Scientia Iranica (2025) **32**(14): 7179

Sharif University of Technology
Scientia Iranica
Transactions on Mechanical Engineering
https://scientiairanica.sharif.edu

Temperature-dependent heat generation and variable viscosity features for viscoelastic fluid with homogeneous and heterogeneous chemical reactions

Munazza Saeed [a], Bilal Ahmad [a], Chemseddine Maatki [b], Tasawar Abbas [a], Bilel Hadrich [c], Sami Ullah Khan [d,*], Karim Kriaa [c], Qazi Mehmood Ul-Hassan [a], Lioua Kolsi [e,f]

a. Department of Mathematics, University of Wah, Wah Cantt, Pakistan.
b. Department of Mechanical Engineering, College of Engineering, Imam Mohammad Ibn Saud Islamic University (IMSIU), Saudi Arabia.
c. Department of Chemical Engineering, College of Engineering, Imam Mohammad Ibn Saud Islamic University (IMSIU), Saudi Arabia.
d. Department of Mathematics, Namal University, Mianwali, Pakistan.
e. Department of Mechanical Engineering, College of Engineering, University of Ha'il, Ha'il, Saudi Arabia.
f. Laboratory of Metrology and Energy Systems, Department of Energy Engineering, University of Monastir, Tunisia.

* Corresponding author: sk_iiu@yahoo.com (S.U. Khan)

Received 26 September 2022; Received in revised form 12 July 2023; Accepted 15 October 2023

Keywords

Heat transfer;
Viscoelastic fluid;
Temperature-dependent viscosity;
Homogeneous and Heterogeneous (HH) reactions;
Numerical solution.

Abstract

This investigation presents the heat and mass transfer phenomenon for the chemically reactive flow of second-grade fluid subject to homogeneous and Heterogeneous (HH) chemical reactions. The viscosity of the fluid is assumed to be temperature-dependent instead of constant. The motivation for considering the viscosity as a function of temperature is justified by applications in metallurgical processes, crude oil extraction, geothermal systems, and machinery lubrication. Additionally, viscous dissipation and temperature-dependent heat generation and absorption effects are also introduced to improve the thermal transport phenomenon. The interaction of different new variables facilitates the problem into dimensionless form. The numerical results are predicted by implementing the Runge-Kutta (RK4) method. The physical meaning behind the parameters has been reported. The tabular quantitative analysis is performed for different physical quantities.

1. Introduction

Widespread attention is devoted by investigators towards the rheology of non-Newtonian liquids in current research problems in many processing, engineering and industrial processes, the contribution of non-Newtonian materials is widely justified. To clarify the genuine performance of non-Newtonian fluids, diverse sorts of models are established in the past. A second-grade model is assumed to be an updated class of linear materials attaining diverse rheology. The shear thinning along with shear thickening outcomes are predicted, preferably by using this model. In recent contributions, the interest of scientists in grown exclusively in non-Newtonian materials. The reason behind their interest is the wide range of its applications in manufacturing processes. In many processes, these fluids are considered to flow over continuously moving surfaces. An individual fluid model is not enough to describe all the properties and qualities of each fluid. To overcome this deficiency and to describe its characteristics, various non-Newtonian fluid models are proposed which are delineated by rate, differential and integral type fluids. Their immense applications involve in industries such as in polymer processing industries, bio fluids dynamics and petroleum drilling and many others. The second-grade fluid properties were elaborated in the literature by scientistic with enrolment of many diverse consequences. Riaz et al. [1] expressed the optimized significance in heat transfer due to viscoelastic material. Ismail et al. [2] reported the inspiration of transient flow for second-grade material numerically. Rehman et al. [3] observed the aspect of surface tension while exploring the different rheological mechanism of viscoelastic fluid. The joule heating determination regarding the optimized features of viscoelastic fluid has been visualized by Abbasi et al. [4]. Loganathan et al. [5] intended

To cite this article:
M. Saeed, B. Ahmad, Ch. Maatki, T. Abbas, B. Hadrich, S.U. Khan, K. Kriaa, Q.M. Ul-Hassan, L. Kolsi "Temperature-dependent heat generation and variable viscosity features for viscoelastic fluid with homogeneous and heterogeneous chemical reactions", *Scientia Iranica* (2025) **32**(14); 7179. https://doi.org/10.24200/sci.2023.61173.7179

the mathematical exploration of Riga surface flow of second-grade nanofluid numerically. Waqas et al. [6] investigated the bioconvection application for viscoelastic nanofluid with biofuel applications.

Heat transfer is a fundamental process in various engineering and industrial phenomena. The source of heat transfer is associated to the exchange of energy in various systems or objects due to a temperature difference. The heat transfer phenomenon is key in automotive engineering, cooling objects, Heating, Ventilation, and Air Conditioning (HVAC) systems, air conditioning, sterilization, industrial processes, aerospace engineering, the petroleum industry etc. Different studies have been reported in recent years to enhance and control the heat transfer phenomenon. For instance, Salawu et al. [7] predicted the heat transfer through the contribution of ferromagnetic particles via the von Karman approach. Shamshuddin and Eid [8] examined the contribution of variable viscosity while reporting the heat transfer aspects in nanofluid flow via a rotating disk. Ferdows et al. [9] explained the assessment of internal heat transfer applications for power-law fluid with attribution of exponential fluid viscosity. The heat transfer analysis in the MOS_2 and Fe_3O_4 nanoparticles with ethylene glycol-based liquid has been treated by Shamshuddin et al. [10]. Farajollahi et al. [11] suggested the heat transfer mechanism in conical turbulators by utilizing the tiny particles. Zheng et al. [12] performed numerical computations for the melting heat transfer problem due to shrinking space.

In fluid flow, the role of viscosity conveys an important role. It has been commonly noticed that the viscosity of a fluid is subsequently fluctuates with the phenomenon of heating, power pumping and pressure gradient. The dependence of viscosity of such factors is important and has many applications in aerodynamic processes, processing materials, extraction of petroleum etc. The process of thermos fluidic bearing with heat transfer is slightly dependent upon the viscosity of the material. Usually, non-Newtonian materials possess variable viscosity. In a more comprehensive way, the viscosity can be a function of pressure, temperature and shear force. However, the consideration of viscosity with temperature dependence is interesting and important to investigate. Numerous studies have been conducted to understand the effect of various models connected to varying viscosity. The inclined direction flows due to second grade material confining the Reynolds viscosity model was explored by Massoudi and Phuoc [13]. Aforesaid model was utilized for second grade material via vertical surface configuration have been evaluated by Massoudi et al. [14].

In a permeable medium, the impact of heat transfer in a viscoelastic fluid and temperature dependent viscosity effects on flow was considerate by Ramya et al. [15]. The viscosity was taken by under the inverse relation of temperature distribution. Refs. [16-19] refer to different flow models with variable viscosity. However, the mutable viscosity effects to an uninterrupted uniform plate were described by Pop et al. [20]. Two different variable viscosity models are discussed by Chu et al. [21]. Djebali et al. [22] observed the formulation of thermal boundary prediction for different similarity solutions.

