Improved CEM-RBS Control Charts for Monitoring the Process Mean using Ranked-Based Sampling Designs

Muhammad Tayyab[1], Wasif Yasin[2], Asma Arshad[3], Muhammad Noor-ul-Amin[4,*], Muhammad Hanif[5]

[1,2,3,5]Department of Statistics, National College of Business Administration and Economics, Lahore, Pakistan;
[4]Department of Statistics, COMSATS University Islamabad-Lahore Campus, Pakistan
[*]Email: nooramin.stats@gmail.com

Complete addresses of authors:

1. Dr. Muhammad Tayyab, Department of Statistics, National College of Business Administraiton and Economics, 40, E-1, Gulberg-III, Lahore, Pakistan. Email: m_tayyab82@hotmail.com; Phone: +92-333-6934863.
2. Wasif Yasin, Department of Statistics, National College of Business Administraiton and Economics, 40, E-1, Gulberg-III, Lahore, Pakistan. Email: wasif.yasin988@gmail.com ; Phone: +92-305-4444988.
3. Asma Arshad, Department of Statistics, National College of Business Administration and Economics, 40, E-1, Gulberg-III, Lahore, Pakistan. E-mail: aarshadstat@gmail.com ; Phone: +92-321-8828212.
4. Dr. Muhammad Noor-ul-Amin, Associate Professor, Department of Statistics, COMSATS University Islamabad, Lahore Campus, Defense Road, Off Raiwind Road, Lahore, Pakistan. Email: nooramin.stats@gmail.com ; Phone: +92-334-4151809.
5. Dr. Muhammad Hanif, Vice Rector (Research), National College of Business Administration and Economics, 40, E-1, Gulberg-III, Lahore, Pakistan. E-mail: drhanif@ncbae.edu.pk ; Phone: +92-307-4747257.

ABSTRACT

In this study, different ranked-based sampling (RBS) schemes are used to design a sensitive control chart to monitor the small or moderate shifts in the process mean and named combined exponentially weighted moving average (EWMA) moving average (MA) RBS (CEM-RBS) control charts. The average run-length (*ARL*) and the standard deviation of the run-length (*SDRL*) through Monte Carlo simulation runs are computed to evaluate the performance of the proposed charts in comparison with the existing charts such as MA, EWMA, exponentially weighted moving average-moving average (EWMA-MA), and the EWMA under RBS control charts. It is proved through a comparative study that the proposed CEM-RBS charts indicate a significant improvement in the performance of the EWMA-MA chart by using the RBS concept. A real dataset-based example is also included to explain the concept in detail.

Keywords: Average run length, control chart, exponentially weighted moving average, ranked set sampling, Monte Carlo simulation.

1. Introduction

Improvement in the quality of a product or production process is one of the main objectives of the manufacturing industries of the world. The existence of an industry in the global market competition cannot be guaranteed until the capitalist/industrialist pays attention to the gradual enhancement in the quality characteristics of the product. To monitor and improve the quality of a product, a control chart is one of the widely used structures for manufacturing industries in the era of the rapid development of technology. A control chart as a statistical process control (SPC) device is a graphical display of three parallel lines, known as lower-control-limit (*LCL*), central-limit (*CL*), and upper-control-limit (*UCL*). It is applied to detect undesirable disturbances in the production process as early as possible before faulty items are produced. A process is generally said to be in-control (IC) when all plotting statistics fall between *UCL* and *LCL* while it is said to be out-of-control (OC) when at least one plotting statistic falls outside of *UCL* or *LCL*. Shewhart [1] was the first who introduced the basic idea of a control chart under simple random sampling (SRS) for improving process capability by reducing variations of the process. To monitor the quality characteristic of a product, the Shewhart mean control chart has been very beneficial in detecting large shifts in the process mean. But when we are interested to identify and remove small/moderate shifts from the process then the cumulative sum (CUSUM) control chart (Page [2]) and exponentially weighted moving average (EWMA) control chart (Roberts [3]) are used which are an efficient alternative of Shewhart control chart for monitoring the process mean. Later, Wong et al. [4] presented a moving average (MA) control chart to detect a shift in the process mean.

The idea of ranked set sampling (RSS) design was delivered by McIntyre [5] and its further mathematical properties were introduced by Takahasi and Wakimoto [6]. A modified form of RSS named extreme RSS (ERSS) for estimating the population mean was introduced by Samawi et al. [7]. The next modification in RSS was suggested by Muttlak [8] for the estimation of population parameters, called median RSS (MRSS). Muttlak [9] developed quartile RSS (QRSS) as an efficient alternative to the RSS and also compared the performance of the QRSS estimator of the population mean with its counterparts in SRS, RSS, and MRSS and found it more competent for some distributions. In recent times, advanced sampling designs like RSS are becoming popular among investigators in control charting structures for quick detection of the shift in the process mean. For instance, Haq et al. [10] have introduced a new mixed RSS based EWMA chart as an alternative to the RSS based EWMA (EWMA-RSS) by encompassing both SRS and RSS with perfect and imperfect rankings methodologies, Awais and Haq [11] developed a new EWMA chart based on the perfect and imperfect rankings with varied RSS methodology as a generalized cost-effective scheme, Tayyab et al. [12] has given the paired RSS scheme based EWMA chart, again Tayyab et al. [13] presented the improved mean control chart by using the quartile paired RSS scheme, Noor-ul-Amin et al. [14] introduced the paired double RSS scheme based monitoring chart and again Noor-ul-Amin and Tayyab [15] has introduced a more sensitive chart by using the EWMA concept in collaboration with paired double RSS schemes. Also, for some recent researches, the readers can refer to Taboran et al. [16], Awais and Haq [17], and Mohamadkhani and Amiri [18].

Recently, Sukparungsee et al. [19] introduced a new combination of exponentially weighted moving average-moving average (EWMA-MA) charts to monitor the process location for some symmetrical and non-symmetrical distributions. The average run-length (*ARL*), the standard deviation of the run-length (*SDRL*), and median run-length (*MRL*) were used to compare the performance of the suggested control chart with Shewhart, MA, EWMA, and moving average-exponentially weighted moving average (MA-EWMA) charts. The results showed that the EWMA-MA chart is better in shift detection ability than the Shewhart, MA, and EWMA charts. By searching the literature on control charting structures, we observe that most of the control charts that utilize SRS is considerably less sensitive in detecting a shift in the process mean than those that use efficient RBS designs. This fact motivated us to develop improved combined EWMA-MA control charts using the RBS designs (RSS, ERSS, MRSS, and QRSS). These control charts can be effectively applied in such scenarios where taking a real measurement of quality characteristic of interest is pricey, destructive, and time-consuming but the ranking of a small random set of observations is relatively easy by exploiting professional judgment or auxiliary information. To compare the proposed control chart with its competitors, a comprehensive simulation study in R-Language is conducted by using Monte Carlo (MC) simulations for computing *ARLs* and *SDRLs* as performance measures. The upcoming structure of the paper is followed by some sections as section 2 which is being provided by a brief review of RBS designs and some basic control charts. Section 3 is based on operational detail about the designing of the proposed CEM-RBS charts. Section 4 has elaborated on the performance evaluation and section 5 gives the comparative analysis. The application of the proposed concept in some real-life industrial dataset is given in section 6. Further research study along with the concluding remarks has been discussed in the last section 7.

2. Brief Review of RBS Designs and Control Charts

In this section, RBS designs and some available control charts utilized in this paper for valid comparative studies are provided.

