Comparative thermal applications of titanium dioxide and molybdenum nanoparticles subject to blood and water base fluids: Caputo-Fabrizio and Atangana-Baleanu model

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Abstract: The aim to current investigation is to explore the thermal analysis for hybrid nanofluid with help of fractional model. The properties of hybrid nanofluid have been observed with interaction of titanium dioxide (TiO_2) and molybdenum disulfide (MoS_2) nanoparticles. Water and blood are used to characterize the properties of base liquid. The flow pattern is based on natural convective flow due to inclined surface. Two fractional algorithms namely Caputo-Fabrizio (CF) and Atangana-Baleanu (AB) are used to perform the analytical simulations. A comparative analysis between both AB and CF operators is presented to justify the accuracy of these fractional techniques. The flow model contains comparative impact of water based hybrid nanofluid $(TiO_2-MoS_2)/H_2O$ and blood based hybrid nanofluid $(TiO_2-MoS_2)/blood$. The numerical values of skin friction and Nusselt number are also calculated. Thermal observations are presented for both nanoparticles and base fluids. The computations reveal that heat transfer declined for hybrid nanofluid when fractional parameters have been considered. The velocity profile declined due to inclination angle and fractional parameters. Furthermore, Nusselt number enhances over time due to fractional effects. The claimed findings conveying applications in the thermal management systems, energy efficient systems, MHD technologies, industrial heat transfer, solar thermal collectors, heat transfer devices etc.

Keywords: Hybrid nanofluid, Fractional model, Heat transfer, Atangana-Baleanu simulations, Caputo-Fabrizio approach.

1. Introduction

With high thermal performances and effective physical properties, the nanomaterials are assumed to be enhanced class of regular fluids attaining improved heat transfer capacity. The nanofluids are combination of nanoparticles with some base liquids. Due to ultra-peak thermal performances, the nanofluids ensure various applications in the medical science, vehicle engine, nuclear systems, heating and cooling devices, chemical processes etc. Based on available literature source, it is observed that various studies are contributed on nanofluids in recent year. In fact, scientists are continuously incorporating different thermal sources to inspect the conducting performances of nanofluids. Sheikholeslami and Ellahi [1] studied the heat transfer analysis for natural convective flow of nanofluid with implementation of magnetic field. Khan et al. [2] discussed the Burgers nanofluid in improving heat transfer due to bidirectional flow. The significance of nanofluids in inspection of heat transfer subject to CVFEM was analyzed by Sheikholeslami [3]. Hejazi et al. [4] predicted the convection aspects for addressing the heat transfer enhancement due to nanoparticles. Gangadhar et al. [5] examined the squeezing aspects of nanofluid by incorporating the gold nanoparticles. Sheikholeslami [6] focused to improvement of heat transfer by using the aluminum oxide nanoparticles. The Maxwell nanofluid flow due to rotating disk has been conveyed by Li et al. [7]. Basha [8] discussed the heat absorption contribution in stretching flow associated to the nanofluid. Murugan et al. [9] reported the thin film of nanofluid via inclined surface. Sheikholeslami [10] focused to applications of multi-walled carbon nanotubes (MWCNT) in concentrated photovoltaic solar system. Adnan et al. [11] addressed the unsteady transport driven by nanofluid by developing a KK model. The applications of ternary nanoparticles for the radiative surface was analyzed by Aich et al. [12]. Sheikholeslami et al. [13] explained the nanofluid thermal performances in spectral splitter configuration.

