



Entropy generation on magneto-hydro-dynamics Eyring-Powell hybrid nanofluid flow over a curved stretching sheet with shape factors and the Cattaneo-Christov heat flux model: A comparative study

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Abstract

This article focuses on the influence of the shape factor of entropy generation on the Magneto-Hydro-Dynamics (MHD) flow of an Eyring-Powell hybrid nanofluid past a permeable, curved stretched sheet with Cattaneo-Christov heat flux. Using the Homotopy Perturbation Method (HPM) and the shooting method, the governing nonlinear coupled Partial Differential Equations (PDEs) are converted into Ordinary Differential Equations (ODEs) with similarity variables and solved (Runge-Kutta (R-K) 4th order). Magnetic field (M), mixed convection (λ), Eyring-Powell fluid, thermal relaxation (γ), curvature, and thermal radiation are studied and represented in terms of velocity, temperature, entropy production, Bejan number, heat transfer, and coefficients of skin friction. To compare outcomes, we employ the HPM. The HPM produces more precise and reliable results than the numerical method. When M affected the hybrid nanofluid as it increased over a curved stretching sheet, the velocity profile decreased. In actuality, the Lorentz force increases as the M value increases. In the presence of a curved stretching sheet, the velocity profile also decreases as a result of increased magnetic parameters. In the three shapes, the temperature profile rises with increasing thermal radiation values. This model is utilized in biological applications such as Magnetic Resonance Imaging (MRI), Radio-Frequency Ablation (RFA), and cancer therapy.

1. Introduction

The use of nanoparticles (metallic, oxide, non-metallic, and ceramic) has garnered much attention in modern times due to its great applicability in various fields, material science, the food industry, and agriculture. Choi [1] produced nanofluids by adding 1-100 nm-sized particles to regular fluids (blood, ethylene glycol, mineral oil, biofluids, etc.). It was first proposed. Many researchers have discussed applications of nanofluids in many areas, including engineering, medical apparatus, paper production, mechanical, architecture, drug delivery, scientific processes biomedical, etc. Additionally, a brand-new kind of nanofluids called mixed nanofluids transmit heat more effectively than conventional heat transfer fluid and nanofluid with a single nanoparticle. When more nanoparticles are mixed in with a base

fluid, this makes a hybrid nanofluid. Hybrid nanofluids receive significant attention given their extensive applications in electronic cooling, machining, electronic cooling, lubrication, solar heating, warming procedures in buildings, nuclear cooling systems, generator cooling, welding, biomedicine, and targeted drug delivery [2,3]. The silver carriers are then modified in two steps to achieve dual functionalities. Silver nanoparticles are crosslinked with iron oxide nanoparticles (Fe_3O_4) to form Ag- Fe_3O_4 nanocomposites (Fe-Ag). Overall, the superparamagnetic component of Fe_3O_4 nanoparticles will allow the nanocomposites to be magnetized in a Magnetic field (M), providing a preferred magnetic moment that can be carefully

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controlled against circulation shearing for drug delivery, medical business as anti-microbial and anti-cancer agents, photothermal therapy, pictures, and catalysis. Shah et al. [4] explained the uses of gold nanoparticles in biology and heating. Lacerda et al. [5] examined the interaction of gold nanoparticles with common human blood proteins. Numerical research was done on heat transfer analysis using nanofluid flow in a porous medium by Kumar et al. [6]. Abbas et al. [7] conducted a study to examine the Magneto-Hydro-Dynamics (MHD), thermal radiation, heat generation, and hybrid (Ag-Cu/pure water) nanofluid flow past a porous media curved sheet with a nonlinear stretching sheet. Babu et al. [8] studied the squeezed flow of polyethylene glycol and water-based hybrid nanofluids over a magnetized sensor surface. The Electro-Magneto-Hydro-Dynamic (EMHD) flow of a nanofluid with a hyperbolic tangent across a stretched sheet was investigated by Asogwa and Shankar Goud [9]. Sajjan et al. [10] studied the influence of linear, nonlinear, and quadratic Rosseland approximations on the 3D flow behavior of ternary hybrid nanoparticles of different shapes.

The importance of analyzing MHD flow phenomena and their applications in industries, geology, including geology, attractive medication focuses, astrophysics, geophysics, the drug industry, engineering, magnetic drug targeting, pharmaceuticals, and mechanical engineering-moreover, biomedical, magnetic endoscopy, and cancer tumour treatment. The influence of thermal radiation and heat transfer on Carbon Nanotubes (CNTs) on MHD flow is currently being examined by Mahabaleshwar et al. [11]. Islam et al. [12] discussed optimizing entropy in MHD nanofluid flow over a curved stretching surface. The entropy generation of MHD and radiation hybrid (CuO-MgO/water (50%)+EG (50%)) nanofluid flow over a curved stretched sheet was studied by Sakkaravarthi and Reddy [13]. Jalili et al. [14] investigated the effect of magnetic and boundary parameters on the flow characteristics analysis of a shrinking sheet of micropolar ferrofluid. The MHD flow of second-grade nano liquid across a convectively heated curved stretched surface was investigated by Reddy et al. [15]. Rauf et al. [16] investigated the effects of nonlinear surface extension on heat transfer and MHD micropolar ferrofluid flow. The magnetohydrodynamic convective streams of tangent hyperbolic nanofluid crossing an elastic texture that is nonlinearly elongating were studied by Ashraf et al. [17].

The process of heat transmission, also known as thermal radiation, involves the transfer of heat energy between two surfaces. In modern-day cycles, MHD production plays an integral part in manufacturing electronic chips and paper plates. The effect of heat radiation in magnetohydrodynamic flow is enormous and becoming more relevant. Research on viscous flow with thermal radiation has a lot of uses in

human activities, especially when it comes to heating and cooling, as well as biomedical and clinical therapy. It is also essential in the context of space technology and high-temperature operations. The effects of ferromagnetic fluid flow across a porous stretched sheet were investigated by Jakeer and Bala Anki Reddy [18]. The impact of the chemical reaction and nonlinear thermal radiation hybrid nanofluids on a curved expanding surface has been studied by Ahmed et al. [19]. Sakkaravarthi and Reddy [20] investigated the MHD flow behavior of two- a dimensional Casson hybrid nanofluid over a porous curved stretching sheet. Jalili et al. [21] investigated thermal analysis and pressure drop in non-continuous helical baffles with different helix angles and hybrid nanoparticles.

Many scientists, engineers, and research scholars have worked on the entropy of thermodynamic systems in recent years. Entropy generation, also known as irreversibility, is a critical measure to calculate the effectiveness of an energy system. The analysis of these irreversible effects (heat transfer and flow) uses the standard measurement of entropy creation and depreciation. Entropy generation occurs through transferring heat, thermal radiation, Joule heating, porosity, and viscous dissipation. Utilizes legal entropy minimization and creation measurement methods. In 1979, Bejan [22] investigated the concepts of entropy generation in convective heat transfer and fluid friction issues. He validated that reducing entropy generation may strengthen a thermal system's mechanical design. Due to its wide variety of heat transfer applications, including maximum heat pump system efficiency, thermal exchangers, maximum heat transfer rate, geothermal applications, biological sectors, and filter design, entropy generation is essential. The irreversibility of the Darcy-Forchheimer presence of CNTs by Joule heating effects was investigated using a curved stretching sheet by Hayat et al. [23].

The main intention of this study is no one has been done on the effect of entropy generation on MHD Eyring-Powell hybrid nanofluid over a porous curved stretching surface in the presence of non-linear radiation, heat generation, different shape factors, and the Cattaneo-Christov heat flux model. So that's why, all effects are considered in this model. In this study, blood was used as the base fluid, and Ag and Fe_3O_4 were the nanoparticles. Non-linear Partial Differential Equations (PDEs) are self-similarly transformed into Ordinary Differential Equations (ODEs) by using a set of transformations. To solve the Homotopy Perturbation Method (HPM) is applied. This model has an extensive variety of biomedical applications, including hyperthermia therapeutics and cancer treatment therapeutics. They are advantageous for incorporating Magnetic Resonance Imaging (MRI), especially Radio-Frequency Ablation (RFA). A magnetic force field is used to diagnose or treat diseases.

2. Mathematical formulation

Consider the steady, incompressible, two-dimensional MHD flow of an Eyring-Powell hybrid nanofluid with blood as the base fluid Ag and Fe_3O_4 are nanoparticles over a porous flow on a curved stretched sheet. We have computed the boundary layer of a hybrid nanofluid traveling over a curved extended surface with a radius R . Putting two equal and opposing pressures in the s direction while maintaining the origin and the velocity the $u_w = as(a > 0)$, where a represents the constant stretch. A M of strength (B_0) is applied in the perpendicular direction to the flow. As seen in Figure 1, the radius of curvature normalizes the surface's shape. The influences of MHD, nonlinear thermal radiation, and porous media, Joule heating are all being investigated. Furthermore, the Cattaneo-Christov theory, which is based on thermal relaxation (γ), is also explored for heat transfer. The following are the governing equations [24]:

$$\frac{\partial}{\partial r}(r+R)v + R\frac{\partial}{\partial s}u = 0, \quad (1)$$

$$\frac{u^2}{r+R} = \frac{1}{\rho_{hnf}} \frac{\partial}{\partial r} \hat{p}, \quad (2)$$

$$\begin{aligned} v \frac{\partial u}{\partial r} + \left(\frac{R}{r+R} \right) u \frac{\partial u}{\partial s} + \frac{uv}{r+R} = \\ - \frac{1}{\rho_{hnf}} \left(\frac{R}{r+R} \right) \frac{\partial \hat{p}}{\partial s} + \left(\gamma_{hnf} + \frac{1}{\rho_{hnf} \beta c} \right) \left(\frac{\partial^2 u}{\partial r^2} + \left(\frac{1}{r+R} \right) \frac{\partial u}{\partial r} \right) \\ - \frac{1}{6\rho_{hnf} \beta c^3} \left(3 \frac{\partial^2 u}{\partial r^2} \left(\frac{\partial u}{\partial r} \right)^2 - \left(\frac{1}{r+R} \right) \left(\frac{\partial u}{\partial r} \right)^3 \right) \\ + \frac{3u}{(r+R)^2} \left(\frac{\partial u}{\partial r} \right)^2 + \frac{3u^2}{(r+R)^2} \frac{\partial^2 u}{\partial r^2} + \frac{u^3}{(r+R)^4} \\ - \frac{6u}{r+R} \frac{\partial u}{\partial r} \frac{\partial^2 u}{\partial r^2} - \frac{3u^2}{(r+R)^3} \frac{\partial u}{\partial r} \\ - \frac{\sigma_{hnf}}{\rho_{hnf}} B_0^2 u + \frac{(\rho\beta)_{hnf}}{\rho_{hnf}} g(T - T_\infty) - \frac{\mu_{hnf}}{\rho_{hnf}} \frac{u}{k_p}, \end{aligned} \quad (3)$$

$$\begin{aligned} v \frac{\partial T}{\partial r} + u \left(\frac{R}{r+R} \right) \frac{\partial T}{\partial s} + \lambda_E \psi_E = \\ \left(\frac{k_{hnf}}{(\rho c_p)_{hnf}} \left(\left(\frac{1}{r+R} \right) \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \right) \right. \\ \left. - \frac{1}{(\rho c_p)_{hnf}} \left(\frac{1}{r+R} \right) \frac{\partial}{\partial r} ((r+R)q_r) \right. \\ \left. + \frac{\mu_{hnf}}{(\rho c_p)_{hnf}} \left(\frac{\partial u}{\partial r} + \frac{u}{r+R} \right)^2 \right. \\ \left. + \frac{\sigma_{hnf}}{(\rho c_p)_{hnf}} B_0^2 u^2 \right. \\ \left. + \frac{1}{(\rho c_p)_{hnf}} Q(T - T_\infty) \exp \left(-\sqrt{\frac{a}{\nu}} r \right) \right). \end{aligned} \quad (4)$$

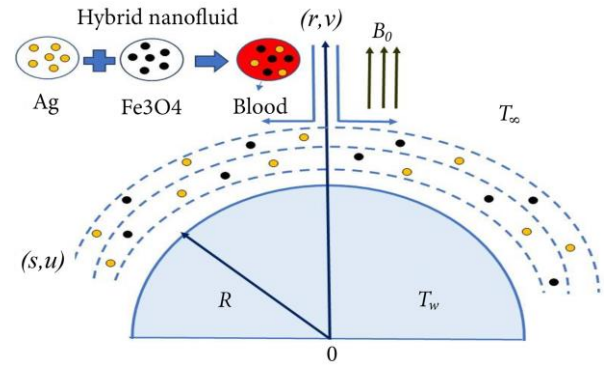


Figure 1. Hybrid nanofluid flow is shown on a curved stretching sheet.

