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Impact of measurement error on maximum hybrid exponentially weighted moving average control chart

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Abstract

Statistical process control provides various types of control charts for monitoring mean and variance shifts in the industrial production process individually as well as jointly to improve and maintain the quality of products. Authors proposed control charts based on sample values selected to calculate the desired statistics, assuming that these values are measured correctly. But in a real-life situation, measurements of the values may suffer from errors, ultimately affecting the efficiency of control charts. A few of the researchers in the field of control charts also discussed the problem of measurement error during process monitoring and proposed solutions to avoid losses for producers. We also present a Hybrid Exponentially Weighted Moving Average (HEWMA) control chart for joint monitoring of mean and variance, with the effect of measurement error on the efficiency of this control chart, and name it the Maximum Hybrid Exponentially Weighted Moving Average with Measurement Error (Max-HEWMAME) control chart. The impact of measurement error has been shown in the calculations and presented in the form of Average Run Lengths (ARLs) and Standard Deviations of Run Lengths (SDRLs) using the Monte Carlo simulation method. A real-life example is also included to support the simulation results.

1. Introduction

Production of quality products and services has been more emphasized since the concept of ISO-9000 has been implemented not only in industrial production but also in the services sector. The industrial sector is not only manufacturing quality products, but industrialists are also planning to enhance and maintain the quality of their production. Statistical process control provides techniques to support industrial production for enhancement and maintenance of its quality. Control charts have been proven an effective technique to maintain the quality of a product through detection of any fault causing a defective production process. The concept of control charts is almost one century old, when Shewhart considered that checking the quality of a specific product on a sample basis is a statistical phenomenon. He developed independent an separate control charts for shifts in the mean and variance of the industrial production process. Those control charts were based on the information from the last sample, and past information of the production process was not taken into the calculations. However, researchers started utilizing past information for the development of control charts, e.g., Cumulative Sum (CUSUM) and Exponentially Weighted Moving Average (EWMA) control charts, which were proposed by Page [1] and Roberts [2], respectively. As more information was used for the development of these latest charts, therefore CUSUM and EWMA charts became more sensitive for early detection of small and moderate process shifts as compared to those of classical control charts by Shewhart. Researchers are developing memory type EWMA and CUSUM charts for

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monitoring of process shift in mean or variance or for joint monitoring of mean as well as variance shifts during the production process. In addition to many others, Lucas and Saccucci [3], Chen et al. [4], Zhang and Chen [5], Khoo et al. [6], Abbas et al. [7], Abujiya et al. [8], Haq [9], Ajadi and Riaz [10], Noor-ul-Amin et al. [11], Hussain et al. [12], Tayyab et al. [13], Awais and Haq [14], and Raza et al. [15] also developed memory based control charts for monitoring mean and/or variance shifts in the production process. However, measurement error is not considered by the authors when they developed efficient control charts for joint monitoring of mean and variance shifts during the production process.

Measurement error usually refers to the amount or value of the quality characteristic of a production process that is collected or observed by the experimenter or quality expert, however, this amount or value is randomly different from the actual one which is required to arrive at the right decision. In the real world, there are chances of these differences between the observed or collected and the actual real values or amounts of our interest which refer to the measurement error. In the application of control charts, measurement error has a significant effect on the efficiency of these charts. To study this issue, the additive measurement error model was used by Bennet [16] i.e., $Y = X + \varepsilon$, where X is the actual value of the variable of interest Y and ε is the random error due to incorrect measurements of the variable. The same type of model was also utilized by Mittag and Stemann [17] to study the effect of contamination of gauge imprecision on the control charts. Montgomery and Runger [18], Sanders [19], Linna and Woodall [20], Maravelakis et al. [21], Huwang and Hung [22], Li and Huang [23], and Wu [24] studied the effect of measurement error on mean and/or variance control charts like $\bar{X}$, S, CUSUM, EWMA, and Maximum Hybrid Exponentially Weighted Moving Average with Measurement Error (Max-HEWMAME) etc. For more literature to study the impact of measurement error, interested readers can refer to [25-38] given in the references.

The CUSUM, EWMA, and Max-EWMA charts have been studied for joint monitoring of mean as well as variance shifts in the production process and contamination of measurement error has also been analyzed by various authors. However, some work has been observed on hybrid control charts which are more sensitive as compared with non-hybrid control charts. Authors like Haq [39], Azam et al. [40], Haq [41], Noor-ul-Amin et al. [42], Javaid et al. [43], and Noor-ul-Amin et al. [44] analyzed and proved that hybrid control charts are more sensitive than those which are not hybrid. However, the sensitivity of these charts is affected if measurements of quality characteristics are not precise. Montgomery [45] highlighted that inadequate measurement can cause losses of time and money resulting in defective products and making unhappy customers. Measurement variability adds to the overall variability of the production process and hurts the quality of the product. In the light of the importance of measurement accuracy to ensure the quality of product, we study the effect of measurement error on the Hybrid Exponentially Weighted Moving Average (HEWMA) control chart and call it Max-HEWMAME control chart for this research endeavor.

The sequence of this article is that after introduction, it comprises five more sections. Section 2 comprises of proposed Max-HEWMA control chart with measurement error and named it Max-HEWMAME control chart. Performance of Max-HEWMAME chart is highlighted in Section 3 and all the tables presented in this article for this control chart are generated through simulations using the Monte Carlo method, showing Average Run Lengths (ARLs) and Standard Deviations of Run Lengths (SDRLs), to study the impact of measurement error. The main findings are highlighted in Section 4, while Section 5 elaborates real life example to study the impact of measurement error. The conclusion is drawn in Section 6 while references are given at the end of the article.

2. Proposed Max-HEWMAME

EWMA control charts are very common for joint monitoring of mean and/or variance for a production process. These charts are very famous to detect small shifts in the parameters. If Y is a variable of interest and follows the normal distribution with mean μ and variance σ^2, then the EWMA statistic for individual observation or sample is defined as:

$$Z_t = \lambda \bar{Y} + (1-\lambda) Z_{t-1}, \tag{1}$$

where $\bar{Y}$ is the sample mean, λ is smoothing constant which is a rate assigned to past observations and is a weight for current information, while it lies between $0 < \lambda \leq 1$, whereas $Z_0 = 0$ for the first sample i.e., $t=1$. The efficiency of the EWMA chart can be increased by making it hybrid. For an in-control production process, the HEWMA statistic for joint monitoring of process mean and variance can be written as:

$$E_t = \lambda_2 \bar{Y} + (1-\lambda_2) E_{t-1}, \tag{2}$$

$$HE_t = \lambda_1 E_t + (1-\lambda_1) HE_{t-1}, \tag{3}$$

where $0 < \lambda_1$ and $\lambda_2 \leq 1$, $E_0 = HE_0 = 0$, $E\ (HE_t) = 0$. Variance for this hybrid statistic was derived by Haq [41] as:

$$Var(HE_t) = \frac{\lambda_1^2 \lambda_2^2}{(\lambda_2 - \lambda_1)^2} \sigma^2 \left[\begin{array}{l} \dfrac{(1-\lambda_1)^2 \left\{1-(1-\lambda_1)^{2t}\right\}}{1-(1-\lambda_1)^2} + \dfrac{(1-\lambda_2)^2 \left\{1-(1-\lambda_2)^{2t}\right\}}{1-(1-\lambda_2)^2} \\ -\dfrac{2(1-\lambda_1)(1-\lambda_2)\left\{1-(1-\lambda_1)^t (1-\lambda_2)^t\right\}}{1-(1-\lambda_1)(1-\lambda_2)} \end{array} \right]. \tag{4}$$

However, the efficiency of the HEWMA chart for joint monitoring is affected, if the quality characteristic is not precisely measured due to any reason and is suffering from measurement error. For an in control production process, if quality characteristic X follows the normal distribution with mean μ and variance σ_p^2 but is not properly measured and we are unable to get its true value. However, we can get another related value Y having covariance with X, and can be explained in linear covariate model as:

$$Y = A + BX + \varepsilon, \tag{5}$$

where A and B are constants while error term $\varepsilon \sim N(\mu_x, \sigma_m^2)$. We can say that covariate Y also follows the normal distribution as $Y \sim N\left(\mu_y, \sigma_y^2\right)$, where $\mu_y = A + B\mu$ and $\sigma_y^2 = B^2\sigma_p^2 + \sigma_m^2$ are known mean and variance respectively. The process variance also includes the variance of the error component i.e., σ_m^2 to constitute the overall variance.

It is assumed that $X \sim N(\mu, \sigma_p^2)$ for in control process. If a shift occurs in mean of the production process from μ to $\mu + a\sigma_p$, then the mean of covariate Y will become $A + B(\mu + a\sigma_p)$. However, if a shift occurs in process variance from σ_p^2 to $b^2\sigma_p^2$, where $b>0$, then the variance of covariate Y will be $(Bb\sigma_p)^2 + \sigma_m^2$. For in control production process $a=0$ and $b=1$.

Keeping in view the covariate model of Eq. (5) for Y, if we select jth sample of size n, then the transformed statistics for mean and variance can be written as:

$$M_j = \frac{\bar{Y} - (A + B\mu)}{\sqrt{B^2\sigma_p^2 + \sigma_m^2 \big/ n}}, \tag{6}$$

and

$$V_j = \phi^{-1}\left[G\left\{\frac{(n-1)S^2}{B^2\sigma_p^2 + \sigma_m^2}, (n-1)\right\}\right], \tag{7}$$

where $\bar{Y} = \frac{\sum_{j=1}^{n} Y_j}{n}$, and $S^2 = \frac{\sum_{j=1}^{n}(Y_j - \bar{Y})^2}{n-1}$, while $G(\omega, \upsilon)$ follows chi-square distribution having υ as degrees of freedom, and ϕ^{-1} is the inverse of standard normal distribution function. As Y follows the normal distribution, therefore, M_j and V_j also follow the normal distribution.

Using transformed statistics for mean in Eq. (6) and variance in Eq. (7), we propose Max-HEWMAME for joint monitoring of mean and variance for the jth sample. First, we write HEWMAME statistic for mean as:

$$E_{mj} = \lambda_2 M_j + (1 - \lambda_2)E_{m(j-1)}, \tag{8}$$

$$HE_{mj} = \lambda_1 E_{mj} + (1 - \lambda_1)HE_{m(j-1)}, \tag{9}$$

and then the HEWMAME statistic for variance is given as:

$$E_{vj} = \lambda_2 V_j + (1 - \lambda_2)E_{v(j-1)}, \tag{10}$$

$$HE_{vj} = \lambda_1 E_{vj} + (1 - \lambda_1)HE_{v(j-1)}, \tag{11}$$

where, values of above proposed four statistics for the first sample when $j=1$ are zero i.e., $E_{m(0)} = E_{v(0)} = HE_{m(0)} = HE_{v(0)} = 0$ and smoothing constants are $0 < \lambda_1 \,\&\, \lambda_2 \le 1$. Both the HEWMAME statistics for mean and variance also follow the normal distribution as M_j and V_j follow the normal distribution for in control process. Mean and variance for HEWMAME statistics HE_{mj} and HE_{vj} are:

$E(HE_j) = 0$, and

$$Var(HE_j) = \frac{\lambda_1^2\lambda_2^2}{(\lambda_2 - \lambda_1)^2}\sigma^2 \left[\begin{array}{l} \dfrac{(1-\lambda_1)^2\left\{1-(1-\lambda_1)^{2j}\right\}}{1-(1-\lambda_1)^2} + \dfrac{(1-\lambda_2)^2\left\{1-(1-\lambda_2)^{2j}\right\}}{1-(1-\lambda_2)^2} \\ -\dfrac{2(1-\lambda_1)(1-\lambda_2)\left\{1-(1-\lambda_1)^j(1-\lambda_2)^j\right\}}{1-(1-\lambda_1)(1-\lambda_2)} \end{array}\right]. \tag{12}$$

For joint monitoring of mean and variance shifts with a single hybrid control chart rather than use separate hybrid charts for mean and variance as written in Eqs. (9) and (11), we utilize the concept of maximum out of absolute values of both the hybrid statistics for mean and variance. Therefore, our proposed Max-HEWMAME chart is:

$$Max - HEWMAME = \max\left[\left|HE_{mj}\right|, \left|HE_{vj}\right|\right]. \tag{13}$$

To observe whether the production process is going out of control or not, we plot the Max-HEWMAME statistic on the control limits. As statistic is the absolute value, so Upper Control Limit (UCL) is sufficient to observe the process control. The UCL for Max-HEWMAME control chart for jth sample can be written as:

$$UCL_j = (1.128379 + 0.602810 \times L)\sqrt{Var(HE_j)}, \tag{14}$$

where L is the width of UCL and is determined to achieve the desired ARL_0 for in control production process of a quality characteristic.

