mentioned deficiencies are decreased.

The steps are summarized as follows:

Step 1. Replace the crisp parameters
$w_{i,q}(i=0,1,2, \ldots ., n ; q=1,2,3 ., \ldots, Q)$,

and,

$W_q(q=0,1,2, \ldots ., Q)$,

with fuzzy parameters in the form of triangular fuzzy numbers:

$\tilde{w}_{i,q}(i=0,1,2, \ldots ., n ; q=1,2,3 ., \ldots, Q)$ and
$\tilde{w}_q(q=0,1,2, \ldots ., Q)$,

according to Eq. (2) as Eq. (7):

$$\tilde{y}_m = f(\tilde{w}_0 + \sum_{q=1}^{Q} \tilde{w}_q . g(\tilde{w}_{0,q} + \sum_{i=1}^{n} \tilde{w}_{i,q} . x_{mi}))$$
$$= f(\tilde{w}_0 + \sum_{q=1}^{Q} \tilde{w}_q . \tilde{X}_{mq}) = f(\sum_{q=0}^{Q} \tilde{W}_q . \tilde{X}_{mq}). \quad (7)$$

Step 2. Determine the membership function of the fuzzy parameters $\tilde{w}_{i,q} = (a_{iq}, b_{iq}, c_{iq})$ and $\tilde{w}_q = (d_q, e_q, f_q)$ in the form of triangular fuzzy numbers as shown in Eqs. (8) and (9), respectively, where $a<b<c$ are triangular fuzzy numbers and membership $b = 1$, while b need not be in the "middle" of a and c. Basic operations and functions on this fuzzy logic can be found in [33]:

$$\mu_{\tilde{w}_{i,q}} = \begin{cases} \dfrac{(w_{iq} - a_{iq})}{(b_{iq} - a_{iq})} & \text{if } a_{iq} \le w_{i,q} \le b_{iq} \\ \dfrac{(w_{iq} - c_{iq})}{(b_{iq} - c_{iq})} & \text{if } b_{iq} \le w_{i,q} \le c_{iq} \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

$$\mu_{w_q} = \begin{cases} \dfrac{(w_q - d_q)}{(e_q - d_q)} & \text{if } d_q \le w_q \le e_q \\ \dfrac{(w_q - f_q)}{(e_q - f_q)} & \text{if } e_q \le w_q \le f_q \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

Step 3. Calculate membership function of $\tilde{X}_{mq}$ in terms of $\mu_{\tilde{w}_{i,q}} (i = 0,1,2,....,n)$; $(q = 0,1,2,....,Q)$,

and,

$\mu_{\tilde{w}_q} (q = 0,1,2,....,Q)$,

through applying the extension principle [37,38], Eq. (5) can be reformulated as Eq. (10):

$$\mu_{\tilde{X}_{mq}} = \begin{cases} \dfrac{X_{mq} - \sum_{i=0}^{n} a_{iq} \cdot x_i}{\sum_{i=0}^{n} b_{iq} \cdot x_i - \sum_{i=0}^{n} a_{iq} \cdot x_i} \\ \text{if } \sum_{i=0}^{n} a_{iq} \cdot x_i \le X_{mq} \le \sum_{i=0}^{n} b_{iq} \cdot x_i \\ \dfrac{X_{mq} - \sum_{i=0}^{n} c_{iq} \cdot x_i}{\sum_{i=0}^{n} b_{iq} \cdot x_i - \sum_{i=0}^{n} c_{iq} x_i} \\ \text{if } \sum_{i=0}^{n} b_{iq} \cdot x_i \le X_{mq} \le \sum_{i=0}^{n} c_{iq} \cdot x_i \\ 0, \quad \text{otherwise} \end{cases} \quad (10)$$

The TFs (f and g) are assumed to be saturated linear TFs (Satlin), and the connected weights between the input and the hidden layers are considered to be of a crisp form for simplicity.