Here,

$$\begin{aligned} \psi_E = \left(\frac{R}{r+R} u \right) \frac{\partial v}{\partial s} \frac{\partial T}{\partial r} + \left(\frac{R^2}{(r+R)^2} u \right) \frac{\partial u}{\partial s} \frac{\partial T}{\partial s} \\ + v \frac{\partial v}{\partial r} \frac{\partial T}{\partial r} + 2 \frac{uv}{r+R} \frac{\partial^2 T}{\partial r \partial s} + \frac{vR}{r+R} \frac{\partial u}{\partial r} \frac{\partial T}{\partial s} \\ - \frac{uv}{(r+R)^2} \frac{\partial T}{\partial s} + \frac{u^2}{(r+R)^2} \frac{\partial^2 T}{\partial s^2} + v^2 \frac{\partial^2 T}{\partial r^2}, \end{aligned}$$

boundary conditions,

$$\begin{aligned} v = v_w, \quad u = as + L \left(\frac{\partial}{\partial r} u - \frac{1}{r+R} u \right) \\ - k \left(\frac{\partial T}{\partial r} \right) = h_f (T_w - T) \quad \text{as } r \rightarrow 0, \\ \frac{\partial u}{\partial r} \rightarrow 0, \quad u \rightarrow 0, \quad T = T_\infty \quad \text{as } r \rightarrow \infty. \end{aligned} \quad (5)$$

Thermophysical models follow as Ref. [25]:

$$\begin{aligned} \rho_{hnf} &= (1 - \phi_2) \left((1 - \phi_1) \rho_{bf} + \phi_1 \rho_{1s} \right) + \phi_2 \rho_{2s}, \\ (\rho c_p)_{hnf} &= (1 - \phi_2) \left((\rho c_p)_{bf} (1 - \phi_1) + \phi_1 (\rho c_p)_{1s} \right) \\ &\quad + (\rho c_p)_{2s} \phi_2, \\ (\rho \beta)_{hnf} &= (1 - \phi_2) (1 - \phi_1) (\rho \beta)_{bf} + \phi_1 (\rho \beta)_{1s} \\ &\quad + \phi_2 (\rho \beta)_{2s}, \end{aligned}$$

$$\begin{aligned} \mu_{hnf} &= \frac{\mu_f}{(1 - \phi_1)^{2.5} (1 - \phi_2)^{2.5}}, \quad \alpha_{hnf} \\ &= \frac{k_{hnf}}{(\rho c_p)_{hnf}}, \quad \nu_{hnf} = \frac{\mu_{hnf}}{\rho_{hnf}}, \end{aligned}$$

$$\begin{aligned} \frac{\sigma_{hnf}}{\sigma_f} = 1 + \frac{3 \left(\frac{(\phi_2 \sigma_{2s} + \phi_1 \sigma_{1s})}{\sigma_{bf}} - (\phi_2 + \phi_1) \right)}{\left(\frac{(\sigma_{2s} \phi_2 + \sigma_{1s} \phi_1)}{\phi \sigma_{bf}} \right) - \left(\frac{(\sigma_{2s} \phi_2 + \sigma_{1s} \phi_1)}{\sigma_{bf}} \right) + 2} \\ \left(\frac{(\sigma_{2s} \phi_2 + \sigma_{1s} \phi_1)}{\phi \sigma_{bf}} \right) - \left(\frac{(\sigma_{2s} \phi_2 + \sigma_{1s} \phi_1)}{\sigma_{bf}} \right) + 2 \end{aligned}$$

$$\begin{aligned} \frac{k_{hnf}}{k_{bf}} &= \frac{k_{2s} + (m-1)k_{bf} - (m-1)\phi_2 + (k_{bf} - k_{2s})}{k_{2s} + (m-1)k_{bf} + \phi_2(k_{bf} - k_{2s})}, \\ \frac{k_{bf}}{k_f} &= \frac{k_{1s} + (m-1)k_f - (m-1)\phi_1 + (k_f - k_{1s})}{k_{1s} + (m-1)k_f + \phi_1(k_f - k_{1s})}. \end{aligned} \quad (6)$$

The following similarity transmutations are taken into consideration:

$$v = -\left(\frac{R}{r+R}\right)\sqrt{av_f} f(\eta), \quad u = saf'(\eta),$$

$$T = T_\infty [1 + (\theta_w - 1)\theta(\eta)], \quad \hat{p} = \rho_f a^2 s^2 P(\eta),$$

$$\eta = \sqrt{\frac{a}{v_f}} r. \quad (7)$$

Eqs. (2)-(5) are simplified by using Eq. (7) as follows:

$$P' = A_3 \frac{f'^2}{(\eta + K)}, \quad (8)$$

$$\frac{2K}{\eta + K} \frac{P}{A_3} = \frac{1}{A_2} (1 + \chi_1) \left(f''' - \frac{f'}{(\eta + K)^2} + \frac{f''}{\eta + K} \right)$$

$$- \chi_1 \chi_2 \left(\frac{3f'^2 f''}{(\eta + K)^2} + \frac{3f'^2 f'}{(\eta + K)^2} - \frac{3f'^2 f''}{(\eta + K)^3} - \frac{6f' f'' f''}{(\eta + K)} - \frac{f'^3}{\eta + K} + \frac{f'^3}{(\eta + K)^4} \right)$$

$$- \frac{f' f'''}{\eta + K} - \frac{K(f'^2)}{(\eta + K)} + \frac{K(ff'')}{(\eta + K)} + \frac{K(ff')}{(\eta + K)^2}$$

$$- \frac{A_4}{A_3} M f' + \frac{A_5}{A_3} \lambda \theta - \frac{S_k}{A_2} f'. \quad (9)$$

Here radiative heat flux is q_r . The Taylor series linear function T^4 continues to be a temperature. Higher orders are not considered T .

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4. \text{ If } S > 0,$$

is the suction and $S < 0$ is the injection, M is the Magnetic field parameter, λ is the mixed convection parameter, K is the Curvature parameter, Pr is the Prandtl number, λ_1 is the heat generation/absorption parameter, Ec is the Eckert number, Rd is the thermal Radiation parameter, S_k is the porosity parameter, γ is the thermal relaxation parameter and χ_1, χ_2 Eyring-Powell parameters are given mathematically as follows:

$$q_r = \frac{-4\sigma^*}{3k^*} \frac{\partial T^4}{\partial r}, \quad M = \frac{\sigma_f B_0^2}{a\rho_f}, \quad \lambda = \frac{g(T_w - T_\infty)}{a^2 s},$$

$$K = \sqrt{\frac{a}{v_f}} R, \quad Ec = \frac{a^2 s^2}{c_p (T_w - T_\infty)}, \quad Pr = \frac{\nu_f}{\alpha},$$

$$\alpha = \frac{k_f}{(\rho c_p)_f}, \quad \lambda_1 = \frac{Q}{a(\rho c_p)_f}, \quad Rd = \frac{16\sigma^* T_\infty^3}{k_f 3k^*},$$

$$EcPr = Br, \quad S_k = \frac{\nu_f}{k_p}, \quad \gamma = \lambda_E a,$$

$$\theta_w = \frac{T_w}{T_\infty}, \quad \chi_1 = \frac{1}{(\mu\beta c)}, \quad \chi_2 = \frac{a^3 s^2}{6(\nu_f c^2)}. \quad (10)$$

Eqs. (8) and (9) can be resolved without taking the pressure into account:

$$(1 + \chi_1) \left(f^{iv} + 2 \frac{f'''}{\eta + K} - \frac{f''}{(\eta + K)^2} + \frac{f'}{(\eta + K)^3} \right)$$

$$+ A_2 \left(\frac{K}{\eta + K} (f''' f - f'' f') + \left(\frac{K}{(\eta + K)^2} \right) (f'' f - (f')^2) - \frac{K}{(\eta + K)^3} f' f \right)$$

$$+ 3A_2 \chi_1 \chi_2 \left(\frac{2f' f'' f^{iv}}{K + \eta} - \frac{f'^2 f^{iv}}{(K + \eta)^2} - f'^2 f^{iv} \right)$$

$$+ \frac{A_2 \chi_1 \chi_2}{(K + \eta)} (6f'' f'^2 + 6(f'')^2 f')$$

$$- \frac{A_2 \chi_1 \chi_2}{(K + \eta)^2} (12f' f' f'' + 3(f'')^3)$$

$$- \frac{9A_2 \chi_1 \chi_2}{(K + \eta)^4} f'^2 + \frac{3A_2 \chi_1 \chi_2}{(K + \eta)^5} f'^3$$

$$+ \frac{9A_2 \chi_1 \chi_2}{(K + \eta)^3} (6f'' f'^2 + 9f' f'^2)$$

$$- 6A_2 \chi_1 \chi_2 (f'')^2 f' - A_4 A_5 M \left(\frac{f'}{(\eta + K)} + f'' \right)$$

$$- S_k \left(\frac{f'}{(\eta + K)} + f'' \right) + A_1 A_3 \lambda \left(\theta' + \frac{1}{(\eta + K)} \theta \right) = 0, \quad (11)$$

$$\left(\theta'' + \frac{\theta'}{\eta + K} \right) \left(1 + \frac{Rd}{A_7} (1 + (\theta_w - 1)\theta)^3 \right)$$

$$+ \frac{3Rd}{A_7} [(1 + (\theta_w - 1)\theta)^2] \theta^2$$

$$+ \frac{A_6}{A_7} Pr \left(\frac{K}{\eta + K} \right) f' \theta' + \frac{Br}{A_4 A_7} \left(f'' + \frac{f'}{\eta + K} \right)^2$$

$$- \frac{Pr}{A_7} \gamma \left(\frac{K^2}{(\eta + K)^2} (f'^2 \theta'' + ff' \theta') - K^2 f'^2 \theta' \right)$$

$$+ \frac{A_4}{A_7} Br M (f')^2 + \frac{Pr}{A_7} \lambda_1 \exp(-\eta) \theta = 0. \quad (12)$$

