



# Rotational stagnation point non-Newtonian second-grade fluid flowing over spiraling disk

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## Keywords

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Keller-Box method.

## Abstract

A numerical study is conducted to execute the analysis of rotational stagnation point second-grade liquid flowing over the spiraling rotatory disk. Heat transmission analysis is accounted. The problem is formulated in the coupled partial differential equation forms which are later simplified in view of similar variables. The Keller-Box (KB) procedure is adopted for the execution of numerical solutions. The influence of involved parameters on the velocity and temperature profiles is presented and interpreted. The skin-frictions and Nusselt number are reported in the forms of numerical data. The present results are verified through comparison with already available material in the literature. This study addressed that the thickness layer of boundary augmented against the incrementing viscoelastic and rotational parameters. Both rotational and viscoelasticity resist the temperature. The rotational parameter rises radial skin-friction and heat transmission rate while is diminishes the swirl skin-friction.

## 1. Introduction

The stagnation point flow investigation is concerned with the motion in surrounding the stagnation point of a rigid surface or moving fluid. Such flows have great physical significance because they forecast the heat transportation and skin-friction in the stagnation region. Due to wide applications of stagnation point flow, several theoretical investigations have been reported by the many researchers. Hiemenz [1] was probably first who analyzed the flow of viscous liquid impinging on infinitely long flat surface. Homann [2] deliberated the axisymmetric Newtonian liquid flow in the neighborhood of stagnation point over a flat surface. Hannah [3] extended this research by adopting the flow generation due to disk rotation. Weidman and Mahalingam [4] studied the axisymmetric flow of incompressible viscous fluid sticking on oscillating porous plate. Mahapatra and Gupta [5] discussed the stagnation-point viscous liquid flowing over

the linear stretched sheet. Lok et al. [6] addressed the Newtonian liquid flow in stagnation regime of a stretched sheet. Sajid et al. [7] numerically inspected the stagnant-point Newtonian liquid flow flowing over a power-law lubricant surface. The time-dependent stagnant point viscous liquid flowing over an impulsively rotating circular disk is executed by Hayat and Nawaz [8]. The above researchers adopted the Hiemenz and Homann flows of viscous fluid where the far field flow is irrotational. Turner and Weidman [9] studied the impingement axisymmetric stagnant point viscous material flow over a surface executed planar and perpendicular. Takhar et al. [10] discussed the axisymmetric flow of fluid striking the surface circular cylinder. Mabood et al. [11] described the non-linear radiative influences in stagnation point Oldroyd-B nanomaterial flow. Mathew et al. [12] elucidated the magnetized stratification effects on

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stagnation point ferro-nanomaterial flow through response surface technique. Raman et al. [13] executed the multiple slip factors on magnetic stagnation point Casson material flow.

Heat transfer is a phenomena that transmits energy from one place to another and this phenomena is prevalent in our daily life and has ample applications in engineering like gas turbine, power plants, cooling of electronic machines and aircraft engine, thermal production of space shuttles, etc. The stagnation zone sensitizes that the heat transportation rate is maximum and the efficiency of cooling is strongly influenced due to numerous reasons. Motivated by such facts, several theoretical studies have been developed to address the energy transportation features in stagnant-point flows. Bhattacharyya et al. [14] examined the heat transmission nature in the stagnant-point neighborhood and reported that the rate of heat transmission reduced against the higher velocity rotation constraint. The axisymmetric heat transportation of compressible viscous liquid impinging on cylinder is illustrated by Mohammadiun et al. [15]. Dero et al. [16] discussed the electrically conducting water-based nanomaterial flow induced by the stagnant point exponential shrunk permeable surface. Alizadeh et al. [17] deliberated the time-independent heat transmission of Newtonian fluid in non-axisymmetric stagnation point zone. Abbas et al. [18] addressed the heterogeneous-homogenous chemically reactive and magnetized Newtonian fluid flow near a stagnant-point. The energy transport analysis of three-dimensional stagnation-point viscous liquid flowing in the zone of linear moving surface is examined by Rehman et al. [19]. Mabood and Khan [20] studied the heat transport behavior in Newtonian material flowing through the Darcy's porous space. Kamal et al. [21] elaborated the analysis of energy transport and liquid motion for Newtonian fluid flowing over the permeable disk and highlighted the heat source influences. Yasin et al. [22] examined the Newtonian heating, magnetic field, viscous dissipation, and partial slip influences on the energy transport characteristics of Newtonian fluid. Shateyi and Makinde [23] presented the heat and flow features behavior of viscous fluid impinging on a convective stretched disk. Khan et al. [24] numerically investigate the Darcy- Forchheimer flow of viscous fluid over a rotating disk incorporating the partial slip conditions. Chu et al. [25] illustrated the Newtonian nanofluid through porous space. Chu et al. [26] analyzed the influence of solid volume fractions and heat generation on the flow and thermal characteristics. Mustafa et al. [27] inspected the heat transportation in the boundary-driven ferrofluid flowing on a rotating disk. Roşca and Pop [28] addressed the rotational stagnation-point viscous material flow with normal impingement on a radially porous stretching surface. Abbasi et al. [29] numerically reported the thermally radiated activation energy influences on the rotational stagnant-point hybrid fluid flow. Farooq et al. [30] explore the rotational

stagnation point flow of carbon nanotubes executed by bi-directional stretching sheet.

Motion of fluid attracts attention in many real-life problems such as wings of air crafts and oscillating bodies. Moreover, this phenomenon is well known for its applications in engineering and industries. Natural, industrial, and technological fluids that exhibit shear dependent viscosity cannot be analyzed through Navier-Stokes equations. To categorize the rheology of such fluids, various models have been proposed. Viscoelastic fluid models is a paramount subclass of the differential type model of non-Newtonian fluid in which grade fluid proposed by Rivlin [31] is simple and it covers linear viscoelastic behavior that has broader range of implications in the polymer sector. Beard and Walters [32] addressed the elastico-viscous liquid flowing near the stagnation zoon and show that the viscoelasticity enhance the velocity in boundary layer region also stress on the solid boundary. Ariel [33] presented an algorithm for the numerical approximation of the formulated equations of two dimensional second-grade non-Newtonian liquid flowing over a circular disk. Barış and Dokuz [34] probed the flow of viscoelastic fluid over a moving surface. Labropulu et al. [35] highlighted the time dependent flow of viscoelastic fluid near a stagnation zoon. Analytic and numeric computations for non-Newtonian second grade material flow under viscous heating induced by the convected moving surface is illustrated by Bhuvaneswari et al. [36]. Krishna et al. [37] addressed the Ion and Hall flow of time-dependent rotatory second-grade liquid under convective heat absorption. Xia et al. [38] considered the Forchheimer-Darcy and Lorentz forces to describe the Couette-Taylor second-grade flow situation. Siddique et al. [39] discussed the time-dependent second grade model of fractional heat transportation. Mabood et al. [40] highlighted the effects of Newtonian heating, suction and mixed convection on the flow of second grade fluid. Chu et al. [41] simulate the hydromagnetized non-Newtonian fluid near stagnation point. Zhao et al. [42] reported the thermal transmission in the stagnation point flow of electrically conducting fluid. Furthermore, Abbas et al. [43], Yavuz [44], Nadeem et al. [45], Nisa et al. [46], Alamri et al. [47], Khan et al. [48], Saif et al. [49] and Tariq et al. [50] examined the different flows of second-grade fluid.

