Scientia Iranica (2025) **32**(8): 5100

Sharif University of Technology
Scientia Iranica
Transactions on Industrial Engineering
https://scientiairanica.sharif.edu

A new class of robust ratio estimators for finite population variance

Tolga Zaman [a,*], Hasan Bulut [b]

a. Department of Mathematical Engineering, Faculty of Engineering and Natural Science, Gümüşhane University, Gümüşhane 29100, Turkey.
b. Department of Statistics, Faculty of Science, Ondokuz Mayıs University, 55139 Samsun, Turkey.

* Corresponding authors: zamantolga@gmail.com (T. Zaman); hasan.bulut@omu.edu.tr (H. Bulut)

Received 15 November 2020; received in revised form 12 October 2021; accepted 3 January 2022

Keywords

Finite population variance;
Robust covariance estimates;
Auxiliary information;
Mean square error;
Efficiency,
Simple random sampling.

Abstract

It is a general practice to use robust estimates to improve ratio estimators using functions of the parameters of an auxiliary variable. In this study, a new class of robust estimators based upon the Minimum Covariance Determinant (MCD) and the Minimum Volume Ellipsoid (MVE) robust covariance estimates have been suggested for estimating population variance in the presence of outlier values in the data set for the simple random sampling. The expression for the Mean Square Error (MSE) of the proposed class of estimators is derived from the first degree of approximation. The efficiency of the proposed class of robust estimators is compared with some competing estimators discussed in the literature, and found that proposed estimators are better than other mentioned estimators here. In addition, real data set and simulation studies are performed to present the efficiencies of the estimators. We demonstrate theoretically and numerically that the proposed class of estimators performs better than all other competitor estimators under all situations.

1. Introduction

The use of auxiliary variables can increase the precision of estimators. The ratio, product, and regression estimators are good examples for improving the performance of estimators. Estimating the population variance has great significance in various fields such as industry, agriculture, medical, economic, and biological sciences. Efficient estimators for the population variance has been discussed by various authors referred to Kadilar and Cingi [1], Khan and Shabbir [2], Singh et al. [3], Yadav et al. [4], Yaqub and Shabbir [5], Singh and Pal [6], Sanaullah et al. [7], Muneer et al. [8], Housila et al. [9] and Sharma et al. [10]. However, in the presence of unusual observations in the data, since the classical estimators are sensitive to these extreme values, their efficiencies decrease [11]. Therefore, to reduce the negative effect of the unusual observation problem in the data, it is suggested to use the robust regression estimate, the Minimum Covariance Determinant (MCD), and the Minimum Volume Ellipsoid (MVE) estimators instead of the classical ones. Abid et al. [12] presented the ratio estimators of variance-based using robust measures in the presence of unusual values. Naz et al. [13] proposed the ratio-type estimators developing the efficiency of the ratio-type estimators of population variance using robust location measures. Zaman and Bulut [14] proposed ratio-type estimators using robust regression estimators and robust covariance matrices for stratified random sampling. Bulut and Zaman [15] presented the ratio-type estimators utilizing MCD estimates. Zaman and Bulut [16] provided the ratio estimators for population variance considering MCD and MVE robust covariance estimates, both simple and stratified random sampling. Zaman et al. [17] presented the robust regression-ratio-type estimators of the mean utilizing two auxiliary variables. Grover and Kaur [18] developed the

To cite this article:
T. Zaman and H. Bulut "A new class of robust ratio estimators for finite population variance", *Scientia Iranica* (2025) **32**(8): 5100 https://doi.org/10.24200/sci.2022.57175.5100.

regression-type estimators of population mean with two auxiliary variables using the robust regression technique. Zaman and Bulut [19] proposed the robust ratio double sampling estimator of finite population mean in the presence of outliers. Unlike other studies, this study proposes regression-ratio-type estimators of population variance-based MCD and MVE covariance estimates for simple random sampling.

We give notations used in Subsection 1.1 and some estimators in Subsection 1.2.

1.1. Notations

Consider a finite population $U = (U_1, U_2, \ldots, U_N)$ having N units. Let y_i and x_i be the values of the study variable and the auxiliary variable, respectively. The notations used in this paper can be described as follows:

$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_i$: the population mean of the auxiliary variable x;

$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$: the population mean of the study variable y;

$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \bar{X}\right)^2$: the population variance of the auxiliary variable x;

$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2$: the population variance of the study variable y;

$\lambda_{rq} = \frac{\mu_{rq}}{\left(\mu_{20}^{\frac{r}{2}}\mu_{02}^{\frac{q}{2}}\right)}$ and $\mu_{rq} = \frac{1}{N-1}\sum_{i=1}^{N},\left(y_i - \bar{Y}\right)^r\left(x_i - \bar{X}\right)^q$, r and q are non-negative integers;

$\lambda_{04} = \beta_{2x}$: population coefficient of kurtosis of the auxiliary variable x;

$\lambda_{40} = \beta_{2y}$: population coefficient of kurtosis of the study variable y;

$b = \frac{S_y^2\left(\lambda_{22} - 1\right)}{S_x^2\left(\lambda_{04} - 1\right)}$: the sample regression coefficient;

$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$: the sample mean of the auxiliary variable x;

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$: the sample mean of the study variable y;

$s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$: the sample variance of the auxiliary variable x;

$s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2$: the sample variance of the study variable y.

The sample means $(\bar{y}, \bar{x})$ are unbiased estimators of the population means $(\bar{Y}, \bar{X})$, respectively, and $\left(s_y^2, s_x^2\right)$ are unbiased estimators of population variances $\left(S_y^2, S_x^2\right)$, respectively.

1.2. Some existing estimators

We discuss the following estimators, and we show which estimators are more efficient under what conditions. We assume that the population variance S_x^2 of the auxiliary variable x is available in this study.

The variance of the unbiased estimator $\left(t_0 = s_y^2\right)$ as:

$$V\left(t_0\right) = \frac{S_y^4}{n}\left(\lambda_{40} - 1\right). \tag{1}$$

If the population variance S_x^2 of the auxiliary variable x is known, Isaki [20] introduced the ratio estimator for s_y^2 as:

$$t_R = s_y^2 \frac{S_x^2}{s_x^2}. \tag{2}$$

The Mean Square Error (MSE) of the estimator t_R, is given by:

$$MSE\left(t_R\right) = \frac{S_y^4}{n}\left[\left(\lambda_{40} - 1\right) + \left(\lambda_{04} - 1\right)\left(1 - 2C\right)\right]. \tag{3}$$

By examining Eqs. (1) and (3), Isaki's [20] estimator provides a lower MSE than the unbiased estimator under the condition $C > 0.5$.

If the population variance S_x^2 of the auxiliary variable x is known and when s_y^2 in Eq. (2) is replaced with t_R, then Singh et al. [21] provided the chain ratio estimator as:

$$t_{CR} = t_R \frac{S_x^2}{s_x^2}. \tag{4}$$

We can rewrite Eq. (4) by using Eq. (2) as:

$$t_{CR} = s_y^2 \frac{S_x^4}{s_x^4}. \tag{5}$$

The MSE of the estimator t_R, is given by:

$$MSE\left(t_{CR}\right) = \frac{S_y^4}{n}\left[\left(\lambda_{40} - 1\right) + 4\left(\lambda_{04} - 1\right)\left(1 - C\right)\right],$$

$$C = \frac{\lambda_{22} - 1}{\lambda_{04} - 1}. \tag{6}$$

By examining Eqs. (1) and (6), Singh et al. [21] estimator provides a lower MSE than the unbiased estimator under the condition $C > 1$. From Eqs. (3) and (6), Singh's et al. [21]

estimator provides a lower MSE than the Isaki's [20] estimator under the condition $C > 1.5$.

If the population variance S_x^2 of the auxiliary variable x is known, Isaki [20] defined the following regression estimator for S_y^2, is given by

$$t_{reg} = s_y^2 + b\left(S_x^2 - s_x^2\right), \tag{7}$$

where b is the sample regression coefficient.

The MSE of the estimator t_R, is given by:

$$MSE\left(t_{reg}\right) = \frac{S_y^4}{n}\left(\lambda_{40} - 1\right)\left(1 - \rho^2\right), \tag{8}$$

where: $\rho = \dfrac{\left(\lambda_{22} - 1\right)}{\sqrt{\left(\lambda_{40} - 1\right)\left(\lambda_{04} - 1\right)}}$.

By examining Eqs. (1) and (8), Isaki's [20] regression ratio-type estimator provides a lower MSE than the unbiased estimator under the condition $\rho^2 > 0$, because the condition is always satisfied.