The contemplation of electrically conducting material with the impulsion of magnetic force acquires great significant from a technical perspective. Aforesaid problems have received abundant attention by several researchers. Magnetohydrodynamics (MHD) flows has significant applications in metallurgical processes, MHD pumps, plasma, chemical processes, engineering devices, generators etc. The term MHD was first introduced by Hannes Alfven in 1942. However, interest in MHD flow began in 1918, when Hartmann invented an electromagnetic pump. MHD 2nd grade fluid was explored by Kia-Long [23]. Pal and Mandal [24] investigated the effect of nonmaterial for updating the heating performance of electrically conducting material. Warke et al. [25] reported the behavior of magneto-micropolar suspension in a heated porous layer.

Flow owing to a stretching sheet also arises in the thermal and moisture behavior of materials, chiefly in processes such as in metallurgy, textile, paper production, glass and polymer sheets etc. Natural convection accompanied by variable viscosity and thermal conductivity due to wavy space was exhibited in research of Hossain et al. [26]. Swain et al. [27] examined the magnetized flow regarding nanomaterials, keeping variable effects of thermal phenomena.

The involvement of different chemical reactions such as Homogeneous and Heterogeneous (HH), has great significance in chemical reacting processes. Basha et al. [28] scrutinized chemical features in viscoelastic material via a numerical approach. The smooth conduction of these processes requires a special agent called a catalyst. A complex structure develops between HH species near the surface of the catalyst. The application of these reactions involves combustion and biochemical systems and at special rate they engage in the manufacturing and utilization on catalytic surfaces and within the fluid. Initially, in fluid flow problems, these reactions were analyzed by Chaudhary and Merkin [29] by considering the stagnation point flow. The homogeneous reaction is applied in bulk through isothermal cubic autocatalator kinetics, whereas the other one on the surface by first-order kinetics. Shaw et al. [30] investigated these reaction effects on micropolar fluid in which they considered the flow over the permeable stretched surface. Sheikh and Abbas [31] investigated its behavior on Casson fluid when the fluid is experiencing the uniform suction and depicted that its concentration increases due to uniform suction. Flow with fluid elasticity for an incompressible homogeneous second-grade fluid was investigated by Cortell [32]. Due to the elastic properties of fluid, the strain energy which is produced due to viscous dissipation is stored in the fluid and is reflected in the energy equation. Abdul Hakeem et al. [33] discussed the effects of elastic deformation of Walters B fluid where the fluid is flowing through permeable sheet, and they show the inverse behavior of fluid temperature due to elastic deformation. Ramzan et al. [34] addressed the role of autocatalytic chemical reactive species for nonlinear thermally developed flow due to curved surface. Patil et al. [35] presented the role of HH chemical reaction while focusing the thermal aspect of hybrid nanofluid. The applications of HH chemical reaction in optimized flow of hybrid nanofluid was visualized by Siddiqui et al. [36]. Vaidya et al. [37] discussed the HH chemical significance for hemodynamic flow. Iqbal et al. [38] addressed the viscous dissipation assessment of viscoelastic fluid by presenting the exact solution.

This communication aims to address the applications of HH chemical reaction for viscoelastic fluid flow with collective variable thermal features. The analysis has been performed when the viscosity of viscoelastic fluid is fluctuated with temperature change. The contribution of heat absorption and generation features are treated as exponential function of temperature gradient. Moreover, the analysis has been further extended by incorporating the viscous dissipation and magnetic force impact. The induced flow is considered due to linearly porous saturated space subject to the convective boundary conditions. The numerical outcomes are presented for flow model by using the BVP 4c approach. The onset of set of parameters is physically observed.

2. Physical description of problem

A second-grade material due to two-dimensional motion against the stretching surface is observed. In the county $(y > 0)$, the flow is restrained in the direction of a surface corresponding to the plane $(y = 0)$ and the fluid is allowed to pass through the porous medium. Both the axis viz. x-axis and y-axis are considered as horizontal and vertical plane, respectively. The viscosity of fluid is supposed to be variable which changes due to temperature. The electrically conductive second grade fluid is inspired by the influence of magnetic field which is applied perpendicularly to sheet. Figure 1 illustrating the flow pattern of current research model. The governing flow-associated expressions [38]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}\right)=$$
$$\frac{\partial}{\partial y}\left(\mu^{*}(T)\frac{\partial u}{\partial y}\right)+k_1\left[\begin{array}{l}\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+u\frac{\partial^3 u}{\partial x\partial y^2}\\ -\frac{\partial^2 v}{\partial y^2}\frac{\partial u}{\partial y}+v\frac{\partial^3 u}{\partial y^3}\end{array}\right]-\sigma B_0^2 u-\frac{\upsilon}{\Gamma}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\mu^{*}(T)}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2$$
$$+\frac{\xi'''}{\rho c_p}-\frac{\delta k_1}{\rho c_p}\left[\frac{\partial u}{\partial y}\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)\right], \tag{3}$$

with associated boundary conditions:

$$u=u_w(x)=cx, -k\frac{\partial T}{\partial y}=h_1\left(T_f-T\right),$$
$$v=0, \quad \text{at} \quad y=0,$$
$$u,v\to 0,\ T\to T_\infty \ \text{at}\ y\to\infty. \tag{4}$$

The defined boundary constraints are associated to stretching surface phenomenon. The flow associated to the stretched surface has important applications in manufacturing systems, chemical processes, coating production, polymer extrusions, fiber spinning, molten polymers, forming of sheet metals, biodiesel applications etc. In above defined problem, c is positive constant, B_0, k_1 represents magnetic field and second grade parameter; c_p is specific heat; α for thermal diffusivity; Γ is the medium permeability; δ represents elastic deformation parameter; h_1 stands for heat transfer coefficient. Fluid temperature is represented by T_f whereas ambient fluid temperature is represented by T_∞. ℓ_1 and ℓ_s be strength of HH chemical species, respectively. The heat source/sink with temperature dependent convective relations ξ''' is [38]:

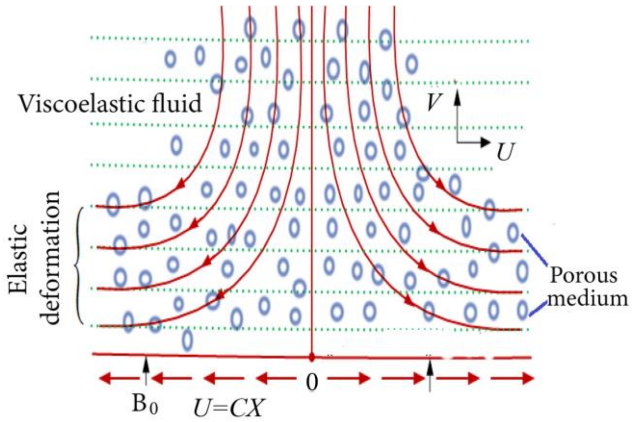

Figure 1. Flow illustration of problem.

$$\xi'''=\left(\frac{k\bar{\bar{u}}_w}{x\nu}\right)\left[H\left(T_f-T_\infty\right)+E\left(T-T_\infty\right)\right], \tag{5}$$

where H and E be temperature dependent heat source/sink relations. If H, $E > 0$ then it relates to heat generation; otherwise then heat absorption. The expression defines the chemical reaction (HH) are:

$$H+2E\to 3E,\ \text{rate}=k_c\alpha\beta^2. \tag{6}$$

The isothermal, first-order reaction confederated with a catalyst surface is characterized as:

$$H\to E,\ \text{rate}=l_s a, \tag{7}$$

where a and b are the amount of concentration in H and E (chemical species) and the term l_s shows rate constants. The governing equations which represent the current flow problem are listed as [38]:

$$u\frac{\partial a}{\partial x}+v\frac{\partial a}{\partial y}=D_H\frac{\partial^2 a}{\partial y^2}-l_1 ab^2, \tag{8}$$

$$u\frac{\partial b}{\partial x}+v\frac{\partial b}{\partial y}=D_E\frac{\partial^2 b}{\partial y^2}+l_1 ab^2, \tag{9}$$

$$\left.\begin{array}{l} D_H\frac{\partial a}{\partial y}=\ell_s a,\ D_E\frac{\partial b}{\partial y}=-\ell_s a, \ \text{at} \ y=0\\ \quad a\to a_0,\ b\to 0, \ \text{at} \ y\to\infty.\end{array}\right\} \tag{10}$$

where D_H and D_E show the diffusion coefficients of H and E.

