Quantifying the Bullwhip Effect in Two-Echelon Competitive Supply Chains Considering Revenue Sharing Contract, Price Volatilities, and Commodity Substitution Policy

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Abstract

This study investigates the bullwhip effect (BWE) under the revenue sharing contract (RSC) in a competitive supply chain network consisting of two or more main and rival suppliers and another layer consisting of several retailers. This study examines, the largest lower bound for the variance ratio of demand estimation from the suppliers' perspective and a bi-objective nonlinear programming model was presented to reduce the BWE and determine the optimal contract parameters. Analysis of the variance changes in retailers' demand estimates by suppliers showed that BWE decreases when suppliers increase the percentage of contract profit for retailers in the intervals of 0.4-0.5 and 0.6-0.8. From a management perspective, the BWE will be minimized in the network if the final price is set within the 50-70 range. Also, the profits of both suppliers remain relatively constant at all points except at point 0.8. Therefore, suppliers can ensure their profits and prevent BWE from aggravating the network by choosing points in the ranges of 0.4-0.5 and 0.6-0.8 while considering the BWE phenomenon. Furthermore, the results show that suppliers can adjust the delivery time in the range of 1-4 range based on their available resources to better meet retailers' needs.

Keywords: Revenue Sharing Contract, Bullwhip Effect, Commodity Substitution Policy, Quantitative Analysis, Game theory.

1. Introduction

BWE is one of the most detrimental phenomena that negatively impacts SCs performance, leading to overproduction and excessive orders, increased inventory, waste and consumption of raw materials/energy, poor customer service, and inaccurate demand forecasting, which incurs significant costs for the SC [1].

The bullwhip effect is amplified at each stage in the supply chain. The effect discovered by Procter & Gamble (P&G) and then brought to extensive academic attention by [2], [3], [4], and [5]. Fig. 1 shows the volatilities of demand and BWE in the SC. It demonstrated several high volatilities in the middle of the time span. When an order is placed by the retailer, there may be a small change in retailers' orders. The volatilities are shown bigger when orders are transferred from retailers to suppliers [3].

[Figure 1]

One approach for analyzing the BWE is the statistical method, which is derived from the variance in orders made by suppliers in the time domain. this approach has been adopted to investigate BWE under lead time, information sharing, batching, and operational deviations. BWE were determined in the supply chain by building a lower bound on the profitability in [2], [6], [7], and [8] . The results indicated that the magnitude of the BWE to a firm varies primarily based on the specific business environment and it could change from 10 to 30 percent. [9] built upon the control-theoretic approach that differs from existing related work on characterizing the BWE of SC.

Multi-layer network modeling helps to understand how contracts that focus on demand transparency or price stability affect the bullwhip effect [10]. Recently, machine learning-based forecasting models, integrated with contractual obligations to minimum order quantities or price discount agreements, can help reduce the bullwhip effect by improving forecast accuracy and providing better alignment between orders and actual demand patterns. Forecast accuracy is key to reducing unnecessary ordering behavior and stabilizing the supply chain[11]. Also, in a study, authers presented a two-stage stochastic optimization model aiming to minimize the overall cost of dealing with unknown demand uncertainty and relying on economic and environmental sustainability to investigate how developing the operational flexibility of the medical supply chain (MSC) can maintain healthcare in the face of disruption risks, such as the COVID-19 pandemic. The authors analyzed the sensitivity of their model responses to different scenarios including seven cases based on demand uncertainty intervals with seven different reliability levels. Their findings showed that regardless of the degree of disruption, their method led to improvements of up to 40% in terms of overall cost, order delivery time, release quantity, and inventory shortage metrics [12]. Also, in a study [13],presented a two-stage stochastic programming model for designing flexible HSC humanitarian supply chains considering viability under unknown demand and capacity uncertainties. Their computational analysis showed that integrating the machine learning algorithm into GA yields superior results in all risk level settings, leading to a win-win situation for all stakeholders in HSCs.

When the downstream sector of supply chain sends an order, the upstream sector processes a part of the data as a signal for prospect demand. The upstream managers reset the demand forecasts based on this signal and orders are sent to the upstream supplier. The demand signal is the main factor creating BWE. The order sent to the supplier indicates the amount required for warehouse replenishment to meet future orders and precautionary reserve. The long schedule may cause weeks of the precautionary reserve, which leads to a BWE [2].

[16] studied a case in which suppliers' capacity is rationed and a shortage occurs. Lin et al. [17] examined the nonlinear theory and dynamic behavior of BWE to analyze customer behavior in a competitive economy and explained that BWE could not explain the dynamic and complex behavior of supply the chain system.

The BWE is strongly influenced by the using forecasting methods, and developments in forecasting methods and data processing cause to have better analysis for the customers' behavior in different aspects. For example, better forecasting for customers' locations, the number of orders, and duration time between two orders are possible using suitable techniques for supply chains [18].

Nguyen et al (2025) developed a hybrid ARIMAX-LSTM forecasting model to improve the ability to capture different combinations of linear and nonlinear patterns in a case study of coffee demand in Vietnam using time series [19].

Recently, some methods for simulating and predicting the BWE have been suggested to refer to for further study. In another paper, the authors examined revenue sharing contracts in competitive supply chain network coordination: Location based on VMI policy and the bullwhip effect [20]. Another approach introduced by [21], is control-theoretical, which is to quantify the BWE of the SC in the frequency domain. This approach employs the transformation to derive the BWE based on the transfer function that maps the output to the input. Yü et al. Therefore, a weak view of the actual demand of customers appears that leading to BWE.

Few studies have examined and analyzed BWE under the SC contracts. This study aims to quantify BWE in the supply chain under the profit-sharing contract. The supply chain of this study comprises one major supplier, one competitor supplier, and multiple retailers. Regarding the competition between suppliers, it is normal to see the substitution of the same commodities by retailers and, subsequently the suppliers' sensitivity to minor volatilites in demand for retailers. Besides the abovementioned cases, price volatilites, delayed order delivery, and precautionary reserve to meet future orders of retailers intensify the BWE, which is investigated in this paper. The novelty aspects of this study are:

- Contract mechanism is used to examine and reduce BWE in the competitive network (in a general supply chain network, participants of the supply chain make diverse decisions; some retailers sign a contract with other suppliers of their chains, while others cooperated without a contract. This paper studies the effect of contract presence in the competitive network on the BWE).

- Creating a network composed of two or more suppliers (major and competitors), and multiple retailers.
- Competition between suppliers to substitute the same commodities and achieve higher utility.
- Estimating the demand depends on the communication between retailers in a network by suppliers.

Because retailers' demand is estimated by major and competitor suppliers, it can be stated that the estimator is sensitive to minor volatilites in retailers' demand, and creates a kind of correlation between estimates. Estimators indeed create a subjective correlation between estimates based on their subjective background of former demand of retailers considering event minor volatilites in these demands. However, such correlation in the actual space of a competitive market is a common case. For instance, suppliers may use various methods, such as moving averages to forecast retailers' demand, but they compare retailers' behaviors considering the prevailing atmosphere in the market, and subsequently, changes in retailers' behavior are confirmed for suppliers when the behavior of the whole network is changed. Therefore, retailers' behavior is not independent of the suppliers' viewpoint.

The next part of the study presents the statement of the problem and its mathematical modeling of it, and then computational and numerical analyses are done. Finally, the conclusion and recommendations for further studies are presented.

2. Statement of Problem

Consider a two-echelon supply chain that comprises of major suppliers (s_1) and competitor supplier (s_1) l= 2, 3,…,L at supplier echelon and includes n retailer at retailer echelon. In this chain, retailers are set in a network with n nodes $N=\{v_i \,|\, i=1,2,...,n\}$. Set L includes edges of this network the weight of each edge is assigned based on the demand dependence and demand cross impacts between retailers from the viewpoint of suppliers. From the viewpoint of suppliers, indeed, a network of retailers is created and its edges represent the coefficient of correlation between network retailers. Because the demand of network retailers is satisfied by one or more suppliers, the supplier perceives a kind of dependence and interaction between retailers based on the retailers' background. This demand correlation and order volatilites between retailers can be seen from the viewpoint of an audience in the real-world and local markets. Each node i has the equivalent weight of W_{i,t,s_l} (weight of possible loss if the same commodity of the competitor supplier is substituted in the retailer's order under the contract with the competitor supplier in period t). The network in fact has undirected edges and one edge exists between two pairs of nodes in a network. It is worth noting, however, that retailers' demands are independent of each other. Before retailers take the order, suppliers take orders from their upper echelon to respond to their retailers' orders and provide their inventories before any order comes from retailers. According to this system, suppliers must estimate their desired retailers' demand. If $Q_{t,i}^{(s_l)}$ indicates the estimate of demand of retailer *i* from the view of the supplier s_l, $corr_{\left(Q_{i,t}^{s_l},Q_{j,t}^{s_l}\right)} = \frac{\operatorname{cov}\left(Q_{i,t}^{s_l},Q_{j,t}^{s_l}\right)}{\sqrt{\operatorname{var} Q_{i,t}^{(s_1)}.\operatorname{var} Q_{j,t}^{(s_1)}}}$ $\forall l$ represents the correlation coefficient demand estimate of retailer *i* from the view of supplier *l* and demand estimate of retailer *j* from the view of the same supplier.

For example, Figure 2 depicts a network with three nodes 1, 2, and 3 in which, demands are responded to by two suppliers. In Figure 2, the continuous line indicates the connection edge between retailers whose demand has been satisfied by two major and competitive suppliers, while the dashed line shows the connection edge between two retailers whose demand is at least satisfied by one supplier. As shown in Figure 2, the demand of Retailers 1 and 2 is estimated by both suppliers, but the demand of Retailer 3 is satisfied only by Supplier 2. Therefore, the edge of connection between retailers 1 and 2 is shown as a continuous line with a weight of $corr\left(Q_{1,t}^{s_1}+Q_{1,t}^{s_2},Q_{2,t}^{s_1}+Q_{2,t}^{s_2}\right)$, while the connection edge between retailers 1 and 3 and retailers 2 and 3 is shown as a dashed line. The weight of the edge between nodes 2 and 3 equals $corr\left(Q_{2,t}^{s_1}+Q_{2,t}^{s_2},Q_{3,t}^{s_1}\right)$ and the weight of the edge between nodes 1 and 3 equals $corr\left(Q_{1,t}^{s_1}+Q_{1,t}^{s_2},Q_{3,t}^{s_1}\right)$. For example, to explain the figure, the correlation coefficient between retailers 1 and 2 in t equal 10 is equal to 0.5, as well as the correlation coefficient between retailers 1 and 3 is equal to 0.4 and the correlation coefficient between retailers 2 and 3 is equal to 0.6, in which case it represents the correlation between retailers 2 and 3 is greater than retailers 1 and 2 as well as retailers 1 and 3.