When the population variance S_x^2 of the auxiliary variable x is known, Upadhyaya and Singh [22] introduced the ratio estimator for S_y^2 as:

$$t_{US} = s_y^2\left(\frac{S_x^2 + \lambda_{04}}{s_x^2 + \lambda_{04}}\right). \tag{9}$$

The MSE of the estimator t_{US}, is given by

$$MSE\left(t_{US}\right) \cong \frac{S_y^4}{n}\left[\left(\lambda_{40} - 1\right) + g_0^2\left(\lambda_{04} - 1\right) - 2g_0\left(\lambda_{22} - 1\right)\right], \tag{10}$$

where: $g_0 = \dfrac{S_x^2}{S_x^2 + \lambda_{04}}$.

By examining Eqs. (1) and (10), Upadhyaya and Singh's [22] estimator provides a lower MSE than the unbiased estimator under the condition:

$$g_0 < \frac{2\left(\lambda_{22} - 1\right)}{\left(\lambda_{04} - 1\right)}.$$

When the population variance S_x^2 of the auxiliary variable x is known, Kadilar and Cingi [1] provided the ratio estimator for S_y^2 as:

$$t_{KC1} = s_y^2\left(\frac{S_x^2 + C_x}{s_x^2 + C_x}\right), \tag{11}$$

$$t_{KC2} = s_y^2\left(\frac{\lambda_{04}S_x^2 + C_x}{\lambda_{04}s_x^2 + C_x}\right), \tag{12}$$

$$t_{KC3} = s_y^2\left(\frac{C_xS_x^2 + \lambda_{04}}{C_xs_x^2 + \lambda_{04}}\right), \tag{13}$$

where $C_x = \dfrac{S_x}{\bar{X}}$ is the population coefficient of variation.

The MSEs of the estimators $t_{KCi}\left(i = 1,2,3\right)$, is given by

$$MSE\left(t_{KCi}\right) \cong \frac{S_y^4}{n}\left[\left(\lambda_{40} - 1\right) + g_i^2\left(\lambda_{04} - 1\right) - 2g_i\left(\lambda_{22} - 1\right)\right], \tag{14}$$

where:

$$g_1 = \frac{S_x^2}{S_x^2 + C_x},\ g_2 = \frac{\lambda_{04}S_x^2}{\lambda_{04}S_x^2 + C_x},\ g_3 = \frac{C_xS_x^2}{C_xS_x^2 + \lambda_{04}}$$ [4].

By examining Eqs. (1) and (14), Kadilar and Cingi's [1] estimators provide a lower MSE than the unbiased estimator under the condition

$$g_i < \frac{2\left(\lambda_{22} - 1\right)}{\left(\lambda_{04} - 1\right)}, \qquad i = 1,2,3.$$

From Eqs. (10) and (14), Kadilar and Cingi's [1] estimators provide a lower MSE than the Upadhyaya and Singh's [22] estimator under the condition

$$g_i < \frac{2\left(\lambda_{22} - 1\right)}{\left(\lambda_{04} - 1\right)} - g_0, \qquad i = 1,2,3.$$

In this paper, we have suggested proposed regression-ratio-type estimators and the proposed class of robust estimators for simple random sampling in Section 2. The expression for the MSEs of the proposed regression-ratio-type estimators the proposed class of robust estimators are provided in Section 3. The performance comparisons of various estimators are demonstrated in Section 4. Application and simulation studies are given in Sections 5 and 6, respectively. Conclusion is presented in Section 7.

2. Robust estimators for the mean vector and the scatter matrix

MVE and MCD estimators are the most commonly used estimators in the literature for multivariate location and scatter parameters. The MVE estimator selects *h* observations out of *n* observation units that will make the volume of the ellipsoid the smallest and takes the sample mean vector of these *h* observations and the covariance matrix as the MVE estimator of the location and scatter parameter of the multivariate data. Similarly, the MCD estimator chooses *h* observations with the smallest determinant of the covariance matrix among *n* observations and takes the sample mean vector and covariance matrix of these *h* observations as the MCD estimator of the location and scatter parameter of the multivariate data [23]. To calculate MVE and MCD estimators in the *R* program, MASS package is used [24].

3. The proposed estimators

In this section, we propose regression-type estimators of population variance for simple random sampling. However, the effectiveness of these classical estimators decreases when there is an outlier in the data set. Therefore, the classical ratio estimators proposed to eliminate the negative effect of the outlier problem have been extended to robust ratio-type estimates in Subsection 3.2.

3.1. The proposed regression-type estimators

We consider the following regression-ratio-type estimators for the population variance s_y^2 as:

$$t_{rZB} = \frac{s_y^2 + b\left(S_x^2 - s_x^2\right)}{\left(\zeta s_x^2 + \xi\right)}\left(\zeta S_x^2 + \xi\right) \tag{15}$$

where ζ and ξ are either constant or the functions of the parameters of auxiliary variable such as C_x, β_{2x} and ρ. To obtain the MSE of the estimator in Eq. (15), the terms with e's are defined as follows:

Let $e_0 = \left(s_y^2 - S_y^2\right)/S_y^2$ and $\left(s_x^2 - S_x^2\right)/S_x^2$, such that $E(e_i) = 0,\ i = 0,1.$ $E(e_0^2) = \frac{(\lambda_{40}-1)}{n}$, $E(e_1^2) = \frac{(\lambda_{04}-1)}{n}$, and $E(e_0 e_1) = \frac{(\lambda_{22}-1)}{n}$.

Following Singh and Malik [25], the expressing Eq. (15) in terms of e's, we have:

$$t_{rZB} = \left[S_y^2(1+e_0) - bS_x^2 e_1\right]\left[1 + A_1 e_1^2\right]^{-1}.$$

Up to first order of approximation, the expressions of MSE for t_{ZB} is given by:

$$\left(t_{rZB} - S_y^2\right)^2 = \left(S_y^2 e_0 - bS_x^2 e_1 - A_1 S_y^2 e_1\right)^2,$$

$$\left(t_{rZB} - S_y^2\right)^2 \cong [\ S_y^4 e_0^2 + \left(b^2 S_x^4 + A_1^2 S_y^4 + 2A_1 b S_y^2 S_x^2\right) e_1^2 - 2S_y^2 e_0 e_1 \left(bS_x^2 + A_1 S_y^2\right)],$$

$$MSE\left(t_{rZB}\right) \cong \frac{1}{n}\ \{\ S_y^4\left(\lambda_{40}-1\right) + \left(\lambda_{04}-1\right) \left[B^2 S_x^4 + A_1^2 S_y^4 + 2A_1 B S_y^2 S_x^2\right] - 2S_y^2\left(\lambda_{22}-1\right)\left[BS_x^2 + A_1 S_y^2\right]\}, \tag{16}$$

where $A_1 = \dfrac{\zeta S_x^2}{\zeta S_x^2 + \xi}$.

Table 1 presents some of the estimators for the population variance s_y^2, which can be obtained by suitable choice of constants ζ and ξ.

3.2 The proposed class of Robust estimators

We define to apply the following ratio estimators for the population variance s_y^2 using Robust covariance estimates to data which have outliers.

$$t_{rZB(j)} = \frac{s_{y(j)}^2 + b_j\left(S_{x(j)}^2 - s_{x(j)}^2\right)}{\left(\zeta_{(j)} s_{x(j)}^2 + \xi_{(j)}\right)}\left(\zeta_{(j)} S_{x(j)}^2 + \xi_{(j)}\right), \tag{17}$$

where $s_{y(j)}^2$, b_j, $S_{(j)}^2$, $s_{x(j)}^2$, $\zeta_{(j)}$ and $\xi_{(j)}$ are obtained by considering MCD and MVE covariance estimates, respectively.

Table 1. Suggested estimators Eq. (15).