Step 4. Similar to Step 3, the membership function of $\tilde{y}_m$ is given as Eq. (11) and is modified considering Eq. (4) as Eq. (12).

$$\mu_{\tilde{y}_m} = \begin{cases} \dfrac{y_m - \sum_{q=0}^{Q} d_q \cdot X_{mq}}{\sum_{q=0}^{Q} e_q \cdot X_{mq} - \sum_{q=0}^{Q} d_q \cdot X_{mq}} \\ \text{if } \sum_{q=0}^{Q} d_q \cdot X_{mq} \le y_m \le \sum_{q=0}^{Q} e_q \cdot X_{mq} \\ \dfrac{y_m - \sum_{q=0}^{Q} f_q \cdot X_{mq}}{\sum_{q=0}^{Q} e_q \cdot X_{mq} - \sum_{q=0}^{Q} f_q \cdot X_{mq}} \\ \text{if } \sum_{q=0}^{Q} e_q \cdot X_{mq} \le y_m \le \sum_{q=0}^{Q} f_q \cdot X_{mq} \end{cases} \quad (11)$$

$$\mu_{\tilde{y}_m} = \begin{cases} 1 - \dfrac{\left| y_m - \sum_{q=0}^{Q} e_q \cdot X_{mq} \right|}{\sum_{q=0}^{Q} c_q \cdot \left| X_{mq} \right|} \\ 0 \end{cases} \quad (12)$$

Step 5: Finally, the criteria of minimizing total vagueness, S (the sum of individual spreads of the fuzzy parameters) is used according to Eq. (5) to develop the LP problem as follows where h is the threshold of the membership function of $\tilde{Y}$:

Minimize $S = \sum_{m=1}^{k} \sum_{q=0}^{Q} (e_q - d_q) \cdot \left| X_{mq} \right|$

Subject to:

$$\begin{cases} \sum_{q=0}^{Q} e_q . X_{mq} + (1-h)(\sum_{q=0}^{Q} (e_q - d_q) \cdot \left| X_{mq} \right|) \ge y_m \\ m = 1,2,....,k \\ \sum_{q=0}^{q} e_q . X_{mq} - (1-h)(\sum_{q=0}^{Q} (e_q - d_q) \cdot \left| X_{mq} \right|) \le y_m \\ q = 0,1,2,....,Q \end{cases} \quad (13)$$

3. Experiments

In recent decades, the popularity of natural gas has increased considerably because it causes no local pollutants (that is, being a "clean fuel"), and because it generates relatively lower carbon emissions compared to coal. Rapid rise in global gas demand has turned the gas markets from a "buyer's market" into a "seller's market". Therefore, the study and forecasting of gas prices based upon actual data and market information, using efficient modeling and forecasting approaches has become a timely and important area of study.

In this section, we study the LNG price in the Japanese market to test our proposed forecasting model. Japan is the world's largest LNG importer and one of the three largest gas markets worldwide. Nonlinearity and complexity of the

LNG price function and the uncertainty due to lack of data on influencing variables cause LNG price forecasting to be a suitable candidate for using the fuzzy logic approach [39].

Based on the related literature [18,40-43], 15 direct and indirect (auxiliary) determinants were identified, which are provided in Table 1 together with their statistical summary information. The related data were collected from February 2008 to 2020 in monthly terms from the International Energy Consortium (IEC) database. The data from 2008 to 2017 was used for training and the rest of the dataset was used for testing. Outliers were removed from the training dataset.

To estimate the weights of the related ANN model, five well-known data mining approaches (dynamic, multiple, prune, exhaustive prune, and RFBN) were applied. To reach the best network configuration using the Neuro Toolbox in MATLAB software, different networks including 1-3 hidden layers, a variable number of neurons in hidden layers, and different activation functions were tested. Finally, the network with the least Mean Square Error (MSE) for each approach was selected as the optimum network.

The values of the training termination criteria are: $1 * E(-5)$ for MSE; 200 for the maximum number of epochs; and $1 * E(-10)$ for the minimum error gradient of each epoch. Table 2 provides further information to compare the relative importance of the input factors for each neural network model.