To study the impact of measurement error on the efficiency of the Max-HEWMAME chart, We prepared four major tables (Tables 1–4), using covariate model with different levels of measurement error showing ARLs and SDRLs. Further to reduce the effect of measurement error, strategy of taking multiple measurements of the same sample is also used in Section 3.

3. Performance of Max-HEWMAME chart

Tables 1–4 are generated by simulations using Monte Carlo method in R-Language, in order to study the impact of measurement error on the performance of proposed control chart.

Table 1. ARLs and SDRLs for Max-HEWMAME chart with different values of $\sigma_m^2 \div \sigma^2$.

b	a	$\sigma_m^2 \div \sigma^2$											
		No error		0.1		0.2		0.3		0.5		1	
		ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
0.25	0.00	1.11	0.31	1.16	0.37	1.32	0.51	1.61	0.71	2.63	1.45	9.30	6.71
	0.25	1.11	0.31	1.16	0.37	1.32	0.51	1.61	0.72	2.61	1.46	8.43	6.09
	0.50	1.11	0.31	1.16	0.37	1.32	0.50	1.59	0.69	2.42	1.25	5.63	3.86
	0.75	1.10	0.29	1.13	0.34	1.26	0.45	1.44	0.56	1.89	0.88	3.49	2.25
	1.00	1.01	0.11	1.03	0.16	1.07	0.26	1.15	0.36	1.39	0.56	2.26	1.33
	1.25	1.00	0.00	1.00	0.02	1.00	0.04	1.01	0.12	1.11	0.32	1.65	0.84
	1.50	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.01	1.02	0.13	1.31	0.56
	1.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.03	1.13	0.36
	2.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.05	0.22
	2.25	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.01	0.11
	2.50	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.06
	2.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.02
	3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.01
0.50	0.00	2.64	1.46	2.73	1.51	2.96	1.70	3.43	2.09	5.04	3.32	15.07	11.68
	0.25	2.63	1.46	2.69	1.50	2.95	1.69	3.37	2.03	4.79	3.09	12.35	9.43
	0.50	2.30	1.15	2.37	1.22	2.51	1.32	2.78	1.50	3.53	2.12	6.69	4.93
	0.75	1.70	0.74	1.72	0.77	1.79	0.82	1.89	0.91	2.24	1.20	3.72	2.58
	1.00	1.23	0.44	1.24	0.45	1.28	0.49	1.33	0.53	1.53	0.72	2.40	1.51
	1.25	1.04	0.19	1.04	0.20	1.05	0.23	1.09	0.29	1.18	0.41	1.71	0.93
	1.50	1.00	0.05	1.00	0.05	1.01	0.07	1.01	0.10	1.04	0.21	1.35	0.61
	1.75	1.00	0.00	1.00	0.01	1.00	0.01	1.00	0.03	1.01	0.08	1.16	0.41
	2.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.02	1.07	0.26
	2.25	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.02	0.15
	2.50	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.07
	2.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.04
	3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.02
0.75	0.00	10.78	7.98	11.04	8.19	11.83	8.83	12.99	9.86	17.26	13.49	40.20	35.50
	0.25	8.19	5.97	8.36	6.14	8.71	6.42	9.34	6.95	11.58	8.89	20.68	17.60
	0.50	3.88	2.59	3.95	2.65	4.07	2.76	4.29	2.99	4.98	3.64	7.87	6.38
	0.75	2.13	1.24	2.15	1.27	2.19	1.30	2.29	1.40	2.61	1.68	4.00	3.01
	1.00	1.42	0.66	1.42	0.66	1.47	0.70	1.50	0.75	1.71	0.94	2.50	1.70
	1.25	1.14	0.37	1.14	0.37	1.16	0.39	1.19	0.44	1.27	0.54	1.78	1.05
	1.50	1.03	0.18	1.03	0.18	1.04	0.20	1.05	0.23	1.10	0.31	1.41	0.69
	1.75	1.00	0.07	1.01	0.08	1.01	0.07	1.01	0.10	1.03	0.16	1.21	0.47
	2.00	1.00	0.02	1.00	0.02	1.00	0.04	1.00	0.04	1.00	0.07	1.09	0.30
	2.25	1.00	0.00	1.00	0.00	1.00	0.01	1.00	0.01	1.00	0.02	1.03	0.18
	2.50	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.01	0.10
	2.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.06
	3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.04
0.90	0.00	50.36	45.52	51.80	48.13	53.70	48.83	57.35	52.62	70.15	68.77	134.9	149.8
	0.25	13.76	11.69	13.98	11.82	14.25	12.17	14.91	12.70	16.68	14.74	25.24	23.06
	0.50	4.50	3.42	4.57	3.53	4.62	3.62	4.87	3.82	5.47	4.36	8.37	7.10
	0.75	2.31	1.53	2.34	1.57	2.36	1.60	2.49	1.73	2.79	1.97	4.16	3.31
	1.00	1.52	0.81	1.52	0.80	1.56	0.86	1.60	0.89	1.77	1.06	2.56	1.81
	1.25	1.20	0.45	1.20	0.47	1.21	0.48	1.25	0.52	1.34	0.63	1.84	1.14
	1.50	1.06	0.24	1.07	0.26	1.07	0.27	1.09	0.30	1.14	0.39	1.45	0.75
	1.75	1.02	0.14	1.02	0.13	1.02	0.13	1.03	0.16	1.05	0.22	1.22	0.49
	2.00	1.00	0.06	1.00	0.06	1.00	0.07	1.00	0.07	1.01	0.11	1.11	0.33
	2.25	1.00	0.02	1.00	0.01	1.00	0.02	1.00	0.03	1.00	0.06	1.05	0.22
	2.50	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.02	1.02	0.14
	2.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.01	0.08
	3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.05
1.00	0.00	370.4	510.4	369.8	501.1	368.7	499.2	371.9	508.2	369.0	511.5	371.1	497.8
	0.25	14.68	13.64	14.86	13.89	15.20	14.25	16.08	14.81	17.59	16.58	26.29	25.54
	0.50	4.64	3.92	4.71	3.91	4.76	4.04	5.04	4.26	5.63	4.87	8.38	7.42
	0.75	2.41	1.74	2.43	1.74	2.49	1.80	2.59	1.86	2.83	2.13	4.18	3.39
	1.00	1.63	0.91	1.64	0.92	1.65	0.96	1.69	1.00	1.86	1.16	2.66	1.95
	1.25	1.29	0.52	1.29	0.52	1.30	0.54	1.33	0.59	1.43	0.70	1.89	1.22
	1.50	1.13	0.29	1.13	0.30	1.14	0.32	1.15	0.35	1.19	0.43	1.50	0.81
	1.75	1.05	0.16	1.05	0.17	1.05	0.18	1.05	0.18	1.09	0.25	1.27	0.54
	2.00	1.02	0.08	1.02	0.08	1.02	0.09	1.02	0.09	1.03	0.14	1.15	0.37
	2.25	1.00	0.03	1.00	0.03	1.00	0.03	1.00	0.05	1.01	0.07	1.08	0.23
	2.50	1.00	0.01	1.00	0.01	1.00	0.01	1.00	0.01	1.00	0.04	1.03	0.16
	2.75	1.00	0.01	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.02	1.01	0.08
	3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.05
1.10	0.00	41.90	48.09	42.90	49.89	43.65	49.90	47.41	55.32	57.24	67.63	107.1	132.9
	0.25	12.86	13.06	12.94	13.16	12.99	13.36	13.71	14.09	15.68	16.12	23.32	23.78
	0.50	4.54	4.11	4.55	4.12	4.65	4.25	4.95	4.53	5.40	4.93	8.08	7.60
	0.75	2.40	1.85	2.41	1.87	2.48	1.94	2.57	2.00	2.80	2.25	4.15	3.57
	1.00	1.63	0.99	1.64	1.01	1.65	1.03	1.68	1.08	1.84	1.22	2.65	2.01
	1.25	1.29	0.57	1.29	0.59	1.30	0.61	1.32	0.63	1.41	0.74	1.89	1.28
	1.50	1.13	0.34	1.13	0.35	1.13	0.39	1.13	0.38	1.19	0.47	1.49	0.84

Table 1. ARLs and SDRLs for Max-HEWMAME chart with different values of $\sigma_m^2 \div \sigma^2$ (continued).