Estimators	Values ζ	Values ξ
$t_{rZB1} = \dfrac{s_y^2 + b\left(S_x^2 - s_x^2\right)}{s_x^2} S_x^2$	1	0
$t_{rZB2} = \dfrac{s_y^2 + b\left(S_x^2 - s_x^2\right)}{\left(s_x^2 + \beta_2(x)\right)}\left(S_x^2 + \beta_2(x)\right)$	1	$\beta_2(x)$
$t_{rZB3} = \dfrac{s_y^2 + b\left(S_x^2 - s_x^2\right)}{\left(s_x^2 + C_x\right)}\left(S_x^2 + C_x\right)$	1	C_x
$t_{rZB4} = \dfrac{s_y^2 + b\left(S_x^2 - s_x^2\right)}{\left(s_x^2 + \rho\right)}\left(S_x^2 + \rho\right)$	1	ρ
$t_{rZB5} = \dfrac{s_y^2 + b\left(S_x^2 - s_x^2\right)}{\left(s_x^2\beta_2(x) + C_x\right)}\left(S_x^2\beta_2(x) + C_x\right)$	$\beta_2(x)$	C_x
$t_{rZB6} = \dfrac{s_y^2 + b\left(S_x^2 - s_x^2\right)}{\left(s_x^2 C_x + \beta_2(x)\right)}\left(S_x^2 C_x + \beta_2(x)\right)$	C_x	$\beta_2(x)$
$t_{rZB7} = \dfrac{s_y^2 + b\left(S_x^2 - s_x^2\right)}{\left(s_x^2 C_x + \rho\right)}\left(S_x^2 C_x + \rho\right)$	C_x	ρ
$t_{rZB8} = \dfrac{s_y^2 + b\left(S_x^2 - s_x^2\right)}{\left(s_x^2\rho + C_x\right)}\left(S_x^2\rho + C_x\right)$	ρ	C_x
$t_{rZB9} = \dfrac{s_y^2 + b\left(S_x^2 - s_x^2\right)}{\left(s_x^2\beta_2(x) + \rho\right)}\left(S_x^2\beta_2(x) + \rho\right)$	$\beta_2(x)$	ρ
$t_{rZB10} = \dfrac{s_y^2 + b\left(S_x^2 - s_x^2\right)}{\left(s_x^2\rho + \beta_2(x)\right)}\left(S_x^2\rho + \beta_2(x)\right)$	ρ	$\beta_2(x)$

Using Eq. (17), the following MSE for all suggested estimators belonging to Robust covariance estimates in interest are obtained as below:

$$MSE\left(t_{rZB(j)}\right) \cong \frac{1}{n}\{S_{y(j)}^4\left(\lambda_{40(j)}-1\right) + \left(\lambda_{04(j)}-1\right) \left[B_j^2 S_{x(j)}^4 + A_{1(j)}^2 S_{y(j)}^4 + 2A_{1(j)} B_j S_{y(j)}^2 S_{x(j)}^2\right] - 2S_{y(j)}^2\left(\lambda_{22(j)}-1\right)\left[B_j S_{x(j)}^2 + A_{1(j)} S_{y(j)}^2\right]\}; \quad j = \text{MCD and MVE}. \tag{18}$$

We remark that the expression for the MSE of the proposed class of robust estimators is in the same form as expression for the MSE presented in Eq. (16), but it is clear that S_y^4, λ_{40}, B, S_x^4, λ_{22}, and A_1 in Eq. (16) should be replaced by $S_{y(j)}^4$, $\lambda_{40(j)}$, B_j, $S_{x(j)}^4$, $\lambda_{22(j)}$, and $A_{1(j)}$, whose values as obtained by robust covariance estimates ($j =$ MCD and MVE).

Table 2. Suggested estimators using robust covariance estimates Eq. (17).

Estimators	Values $\zeta_{(j)}$	Values $\xi_{(j)}$
$t_{rZB1(j)}=\dfrac{s^2_{y(j)}+b_j\left(S^2_{x(j)}-s^2_{x(j)}\right)}{s^2_{x(j)}}S^2_{x(j)}$	1	0
$t_{rZB2(j)}=\dfrac{s^2_{y(j)}+b_j\left(S^2_{x(j)}-s^2_{x(j)}\right)}{\left(s^2_{x(j}+\beta_{2j}(x)\right)}\left(S^2_{x(j)}+\beta_{2j}(x)\right)$	1	$\beta_{2j}(x)$
$t_{rZB3(j)}=\dfrac{s^2_{y(j)}+b_j\left(S^2_{x(j)}-s^2_{x(j)}\right)}{\left(s^2_{x(j)}+C_{x(j)}\right)}\left(S^2_{x(j)}+C_{x(j)}\right)$	1	C_x
$t_{rZB4(j)}=\dfrac{s^2_{y(j)}+b_j\left(S^2_{x(j)}-s^2_{x(j)}\right)}{\left(s^2_{x(j)}+\rho_{(j)}\right)}\left(S^2_{x(j)}+\rho_{(j)}\right)$	1	$\rho_{(j)}$
$t_{rZB5(j)}=\dfrac{s^2_{y(j)}+b_j\left(S^2_{x(j)}-s^2_{x(j)}\right)}{\left(s^2_{x(j)}\beta_{2j}(x)+C_{x(j)}\right)}\left(S^2_{x(j)}\beta_{2j}(x)+C_{x(j)}\right)$	$\beta_{2j}(x)$	$C_{x(j)}$
$t_{rZB6(j)}=\dfrac{s^2_{y(j)}+b_j\left(S^2_{x(j)}-s^2_{x(j)}\right)}{\left(s^2_{x(j)}C_{x(j)}+\beta_{2j}(x)\right)}\left(S^2_{x(j)}C_{x(j)}+\beta_{2j}(x)\right)$	$C_{x(j)}$	$\beta_{2j}(x)$
$t_{rZB7(j)}=\dfrac{s^2_{y(j)}+b_j\left(S^2_{x(j)}-s^2_{x(j)}\right)}{\left(s^2_{x(j)}C_{x(j)}+\rho_{(j)}\right)}\left(S^2_{x(j)}C_{x(j)}+\rho_{(j)}\right)$	$C_{x(j)}$	$\rho_{(j)}$
$t_{rZB8(j)}=\dfrac{s^2_{y(j)}+b_j\left(S^2_{x(j)}-s^2_{x(j)}\right)}{\left(s^2_{x(j)}\rho_{(j)}+C_{x(j)}\right)}\left(S^2_{x(j)}\rho_{(j)}+C_{x(j)}\right)$	$\rho_{(j)}$	$C_{x(j)}$
$t_{rZB9(j)}=\dfrac{s^2_{y(j)}+b_j\left(S^2_{x(j)}-s^2_{x(j)}\right)}{\left(s^2_{x(j)}\beta_{2j}(x)+\rho_{(j)}\right)}\left(S^2_{x(j)}\beta_{2j}(x)+\rho_{(j)}\right)$	$\beta_{2j}(x)$	$\rho_{(j)}$
$t_{rZB10(j)}=\dfrac{s^2_{y(j)}+b_j\left(S^2_{x(j)}-s^2_{x(j)}\right)}{\left(s^2_{x(j)}\rho_{(j)}+\beta_{2j}(x)\right)}\left(S^2_{x(j)}\rho_{y,x(j)}+\beta_{2j}(x)\right)$	$\rho_{(j)}$	$\beta_{2j}(x)$

Table 2 presents the proposed class of robust estimators for the population variance s_y^2, which can be obtained by suitable choice of constants $\zeta_{(j)}$ and $\xi_{(j)}$.

4. Efficiency comparisons

We compare the proposed class of robust estimators with the other competing estimators.

4.1 With the proposed class of robust estimators

(i) *With the MSE of estimators given in Eqs. (16) and (18):*

$$MSE\left(t_{rZB(j)}\right)<MSE\left(t_{rZB}\right)$$

$$\frac{1}{n}\ \{\ S^4_{y(j)}\left(\lambda_{40(j)}-1\right)+\left(\lambda_{04(j)}-1\right)$$

$$\left[B_j^2S^4_{x(j)}+A^2_{1(j)}S^4_{y(j)}+2A_{1(j)}B_jS^2_{y(j)}S^2_{x(j)}\right]$$

$$-2S^2_{y(j)}\left(\lambda_{22(j)}-1\right)\ \left[B_jS^2_{x(j)}+A_{1(j)}S^2_{y(j)}\right]\},$$

$$<\frac{1}{n}\{\ S_y^4\left(\lambda_{40}-1\right)+\left(\lambda_{04}-1\right)\ \left[B^2S_x^4+A_1^2S_y^4+2A_1BS_y^2S_x^2\right]$$

$$-2S_y^2\left(\lambda_{22}-1\right)\left[BS_x^2+A_1S_y^2\right]\}. \tag{19}$$

Let,

$$L_{(j)}=\left(\lambda_{04(j)}-1\right)\left[B_j^2S^4_{x(j)}+A^2_{1(j)}S^4_{y(j)}+2A_{1(j)}B_jS^2_{y(j)}S^2_{x(j)}\right],$$

and $M_{(j)}=S^2_{y(j)}\left(\lambda_{22(j)}-1\right)\left[B_jS^2_{x(j)}+A_{1(j)}S^2_{y(j)}\right]$;

j=MCD and MVE ,

$K=S_y^4\left(\lambda_{40}-1\right)$, $L=\left(\lambda_{04}-1\right)\left[B^2S_x^4+A_1^2S_y^4+2A_1BS_y^2S_x^2\right]$,

$M=S_y^2\left(\lambda_{22}-1\right)\left[BS_x^2+A_1S_y^2\right]$, $N=S_y^4\left(\lambda_{04}-1\right)$.