The values of performance indicators Mean Absolute Percentage Error (MAPE), Root Mean Square Error (RMSE) and coefficient of determination (R^2) for the five networks are provided in Table 3. One of these models is chosen to examine the proposed approach, where MAPE shows the mean ratio between the error and the actual values are calculated as Eq. (14):

$$\text{MAPE} = \frac{1}{m}\sum_{i=1}^{m}\left|100\frac{(y_i - \hat{y}_i)}{y_i}\right|, \tag{14}$$

where $\hat{y}_i$ is the predicted value by the ANN model y_i is the actual value of the response process and m is the number of points in the data set. The RMSE is calculated by Eq. (15):

Table 1. Statistical information on influencing variables of LNG price in the Japanese market.

Influencing variables	Unit	Mean value	Std. dev	Interval		Mean growth rate
				Max	Min	
British ICE natural gas price	MMBtu/$	4.95	2.80	14.33	1.66	0.10
Henry Hub natural gas price	MMBtu/$	4.33	2.81	13.71	1.41	0.13
WTI oil price	MMBtu/$	6.65	4.46	23.16	1.90	0.11
Brent oil price	MMBtu/$	6.45	4.46	22.96	1.70	0.12
Mean price of Japan's imported oil	MMBtu/$	6.65	4.65	22.78	1.94	0.16
Mean oil price of Oman and Dubai	MMBtu/$	6.44	4.65	22.57	1.73	0.16
Price of Liquified Petroleum Gas (LPG)	MMBtu/$	6.96	3.90	19.18	2.17	0.09
Oil fired in oven (1%)	MMBtu/$	4.63	3.02	17.37	1.48	0.11
Oil fired in oven (3.5%)	MMBtu/$	4.09	2.80	15.79	1.32	0.10
Gasoline price (0.2%)	MMBtu/$	7.90	5.58	28.18	2.16	0.13
Oil fired oven (180)	MMBtu/$	3.91	2.71	14.89	1.09	0.12
International gas consumption	TCM	0.210	0.020	0.25	0.18	0.02
International gas production	BCM	0.213	0.023	0.25	0.18	0.025
International oil consumption	MMbl/d	6.543	0.454	7.32	5.79	0.017
International oil production	MMbl/d	6.359	0.402	7.01	5.68	0.015

Table 2. Comparison of the relative importance of input variables for each neural network model.

Input variables	Dynamic	Multiple	Prune	Exhaustive prune	RFBN
British ICE natural gas price	0.098	0.185	0.056	-	0.109
Henry Hub natural gas price	0.158	0.505	0.434	-	0.117
WTI oil price	0.042	0.042	0.068	-	0.110
Brent oil price	0.085	0.056	0.102	0.164	0.112
Mean price of Japan's imported petroleum	0.177	0.174	0.183	0.285	0.115
Mean oil price of Oman and Dubai	0.021	0.089	-	0.152	0.116
Price of Liquid Petroleum Gas (LPG)	0.032	0.054	0.079	-	0.110
Oil fired oven (1%)	0.114	0.055	0.118	-	0.112
Oil fired oven (3.5%)	0.234	0.182	0.138	-	0.129
Gasoline price (0.2%)	0.009	0.174	-	0.549	0.116
Oil fired oven (180)	0.024	0.115	-	-	0.140
International gas consumption	0.021	0.179	0.159	-	0.140
International gas production	0.071	0.103	-	-	0.138
International oil consumption	0.078	0.149	-	-	0.137
International oil production	0.0175	0.194	0.16	-	0.115

Table 3. Numerical results of performance indicators R^2, RMSE, and MAPE for ANN models.

Network \ Indicator	R^2	RMSE	MAPE
Dynamic	0.8441	0.017	1.812
Multiple	0.8743	0.012	1.211
Prune	0.8722	0.012	1.287
Exhausted prune	0.8675	0.016	1.521
REBN	0.8125	0.019	1.803

$$\text{RMSE} = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(y_i-\hat{y}_i)^2}\,. \tag{15}$$

Finally, R^2 or the coefficient of determination, as a statistical criterion that can be applied to multiple regression analysis, is calculated by Eq. (16):

$$R^2 = 1-\left(\frac{\sum_{i=1}^{m}(y_i-\hat{y}_i)^2}{\sum_{i=1}^{m}(y_i-\bar{y})^2}\right), \tag{16}$$

R^2 takes a value between zero and one, where values closer to one indicate a good fit.