In the above equations, we regard:

$$A_1 = \frac{\mu_{hnf}}{\mu_f}, \quad A_2 = \frac{\mu_{hnf}}{\mu_f} \cdot \frac{\rho_{hnf}}{\rho_f},$$

$$A_3 = \frac{\rho_{hnf}}{\rho_f}, \quad A_4 = \frac{\sigma_{hnf}}{\sigma_f},$$

$$A_5 = \frac{(\rho\beta)_{hnf}}{(\rho\beta)_f}, \quad A_6 = \frac{(\rho c_p)_{hnf}}{(\rho c_p)_f},$$

$$A_7 = \frac{k_{hnf}}{k_f},$$

boundary conditions,

$$f(0) = S, \quad -\theta'(0) = Bi(1 - \theta(0)),$$

$$f'(0) = 1 + L_s \left(f''(0) - \frac{f'(0)}{K} \right)$$

$$f''(\infty) = 0, \quad f'(\infty) = 0, \quad \theta(\infty) = 0. \quad (13)$$

3. Development of entropy generation

The second law of thermodynamics defines entropy formation as the irreversible process of hybrid nanofluid flow over the curved stretching sheet. Many organizations, including viscous forces, thermal radiation, porosity, and low-driven force, create normal entropy generation. The creation of entropy may be used to calculate the amount of energy wasted during a task. There are several liquid models. Entropy-reducing systems. The model optimizes entropy in the current flow issue:

3.1. Entropy generation

The entropy development in a dimensional form is:

$$E_G = \frac{k_{hnf}}{T_\infty^2} \left(1 + \frac{16\sigma^* T^3}{k_f 3k^*} \right) \left(\frac{\partial T}{\partial r} \right)^2 + \frac{\mu_{hnf}}{T_\infty} \left(\frac{\partial u}{\partial r} + \frac{u}{r+R} \right)^2 + \frac{\mu_{hnf}}{T_\infty k_p} u^2 + \frac{\sigma_{hnf}}{T_\infty} B_0^2 u^2. \quad (14)$$

The dimensionless form is:

$$S_G = A_7 \alpha_1 \theta'^2 \left(1 + Rd \left(1 + (\theta_w - 1) \theta \right)^3 \right) + \frac{Br}{A_1} \left(f'' + \frac{f'}{\eta + K} \right)^2 + A_4 Br M (f')^2 + \frac{S_k Br f'^2}{A_1}. \quad (15)$$

3.2. Bejan number

The Bejan number, which also has to be written as:

$$Be = \frac{A_7 \alpha_1 \theta'^2 \left(1 + Rd \left(1 + (\theta_w - 1) \theta \right)^3 \right)}{A_7 \alpha_1 \theta'^2 \left(1 + Rd \left(1 + (\theta_w - 1) \theta \right)^3 \right) + \frac{Br}{A_1} \left(f'' + \frac{f'}{\eta + K} \right)^2 + A_4 Br M (f')^2 + \frac{S_k Br f'^2}{A_1}}. \quad (16)$$

Designates the Brinkman number (Br);

$$Br = \frac{\mu_{hnf} (as)^2}{k_f (T_w - T_\infty)},$$

total entropy rate;

$$S_G = \frac{E_G T_\infty v_f}{ak_f (T_w - T_\infty)},$$

temperature ratio parameter;

$$\alpha_1 = \frac{\Delta T}{T_\infty} = \frac{T_w - T_\infty}{T_\infty}.$$

4. Physical quantities

4.1. A measure of the skin friction coefficient

One of the physical factors of importance is the skin friction coefficient along the s -directions;

$$C_{fs} = \frac{\tau_{rs}}{\rho (u_w)^2}.$$

As follows is a representation of shear stress:

$$\tau_{rs} = \left(\left(\mu + \frac{1}{\beta c} \right) \left(\frac{\partial u}{\partial r} - \frac{u}{r+R} \right) - \frac{1}{6\beta c^3} \left(\left(\frac{\partial u}{\partial r} \right)^3 - \frac{u^3}{(r+R)^3} \right) + \frac{3u^2}{(r+R)^2} \left(\frac{\partial u}{\partial r} \right) - \frac{3u}{r+R} \left(\frac{\partial u}{\partial r} \right)^2 \right) \Bigg|_{r=0} \quad (17)$$

Then we have:

$$C_{fs} Re_s^{1/2} = (1 + \chi_1) \left[f''(0) - \frac{f'(0)}{K} \right] - \chi_1 \chi_2 \left(f'''(0) - \frac{3f''(0)f'(0)}{K} + \frac{3f'(0)f'^2(0)}{K^2} - \frac{f'^3(0)}{K^3} \right). \quad (18)$$

4.2. Nusselt number

The physical quantities of interest are the rate of heat transfer along the s -directions, which are given as:

$$Nu_s = \frac{sq_w}{k_f (T_w - T_\infty)}, \quad (19)$$

where q_w is the heat flux at the surface in the s -direction, which is given by:

$$q_w = \left((q_r)_w - \frac{\partial T}{\partial r} \right) \Bigg|_{r=0},$$

then we have:

$$Nu_s Re_s^{-1/2} = - \left(\frac{k_{hnf}}{k_f} + Rd \left(1 + (\theta_w - 1) \theta(0) \right)^3 \right) \theta'(0), \quad (20)$$

where the local Reynolds number is:

$$Re_s^{1/2} = \sqrt{\frac{a}{\nu_f}} s.$$

5. Methodology

This section will present the Homotopy approach before going through the suggested issue. This approach was first introduced by He [26] in 2004, and it has been utilized to solve nonlinear ODEs. The technique of the generic boundary element was created in 2004 when he combined this approach with the boundary element method. The process is as follows: A series of guesses may be obtained via Homotopy perturbation, and a further series of replies can be obtained that are incrementally closer and closer to the correct answer. This approach takes advantage of, among other things, traits like action flexibility when choosing the linear operator and initial function. A big nonlinear issue may be solved and split into smaller, simpler linear problems with the same level of action and beginning option flexibility. The controllability of the convergence region is another

advantage of this methodology, making it stand out from other approaches addressed by Shqair [27].

5.1. HPM approach

Consider the following non-linear ODEs of the system to determine the HPM concept:

$$E(\omega) - T(z) = 0, z \in \varphi, \quad (21)$$

boundary conditions are:

$$\psi\left(\omega, \frac{\partial \omega}{\partial n}\right) = 0, z \in \partial \varphi. \quad (22)$$

The formation of the HPM is given as follows:

$$H(\omega, p) = (1-p)(L(\omega) - L(\omega_0)) + p[E(\omega) - T(z)] = 0, \quad (23)$$

where,

$$\omega(z, p) : \varphi \times [0, 1] \rightarrow R_1. \quad (24)$$

Here, $p \in [0, 1]$ is a fixed parameter, and the boundary conditions are incorporated in (ω_0) . Eq. (21) is important.

To express the previous solution in the power series in p :

$$\omega = \omega_0 + p\omega_1 + p^2\omega_2 + p^3\omega_3 + \dots \quad (25)$$

As an exact approximation, the solution is $p=1$.

$$\omega = \omega_0 + \omega_1 + \omega_2 + \omega_3 + \dots \quad (26)$$

5.2. HPM is implemented in this method

Eqs. (11) and (12) of the HPM model can be expressed as follows by using the equation above:

$$\begin{aligned} & (1-p)f^{iv} + p \\ & \left((1+\chi_1) \left(f^{iv} + 2 \frac{f'''}{\eta+K} - \frac{f''}{(\eta+K)^2} + \frac{f'}{(\eta+K)^3} \right) + \right. \\ & A_2 \left(\frac{K}{\eta+K} (f'''f - f''f') + \left(\frac{K}{(\eta+K)^2} \right) (f''f - f'^2) \right) \\ & \left. - \frac{K}{(\eta+K)^3} f'f \right) \\ & + 3A_2\chi_1\chi_2 \left(\frac{2f'f'f^{iv}}{K+\eta} - \frac{f'^2f^{iv}}{(K+\eta)^2} - f'^2f^{iv} \right) \\ & + \frac{A_2\chi_1\chi_2}{(K+\eta)} (6f''f'^2 + 6f'^2f') - \\ & \frac{A_2\chi_1\chi_2}{(K+\eta)^2} (12f''f'f'' + 3f'^3) \\ & - \frac{9A_2\chi_1\chi_2}{(K+\eta)^4} f'^2 + \frac{3A_2\chi_1\chi_2}{(K+\eta)^5} f'^3 \\ & + \frac{9A_2\chi_1\chi_2}{(K+\eta)^3} (6f''f'^2 + 9f'f'^2) - 6A_2\chi_1\chi_2 f''^2 f' \\ & - A_4A_5M \left(\frac{f'}{(\eta+K)} + f'' \right) \\ & \left. - S_k \left(\frac{f'}{(\eta+K)} + f'' \right) + A_4A_5\lambda \left(\theta' + \frac{1}{(\eta+K)}\theta \right) \right) = 0, \end{aligned} \quad (27)$$

$$\begin{aligned} & (1-p) \left(1 + \frac{1}{A_7} Rd \right) \theta'' + p \\ & \left(\left(\theta'' + \frac{\theta'}{\eta+K} \right) \left(1 + \frac{Rd}{A_7} (1 + (\theta_w - 1)\theta)^3 \right) \right. \\ & + \frac{3Rd}{A_7} \left[(1 + (\theta_w - 1)\theta)^2 \right] \theta'^2 \\ & + \frac{A_6}{A_7} Pr \left(\frac{K}{\eta+K} \right) f\theta' \\ & - \frac{Pr}{A_7} \gamma \left(\frac{K^2}{(\eta+K)^2} (f^2\theta' + ff'\theta') - K^2 f^2\theta' \right) \\ & + \frac{Br}{A_4A_7} \left(f'' + \frac{f'}{\eta+K} \right)^2 \\ & + \frac{A_4}{A_7} BrM (f')^2 \\ & \left. + \frac{Pr}{A_7} \lambda_1 \exp(-\eta)\theta \right) = 0. \end{aligned} \quad (28)$$

Boundary conditions:

$$\left. \begin{aligned} f(0) &= S, -\theta'(0) = Bi(1 - \theta(0)) \\ f'(0) &= 1 + L_s \left(f''(0) - \frac{f'(0)}{K} \right) \\ f''(\infty) &= 0, f'(\infty) = 0, \theta(\infty) = 0 \end{aligned} \right\} \quad (29)$$

$f(\eta)$ and $\theta(\eta)$ are considered in the standard power series:

$$\begin{aligned} f(\eta) &= f_0(\eta) + pf_1(\eta) + p^2f_2(\eta) + p^3f_3(\eta) + \dots \\ \theta(\eta) &= \theta_0(\eta) + p\theta_1(\eta) + p^2\theta_2(\eta) + p^3\theta_3(\eta) + \dots \end{aligned} \quad (30)$$

Eqs. (30) and (31) should be replaced with Eqs. (27) and (29), and the linear equation solution is obtained by comparing the coefficients of similar p -term powers, with the following effect:

$$\begin{aligned} f_0 &= f_0'''\chi_1 + f_0''' = 0, \\ \theta_0 &= \frac{\theta_0'' Rd \theta_w^3}{A_7} - \frac{\theta_0'' \gamma Pr}{A_7} \\ &+ 4\theta_0'' = 0. \end{aligned} \quad (31)$$

When conditions of zero-order apply,

$$\begin{aligned} f_0(0) &= S, f_0'(0) = 1 + L_s \left(f_0''(0) - \frac{f_0'(0)}{K} \right), \\ f_0''(1) &= 0, f_0'(1) = 0, \\ \theta_0(0) &= -Bi(1 - \theta_0(0)), \theta_0(1) = 0. \end{aligned} \quad (32)$$