2.1 RBS Designs

Estimation of mean remains always a questionable discussion in sampling theory. For this purpose, many authors presented their studies to estimate the mean by improving the sampling techniques. One of these improved sampling techniques is RSS which is based on ranking the observations to get a more precise estimate of the population mean. The RSS technique was originally presented by McIntyre [5]. The RSS procedure, with one cycle, for sample selection, was described by Patil [20] as:

- Choose n^2 units randomly from the underlying population of interest, here n represents the set size.
- Allocate these n^2 units into n sets each with n set size.
- In this step, the n units of every set are then arranged in increasing or decreasing order through visual ranking, by any cost-free method or using the auxiliary variable.
- After the ranking mechanism, firstly choose a higher-order ranked unit from the first set. Choose the second higher-ranked unit from the second set. Continue the same procedure of selecting the units until from the n^{th} higher ranked unit is chosen from the n^{th} set. These units represent the RSS of size n.

It is to be noted that the respective ERSS, MRSS, and QRSS procedures may be found by transforming the last units' selection step of RSS procedure as:

- After the ranking mechanism, if n is even then divide the number of sets into two halves, each half consisting of $n/2$ sets. Choose the lowest unit of each set from the first half and the highest-ranked unit for measurement from the set of the second half. For odd n, choose the lowest units from the first half consisting of $(n-1)/2$ sets and choose the largest ranked units for measurement from the other half consisting of $(n-1)/2$ sets. Take the median sample unit from the last set. These units represent the ERSS of size n.
- After the ranking mechanism, if n is even, divide the total sets into two halves, each half contains $n/2$ sets. From the first half pick the lowest-ranked units from the two middle sampling units of the first half and select the largest ranked units from the two middle sampling units of the remaining half. For odd n, select $((n+1)/2)^{th}$ units from each set. These units represent the MRSS of size n.
- After the ranking mechanism, if n is even then dividing the data set into two halves and picking from the first half consisting of $n/2$ sets the $((n+1)/4)^{th}$ ranked units and from the remaining half consisting of remaining $n/2$ sets the $(3(n+1)/4)^{th}$ ranked units. For odd n pick from first $(n-1)/2$ ranked sets the $((n+1)/4)^{th}$ units and from next $(n-1)/2$ ranked sets the $(3(n+1)/4)^{th}$ units and take the median unit from the last set. These units represent the QRSS of size n.

The mean estimators and their variance expressions based on RSS, ERSS, MRSS, and QRSS schemes are given in Tayyab [21].

2.2 EWMA Control Chart

To identify small/moderate shifts in the process, the EWMA control chart (Roberts [3]) is used. This chart uses the past as well as current information to identify the shift in the process mean. Let Y be the variable of interest then Y_t; $\{t = 1, 2, ...\}$ be the identically and independently distributed (IID) random variables. Assume Y_t is a normal random variable and $\bar{Y}_{SRS,t} = \sum_{i=1}^{n} Y_{i,t} \Big/ n$ with $\bar{Y}_{SRS,t} \sim N\left(\mu_Y, \frac{\sigma_Y^2}{n}\right)$ where n is the sample size. The EWMA statistic may be defined as:

$$E_t = \lambda \bar{Y}_{SRS,t} + (1-\lambda) E_{t-1} \qquad 0 < \lambda \le 1 \tag{1}$$

with respective *LCL*, *CL* and *UCL*, such as

$$LCL_t = \mu_Y - L\left(\frac{\sigma_Y}{\sqrt{n}}\right)\sqrt{\frac{\lambda\left(1-(1-\lambda)^{2t}\right)}{2-\lambda}};\ CL = \mu_Y;\ UCL_t = \mu_Y + L\left(\frac{\sigma_Y}{\sqrt{n}}\right)\sqrt{\frac{\lambda\left(1-(1-\lambda)^{2t}\right)}{2-\lambda}} \tag{2}$$

where t denotes sample number, λ is a constant and L is a control coefficient. The $E_0 = \mu_Y$ or the average from the phase-I data. The λ and L are the two parameters of the EWMA chart, L determines the width of control limits while λ determines the decline of weights.

2.3 Moving Average Control Chart

The EWMA control chart is time-weighted. The statistic used in the EWMA chart is a weighted average. The moving average (MA) control chart (Wong et al. [4]) is also a time-weighted type control chart but it is simple and uses an unweighted moving average. The MA uses for smoothing the time series by averaging a fixed number of consecutive terms. The MA statistic is defined for span w at time t as

$$MA_t = \begin{cases} \dfrac{\bar{Y}_t + \bar{Y}_{t-1} + \bar{Y}_{t-2} + \cdots}{t}, & t < w \\[2ex] \dfrac{\bar{Y}_t + \bar{Y}_{t-1} + \bar{Y}_{t-2} + \cdots + \bar{Y}_{t-w+1}}{w}, & t \ge w \end{cases} \tag{3}$$

Where $\bar{Y}_t = \dfrac{Y_{t1} + Y_{t2} + Y_{t3} + \cdots + Y_{tn}}{n}$ (4)

The respective mean and variance of the statistic MA_t are

$$E(MA_t) = E(\bar{Y}_t) = \mu_Y \tag{5}$$

and

$$\sigma_{MA}^2 = Var\left(MA_t\right) = \begin{cases} \dfrac{\sigma^2}{nt} & , \quad t < w \\ \dfrac{\sigma^2}{nw} & , \quad t \geq w \end{cases} \tag{6}$$

The control limits of the MA control chart are given by

$$LCL_t = \mu_Y - L\sqrt{Var\left(MA_t\right)}; CL = \mu_Y \, ; UCL_t = \mu_Y + L\sqrt{Var\left(MA_t\right)} \tag{7}$$

where L determines the width of MA control limits. For more detail, see Montgomery [22].

2.4 EWMA-MA Control Chart

Similar to the MA-EWMA chart, Sukparungsee et al. [19] introduced a new combination of EWMA and MA charts called the EWMA-MA chart to monitor the process mean. The EWMA-MA statistic similar to the EWMA chart may be defined as:

$$\varepsilon_t = \lambda MA_t + (1-\lambda)\varepsilon_{t-1} \, , \, t = 1, 2, ... \tag{8}$$

The limiting form of the control limits of the EWMA-MA control chart may be presented as

$$LCL_t = \mu_Y - L\sqrt{Var\left(MA_t\right)\left(\frac{\lambda}{2-\lambda}\right)}; CL = \mu_Y \, ; UCL_t = \mu_Y + L\sqrt{Var\left(MA_t\right)\left(\frac{\lambda}{2-\lambda}\right)} \tag{9}$$

where L determines the width of EWMA-MA control limits.

3. Proposed CEM-RBS Control Chart

In this section, new and efficient combined exponentially weighted moving average moving average RBS (CEM-RBS) control charts are designed by combining the EWMA and MA charts with RBS designs such as RSS, ERSS, MRSS, and QRSS. These charts are a valuable alternative to the EWMA-MA chart for detecting the shift in the mean. The CEM-RBS control chart is also effectively applicable in such scenarios where taking a real measurement of quality characteristic of interest is pricey, destructive, and time-consuming but the ranking of a small random set of observations is relatively easy by exploiting professional judgment or auxiliary information. Let a sample size n using RBS design (d) at each time point t is selected where $d = ERSS, RSS, QRSS, MRSS$. Assume Y_t be the sequence of IID normal random variables for $t = 1, 2, ...$ and $\overline{Y}_{(d)t}$ be the mean of the sample at t time based on d . Similar to the EWMA chart, the plotting statistic of the CEM-RBS chart is described as:

$$\varepsilon_{(d)t} = \lambda MA_{(d)t} + (1-\lambda)\varepsilon_{(d)t-1} \, , \, t = 1, 2, ... \tag{10}$$

where

$$MA_{(d)t} = \begin{cases} \dfrac{\overline{Y}_{(d)t} + \overline{Y}_{(d)t-1} + \overline{Y}_{(d)t-2} + \cdots}{t} & , \quad t < w \\ \dfrac{\overline{Y}_{(d)t} + \overline{Y}_{(d)t-1} + \overline{Y}_{(d)t-2} + \cdots + \overline{Y}_{(d)t-w+1}}{w} & , \quad t \geq w \end{cases} \tag{11}$$

