



Coordination of a single-supplier multi-retailer supply chain via joint ordering policy considering incentives for retailers and utilizing economies of scale

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Abstract

Lead-time fluctuations cause a low supply chain service level through increasing stock-outs. Lack of the supplier's awareness of the retailers' ordering policy is one of the main reasons for the lead-time fluctuations. In this paper, a two-echelon supply chain including single supplier, multiple retailers is studied under two scenarios of decentralized and centralized decision-making. In the first scenario, each retailer independently uses a continuous review inventory policy and the supplier does not know when each retailer will order. This policy prolongs order fulfillment by the supplier and increases order-processing costs. In the second scenario, retailers are encouraged to enter into a joint cooperation plan and change their ordering policy from independent continuous review policies to a joint periodic review policy. In this case, the supply chain can utilize the benefits of economies of scale via integrating and shipping several retailers' orders. The study also determines range of the acceptable lead-time reduction by supplier and retailers for participating in the joint cooperation plan. The results show that joint cooperation plan creates more benefits for the supply chain in terms of cost and service level.

1. Introduction

Lead-time is one of the most important quantitative indices to evaluate supply chain performance. Lead-time management is an effective way to reduce the impact of demand uncertainty on supply chain decisions [1]. Ouyang and Wu [2] showed that reducing the lead-time can decrease the safety stock and loss caused by stock-out, improve the customer's service level, and the competitive ability in business. Furthermore, due to the serious impact of lead-time on inventory costs, bullwhip effect, and product availability, lead-time management has attracted much attention [3]. Coordination of supply chain members' decisions can play an important role in reducing lead-time fluctuations. Since supply chain members are often independent economic entities with conflicting benefits, centralized decision-making is a challenging issue [4].

In a two-stage supply chain with one supplier and multiple retailers, each retailer independently decides on its own ordering policy. Based on the retailer's order quantity, the supplier uses Less Than Truckload (LTL) or Full Truckload (TL) shipment services. LTL service imposes higher costs on suppliers. Therefore, suppliers prefer to aggregate retailers' order (if possible) and use TL service. Retailers usually face lead-time uncertainties. Lead-time uncertainty can lead to the low supply chain service level because of increasing stock-outs. Thus, the supplier's profitability is influenced by retailers' decisions [5]. Although a low service level has a serious impact on the supplier's profit, in decentralized decision-making, each retailer's service level is determined by its own decision (without other retailers and supplier's participation). In the decentralized mode of supply chain operation, the bullwhip effect gets intensified because of the delay in receiving information by the supplier. In fact, a minor fluctuation in

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retailer's demand can cause a major fluctuation in the supplier ordering. On the other hand, a subtle advantage of the centralized systems is decrease of the bullwhip effect.

Since we will be studying the given supply chain under both centralized and decentralized conditions, the major research questions of this work can be stated in the following manner:

- What is the total supply chain profit in the decentralized condition in which the retailers utilize independent continuous review ordering policy?
- What is the total profit of the supply chain in the centralized condition in which the retailers agree on a joint periodic review ordering policy?
- What is the retailer's condition for moving from decentralized to centralized supply chain coordination?
- What is the supplier's condition for moving from decentralized to centralized supply chain coordination?

Various mechanisms have been proposed to facilitate the coordination of different decisions in the supply chain. This paper provides a coordination mechanism based on lead-time variations control, discounts and retailers' ordering policy. In the proposed mechanism, the supplier can control lead-time variations and offer discounts in specific periods of time. This way, retailers get motivated to change their ordering policy from continuous review to joint periodic review policy in a coordinated manner. The remainder of this paper is organized as follows. The related literature is reviewed in Section 2. Section 3 describes the research problem and model assumptions. In Section 4, a two-stage supply chain is modeled under two scenarios of decentralized and centralized decision-making. Supply chain members' conditions for cooperation in centralized decision-making is discussed in Section 5. Section 6 presents the results of numerical experiments and sensitivity analysis. Finally, Section 7 summarizes the obtained results and suggests directions for future research.

2. Literature review

Liao and Shyu [6] presented the first model for lead-time reduction in which the lead-time is controllable and can be reduced by paying the extra crashing cost. Pan and Yang [7] proposed an integrated supplier-purchaser model with controllable lead-time and emphasized on lead-time reduction benefits. Ryu and Lee [8] considered dual-sourcing models with stochastic lead-times in which suppliers can invest in the lead-time reduction. Later, Yang and Pan [9] improved Pan and Yang's model [7] by incorporating the quality-related issue. Chang et al. [10] investigated the impact of the lead-time and ordering cost reduction in the single-vendor single-buyer integrated inventory model. They assumed that the lead-time reduction costs depend on the lead-time length to be reduced and the ordered lot size. Ouyang et al. [11] developed Yang and Pan's [9] model by adding the shortage cost and considering the reorder point as

a decision. Heydari et al. [12] investigated the impact of lead-time variations in a serially connected supply chain with four levels. Results showed that lead-time variations increase inventory fluctuations. Hsu and Lee [13] studied an integrated inventory system with a single manufacturer and multiple retailers by assuming that each retailer has an identical lead-time, which can be reduced with a crashing cost. Jha and Shanker [14] proposed a two-echelon integrated supply chain inventory model with controllable lead-time and service level constraint.

Chaharsooghi and Heydari [3] investigated the relative importance of lead-time mean and variance reduction in a multi-echelon inventory system. They indicated that the supply chain performance is more sensitive to lead-time variance than it is to lead-time mean. Li et al. [15] considered the coordination issue in a decentralized supply chain with controllable lead-time and service level constraint. Huang et al. [16] proposed the lead-time reduction as a coordination mechanism in supply chains with deteriorating products to convince retailers to order in specific periods. Glock [17] investigated different lead-time reduction strategies in a single vendor single buyer integrated inventory model with stochastic demand and lot size-dependent lead-time. This study indicated that lead-time reduction is profitable in case of high demand uncertainty. Li et al. [18] investigated the effect of information sharing on supply chain coordination with controllable lead-times. In the mentioned model, reducing the lead-time to a certain extent lead to lower inventory costs. Arkan and Hejazi [19] designed a coordination mechanism based on a credit period in a two-stage supply chain with one buyer and one supplier. In this model, it was assumed that lead-time and ordering costs are controllable and the buyer was responsible to pay the cost of lead-time reduction. Dey and Chakraborty [20] investigated the effect of variable lead-time on the fuzzy random periodic review inventory system.

Heydari [5] proposed a new coordination mechanism based on reduction of lead-time variation in order to convince the retailer to participate in the coordination of the reorder point decision. Moon et al. [21] considered a fill rate as a service level constraint in a continuous review model with variable stochastic lead-time. Jamshidi et al. [1] studied a five-tier supply chain with controllable lead-time and multiple transportation options. Heydari et al. [22] considered two different shipping modes (fast and slow) for simultaneous coordination of the order quantity and service level in a two-stage supply chain. In the proposed model, the seller can reduce lead-time by spending more and using a fast-shipping mode. Lin [23] studied the effect of investment in lead-time variability reduction in the integrated vendor-buyer supply chain with stochastic lead-time.

Mou et al. [24] developed the Glock model [17] by considering two different safety stocks and adding the transportation time as a decision variable to present a more realistic lead-time crashing cost. Yılmaz and Pardalos [25] considered a two-stage supply chain scheduling problem with multiple manufacturers and multiple customers to minimize the average lead-time. Sarkar and Mahapatra [26] studied a periodic review fuzzy inventory model by

considering lead-time, reorder point, and cycle length as decision variables. Hossain et al. [27] developed an integrated inventory model for a vendor-buyer supply chain where lead-time was a stochastic variable with general distribution function. The vendor delivered goods at a fixed lot size to the buyer who had a constant demand rate. They obtained optimal values of reorder point, order quantity, and number of shipments from the vendor to buyer, to cooperatively operate under a joint contract. Yang et al. [28] extended the Newsvendor model considering stock-out-based consumer switching behavior to include the delivery lead-time. They examined the retailer's optimal order quantity decision in the retail channel and the manufacturer's optimal inventory level decision in the online direct channel. They explored the manufacturer's optimal delivery lead-time decision in the online direct channel and discuss on the impact of the product price and consumer switching behavior on the optimal decisions of supply chain members. They compared two centralized and decentralized scenarios and concluded that consumers in the online direct channel enjoyed a shorter delivery lead-time and hence better service in the decentralized scenario. Sarkar et al. [29] extended Glock's model [17] by considering quality improvement and setup cost reduction in a two-echelon supply chain in which lead-time depends upon lot size and production rate such that lead-time can be reduced by reducing setup time, production time, and transportation time. Udayakumar and Geetha [30] studied supply chain coordination with permissible delay in payments and controllable lead-time.