The latest type of nanofluid with more progressive thermal effects and characteristics is called a hybrid nanofluid. Its ultra-peak thermal properties have made HNFs well-known. Multiple nanoparticles interacting with base fluids is a term used to describe the many characteristics of the HNF model. HNFs should be taken into consideration for their improved thermal efficiency and thermal reliability. HNF is used in many thermal extrusion operations, power plants, metallic chips, electronic devices, nuclear plant engine cooling and other domains. Ikran et al. [14] claimed the thermal improved observations due to hybrid nanofluid in parallel plate flow. Shatnawi et al. [15] addressed the Casson hybrid nanofluid thermal measurements accounted by vertical surface. Ali et al. [16] examined the thermo-fluidic behavior of hybrid nanofluid in couette surface flow. Kumar et al. [17] visualized the thermal prediction of magnesium oxide and titanium hybrid particles for radiator flow. Sharma et al. [18] investigated the fundamental of heat transfer due to hybrid nanofluid in presence of entropy generation phenomenon. Yasir et al. [19] pronounced the hybrid nanofluid contribution for radiative flow under the additional impact of heat source with irregular pattern. The nanofluid flow of hybrid nanofluid via numerical computations was elaborated by Ahmad et al. [20]. Gangadhar et al. [21] reported the squeezing flow of hybrid nanofluid with melting heat transfer. Ige et al. [22] explained the transient flow of hybrid nanofluid with blood as a base fluid via computational approach. Ali et al. [23] deduced the performances of three different nanoparticles (MoS_2, TiO_2, Ag) due to porous stretched cylinder.

An essential role of factional calculus has been observed in various engineering and applied sciences. Different problems proposed in the physical, economics, engineering and many other era of physical discipline are of highly nonlinear as well as of fractional kind. The closed form solution of such problem is effectively suggested by using various kind of fractional operators. Various fractional models have been proposed for different physical systems including therapy of cancerous tumor [24], fall objects with frictional forces [25] and heat transfer problems [26]. Basically, the fractional calculus offers two major kinds of operators namely singular and nonsingular for which the Caputo derivative and Riemann–Liouville are characterizes to singular kind. The Caputo–Fabrizio (CF) derivative and the Atangana–Baleanu (AB) fractional operators are the most famous techniques associated to the non-singular operators. Several fractional simulations are based on implementation of both CF and AB operators with high accuracy solution. The computations performed via such techniques are realistic and more precise. Both AB and CF fractional operators offer memory effects. Various studies are available for which both techniques are used effectively [27-36].

Based on claimed literature record, it is observed that various studies have been performed for judging the thermal effectiveness of both nanofluids and hybrid nanomaterials. Current research presents a fractional model for hybrid nanofluid with definitions of two famous fractional techniques namely Caputo-Fabrizio (CF) and Atangana-Baleanu (AB) definitions. The thermal efficiency of hybrid nanofluid has been endorsed by incorporating the titanium dioxide $\left(TiO_2\right)$ and molybdenum disulfide $\left(MoS_2\right)$ with blood and water base fluid. The flow is driven by natural convective phenomenon with implementation of magnetic force. The comparative thermal visualization of blood based hybrid nanofluid $\left(TiO_2 - MoS_2\right)/blood$ and water based hybrid nanomaterial $\left(TiO_2 - MoS_2\right)/H_2O$ is performed. The results are deduced by employing both AB and CF models. The physical aspects of model are presented.

1.1 Novelty of work

- This analysis presents comparative thermal impact of hybrid nanofluid by using the two advanced fractional schemes, Caputo-Fabrizio (CF) and Atangana-Baleanu (AB). The choice of CF and AB provides more accurate and generalized representation of thermal problem as compared to integer order models.

- This investigation uniquely decomposes the suspension of titanium dioxide $\left(TiO_2\right)$ and molybdenum disulfide $\left(MoS_2\right)$ nanoparticles into blood and water base fluids. The choice of these nanoparticles is subject to peak thermal characteristics and stable features.
- This study provides thermal performances of blood-based water-based hybrid nanomaterials under certain constraints, filling critical gap in literature and providing practical amplifications for thermal management systems, heat transfer devices, chemical processes, drug delivery, cooling phenomenon, energy production etc.
- By implementing the fractional derivatives to current model, the research provides novel insight into memory features and anomalous diffusion impact of hybrid nanomaterials, which are important for precision in thermal system design.