The viscosity relation variable exponential factor is [7,38]:

$$\mu^{*}(T)=\mu_o^{*}\exp\left[-\hat{a}\left(T-T_\infty\right)\right]. \tag{11}$$

Using the relation of the Reynolds model [10]:

$$\mu^{*}(T)=\mu_0^{*}e^{-\lambda\theta}, \tag{12}$$

where λ represents the viscosity parameter. Intervolving new variables [38]:

$$\eta = \sqrt{\left(\frac{c}{\nu}\right)}y, u = cxf'(\eta), v = -\sqrt{c\nu}\, f(\eta),$$
$$\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, a = a_o\varphi(\eta), b = b_o h(\eta). \quad (13)$$

The dimensionless set of equations in view of the above transformations is:

$$(1-\lambda\theta) f''' + \beta\left(2ff''' - f^2 - ff''''\right)$$
$$-(f')^2 + ff'' - Mf' - \lambda f''\theta - a^* f' = 0, \quad (14)$$

$$\theta'' + \Pr f\theta' + Hf' + E\theta + Ec\Pr f''^2(1-\lambda\theta)$$
$$-Ec\Pr\delta\beta f''(ff'' - ff''') = 0, \quad (15)$$

$$\frac{1}{S_c}\varphi'' + f\varphi' - L\varphi h^2 = 0, \quad (16)$$

$$\frac{1}{S_c}h'' + fh' + L\varphi h^2 = 0, \quad (17)$$

associated boundary conditions will be:

$$f(0) = 0,\ f'(0) = 1,\ f'(\infty) \to 0, \theta'(0) = -\gamma[1+\theta(0)],$$
$$\theta(\infty) \to 0, \varphi(\infty) \to 1, \varphi'(0) \to L_s\varphi(0). \quad (18)$$

In the above mathematical expression λ (viscosity parameter), M (Hartman number), Pr (Prandtl number), Sc (Schmidt number), a^* is the representation for permeability. L (chemical reaction parameter), Ec (Eckert number), δ (Elastic deformation parameter), β (second-grade fluid parameter) and γ (conjugate parameter), which are defined as:

$$\lambda = \hat{a}\left(T_f - T_\infty\right), \quad M = \frac{\sigma B_0^2}{c\rho},$$
$$a^* = \frac{\upsilon}{c\Gamma}, \quad Pr = \frac{\nu}{k_1}, \quad S_c = \frac{\nu}{D},$$
$$L = \frac{l_1 a_0^2}{c}, \quad Ec = \frac{u_w^2}{c_p\left(T_f - T\infty\right)},$$
$$\beta = \frac{k_1 c}{\mu_o^?}, \quad \gamma = \frac{h_s}{k}\sqrt{\frac{\nu x}{c}}. \quad (19)$$

The results for viscous fluid are obtained when $\beta = 0$. Since H and E (chemical species) are believed to be of identical size, D_H and D_E are equivalent:

$$\varphi(\eta) + h(\eta) = 1. \quad (20)$$

Therefore, Eqs. (16) and (17) takes the form:

$$\frac{1}{Sc}\varphi'' + f\varphi' - L\varphi(1-\varphi)^2 = 0, \quad (21)$$
$$\varphi'(0) = L_s\varphi(0), \varphi(\eta) \to 1 \text{ as } \eta \to \infty. \quad (22)$$

Defining the stress component (τ_w) and heat flux rate (q_w) as [38]:

$$\tau_w = \mu^*(T)\frac{\partial u}{\partial y} + \alpha_1\left\{\begin{matrix} 2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \nu\frac{\partial^2 u}{\partial y\partial x} \\ +u\frac{\partial^2 u}{\partial y\partial x}\end{matrix}\right\}\Bigg|_{y=0},$$

$$q_w = -k\frac{\partial T}{\partial y}\bigg|_{y=0}. \quad (23)$$

Defining the skin friction coefficient C_{fx} and Nusselt number Nu_x:

$$C_{f_x} = \frac{\tau_w}{\rho u_w^2}, \quad N_{u_x} = \frac{xq_w}{k\left(T_f - T\infty\right)}, \quad (24)$$

with:

$$(Re_x)^{1/2}C_{fx} = (1 - \lambda\theta(0) + 3\beta)f''(0),$$
$$(Re_x)^{-1/2}Nu_x = -\theta'(0) = \gamma\left(1+\theta(0)\right), \quad (25)$$

where $Re_x = U_w(x)x/\nu$ be Reynolds number.

3. Numerical computational procedure

The complicated nature of the dimensionless set of Eqs. (14), (15), and (21) presents nonlinearity in nature. Instead of an exact solution, the numerical algorithm with the RK5 scheme is implemented. The first-order expressions against the formulated equations are attained with assumptions of:

$$(y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8) = (f, f', f'', f''', \theta, \theta', \phi, \phi').$$

These results are shown in Box I, which yields the following along with the initial conditions:

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \\ y_5' \\ y_6' \\ y_7' \\ y_8' \end{pmatrix} = \begin{pmatrix} y_2 \\ y_3 \\ y_4 \\ \left(\frac{1-\lambda y_5+2\beta y_2}{\beta y_1}\right)y_4 + \frac{1}{\beta y_1}y_3(y_1 - \lambda y_5) - \frac{1}{\beta y_1}y_2^2 + \frac{1}{\beta y_1}y_2\left(M + \frac{\upsilon}{c\Gamma}\right) - \frac{y_3^2}{y_1} \\ y_6 \\ -\Pr y_1 y_6 - Hy_2 - Ey_5 - E_c\Pr\left[y_3^2(1-\lambda y_5) - \delta\beta y_3(y_2 y_3 - y_1 y_4)\right] \\ y_8 \\ -Scy_1 y_8 + Ly_7(1-y_7)^2 \end{pmatrix} \quad (26)$$

Box I.

Table 1. Numerical verification of obtained solution with continuation of Cortell [32] when $\lambda = 0$.