b	**a**	$\sigma_m^2 \div \sigma^2$											
		No error		**0.1**		**0.2**		**0.3**		**0.5**		**1**	
		ARL	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**
	1.75	1.05	0.19	1.05	0.20	1.05	0.21	1.05	0.23	1.08	0.29	1.26	0.56
	2.00	1.02	0.11	1.02	0.10	1.02	0.11	1.02	0.13	1.03	0.18	1.14	0.39
1.10	2.25	1.00	0.05	1.00	0.05	1.00	0.06	1.00	0.06	1.01	0.10	1.07	0.26
	2.50	1.00	0.02	1.00	0.02	1.00	0.02	1.00	0.03	1.00	0.05	1.03	0.17
	2.75	1.00	0.00	1.00	0.01	1.00	0.00	1.00	0.01	1.00	0.02	1.01	0.11
	3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.01	1.00	0.06
	0.00	10.18	11.40	10.22	11.51	10.68	12.16	11.54	12.99	14.10	15.80	27.86	31.38
	0.25	7.15	7.76	7.19	7.74	7.60	8.32	7.80	8.41	9.23	10.11	16.03	17.51
	0.50	3.80	3.66	3.82	3.73	3.94	3.86	4.10	4.00	4.74	4.66	7.41	7.33
	0.75	2.31	1.90	2.32	1.94	2.35	1.96	2.44	2.02	2.74	2.34	4.08	3.72
	1.00	1.63	1.07	1.63	1.07	1.65	1.11	1.68	1.17	1.83	1.33	2.63	2.15
	1.25	1.29	0.62	1.29	0.65	1.30	0.69	1.32	0.72	1.40	0.83	1.87	1.29
1.25	1.50	1.13	0.40	1.13	0.39	1.13	0.41	1.13	0.45	1.18	0.52	1.48	0.91
	1.75	1.05	0.25	1.05	0.26	1.05	0.27	1.05	0.28	1.07	0.34	1.25	0.64
	2.00	1.02	0.14	1.02	0.16	1.02	0.16	1.02	0.17	1.03	0.22	1.13	0.42
	2.25	1.00	0.07	1.00	0.08	1.00	0.08	1.00	0.10	1.01	0.14	1.06	0.29
	2.50	1.00	0.04	1.00	0.04	1.00	0.04	1.00	0.05	1.00	0.07	1.02	0.20
	2.75	1.00	0.01	1.00	0.02	1.00	0.03	1.00	0.03	1.00	0.04	1.01	0.13
	3.00	1.00	0.00	1.00	0.01	1.00	0.01	1.00	0.01	1.00	0.02	1.00	0.08
	0.00	3.37	3.51	3.40	3.55	3.63	3.78	3.83	3.96	4.56	4.92	8.77	9.91
	0.25	3.12	3.20	3.12	3.23	3.27	3.38	3.44	3.57	4.04	4.26	7.52	8.25
	0.50	2.53	2.40	2.55	2.34	2.57	2.43	2.72	2.60	3.08	3.05	5.16	5.43
	0.75	1.91	1.56	1.94	1.63	1.95	1.58	2.08	1.74	2.31	2.03	3.53	3.32
	1.00	1.51	0.99	1.52	1.01	1.58	1.09	1.59	1.09	1.74	1.29	2.46	2.09
	1.25	1.29	0.67	1.29	0.68	1.30	0.71	1.32	0.71	1.40	0.84	1.86	1.37
1.50	1.50	1.13	0.46	1.13	0.45	1.13	0.48	1.13	0.50	1.18	0.57	1.48	0.93
	1.75	1.05	0.30	1.05	0.32	1.05	0.31	1.05	0.33	1.07	0.39	1.25	0.68
	2.00	1.02	0.20	1.02	0.21	1.02	0.21	1.02	0.21	1.03	0.26	1.13	0.48
	2.25	1.00	0.13	1.00	0.12	1.00	0.14	1.01	0.14	1.01	0.17	1.06	0.34
	2.50	1.00	0.07	1.00	0.08	1.00	0.08	1.00	0.09	1.00	0.12	1.02	0.24
	2.75	1.00	0.04	1.00	0.04	1.00	0.04	1.00	0.05	1.00	0.07	1.01	0.17
	3.00	1.00	0.01	1.00	0.02	1.00	0.02	1.00	0.03	1.00	0.04	1.00	0.11
	0.00	1.51	1.09	1.52	1.09	1.55	1.14	1.61	1.21	1.76	1.44	2.76	2.72
	0.25	1.48	1.03	1.49	1.05	1.53	1.09	1.59	1.20	1.73	1.41	2.67	2.66
	0.50	1.41	0.91	1.43	0.95	1.46	1.00	1.52	1.08	1.63	1.28	2.45	2.33
	0.75	1.35	0.81	1.36	0.83	1.36	0.83	1.40	0.90	1.51	1.06	2.17	2.00
	1.00	1.25	0.66	1.27	0.66	1.28	0.70	1.30	0.73	1.39	0.86	1.84	1.48
	1.25	1.19	0.54	1.19	0.54	1.20	0.55	1.21	0.57	1.27	0.69	1.59	1.14
2.00	1.50	1.13	0.42	1.13	0.42	1.13	0.44	1.13	0.44	1.18	0.54	1.42	0.89
	1.75	1.05	0.33	1.05	0.31	1.05	0.32	1.05	0.33	1.07	0.39	1.25	0.70
	2.00	1.02	0.24	1.02	0.24	1.02	0.24	1.02	0.27	1.03	0.31	1.13	0.53
	2.25	1.00	0.18	1.00	0.18	1.00	0.19	1.01	0.21	1.01	0.24	1.06	0.40
	2.50	1.00	0.12	1.00	0.14	1.00	0.14	1.00	0.13	1.00	0.17	1.02	0.30
	2.75	1.00	0.08	1.00	0.08	1.00	0.09	1.00	0.10	1.00	0.12	1.01	0.22
	3.00	1.00	0.07	1.00	0.05	1.00	0.05	1.00	0.06	1.00	0.09	1.00	0.17
	0.00	1.18	0.53	1.18	0.54	1.19	0.55	1.21	0.59	1.27	0.71	1.68	1.34
	0.25	1.17	0.51	1.17	0.52	1.18	0.55	1.20	0.57	1.26	0.66	1.64	1.26
	0.50	1.16	0.50	1.16	0.49	1.17	0.52	1.19	0.55	1.25	0.66	1.61	1.23
	0.75	1.14	0.46	1.14	0.45	1.14	0.46	1.16	0.51	1.22	0.60	1.51	1.06
	1.00	1.11	0.40	1.12	0.43	1.13	0.44	1.14	0.44	1.17	0.52	1.45	0.96
	1.25	1.09	0.36	1.09	0.35	1.11	0.39	1.11	0.40	1.15	0.47	1.35	0.82
2.50	1.50	1.07	0.31	1.08	0.31	1.10	0.31	1.08	0.33	1.11	0.39	1.28	0.70
	1.75	1.05	0.28	1.05	0.25	1.05	0.27	1.05	0.28	1.07	0.33	1.21	0.58
	2.00	1.02	0.21	1.02	0.20	1.02	0.23	1.02	0.22	1.03	0.28	1.13	0.47
	2.25	1.00	0.18	1.00	0.17	1.00	0.18	1.01	0.19	1.01	0.21	1.06	0.39
	2.50	1.00	0.13	1.00	0.14	1.00	0.13	1.00	0.15	1.00	0.18	1.02	0.31
	2.75	1.00	0.12	1.00	0.10	1.00	0.11	1.00	0.11	1.00	0.14	1.01	0.26
	3.00	1.00	0.08	1.00	0.09	1.00	0.10	1.00	0.09	1.00	0.11	1.00	0.20
	0.00	1.07	0.30	1.08	0.33	1.08	0.34	1.09	0.34	1.12	0.43	1.31	0.78
	0.25	1.07	0.31	1.08	0.32	1.08	0.33	1.09	0.34	1.12	0.42	1.30	0.75
	0.50	1.07	0.32	1.07	0.31	1.08	0.32	1.09	0.35	1.11	0.40	1.29	0.74
	0.75	1.07	0.29	1.06	0.28	1.07	0.31	1.08	0.32	1.11	0.38	1.25	0.66
	1.00	1.06	0.28	1.06	0.27	1.06	0.28	1.06	0.29	1.09	0.34	1.23	0.62
	1.25	1.05	0.24	1.05	0.25	1.05	0.26	1.06	0.27	1.08	0.33	1.20	0.57
3.00	1.50	1.04	0.22	1.04	0.22	1.04	0.23	1.05	0.25	1.07	0.30	1.16	0.49
	1.75	1.03	0.20	1.03	0.19	1.04	0.22	1.04	0.21	1.06	0.26	1.13	0.44
	2.00	1.02	0.17	1.02	0.18	1.02	0.18	1.02	0.19	1.03	0.23	1.11	0.39
	2.25	1.00	0.14	1.00	0.16	1.00	0.15	1.01	0.17	1.01	0.19	1.06	0.32
	2.50	1.00	0.13	1.00	0.13	1.00	0.13	1.00	0.14	1.00	0.16	1.02	0.30
	2.75	1.00	0.10	1.00	0.11	1.00	0.11	1.00	0.12	1.00	0.13	1.01	0.24
	3.00	1.00	0.08	1.00	0.10	1.00	0.09	1.00	0.09	1.00	0.11	1.00	0.20

Different combinations of shift in mean i.e., a=0.00, 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00, 2.25, 2.50, 2.75 and 3.00, shift in variance i.e., b= 0.25, 0.50, 0.75, 0.90, 1.00, 1.10, 1.25, 1.50, 2.00, 2.50 and 3.00, have been arranged for calculations of ARLs and SDRLs for Max-HEWMAME control chart.

For an in-control production process, the quality characteristic follows the normal distribution, so $\mu_0 = 0$ and σ_0=1. We arrive L=1.397 to achieve ARL_0=370 for in control process where the shift in mean i.e., a=0 and shift in the variance i.e., b=1. The sample size is taken as n=5 and 50,000 replicates have been performed to get each value of ARLs and SDRLs. Two smoothing constants are used for calculations as $\lambda_1 = 0.05$ and $\lambda_2 = 0.01$. If the value of the Max-HEWMAME statistic falls below the UCL, the process remains in control, and another sample is drawn. However, if the value of the Max-HEWMAME statistic goes beyond the UCL, the process is out of control and it shows one run length which stops the process for corrective measures, and then another sample is drawn. This procedure is performed till the completion of the required 50,000 replicates to arrive one ARLs and its SDRLs.

Algorithm for calculations of Max-HEWMAME statistics and UCL for different combinations of mean and variance shifts is given as:

i) Select a sample of size $n=5$;

ii) Determine value of L=1.397 to arrive ARL_0=370 for in control production process;

iii) Fix the values of parameters and smoothing constants $\lambda_1 = 0.05$ and $\lambda_2 = 0.01$;

iv) Calculate the values of Max-HEWMAME statistic and UCL;

v) If statistic remains below the UCL, process is in control and we take another sample;

vi) If statistic is above the UCL, the process is out of control and it is one ARLs, then stop the process and take another sample;

vii) Complete the 50,000 replicates for desired results.

It has been proved by Javaid et al. [43] that Max-HEWMA control chart is more sensitive to detect minor as well as moderate shifts in mean and variance. This article proposes a control chart to study the impact of measurement error on the efficiency of the Max-HEWMA control chart. For this impact first we analyze Table 1.

Table 1 is prepared using covariate model for slope B=1 and different values of error variance ratio with process variance i.e., $\sigma_m^2 \div \sigma^2$ to show that how the measurement error has its impact on the efficiency of the control chart. It is clear from Table 1 that as shift in mean (a) is increased from 0.00 to 3.00, the ARLs are reduced and reached unit very soon at a > 1 or so. However, as shift in variance is deviated from unit to onward or downward, the ARLs are reduced on both sides. Therefore, our proposed chart also supported results of Javaid et al. [43] for small shifts in mean and variance. However, in order to study the impact of measurement error, as the ratio of $\sigma_m^2 \div \sigma^2$ increased from 0.1 to 0.5 and unit, the ARLs are increased with every increase in measurement error. When we compare error columns with the no-error column, the ARLs under error columns are more than those under no error column. It clearly indicates that measurement error causes delay in detecting process shifts in mean as well variance for joint monitoring. Therefore, we can say that although the hybrid EWMA control chart is more efficient and detects minor as well as moderate shifts in mean and variance for joint monitoring but is badly affected by measurement error. After studying the impact of error variance ratio $\sigma_m^2 \div \sigma^2$ in Table 1, now we try to analyze the impact of B, which is slope and coefficient of X in covariate model, on efficiency of proposed control chart, considering that $\sigma_m^2 \div \sigma^2 = 1$ in Table 2.