[Figure 2]

Assume that $\varphi_{i,t}^{(s_l)}$ represents the supplier of total profit between supplier and retailer. This contract is described with two parameters ($c_{s_l}, \varphi_{i,t}^{(s_l)}$) in which the supplier receives a wholesale price C_{s_l} that is less than the final cost of entity (w_{s_l}) from the retailer in exchange for

$\left(1-\varphi_{i,t}^{(s_l)}\right)\%$ of the retailer's revenue. Constraint $\left(w_{s_l} < c_{s_l}\right)$ sets channel coordination (Giannoccaro and Pontrandolfo 2009). Moreover, it is assumed that a retailer can sign a contract with two suppliers, but only one supplier can take the order, and only one contract of suppliers is active. This policy leads to the substitution of the same commodity supplied by the competitor. Commodity substitution by the retailer makes the condition vague for the supplier because the retailer may have taken an order from the major supplier in one period but substitutes the competing commodity with the commodity provided by the major supplier during the period and vice versa. In addition, it is assumed that the competitor supplier signs a new revenue-sharing contract with retailers in the network retailers to substitute its commodity. A kind of discontinuity appears in the retailer's orders taken from a specific supplier, making the supplier's inventory control problematic. The reason is that the supplier is not sure whether the retailer takes an order from him or not in the next period; on the other hand, this case makes the supplier have a weak view of the actual demand of the retailer during different periods.

The length of commodity procurement led by the major supplier is the main reason for commodity substitution by the competitor supplier. Here, the major supplier shortens commodity led length to achieve timely delivery of the commodity and decrease the loss caused by the contract that the competitor supplier signs with retailers. This supplier considers a commodity-based investment (C_{prev_i}) besides an optimal estimate for each retailer in the network to prevent commodity shortage and prospect substitution of competitor supplier's commodity. To make the competitive atmosphere of the network more actual, it is assumed that the major supplier follows the compensation policy instead of timely non-lead, and the competitor supplier knows the order rate of the major supplier; therefore, a game with complete information is drawn between two major and competitor suppliers. In this game, the major supplier must decide to reduce the loss of attacks by the competitor in the network and its similar commodity substitution by determining the optimum rate of ISA parameters and lead periods. Competitor supplier substitutes the commodity by retailers based on the decision of the major supplier to determine the optimum rate for parameters of the revenue sharing contract and commodity lead period. The competitor supplier indeed tries to increase the probability of commodity substitution with retailers and attract retailers by selecting suitable rates for profit-sharing contract parameters and the proper value of commodity lead.

Parameters and variables of the problem are defined before defining the suppliers' strategies:

η_i	Non-utility rate of competitor supplier's contract with retailer *i* for the major supplier (this value does not depend on the commodity shortage) ($\eta_i \geq 0$).

$\rho_{i,t}$	Price of the commodity for retailer *i* in period *t.*
$\varphi_{i,t}^{(s_l)}$	Parameter of revenue sharing contract and equals percentage profit assigned to retailer *i* by supplier *l* in period *t* ($0 \le \varphi_{i,t}^{(s_l)} \le 1$).
$Q_{i,t}$	Size of commodity order in period *t* by retailer *i*.
w_{s_l}	The wholesale price of supplier *l* given to suppliers during the contract period.
W_{i,t,s_l}	Possible loss of major supplier caused by the contract signed between retailer *i* and supplier *l* in period *t.*
n	Number of nodes in network N (network size).
$corr_{\left(Q_{i,t}^{s_l}, Q_{j,t}^{s_l}\right)}$	Highest demand correlation between two retailers *i* and *j* from the view of supplier *l.*
$\delta_i W_{i,t,s_2}$	Utility rate of the contract signed between competitor supplier and retailer *i* in period *t* regarding the weight of expected loss for retailer *i*.
$\eta_i W_{i,t,s_1}$	Non-utility rate of the contract function from competitor up to retailer *i* from viewpoint of competitor supplier in period *t.*
C_{s_l}	Purchase price of the commodity from external sources by supplier *l.*
v_l	The utility function of supplier *l* in the equilibrium state.
M	One large positive number.
L_{i,s_l}	Commodity lead period of the retailer *i* by supplier *l.*
γ	Penalty for late delivery of commodity for competitor supplier.
α	Penalty for late delivery of commodity for a major supplier.

2.1. Players' strategies (major supplier and competitor suppliers)

The major supplier's strategy is to determine optimal values of contract parameters and optimal lead length in the network. The major supplier incurs a cost called the cost of accelerating commodity lead (C_{prev_i}) to prevent possible loss of retailer *i*, which this cost is spent on transportation and timely commodity distribution. The probability of a competitor supplier's success in substituting the same commodity under the contract with a retailer equals $P\left(C_{prev_i}\right)$ in which, *P* represents decreasing continuous convex function; therefore, the strategy of the major supplier is $S_1 = \left(x_1, x_2, ..., x_n\right)$ in which $x_i = \left(\varphi_{i,t}^{(s_l)}, \rho_{i,t}, w_{s_l}, L_{i,s_l}\right)$.

The strategy of competitor suppliers is shown in S_2. Competitor suppliers can draw a mixed strategy by making m possible choices among retailers for contract signing. Here, the competitor supplier's strategy will be a possible vector $S_2 = \left(p_1, p_2, ..., p_n\right)$ in which p_i represents the probability of substitution and selling commodity under a contract signed with retailer *i*. It is assumed that $p_i > \cdot \sum_{i=1}^{n} p_i = 1$ and. Moreover, losing the contract between a competitive supplier and one retailer does not affect the other retailers.

2.2. Measuring the weight of loss for a major supplier under the revenue sharing contract

A loss rate is considered for major suppliers- or a profit rate for competitor suppliers for each retailer- based on the relative profitability of the revenue sharing contract, which is measured in Equation (1):

$$W_{i,t,s_l} = \left(1-\varphi_{i,t}^{(s_l)}\right)\rho_{i,t}Q_{i,t} + \left(w_{s_l} - c_{s_l}\right)Q_{i,t} \qquad \forall l = 1,2,...,L \tag{1}$$

2.3. The utility function of major and competitive suppliers signing the contract with retailers under the RSC

If the contract signed between a competitor supplier and one retailer is successful, the retailer allows the competitor to meet his/her demand during that period. It is assumed the cost (loss) of the major supplier is a linear function (f_i), which is matched with the interval between commodity transfer from order taking to time of retailer delivery i. Here, equation f_i and the expected utility function of the major supplier equals Equation (2). The utility or non-utility of a retailer from the view of a supplier depends on some factors, such as reliability, demand, place, marketing rate of the retailer, etc.

$$f_i = \alpha L_{i,s_1} + \eta_i :$$

$$\begin{aligned} U_{S_1}\left(S_2, S_1\right) &= -P\left(C_{prev}\right)\sum_{i=1}^{n} f_i W_{i,t,s_1} - \sum_i C_{prev_i} \\ &= -P\left(C_{prev}\right)\sum_{i=1}^{n}\left[\left(\alpha L_{i,s_l} + \eta_i\right)\left(\left(1-\varphi_{i,t}^{(s_1)}\right)\rho_{i,t}Q_{i,t} + \left(w_{s_1} - c_{s_1}\right)Q_{i,t}\right)\right] - \sum_i C_{prev_i} \end{aligned} \tag{2}$$

If the same commodity of the competitor is substituted under the contract signed between the competitor supplier and desired retailer, the competitor supplier's profit depends on the parameters of that contract with the retailer. The expected utility function of competitor supplier equals:

$$\begin{aligned} U_{S_l}\left(S_l, S_1\right) &= \sum_{i=1}^{n} P_i\left(\gamma L_{i,s_l} + \delta_i\right)W_{i,t}^{(s_l)} \\ &= \sum_{i=1}^{n} P_i\left(\gamma L_{i,s_l} + \delta_i\right)\left(\left(1-\varphi_{i,t}^{(s_l)}\right)\rho_{i,t}Q_{i,t} + \left(w_{s_l} - c_{s_l}\right)Q_{i,t}\right) \qquad \forall l = 2,3,...,L \end{aligned} \tag{3}$$

According to the points mentioned above, although some tools are like a zero-sum game, we face a non-zero-sum game because the profit obtained by a competitive supplier under the contract signed with a retailer is not necessarily equal to the loss incurred by the major supplier because of missed profit under the contract signed with the same retailer. The reason is that contract parties may have different parameters, so this case may cause asymmetry. It should be noted that this model assumes that symmetric information exists between suppliers, and the major supplier knows the parameters $\gamma\ \delta_i$ of the competitor suppliers. This assumption is obviously possible based on the information about competitors obtained by the major supplier.

2.4. Equilibrium

The major supplier first selects the strategy S_1 then the competitor supplier makes the strategic decision $\{S_l(S_1)\}_{l=2,3,...,L}$. In this game, equilibrium for each S_1 and $S_1(S_1)$ equals [30]:

$$\begin{cases} S_1 \Rightarrow U_{S_1}\left(S_l^*(0), S_1\right) \le U_{S_1}\left(S_l^*(0), S_1^*\right) \\ S_l(S_1) \Rightarrow U_{S_l}\left(S_2(0), S_1^*\right) \le U_{S_l}\left(S_l^*(0), S_1^*\right) \end{cases} \tag{4}$$

Theorem 1. A pure strategy exists for each decision made by the major supplier, which is the best response to the competitor supplier [30].

The major supplier may set the parameters of the contract signed with retailers. If x is the strategy taken by the major supplier, we will have:

$$\underset{x\in N}{Min} U_{S_1}\left(S_l, S_1\right) = \sum_{i=1}^{n} P_i\left(\alpha L_{i,s_1} + \eta_i\right)\left(\left(1-\varphi_{i,t}^{(s_1)}\right)\rho_{i,t}Q_{i,t} + \left(w_{s_1} - c_{s_1}\right)Q_{i,t}\right) \tag{5}$$

Under such circumstances, the utility function of the competitor supplier equals:

$$\underset{0\le p\le 1}{U_{S_l}}\left(S_l, S_1\right) = \sum_{i=1}^{n} P_i\left(\gamma L_{i,s_l} + \delta_i\right)\left(\left(1-\varphi_{i,t}^{(s_l)}\right)\rho_{i,t}Q_{i,t} + \left(w_{s_l} - c_{s_l}\right)Q_{i,t}\right) \quad \forall l = 2,3,...,L \tag{6}$$

2.5. Preventive Costs

If selected values for contract parameters and commodity lead period by the major supplier are shown by x^* in the equation and the retailer of the contract signed with the competitor supplier is indicated with i^*, the utility function of the major supplier equals Equation (7) based on the preventive costs:

$$U_{S_1}\left(x^*, i^*\right) = -P\left(C_{prev_i}\right)\left[\alpha L_{i^*,s_2} + \eta_{i^*}\right]W_{i^*,t,s_1} - C_{prev_{i^*}} \tag{7}$$

3. BWE index used in the study

As it was mentioned before, BWE intensifies demand volatilities in the movement toward upstream nodes in the supply chain. The ratio of created order variance (output) to the demand variance (input) is used to measure the bullwhip effect. This variance ratio (VR) is expressed as $VR = \frac{\sigma_{output}^2}{\sigma_{input}^2}$. If VR>1, BWE occurs. If VR<1 then smoothing occurs, and if VR=1 then input order up to output order policy occurs [11].