Exhaustive prune network is chosen with 4 input variables, which are "Brent oil price", “mean oil price of Oman and Dubai”, “gasoline price”, and “mean price of Japan’s imported petroleum”. As the “Brent oil price” and “mean price of Japan’s imported oil” variables have a correlation coefficient of 67.8%, “Brent oil price” was eliminated and the variable “the related year” was added to the input variables, considering the performance of the network.

The best configuration of the exhaustively pruned network is determined to be $N^{(4\text{-}4\text{-}1)}$ which is shown in Figure 1, in a $N^{(x\text{-}y\text{-}z)}$ network topology, x, y and z stand for the first, the second and the third layer`s neurons, respectively. $N^{(4\text{-}4\text{-}1)}$ refers to a network consisting of 4 neurons for the input layer, 4 neurons for the hidden layer, and one neuron for the output layer. The weights and biases of the mentioned network are given in Table 4.

As mentioned, the input weights and biases of the exhausted pruned model are considered in a crisp form for simplicity. To obtain the center and the interval width for hidden weights (i.e., c_q and e_q) as unknown parameters with the assumption of h = 0.5, the proposed method is formulated as the following LP problem:

Minimize $$S=\sum_{m=1}^{70}\sum_{q=0}^{4}c_q\left|X_{mq}\right|$$

Subject to $$\begin{cases}\sum_{q=0}^{4}e_q.X_{mq}+(0.5)(\sum_{q=0}^{4}c_q\left|X_{mq}\right|)\geq y_m\\ \sum_{q=0}^{4}e_q.X_{mq}-(0.5)(\sum_{q=0}^{4}c_q\left|X_{mq}\right|)\leq y_m\end{cases}$$

$m=1,2,....,70$

$q=0,1,2,....,4$

where m is the number of training data and q is the number of neurons in the hidden layer of exhaustive prune networks. The center and width of hidden weights (c_q and e_q) obtained using the General Algebraic Modeling System (GAMS) Software are presented in Table 5.

To evaluate the performance of the proposed model, the center of hidden weights (c_q) was considered as weights (w_q) for calculating y_m according to Eq. (17) using the test data:

Table 5. Center and width of hidden weights w_q.

	e_q	c_q
w_0	-1.6221	0.478
w_1	-2.6790	0.882
w_2	0.0402	0.016
w_3	14.6723	2.583
w_4	5.2310	1.771

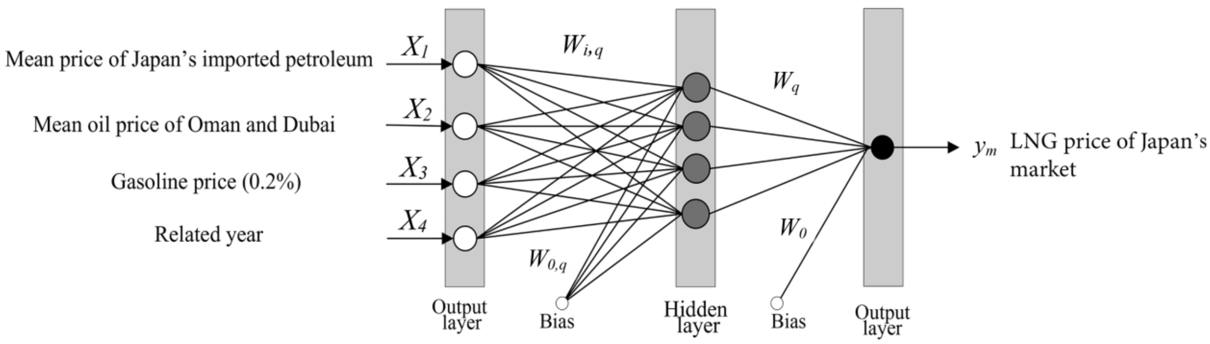

Figure 1. Network structure of the exhaustive prune model.

Table 4. The weights and biases of the exhaustive prune model.