The variance of plotting statistic of the CEM-RBS chart is

$$Var\left(\varepsilon_{(d)t}\right) = Var\left(MA_{(d)t}\right)\left(\frac{\lambda}{2-\lambda}\right) \tag{12}$$

where

$$Var\left(MA_{(d)t}\right) = \begin{cases} \dfrac{Var\left(\overline{Y}_{(d)t}\right)}{t} & , \quad t < w \\ \dfrac{Var\left(\overline{Y}_{(d)t}\right)}{w} & , \quad t \geq w \end{cases} \tag{13}$$

The control limits of the CEM-RBS chart are defined as

$$LCL_t = \mu_Y - L_g\sqrt{Var\left(\varepsilon_{(d)t}\right)}; CL = \mu_Y ; UCL_t = \mu_Y + L_g\sqrt{Var\left(\varepsilon_{(d)t}\right)} \tag{14}$$

where $L_g\left(g = 1,2,3,4\right)$ determines the width of CEM-RBS control limits where L_1 is for the proposed CEM-ERSS chart, L_2 is for the proposed CEM-RSS chart, L_3 is for the proposed CEM-QRSS chart and L_4 is for the proposed CEM-MRSS chart. The complete sampling procedures, estimators, and variance expressions based on RSS, ERSS, MRSS, and QRSS are discussed in section 2.

4. Performance Evaluation

The control charting performance measures *ARL* and *SDRL* are employed in this section to study the sensitivity of suggested CEM-RBS control charts (CEM-ERSS, CEM-RSS, CEM-QRSS, CEM-MRSS). The smaller values of the *ARL* and *SDRL* of a chart indicate that the chart is performing efficiently in detecting a shift in the process. IC *ARL* $\left(ARL_0\right)$ and OC *ARL* $\left(ARL_1\right)$ are the two types of *ARL*. When the process is IC then the average number of points (samples) plotted until a point indicates a false OC signal is known as ARL_0. While the average number of points (samples) required to diagnose a shift, when the process is OC, is known as ARL_1.

The results are computed through MC simulation runs and presented extensively in Tables 1-4 with parametric values associated with schemes i.e. $ARL_0 = 370$, w is taken as $2,5,7$, sample size n is taken as the $4,5,6$ and λ is taken as 0.1 and 0.25 for the shift size f ranges from 0.00 to 2.00 in 10 point breaks. It is evident that the proposed chart is efficiently working for the small to moderate shift sizes and it can be seen through the rapidly decreasing *ARL* and *SDRL* values trend provided in the Tables. The main findings from the results are as follows:

- As n increases from $n=4$ to 5 and then to 6, the shifted ARL_1 along with the $SDRL_1$ is decreasing shows a quick detection of process shifts and this trend can be seen in all proposed CEM-RBS charts. For example at $f=0.05, w=5, \lambda=0.25$, and $n=4,5$ and 6; the $ARL_1\left(SDRL_1\right)$ values (cf. Table 1) of the CEM-ERSS chart are 225.88(226.84), 186.20(188.59), and 164.97(162.52), the $ARL_1\left(SDRL_1\right)$ values (cf. Table 2) of the CEM-RSS chart are 206.98(208.65), 170.86(174.24), and 139.89(140.10), the $ARL_1\left(SDRL_1\right)$ values (cf. Table 3) of the CEM-QRSS chart are 229.70(231.69),149.32(152.19), and 129.02(132.30), at last, the $ARL_1\left(SDRL_1\right)$ values (cf. Table 4) of the CEM-MRSS chart are 196.74(199.18), 147.92(151.43), and 118.88(118.47) respectively.
- The λ has a greater influence on results, as the value of λ increases the run length results also increase, which can be seen in Table 1-4 for all the proposed CEM-RBS charts; such as for $f=0.05, w=5, n=5,$ and $\lambda=0.1$ and 0.25; from Table 1 the $ARL_1\left(SDRL_1\right)$ values of the CEM-ERSS chart are 141.73(134.47) and 186.20(188.59), from Table 2 the $ARL_1\left(SDRL_1\right)$ values of the CEM-RSS chart are 126.96(121.57) and 170.86(174.24), again from Table 3 $ARL_1\left(SDRL_1\right)$ values of the CEM-QRSS chart are 115.53(109.92) and 149.32(152.19), at last from Table 4 $ARL_1\left(SDRL_1\right)$ values of the CEM-MRSS chart are 112.06(106.67) and 147.92(151.43) respectively.
- The w span, the parametric constant of the MA chart efficiently influences the performance of the proposed CEM-RBS charts by decreasing the run-length values. For instance, for $w=2,5$ and 7 with fixed values of $f=0.05, \lambda=0.25$, and $n=5,$ the $ARL_1\left(SDRL_1\right)$ values of the CEM-ERSS chart from Table 1 are 197.69(194.82),186.20(188.59), and 180.19(186.54) respectively. Similarly, the $ARL_1\left(SDRL_1\right)$ values of the CEM-RSS chart from Table 2 are 189.36(186.90), 170.86(174.24), and 162.83(165.57). The $ARL_1\left(SDRL_1\right)$ values of the CEM-QRSS chart from Table 3 are 167.03(166.37), 149.32(152.19), and 144.36(147.94). At last $ARL_1\left(SDRL_1\right)$ values of the CEM-MRSS chart from Table 4 are 162.72(160.83),147.92(151.43), and 140.95(141.54) respectively.
- The overall performance of all proposed CEM-RBS charts for the same parametric values of n, w and λ demonstrated that the CEM-MRSS chart appeared to be an exceptional choice as it detects a shift in the process mean faster than the other proposed CEM-ERSS, CEM-RSS, and CEM-QRSS charts.

[Tables 1- 4 must be here]

5. Performance Comparison for CEM-RBS Control Charts

To compare the performance of the proposed (CEM-ERSS, CEM-RSS, CEM-QRSS, CEM-MRSS) charts with considered (MA, EWMA, EWMA-MA, EWMA-RSS) charts, a new performance measure expected *ARL* (*EARL*) along with *ARL* and *SDRL* is used for detecting a shift in the process mean. Huang et al. [23] discussed that the use of *EARL* is essential when practitioners did not know the size of the process shift in advance. Woon et al. [24], Chan et al. [25], and Wang et al. [26] also employed the *EARL* criterion as a useful performance measure of the charts and it is defined as:

$$EARL = \int_{f_{\min}}^{f_{\max}} g\left(f\right) ARL\, df.$$

Here $g\left(f\right)$ represent the probability density function of the magnitude of the shift in a process, i.e. f .

[Table-5 must be here]

The performance comparison (cf. Table 5) of mentioned charts is evaluated through MC simulations in R-Language with setting $\left(n=5, w=5, \lambda=0.25\right)$ and 16 shift values break to elaborate on the difference in detail. The values of *ARL* and *EARL* are fixed at 370. Here, the shift interval $\left(f_{\min}, f_{\max}\right)=\left(0.01, 2.00\right)$ is considered for *EARL*. It can be easily seen through the simulated results presented in Table 5 that all proposed CEM-RSS, CEM-ERSS, CEM-QRSS, and CEM-MRSS charts detect the shift in the process mean faster than all considered MA, EWMA, EWMA-MA, and EWMA-RSS charts. It is worth mentioning that the CEM-MRSS chart appeared to be an excellent choice as it showed far better performance, in terms of *ARL*, *SDRL*, and *EARL* values, than the other, proposed and competitor charts.