Table 2 reveals that "no error" column remains the same as was in Table 1, in order to make comparison with the same values under no error column. The impact of values of B can be studied on the error effect. For $\sigma_m^2 \div \sigma^2 = 1$, as the values of B increased from 1 to 5, the ARLs are decreased with every increase in B showing reduction in the impact of error. It indicates that B causes reduction in the impact of measurement error which is similar result to the result by Maravelakis et al. [21]. All other details of shift in mean and variance are the same as in Table 1.

We can say that Tables 1 and 2 both indicate negative effect of measurement error on detection of shifts in mean and variance for joint monitoring which guides us that measurement error reduces the efficiency of hybrid Max-EWMA chart by increasing ARLs and causing delay in detection of process shift(s) for joint monitoring. To cope with the issue of measurement error, multiple measurements of the same sample can be considered to reduce the effect of error.

To reduce the effect of measurement error in our proposed control chart, number of samples can be increased or multiple measurements of the same sample can be taken. As the issue is error in measurement of the sampled units, therefore for this study we prefer to exercise multiple measurements of the same sample to reduce the effect of error and increase the efficiency of the control chart as done by Linna and Woodall [20] and Maravelakis et al. [21]. Several measurements of the same sampled unit and taking average of these measurements leads to the precise sample value in case of measurement error. Strategy of several measurements reduces the error variance component of total variance and ideally by increasing infinite measurements the variance of error can be reduced to zero. However, if there is no measurement error, multiple measurements will add nothing to the efficiency of the control chart but the cost of extra measurements. Therefore, error variance along with minimum required cost and time for extra measurements must be carefully examined at the time of decision making for multiple measurements.

Table 2. ARLs and SDRLs for Max-HEWMAME chart with $\sigma_m^2 \div \sigma^2 = 1$.

b	***a***	***B***									
		No error		**1**		**2**		**3**		**5**	
		ARL	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**
	0.00	1.11	0.31	9.30	6.71	2.61	1.45	1.76	0.81	1.33	0.52
	0.25	1.11	0.31	8.43	6.09	2.61	1.43	1.73	0.80	1.32	0.51
	0.50	1.11	0.31	5.63	3.86	2.42	1.25	1.72	0.77	1.31	0.50
	0.75	1.10	0.29	3.49	2.25	1.89	0.86	1.52	0.60	1.26	0.45
	1.00	1.01	0.11	2.26	1.33	1.40	0.56	1.20	0.40	1.07	0.26
	1.25	1.00	0.00	1.65	0.84	1.11	0.31	1.02	0.16	1.00	0.05
0.25	1.50	1.00	0.00	1.31	0.56	1.01	0.11	1.00	0.03	1.00	0.00
	1.75	1.00	0.00	1.13	0.36	1.00	0.02	1.00	0.00	1.00	0.00
	2.00	1.00	0.00	1.05	0.22	1.00	0.00	1.00	0.00	1.00	0.00
	2.25	1.00	0.00	1.01	0.11	1.00	0.00	1.00	0.00	1.00	0.00
	2.50	1.00	0.00	1.00	0.06	1.00	0.00	1.00	0.00	1.00	0.00
	2.75	1.00	0.00	1.00	0.02	1.00	0.00	1.00	0.00	1.00	0.00
	3.00	1.00	0.00	1.00	0.01	1.00	0.00	1.00	0.00	1.00	0.00
	0.00	2.64	1.46	15.07	11.68	5.02	3.28	3.66	2.24	2.96	1.72
	0.25	2.63	1.46	12.35	9.43	4.81	3.12	3.55	2.13	2.95	1.71
	0.50	2.30	1.15	6.69	4.93	3.53	2.09	2.87	1.60	2.51	1.30
	0.75	1.70	0.74	3.72	2.58	2.25	1.22	1.95	0.95	1.79	0.82
	1.00	1.23	0.44	2.40	1.51	1.55	0.72	1.37	0.56	1.29	0.49
	1.25	1.04	0.19	1.71	0.93	1.19	0.42	1.10	0.30	1.06	0.23
0.50	1.50	1.00	0.05	1.35	0.61	1.04	0.21	1.02	0.13	1.01	0.07
	1.75	1.00	0.00	1.16	0.41	1.01	0.09	1.00	0.02	1.00	0.01
	2.00	1.00	0.00	1.07	0.26	1.00	0.02	1.00	0.00	1.00	0.00
	2.25	1.00	0.00	1.02	0.15	1.00	0.00	1.00	0.00	1.00	0.00
	2.50	1.00	0.00	1.00	0.07	1.00	0.00	1.00	0.00	1.00	0.00
	2.75	1.00	0.00	1.00	0.04	1.00	0.00	1.00	0.00	1.00	0.00
	3.00	1.00	0.00	1.00	0.02	1.00	0.00	1.00	0.00	1.00	0.00
	0.00	10.78	7.98	40.20	35.50	17.14	13.54	13.68	10.40	11.69	8.80
	0.25	8.19	5.97	20.68	17.60	11.69	9.10	9.57	7.15	8.63	6.34
	0.50	3.88	2.59	7.87	6.38	4.97	3.64	4.35	3.01	4.09	2.79
	0.75	2.13	1.24	4.00	3.01	2.62	1.70	2.34	1.45	2.21	1.32
	1.00	1.42	0.66	2.50	1.70	1.68	0.92	1.54	0.79	1.46	0.69
	1.25	1.14	0.37	1.78	1.05	1.29	0.55	1.20	0.45	1.15	0.39
0.75	1.50	1.03	0.18	1.41	0.69	1.10	0.32	1.06	0.24	1.04	0.20
	1.75	1.00	0.07	1.21	0.47	1.03	0.17	1.01	0.12	1.01	0.08
	2.00	1.00	0.02	1.09	0.30	1.01	0.08	1.00	0.05	1.00	0.03
	2.25	1.00	0.00	1.03	0.18	1.00	0.03	1.00	0.01	1.00	0.00
	2.50	1.00	0.00	1.01	0.10	1.00	0.00	1.00	0.00	1.00	0.00
	2.75	1.00	0.00	1.00	0.06	1.00	0.00	1.00	0.00	1.00	0.00
	3.00	1.00	0.00	1.00	0.04	1.00	0.00	1.00	0.00	1.00	0.00
	0.00	50.36	45.52	134.9	149.8	71.92	69.41	59.55	56.65	54.14	49.73
	0.25	13.76	11.69	25.24	23.06	17.05	14.92	15.15	12.95	14.19	12.09
	0.50	4.50	3.42	8.37	7.10	5.42	4.38	4.99	3.91	4.66	3.65
	0.75	2.31	1.53	4.16	3.31	2.75	1.95	2.52	1.76	2.40	1.65
	1.00	1.52	0.81	2.56	1.81	1.77	1.06	1.63	0.92	1.57	0.87
	1.25	1.20	0.45	1.84	1.14	1.35	0.64	1.26	0.55	1.22	0.49
0.90	1.50	1.06	0.24	1.45	0.75	1.14	0.38	1.10	0.32	1.07	0.27
	1.75	1.02	0.14	1.22	0.49	1.05	0.22	1.03	0.16	1.02	0.14
	2.00	1.00	0.06	1.11	0.33	1.01	0.12	1.01	0.07	1.00	0.06
	2.25	1.00	0.02	1.05	0.22	1.00	0.05	1.00	0.02	1.00	0.00
	2.50	1.00	0.00	1.02	0.14	1.00	0.03	1.00	0.01	1.00	0.00
	2.75	1.00	0.00	1.01	0.08	1.00	0.01	1.00	0.00	1.00	0.00
	3.00	1.00	0.00	1.00	0.05	1.00	0.00	1.00	0.00	1.00	0.00
	0.00	370.4	510.4	371.1	497.8	372.7	507.2	371.8	505.4	371.1	505.4
	0.25	14.68	13.64	26.29	25.54	17.80	16.84	16.15	15.09	15.39	14.28
	0.50	4.64	3.92	8.38	7.42	5.63	4.87	5.07	4.27	4.80	4.03
	0.75	2.41	1.74	4.18	3.39	2.84	2.16	2.59	1.89	2.46	1.82
	1.00	1.63	0.91	2.66	1.95	1.86	1.16	1.75	1.02	1.67	0.95
	1.25	1.29	0.52	1.89	1.22	1.45	0.71	1.37	0.60	1.33	0.54
1.00	1.50	1.13	0.29	1.50	0.81	1.25	0.42	1.19	0.35	1.16	0.31
	1.75	1.05	0.16	1.27	0.54	1.11	0.25	1.09	0.20	1.07	0.18
	2.00	1.02	0.08	1.15	0.37	1.05	0.14	1.04	0.11	1.02	0.09
	2.25	1.00	0.03	1.08	0.23	1.03	0.08	1.02	0.05	1.01	0.04
	2.50	1.00	0.01	1.03	0.16	1.00	0.03	1.00	0.02	1.00	0.01
	2.75	1.00	0.01	1.01	0.08	1.00	0.01	1.00	0.00	1.00	0.00
	3.00	1.00	0.00	1.00	0.05	1.00	0.01	1.00	0.00	1.00	0.00
	0.00	41.90	48.09	107.1	132.9	56.67	68.19	48.61	57.21	44.05	50.69
	0.25	12.86	13.06	23.32	23.78	15.43	15.80	13.89	14.38	13.01	13.31
	0.50	4.54	4.11	8.08	7.60	5.52	5.11	4.82	4.44	4.66	4.19
	0.75	2.40	1.85	4.15	3.57	2.82	2.27	2.57	2.01	2.45	1.90
	1.00	1.63	0.99	2.65	2.01	1.85	1.25	1.73	1.11	1.66	1.05
	1.25	1.29	0.57	1.89	1.28	1.44	0.75	1.36	0.66	1.32	0.61
1.10	1.50	1.13	0.34	1.49	0.84	1.24	0.46	1.18	0.41	1.15	0.36
	1.75	1.05	0.19	1.26	0.56	1.10	0.29	1.08	0.23	1.06	0.21
	2.00	1.02	0.11	1.14	0.39	1.04	0.16	1.03	0.13	1.02	0.12
	2.25	1.00	0.05	1.07	0.26	1.02	0.09	1.01	0.08	1.01	0.06
	2.50	1.00	0.02	1.03	0.17	1.00	0.05	1.00	0.03	1.00	0.02
	2.75	1.00	0.00	1.01	0.11	1.00	0.02	1.00	0.02	1.00	0.01
	3.00	1.00	0.00	1.00	0.06	1.00	0.01	1.00	0.00	1.00	0.00

Table 2. ARLs and SDRLs for Max-HEWMAME chart with $\sigma_m^2 \div \sigma^2 = 1$ (continued).