M	β	Cortell [32]	Present results	Error
0.5	1.0	0.8660254	0.8660255	1.1×10^{-6}
1.0	1.0	1.0000000	1.0200000	2×10^{-2}
2.0	1.0	1.224745	1.224747	2×10^{-6}

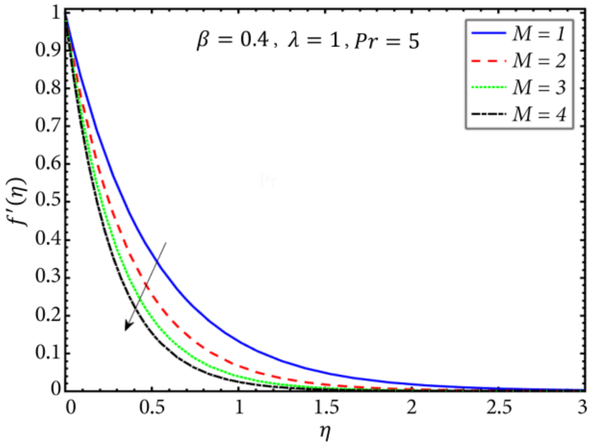

Figure 2. Change in f' with M.

$$\begin{pmatrix} y_1(0) \\ y_2(0) \\ y_3(0) \\ y_4(0) \\ y_5(0) \\ y_6(0) \\ y_7(0) \\ y_8(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ f''(0) \\ \theta(0) \\ -\gamma\left(1+y_4(0)\right) \\ \theta'(0) \\ \varphi(0) \\ Ls\left(y_7(0)\right) \end{pmatrix}. \tag{27}$$

The small step size of $h = 0.001$ and error accuracy 10^{-6} has been followed. MATHEMTICA software is used to perform the computations.

4. Validation of results

For the confirmation and validity obtained numerical data, the comparison of results is worked out with analysis of Cortell [32] in Table 1. The concluded observations are predicted for limiting case. A fine accuracy is noted for both computations.

5. Results and discussion

The findings of second-grade fluid on a stretching sheet in the presence of viscosity and chemical reaction are investigated in this section. The physical structures of the plotted figures and the true mechanism of flow influenced by the dominant dimensionless parameters are also exposed in this section, by keeping $\lambda = 1,\ a^* = \gamma = \delta = 0.5$ fixed. The effects of the Hartmann number, M on axial component of velocity are manifested in Figure 2. The up-surging strength of M is linked to a progressive resistive force known as the Lorentz force, which is activated by the interplay of electric and magnetic fields and resists fluid movement. Second-grade fluid parameter β plays an important role in improving velocity of fluid. Figure 3 relates with the attitude of fluid parameter β on velocity. Wider β, drops the viscosity of the

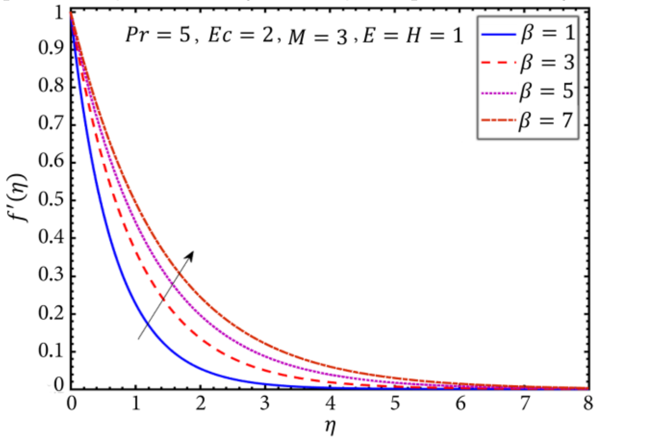

Figure 3. Change in f' with β.

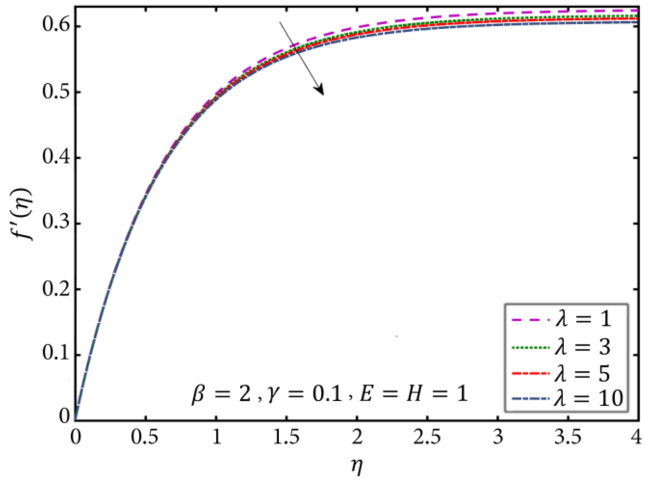

Figure 4. Change in f' with λ.

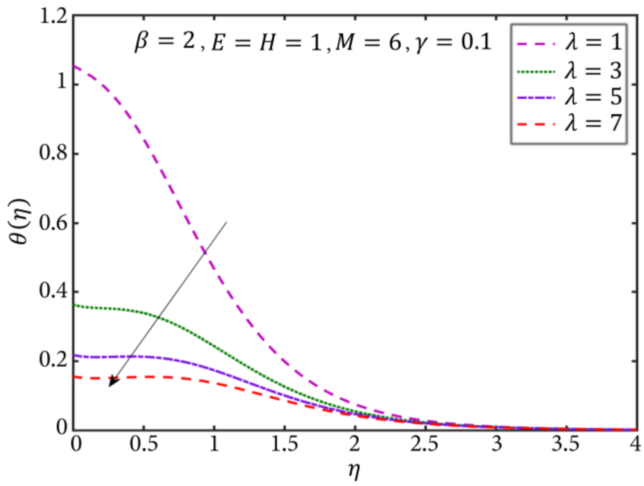

Figure 5. Change in θ with λ.

liquid and accordingly velocity boosted. Such outcomes are physically associated to the rheology of viscoelastic fluid. Figures 4 and 5 display the behavior of velocity and temperature profiles for viscosity parameter λ. The velocity profile and heat transfer pattern gradually reduce when λ is higher. This trend is accounted due to the variable fluid viscosity. Figures 6 and 7 show the impact of coefficient of internal heating generation and absorption H and E on temperature profile, the rise in temperature is due to internal heat generation. The presence of internal heat transfer

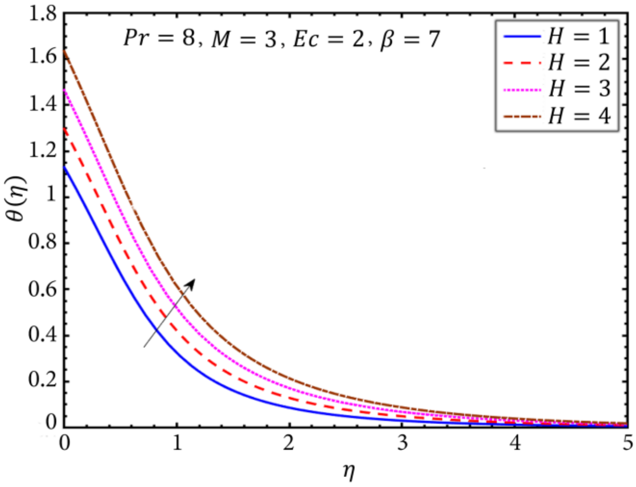

Figure 6. Change in θ with H.

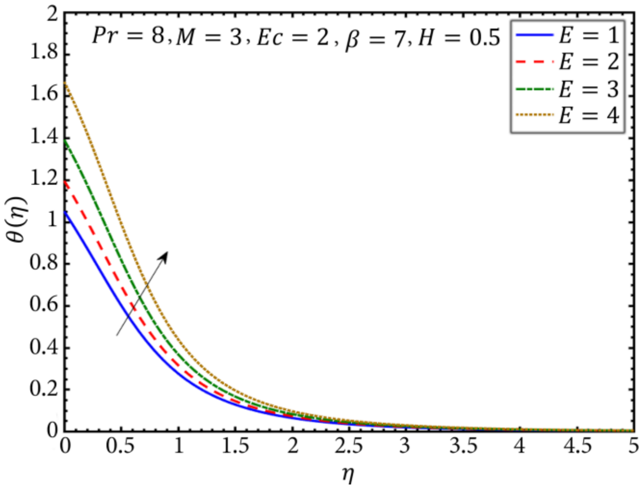

Figure 7. Change in θ with Ec.