Thus, Eq. (20) becomes:

$$\left(K_{(j)}-K\right)+\left(L_{(j)}-L\right)-2\left(M_{(j)}-M\right)<0\ . \tag{20}$$

(ii) *With the MSE of estimators given in Eqs. (1) and (18):*

$$\text{MSE}\left(t_{rZB(j)}\right)<V\left(t_0\right)\ \frac{1}{n}\ \{\ S^4_{y(j)}\left(\lambda_{40(j)}-1\right)+\left(\lambda_{04(j)}-1\right)$$

$$\left[B_j^2S^4_{x(j)}+A^2_{1(j)}S^4_{y(j)}+2A_{1(j)}B_jS^2_{y(j)}S^2_{x(j)}\right]$$

$$-2S^2_{y(j)}\left(\lambda_{22(j)}-1\right)\left[B_jS^2_{x(j)}+A_{1(j)}S^2_{y(j)}\right]<\frac{S_y^4}{n}\left(\lambda_{40}-1\right),$$

$$\left(K_{(j)}-K\right)+L_{(j)}-2M_{(j)}<0\ . \tag{21}$$

(iii) *With the MSE of estimators given in Eqs. (3) and (18):*

$$\text{MSE}\left(t_{rZB(j)}\right)<\text{MSE}\left(t_R\right)\ \frac{1}{n}\ \{\ S^4_{y(j)}\left(\lambda_{40(j)}-1\right)+\left(\lambda_{04(j)}-1\right)$$

$$\left[B_j^2S^4_{x(j)}+A^2_{1(j)}S^4_{y(j)}+2A_{1(j)}B_jS^2_{y(j)}S^2_{x(j)}\right]$$

$$-2S^2_{y(j)}\left(\lambda_{22(j)}-1\right)\left[B_jS^2_{x(j)}+A_{1(j)}S^2_{y(j)}\right]\}$$

$$<\frac{S_y^4}{n}\left[\left(\lambda_{40}-1\right)+\left(\lambda_{04}-1\right)\left(1-2C\right)\right],$$

$$\left(K_{(j)}-K\right)+L_{(j)}-2M_{(j)}-N\left(1-2C\right)<0\,. \quad (22)$$

(iv) *With the MSE of estimators given in Eqs. (6) and (18):*

$$\mathrm{MSE}\left(t_{rZB(j)}\right)<\mathrm{MSE}\left(t_{CR}\right)\ \frac{1}{n}\ \{\ S_{y(j)}^4\left(\lambda_{40(j)}-1\right)+\left(\lambda_{04(j)}-1\right)$$
$$\left[B_j^2S_{x(j)}^4+A_{1(j)}^2S_{y(j)}^4+2A_{1(j)}B_jS_{y(j)}^2S_{x(j)}^2\right]$$
$$-2S_{y(j)}^2\left(\lambda_{22(j)}-1\right)\left[B_jS_{x(j)}^2+A_{1(j)}S_{y(j)}^2\right]\}$$
$$<\frac{S_y^4}{n}\left[\left(\lambda_{40}-1\right)+4\left(\lambda_{04}-1\right)\left(1-C\right)\right],$$

$$\left(K_{(j)}-K\right)+L_{(j)}-2M_{(j)}-4N\left(1-C\right)<0\,. \quad (23)$$

(v) *With the MSE of estimators given in Eqs. (8) and (18):*

$$\mathrm{MSE}\left(t_{rZB(j)}\right)<\mathrm{MSE}\left(t_{reg}\right)\ \frac{1}{n}\ \{\ S_{y(j)}^4\left(\lambda_{40(j)}-1\right)+\left(\lambda_{04(j)}-1\right)$$
$$\left[B_j^2S_{x(j)}^4+A_{1(j)}^2S_{y(j)}^4+2A_{1(j)}B_jS_{y(j)}^2S_{x(j)}^2\right]$$
$$-2S_{y(j)}^2\left(\lambda_{22(j)}-1\right)\left[B_jS_{x(j)}^2+A_{1(j)}S_{y(j)}^2\right]\}$$
$$<\frac{S_y^4}{n}\left(\lambda_{40}-1\right)\left(1-\rho^2\right),$$

$$\left(K_{(j)}-K\right)+L_{(j)}-2M_{(j)}+K\rho^2<0. \quad (24)$$

(vi) *With the MSE of estimators given in Eqs. (10) and (18),*

$$\mathrm{MSE}\left(t_{rZB(j)}\right)<\mathrm{MSE}\left(t_{US}\right)\ \frac{1}{n}\ \{\ S_{y(j)}^4\left(\lambda_{40(j)}-1\right)+\left(\lambda_{04(j)}-1\right)$$
$$\left[B_j^2S_{x(j)}^4+A_{1(j)}^2S_{y(j)}^4+2A_{1(j)}B_jS_{y(j)}^2S_{x(j)}^2\right]$$
$$-2S_{y(j)}^2\left(\lambda_{22(j)}-1\right)\left[B_jS_{x(j)}^2+A_{1(j)}S_{y(j)}^2\right]\}$$
$$<\frac{S_y^4}{n}\left[\left(\lambda_{40}-1\right)+g_0^2\left(\lambda_{04}-1\right)-2g_0\left(\lambda_{22}-1\right)\right],$$

$$\left(K_{(j)}-K\right)+L_{(j)}-2M_{(j)}-g_0N\left(g_0-2C\right). \quad (25)$$

(vii) *With the MSE of estimators given in Eqs. (14) and (18):*

$$\mathrm{MSE}\left(t_{rZB(j)}\right)<\mathrm{MSE}\left(t_{KCi}\right)\ \frac{1}{n}\ \{\ S_{y(j)}^4\left(\lambda_{40(j)}-1\right)+\left(\lambda_{04(j)}-1\right)$$
$$\left[B_j^2S_{x(j)}^4+A_{1(j)}^2S_{y(j)}^4+2A_{1(j)}B_jS_{y(j)}^2S_{x(j)}^2\right]$$
$$-2S_{y(j)}^2\left(\lambda_{22(j)}-1\right)\left[B_jS_{x(j)}^2+A_{1(j)}S_{y(j)}^2\right]\}$$
$$<\frac{S_y^4}{n}\left[\left(\lambda_{40}-1\right)+g_i^2\left(\lambda_{04}-1\right)-2g_i\left(\lambda_{22}-1\right)\right],$$
$$i=1,2,3\,,$$

$$\left(K_{(j)}-K\right)+L_{(j)}-2M_{(j)}-g_iN\left(g_i-2C\right);\quad \mathrm{i}=1,2,3\,. \quad (26)$$

The proposed class of robust estimators $\left(t_{rZB(j)}\right)$ perform better than all other estimators considered here if Conditions (i)-(vii) are satisfied.