b	a	B: No error ARL	No error SDRL	1 ARL	1 SDRL	2 ARL	2 SDRL	3 ARL	3 SDRL	5 ARL	5 SDRL
1.25	0.00	10.18	11.40	27.86	31.38	14.08	15.67	11.66	12.90	10.80	12.03
	0.25	7.15	7.76	16.03	17.51	9.30	10.14	8.09	8.83	7.58	8.32
	0.50	3.80	3.66	7.41	7.33	4.68	4.62	4.21	4.05	3.95	3.83
	0.75	2.31	1.90	4.08	3.72	2.72	2.34	2.49	2.07	2.39	1.99
	1.00	1.63	1.07	2.63	2.15	1.84	1.31	1.73	1.20	1.66	1.10
	1.25	1.29	0.62	1.87	1.29	1.44	0.83	1.35	0.72	1.32	0.68
	1.50	1.13	0.40	1.48	0.91	1.23	0.54	1.17	0.45	1.14	0.41
	1.75	1.05	0.25	1.25	0.64	1.10	0.33	1.08	0.30	1.06	0.26
	2.00	1.02	0.14	1.13	0.42	1.04	0.21	1.03	0.17	1.02	0.15
	2.25	1.00	0.07	1.06	0.29	1.02	0.12	1.01	0.09	1.01	0.09
	2.50	1.00	0.04	1.02	0.20	1.00	0.07	1.00	0.06	1.00	0.04
	2.75	1.00	0.01	1.01	0.13	1.00	0.03	1.00	0.02	1.00	0.02
	3.00	1.00	0.00	1.00	0.08	1.00	0.02	1.00	0.01	1.00	0.00
1.50	0.00	3.37	3.51	8.77	9.91	4.55	5.00	3.87	4.14	3.66	3.92
	0.25	3.12	3.20	7.52	8.25	4.10	4.35	3.55	3.67	3.31	3.44
	0.50	2.53	2.40	5.16	5.43	3.11	3.14	2.77	2.66	2.54	2.40
	0.75	1.91	1.56	3.53	3.32	2.24	1.99	2.08	1.79	1.97	1.63
	1.00	1.51	0.99	2.46	2.09	1.76	1.33	1.62	1.13	1.54	1.04
	1.25	1.29	0.67	1.86	1.37	1.43	0.84	1.34	0.76	1.31	0.70
	1.50	1.13	0.46	1.48	0.93	1.23	0.60	1.16	0.50	1.14	0.47
	1.75	1.05	0.30	1.25	0.68	1.10	0.39	1.08	0.35	1.06	0.31
	2.00	1.02	0.20	1.13	0.48	1.04	0.27	1.03	0.25	1.02	0.20
	2.25	1.00	0.13	1.06	0.34	1.02	0.17	1.01	0.16	1.01	0.13
	2.50	1.00	0.07	1.02	0.24	1.00	0.11	1.00	0.08	1.00	0.08
	2.75	1.00	0.04	1.01	0.17	1.00	0.07	1.00	0.05	1.00	0.04
	3.00	1.00	0.01	1.00	0.11	1.00	0.04	1.00	0.03	1.00	0.02
2.00	0.00	1.51	1.09	2.76	2.72	1.76	1.41	1.62	1.26	1.56	1.17
	0.25	1.48	1.03	2.67	2.66	1.75	1.42	1.61	1.24	1.52	1.08
	0.50	1.41	0.91	2.45	2.33	1.65	1.25	1.50	1.07	1.44	0.98
	0.75	1.35	0.81	2.17	2.00	1.51	1.04	1.43	0.97	1.37	0.87
	1.00	1.25	0.66	1.84	1.48	1.39	0.88	1.31	0.74	1.29	0.73
	1.25	1.19	0.54	1.59	1.14	1.27	0.68	1.22	0.59	1.19	0.55
	1.50	1.13	0.42	1.42	0.89	1.19	0.56	1.16	0.47	1.13	0.44
	1.75	1.05	0.33	1.25	0.70	1.09	0.41	1.08	0.36	1.06	0.31
	2.00	1.02	0.24	1.13	0.53	1.04	0.31	1.03	0.28	1.02	0.24
	2.25	1.00	0.18	1.06	0.40	1.02	0.23	1.01	0.19	1.01	0.17
	2.50	1.00	0.12	1.02	0.30	1.00	0.16	1.00	0.15	1.00	0.14
	2.75	1.00	0.08	1.01	0.22	1.00	0.12	1.00	0.11	1.00	0.11
	3.00	1.00	0.07	1.00	0.17	1.00	0.08	1.00	0.07	1.00	0.08
2.50	0.00	1.18	0.53	1.68	1.34	1.27	0.69	1.22	0.60	1.19	0.53
	0.25	1.17	0.51	1.64	1.26	1.27	0.71	1.21	0.59	1.18	0.55
	0.50	1.16	0.50	1.61	1.23	1.25	0.67	1.20	0.57	1.17	0.52
	0.75	1.14	0.46	1.51	1.06	1.22	0.60	1.17	0.52	1.15	0.47
	1.00	1.11	0.40	1.45	0.96	1.18	0.54	1.15	0.47	1.13	0.45
	1.25	1.09	0.36	1.35	0.82	1.14	0.46	1.12	0.41	1.10	0.35
	1.50	1.07	0.31	1.28	0.70	1.12	0.42	1.09	0.35	1.08	0.32
	1.75	1.05	0.28	1.21	0.58	1.09	0.34	1.06	0.28	1.06	0.26
	2.00	1.02	0.21	1.13	0.47	1.04	0.27	1.03	0.24	1.02	0.21
	2.25	1.00	0.18	1.06	0.39	1.02	0.24	1.01	0.20	1.01	0.18
	2.50	1.00	0.13	1.02	0.31	1.00	0.18	1.00	0.15	1.00	0.15
	2.75	1.00	0.12	1.01	0.26	1.00	0.15	1.00	0.12	1.00	0.11
	3.00	1.00	0.08	1.00	0.20	1.00	0.11	1.00	0.10	1.00	0.09
3.00	0.00	1.07	0.30	1.31	0.78	1.11	0.40	1.10	0.36	1.08	0.32
	0.25	1.07	0.31	1.30	0.75	1.11	0.40	1.09	0.35	1.08	0.32
	0.50	1.07	0.32	1.29	0.74	1.11	0.40	1.09	0.35	1.08	0.31
	0.75	1.07	0.29	1.25	0.66	1.10	0.37	1.08	0.31	1.07	0.30
	1.00	1.06	0.28	1.23	0.62	1.09	0.35	1.07	0.30	1.06	0.26
	1.25	1.05	0.24	1.20	0.57	1.08	0.32	1.06	0.27	1.05	0.25
	1.50	1.04	0.22	1.16	0.49	1.07	0.29	1.05	0.24	1.05	0.24
	1.75	1.03	0.20	1.13	0.44	1.05	0.25	1.04	0.22	1.04	0.20
	2.00	1.02	0.17	1.11	0.39	1.04	0.22	1.03	0.19	1.02	0.18
	2.25	1.00	0.14	1.06	0.32	1.02	0.18	1.01	0.17	1.01	0.17
	2.50	1.00	0.13	1.02	0.30	1.00	0.16	1.00	0.15	1.00	0.13
	2.75	1.00	0.10	1.01	0.24	1.00	0.14	1.00	0.12	1.00	0.11
	3.00	1.00	0.08	1.00	0.20	1.00	0.12	1.00	0.11	1.00	0.09

Let us assume that k is the number of measurements of the same n selected units and successive measurements are independent from each other, such that $n = nk$. Regarding to the Tables 1 and 2, $k=1$, and if we take $k>1$ using covariate model, Eqs. (6) and (7) can be revised for the jth sample of size n, as:

$$M_j = \frac{\bar{Y} - (A + B\mu)}{\sqrt{\frac{B^2\sigma_p^2}{n} + \frac{\sigma_m^2}{nk}}},$$

and

$$V_j = \phi^{-1}\left[G\left\{ \frac{(nk-1)S^2}{B^2\sigma_p^2 + \sigma_m^2}, (nk-1) \right\} \right]. \quad (15)$$

By using these statistics for mean and variance, Tables 3 and 4 with multiple measurements of the same sample have been developed. Rest of the procedures for Max- HEWMAME chart for Tables 3 and 4 are the same just like Tables 1 and 2.

Table 3 is prepared by taking $k=5$ measurements of each sample, $B=1$ and different values of error variance ratio $\sigma_m^2 = \sigma^2$ as used in Table 1. Table 3 reveals that by taking multiple measurements of the same sample, using covariate model, the values of ARLs are sufficiently reduced as compared with Table 1. Reduction in ARLs is clear indication that the multiple measurements technique is strong and capable to indicate a shift in the production process at the earliest possible time and even ARLs have been reduced up to a unit in most cases which means that on average, just after one sample, the process can indicate a shift in mean or variance during joint monitoring using this control chart.

Similarly, Table 4 supported the results of Table 3 that multiple measurements have reduced the ARLs even to unit in most of the cases and Table 4 has been improved as compared with Table 2 regarding measurement error. Both the Tables 3 and 4 reveal that ARLs have been sufficiently reduced by taking multiple measurements, even lower than the ARLs under no error column. Therefore, we can say that multiple measurements of the same sample or observation can reduce the effect of measurement error. However, cost and time for more measurements are other issues that should be taken into consideration when deciding on multiple measurements of the same sample.

For Tables 3 and 4, we consider values of a shift in the variance i.e., b, up to 1.5 keeping in view that the values of ARLs become approximately unit at b=2.00, 2.5, and 3.00, to save the time and paper space.

4. Main findings

Numerical results for the proposed Max-HEWMAME control chart are presented in Tables 1-4. Main findings construed from the calculated values of ARLs and SDRLs are highlighted here:

i. Table 1 presents the effect of error variance while Table 2 highlights the impact of slope or coefficient of X i.e., B used in the covariate model. Tables 3 and 4 uncover that how much the multiple measurements reduce the influence of error component shown in Tables 1 and 2 using the covariate model;

ii. Table 1 shows that measurement error has increased ARLs which means that measurement error causes a delay in shift detection during the production process;

iii. Table 2 tells us that increase in B causes a decrease in ARLs for fixed error $\sigma_m^2 = \sigma^2$ which is in fact decrease in the impact of the error. We can say that for fixed error, the increase in slope B improves the efficiency of the chart;

iv. Tables 3 and 4 uncovered that multiple measurements have reduced the effect of error variance. Both the tables have improved as compared to Tables 1 and 2 due to multiple measurements and ARLs approached unit values in most of the combinations of process shifts in mean and variance for joint monitoring;

v. Javaid et al. [43] proved that the HEWMA chart is more sensitive for the detection of moderate and minor shifts in mean and variance. Tables 1-4 in this study also prove that measurement error affects the efficiency of the HEWMA control chart and multiple measurements reduce this effect of measurement error. However, multiple measurements need more cost and time;

vi. The results of the four tables are similar to the results of Linna and Woodall [20] and Maravelakis et al. [21].

In addition to the above analysis based on the simulations, let us examine and verify the role of measurement error in the proposed Max-HEWMAME control chart through a real-life example.

5. Real life example

We also benefitted for numerical example from real life data set used by Cheng and Wang [35] in their example-1. Data is taken from filling process of milk bottles containing 500 milliliters (ml) milk in each. Quality characteristic is the quantity of milk in ml in each bottle for 20 samples of size $n=5$ each. Parameters estimated from phase 1 are $\mu_0 = 500.023$ and $\sigma_0 = 0.9616$. The authors highlighted that first ten samples are considered as in control, from where we arrived ARL_0 = 370 with L= 5.322, while last ten samples are considered out of control. Rest of the parameters and procedures are the same as adopted for first four tables. We consider these data set as our population which is reproduced as:

500.46, 498.99, 500.22, 500.41, 498.96, 500.06, 500.20, 499.31, 501.07, 499.57, 498.82, 501.55, 499.48, 499.20, 501.56, 502.64, 502.86, 500.06, 499.08, 500.72, 500.06, 500.03, 500.09, 498.88, 497.64, 500.50, 499.54, 499.02, 498.09, 499.87, 498.89, 500.20, 501.10, 502.01, 500.99, 500.37, 499.28, 500.15, 500.87, 500.88, 499.81, 500.62, 500.68, 500.67, 500.00, 499.79, 499.87, 500.98, 499.12, 500.79, 502.39, 500.61, 501.29, 500.32, 500.74, 500.35, 500.57, 501.80, 502.03, 499.56,499.49, 501.09, 499.53, 499.82, 499.90, 500.96, 500.87, 502.71, 500.70, 499.71, 500.85, 499.88, 500.58, 501.62, 501.04, 500.48, 502.82, 501.00, 501.78, 501.73, 502.01, 501.18, 500.67, 501.31, 499.98, 501.36, 501.11, 500.27, 501.12, 501.98, 499.92, 500.13, 501.46, 502.29, 502.78, 501.19, 500.30, 499.03, 500.17, 502.19.