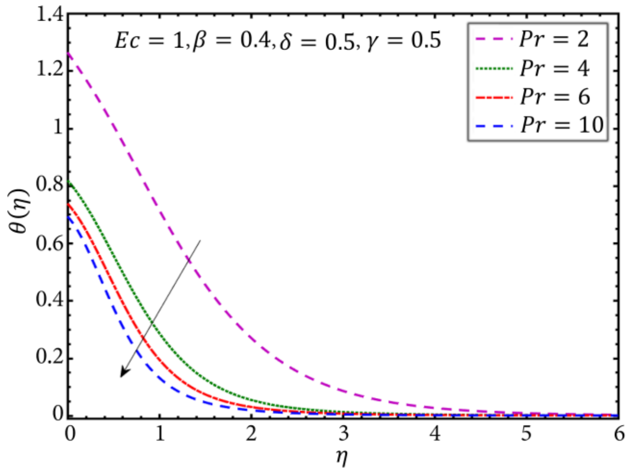

Figure 8. Change in θ with Pr.

enhanced the thermal pattern. Figure 8 displays the temperature contour for distinct values of Pr. It shows that the temperature profile falls as Pr enhanced. The slower rate of thermal diffusivity against larger Pr is depicted which report a declining in heating transportation. The reducing observations in temperature due to larger Pr is associated to the low thermal diffusivity. Figure 9 exhibits uniform rise in Ec causes less significant in heat dissipation means increase in fluid temperature. The physical consequence

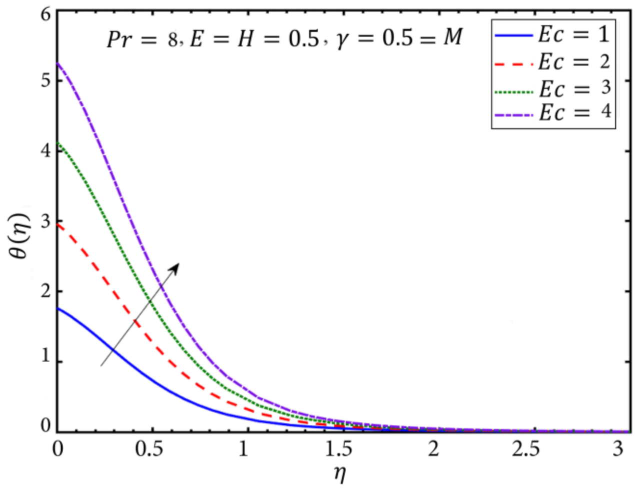

Figure 9. Change in θ with Ec.

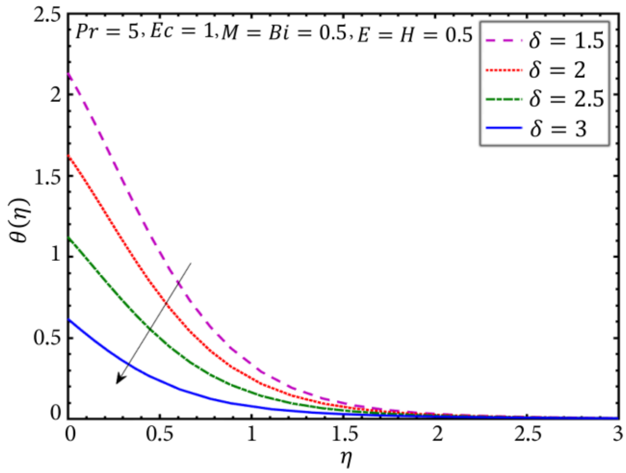

Figure 10. Change in θ with δ.

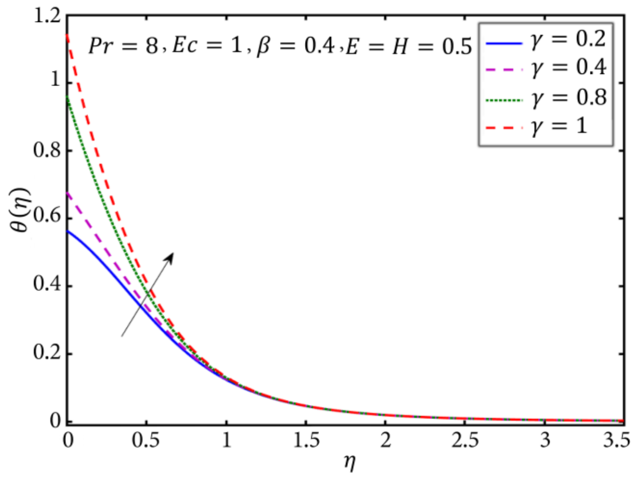

Figure 11. Change in θ with γ.

behind such trend is associated to the fluid kinetic energy. Figure 10 observed the declining assessment in temperature due to elastic deformation increases. The decrement in temperature profile is due to elastic deformation rate which retained larger magnitude. Figure 11 illustrates the influence of conjugate constant for Newtonian heating γ. It is observable that internal temperature of the flow rises for upper values of γ resulting the enhancement in temperature field. The temperature fluctuation due to second grade

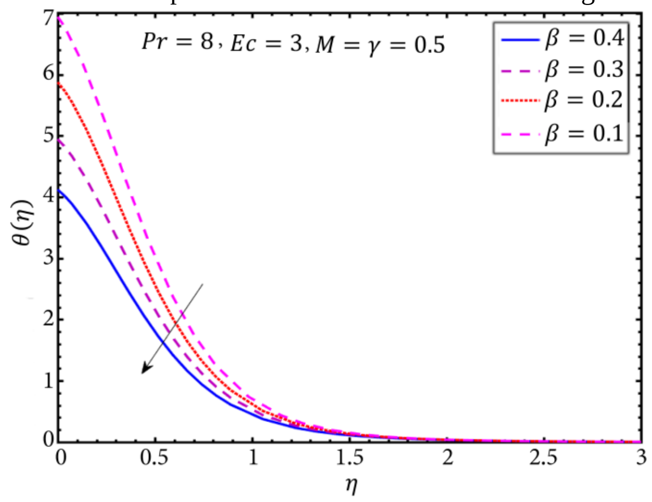

Figure 12. Change in θ with β.