Dominguez et al. [31] focused on understanding how the uncertainty of re-manufacturing lead-times affected the closed-loop supply chain performance. Malik and Sarkar [32] controlled the lead-time variability by considering different transportation modes and proposed a supply chain coordination mechanism based on lead-time crashing. Hellemans et al. [33] examined the impact of lead-time correlation on the inventory distribution, assuming a periodic review base-stock policy. They gave an efficient method to compute the shortfall distribution for any Markovian lead-time process and provided structural results when lead-times are characterized by a 2-state Markov-modulated process. The latter showed how lead-time correlation increased the inventory variance. Slama et al. [34] focused on disassembly lead-time often considered deterministic. They proposed a new scenario-based stochastic linear programming model to deal with a multi-period, single product and two-echelon disassembly lot-sizing problem under lead-time uncertainty. The demand for each component was known for each time period and the real disassembly lead-time of end-of-life product is an independent stochastic discrete variable with a known probability distribution. The proposed model was used to determine the optimal quantity for disassembled end-of-life products. Dziri et al. [35] studied the problem of inventory level optimization in a multi-period two-echelon supply chain with stochastic and lead-time-sensitive demand. The problem focuses on the best service time to end customers and locating inventories along the supply chain to satisfy the addressed service time. The lower the service time is, the higher the demand becomes. Transchel and Hansen

[36] developed a dynamic inventory control policy for a perishable product with a finite shelf life assuming an uncertain replenishment lead-time and a service level constraint. The dynamic inventory control policy gives the optimal replenishment quantity based on the actual composition of the inventory level into different age categories, the demand during the lead-time, and the inventory issuing policy. They studied the impact of not considering lead-time uncertainty on service level and waste rates using a simulation-based optimization technique. Sun and Zhang [37] developed an integrated production-delivery lot sizing model for a single-product manufacturer-retailer supply chain. The manufacturer produced the product at a finite rate less than market demand. The lead-time demand was assumed to be stochastic. The lead-time and the reorder point are decision variables in this model. They determined the optimal ordering quantity, reorder point, lead-time and the delivery number during each production cycle minimizing the expected total cost per unit time. Tang et al. [38] optimized the total profit and customer service level of a supply chain utilizing robust parameter design of inventory policies. They proposed using system dynamics simulation, Taguchi method and Response Surface Methodology (RSM) for modeling a multi-level supply chain. They used RSM to find the optimal combinations of factors for profit maximization and customer service level maximization in continuous levels of parameters.

Li [39] declared that Supply chain managers considered various approaches to improve their performance by lead-time reduction: both the average lead-time and the variance. He quantified the benefits of lead-time reduction for reorder-point batch-ordering inventory policy and presented an exact total cost equation, which was built on relationship between on-hand inventory and backorder. Cui et al. [40] proposed a novel extension of the multi-item joint replenishment problem with lead-time compressing initiatives. They considered a stochastic periodic-review joint replenishment and delivery model in order to investigate the impacts of capital investment on lead-time reduction. They proposed two heuristics and a differential evolutionary algorithm; moreover, their findings gave significant managerial implications, which is proper investment in lead-time reduction not only makes shorten replenishment time, but also can reduce the system cost. Dey et al. [41] studied variable lead-time under controllable production rate and advertisement-dependent variable demand. They explored and quantified the benefits of lead-time reduction for commonly used lot size quantity, production rate, safety factor, reorder point, advertisement cost and vendor's setup cost. Karthick and Uthayakumar [42] considered a two-level integrated vendor-buyer supply chain model that is developed in a fuzzy environment. They investigated the imperfection in the production process with ambiguous demand, reworking, and setup cost reduction under a controllable lead-time.

A categorized form of the literature review papers in terms of ordering policy, lead-time, its controllability, and their pertinent model is shown in Table 1. As can be observed, some researchers only control the lead-time by

Table 1. Categorization of literature review papers.

Ordering policy		Lead-time control		Lead-time Parameter			Model Supply chain			Authors (year), source
Continuous review policy	Periodic review policy	Variance	Mean	Probabilistic	Deterministic	Decision variable	Multiple retailers	One retailer	Inventory control model	
*			*			*			*	Liao & Shyu (1991) [6]
*			*			*			*	Pan & Yang (2002) [7]
*			*			*			*	Ryu & Lee (2003) [8]
*			*			*			*	Yang & Pan (2004) [9]
*			*			*			*	Chang et al. (2006) [10]
*			*			*			*	Ouyang et al. (2007) [11]
	*		*			*	*			Hsu & Lee (2009) [13]
*			*			*		*		Jha & Shanker (2009), [14]
*			*			*		*		Li et al. (2011) [15]
	*		*			*		*		Huang et al. (2011) [16]
*			*			*		*		Glock (2012) [17]
*			*			*		*		Li et al. (2012) [18]
	*		*			*		*		Arkan & Hejazi (2012) [19]
*			*			*			*	Dey & Chakraborty (2012) [20]
*			*			*			*	Moon et al. (2014) [21]
*		*		*				*		Heydari (2014) [5]
	*		*		*		*			Jamshidi et al. (2015) [1]
*			*		*			*		Heydari et al. (2016) [22]
*			*	*				*		Lin (2016) [23]
*			*			*		*		Mou et al. (2017) [24]
*			*				*			Yilmaz & Pardalos (2017) [25]
	*		*			*			*	Sarkar & Mahapatra (2017) [26]
*			*	*				*		Hossain et al. (2017) [27]
*			*			*		*		Sarkar et al. (2018) [29]
*			*			*		*		Udayakumar & Geetha (2018) [30]
	*		*			*		*		Dominguez et al. (2019) [31]
*			*		*			*		Malik & Sarkar (2019) [32]
*			*		*			*		Dziri et al. (2019) [35]
*			*	*					*	Transchel and Hansen (2019) [36]
*		*		*				*		Sun and Zhang (2019) [37]
*		*	*	*	*			*		Li (2020) [39]
	*		*			*			*	Cui et al. (2020) [40]
*		*	*			*		*		Dey et al. (2021) [41]
*			*			*		*		Karthick and Uthayakumar (2021) [42]
*	*	*		*			*			Current research, this paper

using the mean factor while in many practical situations, the variance is much more important in order to assure companies to receive their items in a short period of time. In fact, considering both factors of mean and variance can give a better picture for the supply chain members to plan their operations.

As can be seen from Table 1, most of the studies in the field of lead-time reduction considered lead-time as a random variable in which the average duration is controllable. Lead-time variance control has received less attention. Furthermore, despite the fact that the supplier usually deals with multiple retailers, most previous studies

considered a two-stage supply chain with one supplier and one retailer. The periodic review ordering system has also received less attention in previous studies. In this paper, service level coordination and lead-time variance control are studied in a two-stage supply chain with one supplier and multiple retailers. The order preparation time is assumed to be a component of the lead-time which can be reduced by the supplier's awareness of the ordering periods. Both continuous and periodic ordering review systems are discussed in this paper and simultaneous change of the retailers' ordering policy is considered as part of the coordination mechanism. Overall, result of the literature review shows that there has been no research on the single supplier, multiple retailers supply chain in which the lead-time variance was considered as the control factor and both continuous review and periodic review policies were compared with each other.

3. Problem description

In this paper, a two-stage supply chain with one supplier and multiple retailers is studied under two scenarios: (1) Decentralized decision-making; (2) Centralized decision-making. In the first scenario, it is assumed that each retailer independently uses a continuous review inventory policy and makes replenishments whenever the inventory level reaches a predefined reorder point. In other words, each retailer places orders several time per year on a random basis. The problem with this approach is that the given supplier does not know when each retailer will order. Hence, before the retailer's order, the supplier is unable to prepare a production/supply plan for ensuring on-time delivery. Furthermore, the official processing costs of different orders from different retailers increases the ordering costs and sometimes can prolong the order fulfillment. Lead-time fluctuations can result in loss of the supplier's credibility and sales opportunity. On the other hand, when the retailer's order is less than the truck's full capacity, some/all trucks will become semi-full, consequently imposing an additional cost to the supply chain.