6. An Application

To support the simulated results, this section presents the application of the CEM-RBS charts by using the data (Montgomery [22]) of flow width measurements (microns) in the hard-bake process. For establishing statistical control on the flow width of the resist in this process, Tayyab et al. [12] and Noor-ul-Amin and Tayyab [15] have also used this dataset. To draw wafer samples using RSS, ERSS, MRSS, and QRSS designs, we combine 45 samples in such a way to have a population containing 225 measurements. We applied the Shapiro-Wilk normality test as well as the Q-Q plot displayed in Figure 1 and found that the dataset is normally distributed. The control charting parameters $ARL_0 = 370$, $n=5$, $w=2$ and $\lambda = 0.25$ are used for all proposed CEM-RSS, CEM-MRSS, CEM-ERSS, CEM-QRSS, and their competitors EWMA-SRS, EWMA-RSS, EWMA-MA charts. We select 30 samples, each of size 5, under RSS, ERSS, QRSS, MRSS, and SRS designs from the population. These designs with mean estimators and their variance expressions are presented in Tayyab [21]. By using the plotting-statistics, LCL and UCL, given in sections 2 and 3, the values are computed by utilizing 30 samples for proposed and considered charts. After that, we again select 20 new samples each of size 5 under said charts by adding a shift $f = 0.05$ in all measurements. A similar pattern as discussed above is adopted to get control limits and plotting-statistics for 20 other samples. The values of the plotting-statistics are charted against the control limits of the mentioned charts for 50 samples and the resulting plots are portrayed in Figure 2.

[Figures 1-2 must be here]

Figure 2 indicates that plotting statistics of all proposed and considered charts are lying inside the *UCL* and *LCL* for the first 30 samples which are clearly showing an IC scenario. After introducing a shift in data, it is observed that EWMA-SRS, EWMA-RSS, EWMA-MA, CEM-RSS, CEM-ERSS, CEM-QRSS, and CEM-MRSS charts diagnose the disturbance in the process-mean at $44^{th}, 38^{th}, 41^{th}, 35^{th}, 36^{th}, 34^{th}$ and 33^{rd} sample respectively. It is determined that the proposed CEM-RSS, CEM-ERSS, CEM-QRSS, and CEM-MRSS charts have better shift detection ability than their competitors EWMA-SRS, EWMA-RSS, EWMA-MA charts. Similar to simulation results, it is also discovered that the CEM-MRSS chart offers the best performance in diagnosing a disturbance early in the process-mean among all the proposed and competitor charts.

7. Conclusion and Further Research

In the current study, the EWMA-MA control chart performance is being enhanced by utilizing the RBS schemes and designing the CEM-RBS control charts for monitoring the process mean. It is found that this combination enhanced the performance of the EWMA-MA chart in terms of the smaller *ARL*, *SDRL*, and *EARL* values. The comparison of proposed CEM-ERSS, CEM-RSS, CEM-QRSS and the CEM-MRSS charts has been made with their counterparts such as MA, EWMA, EWMA-MA, and the EWMA-RSS charts. The implementation of the proposed charts on a real-life industrial dataset determines the superiority of the combined chart concept in designing the mean monitoring control charts and found them efficient in terms of the smaller run-length values with a quick detection of small to moderate and large shifts in the process mean value. The simulation- and application-based results concluded that the CEM-MRSS chart appeared to be an excellent choice as it showed far better performance, in terms of *ARL*, *SDRL*, and *EARL* values, than the other proposed (CEM-ERSS, CEM-RSS, CEM-QRSS, CEM-MRSS) and considered (MA, EWMA, EWMA-MA, EWMA-RSS) charts. The proposed study can be extended with auxiliary information and some CUSUM and combined CUSUM and EWMA control charts. The measurement error effects for changing sampling schemes can also be studied to improve the monitoring chart sensitivity.

Acknowledgment

The authors are thankful to learned reviewers for valuable comments that significantly improved this manuscript.

References

1. Shewhart, W.A. "Some applications of statistical schemes to the analysis of physical and engineering data", *Bell Technical Journal*, **3**, pp. 43-87 (1924).
2. Page, E.S. "Continuous inspection schemes", *Biometrika*, **42**, 243-254 (1954).
3. Roberts, S.W. "Control chart tests based on geometric moving averages", *Technometrics*, **1**(3), pp. 239-250 (1959).
4. Wong, H.B., Gan, F.F., and Chang, T.C. "Designs of moving average control chart", *Journal of Statistical Computation and Simulation*, **74** (1), pp. 47-62 (2004).
5. McIntyre, G.A. "A method for unbiased selective sampling using ranked sets", *Australian Journal of Agricultural Research*, **3**, pp. 385-390 (1952).
6. Takahasi, K., and Wakimoto, K. "On unbiased estimates of the population mean based on the sample stratified by means of ordering", *Annals of the Institute of Statistical Mathematics*, **20**, pp. 1-31 (1968).
7. Samawi, H.M., Ahmed, M.S., and Abu-Dayyeh, W. "Estimating the population mean using extreme ranked set sampling", *Biometrical Journal*, **38**(5), pp. 577-586 (1996).
8. Muttlak, H.A. "Median ranked set sampling", *Journal of Applied Statistical Sciences*, **6**(4), pp. 245-255 (1997).
9. Muttlak, H.A. "Investigating the use of quartile ranked set samples for estimating the population mean", *Applied Mathematics and Computation*, **146**, pp. 437-443 (2003).
10. Haq, A., Brown, J., and Moltchanova, E. "A new exponentially weighted moving average control chart for monitoring the process mean", *Quality and Reliability Engineering International*, **31**(8), pp. 1623-1640 (2015).
11. Awais, M., and Haq, A. "An EWMA chart for monitoring the process mean", *Journal of Statistical Computation and Simulation*, **88**(5), pp. 1003-1025 (2018).

12. Tayyab, M., Noor-ul-Amin, M., and Hanif, M. “Exponential weighted moving average control charts for monitoring the process mean using pair ranked set sampling schemes”, *Iranian Journal of Science and Technology, Transactions A: Science*, **43**(4), pp. 1941-1950 (2019).
13. Tayyab, M., Noor-ul-Amin, M., and Hanif, M. “A control chart using quartile pair ranked set sampling for monitoring the process mean”, *Journal of Statistical Theory and Practice*, **14**(2), pp. 1-16 (2020).
14. Noor-ul-Amin, M., Tayyab, M., and Hanif, M. “Efficient control charts for monitoring process mean using paired double ranked set sampling”, *Communications in Statistics: Theory and Methods*, **50**(10), pp. 2211-2223 (2021).
15. Noor-ul-Amin, M., and Tayyab, M. “Enhancing the performance of exponential weighted moving average control chart using paired double ranked set sampling”, *Journal of Statistical Computation and Simulation*, **90**(6), pp. 1118-1130 (2020).
16. Taboran, R., Sukparungsee, S., and Areepong, Y. “Mixed moving average-exponentially weighted moving average control charts for monitoring of parameter change”, *Proceeding of the International MultiConference of Engineers and Computer Scientists*, March 13-15, Hong Kong: Newswood (2019).
17. Awais, M., and Haq, A. “New Shewhart-EWMA and Shewhart-CUSUM control charts for monitoring process mean”, *Scientia Iranica*, **26**(6), pp. 3796-3818 (2019).
18. Mohamadkhani, A., and Amiri, A.H. “Developing mixed EWMA-CUSUM and CUSUM-EWMA control charts based on MRSS and DRSS procedures”, *Published online in Scientia Iranica*, doi. 10.24200/SCI.2020.55632.4328 (2020).
19. Sukparungsee S., Areepong Y., and Taboran, R. “Exponentially weighted moving average-moving average charts for monitoring the process mean”, *PLOS ONE* **15**(2), e0228208. https://doi.org/10.1371/journal.pone.0228208 (2020).
20. Patil, G.P. “Ranked set sampling”, *Encyclopedia of Environmetrics*, **3**, pp. 1684-1690 (2002).
21. Tayyab, M. “Developments and estimation in ranked set sampling”, Ph.D. thesis, Department of Statistics, *National College of Business Administration and Economics*, Lahore, Pakistan (2019).
22. Montgomery, D.C. “Introduction to Statistical Quality Control” *John Wiley & Sons*, New York (2009).
23. Huang, W., Shu, L., and Jiang, W. “A gradient approach to the optimal design of CUSUM charts under unknown mean-shift sizes”, *Journal of Quality Technology*, **48**(1), pp. 68-83 (2016).
24. Woon, Y.H., Chong, M.K.B., Lin, C.Z., et al. “The expected average run length of the EWMA median chart with estimated process parameters”, *Austrian Journal of Statistics*, **49**(3), pp. 19-24 (2020).
25. Chan, K.M., Mukherjee, A., Chong, Z. L., et al. “Distribution-free double exponentially and homogeneously weighted moving average Lepage schemes with an application in monitoring exit rate”, *Computers & Industrial Engineering*, **161**, 107370 (2021).
26. Wang, J., Chong, Z.L., and Qiu, P. “Optimal monitoring of Poisson data with known and unknown shifts”, *Computers & Industrial Engineering*, **154**, 107100 (2021).