First-order,

$$\begin{aligned} f_1 &= \frac{A_4A_5\lambda\theta_0'}{(\eta+K)} - \frac{A_2A_5Kf_0''}{(\eta+K)^2} - A_4A_5Mf_0' + A_4A_5\lambda\theta_0' \\ &+ \frac{6A_2\chi_1\chi_2f_0'f_0''f_0^{iv}}{(\eta+K)^2} - \frac{12A_2\chi_1\chi_2f_0'f_0''f_0'''}{(\eta+K)^2} \end{aligned}$$

$$\begin{aligned}
& -\frac{3A_2\chi_1\chi_2f_0'^2f_0''}{(\eta+K)} + \frac{6A_2\chi_1\chi_2f_0''f_0'''}{(\eta+K)} + \frac{6A_2\chi_1\chi_2f_0''^2f_0'''}{(\eta+K)} \\
& -\frac{9A_2\chi_1\chi_2f_0''^2f_0'''}{(\eta+K)^4} + \frac{6A_2\chi_1\chi_2f_0''f_0''^2}{(\eta+K)^2} + \frac{9A_2\chi_1\chi_2f_0''^2f_0'''}{(\eta+K)^3} \\
& +f_1^{iv} + f_1^{iv}\chi_1 + \frac{2\chi_1f_0'''}{(\eta+K)} - \frac{\chi_1f_0'''}{(\eta+K)^2} + \frac{\chi_1f_0'''}{(\eta+K)^3} - \frac{S_kf_0'''}{(\eta+K)} \\
& -\frac{f_0'''}{(\eta+K)^2} + \frac{f_0'''}{(\eta+K)^3} - S_kf_0'' + \frac{2f_0'''}{(\eta+K)} + \frac{A_2Kf_0f_0'''}{(\eta+K)} \\
& -\frac{A_2Kf_0'f_0''}{(\eta+K)} + \frac{A_2Kf_0f_0''}{(\eta+K)^2} - \frac{A_2Kf_0f_0''}{(\eta+K)^3} - \frac{A_1A_4Mf_0'}{(\eta+K)} \\
& -3A_2\chi_1\chi_2f_0^{iv}f_0''^2 - 6A_2\chi_1\chi_2f_0''^2f_0'' - \frac{3A_2\chi_1\chi_2f_0''^3}{(\eta+K)^2} \\
& + \frac{3A_2\chi_1\chi_2f_0''^3}{(\eta+K)^5} = 0,
\end{aligned} \tag{33}$$

$$\begin{aligned}
\theta_1 = & -\frac{Rd\theta_w^3\theta_0''}{A_7} + \frac{Pr\gamma\theta_0''}{A_7} + \frac{Rd\theta_w^3\theta_1''}{A_7} - \frac{Pr\gamma\theta_1''}{A_7} \\
& + \frac{Rd\theta_0''}{A_7(\eta+K)} + \frac{Brf_0''^2}{A_7A_1} - 3\frac{Rd\theta_0^2\theta_0''}{A_7} - 3\frac{Rd\theta_0\theta_0''}{A_7} \\
& - \frac{Rd\theta_0^3\theta_0''}{A_7} + \frac{3Rd\theta_0^2\theta_0''^2}{A_7} - \frac{6Rd\theta_0\theta_0''^2}{A_7} \\
& + \frac{\lambda_1\exp(-\eta)\theta_0}{A_7} + \frac{PrA_6Kf_0\theta_0''}{A_7(\eta+K)} + \frac{2Brf_0''f_0''}{A_7A_1\beta(\eta+K)} \\
& + \frac{Rd\theta_w^3\theta_0'\theta_0^3}{A_7(\eta+K)} + \frac{Brf_0''^2}{A_7A_1\beta(\eta+K)^2} + \frac{\gamma Pr K^2 f_0^2\theta_0^1}{A_7} \\
& + \frac{3RdT_w\theta_0^2\theta_0'}{A_7(\eta+K)} - \frac{6RdT_w\theta_0^2\theta_0'}{A_7(\eta+K)} - \frac{3RdT_w\theta_0^3\theta_0'}{A_7(\eta+K)} \\
& + \frac{3RdT_w^2\theta_0'}{A_7(\eta+K)} + \frac{3RdT_w\theta_0^3\theta_0'}{A_7(\eta+K)} + \frac{2Brf_0''f_0'}{A_7A_1(\eta+K)} \\
& + 4\theta_1' + \frac{3Rd\theta_0''^2}{A_7} - 3\theta_0'' + \frac{\theta_0''}{\eta+K} + \frac{3RdT_w\theta_0\theta_0''}{A_7} \\
& - \frac{6RdT_w\theta_0'\theta_0^2}{A_7} - \frac{3RdT_w^2\theta_0^3\theta_0''}{A_7} + \frac{3RdT_w\theta_0^3\theta_0'}{A_7} \\
& + \frac{3Rd\theta_0'\theta_0^2}{A_7(\eta+K)} - \frac{3Rd\theta_0\theta_0'}{A_7(\eta+K)} - \frac{6T_wRd\theta_0'\theta_0^2}{A_7} \\
& + \frac{3RdT_w^2\theta_0^2\theta_0^2}{A_7(\eta+K)} - \frac{6RdT_w\theta_0^2\theta_0^2}{A_7} + \frac{RdT_w^2\theta_0^3\theta_0'}{A_7} \\
& + \frac{A_4MBrf_0''^2}{A_7} + \frac{Brf_0''^2}{A_7A_1\beta} - \frac{\gamma Pr K^2 f_0'\theta_0'}{A_7(\eta+K)^2} = 0.
\end{aligned} \tag{34}$$

Boundary conditions are:

$$\begin{aligned}
f_0(0) = S, f_1'(0) = 1 + L_s \left(f_1''(0) - \frac{f_1'(0)}{K} \right), \\
f_1''(1) = 0, f_1'(1) = 0.
\end{aligned}$$

$$\theta_1(0) = -Bi(1 - \theta_1(0)), \theta_1(1) = 0. \tag{35}$$

Second-order,

$$\begin{aligned}
f_2 = & \frac{A_1A_5\lambda\theta_1}{(\eta+K)} + A_1A_5\lambda\theta_1' - A_1A_4Mf_1'' - \frac{f_1''}{(\eta+K)^2} \\
& + \frac{2f_1''}{\eta+K} + \frac{\chi_1f_1'}{(\eta+K)^3} + \frac{2\chi_1f_1''}{(\eta+K)} - \frac{\chi_1f_1''}{(\eta+K)^2} \\
& - S_kf_1'' - \frac{S_kf_1'}{(\eta+K)} + f_2^{iv} + \frac{A_2Kf_1f_0''}{(\eta+K)} + \frac{A_2Kf_0f_1''}{(\eta+K)^2} \\
& - \frac{A_2Kf_0'f_1''}{(\eta+K)} - \frac{A_2Kf_1'f_0''}{(\eta+K)} - \frac{2A_2Kf_0'f_1''}{(\eta+K)^2} \\
& - \frac{A_2Kf_1f_0'}{(\eta+K)^3} + \frac{-A_1A_4Mf_1'}{(\eta+K)} - 3A_2\chi_1\chi_2f_1^{iv}f_0''^2 \\
& - 6A_2\chi_1\chi_2f_0''^2f_1'' + \frac{A_2Kf_0f_1''}{(\eta+K)} + \frac{f_1''}{(\eta+K)^3} \\
& - \frac{3A_2\chi_1\chi_2f_0''^2f_1^{iv}}{(\eta+K)} - 3A_2\chi_1\chi_2f_0^{iv}f_0''f_1'' \\
& + \frac{6A_2\chi_1\chi_2f_0''^2f_1''}{(\eta+K)} + \frac{6A_2\chi_1\chi_2f_0''^2f_1''}{(\eta+K)} + \chi_1f_2^{iv} \\
& - 12A_2\chi_1\chi_2f_0''f_1''f_0'' - \frac{9A_2\chi_1\chi_2f_0''^2f_1''}{(\eta+K)^4} \\
& + \frac{9A_2\chi_1\chi_2f_0''^2f_1'}{(\eta+K)^5} - \frac{12A_2\chi_1\chi_2f_0'f_1''f_0''}{(\eta+K)^2} \\
& - \frac{9A_2\chi_1\chi_2f_0''^2f_1''}{(\eta+K)^2} + \frac{9A_2\chi_1\chi_2f_1'f_0''^2}{(\eta+K)^3} \\
& + \frac{6A_2\chi_1\chi_2f_1'f_0''f_0^{iv}}{(\eta+K)^2} - \frac{6A_2\chi_1\chi_2f_1'f_0'f_0^{iv}}{(\eta+K)} \\
& + \frac{12A_2\chi_1\chi_2f_0''f_1''f_0''}{(\eta+K)} + \frac{12A_2\chi_1\chi_2f_0''f_1''f_0''}{(\eta+K)} \\
& - \frac{12A_2\chi_1\chi_2f_0'f_0''f_1''}{(\eta+K)^2} + \frac{6A_2\chi_1\chi_2f_1''f_0''^2}{(\eta+K)^3} \\
& - \frac{12A_2\chi_1\chi_2f_1'f_0''f_0''}{(\eta+K)^2} - \frac{18A_2\chi_1\chi_2f_0'f_1'f_0''}{(\eta+K)^4} \\
& + \frac{12A_2\chi_1\chi_2f_0''f_0'f_1'}{(\eta+K)^3} + \frac{18A_2\chi_1\chi_2f_0'f_0'f_1''}{(\eta+K)^3} \\
& + \frac{6A_2\chi_1\chi_2f_0'f_0''f_1^{iv}}{(\eta+K)^2} \\
& + \frac{6A_2\chi_1\chi_2f_0'f_1''f_1^{iv}}{(\eta+K)^2} = 0,
\end{aligned} \tag{36}$$

$$\begin{aligned}
\theta_2 = & -\frac{Rd\theta_w^3\theta_1''}{A_7} + \frac{Pr\gamma\theta_1''}{A_7} + \frac{Rd\theta_1''}{A_7(\eta+K)} \\
& - 3\frac{Rd\theta_0^2\theta_1''}{A_7} - 3\frac{Rd\theta_0\theta_1''}{A_7} + \frac{Rd\theta_0^3\theta_1''}{A_7} \\
& - \frac{Rd\theta_w^3\theta_1''}{A_7} - \frac{6Rd\theta_0\theta_1''^2}{A_7} + \frac{\lambda_1\exp(-\eta)\theta_1}{A_7} \\
& + \frac{PrA_6Kf_0\theta_1'}{A_7(\eta+K)} + \frac{PrA_6Kf_1\theta_0'}{A_7(\eta+K)} + \frac{6RdT_w^2\theta_0\theta_1\theta_0'}{A_7(\eta+K)}
\end{aligned}$$