Since each retailer may have less inventory costs when it independently uses a continuous review inventory policy, there should be an incentive strategy to attract the retailers to change their ordering policy or jointly order to the supplier. The second scenario presents an incentive scheme that encourages retailers to a joint cooperation plan by which they change their ordering policy simultaneously from the initial continuous review to the joint periodic review policy. In this case, all retailers review the inventory at regular intervals and an appropriate quantity is ordered after each review. Such approach serves the interest of the supplier since prior knowledge of ordering periods enables the supplier to schedule for on-time delivery. Furthermore, order preparation time, which is one of the lead-time components [43], can be reduced by production planning. If the supplier

guarantees that it can reduce the lead-time variations sufficiently by the jointly periodic review policy, retailers will be persuaded to set the ordering time based on a contract. If retailers who work together in the same geographic area are persuaded to enter this contract and jointly order to the supplier, the supplier could aggregate the several retailers' orders and use the full-truckload shipment. On top of shipping cost reduction, the product cost per unit could also be lower due to the order aggregation. This can enable suppliers to offer time-based price discounts in specific periods and further encourage retailers to change their ordering policy.

The given solution can be applied in different retail industries. For example, it can be applied for different branches of retailing industries like Walmart in the US or Ofogh Koorosh in Iran; the first of which is the world-renowned company while the second one is the biggest retailing company in Iran with around two thousand branches. The branches, which are geographically near each other, can enter to a contract and order jointly in the sale ordering intervals. Another example can be the retailers of home appliances. Since the ordering cost in this industry is high, it is highly recommendable that the retailers order jointly in order to decrease order processing costs, lead-time as well as the operational costs.

3.1. Key assumptions

- ✓ The supplier pays the shipping costs of orders;
- ✓ Unsatisfied order at the supplier is lost; thus, low service level decreases the supplier profit;
- ✓ The supplier uses a lot-for-lot replenishment strategy by means a predetermined order multiplier.

3.2. Notations

3.2.1. Indices

- i The index of retailers ($i=1 \dots I$)
- l The index of a leading retailer in holding the joint cooperation plan among retailers

3.2.2. Parameters

- Q_i i -th retailer's order quantity
- μ_{D_i} Mean of i -th retailer's demand per year
- δ_{D_i} Standard deviation of i -th retailer's demand per year
- λ Mean lead-time
- ξ Standard deviation of lead-time before cooperation
- Standard deviation of lead-time after cooperation.
- ξ_{new} $\xi_{new} = R\xi$ in which $(1-R)$ is the ratio of lead-time variance reduction ($0 \leq R \leq 1$)
- L Maximum truckload capacity
- p Retail price per unit
- w Wholesale price per unit of product before cooperation
- Wholesale price per unit of product after cooperation ($w' < w$)
- m Raw material price per unit of product
- h_b Retailer's inventory holding costs per unit of product per year
- T_b Retailer's ordering costs per order

Table 2. Comparison of the probabilistic continuous and periodic review policies.

	Periodic review policy	Continuous review policy
Safety stock	$SS_i^{FOS} = k_i \sqrt{\mu_{D_i}^2 \xi^2 + \lambda \delta_{D_i}^2}$	$SS_i^{FOI} = k_i \sqrt{\mu_{D_i}^2 \xi^2 + (\lambda + T) \delta_{D_i}^2}$
Average inventory quantity	$\frac{Q_i}{2} + SS_i^{FOS}$	$\frac{\mu_{D_i} T}{2} + SS_i^{FOI}$
i -th retailer's demand during lead-time	$\lambda \mu_{D_i}$	$\mu_{D_i} (\lambda + T)$
Standard deviation of i -th retailer's demand during lead-time	$\sqrt{\mu_{D_i}^2 \xi^2 + \lambda \delta_{D_i}^2}$	$\sqrt{\mu_{D_i}^2 \xi^2 + (\lambda + T) \delta_{D_i}^2}$
i -th retailer's expected shortage per cycle	$S(k_i)^{FOS} = \sqrt{\mu_{D_i}^2 \xi^2 + \lambda \delta_{D_i}^2} \int_{k_i}^{\infty} (x_i - k_i) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i$	$S(k_i)^{FOI} = \sqrt{\mu_{D_i}^2 \xi_{new}^2 + (\lambda + T) \delta_{D_i}^2} \int_{k_i}^{\infty} (x_i - k_i) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i$

- h_s Supplier's inventory holding costs per unit of product per year
 T_s Supplier's ordering costs per order
 B_b Shortage cost per unit of product
 n A positive integer that represents the supplier's replenishment multiplier. The supplier's replenishment size is n times higher than retailer's order quantity (according to the third assumption)
 r_i i -th retailer's reorder point
 α Relative bargaining power of the retailers as compared to the supplier

3.2.3. Decision variable

- k_i i -th retailer's safety factor (i -th retailer's service level is defined as a function of) k_i

3.3. A review on periodic and continuous review policies

Due to the importance of ordering policy in the proposed model, it is necessary to review and make a comparison between the periodic and continuous review policies before presenting the model in further detail. It is assumed that lead-time and i -th retailer's demand are both independent random variables with normal distribution as $N(\lambda, \xi)$ and $N(\mu_{D_i}, \delta_{D_i})$, respectively.

It is assumed that $f(y_i)$ is a probability distribution for a random variable (y_i) which describes i -th retailer's demand during the lead-time. The mean and standard deviation of y_i depend on ordering policy.

3.3.1. Continuous review policy

If each retailer independently uses a continuous review inventory policy, i -th retailer's demand during lead-time follow the normal distribution with mean $\lambda \mu_{D_i}$ and standard deviation $\sqrt{\mu_{D_i}^2 \xi^2 + \lambda \delta_{D_i}^2}$. Therefore, i -th retailer's reorder point can be calculated as follows [44]:

$$r_i = \lambda \mu_{D_i} + k_i \sqrt{\mu_{D_i}^2 \xi^2 + \lambda \delta_{D_i}^2}. \quad (1)$$

Orders may be delayed due to lead-time uncertainty. So, the

i -th retailer's expected shortage per cycle, $S(k_i)$, can be calculated as follows [44]:

$$S(k_i) = \int_{k_i}^{\infty} (y_i - k_i) f(y_i) dy_i. \quad (2)$$

In continuous review policy, the i -th retailer's expected shortage per cycle ($S(k_i)^{FOS}$) is calculated as follows [5]:

$$S(k_i)^{FOS} = \sqrt{\mu_{D_i}^2 \xi^2 + \lambda \delta_{D_i}^2} \int_{k_i}^{\infty} (x_i - k_i) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i, \quad (3)$$

where $x_i = \frac{y_i - \lambda \mu_{D_i}}{\sqrt{\mu_{D_i}^2 \xi^2 + \lambda \delta_{D_i}^2}}$ is the standard normal variable of lead-time demand (y_i).

3.3.2. Periodic review policy

If each retailer independently uses periodic review inventory policy, i -th retailer's demand during lead-time follow the normal distribution with mean $(\lambda + T) \mu_{D_i}$ and standard deviation $\sqrt{\mu_{D_i}^2 \xi^2 + (\lambda + T) \delta_{D_i}^2}$. Maximal inventory of i -th retailer (Q_i^m) in periodic review policies is calculated as follows [44]:

$$Q_i^m = (\lambda + T) \mu_{D_i} + k_i \sqrt{\mu_{D_i}^2 \xi^2 + (\lambda + T) \delta_{D_i}^2}. \quad (4)$$

In periodic review policies, the i -th retailer's expected shortage per cycle ($S(k_i)^{FOI}$) is calculated as follows [44]:

$$S(k_i)^{FOI} = \sqrt{\mu_{D_i}^2 \xi^2 + (\lambda + T) \delta_{D_i}^2} \int_{k_i}^{\infty} (x_i - k_i) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i, \quad (5)$$

where $x_i = \frac{y_i - (\lambda + T) \mu_{D_i}}{\sqrt{\mu_{D_i}^2 \xi^2 + (\lambda + T) \delta_{D_i}^2}}$ is the standard normal variable of lead-time demand (y_i).