2. Mathematical and Physical structure of the proposed modeling

An unsteady, incompressible natural convective flow of hybrid nanofluid (HNF) over inclined surface is assumed. Using water and blood as a base fluid, titanium dioxide $\left(TiO_2\right)$ and molybdenum disulfide $\left(MoS_2\right)$ nanoparticles are used to characterize the hybrid nanofluid properties. It is further assumed that an angled magnetic field with a strength of B_0 is subject to the flow. Initially, the temperature T_∞ of the fluid and plate are both at rest. After some time has passed at $t > 0^+$, natural convection, oscillations, and inclination of porous plate cause static hybrid suspension of various kinds of nanoparticles to begin to move on oscillating plate. The porous plate vibrates with a constant velocity $\sin\left(\omega t\right)$ where ω vibration rate of inclined plate.

The governing equations for current problem are [23, 29]:

$$\rho_{nf}\frac{\partial w\left(\zeta,t\right)}{\partial t}=\mu_{nf}\frac{\partial^2 w\left(\zeta,t\right)}{\partial \zeta^2}-\sigma_{nf}B_0{}^2\sin\left(\theta\right)w\left(\zeta,t\right)+g\left(\rho\beta_T\right)_{nf}\left(T\left(\zeta,t\right)-T_\infty\right)\cos\left(\delta\right), \tag{1}$$

$$\left(\rho C_p\right)_{nf}\frac{\partial T\left(\zeta,t\right)}{\partial t}=-k_{nf}\left(1+\frac{16\sigma^* T_\infty{}^3}{3k_{nf}k^*}\right)\frac{\partial q\left(\zeta,t\right)}{\partial \zeta}, \tag{2}$$

where

$$q(\zeta,t) = -k_{nf}\,\frac{\partial T(\zeta,t)}{\partial \zeta}. \tag{3}$$

with corresponding initial and boundary conditions [23, 29]:

$$w(\zeta,t) = 0, T(\zeta,t) = T_\infty;\ \zeta > 0, t = 0,$$
$$w(0,t) = \sin(\omega t), T(0,t) = T_\infty + T_w\left(1 - ae^{-bt}\right);\ \zeta = 0, t > 0,$$

$$w(\zeta,t) \to 0,\ T(\zeta,t) \to 0\ \ as\ \ \zeta \to \infty, t > 0.$$

with ρ_{nf} (fluid density), w (velocity component), σ_{nf} (electric conductivity), B_0 (magnetic field strength), g (gravity), T (temperature), T_∞ (ambient temperature), $\left(\rho C_p\right)_{nf}$ (specific heat), k_{nf} (thermal conductivity), Stefan Boltzmann constant σ^* and ω (angular frequency).

To analyze the impact of different limitations, introducing the following non-dimensional constraints [28, 29]:

$$t^* = \frac{\nu_f}{L^2}t, \zeta^* = \frac{\zeta}{L}, w^* = \frac{L}{\nu_f}w, T^* = \frac{T - T_\infty}{T_w - T_\infty}, L = \left[\frac{\nu_f^{\ 2}}{g\beta_f\left(T_w - T_\infty\right)}\right]^{1/3}, q^* = \frac{q}{q_0}$$

After working on non-dimensional form, simplified equations will be.

$$\frac{\partial w(\zeta,t)}{\partial t} = \frac{1}{\lambda_1}\frac{\partial^2 w(\zeta,t)}{\partial \zeta^2} - M\sin(\theta)w(\zeta,t) + \lambda_2 Cos(\delta)T(\zeta,t); \zeta, t > 0 \tag{4}$$

$$Pr\,\frac{\partial T(\zeta,t)}{\partial t} = -\frac{\lambda_3}{\lambda_4}\frac{\partial q(\zeta,t)}{\partial \zeta}; \zeta, t > 0 \tag{5}$$

$$q(\zeta,t) = -\lambda_5\frac{\partial T(\zeta,t)}{\partial \zeta} \tag{6}$$

with:

$$w(\zeta,t)=0, T(\zeta,t)=0, \quad \zeta>0, t=0$$
$$w(0,t)=Sin(\omega t),\ T(0,t)=1-ae^{-bt};\ \zeta=0, t>0$$
$$w(\zeta,t)\to 0, T(\zeta,t)\to 0 \ \ as \ \ \zeta\to\infty, t>0$$