4.2 With the proposed regression-type estimators

(i) *With the MSE of estimators given in Eqs. (1) and (16):*

$$\mathrm{MSE}\left(t_{rZB}\right)<V\left(t_0\right)\ \frac{1}{n}\ \{\ S_y^4\left(\lambda_{40}-1\right)+\left(\lambda_{04}-1\right)$$
$$\left[B^2S_x^4+A_1^2S_y^4+2A_1BS_y^2S_x^2\right]$$
$$-2S_y^2\left(\lambda_{22}-1\right)\left[BS_x^2+A_1S_y^2\right]\}$$
$$<\frac{S_y^4}{n}\left(\lambda_{40}-1\right),\ L-2M<0\,. \quad (27)$$

(ii) *With the MSE of estimators given in Eqs. (3) and (16):*

$$MSE\left(t_{rZB}\right)<MSE\left(t_R\right)\ \frac{1}{n}\{\ S_y^4\left(\lambda_{40}-1\right)+\left(\lambda_{04}-1\right)$$
$$\left[B^2S_x^4+A_1^2S_y^4+2A_1BS_y^2S_x^2\right]$$
$$-2S_y^2\left(\lambda_{22}-1\right)\left[BS_x^2+A_1S_y^2\right]\}$$
$$<\frac{S_y^4}{n}\left[\left(\lambda_{40}-1\right)+\left(\lambda_{04}-1\right)\left(1-2C\right)\right]$$
$$L-2M-N\left(1-2C\right)<0\,. \quad (28)$$

(iii) *With the MSE of estimators given in Eqs. (8) and (16):*

$$MSE\left(t_{rZB}\right)<MSE\left(t_{reg}\right)\ \frac{1}{n}\{\ S_y^4\left(\lambda_{40}-1\right)+\left(\lambda_{04}-1\right)$$
$$\left[B^2S_x^4+A_1^2S_y^4+2A_1BS_y^2S_x^2\right]$$
$$-2S_y^2\left(\lambda_{22}-1\right)\left[BS_x^2+A_1S_y^2\right]\}$$
$$<\frac{S_y^4}{n}\left(\lambda_{40}-1\right)\left(1-\rho^2\right)\ K\rho^2+L-2M<0\,. \quad (29)$$

(iv) *With the MSE of estimators given in Eqs. (10) and (16):*

$$MSE\left(t_{rZB}\right)<MSE\left(t_{US}\right)\ \frac{1}{n}\{\ S_y^4\left(\lambda_{40}-1\right)+\left(\lambda_{04}-1\right)$$
$$\left[B^2S_x^4+A_1^2S_y^4+2A_1BS_y^2S_x^2\right]$$
$$-2S_y^2\left(\lambda_{22}-1\right)\left[BS_x^2+A_1S_y^2\right]\}$$
$$<\frac{S_y^4}{n}\left[\left(\lambda_{40}-1\right)+g_0^2\left(\lambda_{04}-1\right)-2g_0\left(\lambda_{22}-1\right)\right]$$
$$L-2M-g_0N\left(g_0-2C\right)<0\,. \quad (30)$$

(v) *With the MSE of estimators given in Eqs. (14) and (16):*

$$MSE\left(t_{rZB}\right)<MSE\left(t_{KCi}\right);\qquad i=1,2,3.$$
$$\frac{1}{n}\{\ S_y^4\left(\lambda_{40}-1\right)+\left(\lambda_{04}-1\right)\left[B^2S_x^4+A_1^2S_y^4+2A_1BS_y^2S_x^2\right]$$
$$-2S_y^2\left(\lambda_{22}-1\right)\left[BS_x^2+A_1S_y^2\right]\}$$
$$<\frac{S_y^4}{n}\left[\left(\lambda_{40}-1\right)+g_i^2\left(\lambda_{04}-1\right)-2g_i\left(\lambda_{22}-1\right)\right]$$
$$L-2M-g_iN\left(g_i-2C\right),$$
$$i=1,2,3\,. \quad (31)$$

The proposed classical ratio estimators $\left(t_{rZB(j)}\right)$ perform better than all other classical estimators considered here if Conditions (i)-(v) are satisfied.

Table 3. Data statistics used for simple random sampling

Statistics	Classical	MCD	MVE	Real values (without outlier)
$\bar{Y}$	36.34234	18.02439	18.35366	29.27928
$\bar{X}$	448.8649	235.3171	231.8519	394.1622
S_y^2	5999.464	70.07523	78.74556	651.3122
S_x^2	476858.7	15063.36	13397.14	160288.2
C_x	1.538435	0.04707897	0.04640262	1.015724
$\lambda_{04} = \beta_{2x}$	48.54609	47.0208	46.86658	8.994593
$\lambda_{40} = \beta_{2y}$	85.99221	84.78439	84.96302	10.26559
λ_{22}	64.25012	62.63676	62.59346	8.8654
C	1.33029067	1.33932396	1.342883206	0.983839953
ρ	0.9949801	0.9926157	0.9925272	0.9138736
B	0.01673668	0.00623058	0.007893181	0.003997717
g_0	0.99989821	0.99688818	0.996513942	0.999943888
g_1	0.99999677	0.99999687	0.999996536	0.999993663
g_2	0.99999993	0.99999993	0.999999926	0.999999295
g_3	0.99993383	0.93781866	0.929895963	0.999944757
n			30	
N			111	

Table 4. Theoretical results for the MSE of estimators.

Estimators	Uncontaminated data	Contaminated data		
		Classical	MCD	MVE
t_0	131017.7843	1.02E+08	--	--
t_R	131010.0481	7244378	--	--
t_{CR}	131018.3011	26606542	--	--
t_{reg}	21596.32214	1021215	--	--
t_{US}	35627.27383	85386937	--	--
t_{KC1}	35628.89487	7328424	--	--
t_{KC2}	35630.48609	7246107	--	--
t_{KC3}	35627.33348	9000031	--	--
t_{rZB1}	134641.8707	144531.9	1565.182	2159.938
t_{rZB2}	134629.1846	144518.2	1563.343	2157.107
t_{rZB3}	134640.438	144530.4	1565.174	2159.925
t_{rZB4}	134640.5816	144530.5	1565.033	2159.712
t_{rZB5}	134641.7114	144531.7	1565.181	2159.936
t_{rZB6}	134629.381	144518.4	1526.682	2100.859
t_{rZB7}	134640.6016	144530.5	1562.001	2155.131
t_{rZB8}	134640.3029	144530.3	1565.172	2159.922
t_{rZB9}	134633.2569	144531.7	1565.168	2159.915
t_{rZB10}	134627.9891	144517.2	1562.973	2156.455

5. Applications

We use the data in Zaman and Bulut [26] and Zaman et al. [27] in order to compare the performances between the proposed classical ratio estimators and the proposed class of robust estimators given in Eqs. (16) and (18), respectively. The statistics of the population are presented in Table 3. We have contaminated the last observation of this data set by multiplying the Y value by 50 and the value of X by 25.

We proposed two different estimators for simple random sampling in the study. The first estimators are as given in Eq. (15). The MSE of these estimators is given in Eq. (16). We obtained the MSE values of proposed classical estimators in Eq. (15), the unbiased estimator, Isaki [20] estimator in Eq. (2), Singh et al. [21] estimator in Eq. (4), the regression estimator in Eq. (7), Upadhyaya and Singh [22] estimator in Eq. (9), and Kadilar and Cingi estimators in Eq. (11). These values are given in the uncontaminated data part of Table 4.

Table 5. The results of condition in Eq. (20)

Estimators	MCD		MVE	
	Condition values	Results	Condition values	Results
t_{rZB1}	-1.7E+09	True	-1.7E+09	True
t_{rZB2}	-1.7E+09	True	-1.7E+09	True
t_{rZB3}	-1.7E+09	True	-1.7E+09	True
t_{rZB4}	-1.7E+09	True	-1.7E+09	True
t_{rZB5}	-1.7E+09	True	-1.7E+09	True
t_{rZB6}	-1.7E+09	True	-1.7E+09	True
t_{rZB7}	-1.7E+09	True	-1.7E+09	True
t_{rZB8}	-1.7E+09	True	-1.7E+09	True
t_{rZB9}	-1.7E+09	True	-1.7E+09	True
t_{rZB10}	-1.7E+09	True	-1.7E+09	True

Table 6. The results of condition in Eq. (21)

Estimators	MCD		MVE	
	Condition values	Results	Condition values	Results
t_{rZB1}	-3.1E+09	True	-3.1E+09	True
t_{rZB2}	-3.1E+09	True	-3.1E+09	True
t_{rZB3}	-3.1E+09	True	-3.1E+09	True
t_{rZB4}	-3.1E+09	True	-3.1E+09	True
t_{rZB5}	-3.1E+09	True	-3.1E+09	True
t_{rZB6}	-3.1E+09	True	-3.1E+09	True
t_{rZB7}	-3.1E+09	True	-3.1E+09	True
t_{rZB8}	-3.1E+09	True	-3.1E+09	True
t_{rZB9}	-3.1E+09	True	-3.1E+09	True
t_{rZB10}	-3.1E+09	True	-3.1E+09	True

The efficiency of these estimators decreases when there are outliers in the data. Here, a second estimator using robust covariance estimates is proposed to eliminate this negative effect of the outlier. The estimators are given in Eq. (17). The MSE of these estimators is given in Eq. (18). In the presence of outliers in the data, we obtained the MSE values of proposed classical estimators in Eq. (15), the proposed class of robust estimators in Eq. (17) and estimators considered here. These values are given in the contaminated data part of Table 4. According to this part, as inferred by the theoretical comparisons, we observe that all of the proposed class of robust estimators have smaller MSE values than the proposed classical estimators and some existing estimators in data with outliers for simple random sampling. These results are expected results because the Conditions (19)-(26) are satisfied for the proposed class of robust estimators. These situations are clearly seen in Tables 5-11. The most efficient estimators are t_{rZB10} robust estimator based on MCD covariance estimate for the dataset. On the other hand, proposed classical estimators in given Eq. (15) do not provide lower MSE than unbiased, and estimators proposed