Table 2 comparatively shows the key indicators of continuous and periodic review policies.

4. Supply chain modeling

4.1. First scenario: Decentralized decision-making (independent continuous review inventory policy)

In decentralized decision-making scenario, retailers are independent economic entities. Each retailer has equal decision-making power and make decisions regardless of other retailers and supplier. It is assumed that each retailer independently makes a replenishment decision under a continuous review inventory policy in order to minimize its own costs. In other words, each retailer considers only its own profitability and makes replenishments whenever the inventory level reaches the reorder point (r_i). The i -th retailer order quantity (Q_i) is fixed due to truckload restrictions and other shipping and storage constraints. In this paper, service level is considered as decision variable. Although sales volume and supplier profitability are influenced by retailers' service level [5], each retailer decides independently on its service level. Before accepting the coordination plan, i -th retailer's expected profit function can be formulated as follows [5]:

$$\pi_b(k_i) = (p - w)\mu_{D_i} - T_b \frac{\mu_{D_i}}{Q_i} - h_b \left[\frac{Q_i}{2} + k_i \sqrt{\mu_{D_i}^2 \xi^2 + \lambda \delta_{D_i}^2} + \sqrt{\mu_{D_i}^2 \xi^2 + \lambda \delta_{D_i}^2} \int_{k_i}^{\infty} (x_i - k_i) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \right] - (B_b + p - w)\mu_{D_i} \frac{\sqrt{\mu_{D_i}^2 \xi^2 + \lambda \delta_{D_i}^2} \int_{k_i}^{\infty} (x_i - k_i) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i}{Q_i}. \quad (6)$$

Here, the first term represents i -th retailer's income from selling products, the second and third terms represent ordering and inventory holding costs, respectively, and the last term indicates the expected shortage cost.

As stated by Heydari [5], the i -th retailer's expected profit function is concave in safety factor (k_i). Hence, by optimizing i -th retailer's profit Function (6) with respect to k_i , the i -th retailer's safety factor and i -th retailer's reorder point (r_i) can be calculated as:

$$\frac{\partial \pi_b(k_i)}{\partial k_i} = 0 \rightarrow \bar{F}(k_i^*) = \frac{h_b Q_i}{(B_b + p - w)\mu_{D_i} + h_b Q_i}, \quad (7)$$

$$r_i^* = \lambda \mu_{D_i} + k_i^* \sqrt{\mu_{D_i}^2 \xi^2 + \lambda \delta_{D_i}^2}. \quad (8)$$

$\bar{F}(k_i^*)$ is the probability that a normal variable takes a value more than k_i^* . This value is easy to calculate from normal distribution tables. Service level is defined as the percentage of customers that do not experience a stock-out. So, the i -th retailer's optimal service level (SL_i^*) is calculated as follows:

$$SL_i^* = 1 - \bar{F}(k_i^*). \quad (9)$$

It is noteworthy that each retailer has a different service level in this scenario. It is assumed that the supplier pays the shipping cost. LTL and TL are two different shipment modes, which have a different pricing structure. Shipping cost in LTL is significantly higher than TL [45]. The truckload is

limited to L . As shown in Eq. (10), shipment modes (TL/LTL) depend on retailer's order quantity (Q_i).

$$Q_i = \begin{cases} Q_i = L & TL \\ Q_i \neq L & LTL \end{cases} \quad (10)$$

Shipping cost is a function of retailer's order quantity, which is determined based on shipment modes (11).

$\sum_i Q_i$	The total retailers' order
$\left\lceil \frac{Q_i}{L} \right\rceil$	The number of full truckloads is shipped.
$Q_i - L \left\lceil \frac{Q_i}{L} \right\rceil$	The number of the product is shipped by less than truckload service.
C_{TL}	Shipping cost per full truck (in this case, shipping cost per unit is $\frac{C_{TL}}{L}$)
C_{LTL}	Shipping cost per unit in less than truckload service. It is assumed that the shipping cost per unit with a semi full service is higher than the full truck service ($C_{LTL} > \frac{C_{TL}}{L}$).

Hence, the shipping cost is calculated as (11):

$$\sum_{i=1}^n \left\lceil \frac{Q_i}{L} \right\rceil C_{TL} + \left(Q_i - L \left\lceil \frac{Q_i}{L} \right\rceil \right) C_{LTL}. \quad (11)$$

The supplier's profit depends on sales volumes. Since the shortage is considered as lost sales, so the supplier's annual sales volume is equal to the total retailers' demand minus

total shortages (i.e., $\sum_i \mu_{D_i} \left(1 - \frac{S(k_i)}{Q_i} \right)$).

Selecting a low service level by each retailer can reduce the supplier sales volume (Note that $\frac{S(k_i)}{Q_i}$ is considered as a percentage.). The supplier's expected profit function can be calculated as follows:

$$\pi_s = (w - m) \sum_i \mu_{D_i} \left(1 - \frac{\sqrt{\mu_{D_i}^2 \xi^2 + \lambda \delta_{D_i}^2} \int_{k_i}^{\infty} (x_i - k_i) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i}{Q_i} \right) - \sum_{i=1}^n \left\lceil \frac{Q_i}{L} \right\rceil C_{TL} + \left(Q_i - L \left\lceil \frac{Q_i}{L} \right\rceil \right) C_{LTL} - \sum_i \left(\frac{\mu_{D_i} \left(1 - \frac{\sqrt{\mu_{D_i}^2 \xi^2 + \lambda \delta_{D_i}^2} \int_{k_i}^{\infty} (x_i - k_i) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i}{Q_i} \right)}{n Q_i} \right) - \frac{h_s (n-1) \sum_i Q_i}{2}. \quad (12)$$

Here, the first term represents supplier's income, the second and third terms represent transportation and ordering costs, respectively, and the last term indicates the holding cost in

which $\frac{(n-1) \sum_i Q_i}{2}$ is average supplier's inventory. Before accepting the coordination plan, the supply chain's expected profit function can be formulated as:

$$\pi_{SC} = \pi_s + \sum_i \pi_{b_i}(k_i). \quad (13)$$

Although k_i^* maximizes the i -th retailer's profit, it is a local optimal from the whole supply chain's viewpoint. The

purpose of this paper is to find k^{**} so that all retailers' service level is significantly improved and coordinated. If i -th retailer selects k_i^* instead of k^{**} , its profitability reduces. So, appropriate incentive plans should be suggested by other members to persuade retailers to participate in centralized decision-making.

4.2. Second scenario: Centralized decision-making (joint periodic review inventory policy)

In the second scenario, joint periodic review inventory policy is used as a mechanism for coordinating retailers' decisions. According to a contract, retailers who work together in the same area are persuaded to reorder inventory in predetermined periods (T) simultaneously. These periods can be determined in several ways (by the supplier or retailers). In this paper, it is assumed that one of the retailers has more decision-making power in the market. In other words, a retailer is the leader and determines the ordering period. It is assumed that the leader retailer will reorder when all products are sold. At T , the leader's inventory reaches zero. Therefore, replenishment cycles are calculated as follows:

$$T = \frac{Q_i}{\mu_{D_i}}. \quad (14)$$

Then ordering periods are informed to the supplier. Awareness of ordering periods enables the supplier to schedule for on-time delivery and reduce the lead-time fluctuations to some extent. The supplier can offer discount to retailers to encourage them to order in these periods ($w' < w$). When all retailers order simultaneously, the supplier can aggregate retailers' orders. Due to the demand aggregation, most trucks become full; thus, the shipping cost incurred by the supplier can be reduced. The supplier is convinced to schedule the production plan based on T . In this scenario, the shipping cost per unit, which depends on ordering time, is considered as Eq. (15).

$$\begin{cases} T_i = T & C_{TL} / L \\ T_i \neq T & C_{LTL} \end{cases} \quad \left(\frac{C_{TL}}{L} \leq C_{LTL} \right) \quad (15)$$

It is assumed that i -th retailer's order quantity (Q_i) depends on mean of i -th retailer's demand per year (μ_{D_i}) and it is not a decision variable. In this case, the i -th retailer's order quantity is calculated as follows (Average demand in T time units):

$$T = \frac{Q_i}{\mu_{D_i}} \rightarrow Q_i = T \mu_{D_i}. \quad (16)$$

After accepting the coordination plan, the i -th retailer's expected profit function can be formulated as in:

$$\begin{aligned} \pi_{b_i}(k_i, \xi_{new}) = & (p - w')\mu_{D_i} - \frac{T_b}{T} \\ & - h_b \left[\frac{\mu_{D_i} T}{2} + k_i \sqrt{\mu_{D_i}^2 \xi_{new}^2 + (\lambda + T)\delta_{D_i}^2} + \right. \\ & \left. \sqrt{\mu_{D_i}^2 \xi_{new}^2 + (\lambda + T)\delta_{D_i}^2} \int_{k_i}^{\infty} (x_i - k_i) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \right] \\ & - \frac{(B_b + p - w')}{T} \\ & \int_{k_i}^{\infty} (x_i - k_i) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \sqrt{\mu_{D_i}^2 \xi_{new}^2 + (\lambda + T)\delta_{D_i}^2}. \end{aligned} \quad (17)$$

Here, the first term represents i -th retailer's income from selling products, the second and third terms represent ordering and inventory holding costs, respectively, and the last term indicates the expected shortage cost.