4. Problem Modeling

In this section, the problem is first examined with two (major and competitor) suppliers and multiple retailers, and then it is generalized and assessed for the mode of multiple suppliers and retailers.

4.1. Two suppliers and multiple retailers

Assume that major suppliers and retailers use the moving average method based on *m* previous periods to forecast demand, and the supplier wants to increase inventory rate greater than the optimal inventory in each period. Demand average is defined $\chi_t = \frac{\sum_{k=1}^{m}\sum_{i=1}^{n} Q_{i,t-k}}{nm}$ for *m* previous network periods with *n* retailers. Here, each network retailer follows an available policy in which, the order point is determined based on the observed demand of end-consumer y^{Q}_{i,t,s_l} shown in Equation (8).

$$y^{Q}_{i,t,s_l} = \mu_{i,t,s_l} + zSD_{i,t,s_l} \tag{8}$$

Where μ_{i,t,s_l} indicate mean order observed by supplier s_1 for retailer *i* in *m* previous periods ($\mu_{i,t,s_l} = L_{i,s_l}\chi_t$), z shows the constant coefficient of normal distribution graph based on the optimal inventory, and SD_{i,t,s_l} is the standard deviation from optimal inventory in period *t* that is ordered by the retailer *i* to supplier s_1.

The following equations are used:

$$W^{(s_l)}_{(i,t)/(Q)} = \left(\left(1-\varphi^{(s_l)}_{i,t}\right)\rho_{i,t} + \left(w_{s_l} - c_{s_l}\right)\right) \qquad \forall l = 1,2 \tag{9}$$

Definition 1. Following equation is defined for threshold limit $\left(\left(1+\frac{2L_{i,s_l}W^{(s_l)}_{(i,t)/(Q)}}{m}+\frac{2\left(L_{i,s_l}W^{(s_l)}_{(i,t)/(Q)}\right)^2}{m^2}\right)\left(\frac{1}{\left(W^{(s_l)}_{(i,t)/(Q)}\right)^2}\right)\text{-}2\frac{\sum_i\sum_{j(i<j)}\operatorname{cov}\left(Q^{(s_l)}_{i,t},Q^{(s_l)}_{j,t}\right)}{\sum_{i=1}^{n}\operatorname{var}\left(\mathrm{Q}_{i,t}\right)}\right)$ per i = 1,2,…,*n* and real number ψ :

$$\limsup_{n\to\infty}\left(\left(1+\frac{2L_{i,s_l}W^{(s_l)}_{(i,t)/(Q)}}{m}+\frac{2\left(L_{i,s_l}W^{(s_l)}_{(i,t)/(Q)}\right)^2}{m^2}\right)\left(\frac{1}{\left(W^{(s_l)}_{(i,t)/(Q)}\right)^2}\right)\text{-}2\frac{\sum_i\sum_{j(i<j)} corr_{\left(Q^{s_l}_{i,t},Q^{s_l}_{j,t}\right)}\sqrt{\operatorname{var}Q^{(s_1)}_{i,t}.\operatorname{var}Q^{(s_1)}_{j,t}}}{\sum_{i=1}^{n}\operatorname{var}\left(\mathrm{Q}_{i,t}\right)}\right)\le\psi \tag{10}$$

Since suppliers and retailers follow RSC, $y^{W_{i,t,s_l}}_{i,t}$ (the profit obtained from the RSC signed with retailer *i* for supplier s_l based on y^{Q}_{i,t,s_l}) is defined as Equation (11). The lead time is considered the same for all periods to simplify calculations.

$$\begin{aligned} y^{W_{i,t,s_l}}_{i,t} &= W^{(s_l)}_{(i,t)/(Q)} y^{Q}_{i,t,s_l} \\ y^{W_{i,t,s_l}}_{i,t} &= W^{(s_l)}_{(i,t)/(Q)}\mu_{i,t,s_l} + z\sqrt{\left(W^{(s_l)}_{(i,t)/(Q)}\right)^2} SD_{i,t,s_l} \quad \forall l = 1,2 \end{aligned} \tag{11}$$

Equation (12) can be written based on the RSC for retailer i in which, $\Delta_{i,t}^{(s_l)}$ indicates the profit obtained from revenue sharing contract based on the $Q_{i,t}^{(s_l)}$.

$$Q_{i,t}^{(s_l)} = y_{i,t,s_l}^{Q} - y_{i,t-1,s_l}^{Q} + \mathrm{Q}_{i,t-1} \qquad \forall l = 1,2$$
$$\Delta_{i,t}^{(s_l)} = W_{(i,t)/(Q)}^{(s_l)} Q_{i,t}^{(s_l)}, \quad \Delta_{i,t}^{(s_l)} = y_{i,t}^{W_{i,t,s_l}} - y_{i,t-1}^{W_{i,t,s_l}} + \mathrm{Q}_{i,t-1} W_{(i,t)/(Q)}^{(s_l)} \qquad \forall l = 1,2 \tag{12}$$

Since n retailers exist in the network. Equation (12) is developed to Equation (13) which, $y_{i,t}^{T,W_{i,t,s_l}}$ indicates the sum profit obtained from RSC for supplier s_l per n retailers based on the y_{i,t,s_l}^{Q}, and $\Delta_{i,t}^{T,(s_l)}$ is the profit obtained from the RSC for supplier s_l per n retailers based on the $Q_{i,t}^{(s_l)}$.

$$y_{i,t}^{T,W_{i,t,s_l}} = \sum_{i=1}^{n} W_{(i,t)/(Q)}^{(s_l)} \mu_{i,t,s_l} + \sum_{i=1}^{n} z W_{(i,t)/(Q)}^{(s_l)} SD_{i,t,s_l} \quad \forall l = 1,2 \tag{13}$$

$$\Delta_{i,t}^{T,(s_l)} = \sum_{i=1}^{n} Q_{i,t}^{(s_l)} W_{(i,t)/(Q)}^{(s_l)}, \quad \Delta_{i,t}^{T,(s_l)} = y_{i,t}^{T,W_{i,t,s_l}} - y_{i,t-1}^{T,W_{i,t,s_l}} + \sum_{i=1}^{n} \mathrm{Q}_{i,t-1} W_{(i,t)/(Q)}^{(s_l)} \qquad \forall l = 1,2$$

$$\Delta_{i,t}^{T,(s_l)} = \sum_{i=1}^{n} \left(\mu_{i,t,s_l} - \mu_{i,t-1,s_l} \right) W_{(i,t)/(Q)}^{(s_l)} - \sum_{i=1}^{n} z W_{(i,t)/(Q)}^{(s_l)} \left(SD_{i,t,s_l} - SD_{i,t-1,s_l} \right) + \sum_{i=1}^{n} \mathrm{Q}_{i,t-1} W_{(i,t)/(Q)}^{(s_l)}$$

$$= \sum_{i=1}^{n} \left(L_{i,s_l} W_{(i,t)/(Q)}^{(s_l)} \left(\frac{\sum_{i=1}^{n} \mathrm{Q}_{i,t-1} - \sum_{i=1}^{n} \mathrm{Q}_{i,t-p-1}}{mn} \right) \right) + \sum_{i=1}^{n} \mathrm{Q}_{i,t-1} W_{(i,t)/(Q)}^{(s_l)} + z \sum_{i=1}^{n} W_{(i,t)/(Q)}^{(s_l)} \left(SD_{i,t,s_l} - SD_{i,t-1,s_l} \right)$$

$$= \sum_{i=1}^{n} \left(\left(W_{(i,t)/(Q)}^{(s_l)} + \frac{L_{i,s_l} W_{(i,t)/(Q)}^{(s_l)}}{mn} \right) \sum_{i=1}^{n} \mathrm{Q}_{i,t-1} \right) - \sum_{i=1}^{n} \left(\left(\frac{L_{i,s_l} W_{(i,t)/(Q)}^{(s_l)}}{mn} \right) \sum_{i=1}^{n} \mathrm{Q}_{i,t-p-1} \right) +$$
$$\mathrm{z} \left(\sum_{i=1}^{n} W_{(i,t)/(Q)}^{(s_l)} \left(SD_{i,t,s_l} - SD_{i,t-1,s_l} \right) \right)$$

According to Equation (13) and pairwise demand independence between retailers, $Q_{i,t}^{T,(s_l)} = \sum_{i=1}^{n} Q_{i,t}^{(s_l)}$ variance can be written as Equation (14):

$$\begin{aligned}
\operatorname{var}\left(\Delta_{i,t}^{T,(s_l)}\right) &= \left(\sum_{i=1}^{n}\left(W_{(i,t)/(Q)}^{(s_l)}+\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)\right)^2 \operatorname{var}\left(\sum_{i=1}^{n}\mathrm{Q}_{i,t-1}\right) \\
&-2\left(\sum_{i=1}^{n}\left(W_{(i,t)/(Q)}^{(s_l)}+\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)\right)\left(\sum_{i=1}^{n}\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)\operatorname{cov}\left(\sum_{i=1}^{n}\mathrm{Q}_{i,t-1},\sum_{i=1}^{n}\mathrm{Q}_{i,t-p-1}\right) \\
&+\left(\sum_{i=1}^{n}\left(\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)\right)^2 \operatorname{var}\left(\sum_{i=1}^{n}\mathrm{Q}_{i,t-p-1}\right) \\
&+\left(\mathrm{z}W_{(i,t)/(Q)}^{(s_l)}\right)^2 \operatorname{var}\left(\sum_{i=1}^{n}\left(SD_{i,t,s_l}-SD_{i,t-1,s_l}\right)\right)+ \\
&2z\left(\sum_{i=1}^{n}\left(W_{(i,t)/(Q)}^{(s_l)}+\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)\right)\operatorname{cov}\left(\sum_{i=1}^{n}\mathrm{Q}_{i,t-1},\sum_{i=1}^{n}SD_{i,t,s_l}\right) \\
&=\left(\left(\sum_{i=1}^{n}\left(W_{(i,t)/(Q)}^{(s_l)}+\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)\right)^2+\left(\sum_{i=1}^{n}\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)^2\right)\operatorname{var}\left(\sum_{i=1}^{n}\mathrm{Q}_{i,t}\right) \\
&+2z\left(1+\frac{2L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{m}\right)\operatorname{cov}\left(\sum_{i=1}^{n}\mathrm{Q}_{i,t-1},\sum_{i=1}^{n}SD_{i,t,s_l}\right)+\left(\mathrm{z}W_{(i,t)/(Q)}^{(s_l)}\right)^2\operatorname{var}\left(\sum_{i=1}^{n}\left(SD_{i,t,s_l}-SD_{i,t-1,s_l}\right)\right)
\end{aligned} \tag{14}$$