Table 7. The results of condition in Eq. (22)

Estimators	MCD		MVE	
	Condition values	Results	Condition values	Results
t_{rZB1}	-3E+09	True	-3E+09	True
t_{rZB2}	-3E+09	True	-3E+09	True
t_{rZB3}	-3E+09	True	-3E+09	True
t_{rZB4}	-3E+09	True	-3E+09	True
t_{rZB5}	-3E+09	True	-3E+09	True
t_{rZB6}	-3E+09	True	-3E+09	True
t_{rZB7}	-3E+09	True	-3E+09	True
t_{rZB8}	-3E+09	True	-3E+09	True
t_{rZB9}	-3E+09	True	-3E+09	True
t_{rZB10}	-3E+09	True	-3E+09	True

Table 8. The results of condition in Eq. (23)

Estimators	MCD		MVE	
	Condition values	Results	Condition values	Results
t_{rZB1}	-3E+09	True	-3E+09	True
t_{rZB2}	-3E+09	True	-3E+09	True
t_{rZB3}	-3E+09	True	-3E+09	True
t_{rZB4}	-3E+09	True	-3E+09	True
t_{rZB5}	-3E+09	True	-3E+09	True
t_{rZB6}	-3E+09	True	-3E+09	True
t_{rZB7}	-3E+09	True	-3E+09	True
t_{rZB8}	-3E+09	True	-3E+09	True
t_{rZB9}	-3E+09	True	-3E+09	True
t_{rZB10}	-3E+09	True	-3E+09	True

Table 9. The results of condition in Eq. (24)

Estimators	MCD		MVE	
	Condition values	Results	Condition values	Results
t_{rZB1}	-6.1E+09	True	-6.1E+09	True
t_{rZB2}	-6.1E+09	True	-6.1E+09	True
t_{rZB3}	-6.1E+09	True	-6.1E+09	True
t_{rZB4}	-6.1E+09	True	-6.1E+09	True
t_{rZB5}	-6.1E+09	True	-6.1E+09	True
t_{rZB6}	-6.1E+09	True	-6.1E+09	True
t_{rZB7}	-6.1E+09	True	-6.1E+09	True
t_{rZB8}	-6.1E+09	True	-6.1E+09	True
t_{rZB9}	-6.1E+09	True	-6.1E+09	True
t_{rZB10}	-6.1E+09	True	-6.1E+09	True

Table 10: The results of condition in Eq. (25)

Estimators	MCD		MVE	
	Condition values	Results	Condition values	Results
t_{rZB1}	-3E+09	True	-3E+09	True
t_{rZB2}	-3E+09	True	-3E+09	True
t_{rZB3}	-3E+09	True	-3E+09	True
t_{rZB4}	-3E+09	True	-3E+09	True
t_{rZB5}	-3E+09	True	-3E+09	True
t_{rZB6}	-3E+09	True	-3E+09	True
t_{rZB7}	-3E+09	True	-3E+09	True
t_{rZB8}	-3E+09	True	-3E+09	True
t_{rZB9}	-3E+09	True	-3E+09	True
t_{rZB10}	-3E+09	True	-3E+09	True

by Isaki [20], Singh et al. [21] the regression, Upadhyaya and Singh [22], and Kadilar and Cingi. This situation is expected because the Conditions (27)-(31) are not satisfied for all estimators under the dataset. This situation is clearly seen in Table 12. Note recall that the proposed classical estimators should be perform better than existing estimators in here if the Conditions (27)-(31) were satisfied.

6. Simulation study

In this section, we use the following simulation study for numerical comparisons. We have used the following models:

$Y_i = 5X_i + \varepsilon_i$ which we generate ε_i and X_i independently and calculate Y_i for $i = 1, 2, . . . , N$. X_i and ε_i have been selected from the following distributions for three different scenarios:

1. X is from $U(0,1)$ and ε is from $N(0,1)$ and independent of X;
2. X is from $Exp(1)$ and ε is from $N(0,1)$ and independent of X;
3. X is from $N(5,1)$ and ε is from $N(0,1)$ and independent of X.

Simulation can be summarized with the steps below. X is generated from above given distributions by taking as $N = 200$. The ratios of outliers are 10 and we have guaranteed that there is the least an outlier in sample selection.

Firstly, classical estimators given in Section 2 are obtained for each sample size, using Simple Random Sampling Without Replacement (SRSWOR).

Then, for each sample taken, the proposed estimators, say s_i^2, such as t_{ZB}, given in Section 3 and the proposed class of robust estimators, $t_{rZB(j)}$, given in Section 4 in simple random sampling are obtained.

The values of MSE for all cases are obtained with the help of Eq. (32).

$$MSE = \frac{1}{10000}\sum_{i=1}^{10000}\left(s_{yi}^2 - S_y^2\right)^2 \tag{32}$$

where S_y^2 is the population variance.

Sample sizes are taken as $n = 20,30$ and 40 under simple random sampling. Tables 13-15 show the values of MSE of the proposed class of robust estimators, proposed regression-type estimators and some existing estimators for the various sample sizes when it comes to uniform, exponential and normal distributions, respectively. These values are computed using Eq. (32). From Tables 13-15 it is concluded that the proposed class of robust estimators are perform better than the proposed classical estimators and some existing estimators for all sample sizes in simple random sampling. All of these findings support the theoretical results in the contaminated data part of Table 4. It is worth to point out that the values of MSE of the proposed class of robust estimators with respect to the classical estimators in Tables 13-15 would decrease notably, when there were more extreme observations in data.

From theoretical and empirical study, the proposed regression-type estimators did not provide a significant advantage over the estimators known in the literature. For example, when X comes from the uniform distribution, the proposed classical estimator t_{rZB6} has a worse result than the estimator t_R, while it has a more efficient results than the estimator t_{US}. In this context, we have developed a new class of robust estimators while there is an outlier in the data. The proposed class of robust estimators provided a significant superiority to the classical estimators considered here in empirical and simulation study. These results are clearly seen in Tables 13-15. The contaminated data part of Table 4 shows these findings clearly.

7. Conclusion

This study has proposed a new class of robust estimators using Minimum Covariance Determinant (MCD) and Minimum Volume Ellipsoid (MVE) estimates in the simple random sampling The expression for Mean Square Error (MSE) of the proposed class of robust estimators are obtained. Conditions are obtained under which the proposed class of robust estimators perform better than the classical estimator and the existing estimators in terms of MSE. In addition, robustness to outliers is a characteristic of the proposed class of estimators. Finally, it is recommended to use the proposed estimators over the classical and other

Table 11. The results of condition in Eq. (26).

	MCD (g_1)		MCD (g_2)		MCD (g_3)		MVE (g_1)		MVE (g_2)		MVE (g_3)	
Estimators	**CV***	**R***	**CV**	**R**	**CV**	**R**	**CV**	**R**	**CV**	**R**	**CV**	**R**
t_{rZB1}	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True
t_{rZB2}	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True
t_{rZB3}	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True
t_{rZB4}	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True
t_{rZB5}	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True
t_{rZB6}	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True
t_{rZB7}	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True
t_{rZB8}	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True
t_{rZB9}	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True
t_{rZB10}	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True	-3E+09	True

*CV: Condition Values; R: Results

Table 12: The results of condition in Eqs. (27)-(31).