$$\begin{aligned}
& -\frac{12RdT_w\theta_0\theta_1'\theta_0'}{A_\gamma(\eta+K)} - \frac{9RdT_w^2\theta_0^2\theta_1'\theta_0'}{A_\gamma(\eta+K)} + \frac{9RdT_w\theta_0^2\theta_1'\theta_0'}{A_\gamma(\eta+K)} \\
& + \frac{2Brf_0''f_1'}{A_\gamma A_\beta(\eta+K)} + \frac{2Brf_1''f_0'}{A_\gamma A_\beta(\eta+K)} + \frac{2Brf_0''f_1'}{A_\gamma A_\beta(\eta+K)} \\
& + \frac{6Rd\theta_0'\theta_1'}{A_\gamma} + \frac{3RdT_w^3\theta_0^2\theta_1'\theta_0'}{A_\gamma(\eta+K)} + \frac{\gamma Pr K^2 f_0^2\theta_1'}{A_\gamma(\eta+K)^2} \\
& - \frac{\gamma Pr K^2 f_0^2\theta_1'}{A_\gamma(\eta+K)^2} + \frac{\gamma Pr K^2 f_0 f_1\theta_0'}{A_\gamma} + \frac{RdT_w^3\theta_0^3\theta_1'}{A_\gamma(\eta+K)} \\
& + \frac{\gamma Pr K^2 f_0^2\theta_1'}{A_\gamma} + \frac{\gamma Pr K^2 f_0^2\theta_1'}{A_\gamma} + \frac{3RdT_w t_1\theta_0'}{A_\gamma(\eta+K)} \\
& + \frac{6Rd\theta_0\theta_1'\theta_0'}{A_\gamma(\eta+K)} - \frac{3Rd\theta_0^2\theta_1'\theta_0'}{A_\gamma(\eta+K)} + \frac{3RdT_w\theta_0\theta_1'}{A_\gamma(\eta+K)} \\
& + \frac{3RdT_w^2\theta_0^2\theta_1'}{A_\gamma(\eta+K)} - \frac{6RdT_w\theta_0^2\theta_1'}{A_\gamma(\eta+K)} - \frac{3RdT_w^2\theta_0^3\theta_1'}{A_\gamma(\eta+K)} \\
& + \frac{3RdT_w\theta_0^3\theta_1'}{A_\gamma(\eta+K)} + \frac{6RdT_w^2\theta_0\theta_1'\theta_0'}{A_\gamma} \\
& - \frac{12RdT_w\theta_0\theta_1'\theta_0''}{A_\gamma} + \frac{3RdT_w^3\theta_0^2\theta_1'\theta_0''}{A_\gamma} - \frac{9RdT_w^2\theta_0^2\theta_1'\theta_0''}{A_\gamma} \\
& + \frac{9RdT_w\theta_0^2\theta_1'\theta_0''}{A_\gamma} + \frac{2Brf_0''f_1'}{A_\gamma A_1(\eta+K)} + \frac{2Brf_1''f_0'}{A_\gamma A_1(\eta+K)} \\
& + \frac{2Brf_0''f_1'}{A_\gamma A_1(\eta+K)^2} + \frac{2Brf_0''f_1'}{A_\gamma A_1(\eta+K)^2} + \frac{2f_0''f_1'}{A_\gamma A_\beta} \\
& + \frac{2A_4 M Brf_0''f_1'}{A_\gamma} + \frac{12RdT_w\theta_0\theta_1'\theta_0'}{A_\gamma} + \frac{6RdT_w^2\theta_0^2\theta_1'}{A_\gamma} \\
& - \frac{12RdT_w\theta_0^2\theta_1'\theta_0'}{A_\gamma} + \frac{6RdT_w^2\theta_0\theta_1'\theta_0'^2}{A_\gamma} - \frac{12RdT_w\theta_0\theta_1'\theta_0'^2}{A_\gamma} \\
& + \frac{Rd\theta_1'}{A_\gamma} - 3\theta_1' + \frac{\theta_1'}{\eta+K} + \frac{3RdT_w\theta_1'\theta_0''}{A_\gamma} + \frac{6Rd\theta_1'\theta_0''}{A_\gamma} \\
& - \frac{3Rd\theta_0^2\theta_1'\theta_0''}{A_\gamma} + \frac{3RdT_w\theta_0\theta_1''}{A_\gamma} + \frac{3RdT_w^2\theta_0^2\theta_1''}{A_\gamma} \\
& - \frac{6RdT_w\theta_0^2\theta_1''}{A_\gamma} - \frac{3RdT_w^2\theta_0^3\theta_1''}{A_\gamma} + \frac{3RdT_w\theta_0^3\theta_1''}{A_\gamma} \\
& + \frac{3Rd\theta_0^2\theta_1''}{A_\gamma(\eta+K)} - \frac{3Rd\theta_0\theta_1''}{A_\gamma(\eta+K)} - \frac{Rd\theta_0\theta_1''}{A_\gamma(\eta+K)} \\
& + \frac{2Brf_0''f_1'}{A_\gamma A_1} + \frac{6Rd\theta_0^2\theta_1'}{A_\gamma} - \frac{12Rd\theta_0\theta_1'}{A_\gamma} + \frac{6RdT_w\theta_1'\theta_0'}{A_\gamma} \\
& + \frac{6RdT_w\theta_1'\theta_0'^2}{A_\gamma} + \frac{6RdT_w^3\theta_0^3\theta_1'}{A_\gamma} - \frac{\gamma Pr K^2 f_0 f_1\theta_0''}{A_\gamma(\eta+K)^2} \\
& - \frac{\gamma Pr K^2 f_0 f_1\theta_1'}{A_\gamma(\eta+K)^2} - \frac{\gamma Pr K^2 f_0 f_1\theta_0'}{A_\gamma(\eta+K)^2} = 0. \quad (37)
\end{aligned}$$

Boundary conditions are:

$$\begin{aligned}
f_2(0) &= S, \quad f_2'(0) = 1 + L_s \left(f_2''(0) - \frac{f_2'(0)}{K} \right), \\
f_2''(1) &= 0, \quad f_2'(1) = 0, \\
\theta_2(0) &= -Bi(1 - \theta_2(0)), \quad \theta_2(1) = 0. \quad (38)
\end{aligned}$$

HPM solutions take $f(\eta)$ and $\theta(\eta)$ into account predetermined dimensionless parameter values in $\phi_1 = 0.02$, $\phi_2 = 0.02$, $Rd = 0.1$, $LS = 0.5$, $\gamma = 0.02$, $M = 1.0$, $\lambda = 0.1$, $Bi = 0.3$, $Pr = 21$, $K = 0.5$, $\lambda_1 = 0.1$, $S_k = 0.2$,

$\chi_1 = 0.2$, $\chi_1 = 0.2$, $\alpha_1 = 0.1$, $S = 0.5$, $\theta_w = 1.5$. The following outcomes are achieved by determining a succession of functions:

$$\begin{aligned}
f(\eta) &= \frac{1}{2\eta+1} (3.858024691 \\
&\times 10^{-17} (1.6661379670 \times 10^{12} \eta^7 \\
&+ 1.181815725 \times 10^{14} \eta^6 + 2.094189598 \\
&\times 10^{16} \eta^4 - 4.347271693 \times 10^{15} \times \ln(2\eta+1) \\
&\times \eta^4 - 1.063806241 \times 10^{15} \eta^5 + 1.915137715 \\
&\times 10^{16} \eta^3 - 2.085340034 \times 10^{16} \ln(2\eta+1) \eta^3 \\
&+ 6.200718999 \times 10^{16} \eta^2 - 3.880199531 \\
&\times 10^{16} \times \ln(2\eta+1) \times \eta^2 + 8.088966250 \times 10^{16} \\
&- 2.533558089 \times 10^{16} \ln(2\eta+1) \eta \\
&- 5.302262184 \times 10^{16} \ln(2\eta+1) \eta \\
&- 5.302262184 \times 10^{15} \ln(2\eta+1) \\
&+ 2.59200000 \times 10^{16})), \quad (39)
\end{aligned}$$

$$\begin{aligned}
\theta(\eta) &= -0.9075255449\eta + 0.3772321588 \\
&+ 0.09968590234\eta^2 + 0.276492273643 \ln(2\eta+1)\eta \\
&+ 0.1511897789 \ln(2\eta+1) - 0.0000001715329004\eta^7 \\
&+ 8.7134714334 \times 10^{-7} \eta^6 + 0.00041194224407\eta^5 \\
&+ 0.0004505548475\eta^4 - 0.01330311444\eta^3 \\
&+ 0.005145987013e^{-\eta}\eta + 1.0720806288 \times 0^{-7} \eta^8. \quad (40)
\end{aligned}$$

5.3. Numerical procedure

Numerical methods are used to solve non-dimensional ODE equations as flows:

$$\begin{aligned}
& (1 + \chi_1) \left(f^{iv} + 2 \frac{f'''}{\eta+K} - \frac{f''}{(\eta+K)^2} + \frac{f'}{(\eta+K)^3} \right) \\
& + A_2 \left[\frac{K}{\eta+K} (f''' f - f'' f') + \left(\frac{K}{(\eta+K)^2} \right) (f'' f - f'^2) \right. \\
& \left. - \frac{K}{(\eta+K)^3} f' f \right] \\
& + 3A_2 \chi_1 \chi_2 \left(\frac{2f' f'' f^{iv}}{K + \eta} - \frac{f''^2 f^{iv}}{(K + \eta)^2} - f''^2 f^{iv} \right) \\
& + \frac{A_2 \chi_1 \chi_2}{(K + \eta)} (6f'' f'^2 + 6f''^2 f') - \frac{A_2 \chi_1 \chi_2}{(K + \eta)^2} (12f'' f' f'' + 3f''^3) \\
& - \frac{9A_2 \chi_1 \chi_2}{(K + \eta)^4} f'^2 + \frac{3A_2 \chi_1 \chi_2}{(K + \eta)^5} f'^3 + \frac{9A_2 \chi_1 \chi_2}{(K + \eta)^3} (6f'' f'^2 + 9f' f'^2) \\
& - 6A_2 \chi_1 \chi_2 f''^2 f' - A_4 A_4 M \left(\frac{f'}{(\eta+K)} + f'' \right) \\
& - S_k \left(\frac{f'}{(\eta+K)} + f'' \right) + A_1 A_3 \lambda \left(\theta' + \frac{1}{(\eta+K)} \theta \right) = 0, \quad (41)
\end{aligned}$$

$$\begin{aligned}
& \left(\theta'' + \frac{\theta'}{\eta+K} \right) \left(1 + \frac{Rd}{A_\gamma} (1 + (\theta_w - 1)\theta)^3 \right) \\
& + \frac{3Rd}{A_\gamma} \left[(1 + (\theta_w - 1)\theta)^2 \right] \theta'^2 + \frac{A_6}{A_\gamma} Pr \left(\frac{K}{\eta+K} \right) f' \theta'
\end{aligned}$$

$$\begin{aligned}
& + \frac{Br}{A_\gamma A_\gamma} \left(f'' + \frac{f'}{\eta + K} \right)^2 - \frac{Pr}{A_\gamma} \gamma \left(\frac{K^2}{(\eta + K)^2} (f^2 \theta'' + f f' \theta') - K^2 f^2 \theta' \right) \\
& + \frac{A_\gamma}{A_\gamma} BrM (f')^2 + \frac{Pr}{A_\gamma} \lambda_1 \exp(-\eta) \theta = 0, \quad (42)
\end{aligned}$$

boundary conditions are:

$$\left. \begin{aligned} f(0) = S, -\theta'(0) = Bi(1 - \theta(0)) \\ f'(0) = 1 + L_s \left(f''(0) - \frac{f'(0)}{K} \right) \\ f''(\infty) = 0, f'(\infty) = 0, \theta(\infty) = 0 \end{aligned} \right\} \quad (43)$$

In the above Eqs. (41) and (42) use the MAPLE software and Runge-Kutta (R-K) 4th order with the shooting method. As a result, the leading equations are solved in Eq. (43), along with their boundary conditions. Limiting asymptotic conditions in Eq. (43), were revived by an imperfect set of efforts η , say η a state in which there is no discernible variation in temperature, velocity, Bejan number, entropy production, or any other characteristic. In boundary layer studies, this is standard. Completed the problem-solving procedure, step scope with $\Delta\eta = 0.01$ It's better to be realistic about the inward converging condition 10^{-6} under all circumstances.