If $\operatorname{cov}\left(\sum_{i=1}^{n}\mathrm{Q}_{i,t-1},\sum_{i=1}^{n}SD_{i,t,s_l}\right)=0$ then we will have the:

$$\begin{aligned}
&\frac{\operatorname{var}\left(\Delta_{i,t}^{T,(s_l)}\right)}{\operatorname{var}\left(\sum_{i=1}^{n}\mathrm{Q}_{i,t}\right)} \geq \left(\sum_{i=1}^{n}\left(W_{(i,t)/(Q)}^{(s_l)}+\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)\right)^2+\left(\sum_{i=1}^{n}\left(\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)\right)^2 \\
&\Rightarrow \frac{\operatorname{var}\left(\sum_{i=1}^{n}W_{(i,t)/(Q)}^{(s_l)}Q_{i,t}^{(s_l)}\right)}{\operatorname{var}\left(\sum_{i=1}^{n}\mathrm{Q}_{i,t}\right)} \geq \left(\sum_{i=1}^{n}\left(W_{(i,t)/(Q)}^{(s_l)}+\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)\right)^2+\left(\sum_{i=1}^{n}\left(\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)\right)^2
\end{aligned} \tag{15}$$

4.2. Problem development to multiple suppliers-retailers model

If over two suppliers exist $\{l = 1,2,\ldots, L\}$, one supplier is taken as the major and the others as competitor suppliers. In the real world, some retailers may sign revenue sharing contracts with some suppliers, while some of them have no contracts. This study assumes that all network retailers have signed a contract with all suppliers. This is a beneficial assumption for study in two ways; first, the most complete scenario is examined in terms of contract presence and quantification of BWE when a contract is signed. Second, the commodity substitution policy taken by retailers and creating a type of non-continuity and irregularity in orders lead to a competitive atmosphere in which, each supplier considers a precautionary reserve if the order is done by the retailer. According to the mentioned points, the sum demand estimate variance by all suppliers ($\sum_{l=1}^{k} Q_{i,t}^{T,(s_l)}$) is measured based on the following equations:

$$
\begin{aligned}
&\sum_{l=1}^{L}\Delta_{i,t}^{T,(s_l)}=\sum_{l=1}^{L}\sum_{i=1}^{n}Q_{i,t}^{(s_l)}W_{(i,t)/(Q)}^{(s_l)},\quad \sum_{l=1}^{L}\Delta_{i,t}^{T,(s_l)}=\sum_{l=1}^{L}\left(y_{i,t}^{T,W_{i,t,s_l}}-y_{i,t-1}^{T,W_{i,t,s_l}}+\sum_{i=1}^{n}\mathrm{Q}_{i,t-1}W_{(i,t)/(Q)}^{(s_l)}\right)\\
&\sum_{l=1}^{L}\Delta_{i,t}^{T,(s_l)}=\sum_{l=1}^{L}\left(\sum_{i=1}^{n}\left(\mu_{i,t,s_l}-\mu_{i,t-1,s_l}\right)W_{(i,t)/(Q)}^{(s_l)}-\sum_{i=1}^{n}zW_{(i,t)/(Q)}^{(s_l)}\left(SD_{i,t,s_l}-SD_{i,t-1,s_l}\right)+\sum_{i=1}^{n}\mathrm{Q}_{i,t-1}W_{(i,t)/(Q)}^{(s_l)}\right)\\
&=\sum_{l=1}^{L}\left(\sum_{i=1}^{n}L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}\left(\frac{\sum_{i=1}^{n}\mathrm{Q}_{i,t-1}-\sum_{i=1}^{n}\mathrm{Q}_{i,t-p-1}}{mn}\right)+\sum_{i=1}^{n}\mathrm{Q}_{i,t-1}W_{(i,t)/(Q)}^{(s_l)}+z\sum_{i=1}^{n}W_{(i,t)/(Q)}^{(s_l)}\left(SD_{i,t,s_l}-SD_{i,t-1,s_l}\right)\right)\\
&=\sum_{l=1}^{L}\left(\sum_{i=1}^{n}\left(W_{(i,t)/(Q)}^{(s_l)}+\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)\sum_{i=1}^{n}\mathrm{Q}_{i,t-1}\right)-\sum_{l=1}^{L}\left(\sum_{i=1}^{n}\left(\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)\sum_{i=1}^{n}\mathrm{Q}_{i,t-p-1}\right)\\
&\quad+\sum_{l=1}^{L}\left(\mathrm{z}\left(\sum_{i=1}^{n}W_{(i,t)/(Q)}^{(s_l)}\left(SD_{i,t,s_l}-SD_{i,t-1,s_l}\right)\right)\right)
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
\mathrm{var}\left(\sum_{l=1}^{L}\Delta_{i,t}^{T,(s_l)}\right)&=\left(\sum_{l=1}^{L}\sum_{i=1}^{n}\left(W_{(i,t)/(Q)}^{(s_l)}+\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)\right)^2\mathrm{var}\left(\sum_{i=1}^{n}\mathrm{Q}_{i,t-1}\right)\\
&-2\left(\sum_{l=1}^{L}\sum_{i=1}^{n}\left(W_{(i,t)/(Q)}^{(s_l)}+\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)\right)\left(\sum_{l=1}^{L}\sum_{i=1}^{n}\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)\mathrm{cov}\left(\sum_{i=1}^{n}\mathrm{Q}_{i,t-1},\sum_{i=1}^{n}\mathrm{Q}_{i,t-p-1}\right)\\
&+\left(\sum_{l=1}^{L}\sum_{i=1}^{n}\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)^2\mathrm{var}\left(\sum_{i=1}^{n}\mathrm{Q}_{i,t-p-1}\right)\\
&+\sum_{l=1}^{L}\left(\left(\mathrm{z}W_{(i,t)/(Q)}^{(s_l)}\right)^2\mathrm{var}\left(\sum_{i=1}^{n}\left(SD_{i,t,s_l}-SD_{i,t-1,s_l}\right)\right)\right)\\
&+2\sum_{l=1}^{L}\left(z\left(\sum_{i=1}^{n}\left(W_{(i,t)/(Q)}^{(s_l)}+\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)\right)\mathrm{cov}\left(\sum_{i=1}^{n}\mathrm{Q}_{i,t-1},\sum_{i=1}^{n}SD_{i,t,s_l}\right)\right)\\
&=\left(\left(\sum_{l=1}^{L}\sum_{i=1}^{n}\left(W_{(i,t)/(Q)}^{(s_l)}+\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)\right)^2+\left(\sum_{l=1}^{L}\sum_{i=1}^{n}\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)^2\right)\mathrm{var}\left(\sum_{i=1}^{n}\mathrm{Q}_{i,t}\right)\\
&+2\sum_{l=1}^{L}\left(z\left(1+\frac{2L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{m}\right)\mathrm{cov}\left(\sum_{i=1}^{n}\mathrm{Q}_{i,t-1},\sum_{i=1}^{n}SD_{i,t,s_l}\right)\right)\\
&+\sum_{l=1}^{L}\left(\left(\mathrm{z}W_{(i,t)/(Q)}^{(s_l)}\right)^2\mathrm{var}\left(\sum_{i=1}^{n}\left(SD_{i,t,s_l}-SD_{i,t-1,s_l}\right)\right)\right)
\end{aligned}
\tag{17}
$$

If $\overline{W}_{(i,t)/(Q)}^{(s_l)}=\max\left\{W_{(i,t)/(Q)}^{(s_l)}\right\}_{i=1,2,\dots,n,\ l=1,2,\dots,L}$, the variance ratio is more intense and Equation (18) can be written:

$$\frac{\left(\overline{W}^{(s_l)}_{(i,t)/(Q)}\right)^2\left(\sum_{l=1}^{L}\left(\sum_{i=1}^{n}\mathrm{var}\left(Q^{(s_l)}_{i,t}\right)\right)+2\sum_{l}\sum_{k(l<k)}\mathrm{cov}\left(\sum_{i=1}^{n}Q^{(s_l)}_{i,t},\sum_{i=1}^{n}Q^{(s_l)}_{j,t}\right)\right)}{\mathrm{var}\left(\sum_{i=1}^{n}\mathrm{Q}_{i,t}\right)} \geq$$

$$\left(\left(\sum_{l=1}^{L}\sum_{i=1}^{n}\left(W^{(s_l)}_{(i,t)/(Q)}+\frac{L_{i,s_l}W^{(s_l)}_{(i,t)/(Q)}}{mn}\right)\right)^2+\left(\sum_{l=1}^{L}\sum_{i=1}^{n}\frac{L_{i,s_l}W^{(s_l)}_{(i,t)/(Q)}}{mn}\right)^2\right) \quad (18)$$

$$\forall i=1,2,\ldots,n \quad ,\forall l,k=1,2,\ldots,L$$

If $A=\max\left\{\frac{\mathrm{var}\left(\sum_{i=1}^{n}Q^{(s_l)}_{i,t}\right)}{\mathrm{var}\left(\sum_{i=1}^{n}Q_{i,t}\right)}\right\}_{l=1,2,\ldots,L}$ and $corr=\max\left\{corr\left(\sum_{i=1}^{n}Q^{(s_l)}_{i,t},\sum_{i=1}^{n}Q^{(s_k)}_{j,t}\right)\right\}_{l,k=1,2,\ldots,L}$ then following steps are taken to achieve a well-ordered equation:

$$\frac{\left(\overline{W}^{(s_l)}_{(i,t)/(Q)}\right)^2\left(\sum_{l=1}^{L}(A)+2\binom{L}{2}\mathrm{cov}\left(\sum_{i=1}^{n}Q^{(s_l)}_{i,t},\sum_{i=1}^{n}Q^{(s_l)}_{j,t}\right)\sqrt{\mathrm{var}\left(\sum_{i=1}^{n}Q^{(s_l)}_{i,t}\right).\mathrm{var}\left(\sum_{j=1}^{n}Q^{(s_l)}_{i,t}\right)}\right)}{\mathrm{var}\left(\sum_{i=1}^{n}\mathrm{Q}_{i,t}\right)\sqrt{\mathrm{var}\left(\sum_{i=1}^{n}Q^{(s_l)}_{i,t}\right).\mathrm{var}\left(\sum_{j=1}^{n}Q^{(s_l)}_{i,t}\right)}} \geq$$

$$\left(\left(\sum_{l=1}^{L}\sum_{i=1}^{n}\left(W^{(s_l)}_{(i,t)/(Q)}+\frac{L_{i,s_l}W^{(s_l)}_{(i,t)/(Q)}}{mn}\right)\right)^2+\left(\sum_{l=1}^{L}\sum_{i=1}^{n}\frac{L_{i,s_l}W^{(s_l)}_{(i,t)/(Q)}}{mn}\right)^2\right) \quad (19)$$

$$\forall i=1,2,\ldots,n \quad ,\forall l,k=1,2,\ldots,L$$

Equation (19) is rewritten:

$$\frac{\left(\overline{W}^{(s_l)}_{(i,t)/(Q)}\right)^2\left(\sum_{l=1}^{L}(A)+2\binom{L}{2}corr\left(\sum_{i=1}^{n}Q^{(s_l)}_{i,t},\sum_{j=1}^{n}Q^{(s_l)}_{j,t}\right)\sqrt{\mathrm{var}\left(\sum_{i=1}^{n}Q^{(s_l)}_{i,t}\right).\mathrm{var}\left(\sum_{j=1}^{n}Q^{(s_l)}_{i,t}\right)}\right)}{\sqrt{\mathrm{var}\left(\sum_{i=1}^{n}Q_{i,t}\right).\mathrm{var}\left(\sum_{j=1}^{n}Q_{i,t}\right)}} \quad (20)$$

$$\geq\left(\overline{W}^{(s_l)}_{(i,t)/(Q)}\right)^2\left(\sum_{l=1}^{L}(A)+2\binom{L}{2}corr\left(\sum_{i=1}^{n}Q^{(s_l)}_{i,t},\sum_{i=1}^{n}Q^{(s_l)}_{j,t}\right)A\right)$$

$$\forall i=1,2,\ldots,n \quad ,\forall l,k=1,2,\ldots,L$$

Equation (20) is then summarized as Equation (21).