	Eq. (27)		Eq. (28)		Eq. (29)		Eq. (30)		Eq. (31 (g_1))		Eq. (31 (g_2))		Eq. (31 (g_3))	
Estimators	**CV***	**R***	**CV**	**R**	**CV**	**R**	**CV**	**R**	**CV**	**R**	**CV**	**R**	**CV**	**R**
t_{rZB1}	108722.5914	F*	3390480.257	F	3700585.629	F	3390469.468	F	3390480.951	F	3390480.334	F	3390486.303	F
t_{rZB2}	108342.0095	F	3390099.675	F	3700205.047	F	3390088.886	F	3390100.369	F	3390099.752	F	3390105.721	F
t_{rZB3}	108679.6106	F	3390437.276	F	3700542.648	F	3390426.487	F	3390437.971	F	3390437.353	F	3390443.322	F
t_{rZB4}	108683.9204	F	3390441.586	F	3700546.958	F	3390430.797	F	3390442.28	F	3390441.663	F	3390447.632	F
t_{rZB5}	108717.8129	F	3390475.478	F	3700580.85	F	3390464.689	F	3390476.173	F	3390475.556	F	3390481.524	F
t_{rZB6}	108347.9007	F	3390105.566	F	3700210.938	F	3390094.777	F	3390106.261	F	3390105.643	F	3390111.612	F
t_{rZB7}	108684.519	F	3390442.185	F	3700547.557	F	3390431.395	F	3390442.879	F	3390442.262	F	3390448.23	F
t_{rZB8}	108675.56	F	3390433.225	F	3700538.598	F	3390422.436	F	3390433.92	F	3390433.303	F	3390439.271	F
t_{rZB9}	108464.1784	F	3390221.844	F	3700327.216	F	3390211.055	F	3390222.538	F	3390221.921	F	3390227.89	F
t_{rZB10}	108306.1456	F	3390063.811	F	3700169.183	F	3390053.022	F	3390064.505	F	3390063.888	F	3390069.857	F

*CV: Condition Values; R: Results; F: False.

Table 13. Simulation results for the MSE of estimators for various sample sizes when it comes to uniform distribution $(X\sim U(0,1))$.

No.	20			30			40		
Estimators	**Classical**	**MCD**	**MVE**	**Classical**	**MCD**	**MVE**	**Classical**	**MCD**	**MVE**
t_R	1.25E+15	--	--	1.37E+13	--	--	1.10E+15	--	--
t_{CR}	4.27E+19	--	--	2.38E+15	--	--	4.02E+19	--	--
t_{reg}	3.36E+12	--	--	2.56E+12	--	--	2.25E+12	--	--
t_{US}	3.79E+12	--	--	2.69E+12	--	--	2.27E+12	--	--
t_{KC1}	9.74E+12	--	--	4.60E+12	--	--	4.35E+12	--	--
t_{KC2}	2.38E+14	--	--	1.19E+13	--	--	1.46E+14	--	--
t_{KC3}	5.18E+12	--	--	3.20E+12	--	--	2.63E+12	--	--
t_{rZB1}	2.14E+15	182403.3	182389.6	2.44E+13	211756.7	211741.2	2.46E+15	306725	306705.4
t_{rZB2}	4.45E+12	182404.9	182392	2.88E+12	211756.3	211741.5	2.44E+12	306725.2	306719.1
t_{rZB3}	1.52E+13	182404.8	182391.9	6.77E+12	211756.3	211741.4	7.21E+12	306725.2	306718.2
t_{rZB4}	1.02E+14	182404.7	182391.8	1.48E+13	211756.3	211741.5	5.91E+13	306725.2	306717.3
t_{rZB5}	4.17E+14	182404	182390.5	2.08E+13	211756.5	211741.2	3.23E+14	306725.1	306711.8
t_{rZB6}	7.03E+12	182404.9	182392	4.03E+12	211756.3	211741.5	3.40E+12	306725.2	306719.1
t_{rZB7}	3.14E+14	182404.6	182391.8	1.95E+13	211756.3	211741.5	2.14E+14	306725.2	306717.2
t_{rZB8}	8.47E+12	182404.9	182391.9	4.84E+12	211756.3	211741.4	4.05E+12	306725.2	306718.6
t_{rZB9}	1.39E+15	182403.7	182390.2	2.34E+13	211756.5	211741.3	1.27E+15	306725	306709.6
t_{rZB10}	3.76E+12	182404.9	182392	2.62E+12	211756.3	211741.4	2.20E+12	306725.2	306719.1

Table 14. Simulation results for the MSE of estimators for various sample sizes when it comes to exponential distribution$(X \sim Exp(1))$

Estimators	Classical	MCD	MVE	Classical	MCD	MVE	Classical	MCD	MVE
t_R	9.06E+17	--	--	4.56E+16	--	--	5.29E+14	--	--
t_{CR}	1.70E+23	--	--	1.94E+21	--	--	4.80E+16	--	--
t_{reg}	2.95E+14	--		1.29E+14	--	--	1.38E+14	--	--
t_{US}	6.50E+14	--	--	2.55E+14	--	--	1.64E+14	--	--
t_{KC1}	1.19E+16	--	--	2.10E+15	--	--	3.52E+14	--	--
t_{KC2}	6.60E+17	--	--	3.43E+16	--	--	5.19E+14	--	--
t_{KC3}	1.93E+15	--	--	6.20E+14	--	--	2.25E+14	--	--
t_{rZB1}	9.39E+17	24190505	24188468	1.55E+17	4975362	4974004	6.74E+14	3406051	3407571
t_{rZB2}	6.83E+14	24190424	24196828	3.58E+14	4975075	4978358	1.76E+14	3406186	3409347
t_{rZB3}	1.27E+16	24190468	24195776	5.49E+15	4975154	4977771	4.23E+14	3406154	3409007
t_{rZB4}	7.71E+17	24190466	24192613	4.40E+16	4975283	4976777	6.30E+14	3406048	3408230
t_{rZB5}	6.85E+17	24190511	24189739	1.16E+17	4975341	4974567	6.58E+14	3406056	3407716
t_{rZB6}	2.07E+15	24190423	24196823	1.16E+15	4975076	4978352	2.54E+14	3406186	3409346
t_{rZB7}	8.84E+17	24190462	24192421	9.42E+16	4975286	4976593	6.56E+14	3406048	3408198
t_{rZB8}	1.01E+15	24190432	24196788	2.19E+15	4975089	4978094	2.39E+14	3406186	3409318
t_{rZB9}	9.36E+17	24190502	24188756	1.47E+17	4975357	4974209	6.72E+14	3406049	3407603
t_{rZB10}	3.06E+14	24190423	24196867	1.85E+14	4975073	4978371	1.40E+14	3406186	3409356

Table 15. Simulation results for the MSE of estimators for various sample sizes when it comes to normal distribution $(X \sim N(5,1))$.

No.	20			30			40		
Estimators	Clsical	MCD	MVE	Classical	MCD	MVE	Classical	MCD	MVE
t_R	9.84E+15	--	--	9.65E+15	--	--	9.91E+15	--	--
t_{CR}	6.48E+16	--	--	1.14E+16	--	--	1.80E+16	--	--
t_{reg}	9.71E+15	--	--	1.02E+16	--	--	9.87E+15	--	--
t_{US}	9.73E+15	--	--	9.65E+15	--	--	9.89E+15	--	--
t_{KC1}	9.82E+15	--	--	9.65E+15	--	--	9.91E+15	--	--
t_{KC2}	9.84E+15	--	--	9.65E+15	--	--	9.91E+15	--	--
t_{KC3}	9.79E+15	--	--	9.65E+15	--	--	9.90E+15	--	--
t_{rZB1}	1.38E+16	9.01E+08	9.01E+08	9.94E+15	9.91E+08	9.91E+08	1.23E+16	1.47E+09	1.47E+09
t_{rZB2}	1.33E+16	9.01E+08	9.01E+08	9.88E+15	9.91E+08	9.91E+08	1.22E+16	1.47E+09	1.47E+09
t_{rZB3}	1.37E+16	9.01E+08	9.01E+08	9.93E+15	9.91E+08	9.91E+08	1.23E+16	1.47E+09	1.47E+09
t_{rZB4}	1.38E+16	9.01E+08	9.01E+08	9.93E+15	9.91E+08	9.91E+08	1.23E+16	1.47E+09	1.47E+09
t_{rZB5}	1.38E6	9.01E+08	9.01E+08	9.94E+15	9.91E+08	9.91E+08	1.23E+16	1.47E+09	1.47E+09
t_{rZB6}	1.36E+16	9.01E+08	9.01E+08	9.91E+15	9.91E+08	9.91E+08	1.23E+16	1.47E+09	1.47E+09
t_{rZB7}	1.38E+16	9.01E+08	9.01E+08	9.94E+15	9.91E+08	9.91E+08	1.23E+16	1.47E+09	1.47E+09
t_{rZB8}	1.37E+16	9.01E+08	9.01E+08	9.92E+15	9.91E+08	9.91E+08	1.23E+16	1.47E+09	1.47E+09
t_{rZB9}	1.38E+16	9.01E+08	9.01E+08	9.94E+15	9.91E+08	9.91E+08	1.23E+16	1.47E+09	1.47E+09
t_{rZB10}	1.31E+16	9.01E+08	9.01E+08	9.87E+15	9.91E+08	9.91E+08	1.22E+16	1.47E+09	1.47E+09

existing estimators, especially in the presence of extreme observations in the data. In future work, we hope to extend the proposed class of robust estimators given in this article to the stratified two-stage sampling.