Owning the tremendous applications of stagnation point flows due to their importance in science and engineering like cooling of electronic devices, nuclear reactors metallic plate, and by fans, crystal puffing, plastic glass and films drawing continuous costing, production of papers, food processing, wire drawing etc. and keeping the fact in mind that most of the industrial fluids are combination of dissimilar stuffs such as oil, water or a mixture of chain molecules, these fluids shows non-Newtonian behavior in the response of applied stresses natural, industrial, and technological fluids that exhibit shear dependent viscosity cannot be analyzed through

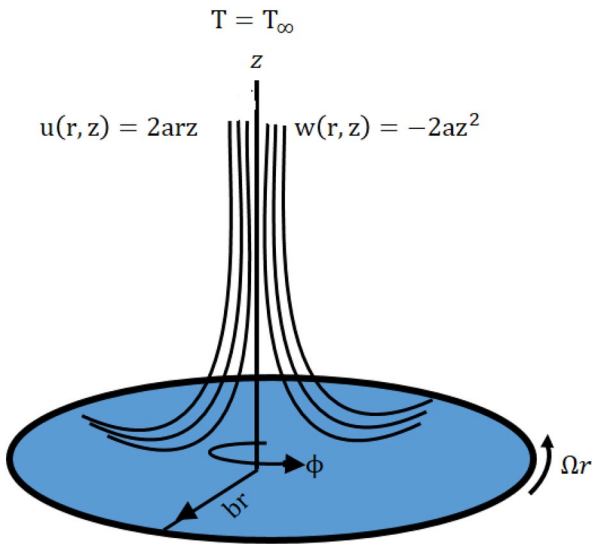


Figure 1. Geometry of the problem.

Navier-Stokes equations. To categorize the rheology of such fluids, the second-grade fluid model is simplest subclass of viscoelastic fluid models in this paper a numerical study is carried out to deliberate the rotational stagnation point flow of second-grade fluid over a circular spiraling disk. The governed physical equations are converted by the use of similarity constraints. This non-dimensional model is treated by the utilization of Keller-Box (KB) scheme. The graphical illustration and numerical benchmarks are produced to address and analyze the behavior of versatile involved parameters on physical quantities.

## 2. Problem formulation

The axisymmetric movement of Rivlin-Erickson type fluid near rotational stagnant-point over spiraling disk. The disk is in rotation through the angular velocity  $\Omega$  about  $z$ -axis. The disk is stretched in radial direction under velocity  $br$  (see Figure 1). The model is reported in cylindrical coordinates  $r, \phi$ , and  $z$  with corresponding velocity components  $u, v$ , and  $w$  respectively. Here,  $z = 0$  represents the disk surface which maintained at temperature  $T_w$  and  $z > 0$  is the space occupied by the fluid and the ambient temperature is assumed to be  $T_\infty$ .

Fundamental conservation laws are utilized for the mathematical modeling of the problem. For an incompressible fluid the continuity equations are:

$$\nabla \cdot \mathbf{V} = 0. \tag{1}$$

In the absence of body forces for steady axisymmetric incompressible liquid flow, the momentum equation in components form can be expressed as:

$$\rho \left[ u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} - \frac{v^2}{r} \right] = -\frac{\partial p}{\partial r} + \frac{1}{r} \frac{\partial (r\tau_{rr})}{\partial r} + \frac{\partial (\tau_{rz})}{\partial z} - \frac{\tau_{\phi\phi}}{r}, \tag{2}$$

$$\rho \left[ u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial z} - \frac{uv}{r} \right] = \frac{1}{r^2} \frac{\partial (r^2 \tau_{\phi r})}{\partial r} + \frac{\partial (\tau_{\phi z})}{\partial z} - \frac{\tau_{\phi r} - \tau_{\phi r}}{r}, \tag{3}$$

$$\rho \left[ u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right] = -\frac{\partial p}{\partial z} + \frac{1}{r} \frac{\partial (r\tau_{zr})}{\partial r} + \frac{\partial (\tau_{zz})}{\partial z}. \tag{4}$$

In above equations  $u, v$ , and  $w$  are velocity components along radial azimuthal and axial direction,  $\rho$  is the density of the fluid  $\tau_{rr}, \tau_{\phi r}, \tau_{rz}, \tau_{\phi\phi}, \tau_{\phi z}$  and  $\tau_{zz}$  are the component of stress tensor  $\tau$ . Most of the industrial fluid are non-Newtonian and have been modeled using relatively simple constitutive equation presented by Rivlin [31]:

$$\tau = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_2^2. \tag{5}$$

In which  $p$  is pressure,  $A_1$  and  $A_2$  are kinematic tensors,  $\mu$  is coefficient of viscosity  $\alpha_1$  is viscoelasticity and  $\alpha_2$  is cross-viscosity of the fluid and for a second grade fluid these constants satisfy the stable thermodynamic conditions defined by Dunn and Fosdick [51] as  $\alpha_1 \geq 0, \mu \geq 0$ , and  $\alpha_1 + \alpha_2 = 0$ . The energy balance equation in the absence of Joule and Ohmic dissipations has the following form:

$$\rho c_p \left( \frac{dT}{dt} \right) = k \nabla^2 T, \tag{6}$$

where  $c_p$  is used to represent the specific heat,  $T$  and  $k$  denotes the temperature and thermal conductivity of the fluid. In view of Weidman [52] the pertinent boundary conditions satisfy by the velocity components along with the surface and ambient temperature conditions are:

$$\begin{aligned} u = br, \quad v = r\Omega, \quad w = 0, \quad T = T_w, \quad \text{when} \\ z \rightarrow 0, \quad u(r, z) = 2arz, \quad w(r, z) = -2az^2, \\ T = T_\infty, \quad \text{when} \quad z \rightarrow \infty. \end{aligned} \tag{7}$$

For incompressible axisymmetric second-grade liquid, the Eqs. (1)-(5) can be expressed as:

$$\frac{u}{r} + \frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} = 0, \tag{8}$$

$$\begin{aligned} u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \xi \left[ 2 \frac{\partial^2 u}{\partial r^2} + \frac{\partial}{\partial z} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right) \right] \\ + \frac{\alpha_1}{\rho} \left\{ 2 \frac{\partial}{\partial r} \left( u \frac{\partial^2 u}{\partial r^2} \right) - \frac{\partial}{\partial r} \left( \frac{\partial u}{\partial z} \right)^2 + \frac{\partial}{\partial r} \left( \frac{\partial v}{\partial r} \right)^2 \right\} \\ + 2 \frac{\partial}{\partial r} \left( w \frac{\partial^2 u}{\partial z \partial r} \right) - \frac{\partial}{\partial r} \left( \frac{v^2}{r^2} \right) + \frac{\partial}{\partial r} \left( \frac{\partial w}{\partial r} \right)^2 + \frac{\partial}{\partial z} \left( u \frac{\partial^2 u}{\partial r \partial z} \right) \\ + \frac{\partial}{\partial z} \left( u \frac{\partial^2 w}{\partial r^2} \right) + \frac{\partial}{\partial z} \left( w \frac{\partial^2 u}{\partial z^2} \right) + \frac{\partial}{\partial z} \left( \frac{\partial u}{\partial z} \frac{\partial u}{\partial r} - \frac{\partial w}{\partial r} \frac{\partial u}{\partial r} \right) \\ + \frac{\partial}{\partial z} \left( \frac{\partial w}{\partial r} \frac{\partial w}{\partial z} + \frac{\partial v}{\partial r} \frac{\partial v}{\partial z} \right) - \frac{\partial u}{\partial z} \frac{\partial w}{\partial z} + w \frac{\partial^2 w}{\partial z \partial r} \end{aligned}$$