List of Tables and Figures

Table 1: The *ARL*s and *SDRL*s of the proposed CEM-ERSS control chart at $ARL_0 = 370$.

Table 2: The *ARL*s and *SDRL*s of the proposed CEM-RSS control chart at $ARL_0 = 370$.

Table 3: The *ARL*s and *SDRL*s of the proposed CEM-QRSS control chart at $ARL_0 = 370$.

Table 4: The *ARL*s and *SDRL*s of the proposed CEM-MRSS control chart at $ARL_0 = 370$.

Table 5: Comparative analysis

Figure 1: Q-Q Plot of the flow width measurements (microns) in the hard-bake process.

Figure 2: The performance comparison of shift detection ability between **(A)** EWMA-SRS, **(B)** EWMA-RSS, **(C)** EWMA-MA, **(D)** CEM-RSS, **(E)** CEM-ERSS, **(F)** CEM-MRSS and **(G)** CEM-QRSS charts

Table 1: The *ARL*s and *SDRL*s of the proposed CEM-ERSS control chart at $ARL_0 = 370$.

w	f	$n=4$				$n=5$				$n=6$			
		$\lambda=0.1$ $L_1=3.6435$		$\lambda=0.25$ $L_1=3.7475$		$\lambda=0.1$ $L_1=3.6175$		$\lambda=0.25$ $L_1=3.7375$		$\lambda=0.1$ $L_1=3.5895$		$\lambda=0.25$ $L_1=3.7375$	
		ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$
2	0.00	371.34	375.83	370.92	367.61	371.85	367.95	371.73	369.03	370.77	369.91	371.77	369.23
	0.03	274.06	265.12	311.99	315.49	241.70	224.17	277.20	275.53	229.57	204.15	277.19	275.92
	0.05	184.02	175.07	234.08	232.75	145.72	135.45	197.69	194.82	129.92	121.79	187.08	184.92
	0.15	36.22	27.21	53.80	49.88	25.55	17.78	35.63	32.02	21.57	14.43	29.79	26.20
	0.25	15.51	9.15	18.45	14.90	11.45	6.02	12.35	9.15	9.97	5.02	10.60	7.59
	0.70	3.97	1.23	3.19	1.29	3.21	0.91	2.57	0.79	2.94	0.79	2.36	0.62
	1.00	2.77	0.70	2.25	0.51	2.33	0.50	2.05	0.28	2.17	0.38	1.99	0.23
	2.00	2.00	0.07	1.47	0.50	1.91	0.29	1.09	0.29	1.74	0.44	1.03	0.16
		$\lambda=0.1$ $L_1=5.1507$		$\lambda=0.25$ $L_1=4.9253$		$\lambda=0.1$ $L_1=5.1597$		$\lambda=0.25$ $L_1=4.8975$		$\lambda=0.1$ $L_1=5.1297$		$\lambda=0.25$ $L_1=4.8823$	
5	0.00	370.09	371.60	371.88	382.03	370.64	368.20	370.03	377.12	373.24	385.63	372.87	374.87
	0.03	266.66	263.24	306.73	316.26	237.85	236.72	274.99	275.57	220.15	221.39	263.89	269.19
	0.05	178.48	175.19	225.88	226.84	141.73	134.47	186.20	188.59	120.14	123.44	164.97	162.52
	0.15	33.72	27.63	46.12	43.73	23.92	18.02	30.46	27.93	20.65	14.85	24.88	22.39
	0.25	14.10	8.93	15.97	13.64	10.52	5.90	10.47	7.79	9.35	4.90	8.94	6.27
	0.70	4.22	1.00	3.50	0.99	3.68	0.74	2.97	0.72	3.47	0.64	2.78	0.65
	1.00	3.30	0.57	2.63	0.61	2.96	0.45	2.27	0.46	2.81	0.47	2.15	0.36
	2.00	2.05	0.21	1.96	0.20	2.00	0.03	1.67	0.47	2.00	0.00	1.42	0.49
		$\lambda=0.1$ $L_1=5.7518$		$\lambda=0.25$ $L_1=5.2452$		$\lambda=0.1$ $L_1=5.7528$		$\lambda=0.25$ $L_1=5.2456$		$\lambda=0.1$ $L_1=5.7528$		$\lambda=0.25$ $L_1=5.2456$	
7	0.00	371.15	377.35	371.49	381.91	373.98	370.03	371.81	380.57	371.55	377.93	372.94	385.56
	0.03	263.43	266.64	294.01	299.74	230.13	235.12	267.99	272.23	211.45	216.56	258.76	265.10
	0.05	172.37	169.98	211.76	216.19	136.53	132.40	180.19	186.54	116.15	118.62	160.39	162.10
	0.15	32.27	26.88	41.30	39.59	23.06	17.66	28.15	25.70	19.96	14.79	23.31	20.39
	0.25	13.84	8.84	14.61	11.90	10.36	5.60	10.09	7.25	9.18	4.58	8.84	5.74
	0.70	4.52	1.03	3.65	1.00	3.94	0.76	3.13	0.74	3.71	0.69	2.93	0.66
	1.00	3.52	0.62	2.78	0.61	3.13	0.44	2.37	0.51	2.99	0.38	2.22	0.42

	2.00	2.15	0.35	1.99	0.12	2.01	0.07	1.84	0.37	2.00	0.01	1.63	0.48

* *ARL= average run-length; * SDRL= standard deviation of the run-length*

Table 2: The *ARL*s and *SDRL*s of the proposed CEM-RSS control chart at $ARL_0 = 370$.