6. Result and discussion

This section studies how different parameters affect velocity, temperature, entropy generation, and Bejan number profiles for the blood as the base fluid mixture with Silver (Ag) and Ferrosferic Oxide (Fe_3O_4) as the nanoparticles over a curved stretching sheet. It looks at how different parameters affect the velocity profile through a curved stretched sheet. The temperature, entropy generation and Bejan number of a hybrid nanofluid over a curved stretched sheet have been studied by comparing shapes (spheres, platelets, and blades). Additionally, investigated and shown in graphs are the skin friction and heat transfer coefficients. To compare results, we use the HPM. The Homotopy Perturbation Technique yields more accurate and consistent results than the numerical method. The results are most likely influenced by many parameters such as the K , Magnetic field parameter (M), porosity parameter (S_k), λ parameter, thermal Radiation parameter (Rd), Brinkman number (Br), thermal relaxation parameter (γ), Eyring-Powell parameters χ_1, χ_2 , heat generation parameter (λ_1), dimensionless Biot number (Bi), Silver (Ag) nanoparticle are volume fraction (ϕ_1), Ferrosferic Oxide (Fe_3O_4) nanoparticles are volume fraction (ϕ_2), dimensionless slip parameter (L_s), the temperature ratio parameter (α_1), velocity profile $f'(\eta)$, temperature profile $\theta(\eta)$, entropy generation (S_G) and Bejan number (Be) is the shown in Figures 2-20. The HPM and NM are compared for temperature and velocity in Figures 2 and 3. The conclusions are based on previous results to validate the results obtained. The values of many dimensionless parameters are fixed in:

$\phi_1 = 0.02, \phi_2 = 0.02, Rd = 0.1, L_s = 0.5, \beta = 2.5, \gamma = 0.02, M = 1.0, \lambda = 0.1, Bi = 0.3, Pr = 21, K = 0.5, \lambda_1 = 0.1, S_k = 0.2, \alpha_1 = 0.1, \chi_1 = 0.2, \chi_2 = 0.2, S = 0.5$ and $\theta_w = 1.5$, are considered. The effects of the shape factors

discussed for shapes like spheres, platelets, and blades are further represented by $m = 3.0, 5.7$, and 8.6 , and hybrid nanoparticles are shown in Table 1.

6.1. Dimensionless velocity profiles

The influences of the M, χ_2 and S_k on the velocity profile are

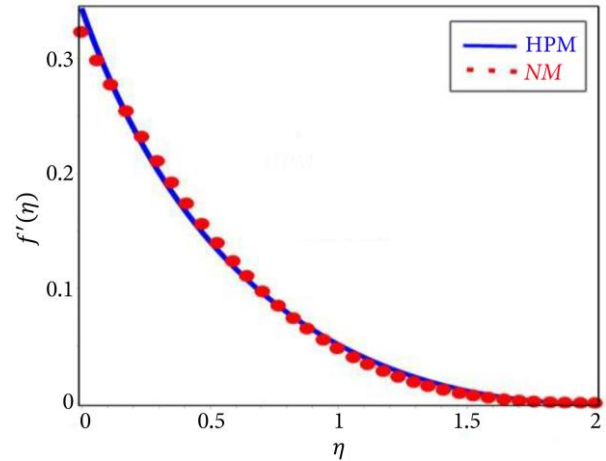


Figure 2. Comparison of the R-K method with the HPM for velocity profile.

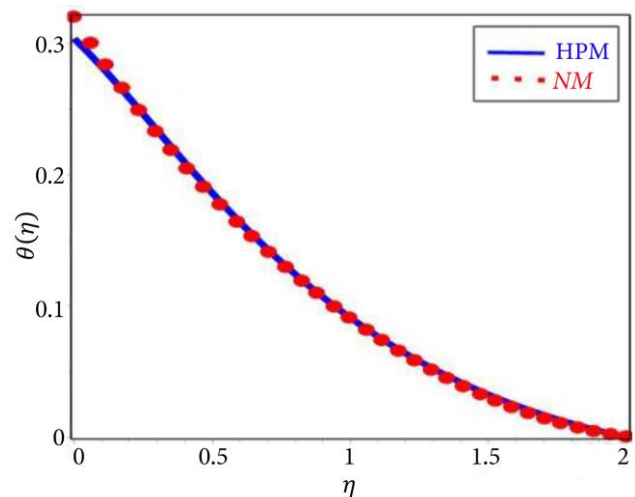


Figure 3. Comparisons of the R-K method with the HPM for temperature profile.

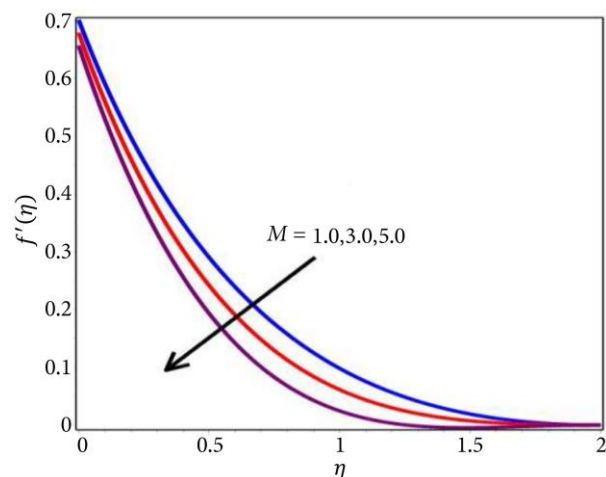


Figure 4. The behavior of $f'(\eta)$ with different values of M .

exposed in Figures 4-6. Figure 4 demonstrates that the velocity profile $f'(\eta)$ is a decreasing function of the M increases over a curved stretching sheet. Physically, when the M effect is increased, the Lorentz force increases, acting in the backward direction of the liquid flow and slowing the arrangement. So, when a curved sheet is stretched, increasing the magnetic parameter slows down the $f'(\eta)$. Figure 5 demonstrates that the velocity profile $f'(\eta)$ is a reducing function of the Eyring-Powell fluid (χ_2). The velocity profile decays gradually for higher values χ_2 .

Table 1. Shows the thermo-physical properties of blood as a base fluid and nanoparticles are Silver (Ag) and Ferrosferic oxide (Fe_3O_4): Divya and Reddy [30].

Property	Blood ($b\eta$)	Silver Ag (1 s)	Ferrosferic oxide Fe_3O_4 (2s)
ρ (kg/m ³)	1063	10500	5180
c_p (J/kgK)	3594	235	670
k (W/mk)	0.492	429	9.7
σ (s/m)	0.8	6.30×10^7	0.025×10^7
$\beta \times 10^{-6}$ (K ⁻¹)	0.18	1.89	1.3
Pr	-	-	-

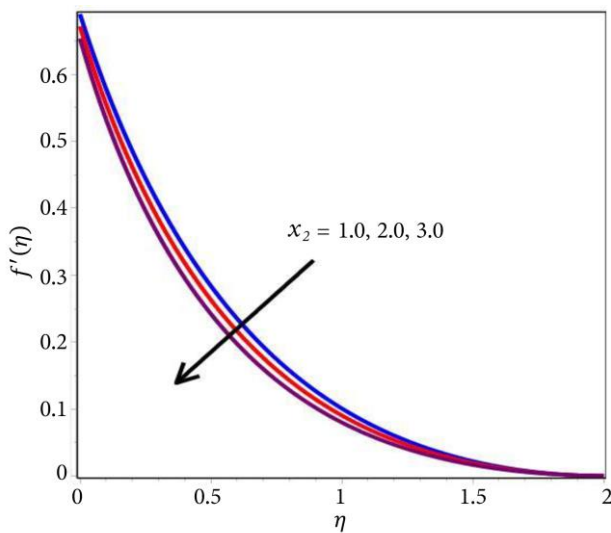


Figure 5. The behavior of $f'(\eta)$ with specific values of χ_2 .

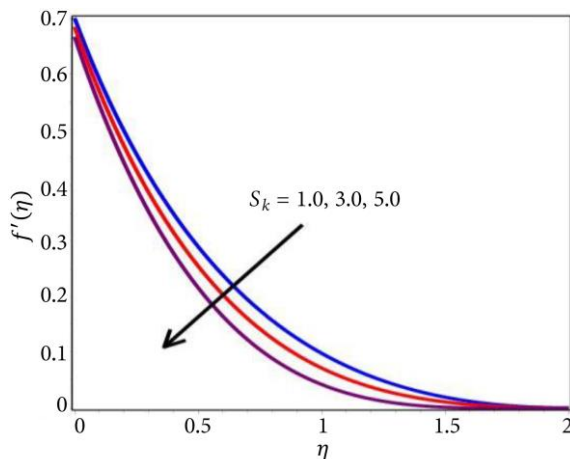


Figure 6. The behavior of $f'(\eta)$ with specific values of S_k .

Figure 6 shows the impact of the porosity (S_k) curved stretching sheet on the hybrid nanofluid velocity profile has decreased. As the porosity variable (S_k) rises, so does the friction force between nanoparticles. Physically, increasing porosity causes the medium's pore size to decrease.

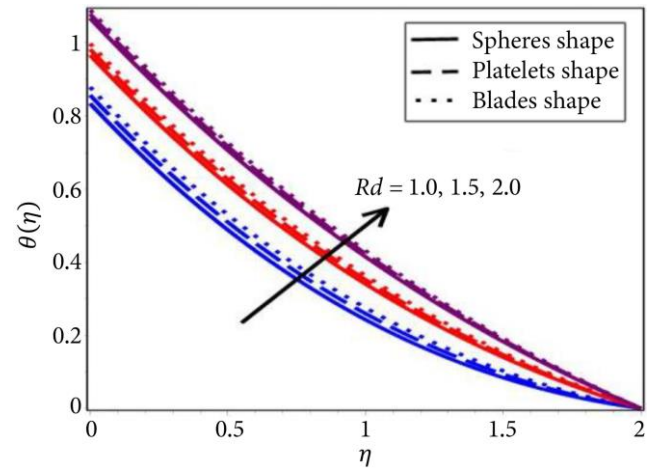


Figure 7. The behavior of $\theta(\eta)$ with specific values of Rd .

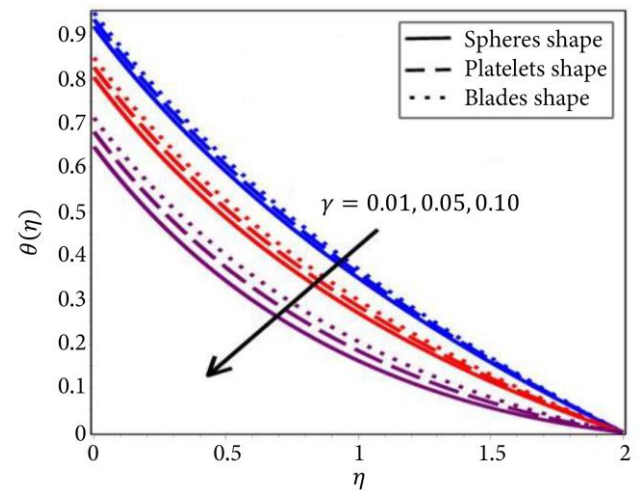


Figure 8. The behavior of $\theta(\eta)$ with specific values of γ .