$$\begin{aligned}
& + \frac{2u}{r} \frac{\partial^2 u}{\partial r^2} + \frac{2w}{r} \frac{\partial^2 u}{\partial z \partial r} - \frac{2u}{r^2} \frac{\partial u}{\partial r} - \frac{2w}{r^2} \frac{\partial u}{\partial z} + \frac{2}{r} \left( \frac{\partial v}{\partial r} \right)^2 \\
& + \frac{1}{r} \left( \frac{\partial w}{\partial r} \right)^2 - \frac{1}{r} \left( \frac{\partial u}{\partial z} \right)^2 + \frac{1}{r} \left( \frac{\partial v}{\partial z} \right)^2 - \frac{2v^2}{r^3} + \frac{2u^2}{r^3} \Big\}, \quad (9)
\end{aligned}$$

$$\begin{aligned}
u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial z} + \frac{uv}{r} = \xi \left[ \frac{\partial^2 v}{\partial z^2} + \frac{\partial}{\partial r} \left( \frac{\partial v}{\partial r} - \frac{v}{r} \right) \right] \\
+ \frac{\alpha_1}{\rho} \left\{ \frac{\partial}{\partial r} \left( u \frac{\partial^2 v}{\partial r^2} \right) - \frac{\partial}{\partial r} \left( \frac{\partial v}{\partial r} \frac{\partial u}{\partial r} \right) - \frac{\partial}{\partial r} \left( \frac{\partial u}{\partial r} v \right) \right. \\
- \frac{\partial}{\partial r} \left( \frac{\partial u}{\partial z} \frac{\partial v}{\partial z} \right) + \frac{\partial}{\partial r} \left( w \frac{\partial^2 v}{\partial z \partial r} \right) - \frac{\partial}{\partial r} \left( \frac{w}{r} \frac{\partial v}{\partial z} \right) \\
+ \frac{\partial}{\partial r} \left( \frac{2uv}{r^2} \right) + \frac{\partial}{\partial z} \left( u \frac{\partial^2 v}{\partial r \partial z} \right) + \frac{\partial}{\partial z} \left( w \frac{\partial^2 v}{\partial z^2} \right) \\
- \frac{\partial}{\partial z} \left( \frac{\partial^2 u}{\partial z^2} v \right) - \frac{\partial}{\partial z} \left( \frac{\partial w}{\partial z} \frac{\partial v}{\partial z} \right) + \frac{\partial}{\partial z} \left( \frac{u}{r} \frac{\partial v}{\partial z} \right) \\
- \frac{\partial}{\partial z} \left( \frac{\partial w}{\partial r} \frac{\partial v}{\partial r} \right) + \frac{u}{r} \frac{\partial^2 v}{\partial r^2} - \frac{\partial u}{\partial r} \frac{v}{r^2} - \frac{1}{r} \frac{\partial v}{\partial r} \frac{\partial u}{\partial r} \\
\left. - \frac{1}{r} \frac{\partial v}{\partial z} \frac{\partial u}{\partial z} + \frac{w}{r} \frac{\partial^2 v}{\partial z \partial r} - \frac{w}{r^2} \frac{\partial v}{\partial z} + \frac{2uv}{r^3} \right\}, \quad (10)
\end{aligned}$$

$$\begin{aligned}
u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} - \frac{v^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \xi \left[ \frac{\partial^2 w}{\partial r^2} \right. \\
+ \frac{\partial}{\partial r} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right) \\
\left. + \frac{1}{r} \frac{\partial u}{\partial z} + \frac{1}{r} \frac{\partial w}{\partial r} \right] \\
+ \frac{\alpha_1}{\rho} \left\{ \frac{\partial}{\partial r} \left( u \frac{\partial^2 w}{\partial r \partial z} \right) + \frac{\partial}{\partial r} \left( u \frac{\partial^2 w}{\partial r^2} \right) + \frac{\partial}{\partial r} \left( w \frac{\partial^2 u}{\partial z^2} \right) \right. \\
+ \frac{\partial}{\partial r} \left( w \frac{\partial w}{\partial z \partial r} \right) + \frac{\partial}{\partial r} \left( \frac{\partial u}{\partial z} \frac{\partial u}{\partial r} \right) - \frac{\partial}{\partial r} \left( \frac{\partial w}{\partial z} \frac{\partial u}{\partial z} \right) \\
+ \frac{\partial}{\partial r} \left( \frac{\partial w}{\partial z} \frac{\partial w}{\partial r} \right) + \frac{\partial}{\partial r} \left( \frac{\partial v}{\partial r} \frac{\partial v}{\partial z} \right) - \frac{\partial}{\partial r} \left( \frac{\partial w}{\partial r} \frac{\partial u}{\partial r} \right) \\
+ 2 \frac{\partial}{\partial r} \left( \frac{\partial^2 w}{\partial z^2} w \right) + \frac{\partial}{\partial z} \left( \frac{\partial u}{\partial z} \right)^2 + \frac{\partial}{\partial z} \left( \frac{\partial v}{\partial z} \right)^2 - \frac{\partial}{\partial z} \left( \frac{\partial w}{\partial r} \right)^2 \\
+ \frac{w}{r} \frac{\partial^2 u}{\partial z^2} + 2 \frac{\partial}{\partial z} \left( u \frac{\partial^2 w}{\partial z \partial r} \right) + \frac{u}{r} \left( \frac{\partial^2 u}{\partial r \partial z} + \frac{\partial^2 w}{\partial r^2} \right) \\
\left. + \frac{1}{r} \left( \frac{\partial v}{\partial z} \frac{\partial v}{\partial r} + \frac{\partial w}{\partial z} \frac{\partial w}{\partial r} - \frac{\partial w}{\partial z} \frac{\partial u}{\partial z} + \frac{\partial u}{\partial z} \frac{\partial u}{\partial r} \right) \right\}, \quad (11)
\end{aligned}$$

$$u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \alpha \left[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right], \quad (12)$$