Table 3. ARLs and SDRLs for Max-HEWMAME chart with $k=5$ and $B=1$.

b	a	$\sigma_m^2 \div \sigma^2$											
		No error		0.1		0.2		0.3		0.5		1	
		ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
0.25	0.00	1.11	0.31	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.04	1.80	0.01
	0.25	1.11	0.31	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.04	1.70	0.92
	0.50	1.11	0.31	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.02	1.40	0.63
	0.75	1.10	0.29	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.09	0.30
	1.00	1.01	0.11	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.01	0.09
	1.25	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.01
	1.50	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	1.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.25	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.50	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.50	0.00	2.64	1.46	1.00	0.05	1.01	0.08	1.02	0.15	1.17	0.41	2.86	1.95
	0.25	2.63	1.46	1.00	0.04	1.01	0.07	1.02	0.14	1.15	0.38	2.48	1.63
	0.50	2.30	1.15	1.00	0.02	1.00	0.04	1.01	0.07	1.05	0.23	1.62	0.86
	0.75	1.70	0.74	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.04	1.14	0.38
	1.00	1.23	0.44	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.02	0.13
	1.25	1.04	0.19	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.02
	1.50	1.00	0.05	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	1.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.25	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.50	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.75	0.00	10.78	7.98	2.11	1.28	2.23	1.41	2.46	1.60	3.21	2.23	8.26	6.65
	0.25	8.19	5.97	1.78	1.01	1.85	1.07	1.98	1.18	2.45	1.58	4.67	3.64
	0.50	3.88	2.59	1.15	0.39	1.18	0.43	1.22	0.47	1.34	0.59	1.97	1.23
	0.75	2.13	1.24	1.01	0.08	1.01	0.09	1.01	0.11	1.03	0.18	1.24	0.51
	1.00	1.42	0.66	1.00	0.00	1.00	0.00	1.00	0.01	1.00	0.03	1.03	0.18
	1.25	1.14	0.37	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.06
	1.50	1.03	0.18	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.01
	1.75	1.00	0.07	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.00	1.00	0.02	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.25	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.50	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.90	0.00	50.36	45.52	10.84	8.90	11.42	9.66	12.34	10.41	15.89	13.70	33.99	32.02
	0.25	13.76	11.69	3.36	2.53	3.44	2.57	3.61	2.74	4.08	3.19	6.47	5.43
	0.50	4.50	3.42	1.33	0.62	1.35	0.64	1.39	0.68	1.52	0.82	2.14	1.42
	0.75	2.31	1.53	1.02	0.15	1.03	0.16	1.04	0.20	1.06	0.25	1.27	0.56
	1.00	1.52	0.81	1.00	0.02	1.00	0.02	1.00	0.03	1.00	0.05	1.05	0.23
	1.25	1.20	0.45	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.01	0.07
	1.50	1.06	0.24	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.01
	1.75	1.02	0.14	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.00	1.00	0.06	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.25	1.00	0.02	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.50	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00

Table 3. ARLs and SDRLs for Max-HEWMAME chart with $k=5$ and $B=1$ (continued).

		$\sigma_m^2 \div \sigma^2$											
b	**a**	**No error**		**0.1**		**0.2**		**0.3**		**0.5**		**1**	
		ARL	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**
	2.50	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
1.00	0.00	370.4	510.4	369.6	504.3	369.1	500.3	372.7	506.9	370.2	500.7	369.8	512.1
	0.25	14.68	13.64	3.83	3.13	3.94	3.22	4.15	3.45	4.58	3.86	6.98	6.11
	0.50	4.64	3.92	1.40	0.69	1.43	0.74	1.47	0.76	1.61	0.93	2.19	1.54
	0.75	2.41	1.74	1.06	0.19	1.07	0.21	1.08	0.23	1.12	0.29	1.32	0.60
	1.00	1.63	0.91	1.00	0.04	1.00	0.04	1.00	0.04	1.01	0.08	1.08	0.24
	1.25	1.29	0.52	1.00	0.00	1.00	0.00	1.00	0.01	1.00	0.01	1.01	0.08
	1.50	1.13	0.29	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.02
	1.75	1.05	0.16	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.00	1.02	0.08	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.25	1.00	0.03	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.50	1.00	0.01	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.75	1.00	0.01	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
1.10	0.00	41.90	48.09	10.42	10.35	10.99	10.83	11.94	11.97	14.70	14.71	30.31	31.68
	0.25	12.86	13.06	3.31	2.80	3.43	2.97	3.49	3.03	4.12	3.64	6.30	5.73
	0.50	4.54	4.11	1.39	0.74	1.42	0.78	1.46	0.81	1.59	0.95	2.18	1.55
	0.75	2.40	1.85	1.05	0.22	1.06	0.25	1.07	0.28	1.10	0.33	1.31	0.61
	1.00	1.63	0.99	1.00	0.05	1.00	0.06	1.00	0.06	1.01	0.09	1.07	0.27
	1.25	1.29	0.57	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.02	1.01	0.09
	1.50	1.13	0.34	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.02
	1.75	1.05	0.19	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.00	1.02	0.11	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.25	1.00	0.05	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.50	1.00	0.02	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
1.25	0.00	10.18	11.40	2.42	1.96	2.57	2.12	2.70	2.26	3.28	2.87	6.56	6.35
	0.25	7.15	7.76	1.90	1.40	1.97	1.46	2.10	1.62	2.41	1.95	4.12	3.82
	0.50	3.80	3.66	1.28	0.61	1.30	0.64	1.34	0.69	1.44	0.82	1.99	1.42
	0.75	2.31	1.90	1.05	0.24	1.06	0.26	1.07	0.28	1.10	0.34	1.30	0.64
	1.00	1.63	1.07	1.00	0.07	1.00	0.08	1.00	0.08	1.01	0.11	1.07	0.28
	1.25	1.29	0.62	1.00	0.02	1.00	0.00	1.00	0.02	1.00	0.03	1.01	0.12
	1.50	1.13	0.40	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.03
	1.75	1.05	0.25	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.01
	2.00	1.02	0.14	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.25	1.00	0.07	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.50	1.00	0.04	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.75	1.00	0.01	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
1.50	0.00	3.37	3.51	1.19	0.49	1.20	0.51	1.23	0.56	1.36	0.74	2.13	1.66
	0.25	3.12	3.20	1.14	0.42	1.16	0.44	1.19	0.49	1.28	0.64	1.93	1.41
	0.50	2.53	2.40	1.07	0.29	1.08	0.31	1.09	0.34	1.15	0.43	1.52	0.93
	0.75	1.91	1.56	1.02	0.15	1.03	0.16	1.03	0.19	1.05	0.22	1.20	0.51
	1.00	1.51	0.99	1.00	0.06	1.00	0.06	1.00	0.07	1.01	0.10	1.06	0.26
	1.25	1.29	0.67	1.00	0.02	1.00	0.02	1.00	0.02	1.00	0.03	1.01	0.12
	1.50	1.13	0.46	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.05
	1.75	1.05	0.30	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.02
	2.00	1.02	0.20	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.25	1.00	0.13	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.50	1.00	0.07	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.75	1.00	0.04	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	3.00	1.00	0.01	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00

Table 4. ARLs and SDRLs for Max-HEWMAME chart with $k = 5$ and $\sigma_m^2 = \sigma^2$.

b	a	B									
		No error		1		2		3		5	
		ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
0.25	0.00	1.11	0.31	1.80	0.01	1.00	0.04	1.00	0.00	1.00	0.00
	0.25	1.11	0.31	1.70	0.92	1.00	0.03	1.00	0.00	1.00	0.00
	0.50	1.11	0.31	1.40	0.63	1.00	0.02	1.00	0.00	1.00	0.00
	0.75	1.10	0.29	1.09	0.30	1.00	0.00	1.00	0.00	1.00	0.00
	1.00	1.01	0.11	1.01	0.09	1.00	0.00	1.00	0.00	1.00	0.00
	1.25	1.00	0.00	1.00	0.01	1.00	0.00	1.00	0.00	1.00	0.00
	1.50	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	1.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.25	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.50	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.50	0.00	2.64	1.46	2.86	1.95	1.16	0.39	1.03	0.18	1.01	0.09
	0.25	2.63	1.46	2.48	1.63	1.15	0.38	1.03	0.18	1.01	0.08
	0.50	2.30	1.15	1.62	0.86	1.05	0.23	1.01	0.10	1.00	0.03
	0.75	1.70	0.74	1.14	0.38	1.00	0.04	1.00	0.00	1.00	0.00
	1.00	1.23	0.44	1.02	0.13	1.00	0.00	1.00	0.00	1.00	0.00
	1.25	1.04	0.19	1.00	0.02	1.00	0.00	1.00	0.00	1.00	0.00
	1.50	1.00	0.05	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	1.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.25	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.50	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.75	0.00	10.78	7.98	8.26	6.65	3.26	2.27	2.56	1.65	2.21	1.38
	0.25	8.19	5.97	4.67	3.64	2.46	1.58	2.04	1.22	1.85	1.05
	0.50	3.88	2.59	1.97	1.23	1.33	0.60	1.23	0.48	1.18	0.42
	0.75	2.13	1.24	1.24	0.51	1.03	0.18	1.01	0.10	1.01	0.09
	1.00	1.42	0.66	1.03	0.18	1.00	0.04	1.00	0.01	1.00	0.00
	1.25	1.14	0.37	1.00	0.06	1.00	0.00	1.00	0.00	1.00	0.00
	1.50	1.03	0.18	1.00	0.01	1.00	0.00	1.00	0.00	1.00	0.00
	1.75	1.00	0.07	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.00	1.00	0.02	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.25	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.50	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.90	0.00	50.36	45.52	33.99	32.02	15.86	13.79	12.92	11.13	11.43	9.63
	0.25	13.76	11.69	6.47	5.43	4.15	3.21	3.70	2.83	3.44	2.57
	0.50	4.50	3.42	2.14	1.42	1.50	0.80	1.41	0.71	1.33	0.62
	0.75	2.31	1.53	1.27	0.56	1.06	0.25	1.04	0.20	1.03	0.17
	1.00	1.52	0.81	1.05	0.23	1.00	0.05	1.00	0.03	1.00	0.03
	1.25	1.20	0.45	1.01	0.07	1.00	0.00	1.00	0.00	1.00	0.00
	1.50	1.06	0.24	1.00	0.01	1.00	0.00	1.00	0.00	1.00	0.00
	1.75	1.02	0.14	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.00	1.00	0.06	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.25	1.00	0.02	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.50	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
1.00	0.00	370.4	510.4	369.8	512.1	372.5	502.5	372.3	503.3	370.7	499.6
	0.25	14.68	13.64	6.98	6.11	6.64	3.85	4.21	3.45	3.95	3.17
	0.50	4.64	3.92	2.19	1.54	1.59	0.91	1.49	0.81	1.42	0.72
	0.75	2.41	1.74	1.32	0.60	1.11	0.30	1.08	0.24	1.06	0.22
	1.00	1.63	0.91	1.08	0.24	1.01	0.07	1.01	0.06	1.00	0.04
	1.25	1.29	0.52	1.01	0.08	1.00	0.02	1.00	0.00	1.00	0.00
	1.50	1.13	0.29	1.00	0.02	1.00	0.00	1.00	0.00	1.00	0.00
	1.75	1.05	0.16	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.00	1.02	0.08	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.25	1.00	0.03	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.50	1.00	0.01	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.75	1.00	0.01	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00

Table 4. ARLs and SDRLs for Max-HEWMAME chart with $k = 5$ and $\sigma_m^2 = \sigma^2$ (continued).