Input weights				Hidden weights	Biases	
$w_{i,1}$	$w_{i,2}$	$w_{i,3}$	$w_{i,4}$	w_q	$w_{0,j}$	w_0
-2.5677	10.3961	-2.0112	16.0023	-2.6790	-13.2756	-1.6221
1.3113	17.2834	8.5331	-0.2344	0.0402	3.0111	
4.6701	-1.8766	-11.9012	14.7558	14.6723	-8.3783	
0.2349	20.9123	-2.7663	5.03777	5.2310	-5.2311	

Table 6. Numerical results of performance indicators R^2, RMSE, and MAPE for the proposed model and the alternative models.

Indicator \ Model	Proposed	Exhausted prune ANN	Fuzzy regression	Linear regression
R^2	0.8701	0.8675	0.7901	0.7733
MAPE	1.491	1.521	2.2988	2.4611
RMSE	0.014	0.016	0.019	0.021

$$y_m = f(0.478+\sum_{q=1}^{4} c_q.X_{mq}) = Satlin(\sum_{q=0}^{4} c_q.X_{mq}) = \sum_{q=0}^{4} c_q.X_{mq}. \quad (17)$$

The results of performance evaluation of the proposed models using 30 test data are provided in Table 6.

Note that considering the center of hidden weights (c_q) leads to the crisp y_m. Also, Satlin is a neural transfer function. This transfer function calculates a layer's output from its net input. It clips the input to [-1,1]. This function is chosen based on trial and error.

Comparing the values of performance indicators in Table 6, we observe that the proposed model achieves an improvement in R^2 of 0.26%, a decrease in MAPE of 3%, and a decrease in RMSE of 2 US Dollars (USD) compared with the exhausted prune ANN model. These results validate that reformulating the ANN model based on the fuzzy regression model tends to improve the performance of the ANN model. The ANN model, in turn, outperforms both the linear and the fuzzy regression models. The proposed model also offers the advantage of addressing uncertainty and complexity as a clear box model.

4. Conclusion and final remarks

Emerging computational intelligent approaches have led to significant advances in forecasting energy price time series. In this regard, Artificial Neural Network (ANN) models have been extensively used; however, these models are identified as black box techniques, and they are not capable of handling problems that involve uncertainty. Fuzzy regression models, on the other hand, are a good choice for addressing uncertainty; however, they cannot map functions with nonlinear behavior.

In this study, we reformulated an ANN model as a fuzzy Non-Linear Regression (NLR) model that can deal with both uncertain situations and nonlinear functions. This novel modeling approach can be applied to uncertain, ambiguous, and complex environments due to its flexibility. As a case study, the price function of Liquefied Natural Gas (LNG) in the Japanese market was formulated based on the proposed modeling approach. The performance of the proposed model was compared to those of conventional approaches such as the selected neural network, fuzzy regression, and linear autoregressive models. Based on a number of performance indicators, the proposed model is shown to have the best performance.

To sum up, the proposed approach is applicable to complex, nonlinear, and ambiguous environments due to embedded ANN and fuzzy mechanisms. It is robust to inconsistency and noise in data as well as the presence of high dimensionality and collinearity. Besides, it performs as a clear box model and provides higher generalization capability. The ability to deal with uncertain, limited, and non-crisp data would allow this approach to be preferred over conventional alternatives.

Funding
This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Conflicts of interest
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Authors contribution statement

First author: Conceptualization, Data curation, Formal analysis, Methodology, Visualization, Writing - original draft.

Second author: Supervision, Validation, Investigation.

Third author: Conceptualization, Writing-Review and editing.

References

1. Chen, M.-Y. and Chen, B.-T. "A hybrid fuzzy time series model based on granular computing for stock price forecasting", *Information Sciences*, **294**, pp. 227-241 (2015). https://doi.org/10.1016/j.ins.2014.09.038

2. Lu, H., Ma, X., Ma, M., et al. "Energy price prediction using data-driven models: A decade review", *Computer Science Review*, **39**, 100356 (2021). https://doi.org/10.1016/j.cosrev.2020.100356

3. Ryu, S., Noh, J., and Kim, H. "Deep neural network-based demand side short term load forecasting", *Energies*, **10**(1), 3 (2016). https://doi.org/10.3390/en10010003