In which  $\xi = \mu/\rho$  represents kinematic viscosity and  $\alpha = k/c_p$  is the thermal diffusivity. In all above cited literature

about second grade fluid the similarity variables for which the far field flow is irrotational. Weidman [52] suggested that the similarity variables which give rise the rotational stagnation point in the obey the mass conservation law are provided as below:

$$\begin{aligned}
u(r, \eta) = a^{2/3} r \xi^{1/3} f'(\eta), v(r, \eta) = r g(\eta) \Omega, \\
w(\eta) = -2a^{1/3} r \xi^{2/3} g(\eta), \quad (13)
\end{aligned}$$

where  $\eta = \left(\frac{a}{\xi}\right)^{1/2} z$ ,  $a$  is constant. The temperature equation can be transformed by introducing  $T = \theta(\eta)(T_w - T_\infty) + T_\infty$ . By invoking the above similarity constraints, the continuity is automatically satisfied and pair of differential equations is obtained for second grade fluid which along with energy equation takes the form:

$$f''' - f'^2 + 2ff'' + \sigma^2 g^2 + We \begin{pmatrix} f''^2 + 2f'f''' \\ -2ff^{iv} \\ +\sigma^2(g''^2 + gg''') \end{pmatrix} = 0, \quad (14)$$

$$g'' + 2(fg' - f'g) + We \begin{pmatrix} 2f'g'' - 2fg''' \\ -f'''g \end{pmatrix} = 0, \quad (15)$$

$$\theta'' + 2Prf\theta' = 0. \quad (16)$$

The non-dimensional conditions are:

$$f(\eta) = S, f'(\eta) = \beta, g(\eta) = 1 = \theta(\eta) \quad \text{when } \eta \rightarrow 0, \quad (17)$$

$$f''(\eta) = 2, f'''(\eta) = 0, g(\eta) = 0 = g'(\eta) = \theta(\eta) \quad \text{when } \eta \rightarrow \infty. \quad (18)$$

In above over prime is the derivative with respect to the  $\eta$ . The dimensionless numbers  $We = \alpha_1 a^{2/3} / \rho \xi^{2/3}$  depicts the Weissenberg number,  $\sigma = \Omega / a^{2/3} \xi^{1/3}$  is dimensionless rotation rate parameter,  $Pr = \xi/\alpha$  is Prandtl number,  $\beta$  is stretching parameter and  $S$  is suction/ injection parameter.

The skin friction coefficients along radial and azimuthal directions are expressed as:

$$C_{fr} = \frac{\tau_{wr}|_{z=0}}{\rho U_w^2}, \quad C_{f\phi} = \frac{\tau_{w\phi}|_{z=0}}{\rho (r\Omega)^2}. \quad (19)$$

Elucidation of wall shear stresses  $\tau_{wr}$  and  $\tau_{w\phi}$  along radial and azimuthal directions are  $\theta'(0)$ .

Skin friction coefficients in dimensionless form are given as:

$$\sqrt{Re} C_{fr} = \left[ f'' + We(4f'f'' - 2ff''' + \sigma^2 gg') \right]_{\eta=0}, \quad (20)$$

$$\sqrt{Re_r} C_{f\phi} = \left[ g' + We(4f'g' - 2ffg'' - f''g) \right]_{\eta=0}, \quad (21)$$

where  $\sqrt{Re}$  and  $\sqrt{Re_r}$  are the local and rotating Reynold numbers.

Heat transfer rate by Fourier law of conduction at the surface of disk is  $q_w = -k \left( \frac{\partial T}{\partial z} \right)_{z=0}$ .

### 3. Solution methodology

The transformed Eqs. (14)-(16) subject to Conditions (17) and (18) are approximated numerically by employing finite difference method which is commonly titled by KB method in literature. The whole numerical procedure is implemented in four steps:

**Step 1:** Reducing Eqs. (14)-(16) into first order system of equations. For this let:

$$f' = U, U' = V, V' = W, g' = P, P' = q, \theta' = Y, \quad (22)$$

we get:

$$V' - U^2 + 2fV + \sigma^2 g^2 + We \begin{pmatrix} V^2 + 2UW \\ -2fW' \\ +\sigma^2 P^2 \\ +\sigma^2 qg \end{pmatrix} = 0, \quad (23)$$

$$P' + 2(fP - Ug) + We(2Uq - 2fq' - Wg) = 0, \quad (24)$$

$$Y' + 2PrfY = 0. \quad (25)$$

The associated boundary conditions are:

$$f(\eta) = S, U(\eta) = \beta, g(\eta) = l = \theta(\eta) \text{ when } \eta \rightarrow 0, \quad (26)$$

$$V(\eta) = 2, W(\eta) = 0, g(\eta) = 0 = P(\eta) = \theta(\eta) \text{ when } \eta \rightarrow \infty. \quad (27)$$

**Step 2:** Computing the derivative by centered difference and rest of dependent variables by taking averaging at the midpoint of the net. The derivatives are approximated by the relation:

$$\eta_0 = 0, \eta_j = \eta_{j-1} + h_j, \quad j = 1, 2, 3, \dots, J, \eta_J = \eta_\infty.$$

So the Eqs. (22)-(25) take the form:

$$\left. \begin{aligned} \frac{f_j - f_{j-1}}{h_j} &= U_{j-\frac{1}{2}} & \frac{U_j - U_{j-1}}{h_j} &= V_{j-\frac{1}{2}} \\ \frac{g_j - g_{j-1}}{h_j} &= P_{j-\frac{1}{2}} & \frac{V_j - V_{j-1}}{h_j} &= W_{j-\frac{1}{2}} \\ \frac{\theta_j - \theta_{j-1}}{h_j} &= Y_{j-\frac{1}{2}} & \frac{P_j - P_{j-1}}{h_j} &= q_{j-\frac{1}{2}} \end{aligned} \right\} \quad (28)$$

$$\begin{aligned} &\frac{V_j - V_{j-1}}{h_j} - U_{j-\frac{1}{2}}^2 + 2f_{j-\frac{1}{2}}V_{j-\frac{1}{2}} + \sigma^2 g_{j-\frac{1}{2}}^2 \\ &+ We \begin{pmatrix} V_{j-\frac{1}{2}}^2 + 2U_{j-\frac{1}{2}}V_{j-\frac{1}{2}} \\ -2f_{j-\frac{1}{2}}\frac{W_j - W_{j-1}}{h_j} + \sigma^2 P_{j-\frac{1}{2}}^2 \\ +\sigma^2 g_{j-\frac{1}{2}}q_{j-\frac{1}{2}} \end{pmatrix} = 0, \quad (29) \\ &\frac{P_j - P_{j-1}}{h_j} + 2 \left( f_{j-\frac{1}{2}}P_{j-\frac{1}{2}} - U_{j-\frac{1}{2}}g_{j-\frac{1}{2}} \right) \\ &+ We \left( 2U_{j-\frac{1}{2}}q_{j-\frac{1}{2}} - 2f_{j-\frac{1}{2}}\frac{q_j - q_{j-1}}{h_j} - W_{j-\frac{1}{2}}g_{j-\frac{1}{2}} \right) = 0, \quad (30) \end{aligned}$$

$$\frac{Y_j - Y_{j-1}}{h_j} + 2Prf_{j-\frac{1}{2}}Y_{j-\frac{1}{2}} = 0, \quad (31)$$

where  $f_{j-\frac{1}{2}} = \frac{f_j + f_{j-1}}{2}$  etc.