		B									
b	***a***	**No error**		**1**		**2**		**3**		**5**	
		ARL	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**
	0.00	41.90	48.09	30.31	31.68	14.59	14.78	12.15	12.00	11.00	10.96
	0.25	12.86	13.06	6.30	5.73	4.09	3.63	3.65	3.17	3.38	2.97
	0.50	4.54	4.11	2.18	1.55	1.58	0.95	1.48	0.82	1.41	0.76
	0.75	2.40	1.85	1.31	0.61	1.10	0.33	1.07	0.27	1.06	0.24
	1.00	1.63	0.99	1.07	0.27	1.01	0.09	1.01	0.08	1.01	0.05
	1.25	1.29	0.57	1.01	0.09	1.00	0.02	1.00	0.00	1.00	0.01
1.10	1.50	1.13	0.34	1.00	0.02	1.00	0.00	1.00	0.00	1.00	0.00
	1.75	1.05	0.19	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.00	1.02	0.11	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.25	1.00	0.05	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.50	1.00	0.02	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	0.00	10.18	11.40	6.56	6.35	3.27	2.85	2.78	2.34	2.56	2.10
	0.25	7.15	7.76	4.12	3.82	2.40	1.97	2.11	1.64	1.99	1.50
	0.50	3.80	3.66	1.99	1.42	1.43	0.82	1.35	0.71	1.30	0.63
	0.75	2.31	1.90	1.30	0.64	1.10	0.32	1.07	0.28	1.05	0.24
	1.00	1.63	1.07	1.07	0.28	1.01	0.12	1.01	0.09	1.01	0.08
1.25	1.25	1.29	0.62	1.01	0.12	1.00	0.03	1.00	0.01	1.00	0.01
	1.50	1.13	0.40	1.00	0.03	1.00	0.00	1.00	0.00	1.00	0.00
	1.75	1.05	0.25	1.00	0.01	1.00	0.00	1.00	0.00	1.00	0.00
	2.00	1.02	0.14	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.25	1.00	0.07	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.50	1.00	0.04	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.75	1.00	0.01	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	3.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	0.00	3.37	3.51	2.13	1.66	1.33	0.71	1.25	0.59	1.20	0.51
	0.25	3.12	3.20	1.93	1.41	1.28	0.64	1.20	0.52	1.16	0.46
	0.50	2.53	2.40	1.52	0.93	1.15	0.43	1.10	0.35	1.08	0.31
	0.75	1.91	1.56	1.20	0.51	1.05	0.24	1.03	0.18	1.02	0.15
	1.00	1.51	0.99	1.06	0.26	1.01	0.10	1.01	0.07	1.01	0.08
1.50	1.25	1.29	0.67	1.01	0.12	1.00	0.04	1.00	0.02	1.00	0.03
	1.50	1.13	0.46	1.00	0.05	1.00	0.01	1.00	0.00	1.00	0.00
	1.75	1.05	0.30	1.00	0.02	1.00	0.00	1.00	0.00	1.00	0.00
	2.00	1.02	0.20	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.25	1.00	0.13	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.50	1.00	0.07	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	2.75	1.00	0.04	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
	3.00	1.00	0.01	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00

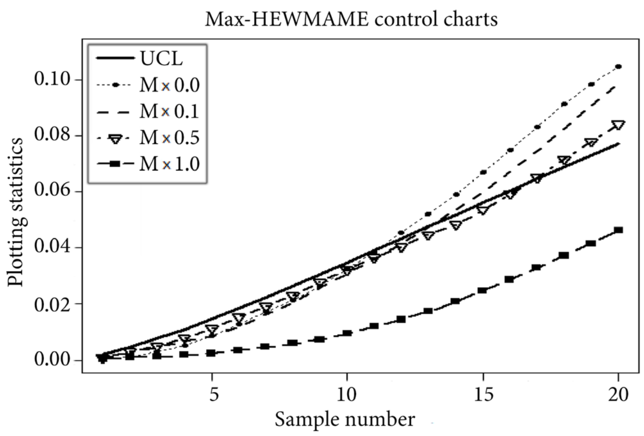

Figure. 1. Max-HEWMAME control chart for real life example.

Considering it population, we selected 20 samples of size 5 each out of this population, first without error and then introducing error as $\sigma_m^2 \div \sigma^2 = 0.1$, 0.5, and 1.0. The calculated plotting statistics and UCL from these samples, are given in Table 5 and graphically presented in Figure 1.

Table 5 indicates the first 10 samples selected from the first 50 in control observations and the last 10 samples selected from the last 50 shifted values. When there is no shift or measurement error in the filling of bottles selected from the first 10 samples, the process remains in control till the 10th sample. However, when a shift occurs after the 10th sample, the process goes out of control at the 12th sample. By introducing measurement error i.e., $\sigma_m^2 \div \sigma^2 = 0.1$ during measurement of bottles, the statistic of the 14th sample goes beyond UCL which causes a delay in process shift detection. For an error of 0.5, the process goes out of control at the 17th sample which further delays the shift detection. However, for an error of 1.0, the detection of process shift is too late to show the statistic beyond UCL even at the 20th sample, which may be detected even after 20 samples, if we can

Table 5. Plotting statistics and UCL.

Sample no.	$\sigma_m^2 \div \sigma^2$				UCL
	No error	0.1	0.5	1.0	
1	0.0005522734	0.0007722727	0.0008675454	0.0004085868	0.002168267
2	0.0012493007	0.0023590225	0.0030993756	0.0010110548	0.004732388
3	0.0029145999	0.0047100260	0.0049668235	0.0012249961	0.007737248
4	0.0054362428	0.0067372659	0.0079224469	0.0018728649	0.011071414
5	0.0088038729	0.0085726022	0.0114378811	0.0024240309	0.014657757
6	0.0128771062	0.0119880645	0.0154037868	0.0035861537	0.018439433
7	0.0171863815	0.0162426372	0.0191559345	0.0046794663	0.022372572
8	0.0213271642	0.0205794056	0.0231219188	0.0060105417	0.026422229
9	0.0264724156	0.0257661284	0.0279183506	0.0073856614	0.030559948
10	0.0323644668	0.0310303315	0.0321235028	0.0095036850	0.034762181
11	0.0385722524	0.0360756615	0.0365647585	0.0119913927	0.039009212
12	**0.0454552417**	0.0414168286	0.0404397969	0.0144483875	0.043284386
13	0.0522117506	0.0473411741	0.0447273464	0.0174119917	0.047573547
14	0.0591342713	**0.0538318528**	0.0482680613	0.0208989337	0.051864600
15	0.0668698676	0.0601626815	0.0535164049	0.0247844598	0.056147183
16	0.0750593459	0.0671818975	0.0591509034	0.0286786489	0.060412395
17	0.0832135093	0.0746946943	**0.0650818064**	0.0329159007	0.064652587
18	0.0913675877	0.0825615546	0.0715917922	0.0372335290	0.068861183
19	0.0984176129	0.0908268649	0.0777993310	0.0416052511	0.073032538
20	0.1047047946	0.0983549110	0.0840556044	0.0462239117	0.077161811

afford to take more samples. These results supported the results of Table 1 already discussed. It indicates that error in measurement of milk bottles can damage the quality control process of production which may cause loss to the producer.

Graph of Table 5 is presented in Figure 1, where UCL line is the Upper Control Limit, graphs of $\mathrm{M}\times 0.0$ is for no measurement error, $\mathrm{M}\times 0.1$ is for $\sigma_m^2 \div \sigma^2 = 0.1$, $\mathrm{M}\times 0.5$ is for $\sigma_m^2 \div \sigma^2 = 0.5$ and $\mathrm{M}\times 1.0$ is for $\sigma_m^2 \div \sigma^2 = 1.0$. When there is no measurement error, the graph of $\mathrm{M}\times 0.0$ indicates that the process of filling milk in bottles remains in control till the 10th sample while due to some shift, the process goes out of control at the 12th sample due to some assignable causes. By introducing error, the process goes out of control at the 14th sample and line of $\mathrm{M}\times 0.1$ shows delay in detection of the process shift. For an increase in error from 0.1 to 0.5, the line for $\mathrm{M}\times 0.5$ goes above UCL at the 17th sample which further delays the shift detection due to increased measurement error. When an error is further increased up to the unit, the line of $\mathrm{M}\times 1.0$ shows that all the 20 sample statistics are below the UCL and there is no indication of process shift which is misleading for decision makers due to measurement error.

To further elaborate the graph of Figure 1, we can say that an in-control production process was running smoothly till the 10th sample, while suddenly a shift occurred in the process which was detected at the 12th sample. However, the measurement error of 0.1 affected the sample values, and the shift was detected at the 14th sample which caused loss to the producer, as after the 12th sample, defective units were produced by the production unit. Similarly, when the measurement error severely increased from 0.1 to 0.5, the shift was detected at the 17th sample further increasing the producer's loss. When this error increased so much that the error variance becomes equal to the process variance, the control chart could not detect the process shift even after the 20th sample and added further loss to the producer. Therefore, it is clear from Figure 1 that measurement error is deteriorating the efficiency of the HEWMA control chart by delaying the indication of process shifts.

6. Conclusion

Statistical process control provides different control charts for monitoring process shifts in the mean and variance separately as well as jointly during industrial production. Ample research is available in the literature and various control charts have been developed by different researchers for process monitoring. However, the effect of measurement error on the efficiency of control charts is not discussed by many authors during the development of new and efficient control charts in the field of quality control. We highlighted the impact of measurement error on the efficiency of control charts for joint monitoring, even for the hybrid Exponentially Weighted Moving Average (EWMA) control chart, which is considered more efficient than the simple EWMA chart. We proposed a Maximum Hybrid Exponentially Weighted Moving Average with Measurement Error (Max-HEWMAME) control chart and studied the impact of measurement error. We proved through analysis of calculated values of Average Run Lengths (ARLs) and Standard Deviations of Run Lengths (SDRLs) that measurement error harms the efficiency of the hybrid EWMA control chart using a covariate model. This research work also included the diagnostic aspect to reduce the impact of measurement error. It is shown that multiple measurements of the sample values can reduce the effect of measurement error, but of course, multiple measurements involve cost and time, which should be taken into consideration at the time of planning. The concept of measurement error can be applied to other efficient control charts to avoid late indication of process shifts. The proposed chart, Max-HEWMAME, can be implemented for the actual application of control charts in real life for process control in industrial production. The issue of measurement error can be discussed and implemented by future researchers, particularly for control charts.