w	f	$n=4$				$n=5$				$n=6$			
		$\lambda=0.1$ $L_2=3.6175$		$\lambda=0.25$ $L_2=3.7301$		$\lambda=0.1$ $L_2=3.6195$		$\lambda=0.25$ $L_2=3.7495$		$\lambda=0.1$ $L_2=3.6199$		$\lambda=0.25$ $L_2=3.7495$	
		ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$
2	0.00	372.69	370.01	370.24	373.43	370.84	364.54	371.89	378.27	371.82	369.81	373.05	372.18
	0.03	267.32	258.94	297.86	295.69	226.83	215.42	282.28	279.53	198.40	193.17	255.58	249.58
	0.05	169.40	159.04	222.78	220.46	135.91	126.18	189.36	186.90	108.24	97.32	157.90	153.94
	0.15	31.75	23.79	45.47	41.92	23.00	15.65	31.03	26.92	17.69	11.21	22.31	18.99
	0.25	13.79	7.81	16.10	12.54	10.25	5.13	10.92	7.78	8.30	3.81	7.99	5.16
	0.70	3.69	1.11	2.94	1.07	3.02	0.83	2.42	0.66	2.59	0.64	2.16	0.41
	1.00	2.58	0.63	2.16	0.41	2.22	0.42	2.01	0.24	2.05	0.22	1.91	0.30
	2.00	1.99	0.12	1.32	0.47	1.80	0.40	1.04	0.19	1.32	0.47	1.00	0.04
		$\lambda=0.1$ $L_2=5.1295$		$\lambda=0.25$ $L_2=4.8795$		$\lambda=0.1$ $L_2=5.1395$		$\lambda=0.25$ $L_2=4.8899$		$\lambda=0.1$ $L_2=5.1395$		$\lambda=0.25$ $L_2=4.8899$	
5	0.00	370.31	369.67	372.64	374.94	370.64	373.93	370.69	380.78	371.03	370.04	372.28	377.24
	0.03	255.54	256.70	291.03	293.83	222.58	220.10	260.73	264.11	194.31	189.14	235.41	238.63
	0.05	166.83	163.92	206.98	208.65	126.96	121.57	170.86	174.24	102.87	97.04	139.89	140.10
	0.15	30.02	23.57	38.70	37.39	21.11	15.29	25.93	23.11	16.32	10.91	18.69	16.20
	0.25	12.59	7.67	13.51	10.97	9.50	5.01	9.14	6.34	7.71	3.60	7.01	4.26
	0.70	4.00	0.90	3.29	0.86	3.51	0.66	2.84	0.66	3.17	0.50	2.49	0.56
	1.00	3.17	0.50	2.49	0.56	2.83	0.46	2.17	0.39	2.51	0.51	2.04	0.19
	2.00	2.01	0.11	1.89	0.31	2.00	0.01	1.47	0.50	2.00	0.03	1.09	0.29
		$\lambda=0.1$ $L_2=5.7515$		$\lambda=0.25$ $L_2=5.2401$		$\lambda=0.1$ $L_2=5.7551$		$\lambda=0.25$ $L_2=5.2355$		$\lambda=0.1$ $L_2=5.7551$		$\lambda=0.25$ $L_2=5.2399$	
7	0.00	371.87	374.97	374.14	379.77	371.15	376.62	371.84	379.10	370.92	372.36	377.59	387.95
	0.03	247.89	250.76	289.33	291.03	218.81	220.89	258.24	265.15	185.10	184.87	229.96	234.02
	0.05	161.97	159.49	203.37	205.18	125.20	121.46	162.83	165.57	99.00	95.75	133.84	136.12
	0.15	28.82	22.90	36.18	34.81	20.70	15.50	24.28	22.00	15.53	10.55	17.30	14.64
	0.25	12.38	7.55	12.59	9.83	9.40	4.81	8.86	5.86	7.75	3.31	6.97	3.86
	0.70	4.31	0.91	3.45	0.89	3.75	0.71	2.96	0.67	3.36	0.55	2.62	0.58
	1.00	3.37	0.55	2.62	0.58	3.02	0.39	2.26	0.45	2.76	0.45	2.06	0.24

	2.00	2.06	0.24	1.97	0.18	2.00	0.03	1.68	0.47	2.00	0.00	1.20	0.40

Table 3: The *ARL*s and *SDRL*s of the proposed CEM-QRSS control chart at $ARL_0 = 370$.

w	f	$n=4$				$n=5$				$n=6$			
		$\lambda=0.1$ $L_3=3.6135$		$\lambda=0.25$ $L_3=3.7375$		$\lambda=0.1$ $L_3=3.6095$		$\lambda=0.25$ $L_3=3.7125$		$\lambda=0.1$ $L_3=3.6215$		$\lambda=0.25$ $L_3=3.7395$	
		ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$
2	0.00	371.51	369.84	370.19	369.68	371.24	364.10	370.71	372.92	370.92	360.21	370.47	371.72
	0.03	272.04	264.55	318.93	310.94	214.60	210.34	261.92	260.75	190.81	182.72	238.66	235.06
	0.05	183.83	174.46	235.70	233.46	120.83	110.54	167.03	166.37	99.09	89.22	144.87	142.47
	0.15	36.00	27.70	51.88	48.28	20.19	13.05	25.66	21.98	16.09	9.68	19.55	15.83
	0.25	15.37	9.22	18.66	15.33	9.31	4.40	9.26	6.34	7.66	3.35	7.34	4.54
	0.70	3.95	1.23	3.20	1.27	2.79	0.72	2.27	0.53	2.47	0.58	2.10	0.34
	1.00	2.77	0.71	2.26	0.52	2.12	0.32	1.95	0.26	2.03	0.16	1.85	0.36
	2.00	2.00	0.07	1.47	0.50	1.60	0.49	1.01	0.09	1.17	0.38	1.00	0.00
		$\lambda=0.1$ $L_3=5.1295$		$\lambda=0.25$ $L_3=4.9175$		$\lambda=0.1$ $L_3=5.1055$		$\lambda=0.25$ $L_3=4.8399$		$\lambda=0.1$ $L_3=5.1395$		$\lambda=0.25$ $L_3=4.8811$	
5	0.00	371.70	373.40	370.32	378.39	370.35	371.69	370.20	364.23	370.29	372.11	370.39	376.67
	0.03	266.83	274.41	304.92	316.14	209.24	203.30	245.74	247.27	181.58	177.06	223.69	225.51
	0.05	178.46	175.27	229.70	231.69	115.53	109.92	149.32	152.19	93.53	88.29	129.02	132.30
	0.15	33.35	27.20	45.77	44.64	18.40	12.89	21.42	18.62	14.87	9.54	16.41	13.73
	0.25	14.19	9.04	15.68	13.13	8.49	4.31	8.05	5.28	7.14	3.16	6.45	3.67
	0.70	4.22	1.00	3.49	0.99	3.32	0.57	2.65	0.62	3.06	0.46	2.39	0.53
	1.00	3.31	0.57	2.65	0.60	2.68	0.49	2.08	0.28	2.38	0.49	2.01	0.13
	2.00	2.04	0.21	1.96	0.20	2.00	0.01	1.23	0.42	2.00	0.07	1.03	0.17
		$\lambda=0.1$ $L_3=5.7505$		$\lambda=0.25$ $L_3=5.2315$		$\lambda=0.1$ $L_3=5.7351$		$\lambda=0.25$ $L_3=5.2014$		$\lambda=0.1$ $L_3=5.7491$		$\lambda=0.25$ $L_3=5.2315$	
7	0.00	373.98	383.21	371.25	380.89	373.69	377.90	371.16	376.34	371.88	377.37	371.55	380.94
	0.03	258.93	260.36	293.11	296.01	205.54	203.58	235.71	248.78	175.31	180.42	214.03	219.41
	0.05	170.03	170.90	214.01	218.77	113.01	109.35	144.36	147.94	92.34	89.16	120.05	124.66
	0.15	32.22	26.92	41.20	39.74	18.00	12.87	20.07	17.63	14.42	9.39	15.16	12.39

	0.25	13.69	8.71	14.45	11.92	8.53	3.93	7.86	4.78	7.24	2.86	6.47	3.37
	0.70	4.56	1.02	3.66	1.00	3.56	0.63	2.78	0.62	3.25	0.50	2.50	0.55
	1.00	3.53	0.62	2.77	0.61	2.90	0.39	2.14	0.35	2.63	0.49	2.03	0.17
	2.00	2.16	0.36	1.98	0.13	2.00	0.00	1.42	0.49	2.00	0.00	1.09	0.29

Table 4: The *ARL*s and *SDRL*s of the proposed CEM-MRSS control chart at $ARL_0 = 370$.