Funding

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Authors contribution statement

Tolga Zaman: Data curation; Formal analysis; Methodology; Validation; Visualization; Writing – original draft.

Hasan Bulut: Resources; Software; Investigation; Software Supervision; Writing – review and editing

References

1. Kadilar, C., and Cingi, H. "Ratio estimators for the population variance in simple and stratified random sampling", *Applied Mathematics and Computation*, **173**(2), pp. 1047-1059 (2006). https://doi.org/10.1016/j.amc.2005.04.032

2. Khan, M., and Shabbir, J. "A ratio type estimator for the estimation of population variance using quartiles of an auxiliary variable", *Journal of Statistics Applications and Probability*, ***2***(3), pp. 319-325, (2013). http://dx.doi.org/10.12785/jsap/020314

3. Singh, H. P., Pal, S. K., and Solanki, R. S. "A new procedure for estimation of finite population variance using auxiliary information", *Journal of Reliability and Statistical Studies*, *7*(2), pp. 149-160, (2014).

4. Yadav, S.K., Kadilar, C., Shabbir, J., et al. "Improved family of estimators of population variance in simple random sampling", *Journal of Statistical Theory and Practice*, ***9***(2), pp. 219-226 (2015). https://doi.org/10.1080/15598608.2013.856359

5. Yaqub, M., and Shabbir, J. "An improved class of estimators for finite population variance", *Hacettepe Journal of Mathematics and Statistics*, **45**(5), pp. 1641-1660 (2016). http://dx.doi.org/10.15672/HJMS.20156310746

6. Singh, H. P., and Pal, S. K. "Estimation of population variance using known coefficient of variation of an auxiliary variable in sample surveys", *Journal of Statistics and Management Systems*, **20**(1), pp. 91-111, (2017). http://dx.doi.org/10.1080/09720510.2016.1220100

7. Sanaullah, A., Asghar, A., and Hanif, M. "General class of exponential estimator for estimating finite population variance", *Journal of Reliability and Statistical Studies*, ***10***(2), pp. 1-16 (2017).

8. Muneer, S., Khalil, A., Shabbir, J., et al. "A new improved ratio-product type exponential estimator of finite population variance using auxiliary information", *Journal of Statistical Computation and Simulation*, **88**(16), pp. 3179-3192 (2018). https://doi.org/10.1080/00949655.2018.1504947

9. Housila, P., Singh, Surya, K.P., and Yadav, A. "A study on the chain ratio-ratio-type exponential estimator for finite population variance", *Communications in Statistics-Theory and Methods*, **47**(6), pp.1442-1458 (2018). https://doi.org/10.1080/03610926.2017.1321124

10. Sharma, P., Verma, H.K., Singh, R., and Bouza, C.N. "Estimators for population variance using auxiliary information on quartile", *Investigation Operational*, **39**(4), pp. 528-535 (2019).

11. Abid, M., Nazir, H. Z., Riaz, M., Lin, Z., and Tahir, H. M. "Improved ratio estimators using some robust measures", *Hacettepe Journal of Mathematics and Statistics*, **47**(5), pp. 1375-1393 (2018). https://doi.org/10.15672/HJMS.2017.442

12. Abid, M., Ahmed, S., Tahir, M., et al. "Improved ratio estimators of variance based on robust measures", *Scientia Iranica*, **26**(4), pp. 2484-2494 (2019). https://doi.org/10.24200/sci.2018.20604

13. Naz, F., Abid, M., Nawaz, T., et al. "Enhancing the efficiency of the ratio-type estimators of population variance with a blend of information on robust location measures", *Scientia Iranica*, **27**(4), pp. 2040-2056 (2019). https://doi.org/10.24200/sci.2019.5633.1385

14. Zaman, T. and Bulut, H. "Modified regression estimators using robust regression methods and covariance matrices in stratified random sampling", *Communications in Statistics-Theory and Methods*, **49**(14), pp. 3407-3420 (2020). https://doi.org/10.1080/03610926.2019.1588324

15. Bulut, H. and Zaman, T. "An improved class of robust ratio estimators by using the minimum covariance determinant estimation", *Communications in Statistics-Simulation and Computation*, **51**(5), pp. 2457-2463 (2022). https://doi.org/10.1080/03610918.2019.1697818

16. Zaman, T. and Bulut, H. "An efficient family of robust-type estimators for the population variance in simple and stratified random sampling", *Communications in Statistics-Theory and Methods*, **52**(8), pp. 2610-2624 (2023). https://doi.org/10.1080/03610926.2021.1955388

17. Zaman, T., Dünder, E., Audu, A., Alilah, D. A., Shahzad, U., and Hanif, M. "Robust regression-ratio-type estimators

of the mean utilizing two auxiliary variables: A simulation study", *Mathematical Problems in Engineering*, **2021,** pp. 1-9 (2021). https://doi.org/10.1155/2021/6383927

18. Grover, L.K. and Kaur, A. "An improved regression type estimator of population mean with two auxiliary variables and its variant using robust regression method", *Journal of Computational and Applied Mathematics*, **382**, pp. 1-18 (2021). https://doi.org/10.1016/j.cam.2020.113072

19. Zaman, T. and Bulut, H. "A simulation study: Robust ratio double sampling estimator of finite population mean in the presence of outliers", *Scientia Iranica*, **31**(15), pp. 1330-1341 (2024). https://doi.org/10.24200/sci.2021.55813.4418

20. Isaki, C. T. "Variance estimation using auxiliary information", *Journal of the American Statistical Association,* **78**, pp. 117-123 (1983). https://doi.org/10.1080/01621459.1983.10477939

21. Singh, H.P., Pal, S.K., and Yadav, A. "A study on the chain ratio-ratio-type exponential estimator for finite population variance", *Communications in Statistics-Theory and Methods*, **47**(6), pp. 1442-1458 (2018). https://doi.org/10.1080/03610926.2017.1321124

22. Upadhyaya, L. N., and Singh, H. P. "An estimator for population variance that utilizes the kurtosis of an auxiliary variable in sample surveys", *Vikram Mathematical Journal*, **19**(1), pp. 14-17 (1999).

23. Bulut, H., *Multivariate Statistical Methods with R Applications*, Nobel, Ankara (2018).

24. Venables, W.N. and Ripley, B.D. *Modern Applied Statistics with S*, Fourth Edition, Springer, New York (2002).

25. Singh, R. and Malik, S. "Improved estimation of population variance using information on auxiliary attribute in simple random sampling", *Applied Mathematics and Computation*, **235**, pp. 43-49 (2014). https://doi.org/10.1016/j.amc.2014.03.002

26. Zaman, T., and Bulut, H. "Modified ratio estimators using robust regression methods", *Communications in Statistics-Theory and Methods,* **48**(8), pp.2039-2048, (2019).https://doi.org/10.1080/03610926.2018.1441419

27. Zaman, T., Sağlam, V., Sağır, M., Yücesoy, E., and Zobu, M. "Investigation of some estimators via taylor series approach and an application", *American Journal of Theoretical and Applied Statistics*, **3**(5), pp. 141-147 (2014). https://doi.org/10.11648/j.ajtas.20140305.14

Biographies

Tolga Zaman is an Associate Professor at the Department of Mathematical Engineering in Gümüşhane University, Gümüşhane, Turkey. He received his MS and PhD degrees in Statistics from Ondokuz Mayis University Samsun, Turkey in 2013 and 2017, respectively. His research interests are sampling theory, resampling methods, robust statistics, and statistical inference. He has published more than 60 research papers in international/national journals and conferences. He has papers published in journals like Applied Mathematics and Computations, Communication in Statistics: Simulation and Computation, Communication in Statistics: Theory and Methods, Mathematical Population Studies, RevStat-Statistical Jounal, Scientia Irenica.

Hasan Bulut is working as an Associate Professor in Department of Statistics at Ondokuz Mayıs University, where he received his Doctor degree in 2017 based on robust clustering and robust multivariate analyses. His main research interests have been the fields of socio- economic development, robust principal component analysis, robust clustering analysis, multivariate statistical methods, and applied statistics. He has papers published in journals like Socio-economic Planning Sciences, the Journal of Applied Statistics, Communication in Statistics: Theory and Methods, and Communication in Statistics: Simulation and Computation, Scientia Irenica. Moreover, he has a book about multivariate statistical methods with R applications.