**Step 3:** Eqs. (28)-(31) are non-linear before factorization we apply the Newton's method for which we write the Newton's iterates as follows: For the  $(i + 1)$ th, iterates, we write  $f_j^{i+1} = f_j^i + \delta f_j^i$ , for all dependent variables. After using the above procedure of linearization, we write the linear tri-diagonal system of algebraic equations as follows:

$$\delta f_j - \delta f_{j-1} - \frac{h_j}{2}(\delta U_j + \delta U_{j-1}) = (r_1)_{j-\frac{1}{2}}, \quad (32)$$

$$\delta g_j - \delta g_{j-1} - \frac{h_j}{2}(\delta P_j + \delta P_{j-1}) = (r_2)_{j-\frac{1}{2}}, \quad (33)$$

$$\begin{aligned} \xi_1 \delta f_j + \xi_2 \delta f_{j-1} + \xi_3 \delta U_j + \xi_4 \delta U_{j-1} + \xi_5 \delta V_j + \xi_6 \delta V_{j-1} \\ + \xi_7 \delta W_j + \xi_8 \delta W_{j-1} + \xi_9 \delta g_j + \xi_{10} \delta g_{j-1} + \xi_{11} \delta P_j \\ + \xi_{12} \delta P_{j-1} + \xi_{13} \delta q_j + \xi_{14} \delta q_{j-1} = (r_3)_{j-\frac{1}{2}}, \end{aligned} \quad (34)$$

$$\begin{aligned} \psi_1 \delta f_j + \psi_2 \delta f_{j-1} + \psi_3 \delta U_j + \psi_4 \delta U_{j-1} + \psi_5 \delta W_j \\ + \psi_6 \delta W_{j-1} + \psi_7 \delta g_j + \psi_8 \delta g_{j-1} + \psi_9 \delta P_j \\ + \psi_{10} \delta P_{j-1} + \psi_{11} \delta q_j + \psi_{12} \delta q_{j-1} = (r_4)_{j-\frac{1}{2}}, \end{aligned} \quad (35)$$

$$\lambda_1 \delta f_j + \lambda_2 \delta f_{j-1} + \lambda_3 \delta Y_j + \lambda_4 \delta Y_{j-1} = (r_5)_{j-\frac{1}{2}}, \quad (36)$$

$$\delta U_j - \delta U_{j-1} - \frac{h_j}{2}(\delta V_j + \delta V_{j-1}) = (r_6)_{j-\frac{1}{2}}, \quad (37)$$

$$\delta V_j - \delta V_{j-1} - \frac{h_j}{2}(\delta W_j + \delta W_{j-1}) = (r_7)_{j-\frac{1}{2}}, \quad (38)$$

$$\delta P_j - \delta P_{j-1} - \frac{h_j}{2}(\delta q_j + \delta q_{j-1}) = (r_8)_{j-\frac{1}{2}}, \quad (39)$$

$$\delta \theta_j - \delta \theta_{j-1} - \frac{h_j}{2}(\delta Y_j + \delta Y_{j-1}) = (r_9)_{j-\frac{1}{2}}, \quad (40)$$

where,

$$\xi_1 = \frac{h_j}{2}V_{j-\frac{1}{2}} - We(W_j - W_{j-1}), \quad \xi_2 = \xi_1,$$

$$\xi_4 = \xi_3, \quad \xi_3 = -h_jU_{j-\frac{1}{2}} + Weh_jW_{j-\frac{1}{2}},$$

$$\xi_5 = 1 + \frac{h_j}{2}f_{j-\frac{1}{2}} + Weh_jV_{j-\frac{1}{2}}, \quad \xi_6 = -1 + \frac{h_j}{2}f_{j-\frac{1}{2}} + Weh_jV_{j-\frac{1}{2}},$$

$$\xi_7 = Weh_jU_{j-\frac{1}{2}} - 2Wef_{j-\frac{1}{2}}, \quad \xi_8 = Weh_jU_{j-\frac{1}{2}} - 2Wef_{j-\frac{1}{2}},$$

$$\xi_9 = h_j\sigma^2 g_{j-\frac{1}{2}} + We\sigma^2 \frac{h_j}{2}q_{j-\frac{1}{2}}, \quad \xi_{10} = \xi_9, \quad \xi_{11} = We\sigma^2 h_j P_{j-\frac{1}{2}},$$

$$\xi_{12} = \xi_{11},$$

$$\xi_{13} = We\sigma^2 h_j g_{j-\frac{1}{2}}, \quad \xi_{14} = We\sigma^2 h_j g_{j-\frac{1}{2}},$$

$$\psi_1 = \frac{h_j}{2} P_{j-\frac{1}{2}} - We(q_j - q_{j-1}),$$

$$\psi_2 = \psi_1,$$

$$\psi_3 = -h_j g_{j-\frac{1}{2}} + We h_j q_{j-\frac{1}{2}},$$

$$\psi_4 = \psi_3,$$

$$\psi_5 = -We \frac{h_j}{2} g_{j-\frac{1}{2}},$$

$$\psi_6 = \psi_5,$$

$$\psi_7 = -We \frac{h_j}{2} W_{j-\frac{1}{2}} - h_j U_{j-\frac{1}{2}},$$

$$\psi_8 = \psi_7,$$

$$\psi_9 = 1 + h_j f_{j-\frac{1}{2}}, \quad \psi_{10} = -1 + h_j f_{j-\frac{1}{2}},$$

$$\psi_{11} = We h_j U_{j-\frac{1}{2}} - 2We f_{j-\frac{1}{2}}, \quad \psi_{12} = We h_j U_{j-\frac{1}{2}} + 2We f_{j-\frac{1}{2}},$$

$$\lambda_1 = Pr h_j Y_{j-\frac{1}{2}}, \quad \lambda_2 = \lambda_1, \quad \lambda_3 = 1 + Pr h_j f_{j-\frac{1}{2}},$$

$$\lambda_4 = -1 + Pr h_j f_{j-\frac{1}{2}},$$

also,

$$(r_1)_{j-\frac{1}{2}} = f_{j-1} - f_j + \frac{h_j}{2}(U_j + U_{j-1}),$$

$$(r_2)_{j-\frac{1}{2}} = g_{j-1} - g_j + \frac{h_j}{2}(P_j + P_{j-1}),$$

$$(r_3)_{j-\frac{1}{2}} = \begin{pmatrix} V_{j-1} - V_j + h_j U_{j-\frac{1}{2}}^2 \\ -h_j f_{j-\frac{1}{2}} V_{j-\frac{1}{2}} - h_j \sigma^2 g_{j-\frac{1}{2}}^2 \end{pmatrix} + We \begin{pmatrix} -h_j \sigma^2 P_{j-\frac{1}{2}}^2 V_{j-\frac{1}{2}}^2 - 2h_j U_{j-\frac{1}{2}} W_{j-\frac{1}{2}} \\ 2f_{j-\frac{1}{2}}(W_j - W_{j-1}) - h_j \sigma^2 g_{j-\frac{1}{2}} q_{j-\frac{1}{2}} \end{pmatrix},$$

$$(r_4)_{j-\frac{1}{2}} = \begin{pmatrix} q_{j-1} - q_j - 2h_j f_{j-\frac{1}{2}} P_{j-\frac{1}{2}} \\ +2h_j g_{j-\frac{1}{2}} U_{j-\frac{1}{2}} \end{pmatrix} + We \begin{pmatrix} -2h_j U_{j-\frac{1}{2}} q_{j-\frac{1}{2}} + h_j W_{j-\frac{1}{2}} g_{j-\frac{1}{2}} \\ +2f_{j-\frac{1}{2}}(q_j - q_{j-1}) \end{pmatrix},$$

$$(r_5)_{j-\frac{1}{2}} = Y_{j-1} - Y_j - 2h_j Pr f_{j-\frac{1}{2}} Y_{j-\frac{1}{2}},$$

$$(r_6)_{j-\frac{1}{2}} = U_{j-1} - U_j + \frac{h_j}{2}(V_j + V_{j-1}),$$

$$(r_7)_{j-\frac{1}{2}} = V_{j-1} - V_j + \frac{h_j}{2}(W_j + W_{j-1}),$$

**Table 1.** Convergence of  $f''(0), g''(0)$  and  $-\theta'(0)$  for various number at grid points when  $We = 0.1, \sigma = 1.0, Pr = 4.0, S = 0.1$  and  $\beta = 0.0$ .