w	f	$n=4$				$n=5$				$n=6$			
		$\lambda=0.1$ $L_4=3.6102$		$\lambda=0.25$ 3.7312		$\lambda=0.1$ $L_4=3.6101$		$\lambda=0.25$ $L_4=3.7235$		$\lambda=0.1$ $L_4=3.6179$		$\lambda=0.25$ $L_4=3.7453$	
		ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$
2	0.00	371.12	368.16	371.50	372.32	370.86	367.17	371.65	372.65	373.20	361.82	372.66	373.44
	0.03	244.72	239.34	293.05	290.59	209.67	200.82	261.34	256.79	175.10	169.42	231.93	229.22
	0.05	153.86	145.62	207.49	205.52	116.32	106.97	162.72	160.83	90.63	82.93	134.10	130.38
	0.15	27.80	20.13	38.65	35.40	18.85	12.06	24.59	20.73	14.61	8.45	17.40	13.75
	0.25	12.20	6.59	13.61	10.36	8.86	4.10	8.75	5.79	7.03	2.97	6.49	3.83
	0.70	3.39	0.98	2.68	0.87	2.71	0.69	2.23	0.49	2.33	0.51	2.05	0.29
	1.00	2.40	0.55	2.08	0.32	2.09	0.28	1.94	0.26	2.01	0.10	1.77	0.42
	2.00	1.95	0.22	1.15	0.35	1.49	0.50	1.00	0.05	1.06	0.23	1.00	0.00
		$\lambda=0.1$ $L_4=5.1189$		$\lambda=0.25$ $L_4=4.8843$		$\lambda=0.1$ $L_4=5.1299$		$\lambda=0.25$ $L_4=4.8653$		$\lambda=0.1$ $L_4=5.1753$		$\lambda=0.25$ $L_4=4.8995$	
5	0.00	371.01	374.96	371.65	378.38	372.69	372.78	371.38	377.38	370.38	377.15	371.22	373.82
	0.03	241.64	239.73	280.58	285.92	204.37	201.08	243.34	252.62	170.87	169.01	219.04	219.81
	0.05	148.08	143.85	196.74	199.18	112.06	106.67	147.92	151.43	86.82	79.93	118.88	118.47
	0.15	25.82	19.56	33.20	30.72	17.48	12.08	20.03	17.15	13.48	8.28	14.62	11.99
	0.25	11.06	6.36	11.61	8.95	8.22	4.04	7.65	5.00	6.68	2.80	5.87	3.17
	0.70	3.78	0.79	3.08	0.76	3.27	0.55	2.58	0.59	2.97	0.45	2.28	0.46
	1.00	3.03	0.45	2.35	0.50	2.61	0.50	2.06	0.24	2.26	0.44	2.00	0.11
	2.00	2.00	0.04	1.77	0.42	2.00	0.01	1.17	0.37	1.98	0.15	1.01	0.08
		$\lambda=0.1$ $L_4=5.7183$		$\lambda=0.25$ $L_4=5.2113$		$\lambda=0.1$ $L_4=5.7356$		$\lambda=0.25$ $L_4=5.2016$		$\lambda=0.1$ $L_4=5.7553$		$\lambda=0.25$ $L_4=5.2453$	
7	0.00	370.98	377.87	370.36	378.62	373.50	373.93	370.92	378.01	371.92	379.51	372.41	379.20

	0.03	235.86	236.37	273.81	278.92	199.89	199.08	233.53	236.54	166.59	168.45	207.25	213.44
	0.05	141.26	138.49	185.39	184.59	109.55	105.20	140.95	141.54	83.87	77.88	110.02	111.32
	0.15	24.81	19.55	30.29	28.28	16.91	11.51	18.67	15.88	12.88	8.15	13.40	10.64
	0.25	10.92	6.14	10.66	7.81	8.16	3.67	7.43	4.46	6.79	2.53	5.94	2.83
	0.70	4.06	0.82	3.22	0.76	3.48	0.60	2.70	0.60	3.14	0.43	2.38	0.51
	1.00	3.20	0.47	2.44	0.53	2.85	0.41	2.10	0.30	2.48	0.50	2.01	0.11
	2.00	2.01	0.11	1.89	0.31	2.00	0.01	1.32	0.47	2.00	0.03	1.02	0.15

Table 5: Comparative analysis

f	Existing Control Charts								Proposed CEM-RBS Control Charts							
	MA		EWMA		EWMA-MA		EWMA-RSS		CEM-ERSS		CEM-RSS		CEM-QRSS		CEM-MRSS	
	$n=5$ $w=5$ $L=2.8781$		$n=5$ $\lambda=0.25$ $L=2.8981$		$n=5$ $w=5$ $\lambda=0.25$ $L=4.8781$		$n=5$ $\lambda=0.25$ $L=2.9075$		$n=5$ $w=5$ $\lambda=0.25$ $L_1=4.8975$		$n=5$ $w=5$ $\lambda=0.25$ $L_2=4.8899$		$n=5$ $w=5$ $\lambda=0.25$ $L_3=4.8399$		$n=5$ $w=5$ $\lambda=0.25$ $L_4=4.8653$	
	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$	ARL_1	$SDRL_1$
0.00	370.53	368.72	370.43	364.38	370.70	376.76	370.15	368.76	370.03	377.12	370.69	380.78	370.20	364.23	371.38	377.38
0.03	338.95	337.19	333.17	329.96	325.37	329.34	288.03	287.00	274.99	275.57	260.73	264.11	245.74	247.27	243.34	252.62
0.05	296.79	296.86	281.44	277.55	261.82	264.06	195.30	190.56	186.20	188.59	170.86	174.24	149.32	152.19	147.92	151.43
0.10	181.62	181.06	155.25	154.14	134.58	134.94	72.56	67.32	67.47	66.68	58.45	57.32	48.60	48.18	47.42	44.72
0.15	107.54	106.70	84.98	80.35	71.59	70.34	32.82	28.72	30.46	27.93	25.93	23.11	21.42	18.62	20.03	17.15
0.20	64.43	62.83	51.25	46.49	40.79	38.80	18.43	14.44	16.41	13.60	14.34	11.75	12.15	9.46	11.33	8.64
0.25	40.88	38.78	32.86	28.61	25.77	23.51	11.88	8.16	10.47	7.79	9.14	6.34	8.05	5.28	7.65	5.00
0.30	27.39	25.15	22.73	18.67	17.67	15.09	8.40	5.04	7.59	4.84	6.85	4.13	5.93	3.19	5.74	2.96
0.40	14.15	12.00	12.68	8.80	9.89	7.20	5.24	2.54	5.00	2.30	4.62	1.89	4.20	1.51	4.10	1.43
0.50	8.56	6.71	8.35	5.09	6.85	4.16	3.82	1.55	3.94	1.31	3.71	1.12	3.43	0.94	3.35	0.91
0.60	5.76	4.08	6.16	3.28	5.25	2.50	3.02	1.07	3.36	0.89	3.19	0.81	2.97	0.72	2.91	0.70
0.70	4.23	2.68	4.85	2.27	4.37	1.67	2.54	0.80	2.97	0.72	2.84	0.66	2.65	0.62	2.58	0.59

0.80	3.35	1.96	4.02	1.68	3.83	1.21	2.22	0.63	2.68	0.63	2.55	0.58	2.37	0.51	2.34	0.50
0.90	2.74	1.48	3.42	1.31	3.45	0.96	1.98	0.53	2.46	0.55	2.33	0.49	2.19	0.40	2.16	0.37
1.00	2.32	1.18	3.02	1.08	3.17	0.81	1.80	0.50	2.27	0.46	2.17	0.39	2.08	0.28	2.06	0.24
2.00	1.05	0.23	1.46	0.50	2.02	0.16	1.00	0.03	1.67	0.47	1.47	0.50	1.23	0.42	1.17	0.37
$(f_{\min}, f_{\max}) = (0.01, 2.00)$																
EARL	70.01		65.34		59.87		40.45		39.03		35.12		32.53		29,74	