The i -th retailer's expected profit function is concave in safety factor (k_i). Hence, by optimizing i -th retailer's profit function (16) with respect to k_i , the i -th retailer's safety factor can be calculated as:

$$\frac{\partial \pi_{b_i}(k_i, \xi_{new})}{\partial k_i} = 0 \rightarrow \bar{F}(k_i^{**}) = \bar{F}(k^{**}) = \frac{h_b T}{(B_b + p - w') + h_b T}. \quad (18)$$

So, in jointly decision-making, all retailers have similar service level.

$$SL_i^{**} = SL^{**} = 1 - \bar{F}(k^{**}). \quad (19)$$

After accepting the coordination plan, the supplier's expected profit function can be calculated as Eq. (20). Here, the first term represents the supplier's income, the second and third terms represent ordering and holding costs respectively, and the last term indicates the transportation cost.

$$\begin{aligned} \pi_s(k^{**}, \xi_{new}) = & (w' - m) \sum_i \mu_{D_i} \\ & \left(1 - \left(\sqrt{\mu_{D_i}^2 \xi_{new}^2 + (\lambda + T)\delta_{D_i}^2} \int_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i / \mu_{D_i} T \right) \right) \\ & - T_s \left[\sum_i \left(\frac{1 - \left(\sqrt{\mu_{D_i}^2 \xi_{new}^2 + (\lambda + T)\delta_{D_i}^2} \int_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i / \mu_{D_i} T \right)}{nT} \right) \right] \\ & - \frac{h_s(n-1)T \sum_i \mu_{D_i}}{2} - \sum_{i=1}^I \frac{\mu_{D_i} TC_{TL}}{L} \end{aligned} \quad (20)$$

After accepting the coordination plan, the supply chain's expected profit function can be formulated as follows:

$$\pi_{SC} = \pi_s(k^{**}, \xi_{new}) + \sum_i \pi_{b_i}(k_i^{**}, \xi_{new}). \quad (21)$$

5. Supply chain members' condition for participation

5.1. i -th retailer's condition for participation

The i -th retailer participates in the jointly periodic review system only if its profitability does not decrease with respect to independent continuous review inventory policy. From the mathematical point of view, i -th retailer's participation constraint is:

$$\pi_{b_i}(k^{**}, \xi_{new}) \geq \pi_{b_i}(k_i^*). \quad (22)$$

Based on Constraint (22), the maximum acceptable R from i -th retailer's view point (R_{max}^i) is calculated as in Eq. (23) is shown in Box I.

In other words, i -th retailer contributes to this plan if and only if the lead-time fluctuations are reasonably reduced.

Since the aim is to coordinate all members of the whole supply chain, R should be determined in such a way that is acceptable to all of them. In order to achieve this, initially

$$R_{max}^i = \sqrt{\frac{\left((w-w')\mu_{D_i} + T_b \left(-\frac{1}{T} + \frac{\mu_{D_i}}{Q_i} \right) + h_b \left(-\frac{\mu_{D_i}T}{2} + \frac{Q_i}{2} \right) + \sqrt{\mu_{D_i}^2 \xi^2 + \lambda \delta_{D_i}^2} \left(h_b k_i^* + \left(h_b + \frac{(B_b + p - w)\mu_{D_i}}{Q_i} \right) \int_{k_i^*}^{\infty} (x_i - k_i^*) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \right) \right)^2}{\xi \mu_{D_i} \left(h_b k_i^{**} + \left(h_b + \frac{(B_b + p - w')}{T} \right) \int_{k_i^{**}}^{\infty} (x_i - k_i^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \right)}} - \frac{(\lambda + T)\delta_{D_i}^2}{\xi^2 \mu_{D_i}^2} \quad (23)$$

Box I

$$\left\{ \begin{array}{l} A \geq 0 \rightarrow R_{max}^S = \frac{B - \sum_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \left(\frac{T_s}{\mu_{D_i} n T^2} - \frac{(w' - m)}{T} \right) \sqrt{(\lambda + T)\delta_{D_i}^2}}{\sum_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \left(\frac{T_s}{\mu_{D_i} n T^2} - \frac{(w' - m)}{T} \right) \sqrt{\mu_{D_i}^2 \xi^2}} \\ A < 0 \rightarrow R_{min}^S = \frac{B - \sum_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \left(\frac{T_s}{\mu_{D_i} n T^2} - \frac{(w' - m)}{T} \right) \sqrt{(\lambda + T)\delta_{D_i}^2}}{\sum_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \left(\frac{T_s}{\mu_{D_i} n T^2} - \frac{(w' - m)}{T} \right) \sqrt{\mu_{D_i}^2 \xi^2}} \end{array} \right. \quad (26)$$

$$R_{min} = 0$$

$$R_{max} = \min(R_{max}^S, R_{max}^R)$$

$$A = \sum_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \left(\frac{T_s}{\mu_{D_i} n T^2} - \frac{(w' - m)}{T} \right) \sqrt{\mu_{D_i}^2 \xi^2}$$

$$B = \sum_i \left(\frac{T_s \mu_{D_i}}{n Q_i^2} - \frac{\mu_{D_i} (w - m)}{Q_i} \right) \sqrt{\mu_{D_i}^2 \xi^2 + \lambda \delta_{D_i}^2} \int_{k_i}^{\infty} (x_i - k_i) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i - \sum_{i=1}^n \left[\frac{Q_i}{L} \right] C_{TL} + \left(Q_i - L \left[\frac{Q_i}{L} \right] \right) C_{LTL} + \frac{T_s}{n} \sum_i \left(\frac{1}{T} - \frac{\mu_{D_i}}{Q_i} \right) + \frac{h_b (n-1) (T \sum_i \mu_{D_i} - \sum_i Q_i)}{2}$$

$$+ \sum_{i=1}^I \frac{\mu_{D_i} T C_{TL}}{L} - \sum_i \mu_{D_i} (w' - w).$$

Box II

maximum acceptable R from each retailers' viewpoint (R_{max}^i) is calculated, and then their minimum value (R_{max}^R) is considered as the acceptable R from all retailers' viewpoint as:

$$R_{max}^R = \min(R_{max}^i). \quad (24)$$

5.2. Supplier's condition for participation

The supplier will only participate in jointly periodic review system if its profitability does not decrease with respect to the first scenario. From the mathematical point of view, participation constraint of the supplier is:

$$\pi_s(k^{**}, \xi_{new}) \geq \pi_s. \quad (25)$$

Under this condition (25), we will have (Refer to the Appendix A for the detailed proof). Eq. (26) is shown in Box II.