Grid points	$f''(0)$	$g''(0)$	$-\theta'(0)$
200	3.1363328	-1.4013549	2.2212672
400	3.1359176	-1.4010349	2.220381
500	3.1358678	1.4009965	2.2202746
1000	3.1358014	-1.4009452	2.2201328
2000	3.1357848	-1.4009324	2.2200974
3000	3.1357817	-1.4009301	2.2200908
5000	3.1357801	-1.4009289	2.2200875
10000	3.1357794	-1.4009283	2.2200861
20000	3.1357793	-1.4009282	2.2200857
30000	3.1357792	-1.4009282	2.2200856
50000	3.1357792	-1.4009282	2.2200856

**Table 2.** Comparison of  $f''(0)$  and  $g'(0)$  when  $S = \beta = 0$  and  $We = 0$ .

$\sigma^2$	$f''(0)$		$g'(0)$	
	Present	Lok et al. [53]	Present	Lok et al. [53]
0	2.000011	2.00000	-1.339197	-1.33919
25	6.819139	6.81915	-1.7080642	-1.70806
100	17.05556	17.05561	-2.167531	-2.16752
225	30.48817	30.48822	-2.5617991	-2.56178
400	46.44322	46.44340	-2.906941	-2.90692
625	64.56087	64.56092	-3.216604	-3.21660

$$(r_8)_{j-\frac{1}{2}} = P_{j-1} - P_j + \frac{h_j}{2}(q_j + q_{j-1}),$$

$$(r_9)_{j-\frac{1}{2}} = \theta_{j-1} - \theta_j + \frac{h_j}{2}(q_j + q_{j-1}).$$

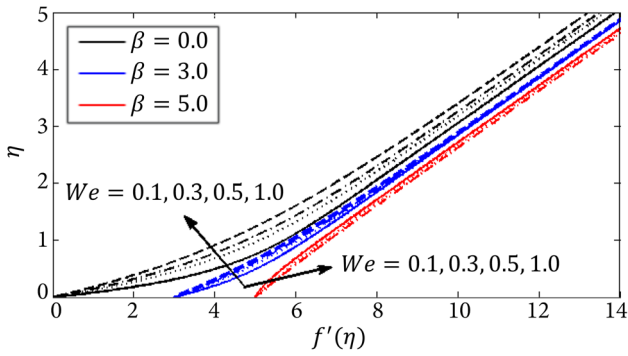
**Step 4:** After writing Eqs. (32)-(40) in matrix form having the block elements of  $9 \times 9$  matrices, which are computed by block-tridiagonal elimination technique which consists of forward and backward sweeps.

#### 4. Validation of numerical results

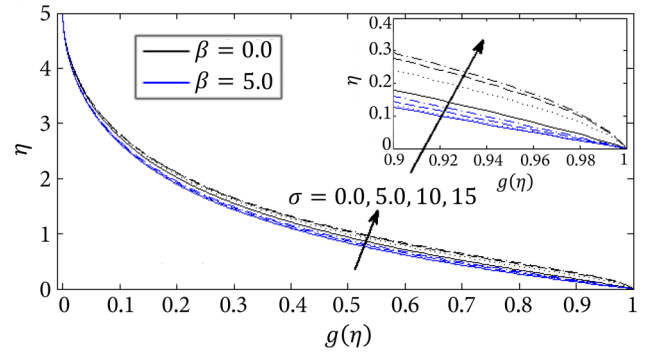
Table 1 provides the convergence of  $f''(0), g''(0)$  and  $-\theta'(0)$  at multiple grid points by fixing.  $We = 0.1, \sigma = 1.0, Pr = 4.0, S = 0.1$  and  $\beta = 0.0$ . In order to validate the present numerical scheme we compare the results of  $f''(0)$  and  $g'(0)$  for viscous fluid on a rigid rotating disk (see Table 2). The deep analysis of the results show that the results obtained by finite difference show a good agreement 3 to 4 decimal places.

#### 5. Results and discussion

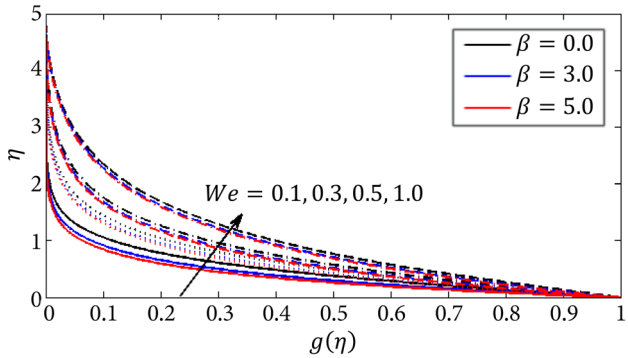
The variation of radial velocity  $f'(\eta)$  against the rheological parameter  $We$ , for several values of stretching parameter is examined in Figure 2. The rheological parameter which demonstrates the viscoelasticity of the fluid reduces the radial velocity  $f'(\eta)$  of the fluid for rising values of stretching parameter. Furthermore, when only rotating disk is considered the decline in velocity is large on the surface of



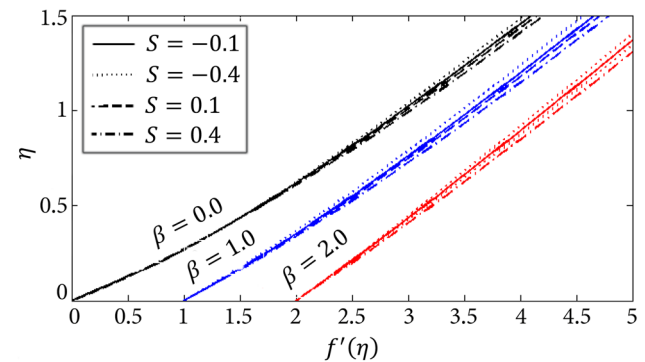
**Figure 2.** Variation of radial velocity  $f'(\eta)$  with  $We$  when  $\sigma = 0.1$  and  $S = 5.0$ .