4. Ziel, F. and Steinert, R. "Probabilistic mid- and long-term electricity price forecasting", *Renewable and Sustainable Energy Reviews*, **94**, pp. 251-266 (2018). https://doi.org/10.1016/j.rser.2018.05.038

5. Lago, J., De Ridder, F., and De Schutter, B. "Forecasting spot electricity prices: Deep learning approaches and empirical comparison of traditional algorithms", *Applied Energy*, **221**, pp. 386-405, (2018). https://doi.org/10.1016/j.apenergy.2018.02.069

6. Nogales, F.J. and Conejo, A.J. "Electricity price forecasting through transfer function models", Journal of the Operational Research Society, **57**(4), pp. 350-356 (2017). https://doi.org/10.1057/palgrave.jors.2601995

7. Keynia, F. and Heydari, A. "A new short-term energy price forecasting method based on wavelet neural network", *IJMOR*, **14**(1), pp. 1-14 (2019). https://doi.org/10.1504/IJMOR.2019.096975

8. Kumar, V., Singh, N., Singh, D., et al. "Short-term electricity price forecasting using hybrid SARIMA and GJR-GARCH model", in *Networking Communication and Data Knowledge Engineering*, pp. 299-310 (2018). https://doi.org/10.1007/978-981-10-4585-1_25

9. Yang, Z., Ce, L., and Lian, L. "Electricity price forecasting by a hybrid model, combining wavelet transform, ARMA and kernel-based extreme learning machine methods", *Applied Energy*, **190**, pp. 291-305 (2017). https://doi.org/10.1016/j.apenergy.2016.12.130

10. Lin, L., Jiang, Y., Xiao, H., et al. "Crude oil price forecasting based on a novel hybrid long memory GARCH-M and wavelet analysis model", *Physica A: Statistical Mechanics and its Applications*, **543**, 123532 (2020). https://doi.org/10.1016/j.physa.2019.123532

11. Zhao, Z., Wang, C., Nokleby, M., et al. "Improving short-term electricity price forecasting using day-ahead LMP with ARIMA models", *2017 IEEE Power & Energy Society General Meeting*, Chicago, USA, pp 1-5 (2018). https://doi.org/10.1109/PESGM.2017.8274124

12. Trespalacios, A., Cortés, L.M., and Perote, J. "Uncertainty in electricity markets from a semi-nonparametric approach", *Energy Policy*, **137**, 111091 (2020). https://doi.org/110.1016/j.enpol.2019.111091

13. Zhang, J., Tan, Z., and Wei, Y. "An adaptive hybrid model for short term electricity price forecasting", *Applied Energy*, **258**, 114087 (2020). https://doi.org/10.1016/j.enpol.2019.111091

14. Chen, Y., Zhang, C., He, K., et al. "Multi-step-ahead crude oil price forecasting using a hybrid grey wave model", *Physica A: Statistical Mechanics and its Applications*, **501**, pp. 98-110 (2018). https://doi.org/10.1016/j.physa.2018.02.061

15. Lei, M. and Feng, Z. "A proposed grey model for short-term electricity price forecasting in competitive power markets", *International Journal of Electrical Power & Energy Systems*, **43**(1), pp. 531-538 (2012). https://doi.org/ 10.1016/j.ijepes.2012.06.001

16. Lei, M. and Feng, Z. *A Novel Grey Model to Short-Term Electricity Price Forecasting for Nord Pool Power Market*, *2009 IEEE International Conference on Systems, Man and Cybernetics*, San Antonio, USA, pp. 4347-4352 (2009). https://doi.org/ 10.1109/ICSMC.2009.5346948

17. Shrivastava, N. and Panigrahi, B. "A hybrid wavelet-ELM based short term price forecasting for electricity markets", *International Journal of Electrical Power & Energy Systems*, **55**, pp. 41-50 (2014). https://doi.org/10.1016/j.ijepes.2013.08.023

18. Su, M., Zhang, Z., Zhu, Y., et al. "Data-driven natural gas spot price forecasting with least squares regression boosting algorithm", *Energies*, **12**(6), 1094 (2019). https://doi.org/10.3390/en12061094