$$LA\left(\left(\overline{W}_{(i,t)/(Q)}^{(s_l)}\right)^2\left(1+\left(L-1\right)corr_{\left(Q_{i,t}^{s_l},Q_{j,t}^{s_l}\right)}\right)\right)\qquad \forall i=1,2,\ldots,n\ \ ,\forall l,k=1,2,\ldots,L \tag{21}$$

4.3. Simultaneous contracts signed by competitor suppliers in the network

If a competing supplier tries to impose more loss, he/she can do this by setting contract parameters and considering the suitable lead time with *b* retailers. Here, we will have:

$$\sum_{i=1}^{n} e_{i,s_l}=b\qquad \forall l=2,\ldots,L \tag{22}$$

Under such circumstances, the maximum lower bound of variance ratios is shown as follows:

$$\max\left(\left(\left(\sum_{l=1}^{L}\sum_{i=1}^{n}\left(W_{(i,t)/(Q)}^{(s_l)}+\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)\right)^2+\left(\sum_{l=1}^{L}\sum_{i=1}^{n}\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)^2\right)\left(\frac{1}{\left(\left(\overline{W}_{(i,t)/(Q)}^{(s_l)}\right)^2\left(1+\left(L-1\right)corr_{\left(Q_{i,t}^{s_l},Q_{j,t}^{s_l}\right)}\right)\right)}\right)\right)=LB \tag{23}$$

4.4. The nonlinear programming model for reducing BWE in the competitive network and RSC

This part of the study considers competition between major and competitor suppliers to present a new multi-objective mathematical model by consideration of RSC, price volatilities, and commodity substitution policy adopted by network retailers to decrease BWE as follows:

$$\min\left(\left|\frac{\sum_{l=1}^{L}\left(\operatorname{var}\left(\sum_{i=1}^{n}Q_{i,t}^{(s_l)}\right)\right)}{\operatorname{var}\left(\sum_{i=1}^{n}\mathrm{Q}_{i,t}\right)}-LB\right|\right)\qquad \forall i,j=1,2,\ldots,n\ \ ,l=1,2,\ldots,L \tag{24}$$

$$\min\ \ v_l\qquad \forall l=2,\ldots,L \tag{25}$$

$$s.t:\quad\left(\left(\sum_{l=1}^{L}\sum_{i=1}^{n}\left(W_{(i,t)/(Q)}^{(s_l)}+\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)\right)^2+\left(\sum_{l=1}^{L}\sum_{i=1}^{n}\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)^2\right)\left(\frac{1}{\left(\left(\overline{W}_{(i,t)/(Q)}^{(s_l)}\right)^2\left(1+\left(L-1\right)corr_{\left(Q_{i,t}^{s_l},Q_{j,t}^{s_l}\right)}\right)\right)}\right)\le LB \tag{26}$$

$\forall i,j=1,2,\ldots,n\ ,l=1,2,\ldots,L$

$$\left(\left(\sum_{l=1}^{L}\sum_{i=1}^{n}\left(W_{(i,t)/(Q)}^{(s_l)}+\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)\right)^2+\left(\sum_{l=1}^{L}\sum_{i=1}^{n}\frac{L_{i,s_l}W_{(i,t)/(Q)}^{(s_l)}}{mn}\right)^2\right)\left(\frac{1}{\left(\left(\overline{W}_{(i,t)/(Q)}^{(s_l)}\right)^2\left(1+\left(L-1\right)corr_{\left(Q_{i,t}^{s_l},Q_{j,t}^{s_l}\right)}\right)\right)}\right)>0 \tag{27}$$

$$\sum_{i=1}^{n} e_{i,s_l}=b\qquad \forall l=2,\ldots,L \tag{28}$$

$$\sum_{i=1}^{n}P_i\left(\gamma L_{i,s_l}+\delta_i\right)\left(\left(1-\varphi_{i,t}^{(s_l)}\right)\rho_{i,t}Q_{i,t}+\left(w_{s_l}-c_{s_l}\right)Q_{i,t}\right)e_{i,s_l}=v_l\qquad \forall l\ne 1 \tag{29}$$

$$\sum_{i=1}^{n} P_i\left(\alpha L_{i,s_1}+\eta_i\right)\left(\left(1-\varphi_{i,t}^{(s_1)}\right)\rho_{i,t}Q_{i,t}+\left(w_{s_1}-c_{s_1}\right)Q_{i,t}\right)\leq$$
$$\frac{\alpha}{\gamma}v_l+\sum_{i=1}^{n} e_{i,s_l}\left(\left(1-\varphi_{i,t}^{(s_l)}\right)\rho_{i,t}Q_{i,t}+\left(w_{s_l}-c_{s_l}\right)Q_{i,t}\right)\left(\eta_i-\frac{\alpha\delta_i}{\gamma}\right) \qquad \forall l\neq 1 \tag{30}$$

$$\left(\rho_{i,t}-c_{s_l}\right)Q_{i,t}\leq\left(\varphi_{i,t}^{(s_l)}\rho_{i,t}-w_{s_l}\right)Q_{i,t} \qquad \forall l=1,2,\ldots,L, i=1,2,\ldots,n \tag{31}$$

$$W_{(i,t)/(Q)}^{(s_l)}=\left(\left(1-\varphi_{i,t}^{(s_l)}\right)\rho_{i,t}+\left(w_{s_l}-c_{s_l}\right)\right) \qquad \forall l=1,2,\ldots,L \tag{32}$$

$$w_{s_l}>c_{s_l} \qquad \forall l=1,2,\ldots,L \tag{33}$$

$$\rho_{i,t}>w_{s_l} \qquad \forall l=1,2,\ldots,L \tag{34}$$

$$\xi_l<\varphi_{i,t}^{(s_l)}<\xi_u \qquad \forall l,i,t \quad \zeta_l,\zeta_u\in\square \tag{35}$$

Equation (24) minimizes the demand estimate variance ratio from the view of suppliers to the sum demand of retailers, which minimizes BWE. It aims to minimize the absolute value of the difference between the variance ratio of demand estimate from the view of suppliers to the sum demand of retailers and the lower bound of variance ratios. Equation (25) ensures minimizing the loss of major suppliers in competition with competitor suppliers. Equation (26) ensures that the lower bound of variance ratios becomes less than its maximum rate at the equilibrium point. Equation (27) ensures that the lower bound of variance ratios is not less than zero. Equation (28) indicates the number of contracts signed between competitor suppliers and retailers. Constraint (32) confirms that in different values of the RSC parameters, the desired retailer of competitor suppliers proposes the shortest lead time. Constraint (33) ensures that the maximum rate of the competitor supplier's utility function occurs at the equilibrium point. Equation (34) ensures that the retailer obtains the same profit with or without signing the contract. Equation (35) indicates the $W_{(i,t)/(Q)}^{(s_l)}$ based on the variables and parameters of the problem. Equations (33) and (34) ensure that wholesale prices be greater than the price of external resources-based procurement, and the end price is greater than the wholesale price, respectively. Equation (35) shows upper and lower bounds for demand estimates, which can be measured based on the suppliers' consideration in terms of capacity, cost, etc.

4.5. The solution algorithm used in the study

Nonlinear programming problem with nonlinear constraints is a programming problem widely used in engineering sciences (For example, refer to [36]). These problems are usually solved based on three approaches: 1) gradient project and related techniques, 2) the complex method of the box, and 3) penalty-function techniques. This study used MATLAB optimization box to solve the programming model in which, an interior-point algorithm that is based on the second algorithm is used.

5. Analysis of numerical analyses

In this section, the demands related to retailers 1 and 2 were used from the article [12]. However, considering that some of the necessary data were not available in that article, the random data in Table 1 was used in addition to the data from that study. the stochastic data of Table 1 were also used because some required data were not available in the mentioned paper.

[Table1]

5.1. Optimal results with and without RSC

The problem is solved through MATLAB software by using the interior-point algorithm. In this solution, demand estimates of suppliers have been taken as variable values. Table 2 shows optimal values of demand forecasting with/without a contract.

[Table 2]

According to Table 3, the largest lower bound of the estimated demand variance ratio for the supplier to the retailer's demand variance equaled 0.026. Lead time equals 1 in the optimum model.

[Table 3]

When no contract is signed between supplier and retailer, the supplier provides commodity for the retailer at a wholesale price, and price values are not calculated for suppliers. According to Table 4, the largest lower bound of the suppliers-to-retailers variance ratio equals 0.100, which is greater than the case in which a revenue sharing contract has been signed. Therefore, BWE is amplified when no contract has been signed.

[Table 4]

5.2. Comparing optimal results of demand estimate based on other demand estimation methods

This part of the study compares the results obtained from the studied model with common demand estimate methods. Table 5 reports suppliers' demand estimate for orders of both retailers by the studied model and Naïve methods, exponential smoothing, and moving average per steps 3, 4, 5, and 6. According to the data reported in this table, the variance ratios of the method proposed in this study outperform other techniques. Therefore, this method can decrease BWE more effectively than other techniques.

[Table 5]

This part of the study examines variance changes of sum estimates relative to percent profit variances ($\varphi_{i,t}^{(s_l)}$), price variations($\rho_{i,t}$) , and lead time variations (L_{i,s_l}).

5.3. Taguchi-style test design to check the parameters of the problem

In this design, which was done in the mini-tab software, the results of which are shown in Figure 3. In this graph, parameter A is the same as the subscription percentage, parameter B is the same price and the supply period parameter is the same as C. The first to third modes have a subscription percentage of 0.5, 0.6 and 0.7, and the first to third modes have a price equal to 30, 40 and 50, and the first to third modes have a supply period equal to 2, 4 and 5. The corresponding values are approximately equal to the following image in the Minitab software.after analyzing the table above in the software, the results are obtained as follows.

In this section, a test design with three parameters of contract profit percentage, price and procurement period is designed in three levels. The minitab17 software was used to perform this test. The first to third modes of the subscription percentage are 0.5, 0.6 and 0.7, the first to third modes of the price are equal to 30, 40 and 50, and the first to third modes of the supply period are equal to 2, 4 and 5. Following the implementation of this test in the minitab17 software environment, the following results have been obtained, which will be discussed below. According to the results of the test, the price parameter has the highest impact and the supply period parameter has the lowest impact variance variance total estimates of retail demand 1 and 2 by the main supplier and competitor.

Graph 7 shows the ratio of S/N as well as the averages of the response. According to this figure, S/N ratio charts, as well as response averages, confirm each other in all cases.