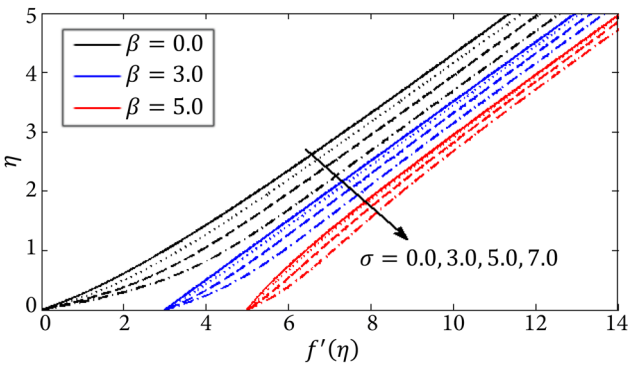
**Figure 5.** Variation of swirl velocity  $g(\eta)$  with  $\sigma$  when  $We = 2.0$  and  $S = 0.1$ .



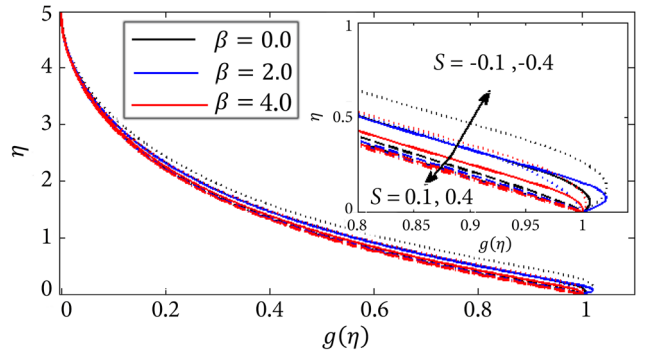
**Figure 3.** Variation of swirl velocity  $g(\eta)$  with  $We$  when  $\sigma = 0.1$  and  $S = 0.2$ .



**Figure 6.** Variation of radial velocity  $f'(\eta)$  with  $S$  when  $We = 2.0$  and  $\sigma = 0.2$ .



**Figure 4.** Variation of radial velocity  $f'(\eta)$  with  $\sigma$  when  $We = 2.0$  and  $S = 0.2$ .



**Figure 7.** Variation of swirl velocity  $g(\eta)$  with  $S$  when  $We = 2.0$  and  $\sigma = 0.2$ .

the disk and when stretching of the disk is involved, the decline in velocity decreases and it is observed that the effects of viscoelasticity dismissed for large stretching of the disk. In many industrial processes where the speed of the transporting fluids needs to be improved stretching is an important factor. The rise in Azimuthal velocity against viscoelasticity of the fluid for the different values of  $\beta$  is observed in Figure 3. Also, it is noted that the velocity profile of the non-Newtonian fluid has maximum amplitude over rotating disk and it declines over stretching disk. Figures 4 and 5 are plotted to compute the effects of rotation parameter  $\sigma$  on radial velocity  $f'(\eta)$  and Azimuthal velocity  $g(\eta)$  for various values of stretching parameter  $\beta$ . The rise in radial velocity  $f'(\eta)$  is noted against the incrementing rotation constraint. It is noted that rise is minimum corresponding

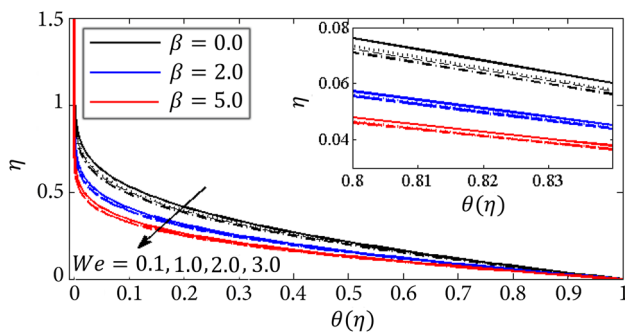
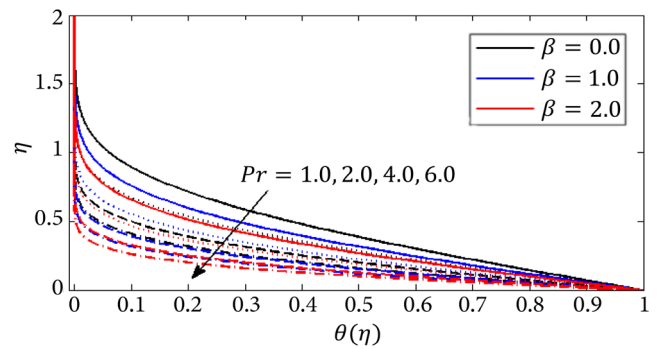
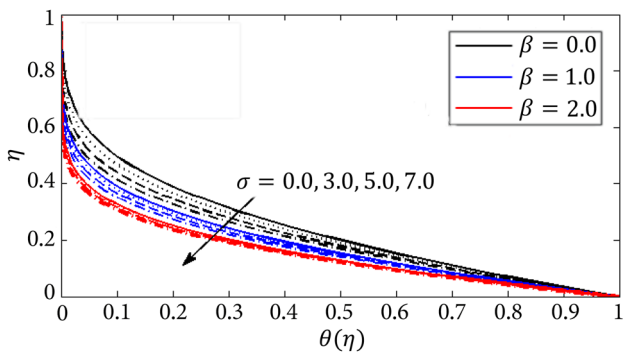
to the rotation parameter over only rotating disk and rising rate is maximum as stretching of the disk is enhanced. The similar response is for radial velocity is reported by Weidman [52] for the Agarwal stagnation point flow of Newtonian fluid over rotating disk. The rotation parameter  $\sigma$  rises the azimuthal velocity of the viscoelastic fluid over both stretching and stationary disk. Figures 6 and 7 displayed similarity velocities  $f'(\eta)$  and  $g(\eta)$  for both suction ( $S = 0.1, 0.4$ ) and blowing ( $S = -0.1, -0.4$ ) cases. The flow behavior is also analyzed for several values of stretching parameter  $\beta$ . The decreasing trend is followed for blowing case and the decreasing trend is boosted with rising the stretching velocity of the rotating disk. On the other hand, increasing trend is noted for the suction case and increasing

**Table 3.** Variation of radial and swirl skin frictions against rotational parameter with  $\beta = 0.5$  and  $S = 0.1$ .

$\sigma$	Radial skin friction			Swirl skin friction		
	$We = 0.0$	$We = 0.5$	$We = 1.0$	$We = 0.0$	$We = 0.5$	$We = 1.0$
0.0	2.131884	1.774703	0.782649	-1.879153	-1.363186	-1.036682
0.5	2.177293	2.131468	1.235408	-1.882072	-1.346105	-1.012119
1.0	2.312618	3.172399	2.541887	-1.890696	-1.296779	-0.941905
1.5	2.535295	4.816952	4.561773	-1.904654	-1.220392	-0.835389
2.0	2.841467	6.952039	7.103464	-1.923399	-1.124033	-0.705161

**Table 4.** Variation of axial and swirl skin frictions against stretching parameter with  $\sigma = 0.5$  and  $S = 0.1$ .