19. Saini, D., Saxena, A., and Bansal, R.C. "Electricity price forecasting by linear regression and SVM", *International Conference on Recent Advances and Innovations in Engineering (ICRAIE)*, Jaipur, India, pp. 1-7 (2016). https://doi.org/10.1109/ICRAIE.2016.7939509

20. Atef, S. and Eltawil, A. "A comparative study using deep learning and support vector regression for electricity price forecasting in smart grids", in *2019 IEEE 6th International Conference on Industrial Engineering and Applications (ICIEA)*, pp. 603-607 (2019). https://doi.org/10.1109/IEA.2019.8715213

21. Kuo, P.-H. and Huang, C.-J. "An electricity price forecasting model by hybrid structured deep neural networks", *Sustainability*, **10**(4), 1280 (2018). https://doi.org/10.3390/su10041280

22. Misiorek, A., Trueck, S., and Weron, R. "Point and interval forecasting of spot electricity prices: Linear vs. non-linear time series models", *Studies in Nonlinear Dynamics & Econometrics*, **10**(3), pp. 1-36 (2007). https://doi.org/110.2202/1558-3708.1362

23. Zareipour, H., Bhattacharya, K., and Canizares, C. "Forecasting the hourly Ontario energy price by multivariate adaptive regression splines", in *2006 IEEE Power Engineering Society General Meeting* (2006). https://doi.org/10.1109/PES.2006.1709474

24. Tahir, A., Khan, Z., Javaid, N., et al. "Load and price forecasting based on enhanced logistic regression in smart grid", in *Advances in Internet, Data and Web Technologies*, Cham, pp. 221-233 (2019). https://doi.org/10.1007/978-3-030-12839-5_21

25. Itaba, S. and Mori, H. "A Fuzzy-Preconditioned GRBFN Model for Electricity Price Forecasting", *Procedia Computer Science*, **114**, pp. 441-448 (2017). https://doi.org/10.1016/j.procs.2017.09.010

26. Meier, J.H., Schneider, S., Schmidt, I. et al. "ANN-based electricity price forecasting under special consideration of time series properties", *In International Conference on Information and Communication Technologies in Education, Research, and Industrial Applications*, pp. 262-275 (2019). https://doi.org/10.1007/978-3-030-13929-2_13

27. Gao, G., Lo, K., and Fan, F. "Comparison of ARIMA and ANN models used in electricity price forecasting for power market", *Energy and Power Engineering*, **9**(4) (2017). https://doi.org/10.4236/epe.2017.94B015

28. Yadav, A., Sahay, A., Yadav, M., et al. "One hour ahead short-term electricity price forecasting using ANN algorithms", in *2018 International Conference and Utility Exhibition on Green Energy for Sustainable Development (ICUE)*, pp. 1-4 (2018). https://doi.org/10.23919/ICUE-GESD.2018.8635596

29. Safarinejadian, B., Gharibzadeh, M., and Rakhshan, M. "An optimized model of electricity price forecasting in the electricity market based on fuzzy timeseries", *Systems Science & Control Engineering*, **2**(1), pp. 677-683 (2014). https://doi.org/10.1080/21642583.2014.970733

30. Gao, W., Sarlak, V., Parsaei, M., et al. "Combination of fuzzy based on a meta-heuristic algorithm to predict electricity price in an electricity market", *Chemical Engineering Research and Design*, **131**, pp. 333-345 (2018). https://doi.org/10.1016/j.cherd.2017.09.021

31. Alshejari, A., Kodogiannis, V., and Leonidis, S. "Development of Neurofuzzy Architectures for Electricity Price Forecasting", *Energies*, **13**(5), 1209 (2020). https://doi.org/10.3390/en13051209

32. Azadeh, A., Moghaddam, M., Khakzad, M., et al. "A flexible neural network-fuzzy mathematical programming algorithm for improvement of oil price estimation and forecasting", *Computers & Industrial Engineering*, **62**(2), pp. 421-430 (2012). https://doi.org/10.1016/j.cie.2011.06.019

33. Zadeh, L.A., Klir, G.J., and Yuan, B., *Fuzzy Sets, Fuzzy Logic, and Fuzzy Systems*, Advances in Fuzzy Systems, Singapore: World Scientific Publishing, **6** (1996).