[Figure 3]

Taking into account the results of this experiment, below the optimal levels of each parameter, the analysis and examination of changes in the total variance of the estimate is given in detail. The following changes in variance will be examined relative to the changes in each of the parameters. This study confirms the results of the Taguchi method.

5.4. Sum estimation variance per percent profit

According to data reported in Appendix I, the sum estimation variance of demand of retailers 1 and 2 by major and competitor suppliers is shown per percent of profit variations. Contract profit percent has been considered between 0.4-0.8 to achieve a real understanding of the problem. A constant profit percent is considered for the contract signed between major and competitor suppliers and retailers. The minimum sum variance ratio occurs at a point of 0.8. The maximum variance ratio occurs at a point of 0.6. the sum variance ratio is descending in the 0.4-0.5 interval, while ascending in the 0.5-0.6 interval, and again becomes descending in the 0.6-0.8 interval. Therefore, a contract profit percent of 0.5-0.6 must not be taken to prevent BWF amplification in the network. Although the minimum sum variance ratio occurs at the point of 0.8, suppliers are less willing towards this point because of their low-profit percent for them. Figures 4,5 and 6 depict the variance of demand estimation by suppliers per retailer 1 and 2.

[Figure 4]
[Figure 5]
[Figure 6]

5.5. Sum demand variance per price variation

According to Appendix II and Figure 7, the minimum sum variance of suppliers occurs at the price point of 30. The maximum sum variance of suppliers occurs at the price point of 40. Although the estimation variance of retailer 2's demand by suppliers is higher than other points, following figures 8 and 9, the variance will increase at the point of 40 because the major supplier’s estimation of demand variance of retailer 1 is higher at this point. However, the optimal solution of the model appears at the point of 60 since the values of variables are considered in the optimal solution.

[Figure 7]
[Figure 8]
[Figure 9]

5.6. Sum demand variance per lead time variation

According to Appendix III, Figure 10 depicts the sum variance of suppliers in the points of lead time that have feasible solutions. The minimum sum variance of suppliers occurs in lead time 1, while the maximum sum variance of suppliers occurs in lead time 4. According to Figure 11, the minimum variance rate of retailer 1’s demand estimation by the competitor supplier occurs in lead time 5. Figure 12 indicates that the minimum variance rate of retailer 2's demand estimation by the competitor occurs in lead time 1. Since the competitor supplier has selected retailer 2 to sign the contract, the minimum variance of retailer 2's demand estimation by the competitor supplier occurs in the lead time 1.

[Figure 10]
[Figure 11]
[Figure 12]

5.7. Competitor supplier’s profit variance per percent profit

According to Figure 13, major suppliers’ profit at the point of 0.8 is higher than other points. The profit of both suppliers is almost the same in other points; therefore, the supplier's percent profit does not change the supplier's profit in other points except for the point of 0.8. A major supplier's profit is greater than the profit of a competitor supplier at all available points.

[Figure 13]

5.8. Competitor supplier’s profit variance per price variation

According to Figure 14, the competitor supplier’s profit is almost fixed in the price range of 30-60, while this profit at the price point of 40 is higher than other points.

[Figure 14]

5.9. Competitor supplier's profit variance per lead time variation

According to Figure 15, the major supplier’s profit is almost fixed in the price range of 1-5. The competitor supplier's profit remains constant in the price interval of 1-4, while is double in lead time 5 compared to other points. Although the competitor supplier's profit is greater at a point of 5, this is a considerable point that the competitor supplier cannot select the lead time of 5 because of the competition between suppliers.

6. Conclusion and Recommendations

This study aims to present a programming model to reduce the bullwhip effect and loss of major suppliers when competing with competitor suppliers by using the revenue sharing contract. This study examined the BWE under RSC within a two-echelon supply chain in which one layer comprises two or more major and competitor suppliers, and another layer comprises a network of retailers. In the next step, a lower bound was measured for the variance ratio of demand estimation by suppliers within two specific modes of two suppliers and multiple suppliers (one major supplier and competitors) in presence of RSC to propose a multi-objective nonlinear programming model with nonlinear constraints for the problem. The first objective of the model was to minimize the BWE in the network and another objective was to decrease competitor suppliers' profit (loss of major supplier). A numerical analysis was done to validate the studied model. According to the results of numerical analysis, the demands' variance ratio under the revenue sharing contract was less that the case without a contract. The largest lower bound with a contract equaled 0.026, while this rate equaled 0.1 in the case without a contract; therefore, BWE is amplified in the case without a contract. The results of optimal demand estimation by the proposed model were compared with other methods of demand forecast (naïve method, simple exponential smoothing, and moving average method). The results indicated that the estimation variance of this model outperformed other methods regarding the data presented in this research so that this method could effectively reduce the BWE in the network. Variance changes in retailers' demand estimation by suppliers per contract percent profit were examined in the next step.

The following discusses the managerial insights derived from the research results. the results showed that variances had a descending trend in all points except for some specific points. It can be stated as a management insight that BWE will be lower if suppliers increase contract percent profit for retailers within the 0.4-0.5 and 0.6-0.8 intervals. Demand estimation variance per price variance was examined. In terms of management insight, the BWE will be reduced in the network if the end price is set at the 50-70 interval. After the estimation variance per price variance was examined, the estimation variance per lead time variance was assessed. According to numerical results and customer satisfaction, the shorter the lead time, the lower the BWE in the network will be. Moreover, BWE variance per contract percent profit, lead time, and price variances were assessed, and then the supplier's profit variance per contract profit, price, and lead time variances were measured. According to the results of suppliers' profit variance per contract percent profit variance, the profit of major and competitor suppliers is almost constant in all points except for the point of 0.8. Therefore, suppliers can obtain a profit and prevent BWE amplification in the network by selecting a point at 0.4-0.5 and 0.6-0.8 intervals and considering the BWE phenomenon. According to the results of suppliers' profit variance per price variance, the major supplier's profit is maximum at the price point of 40. This rate almost remains constant at other price points; therefore, suppliers can adjust the price based on the end-consumers' affordability to obtain profit and reduce BWE. According to the results of suppliers' profit variance per lead time variance, major and competitor suppliers' profit is almost constant during all lead times except for lead time 5 for the major supplier. Therefore, suppliers can change the lead time within 1-4 intervals based on their available facilities to satisfy retailers.

The limitation of this research is that, given that this research has addressed the general issue of reducing the bullwhip effect, in order to implement it in different chains, it must first be accurately identified and all the factors affecting its disorders must be discovered in order to obtain more accurate results.

It is recommended that further studies use metaheuristic algorithms to solve the large-sized problem. Moreover, more studies can be conducted on the BWE under the other contracts in the network. Other interesting subjects may include investigation of BWE in the network within the generalized mode in which some retailers have signed a contract with some suppliers beside topics about symmetric and asymmetric information of chain participants.

Compliance with Ethical Standards

Funding

Not applicable.

Ethical approval

This article does not contain any studies with human participants performed by any of the authors.

Conflict of interest
The authors have no relevant financial or non-financial interests to disclose.
Data availability
Data are available upon reasonable request.

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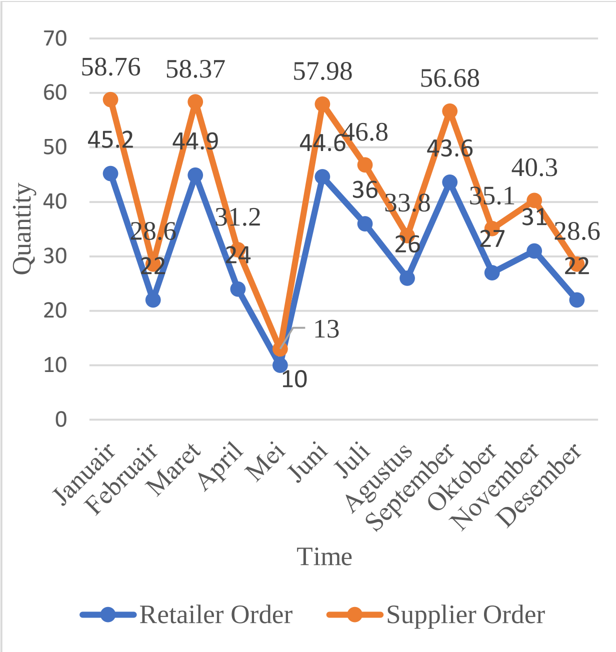

Figure 1. Increased orders volatility toward the supplier [22]