As noted in Appendix A, if A is negative ($A < 0$), the maximum acceptable R from supplier's viewpoint (R_{max}^S) is obtained from Eq. (26). By considering Eq. (24), the maximum acceptable R from the whole supply chain's viewpoint (R_{max}) is $\min(R_{max}^S, R_{max}^R)$. Since, R must take a value in the range $[0, 1]$, minimum R will be zero. If A is positive ($A \geq 0$), the minimum acceptable R from supplier's viewpoint (R_{min}^S) is obtained from (26). In this case, the maximum acceptable R from the whole supply chain's viewpoint (R_{max}) is maximum acceptable R from retailers' viewpoint (R_{max}^R). Note that if the maximum value of R becomes greater than one, the maximum acceptable R will

be replaced by one. If the interval $[R_{min}, R_{max}]$ is non-empty in the range of $[0, 1]$, supply chain decisions are coordinated. Choosing any value of R in the specified interval can make the supply chain members more profitable. The value of R depends on the relative bargaining power of supply chain members and it is calculated as follows:

$$R = \alpha R_{min} + (1 - \alpha) R_{max}, \quad 0 \leq \alpha \leq 1 \quad (27)$$

If R is closer to zero, there will be more control on lead-time fluctuations while if R gets closer to one, the delivery time variations shall be slightly reduced. $R = 0$ means that the supplier guarantees on-time delivery.

6. Numerical examples and sensitivity analysis

To evaluate the performance of the proposed model, a set of numerical experiments inspired by Heydari [5] were generated. In the test problems, a two-stage supply chain with one supplier and three retailers is considered in which third retailer is a leader. Due to the importance of truck capacity, all of the test problems are run for various values of L . Table 3 lists the data used in the investigated test problems.

Results of running the model in the centralized and decentralized decision-making are summarized in Table 4. First, the maximum acceptable R from supplier and i -th retailer's viewpoint is calculated (R_{max}^i, R_{max}^S). Then, maximum acceptable R that is acceptable from all retailers' viewpoint is determined (R_{max}^R). Since R is defined in $[0, 1]$, if $R_{max}^R > 1$, then $R_{max}^R = 1$ (TP#2-4, 7, 8). $\min(R_{max}^S, R_{max}^R)$

Table 3. Test problems.

Test Problem			i	μ_{D_i}	δ_{D_i}	Q_i	p	w	w'	m	h_b	h_s	T_b	T_s	B_b	n	L	C_{TL}	C_{LTL}	λ	ξ
1	TP#1	a	1	28000	5000	4750											1500				
2		b	2	32500	4500	3100	17	10	9	2	8	6	90	60	2	3	3000	0.05L	0.6	5.5	1.2
3		c	3	47500	5500	3750											4500				
4	TP#2	a	1	27000	5000	4500											1500				
5		b	2	30000	4500	3100	23	14	12	3	4	2	80	50	2	1	3000	0.05L	0.6	7	2
6		c	3	25000	3500	3700											4500				
7	TP#3	a	1	28000	2200	4500											1500				
8		b	2	25000	2500	5000	14	10	9.5	2.5	5	3	40	25	2	2	3000	0.05L	0.6	5.5	1.5
9		c	3	25000	3200	3000											4500				
10	TP#4	a	1	35000	2500	4500											1500				
11		b	2	20000	2000	5000	12	10	9.5	2.5	5	3	30	20	2	2	3000	0.05L	0.6	5.5	1.5
12		c	3	25000	3000	3000											4500				
13	TP#5	a	1	35000	3500	4500											1500				
14		b	2	27500	1800	3500	14	10	9.5	2	5	3	30	20	2	4	3000	0.05L	0.6	6	2
15		c	3	25000	3000	1500											4500				
16	TP#6	a	1	25000	4500	4500											1500				
17		b	2	32500	4000	3000	17	10	9	2	8	5	85	60	3	3	3000	0.08L	0.4	5.5	1.5
18		c	3	45000	5500	3750											4500				
19	TP#7	a	1	50000	4500	4000											1500				
20		b	2	35000	3500	3000	19	11	10	3	8	5	75	50	3	3	3000	0.08L	0.4	5	1
21		c	3	45000	5500	5000											4500				
22	TP#8	a	1	45000	3500	4500											1500				
23		b	2	35000	4500	3500	15	10	9	1.5	10	8	90	70	3	2	3000	0.05L	0.5	5	0.5
24		c	3	55500	5500	3000											4500				

Table 4. Results of running the models for different test problems.

	$R_{\max}^t$	$R_{\max}^R$	Supplier		$R_{\max}$	$R_{\min}$	R	Before cooperation				After cooperation				SC
			$R_{\min}^S$	$R_{\max}^S$				π_S	π_{b_i}	π_{SC}	SL^*	π_S	π_{b_i}	π_{SC}	SL^{**}	
1	0.9984								135334.71		0.87		184390.08			
	1.1411	0.9984	0	0.3872	0.3872	0	0.1879	632492.51	147370.77	1115669	0.92	643078.54	220033.49	1374111	0.94	0.23
	1.2755								200471.30		0.93		326609.27			
2	0.9984								135334.71		0.87		187565.42			
	1.1411	0.9984	0	0.3917	0.3917	0	0.0556	631667.51	147370.77	1114844	0.92	652858.29	224597.79	1399299	0.94	0.26
	1.2755								200471.30		0.93		334277.15			
3	0.9984								135334.71		0.87		184963.94			
	1.1411	0.9984	0	0.4053	0.4053	0	0.1709	629192.50	147370.77	1112369	0.92	644798.63	220839.02	1378535	0.94	0.24
	1.2755								200471.30		0.93		327933.60			
4	1.3666								169249.48		0.94		266274.20			
	1.8659	1	0	0.4356	0.4356	0	0.0746	660839.63	141587.82	1121795	0.96	697674.97	300174.77	1515171	0.96	0.35
	1.4593								150117.73		0.95		251047.16			
5	1.3666								169249.48		0.94		258205.11			
	1.8659	1	0	0.4404	0.4404	0	0.2510	660014.63	141587.82	1120970	0.96	679726.70	289937.89	1470044	0.96	0.31
	1.4593								150117.73		0.95		242174.15			
6	1.3666								169249.48		0.94		264625.86			
	1.8659	1	0	0.4547	0.4547	0	0.1259	657539.63	141587.82	1118495	0.96	693881.64	297995.59	1505634	0.96	0.35
	1.4593								150117.73		0.95		249130.45			

Table 4. Results of running the models for different test problems.

	$R_{\max}^I$	$R_{\max}^R$	Supplier		$R_{\max}$	$R_{\min}$	R	Before cooperation			After cooperation					SC
			$R_{\min}^S$	$R_{\max}^S$				π_S	π_{b_i}	π_{SC}	SL^*	π_S	π_{b_i}	π_{SC}	SL^{**}	
7	1.2007								68599.60		0.88		101428.29			
	1.2192	1	0	0.4096	0.4096	0	0.3711	491353.50	60402.63	678049.2	0.86	491819.41	90056.61	772610.9	0.92	0.14
	1.2883								57693.46		0.91		89306.55			
8	1.2007								68599.60		0.88		111606.48			
	1.2192	1	0	0.4268	0.4268	0	0.0542	489703.50	60402.63	676399.2	0.86	514211.36	98420.52	821063.2	0.92	0.22
	1.2883								57693.46		0.91		96824.85			
9	1.2007								68599.60		0.88		105203.02			
	1.2192	1	0	0.4268	0.4268	0	0.2699	489703.50	60402.63	676399.2	0.86	500444.26	93287.26	791273.6	0.92	0.17
	1.2883								57693.46		0.91		92339.03			
10	1.4046								20905.37		0.86		63473.46			
	1.8368	1	0	0.4163	0.4163	0	0.3154	508366.75	2000.26	545918.5	0.77	514412.93	35760.62	657978.7	0.88	0.21
	1.4040								14646.09		0.87		44331.71			
11	1.4046								20905.37		0.86		63174.50			
	1.8368	1	0	0.4352	0.4352	0	0.3234	506716.75	2000.26	544268.5	0.76	513782.23	35598.22	656691.9	0.88	0.21
	1.4040								14646.09		0.87		44136.96			
12	1.4046								20905.37		0.86		68639.76			
	1.8368	1	0	0.4352	0.4352	0	0.1707	506716.75	2000.26	544268.5	0.76	525066.98	38482.01	679716	0.88	0.25
	1.4040								14646.09		0.87		47527.24			
13	0.5976								68181.14		0.90		85200			
	0.6047	0.5976	0	0.5163	0.5163	0	0.47	435668.85	53470.77	562768.2	0.90	449409.41	67803.77	662460.9	0.96	0.18
	1.0403								5447.407		0.95		60047.76			
14	0.5976								68181.14		0.90		125171.62			
	0.6047	0.5976	0	0.5207	0.5207	0	0.15	435118.85	53470.77	562218.2	0.90	560053.81	100800.16	873678.9	0.96	0.55
	1.0403								5447.407		0.95		87653.27			
15	0.5976								68181.15		0.90		98337.95			
	0.6047	0.5976	0	0.5229	0.5229	0	0.37	434293.85	53470.77	561393.2	0.90	485635.23	78370.91	731615.3	0.96	0.30
	1.0403								5447.407		0.95		69271.21			
16	0.8755								110861.47		0.87		149190.85			
	1.1109	0.8755	0	0.4818	0.4818	0	0.3850	573053.32	121039.11	960287.2	0.93	581881.16	199561.19	1207435	0.94	0.26
	1.1934								155333.28		0.94		276802.04			
17	0.8755								110861.47		0.87		162690.94			
	1.1109	0.8755	0	0.4841	0.4841	0	0.1535	572573.32	121039.11	959807.2	0.93	618527.70	220273.48	1307043	0.94	0.36
	1.1934								155333.28		0.94		305550.87			
18	0.8755								110861.47		0.87		166019.01			
	1.1109	0.8755	0	0.4911	0.4911	0	0.0169	571133.32	121039.11	958367.2	0.93	628732.423	226211.25	1334780	0.94	0.39
	1.1934								155333.28		0.94		313817.21			
19	1.7694								259816.52		0.95		403208.95			
	1.6900	1	0	0.3480	0.3480	0	0.1527	792606.98	186669.43	1497823	0.94	801399.69	280832.73	1843080	0.93	0.23
	1.4599								258730.14		0.93		357638.18			