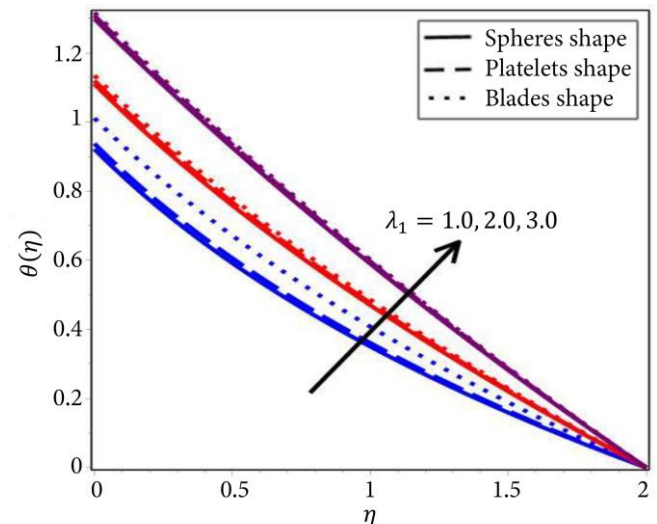


Figure 9. The behavior of $\theta(\eta)$ with specific values of λ .

6.2. Dimensionless temperature profiles

The influences of the thermal radiation (Rd), thermal relaxation (γ), heat generation (λ_1), Brinkman number (Br), and Biot number (Bi) on the comparing shapes (spheres,

platelets, and blades) over a curved stretched sheet at the temperature profile $\theta(\eta)$ are shown in Figures 7-9. The highlights of thermal radiation (Rd) on hybrid nanofuid

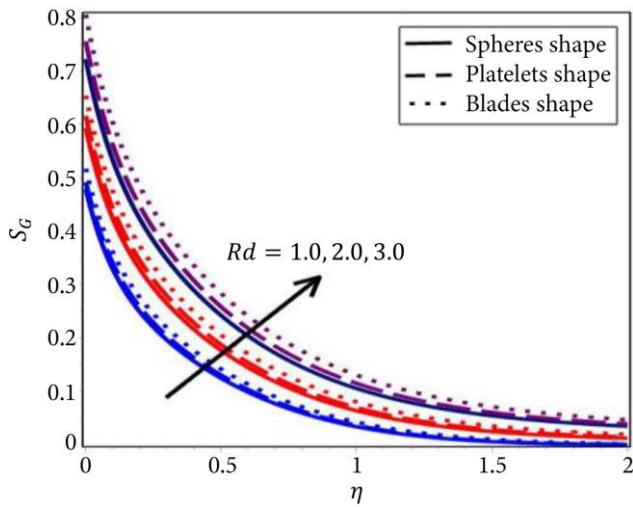


Figure 10. The behavior of S_G with specific values of Rd .

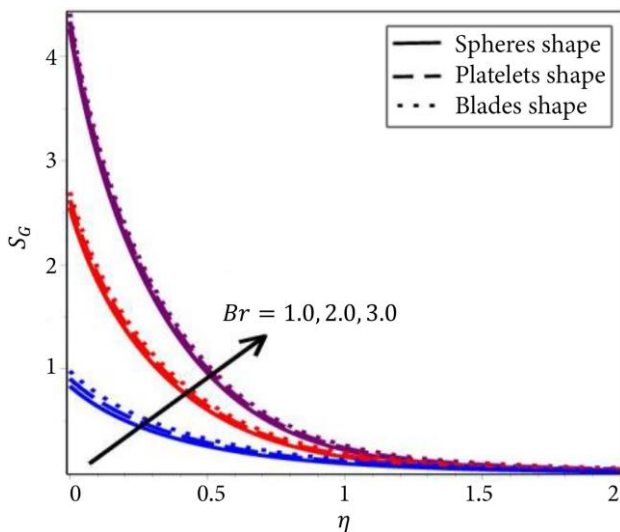


Figure 11. The behavior of S_G with specific values of Br .

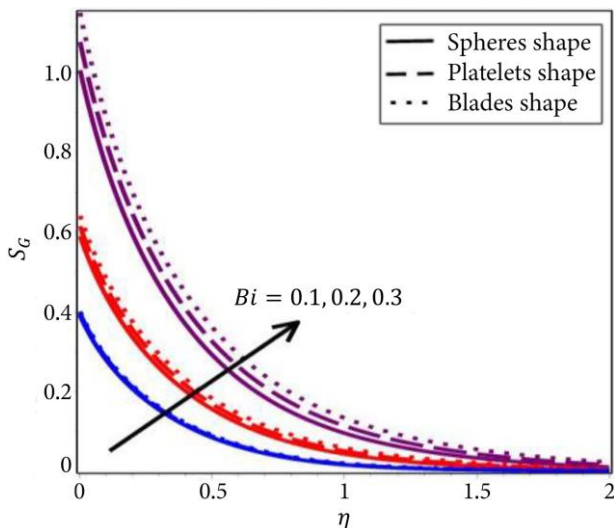


Figure 12. The behavior of S_G with specific values of Bi .

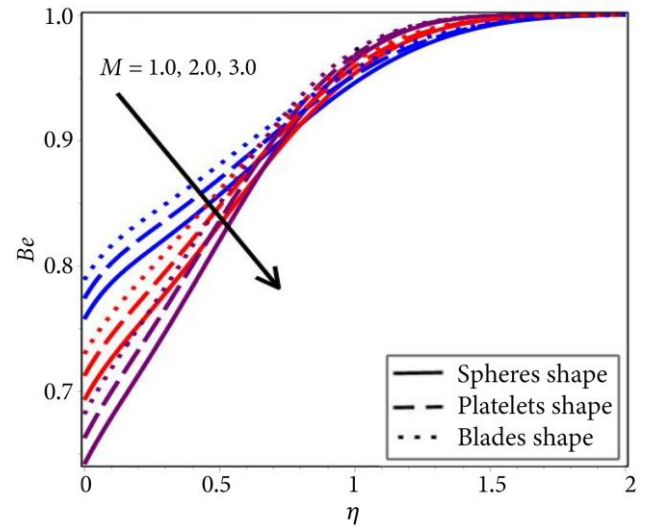


Figure 13. The behavior of Be with specific values of M .

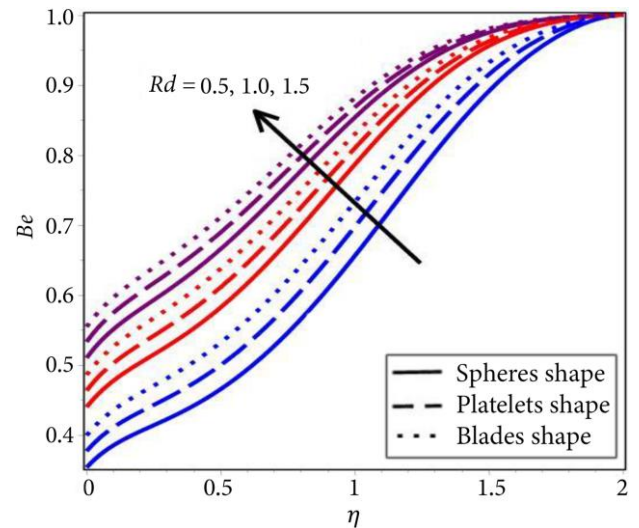


Figure 14. The behaviour of Be with specific values of Rd .

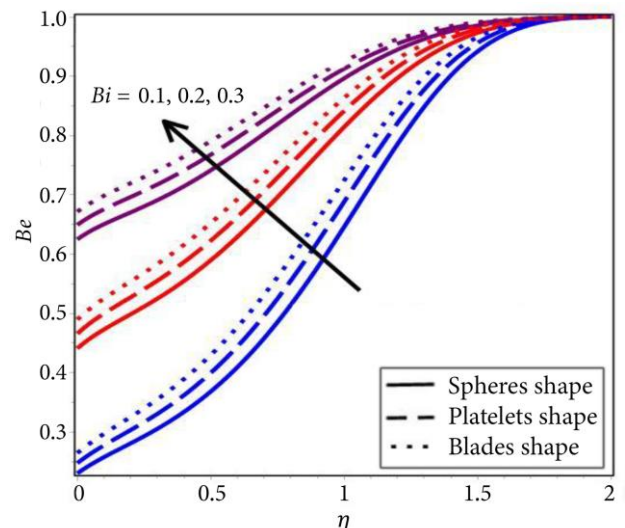


Figure 15. The behavior of Be with specific values of Bi .

temperature profiles are conveyed in Figure 7. As the thermal radiation increases, the temperature profile increases for comparison shapes (spheres, platelets, and blades). When the mean absorption coefficient is reduced due to an increase in the thermal Rd , more heat is transmitted in the fluid direction as the fluid temperature rises. Figure 8 depicts the γ for comparison shapes by increasing the values as the temperature profile $\theta(\eta)$ decreases. Physically, inclined (γ)

material particles need more time to transfer heat to their contiguous particles. Also, inclination in (γ) the material has a non-conducting effect that leads to the loss of a γ . The highlights of the heat generation parameter (λ_1) on hybrid nanofluid flow temperature profiles $\theta(\eta)$ on the shapes (spheres, platelets, and blades) are conveyed in Figure 9. Physically, the fluid will get more heat from the surface. As a result, raising the heat generation value causes the temperature profile $\theta(\eta)$ to rise.

6.3. Entropy generation

In Figures 10-13, the comparison of shapes (spheres, platelets, and blades) over a curved stretched sheet at the entropy generation S_G is shown for several influence parameters such as Rd , Bi , and Br . Figure 10 illustrates the impact of Rd on the entropy generation S_G of hybrid nanofluid for the curved stretching sheet. When the thermal radiation grows, the three shapes increase in entropy generation. Physically, growing the thermal radiation emission increases the disorder in the closed system, causing the radiative constant to rise. As a result, the system's entropy rate increases. The effect of the Br on the S_G profile is seen

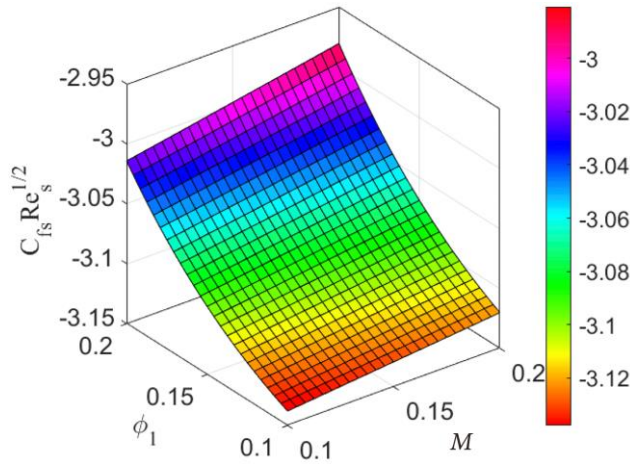


Figure 16. The variant of ϕ_1 and M on $C_{fs} Re_s^{1/2}$.