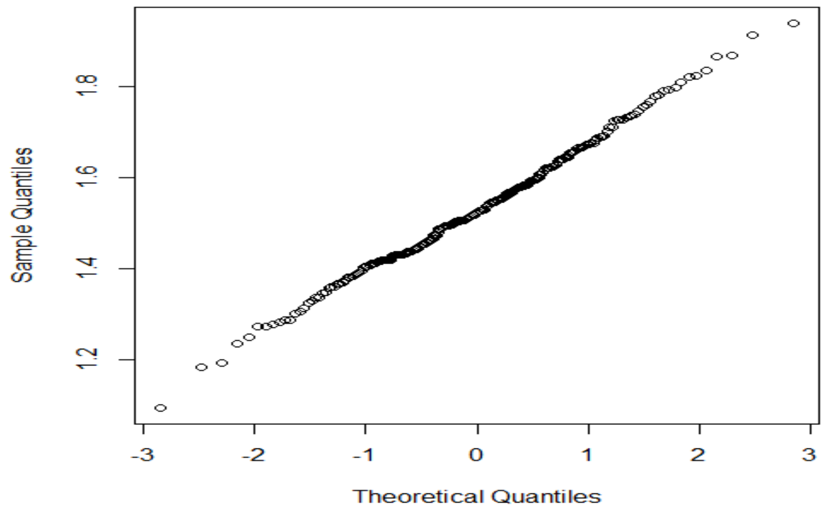

Figure 1: Q-Q Plot of the flow width measurements (microns) in the hard-bake process.

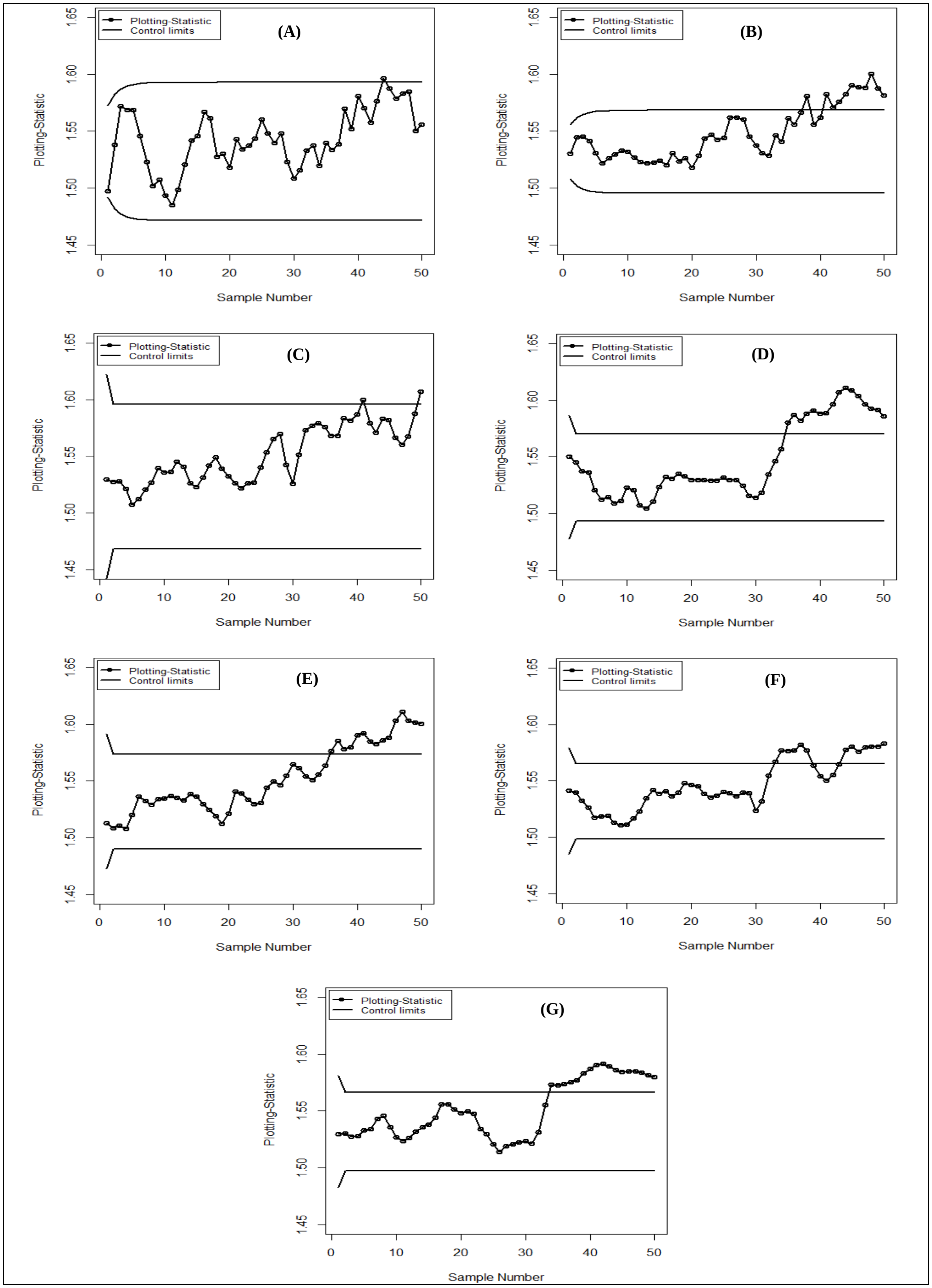

Figure 2: The performance comparison of shift detection ability between **(A)** EWMA-SRS, **(B)** EWMA-RSS, **(C)** EWMA-MA, **(D)** CEM-RSS, **(E)** CEM-ERSS, **(F)** CEM-MRSS and **(G)** CEM-QRSS charts

Biographies

Muhammad Tayyab received the master's degree in Statistics from College of Statistical and Actuarial Sciences, University of the Punjab, Lahore, M. Phil Statistics from Allama Iqbal Open University, Islamabad and philosophy of doctorate degree in Statistics from National College of Business Administration & Economics (NCBA&E), Lahore, Pakistan. He is currently working as Senior Subject Specialist (Statistics) in department of Education. His research areas include mathematical statistics, sampling techniques and control charting structures. He has been serving as a reviewer for many highly-respected journals.

Wasif Yasin is a Ph.D. student at the National College of Business Administration & Economics (NCBA&E), Lahore, Pakistan. He did his M.Phil and M.Sc. degree from the University of the Punjab, Lahore. He is currently working as Secretary Regional Transport Authority, Sahiwal. His research interests include sampling techniques and Statistical Process Control.

Asma Arshad completed her Ph.D. in statistics from National College of Business Administration & Economics, Lahore, Pakistan. She did her M. Phil in Statistics from the Department of Statistics, Govt. College University, Lahore, Pakistan. She is presently serving as Assistant Professor at Higher Education Department, Punjab, Pakistan. Her research interests are survey sampling, sampling techniques, control chart techniques, acceptance sampling plans and decision trees.

Muhammad Noor-ul-Amin did his MSc in Statistics from University of the Punjab, Lahore, Pakistan. He received his PhD degree from National College of Business Administration and Economics, (NCBA & E), Lahore, Pakistan. He is currently working as Associate Professor at COMSATS University Islamabad, Lahore Campus. He has supervised many M.Phil and PhD dissertations. His research interests include Sampling methods and Statistical Quality Control. He is author of many papers published in international journals.

Muhammad Hanif completed his Masters in Statistics from New South Wales University, Australia with major in Multistage Cluster Sampling. He completed his Ph.D. in Statistics from University of the Punjab, Lahore, Pakistan. He has more than forty years research experience. He is an author of more than 300 research articles and ten books. He has served as Professor of Statistics in various parts of the world viz Australia, Libya, Pakistan, and Saudi Arabia. He supervised more than 30 M. Phil. thesis and more than 20 Ph.D. dissertations. His areas of interest are Survey Sampling and Control Charts.

He is presently working as a professor of Statistics and Vice Rector (Research) at National College of Business Administration and Economics, Lahore, Pakistan.