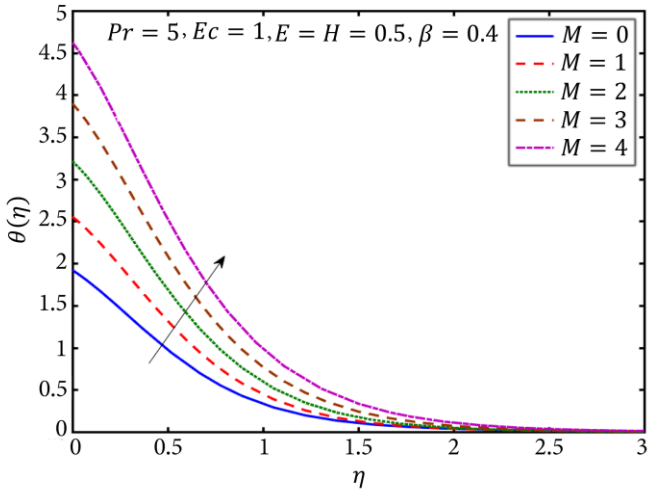

Figure 13. Change in θ with M.

parameter β in the presence of variable viscosity is clearly shown in Figure 12. The slower heating transportation with increasing material parameter is observed. This trend is physically associated with the normal stresses, which control the increment in thermal phenomenon. Figure 13 clarifies that for different values of magnetic force, temperature increases. The Lorentz force, which possesses a resistive nature, appeared due to the implementation of a larger magnetic force. However, an increase in M reduces the concentration level slightly in Figure 14, referred to as momentum diffusivity. The results exhibited for reaction parameter L against ϕ have been reported via Figure 15, which displays for up surging values of L, the concentration field drops. The physical dynamics of the Schmidt number Sc on ϕ have been observed in Figure 16. The decreasing outcomes for ϕ due to larger Sc are reflected. The physical behavior due to Sc is related to the fact of lower mass diffusivity.

The numerical framework presented in Table 2 present the change in skin friction and wall temperature gradient. The wall shear force enhanced with λ, β and a^*. The Nusselt coefficient declined with Pr, Ec, β and λ. The reducing numerical outcomes for skin fraction coefficient are noted for a^* and δ. Also, same performance of δ for Nusselt number. Growing values of γ, display a rise in Nusselt number.

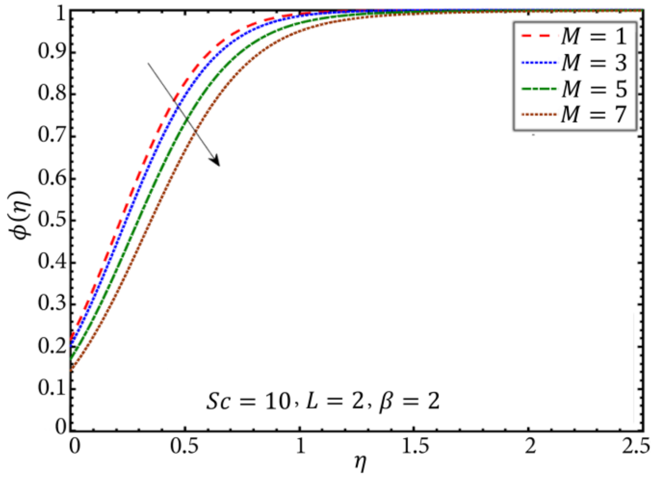

Figure 14. Change in ϕ with M.

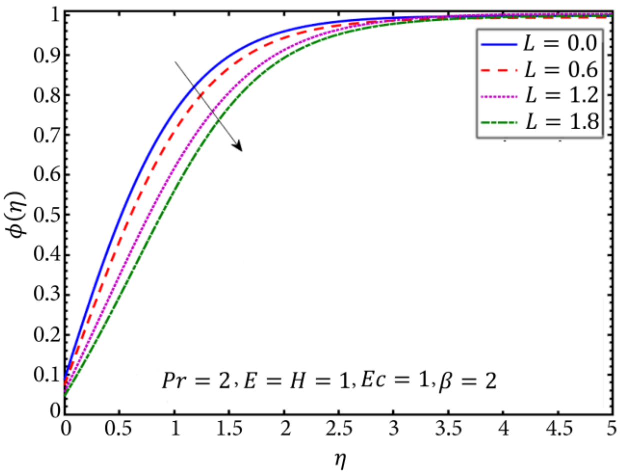

Figure 15. Change in ϕ with L.

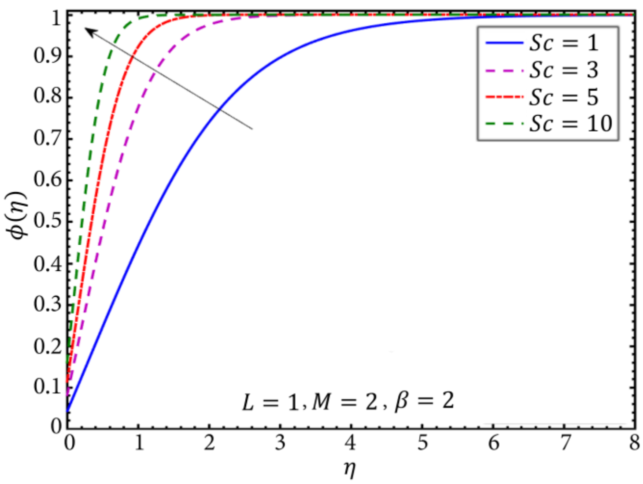

Figure 16. Change in ϕ with Sc.

6. Concluding remarks

The flow due second grade fluid with heat transfer analysis due to elongating surface with elastic deformation have been studied numerically. The Homogeneous and Heterogeneous (HH) reaction effects are also inspected. Runge Kutta (RK4) numerical computations are predicted for formulated flow model. The verification for obtained solution is presented with excellent accuracy. The study reveals following major observations:

Table 2. Numerical values of skin friction coefficient and Nusselt number for emerging parameters.

M	a^*	β	λ	δ	$-f''(0)$	Pr	Ec	β	λ	γ	δ	$-\theta'(0)$
2.0					1.36633	5.0						0.151835
3.0	0.2	0.4	2.0	0.1	1.35041	8.0	2.0	0.3	2.0	0.1	0.1	0.51071
4.0					0.395928	9.0						0.150519
	0.5				1.34382		1.0					0.151432
	1.0				1.35252		2.5					0.150427
	1.5				1.34766		3.0					0.150206
		0.6			1.96456			0.4				0.150482
		0.8			2.56906			0.6				0.149948
		1.0			3.17271			0.8				0.149345
			1.0		2.50479				0.5			0.285026
			1.5		2.55333				1.0			0.19959
			2.5		2.57414				1.5			0.167358
				0.1	2.46129					0.15		0.22704
				0.2	2.3383					0.2		0.304028
				0.3	2.21757					0.3		0.454654
											0.15	0.149963
											0.2	0.149216
											0.3	0.147723

- By increasing magnetic parameter, the velocity and concentration of fluid reduces while temperature escalate;
- The temperature phenomenon drops for upsurging values of *Pr*;
- An increment in Eckert number lead to enhancing the temperature gradient;
- Velocity profile upsurges with escalating values of viscoelastic parameter whereas reverse results shown for temperature profile;
- For growing values of chemical reaction constant, the concentration field reduced;
- Both heat and mass transfer pattern reduce due to viscoelastic parameter;
- The wall shear force reduces due to Hartmann number and viscosity parameter;
- With enhancing viscoelastic parameter, the Nusselt number reduces.

Nomenclature

(u, v)	Velocity components
$\tilde{u}_w(x)$	Surface velocity
Γ	Medium permeability
β	Second grade fluid parameter
β_0	Magnetic field strength
C_{fx}	Skin friction coefficient
Nu_x	Nusselt number
c_p	Specific heat
T	Fluid temperature
h_1	Heat transfer coefficient
k_1	Material parameter
α	Thermal conductivity
T_∞	Ambient fluid temperature
T_f	Fluid temperature
ρ	Density
λ	Viscosity parameter

Funding

This work was supported and funded by the Deanship of Scientific Research at Imam Mohammad Ibn Saud Islamic University (IMSIU) (grant number IMSIU-RG23057).

Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Authors contribution statement

First author: Conceptualization

Second author: Data curation; Formal analysis

Third author: Investigation

Fourth author: Methodology

Fifth author: Project administration; Resources

Sixth author: Software

Seventh author: Visualization

Eighth author: Writing - original draft; Writing

Ninth author: Supervision

References

1. Riaz, A., Gul, A., Khan, I., et al. "Mathematical analysis of entropy generation in the flow of viscoelastic nanofluid through an annular region of two asymmetric annuli having flexible surfaces", *Coatings*, **10**(3), 213 (2020). https://doi.org/10.3390/coatings10030213

2. Ismail, M.A., Kamal, M.H.A., Jiann, L.Y., et al. "Transient free convection mass transfer of second-grade fluid flow with wall transpiration", *CFD Letters*,

13(11), pp. 35-52 (2021). https://dx.doi.org/10.37934/cfdl.13.11.3552

3. Rehman, S., Shah, R.A., Khan, A., et al. "Free surface film flow of an unsteady second grade fluid over a stretching sheet with surface tension linearly varies with temperature", *Physica A: Statistical Mechanics and its Applications*, **551**, 123956 (2020). https://doi.org/10.1016/j.physa.2019.123956

4. Abbasi, A., Farooq, W., Khan, M.I., et al. "Entropy generation applications in flow of viscoelastic nanofluid past a lubricated disk in presence of nonlinear thermal radiation and Joule heating", *Communications in Theoretical Physics*, **73**(9), 095004 (2021). https://doi.org/10.1088/1572-9494/ac0c75

5. Loganathan, K., Alessa, N., and Kayikci, S. "Heat transfer analysis of 3-D viscoelastic nanofluid flow over a convectively heated porous Riga plate with Cattaneo-Christov double flux", *Frontiers in Physics*, **9**, 641645 (2021). https://doi.org/10.3389/fphy.2021.641645

6. Waqas, H., Khan, S.U., Tlili, I., et al. "Significance of bioconvective and thermally dissipation flow of viscoelastic nanoparticles with activation energy features: novel biofuels significance", *Symmetry*, **12**(2), 214 (2020). https://doi.org/10.3390/sym12020214

7. Salawu, S.O., Shamshuddin, M.D., and Beg, O.A. "Influence of magnetization, variable viscosity and thermal conductivity on Von Karman swirling flow of H_2O-FE_3O_4 and H_2O-Mn-$ZNFe_2O_4$ ferromagnetic nanofluids from a spinning DISK: Smart spin coating simulation", *Materials Science and Engineering: B*, **279**, 115659 (2022). https://doi.org/10.1016/j.mseb.2022.115659

8. Shamshuddin, M.D. and Eid, M.R. "Magnetized nanofluid flow of ferromagnetic nanoparticles from parallel stretchable rotating disk with variable viscosity and thermal conductivity", *Chinese Journal of Physics*, **74**, pp. 20-37 (2021). https://doi.org/10.1016/j.cjph.2021.07.038

9. Ferdows, M., Murtaza, M.G., and Shamshuddin, M.D. "Effect of internal heat generation on free convective power-law variable temperature past a vertical plate considering exponential variable viscosity and thermal conductivity", *Journal of the Egyptian Mathematical Society*, **27**, 56 (2019). https://doi.org/10.1186/s42787-019-0062-5

10. Shamshuddin, M.D., Salawu, S.O., Asogwa, K.K., et al. "Thermal exploration of convective transportation of ethylene glycol based magnetized nanofluid flow in porous cylindrical annulus utilizing MOS_2 and Fe_3O_4 nanoparticles with inconstant viscosity" *Journal of Magnetism and Magnetic Materials*, **573**, 170663 (2023). https://doi.org/10.1016/j.jmmm.2023.170663

11. Farajollahi, A., Mokhtari, A., Rostami, M., et al. "Numerical study of using perforated conical turbulators and added nanoparticles to enhance heat transfer performance in heat exchangers", *Scientia Iranica*, **30**(3), pp. 1027-1038 (2023). https://doi.org/10.24200/sci.2022.59717.6394
12. Zheng, K., Shah, S.I., Naveed Khan, M., et al. "Numerical simulation of melting heat transfer towards the stagnation point region over a permeable shrinking surface", *Scientia Iranica,* **30**(3), pp. 1039-1048 (2023). https://doi.org/10.24200/sci.2022.59956.6518

13. Massoudi, M. and Phuoc, T.X. "Fully developed flow of a modified second grade fluid with temperature dependent viscosity", *Acta Mechanica*, **150**, pp. 23-37 (2001). https://doi.org/10.1007/BF01178542

14. Massoudi, M., Vaidya, A., and Wulandana, R., "Natural convection flow of a generalized second grade fluid between two vertical walls", *Nonlinear Analysis: Real World Applications*, **9**(1), pp. 80-93 (2008). https://doi.org/10.1016/j.nonrwa.2006.09.003

15. Ramya, M., Sangeetha, K., and Pavithra, M. "Study of visco-elastic fluid flow and heat transfer over a stretching sheet with variable viscosity and thermal radiation", *IOSR Journal of Mathematics (IOSR-JM)*, **10**(2), pp. 29-34 (2014). https://doi.org/10.1016/S0020-7462(00)00098-6

16. Ahmad, B. and Iqbal, Z. "Framing the performance of variation in resistance on viscous dissipative transport of ferro fluid with homogeneous and heterogeneous reactions", *Journal of Molecular Liquids*, **241**, pp. 904-911 (2017). https://doi.org/10.1016/j.molliq.2017.06.077

17. Deka, R.K., Basumatary, M., and Paul, A. "Effect of variable viscosity on a nanofluid over a porous wedge", *International Journal of Fluid Mechanics Research*, **45**, pp. 355-368 (2018). https://doi.org/10.1615/InterJFluidMechRes.2018019500

18. Mabood, F., Khan, W.A., and Makinde, O.D. "Hydromagnetic flow of a variable viscosity nanofluid in a rotating permeable channel with hall effects", *Journal of Engineering Thermophysics*, **26**, pp. 553-566 (2017). https://doi.org/10.1134/S1810232817040105

19. Asogwa, K.K., Mebarek-Oudina, F., and Animasaun, I.L. "Comparative investigation of water-based Al_2O_3 nanoparticles through water-based CuO nanoparticles over an exponentially accelerated radiative riga plate surface via heat transport", *Arabian Journal for Science and Engineering*, **47**, pp. 8721-8738 (2022). https://doi.org/10.1007/s13369-021-06355-3

20. Pop, I., Gorla, R.S.R., and Rashidi, M. "The effect of variable viscosity on flow and heat transfer to a

continuous moving flat plate", *International Journal of Engineering Science*, **30**(1), pp. 1-6 (1992). https://doi.org/10.1016/0020-7225(92)90115-W

21. Chu, Y.M., Alzahrani, F., and Mopuri, O. "Thermal impact of hybrid nanofluid due to inclined oscillatory porous surface with thermo-diffusion features", *Case Studies in Thermal Engineering*, **42**, 102695 (2023). https://doi.org/10.1016/j.csite.2023.102695

22. Djebali, R., Mebarek-Oudina, F., and Rajashekhar, C. "Similarity solution analysis of dynamic and thermal boundary layers: further formulation along a vertical flat plate", *Physical Scripta*, **96**(8), 085206 (2021). https://doi.org/10.1088/1402-4896/abfe31