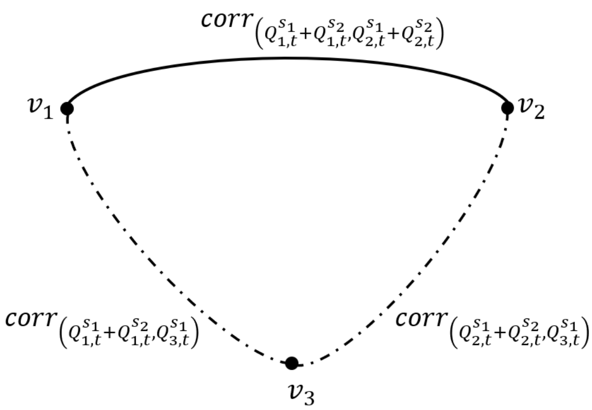

Figure 2. A network of retailers with three nodes

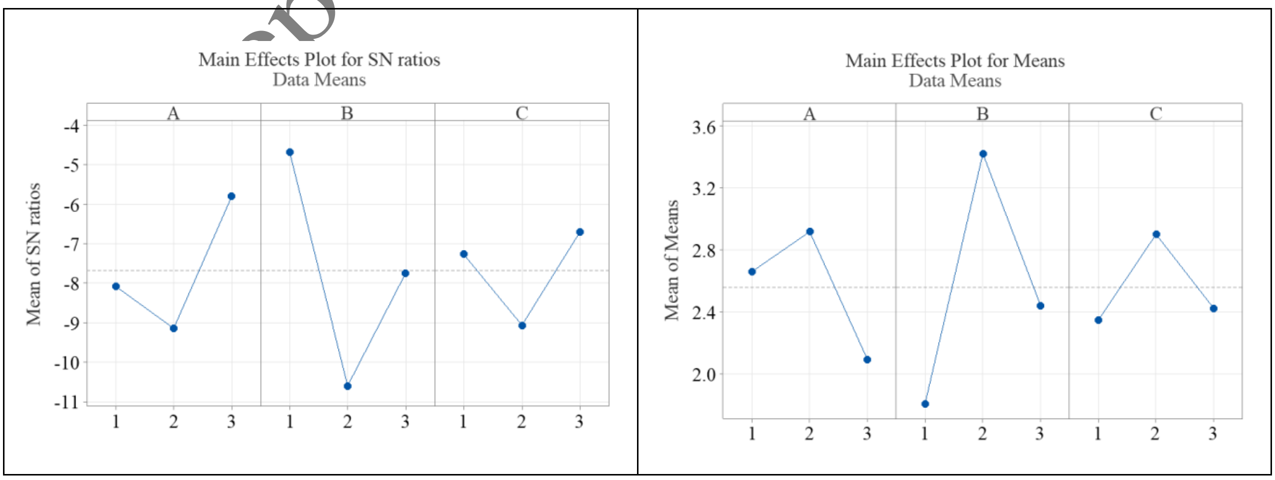

Figure3 : Main Effects Plot for SN rations and Means

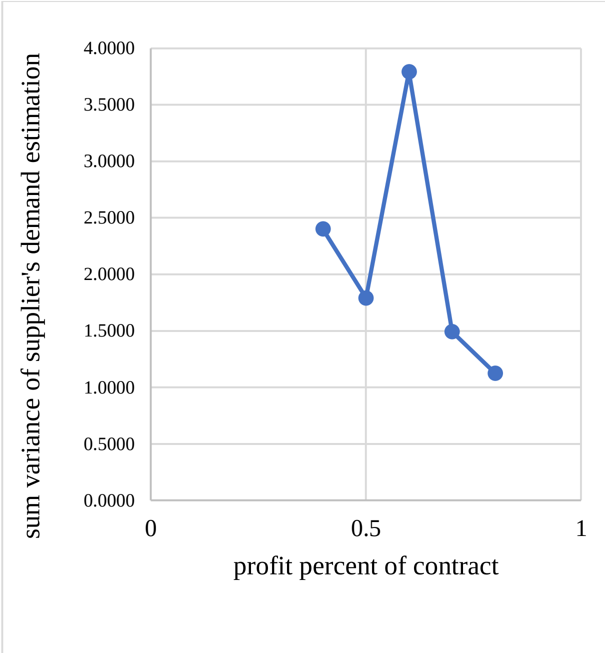

Figure4: sum supplier's estimation variance per percent profit

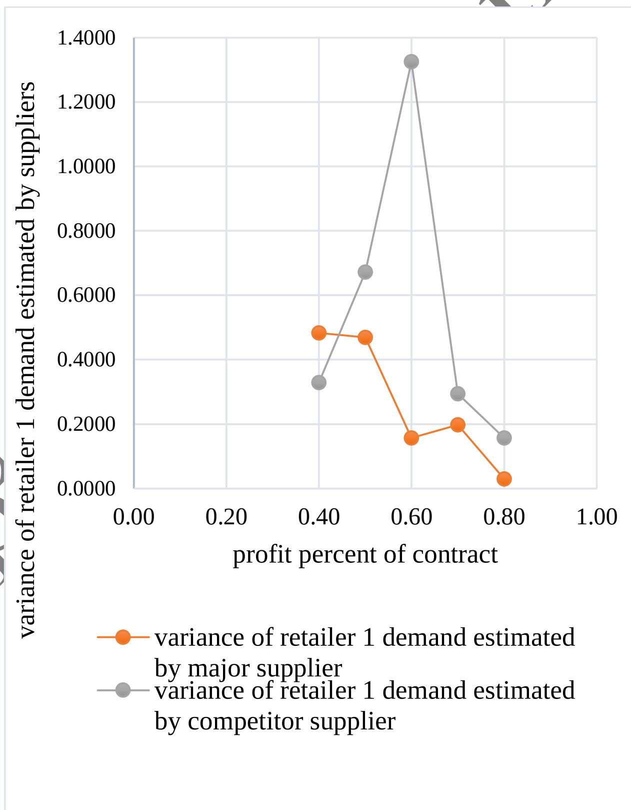

Figure 5: suppliers’ estimations of retailer1 variance per percent profit

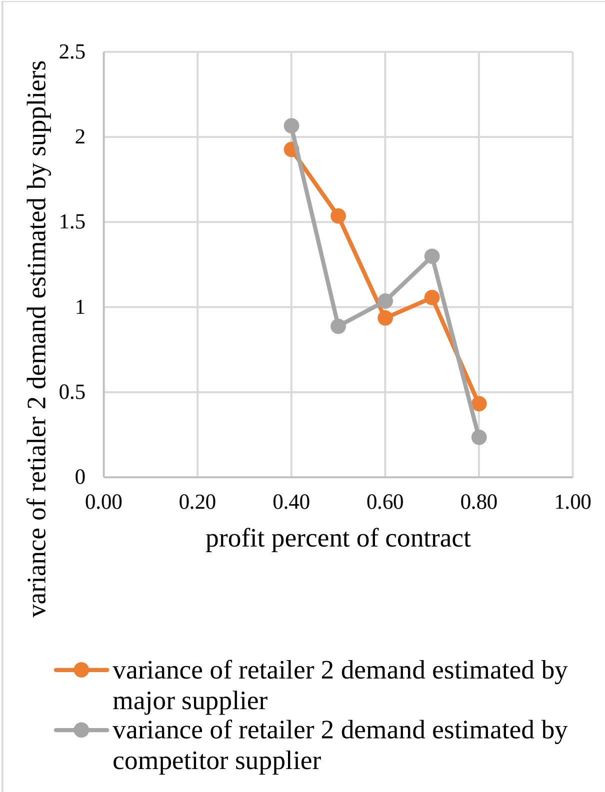

Figure 6: suppliers’ estimations of retailer2 variance per percent profit

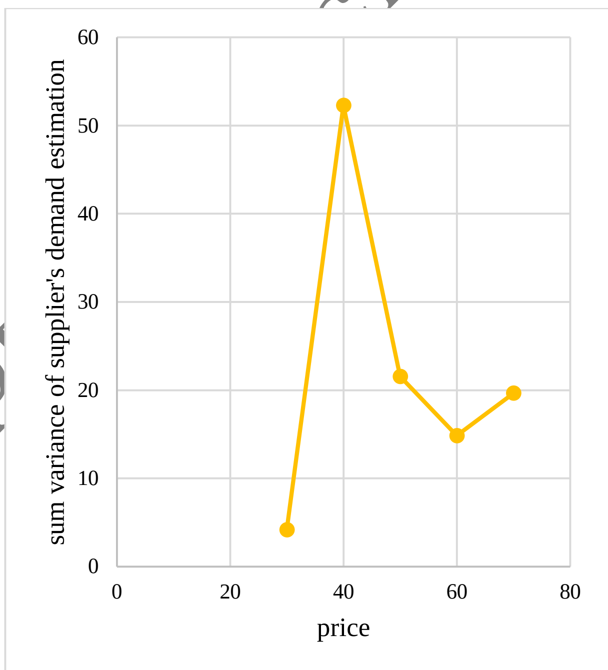

Figure 7: sum suppliers' estimation variance per price.

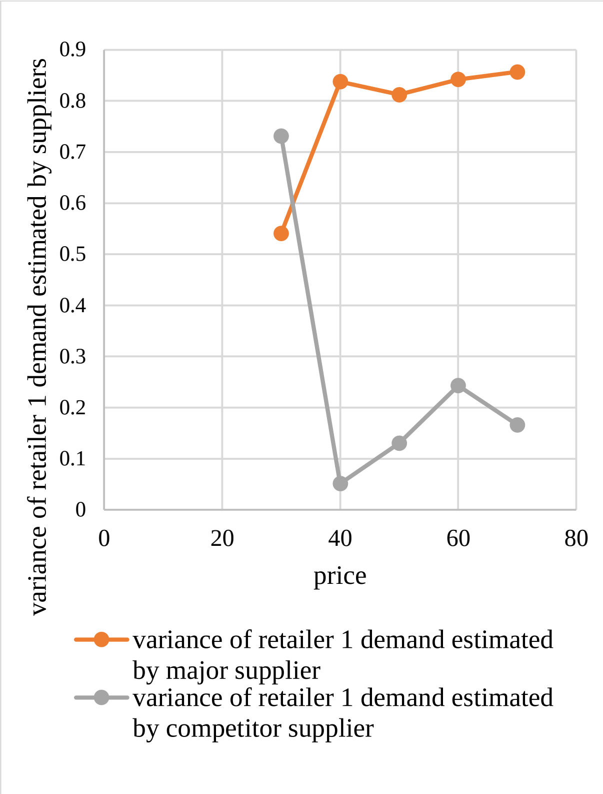

Figure 8: suppliers' estimation of retailer1 s' demand variance per price.

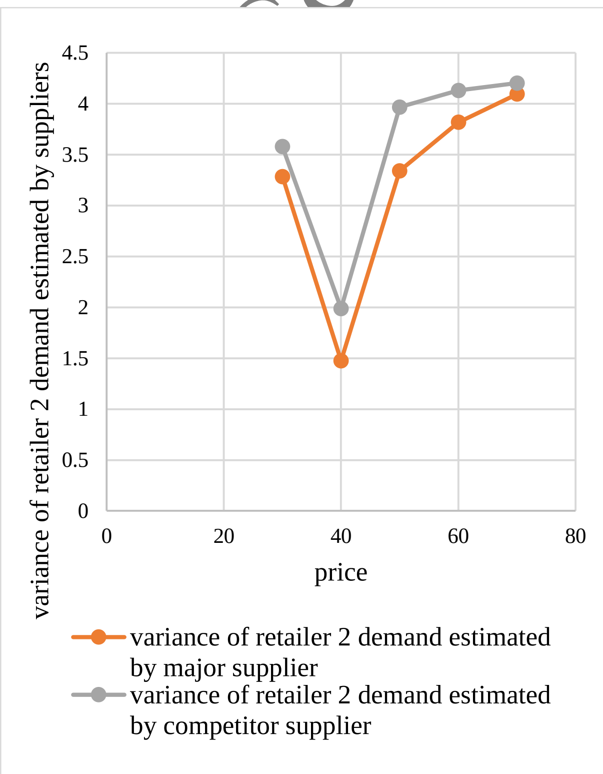

Figure 9 :suppliers' estimation of retailer2 s' demand variance per price.

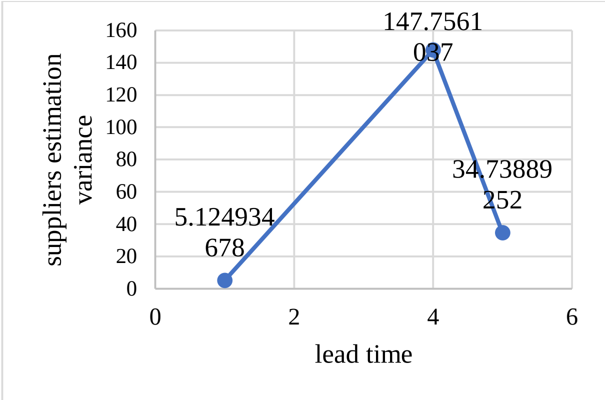

Figure 10: sum supplier's estimation variance per lead time.