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Authors' contribution statement

First Author: Conceptualization, methodology, software supervision, and validation;

Second Author: Data curation, formal analysis, investigation, and writing original draft;

Third Author: Project administration, resources, visualization, and supervision;

Fourth Author: review and editing.

References

1. Page, E.S. "Continuous inspection schemes", *Biometrika,* **41**(1-2), pp. 100-115 (1954). https://doi.org/10.1093/biomet/41.1-2.100

2. Roberts, S.W. "Control chart tests based on geometric moving averages", *Technometrics*, **1**(3), pp. 239-250 (1959). https://doi.org/10.1080/00401706.1959.10489860

3. Lucas, J.M. and Saccucci, M.S. "Exponentially weighted moving average control schemes: Properties and enhancements", *Technometrics*, **32**(1), pp. 1–12 (1990). https://doi.org/10.1080/00401706.1990.10484583

4. Chen, G., Cheng, S.W., and Xie, H. "Monitoring process mean and variability with one EWMA chart", *Journal of Quality Technology,* **33**(2), pp. 223-233 (2001). https://doi.org/10.1080/00224065.2001.11980069

5. Zhang, L. and Chen, G. "An extended EWMA mean chart", *Quality Technology Quantity Management*, **2**, pp. 39-52 (2005). https://doi.org/10.1080/16843703.2005.11673088

6. Khoo, M.B.C., Teh, S.Y., and Wu, Z. "Monitoring process mean and variability with one double EWMA chart", *Communications in Statistics-Theory and Methods,* **39**(20), pp. 3678-3694 (2010). https://doi.org/10.1080/03610920903324866

7. Abbas, N., Riaz, M., and Does, R.J.M.M. "An EWMA-type control chart for monitoring the process mean using auxiliary information", *Communications in Statistics-Theory and Methods,* **43**(16), pp. 3485-3498 (2014). https://doi.org/10.1080/03610926.2012.700368

8. Abujiya, M.R., Lee, M.H., and Riaz, M. "New EWMA S^2 control charts for monitoring process dispersion", *Scientia Iranica,* **24**(1), pp. 378-389 (2017). https://doi.org/10.24200/sci.2017.4041

9. Haq, A. "A new maximum EWMA control chart for simultaneously monitoring process mean and dispersion using auxiliary information", *Quality and Reliability Engineering International,* **33**(7), pp. 1577-1587 (2017). https://doi.org/10.1002/qre.2126

10. Ajadi, J.O. and Riaz, M. "New memory-type control charts for monitoring process mean and dispersion", *Scientia Iranica,* **24**(6), pp. 3423-3438 (2017). https://doi.org/10.24200/sci.2017.4483

11. Noor-ul-Amin, M., Khan, S., and Aslam, M. "An EWMA control chart using two parametric ratio estimator", *Journal of Industrial and Production Engineering,* **35**(5), pp. 298-308 (2018). https://doi.org/10.1080/21681015.2018.1476414

12. Hussain, S., Song, L., Ahmad, S., et al. "On auxiliary information based improved EWMA median control charts", *Scientia Iranica,* **25**(2), pp. 954-982 (2018). https://doi.org/10.24200/sci.2017.4432

13. Tayyab, M., Noor-ul-Amin, M., and Hanif, M. "Exponential weighted moving average control charts for monitoring the process mean using pair ranked set sampling schemes", *Iranian Journal of Science and Technology, Transactions A: Sciences,* **43**(4), pp. 1941-1950 (2019).

https://doi.org/10.1007/s40995-018-0668-8

14. Awais, M. and Haq, A. "New Shewhart-EWMA and Shewhart-CUSUM control charts for monitoring process mean", *Scientia Iranica,* **26**(6), pp. 3796-3818 (2019).
https://doi.org/10.24200/sci.2018.4962.1011

15. Raza, M.A., Nawaz, T., and Aslam, M. "On designing CUSUM charts using ratio-type estimators for monitoring the location of normal processes", *Scientia Iranica,* **27**(3), pp. 1593-1605 (2020).
https://doi.org/10.24200/sci.2018.50429.1688

16. Bennet, C.A. "Effect of measurement error on chemical process control", *Industrial Quality Control*, **10**(4), pp. 17-20 (1954).

17. Mittag, H.J. and Stemann, D. "Gauge imprecision effect on the performance of the *X-S* control chart", *Journal of Applied Statistics*, **25**(3), pp. 307-317 (1998).
https://doi.org/10.1080/02664769823043

18. Montgomery, D.C. and Runger, G.C. "Gauge capability and designed experiments. Part I: basic methods", *Quality Engineering*, **6**(1), pp. 115-135 (1993).
https://doi.org/10.1080/08982119308918710

19. Sanders, F.F. "Handling process disturbances and measurement errors", *ISA Transactions*, **34**, pp. 165-173 (1995).
https://doi.org/10.1016/0019-0578(95)00010-W

20. Linna, K.W. and Woodall, W.H. "Effect of measurement error on Shewhart control charts", *Journal of Quality Technology*, **33**(2), pp. 213-222 (2001).
https://doi.org/10.1080/00224065.2001.11980068

21. Maravelakis, P., Panaretos, J., and Psarakis, S. "EWMA chart and measurement error", *Journal of Applied Statistics,* **31**(4), pp. 445-455 (2004).
https://doi.org/10.1080/0266476041000l681738

22. Huwang, L. and Hung, Y. "Effect of measurement error on monitoring multivariate process variability", *Statistica Sinica,* **17**(2), pp. 749-790 (2007).

23. Li, J. and Huang, S. "Regression-based process monitoring with consideration of measurement errors", *IIE Transaction,* **42**(2), pp. 146-160 (2009).
https://doi.org/10.1080/07408170903232563

24. Wu, C.W. "Using a novel approach to assess process performance in the presence of measurement errors", *Journal of Statistical Computation and Simulation*, **81**(3), pp. 301-314 (2011).
https://doi.org/10.1080/00949650903313761

25. Maravelakis, P.E. "Measurement error effect on the CUSUM control chart", *Journal of Applied Statistics*, **39**(2), pp. 323-336 (2012).
https://doi.org/10.1080/02664763.2011.590188

26. Khurshid, A., and Chakraborty, A.B. "Measurement error effect on the power of the control chart for zero-truncated binomial distribution under standardization procedure", *International Journal of Quality Research*, **8**(4), pp. 495-504 (2014).

27. Hu, X., Castagliola, P., Sun, J., et al. "The effect of measurement errors on the synthetic $\bar{X}$ chart", *Quality and Reliability Engineering International,* **31**(8), pp. 1769-1778 (2015).
https://doi.org/10.1002/qre.1716

28. Baral, A.K. and Anis, M.Z. "Assessment of C_{pm} in the presence of measurement errors", *Journal of Statistical Theory and Applications,* **14**(1), pp. 13-27 (2015).
https://doi.org/10.2991/jsta.2015.14.1.2

29. Tran, K.P., Castagliola, P., and Celano, G. "The performance of the Shewhart-RZ control chart in the presence of measurement error", *International Journal of Production Research*, **54**(24), pp. 7504-7522 (2016).
https://doi.org/10.1080/00207543.2016.1198507

30. Ghashghaei, R., Bashiri, M., Amiri, A., et al. "Effect of measurement error on joint monitoring of process mean and variability under ranked set sampling", *Quality and Reliability Engineering International,* **32**(8), pp. 3035-3050 (2016).
https://doi.org/10.1002/qre.1988

31. Unnati, B. and Raj, S.J. "The Effect of Measurement Error on the Power of $\bar{X}$ Charts with Known Coefficient of variation", *International Journal of Scientific Research in Mathematical and Statistical Sciences*, **4**(5), pp. 33-39 (2017).

32. Daryabari, S.A., Hashemian, S.M., Keyvandarian, A., et al. "The effects of measurement error on the MAX EWMAMS control chart", *Communications in Statistics - Theory and Methods,* **46**(12), pp. 5766-5778 (2017).
https://doi.org/10.1080/03610926.2015.1112911

33. Sabahno, H., and Amiri, A. "The effect of measurement error on the performance of variable sample size and sampling interval $\bar{X}$ control chart", *IJE Transactions A: Basics,* **30**(7), pp. 995-1004 (2017).
https://doi.org/10.5829/ije.2017.30.07a.09

34. Maleki, M.R. and Salmasnia, A. "Joint monitoring of process location and dispersion based on CUSUM

procedure and generalized likelihood ratio in the presence of measurement error", *Quality and Reliability Engineering International,* **33**(7), pp. 1485-1498 (2017). https://doi.org/10.1002/qre.2120

35. Cheng, X.B. and Wang, F.K. "The performance of EWMA median and CUSUM median control charts for a normal process with measurement errors", *Quality and Reliability Engineering International,* **34**, pp. 203–2013 (2018). https://doi.org/10.1002/qre.2248

36. Tang, A., Castagliola, P., Sun, J.S., et al. "The effect of measurement error on the adaptive EWMA $\bar{X}$ chart", *Quality and Reliability Engineering International,* **34**(4), pp. 609-630 (2018). https://doi.org/10.1002/qre.2275

37. Asif, F., Khan, S., and Noor-ul-Amin, M. "Hybrid exponentially weighted moving average control chart with measurement error", *Iranian Journal of Science and Technology, Transections A: Science,* **44**, pp. 801-811 (2020). https://doi.org/10.1007/s40995-020-00879-3

38. Arif, F., Noor-ul-Amin, M., and Hanif, M. "Joint monitoring of mean and variance using likelihood ratio test statistic with measurement error", *Quality Technology and Quantitative Management,* **18**(2), pp. 202-224 (2021). https://doi.org/10.1080/16843703.2020.1819138

39. Haq, A. "A new hybrid exponentially weighted moving average control chart for monitoring process mean", *Quality and Reliability Engineering International,* **29**, pp. 1015-1025 (2013). https://doi.org/10.1002/qre.1453

40. Azam, M., Aslam, M., and Jun, C. H. "Designing of a hybrid exponentially weighted moving average control chart using repetitive sampling", *International Journal of Advanced Manufacturing Technology,* **77**, pp. 1927-1933 (2015). https://doi.org/10.1007/s00170-014-6585-x

41. Haq, A. "A new hybrid exponentially weighted moving average control chart for monitoring process mean, discussion", *Quality and Reliability Engineering International*, **33**(7), pp. 1629-1631 (2017). https://doi.org/10.1002/qre.2092

42. Noor-ul-Amin, M., Khan, S., and Sanaullah, A. "HEWMA control chart using auxiliary information", *Iranian Journal of Science and Technology, Transactions A: Science,* **43**(3), pp. 891-903 (2019). https://doi.org/10.1007/s40995-018-0585-x

43. Javaid, A., Noor-ul-Amin, M., and Hanif, M. "A new Max-HEWMA control chart using auxiliary information", *Communications in Statistics – Simulation and Computation,* **49**(5), pp. 1285-1305 (2020). https://doi.org/10.1080/03610918.2018.1494282

44. Noor-ul-Amin, M., Shabbir, N., Khan, N., et al. "Hybrid exponentially weighted moving average control chart for mean by using different ranked set sampling schemes", *Kuwait Journal of Science,* **47**(4), pp. 19-28 (2020).

45. Montgomery, D.C., *Introduction to Statistical Quality Control,* 6th Ed., John Wiley and Sons, New York (2009).

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