Table 4. Results of running the models for different test problems.

	$R_{\max}^I$	$R_{\max}^R$	Supplier		$R_{\max}$	$R_{\min}$	R	Before cooperation				After cooperation				SC
			$R_{\min}^S$	$R_{\max}^S$				π_S	π_{b_i}	π_{SC}	SL^*	π_S	π_{b_i}	π_{SC}	SL^{**}	
20	1.7694								259816.52		0.95		404205.84			
	1.6900	1	0	0.3519	0.3519	0	0.1343	792126.98	186669.43	1497343	0.94	802599.06	281481.60	1846642	0.93	0.23
	1.4599								258730.14		0.93		358355.72			
21	1.7694								259816.52		0.95		394723.60			
	1.6900	1	0	0.3635	0.3635	0	0.2783	790686.98	186669.43	1495903	0.94	790952.48	275196.18	1812047	0.93	0.21
	1.4599								258730.14		0.93		351174.46			
22	1.3516								164503.71		0.89					
	1.3225	1	0	-0.0016	-0.0016	0	-	983874.94	125165.15	1456165.77	0.89	Channel coordination is not achievable.				
	1.5898								182621.94		0.94					
23	1.3516								164503.71		0.89		233209.69			
	1.3225	1	0	0.0029	0.0029	0	0.0021	983199.94	125165.15	1455490.77	0.89	917077.83	169351.82	1599889.64	0.94	0.09
	1.5898								182621.94		0.94		280250.29			
24	1.3516								164503.71		0.89		233208.16			
	1.3225	1	0	0.0163	0.0163	0	0.0163	981174.94	125165.15	1453465.77	0.89	917074.99	169351.10	1599883.09	0.94	0.10
	1.5898								182621.94		0.94		280248.82			

is considered as maximum acceptable R from the whole supply chain's viewpoint ($R_{\max}$). Based on Eq. (25), minimum R will be zero ($R_{\min} = 0$). After specifying the interval in each problem, R is randomly generated within the specified interval. The profitability and service level of each model are also presented in Table 4. The supply chain profitability improvement in the jointly periodic review system (i.e., centralized condition) compared with independent continuous review inventory policy (i.e., decentralized condition) which is represented by Supply Chain (SC) is shown in the last column of Table 4. As can be seen from Table 4, in all of the test problems, the centralized condition outperforms that of the decentralized condition. Furthermore, the best performance of the centralized condition occurs for test problem 14 with $SC=0.55$ while the lowest performance occurs for the test problem 23 with $SC=0.09$. If the intersection of $[R_{\min}, R_{\max}]$ and $[0, 1]$ is non-empty and $R_{\min} \leq R_{\max}$, supply chain decisions can be coordinated by choosing any value of R in the specified interval. By comparing the results of the model under two scenarios, it is observed that the service level and members' profitability increases after accepting the coordination plan. As demonstrated in example TP#8a, $R_{\min}$ is greater than of $R_{\max}$, so, supply chain coordination could not be achieved. Since the number of full truckloads is shipped ($\lceil \frac{Q_i}{L} \rceil$) depends on the retailer's order size (Q_i), increasing the truck capacity (L) does not necessarily make the supply chain members more profitable.

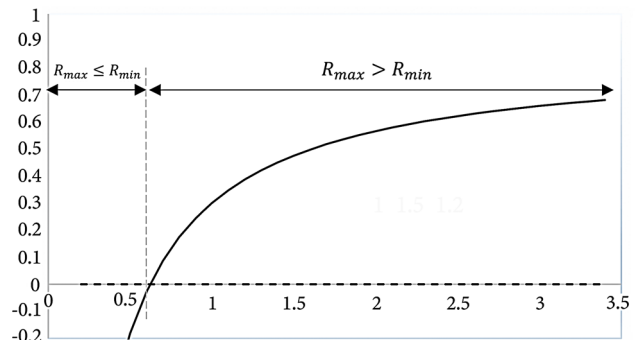


Figure 1. Value of $R_{\min}$ and $R_{\max}$ for ξ changes from 0 to 3.5 in TP#1a.

In this section, sensitivity analysis is performed to illustrate the impact of lead-time variation (ξ) on the proposed model's performance. As demonstrated in Figure 1 the interval $[R_{\min}, R_{\max}]$ becomes wider by increasing the lead-time variations. Therefore, the proposed model is more suitable in the supply chain with high lead-time uncertainty. According to Figure 1, the intersection point between $R_{\min}$ and $R_{\max}$ curves occurs in $\xi = 0.7$. At low levels of lead-time variation ($\xi \leq 0.7$), $R_{\max}$ become negative ($R_{\max} \leq R_{\min}$). So, supply chain coordination could not be achieved. In fact, when lead-time variation is too small, the retailers are not interested to change their ordering policy. So, supply chain coordination could not be achieved.

The profitability of the centralized decision-making model is higher than that of the decentralized decision-making model. Figure 2 shows the improvement of SC profitability in the centralized decision-making compared to the decentralized model by increasing ξ . As demonstrated in

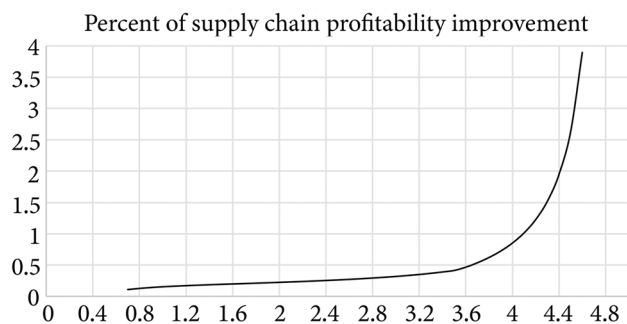


Figure 2. The supply chain profitability improvement with respect to decentralized decision-making for ξ change from 0 to 4.5 in TP#1a.

Figure 2, the centralized decision-making is more suitable when lead-time variability is high.

7. Conclusions and future research

Due to high lead-time variations, the retailer must maintain a service level at a reasonable level by keeping more inventory. Reducing lead-time variations will save a lot of money for the retailers. In this paper, order preparation time is considered as a component of lead-time that can be partially controlled by supplier awareness of retailers' ordering periods. A new mechanism presented to reduce lead-time variations, service level improvement, and supply chain coordination. The supply chain is modeled in two different scenarios: (1) Decentralized decision-making, (2) Centralized decision-making. In the first scenario, it is assumed that each retailer independently uses a continuous review inventory policy and the supplier does not know when each retailer will order. In the second scenario, retailers use a jointly periodic review system in which ordering periods are determined by the leader retailer. Ordering periods are notified to the supplier. Awareness of ordering periods enables the supplier to schedule for on-time delivery to retailers and reduce the lead-time fluctuations to some extent. With synchronized ordering, the supplier can aggregate retailers' order. This way, the supplier can take advantage of full trucks and reduce shipping costs. To further encourage retailers to change their ordering policy from a continuous review system to a periodic review system, the supplier offers discounts in specified periods. The results show that if lead-time fluctuations are reasonably reduced, supply chain members participate in the plan. In this scenario, in addition to service level coordination and improvement, each member's profitability will also increase.