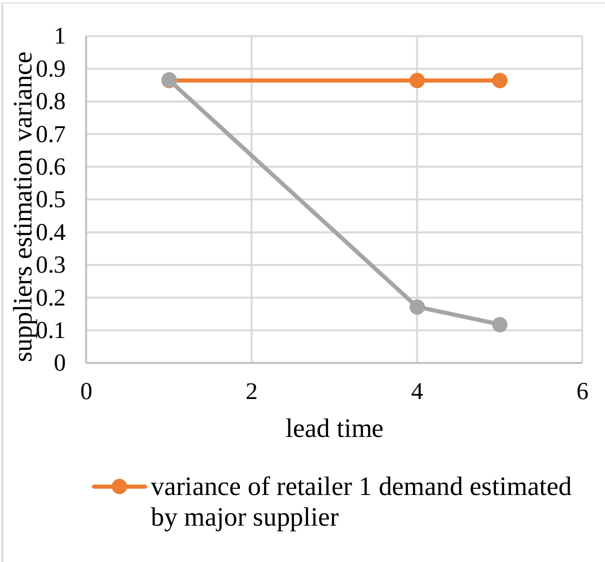

Figure 11: suppliers’ estimation of retailer1s’ demand variance per lead time.

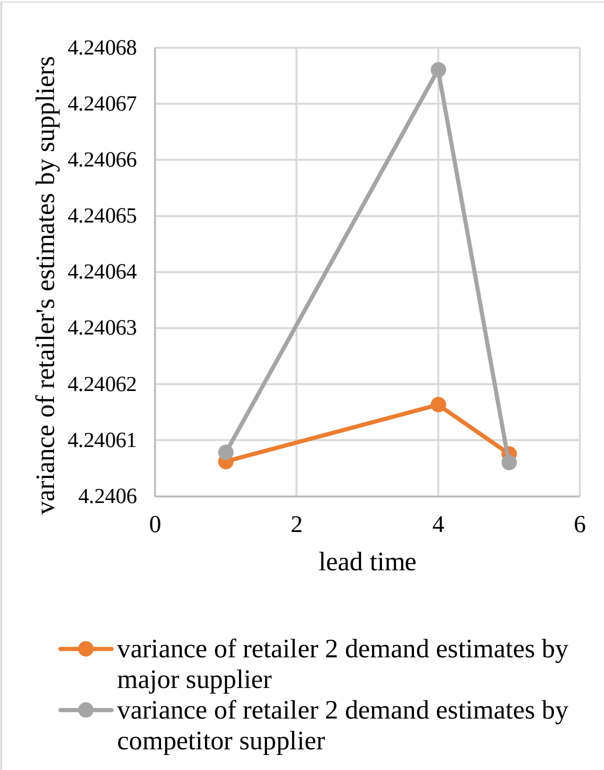

Figure 12: suppliers’ estimation of retailer2s’ demand variance per lead time.

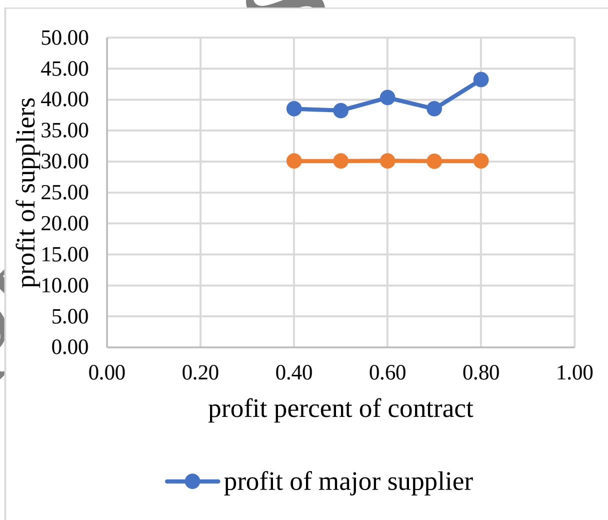

Figure13: suppliers’ profit variance per percept profit of the contract

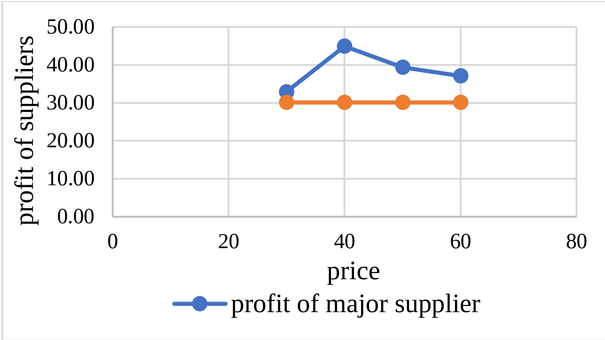

Figure14: suppliers' profit variance per price variance

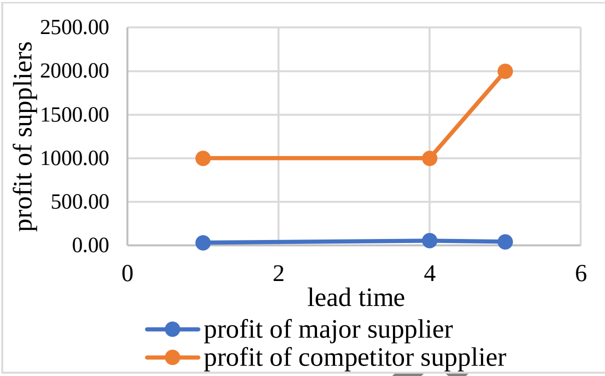

Figure15: suppliers' profit variance per lead time variation

Table 1: retailers' demands

Month	Retailer 1	Retailer 2
Januair	45.2	17.4
February	42.6	13.6
Maret	44.9	14.7
April	43.8	14.4
Mei	45.3	17.6
Juni	44.6	18
Juli	45.7	13.2
Agustus	45.1	16.4
September	43.6	12.1
Oktober	44.8	12.6
November	43.7	16.6
desember	45.4	16.2

Table 2: parameters' values

α	γ	η	θ	b	m	$\operatorname{cov}\left(Q_{1,t}, Q_{2,t}\right)$
1	1	1	0	1	3	0.03
C_{s_1}	C_{s_2}	$\operatorname{var}\left(Q_{1,t}\right)$	$\operatorname{var}\left(Q_{2,t}\right)$	$\overline{W}^{(s_l)}_{(i,t)/(Q)}$	pro_1	pro_2
15	15	0.9	4.3	30	0.5	0.5

Table 3: optimal values of demand forecasting with/without a contract

Month	Optimal values of demand forecasting with contract				Optimal values of demand forecasting without contract			
	Retailer (1)		Retailer (2)		Retailer (1)		Retailer (2)	
	Main supplier	Competitor supplier	Main supplier	Competitor supplier	Main supplier	Competitor supplier	Main supplier	Competitor supplier
Januair	45.2	46.2	17.4	17.4	44.71	44.85	17.91	16.83
February	42.6	43.6	13.6	13.6	42.10	42.05	14.23	13.25
Maret	44.9	45.9	14.7	14.7	44.38	42.87	15.31	14.48
April	43.8	44.8	14.4	14.4	43.29	44.10	15.01	14.11
Mei	45.3	46.3	17.6	17.6	44.80	44.96	18.10	17.06
Juni	44.6	45.6	18.0	18.0	44.09	43.77	18.46	17.67
Juli	45.7	46.7	13.2	13.2	45.19	46.49	13.88	12.89
Agustus	45.1	46.1	16.4	16.4	44.61	45.18	16.96	15.82
September	43.6	44.6	12.1	12.1	43.10	43.36	12.80	11.77
Oktober	44.8	45.8	12.6	12.6	44.29	44.99	13.29	12.27
November	43.7	44.7	16.6	16.6	43.19	43.55	17.10	15.95
Desember	45.4	46.4	16.2	16.2	44.90	45.47	16.77	15.67

Table 4: optimal values of variables with contract

$\varphi^{(s_1)}_{1,t}$	$\varphi^{(s_1)}_{2,t}$	$\varphi^{(s_2)}_{1,t}$	$\varphi^{(s_2)}_{2,t}$	w_{s_1}	w_{s_2}	$\rho_{1,t}$	$\rho_{2,t}$	LB
0.45	0.45	0.27	0.45	25	25	60	60	0.026
T_{1,s_1}	T_{2,s_1}	T_{1,s_2}	T_{2,s_2}	L_{1,s_1}	L_{1,s_2}	L_{2,s_1}	L_{2,s_2}	
0.0	0.0	0.0	1.0	1	1	1	1	

Table 5: optimal values of variables without a contract

L_{1,s_1}	L_{2,s_1}	L_{1,s_2}	L_{2,s_2}	w_{s_1}	LB
1.0	1.0	1.0	1.0	22.13941	0.100
T_{1,s_1}	T_{2,s_1}	T_{1,s_2}	T_{2,s_2}	w_{s_2}	
0.0	0.0	0.0	1.0	17.97097	

Table 6: comparing different methods of forecasting demand.

demand	demand	Main supplier-model sol	Competitor-model	naive	Single exponential smoothing	Moving average (m=3)	Moving average (m=4)	Moving average (m=5)	Moving average (m=6)	demand	Main supplier-model sol	Competitor-model	naive	Single exponential smoothing	Moving average (m=3)	Moving average (m=4)	Moving average (m=5)	Moving average (m=6)

Januair	45.2	45.6	50.6	-	-	-	-	-	-	47.4	18.2	17.8	-	-	-	-	-	-
February	42.6	43.0	49.1	45.2	45.2	-	-	-	-	13.6	14.5	14.0	47.4	47.4	-	-	-	-
Maret	9.9	45.3	50.4	42.6	44.8	32.6	-	-	-	14.7	15.6	15.1	13.6	42.3	25.2	-	-	-
April	43.8	44.2	49.8	9.9	39.6	32.1	35.4	-	-	21.4	15.3	14.8	14.7	38.2	16.6	24.3	-	-
Mei	45.3	45.7	50.7	43.8	40.2	33.0	35.4	37.4	-	17.6	18.4	18.0	21.4	35.7	17.9	16.8	22.9	-
Juni	20.6	45.0	50.3	45.3	41.0	36.6	29.9	32.4	34.6	45.0	18.8	18.4	17.6	33.0	28.0	24.7	22.5	-
Juli	45.7	46.1	50.9	20.6	37.9	37.2	38.8	33.1	34.6	13.2	14.1	13.6	45	34.8	25.3	24.3	22.4	26.6
Agustus	5.1	45.5	50.5	45.7	39.1	23.8	29.2	32.1	28.4	50.4	17.2	16.8	13.2	31.5	36.2	31.5	29.5	20.9
September	43.6	44.0	49.7	5.1	34.0	31.5	28.7	32.0	34.0	7.1	13.0	12.5	50.4	34.4	23.6	28.9	26.7	27.0
Oktober	44.8	45.2	50.4	43.6	35.4	31.2	34.8	32.0	34.2	43.6	13.5	13.0	7.1	30.3	33.7	28.5	31.9	25.8
November	23.7	44.1	49.7	44.8	36.8	37.4	29.3	32.6	30.6	16.6	17.4	17.0	43.6	32.3	22.4	29.4	26.2	29.5
Desember	45.4	45.8	50.7	23.7	34.9	38.0	39.4	32.5	34.7	10.2	17.0	16.6	16.6	29.9	23.5	19.4	25.6	29.3
Variance ratio	-	0.003	0.004	1.00	0.06	0.07	0.07	0.01	0.02	-	0.02	0.02	1.01	0.11	0.14	0.09	0.04	9.4

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