23. Kia-Long, H. "Viscouselastic fluid over a stretching sheet with electromagnetic effects and non-uniform heat source/sink", *Mathematical Problems in Engineering*, **2010**, 740943 (2010). http://dx.doi.org/10.1155/2010/740943

24. Pal, D. and Mandal, G. "MHD convective stagnation-point flow of nanofluids over a non-isothermal stretching sheet with induced magnetic field", *Meccanica*, **50**, pp. 2023-2035 (2015). https://doi.org/10.1007/s11012-015-0153-9

25. Warke, A.S., Ramesh, K., Mebarek-Oudina, F., et al. "Numerical investigation of the stagnation point flow of radiative magnetomicropolar liquid past a heated porous stretching sheet" *Journal of Thermal Analysis and Calorimetry*, **147**, pp. 6901-6912 (2022). https://doi.org/10.1007/s10973-021-10976-z

26. Hossain, Md.A., Munir, Md.S., and Pop, I. "Natural convection flow of a viscous fluid with viscosity inversely proportional to linear function of temperature from a vertical wavy cone", *International Journal of Thermal Sciences*, **40**(4), pp. 366-371 (2001). https://doi.org/10.1016/S1290-0729(01)01226-1

27. Swain, K., Mahanthesh, B., and Mebarek-Oudina, F. "Heat transport and stagnation-point flow of magnetized nanoliquid with variable thermal conductivity, Brownian moment, and thermophoresis aspects", *Heat Transfer*, **50**(1), pp. 754-767 (2021). http://dx.doi.org/10.1002/htj.21902

28. Basha, H., Reddy, G.J., Abhishek, et al. "Numerical modelling of second-grade fluid flow past a stretching sheet", *Heat Transfer-Asian Research*, **48**(5), pp. 1595-1621 (2019). http://dx.doi.org/10.1002/htj.21448

29. Chaudhary, M.A. and Merkin, J.H. "A simple isothermal model for homogeneous-heterogeneous reactions in boundary-layer flow", *Fluid Dynamics Research*, **16**(6), pp. 311-333 (1995). https://doi.org/10.1016/0169-5983(95)00015-6

30. Shaw, S., Kameswaran, P.K., and Sibanda, P. "Homogeneous-heterogeneous reactions in micropolar fluid flow from a permeable stretching or shrinking sheet in a porous medium", *Boundary Value Problems*, **2013**, 77 (2013). https://doi.org/10.1186/1687-2770-2013-77

31. Sheikh, M. and Abbas, Z. "Homogeneous-heterogeneous reactions in stagnation point flow of Casson fluid due to a stretching/shrinking sheet with uniform suction and slip effects", *Ain Shams Engineering Journal*, **8**(3), pp. 467-474 (2017). https://doi.org/10.1016/j.asej.2015.09.010

32. Cortell, R. "Effects of viscous dissipation and work done by deformation on the MHD flow and heat transfer of a viscoelastic fluid over a stretching sheet", *Physics Letters A*, **357**(4-5), pp. 298-305 (2006). https://doi.org/10.1016/j.physleta.2006.04.051

33. Abdul Hakeem, A.K., Vishnu Ganesh, N., and Ganga, B. "Effect of heat radiation in a Walter's liquid B fluid over a stretching sheet with non-uniform heat source/sink and elastic deformation", *Journal of King Saud University- Engineering Sciences*, **26**(2), pp. 168-175 (2014). https://doi.org/10.1016/j.jksues.2013.05.006

34. Ramzan, M., Rafiq, A., Chung, J.D., et al. "Nanofluid flow with autocatalytic chemical reaction over a curved surface with nonlinear thermal radiation and slip condition", *Scientific Reports,* **10**, 18339 (2020). https://doi.org/10.1038/s41598-020-73142-9

35. Patil, P.M., Benawadi, S., Tonannavar, J.R., et al. "Homogeneous and heterogeneous reactions in the mixed convection flow of hybrid nanofluid over a slender cylinder" *Asia-Pacific Journal of Chemical Engineering*, **17**(2), e2740 (2022). https://doi.org/10.1002/apj.2740

36. Siddiqui, B.K., Batool, S., Mahmood Ul-Hassan, Q., et al. "Repercussions of homogeneous and heterogeneous reactions of 3D flow of Cu-water and AL_2O_3-water nanofluid and entropy generation estimation along stretching cylinder", *Ain Shams Engineering Journal*, **13**(1), 101493 (2022). http://dx.doi.org/10.1016/j.asej.2021.05.007

37. Vaidya, H., Choudhari, R., Mebarek-Oudina, F., et al. "Combined effects of homogeneous and heterogeneous reactions on peristalsis of Ree-Eyring liquid: Application in hemodynamic flow", *Heat Transfer*, **50**(3), pp. 2592-2609 (2021). https://doi.org/10.1002/htj.21995

38. Iqbal, Z., Mehmood, R., and Ahmed, B. "Elastic deformation analysis on MHD viscous dissipative Flow of viscoelastic fluid: An exact approach", *Communications in Theoretical Physics*, **67**(5), pp. 561-567 (2017). https://doi.org/10.1088/0253-6102/67/5/561

`

Biographies

Munazza Saeed is working as a Lecturer in the Department of Mathematics, University of Wah, Wah Cantt, Pakistan. She has completed her PhD degree in computation fluid mechanics in 2022. Her area of research is fluid mechanics and nanotechnology. She has published 25 research papers in different journals.

Bilal Ahmad is an Assistant Professor in the University of Wah, Wah Cantt, Pakistan. He is working in nanofluids and heat and mass transfer problems. He is published 755 research papers in well reputed international journals.

Chemseddine Maatki has an overall experience of more than 19 years of teaching at the university level. He has authored and co-authored more than 50 publications in peer-reviewed journals and conference proceedings. His research interest includes thermal engineering, computational fluid mechanics, numerical modeling, renewable energy, design of solar distiller system, and thermal energy storage.

Tasawar Abbas is working in University of Wah, Wah Cantt, Pakistan. He has published 40 research papers with impact factor 60 plus. His area of research is fluid mechanics and nanofluids. He is reviewer of 18 impact factor journals.

Bilel Hadrich is Associate Professor in Imam Muhammad Ibn Saud Islamic University Riyadh, Saudi Arabia. His area of research is numerical modeling, biofuels, heat transfer and bioenergy.

Sami Ullah Khan is an Associate Professor in the Namal University Mianwali, Pakistan. He has published 370 research papers with impact factor 1000 plus. He is reviewer of more than 70 impact factor journals. He is guest editor of 4 impact factor journals. He has been awarded as distinguish researcher from Namal University Mianwali.

Karim Kriaa is an Assistant Professor of Chemical Engineering in Chemical Engineering Department, Faculty of Engineering, Al Imam Mohammad Ibn Saud Islamic University. He works on extraction and separation. His focus is on valorization of food by product and application of new technologies. He is the coauthor of more than 9 publications and chapter in book.

Qazi Mehmood Ul-Hassan is an Associate Professor and head of Mathematics Department at University of Wah, Wah Cantt, Pakistan. He is working in nanofluids and numerical analysis. He is published 80 research papers in well reputed international journals.

Lioua Kolsi is an Associate Professor in the University of Hail, Saudia Arabia. He is working in thermal systems and published more than 300 research papers in different journals. His area of research is nanofluids, thermal engineering and computational fluid mechanics. He is Guest Editor of applied sciences journals.