34. Ishibuchi, H., Nozaki, K., and Tanaka, H. "Distributed representation of fuzzy rules and its application to pattern classification", *Fuzzy Sets and Systems*, **52**(1), pp. 21-32 (1992). https://doi.org/10.1016/0165-0114(92)90032-Y

35. Khashei, M., Rafiei, F., and Bijari, M. "Hybrid Fuzzy Auto-Regressive Integrated Moving Average (FARIMAH) Model for Forecasting the Foreign Exchange Markets", *International Journal of Computational Intelligence Systems*, **6**(5), pp. 954-968 (2013). https://doi.org/10.1080/18756891.2013.809937

36. Sahin, Y. and Duman, E. "Detecting credit card fraud by ANN and logistic regression", in *International Symposium on Innovations in Intelligent Systems and Applications*, pp. 315-319 (2011). https://doi.org/10.1109/INISTA.2011.5946108

37. Souza, P.V.C., and de Oliveira, P.F.A "Regularized fuzzy neural networks based on null neurons for problems of classification of patterns", *IEEE Symposium on Computer Applications & Industrial Electronics (ISCAIE),* Penang, Malaysia (2018). https://doi.org/10.1109/ISCAIE.2018.8405439

38. Torabi, A. Pashapour Nazari, S. and Neshat, N. "A novel approach of artificial neural networks modeling based on fuzzy regression approach for forecasting purposes: The case of liquid gas price in Japan's market," *Advances in Industrial Engineering*, **47**(1), pp. 15-24 (2013). https://doi.org/10.22059/jieng.2013.35507

39. Atil, A., Lahiani, A., and Nguyen, D. "Asymmetric and nonlinear pass-through of crude oil prices to gasoline and natural gas prices" *Energy Policy*, **65**, pp. 567-573 (2014). https://doi.org/10.1016/j.enpol.2013.09.064

40. Hailemariam, A. and Smyth, R. "What drives volatility in natural gas prices?", *Energy Economics*, **80**, pp. 731-742 (2019). https://doi.org/10.1016/j.eneco.2019.02.011

41. Jianglin, H. "Comparing the natural gas pipeline pricing between Europe and America and the revelation to China", *Energy Procedia*, **5**, pp. 659-663 (2011). https://doi.org/10.1016/j.egypro.2011.03.116

42. Maxwell, D. and Zhu, Z. "Natural gas prices, LNG transport costs, and the dynamics of LNG imports", *Energy Economics*, **33**(2), pp. 217-226 (2011). https://doi.org/10.1016/j.eneco.2010.06.012

43. Nick, S. and Thoenes, S. "What drives natural gas prices? A structural VAR approach", *Energy Economics*, **45**, pp. 517-527 (2014). https://doi.org/10.1016/j.eneco.2014.08.010

Biographies

Najmeh Neshat received her PhD degree in Industrial Engineering in 2015 from Tarbiat Modaress University, Tehran, Iran. She now works as an Assistant Professor at the Industrial Engineering Department of Meybod University. She is the author and co-author of more than 40 technical papers and the author of 2 books on topics in the area of energy systems. Recently, she has directed her research interest towards agent-based modelling and artificial neural networks.

Hashem Mahlooji received his PhD degree in Industrial Engineering in 1984 from Texas A&M, USA. He is now a Full Professor at the Sharif University of Technology. He is the author and co-author of more than 60 technical papers and the author of 5 books on the topics in the area of Simulation and Probability. His research area is focused on simulations, Monte Carlo, and Probability and Statistics.

Murat Kaya is currently working as an assistant professor at the Industrial Engineering Program of Sabancı University, Istanbul, Turkey. He holds an MSc and PhD from Stanford University's Management Science and Engineering (MS&E) Department. During his PhD studies, he worked on several projects at Hewlett Packard Research Laboratories in Palo Alto, USA. His recent interest is directed towards applied decision & optimization problems in energy supply chains.