$\beta$	Radial skin friction			Swirl skin friction		
	$We = 0.0$	$We = 0.5$	$We = 1.0$	$We = 0.0$	$We = 0.5$	$We = 1.0$
0.0	2.481262	4.509229	6.142849	-1.522795	-1.010654	-0.354739
0.5	2.177293	2.131468	1.235408	-1.882072	-1.346105	-1.012119
1.0	1.577820	0.447832	-2.036308	-2.176116	-1.518813	-1.274356
1.5	0.752212	-0.698510	-4.305664	-2.431099	-1.623040	-1.405607
2.0	-0.261424	-1.361850	-5.786673	-2.659341	-1.692473	-1.481461

**Figure 8.** Variation of temperature profile  $\theta(\eta)$  with  $We$  when  $S = 0.2 = \sigma$  and  $Pr = 4.0$ .**Figure 10.** Variation of temperature profile  $\theta(\eta)$  with  $Pr$  when  $We = 2.0, S = 0.2$  and  $\sigma = 0.5$ .**Figure 9.** Variation of temperature profile  $\theta(\eta)$  with  $\sigma$  when  $We = 2.0, S = 0.2$  and  $Pr = 4.0$ .

behavior is also improved with stretching of disk. The similar response for radial velocity is reported by Weidman [52] temperature at large scale and fall in temperature reduces as stretching is involved. Figure 10 reflects the Prandtl number analyzing the rotational stagnation point flow over a radial stretching sheet with suction and blowing effects. The swirl velocity  $g(\eta)$  show a similar response for both suction and blowing flow cases as radial velocity but the difference is that the increasing and decreasing behavior of velocity w.r.t blowing and suction minimizes with the stretching of disk. The decline in magnitude of swirl velocity occurs with stret-

-ching parameter is probably due to rushing of fluid in radial direction.

Figure 8 exhibits the decline in similarity temperature profile against the variation of viscoelastic parameter  $We$  for both  $\beta = 0$  and  $\beta > 0$ . The decline in temperature of the fluid over rotatory disk is maximum and it cut off as stretching is involved. The temperature of the viscoelastic fluid also shows fall for several values of  $\beta$  in Figure 9. It is observed that over non-rotating disk the rotation declines the  $Pr$  nature on temperature of the viscoelastic fluid over stretching and non-stretching disk. The decline in temperature against Prandtl number  $Pr$  is noted due to the fact that Prandtl number rises the heat transfer rate which cools the rotational system and as a result the decline in temperature inside the thermal boundary layer is noticed.

In Tables 3 and 4, the radial and swirl skin frictions are computed and presented for various values of rotation parameter  $\sigma$  and stretching parameter for both viscous and viscoelastic fluid. The rotation parameter rises the radial skin friction for viscous as well as non-Newtonian fluid. The rise is reliable with the fact that the viscoelastic boundary layer become thinner in radial direction with boosting rotation. The swirl or azimuthal skin friction is decreasing function of rotation parameter. The decreasing behavior is followed by

**Table 5.** Variation of Nusselt’s number against rotation parameter and stretching parameter with  $S = 0.1$ . and  $Pr = 4.0$ .

$\sigma$	Nusselt’s number			$\beta$	Nusselt’s number		
	$We = 0.0$	$We = 0.5$	$We = 1.0$		$We = 0.0$	$We = 0.5$	$We = 1.0$
0.0	2.633004	2.739910	2.780009	0.0	2.130364	2.299833	2.355965
0.5	2.636729	2.746533	2.785276	0.5	2.636729	2.746532	2.785276
1.0	2.647737	2.765821	2.800629	1.0	3.051624	3.154722	3.188188
1.5	2.665551	2.796206	2.824865	1.5	3.411054	3.528017	3.560684
2.0	2.689470	2.835546	2.856293	2.0	3.732331	3.871926	3.905467

swirl skin friction for viscous fluid  $We = 0$  and decline is strengthen for viscoelastic fluid. Under the influence of rotation, the azimuthal boundary layer in viscoelastic fluid becomes thicker and as a result the azimuthal skin friction decrease. Both radial and azimuthal skin frictions are decreasing function of radial stretching parameter of the disk for several values of  $We$ . The stretching of the disk in radial direction thinner the viscoelastic boundary layer in both radial and azimuthal directions and as a result the corresponding shear stresses reduces. Furthermore, under the influence of strong rheology, radial skin friction has small magnitude with strong rotation and opposite trend is noted for azimuthal skin fraction with strong rotation.

The variation of Nusselt’s number against the rotation parameter  $\sigma$  and stretching parameter  $\beta$  can be seen in Table 5 for various values of  $We$ . Numerical values of Nusselt’s number rises against rotation parameter for viscous fluid  $We = 0$  and viscoelastic fluid  $We > 0$ . Similar trend is noted for Nusselt’s number against the stretching parameter for both viscous and viscoelastic fluid. Furthermore, heat transfer rate increases with viscoelastic parameter  $We$ , because for higher values of viscoelastic parameter  $We$  the rotating fluid over a stretching disk has lower thermal conductivity, which overcomes the conduction and therefore rises the variation in heat transfer rate. Due to this reason, the reduction in thermal boundary layer thickness and rise in heat transfer rate over a spiraling disk is examined.

### 6. Conclusions

Numerical analysis is made to express the rotational stagnation point second-grade liquid flowing over the spiraling rotatory disk. The problem is formulated in the coupled partial differential equations forms which are later simplified in view of similar variables. The Keller-Box (KB) procedure is implemented for the numerical solutions of physical model. Below are the final remarks of this conducted research:

- Both viscoelastic parameter and rotation parameter increase the radial and azimuthal boundary-layer thicknesses;
- The impact of viscoelastic parameter and rotation parameter are strengthened with the stretching of the disk;
- The radial and azimuthal velocities decline for blowing case and suction case.

- The temperature profile is decreasing function of rotation parameter, viscoelastic parameter and Prandtl number;
- The rotation parameter rises the radial skin friction while reduces the swirl skin friction.
- Both rotation and stretching parameters growth the heat transfer rate over spiraling disk.

### List of symbols

$T_\infty$	Ambient temperature
$\Omega$	Angular velocity
$z$	Axial coordinate
$\tau_{f\phi}$	Azimuthal shear stresses
$C_{f\phi}$	Azimuthal skin friction
$\mu$	Coefficient of viscosity
$\alpha_2$	Cross-viscosity of the fluid
$\rho$	Density
$f, g$	Dimensionless similarity variables
$\theta$	Dimensionless temperature
$br$	Disk stretching velocity
$q_w$	Heat flux
$A_1, A_2$	Kinematic tensors
$\xi$	Kinematic viscosity
$Pr$	Prandtl number
$\tau_{wr}$	Radial stress
$r$	Radial coordinate
$C_{fr}$	Radial skin friction
$\sigma$	Rotation rate parameter
$c_p$	Specific heat
$\tau$	Stress tensor
$S$	Suction/injection parameter
$T_w$	Surface temperature
$T$	Temperature
$k$	Thermal conductivity
$\alpha$	Thermal diffusivity
$U, V, P, q, Y$	Variables
$u, v, w$	Velocity components
$V$	Velocity field
$\alpha_1$	Viscoelasticity
$We$	Weissenberg number

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## Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Authors contribution statement

**First author:** Data curation; Formal analysis;

**Second author:** Resources; Software; Conceptualization; Data curation; Formal analysis;

**Third author:** Administration; Resources; Software;

**Fourth author:** Administration; Resources; Software;

**Fifth author:** Investigation; Methodology; Project; Writing original draft; Writing, review and editing;

**Sixth author:** Supervision; Validation; Visualization; Writing original draft; Writing, review and editing.

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