For the managerial implications of the retail industries, the results of this research show that joint ordering of retailers based on a contract can increase the supply chain profitability by more than 50%. Such a contract can be concluded among retailers that are located in nearby geographical locations. This is mainly because of saving on transportation costs since geographically dispersed retailers

would entail higher transportation costs and could hardly reach such a contract.

The limitation of the proposed model is that the order quantity is fixed. Considering that the order quantity may vary based on the inventory level during ordering periods, future studies can extend this model for such consideration. Furthermore, consideration of different types of discounts in ordering periods can also be studied as an incentive factor for supply chain coordination. Another possible research area is to consider the wholesale price after the cooperation as a decision variable and investigate the relation between the wholesale price and lead-time reduction.

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Appendix A

$$\begin{aligned} \pi_s(k^{**}, \xi_{new}) &\geq \pi_s(k_i^*) \rightarrow \\ \sum_i &\sqrt{\mu_{D_i}^2 \xi^2 R^2 + (\lambda + T) \delta_{D_i}^2} \int_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \left(\frac{T_s}{100\mu_{D_i} n T^2} - \frac{(w' - m)}{100T} \right) \\ &\geq \sum_i \left(\frac{T_s \mu_{D_i}}{100n Q_i^2} - \frac{\mu_{D_i}(w - m)}{100Q_i} \right) \sqrt{\mu_{D_i}^2 \xi^2 + \lambda \delta_{D_i}^2} \int_{k_i}^{\infty} (x_i - k_i) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \\ &\quad - \sum_{i=1}^n \left[\frac{Q_i}{L} \right] C_{TL} + \left(Q_i - L \left[\frac{Q_i}{L} \right] \right) C_{LTL} + T_s \sum_i \left(\frac{1}{nT} - \frac{\mu_{D_i}}{nQ_i} \right) + \frac{h_s(n-1)(T \sum_i \mu_{D_i} - \sum_i Q_i)}{2} \\ &\quad + \sum_{i=1}^I \frac{\mu_{D_i} TC_{TL}}{L} - \sum_i \mu_{D_i} (w' - w). \end{aligned}$$

Suppose:

$$\begin{aligned} B &= \sum_i \left(\frac{T_s \mu_{D_i}}{100n Q_i^2} - \frac{\mu_{D_i}(w - m)}{100Q_i} \right) \sqrt{\mu_{D_i}^2 \xi^2 + \lambda \delta_{D_i}^2} \int_{k_i}^{\infty} (x_i - k_i) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i - \sum_{i=1}^n \left[\frac{Q_i}{L} \right] C_{TL} + \left(Q_i - L \left[\frac{Q_i}{L} \right] \right) C_{LTL} \\ &\quad + T_s \sum_i \left(\frac{1}{nT} - \frac{\mu_{D_i}}{nQ_i} \right) + \frac{h_s(n-1)(T \sum_i \mu_{D_i} - \sum_i Q_i)}{2} + \sum_{i=1}^I \frac{\mu_{D_i} TC_{TL}}{L} - \sum_i \mu_{D_i} (w' - w). \end{aligned}$$

$$\sum_i \sqrt{R^2 \mu_{D_i}^2 \xi^2 + (\lambda + T) \delta_{D_i}^2} \int_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \left(\frac{T_s}{100\mu_{D_i} n T^2} - \frac{(w' - m)}{100T} \right) \geq B$$

Given the theorem (28) in mathematics we have:

$$\begin{aligned} \sqrt{N^2 P^2} + \sqrt{OM^2} &\geq \sqrt{N^2 P^2 + OM^2} \\ \sqrt{R^2 \mu_{D_i}^2 \xi^2} + \sqrt{(\lambda + T) \delta_{D_i}^2} &\geq \sqrt{R^2 \mu_{D_i}^2 \xi^2 + (\lambda + T) \delta_{D_i}^2} \rightarrow |R| \sqrt{\mu_{D_i}^2 \xi^2} + \sqrt{(\lambda + T) \delta_{D_i}^2} \geq \sqrt{R^2 \mu_{D_i}^2 \xi^2 + (\lambda + T) \delta_{D_i}^2} \\ &\rightarrow \sum_i \int_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \left(\frac{T_s}{100\mu_{D_i} n T^2} - \frac{(w' - m)}{100T} \right) |R| \sqrt{\mu_{D_i}^2 \xi^2} \\ &\quad + \sum_i \int_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \left(\frac{T_s}{100\mu_{D_i} n T^2} - \frac{(w' - m)}{100T} \right) \sqrt{(\lambda + T) \delta_{D_i}^2} \\ &\geq \sum_i \int_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \left(\frac{T_s}{100\mu_{D_i} n T^2} - \frac{(w' - m)}{100T} \right) \sqrt{R^2 \mu_{D_i}^2 \xi^2 + (\lambda + T) \delta_{D_i}^2} \geq B. \end{aligned} \tag{28}$$

R is positive ($0 \leq R \leq 1$):

$$\begin{aligned} R \sum_i \int_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \left(\frac{T_s}{100\mu_{D_i} n T^2} - \frac{(w' - m)}{100T} \right) \sqrt{\mu_{D_i}^2 \xi^2} + \sum_i \int_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \left(\frac{T_s}{100\mu_{D_i} n T^2} - \frac{(w' - m)}{100T} \right) \sqrt{(\lambda + T) \delta_{D_i}^2} &\geq B \\ A = \sum_i \int_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \left(\frac{T_s}{\mu_{D_i} n T^2} - \frac{(w' - m)}{T} \right) \sqrt{\mu_{D_i}^2 \xi^2} \\ \rightarrow \begin{cases} R \geq \frac{B - \sum_i \int_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \left(\frac{T_s}{100\mu_{D_i} n T^2} - \frac{(w' - m)}{100T} \right) \sqrt{(\lambda + T) \delta_{D_i}^2}}{\sum_i \int_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \left(\frac{T_s}{100\mu_{D_i} n T^2} - \frac{(w' - m)}{100T} \right) \sqrt{\mu_{D_i}^2 \xi^2}} & A > 0 \\ R \leq \frac{B - \sum_i \int_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \left(\frac{T_s}{100\mu_{D_i} n T^2} - \frac{(w' - m)}{100T} \right) \sqrt{(\lambda + T) \delta_{D_i}^2}}{\sum_i \int_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \left(\frac{T_s}{100\mu_{D_i} n T^2} - \frac{(w' - m)}{100T} \right) \sqrt{\mu_{D_i}^2 \xi^2}} & A < 0 \end{cases} \end{aligned}$$

So, we have:

$$\left\{ \begin{array}{ll} A \geq 0 \rightarrow R_{min}^S = \frac{B - \sum_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \left(\frac{T_s}{\mu_{D_i} n T^2} - \frac{(w' - m)}{T} \right) \sqrt{(\lambda + T) \delta_{D_i}^2}}{\sum_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \left(\frac{T_s}{\mu_{D_i} n T^2} - \frac{(w' - m)}{T} \right) \sqrt{\mu_{D_i}^2 \xi^2}} & \begin{array}{l} R_{min} = R_{min}^S \\ R_{max} = R_{max}^R \end{array} \\ A < 0 \rightarrow R_{max}^S = \frac{B - \sum_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \left(\frac{T_s}{\mu_{D_i} n T^2} - \frac{(w' - m)}{T} \right) \sqrt{(\lambda + T) \delta_{D_i}^2}}{\sum_{k^{**}}^{\infty} (x_i - k^{**}) \frac{1}{\sqrt{2\pi}} e^{-\frac{x_i^2}{2}} dx_i \left(\frac{T_s}{\mu_{D_i} n T^2} - \frac{(w' - m)}{T} \right) \sqrt{\mu_{D_i}^2 \xi^2}} & \begin{array}{l} R_{min} = 0 \\ R_{max} = \min(R_{max}^S, R_{max}^R) \end{array} \end{array} \right. \quad (A.1)$$

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