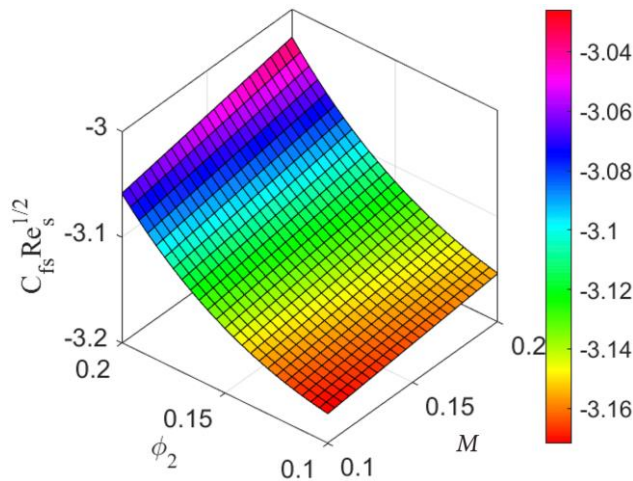


Figure 17. The variant of ϕ_2 and M on $C_{fs} Re_s^{1/2}$.

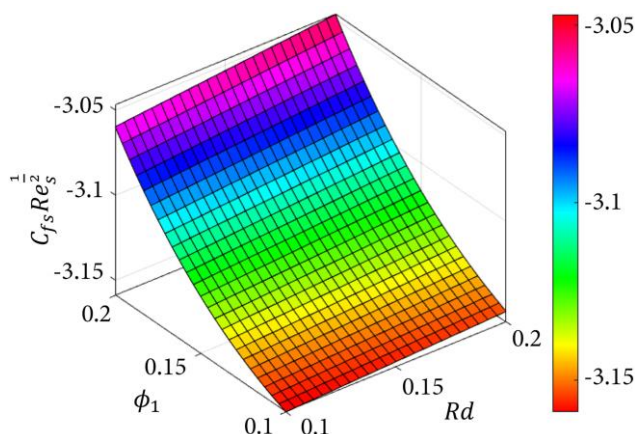


Figure 18. The variant of ϕ_1 and Rd on $C_{fs} Re_s^{1/2}$.

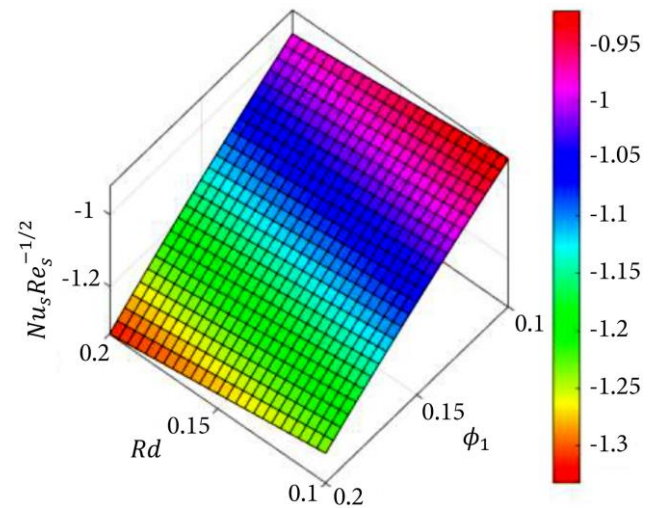


Figure 19. The variant of ϕ_1 and Rd on $Nu_s Re_s^{-1/2}$.

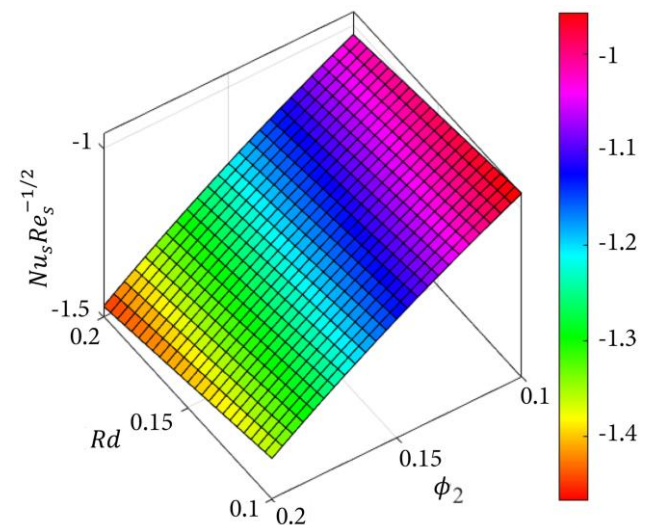


Figure 20. The variant of ϕ_2 and Rd on $Nu_s Re_s^{-1/2}$.

in Figure 11. Entropy generation rises in three different shapes, as Br rises. A higher Br increases the viscous force, resulting in increased collisions between fluid particles. As a result, the entropy rate has increased. Figure 12 shows the effect of the Biot number (Bi) on the entropy generation for the curved stretching sheet. The S_G grows in three shapes as the Biot number increases.

6.4. Bejan number

Figures 13-15 compare the shapes like spheres, platelets, and blades over a curved stretched sheet at the Be for different parameters such as M and Rd . Figure 13 shows the influence of the M parameter on the Bejan number. The Bejan number decreases in three shapes as the M increases. This is because stronger M tend to have more Lorentz force acting on fluid motion. Bejan number (Be) is reduced versus larger values of M . Figure 14 demonstrates the effect of the thermal radiation (Rd) parameter on the Bejan number. As the thermal radiation rises, the Bejan number increases in three shapes. Growing the thermal radiation emission increases the dis-orderedness in the closed system, causing the radiative constant to rise. As a result, the Bejan number of the system rises. Figure 15 depicts the impact of the Biot number on the Bejan number, Be increases in three shapes as the Biot number (Bi) increases. It has been shown that the viscous consequence has a greater impact on physical strength than thermal irreversibility.

6.5. Physical quantities

In Figures 16 and 17 the effect of ϕ_1 , ϕ_2 , and M on $C_{fs}Re_s^{1/2}$ Eyring-Powell hybrid nanofluid is used to investigate the impact of the $C_{fs}Re_s^{1/2}$ different hybrid nanoparticle volume fraction values ϕ_1 , ϕ_2 , and M , respectively. It's been seen that the $C_{fs}Re_s^{1/2}$ is a growing function of ϕ_1 , ϕ_2 , and M . The increase in skin friction is due to the thermal conductivity of nanoparticles, which is improved by growth estimates of the nanoparticle volume fraction. Figure 18 shows that the effect of ϕ_1 and thermal Rd on $C_{fs}Re_s^{1/2}$ Eyring-Powell hybrid nanofluid is used to investigate the impact of the $C_{fs}Re_s^{1/2}$ different hybrid nanoparticle volume fraction values ϕ_1 and thermal Rd respectively. It is seen that the $C_{fs}Re_s^{1/2}$ is enhancing the performance of ϕ_1 and Rd . In Figures 19 and 20 the effect of ϕ_1 , ϕ_2 , and thermal Rd on $Nu_sRe_s^{-1/2}$ Eyring-Powell hybrid nanofluid is used to investigate the impact of the $Nu_sRe_s^{-1/2}$ different hybrid nanoparticle volume fraction values ϕ_1 , ϕ_2 and thermal Rd , correspondingly. It is shown that the $Nu_sRe_s^{-1/2}$ is an increasing productivity function of ϕ_1 , ϕ_2 , and Rd . Table 2 demonstrates that the present approach corresponds more closely with Okechi et al. [28] and Ahmed et al. [29] solutions based on prior work. Which received a good agreement. This suggests that the HPM simulation used produces reliable findings.

Table 2. Adjustments K to compare the Skin-friction coefficient.

The radius of curvature parameters	Okechi et al. [28]	Ahmed et al. [29]	Current result
5	1.41960	1.41970	1.41970
10	1.34670	1.34690	1.34680
20	1.31350	1.31360	1.31350
30	1.30280	1.30280	1.30280
40	1.29750	1.29750	1.29720
50	1.29440	1.29440	1.29460
100	1.28810	1.28810	1.28890

7. Conclusion

This article investigates the entropy generation in a Magneto-Hydro-Dynamics (MHD) Eyring-Powell hybrid nanofluid (Ag- Fe_3O_4 / Blood) over a curved stretched sheet, as well as the effect of shape factors (spheres, platelets, and blades). Investigated were the effects of the porous medium, heat generation, nonlinear thermal radiation, the Cattaneo-Christov heat flux model, mixed convection (λ), and bioconvective boundary conditions. In comparison to the numerical method, the Homotopy Perturbation Method (HPM) generates more dependable and accurate results, which can be represented by graphs and tables. This model is related to radiation therapy and is one of the most effective methods for treating cancer or hyperthermia. This technology employs silver and a magnetic particle to treat cancer without harming other organs. It is reasonable and prudent to draw that conclusion.

- When a Magnetic field (M) influenced the hybrid nanofluid increase over a curved stretching sheet, the velocity profile was reduced. In reality, the Lorentz force increases as the M effect is amplified, acting against the flow of the liquid to slow the system down. In the presence of a curved stretching sheet, the velocity profile also decreases as a result of increased magnetic parameters;
- As Eyring-Powell and porosity are enhanced, the profile of velocity decreases;
- As thermal relaxation increases, the shape of the temperature profile diminishes (spheres, platelets, and blades). The hybrid nanofluid particles that are physically inclined require more time to transfer heat to their contiguous particles. In addition, the inclination of the material in (γ) results in the loss of a thermal relaxation parameter due to a non-conducting effect;
- As thermal radiation increases, the temperature profile for comparing shapes (spheres, platelets, and blades) rises. An increase in the thermal Rd reduces the average absorption coefficient. As fluid temperature rises, more heat is transferred in the fluid's direction;
- As the M parameter increases, three distinct forms of entropy are produced. Physically, increasing the magnetic parameter and Lorentz force at the same time increases abrasion, thereby increasing entropy production;
- As the influence of the thermal Rd and Bi increases, the Bejan number contour rises;
- In conclusion, I offer the following suggestions for the future: Various non-Newtonian fluids can be

considered with a variety of boundary conditions. I attempt to solve base fluid and nanofluid ternary-hybrid.

Nomenclature

A	Constant
B_0	Strengthening magnetic field
Be	Bejan number
Br	Brinkman number
$C_{fs} Re_s^{1/2}$	Skin friction coefficient
c_p	Specific heat
Ec	Eckert number
S_k	Porosity parameter
k	Thermal conductivity
g	Gravity
L	Slip length
L_s	Dimensionless slip parameter
M	Magnetic field parameter
$\hat{p}$	Hybrid nanofluid pressure
P	Hybrid nanofluid dimensionless pressure
Pr	Prandtl number
Q	Heat source/sink
q_r	Heat flux radiative
R	Radius of curvature
Rd	Thermal radiation parameter
S_G	Total entropy rate
T	The temperature at the surface
T_w	Wall temperature
T_∞	Faraway
u	Component of s -direction velocity
v	Component of r -direction velocity

Greek symbols

ρ	Fluid density
μ	Dynamic viscosity
α_1	Temperature ratio parameter
ν	Kinetic viscosity
σ	Electrical conductivity
θ	The dimensionless temperature of a fluid
η	Similarity variable
σ^*	Stefan-Boltzmann constant
λ	Mixed convection parameter
λ_1	Heat generation parameter
k^*	Mean absorption coefficient
ϕ_1, ϕ_2	Hybrid nanoparticles volume fractions

Subscripts

b_f	Base fluid
hnf	Hybrid nanofluid
$1s$	First particle
$2s$	Second particle

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Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Authors contribution statement

First author: Formal analysis; Data curation; Investigation; Methodology; Writing-original draft; Writing-review and editing

Second author: Resources; Software; Conceptualization; Data curation; Formal analysis; Resources; Software Supervision; Validation; Visualization

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