where

$$\lambda_1=\left(1-\varphi+\varphi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}, \lambda_2=\left(1-\varphi+\varphi\frac{(\rho\beta_T)_s}{(\rho\beta_T)_f}\right)\left(1-\varphi+\varphi\frac{\rho_s}{\rho_f}\right)^{-1},$$

$$\lambda_3=\left(\frac{V_0}{V_f}\right)k_f,\ \lambda_4=\frac{1-\varphi+\varphi\frac{(\rho C_p)_s}{(\rho C_p)_f}}{\frac{k_{nf}}{k_f}+Nr}, \lambda_5=\frac{k_{nf}}{k_f}\left(\frac{V_f}{V_0 k_f}\right),$$

$$\lambda=\frac{\lambda_3\lambda_5}{\lambda_4}=\frac{k_{nf}}{k_f}\left(\frac{k_{nf}}{k_f}+Nr\right)\left(1-\varphi+\varphi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)^{-1},$$

$$Pr=\frac{(\mu C_p)_f}{k_f},\ Nr=\frac{16\sigma^* T_\infty^{\ 3}}{3k^* k_f},\ M=\frac{\sigma_f L^2 B_0^{\ 2}}{\mu_f}$$

with radiation parameter Nr, Prandtl number Pr and Hartmann number M. The hybrid nanofluid coefficients are denoted with λ_1, λ_2, λ_3,, λ_4, and λ_5. Table 1 presents mathematical expressions for nanofluid and hybrid nanofluid. The numerical values of thermo-physical properties of nanomaterials have been presented in table 2.

3. Basic definitions of fractional derivatives

ABC-definition: The mathematical definition of AB-fractional derivative in the sense of Caputo sense can be summarized as for a continuous function $f(t)$ [30]

$$ {}^{ABC}{}_0D_t^{\eta} f(t)=\frac{1}{1-\eta}\int_0^t \varepsilon_\eta\left[\frac{-\eta(t-\tau)^\eta}{1-\eta}\right]f'(t)dt \tag{7}$$

Where ${}^{ABC}{}_0D_t^{\eta}$ is the well known as the fractional operator of ABC-definition. Where $\varepsilon_{\beta}(t)$ is the Mittag-Laffler with,

$$\varepsilon_{\eta}\left(-t^{n}\right)=\sum_{j=0}^{\infty}\frac{\left(-t^{n}\right)^{j\eta}}{\Gamma\left(j\eta+1\right)}$$

The respective Laplace is [31]:

$$L\left\{{}^{ABC}{}_0D^{\eta}{}_t f\left(\xi,t\right)\right\}=\frac{q^{\eta}L\left[f\left(\xi,t\right)\right](q)-q^{\eta-1}f\left(\xi,0\right)}{\eta+\delta\left(1-q\right)} \tag{8}$$

CFD-definition: Like ABC-definition, the mathematical notation of CFD can be written as for the continuous function $Q(T)$ [30]:

$${}^{CF}{}_0D^{\delta}{}_{\Upsilon}Q\left(T\right)=\frac{\mathrm{M}\left(\delta\right)}{1-\delta}\int_0^{\Upsilon}\exp\left[\frac{-\delta\left(T-\tau\right)}{1-\delta}\right]D\left(Q\left(\Upsilon\right)\right)d\Upsilon \tag{9}$$

Where δ is the non-integer order of CFD operator ${}^{CF}{}_0D_{\Upsilon}{}^{\delta}$, and the Laplace of ${}^{CF}{}_0D_T{}^{\delta}$ is [31]:

$$L\left\{{}^{CFD}{}_0D_{\Upsilon}{}^{\delta}Q\left(\xi,\Upsilon\right)\right\}=M\left(\delta\right)\left(\frac{qL\left(Q\left(\xi,T\right)\left(q\right)\right)-Q\left(\xi,0\right)}{q+\delta\left(1-q\right)}\right) \tag{10}$$

4. Implementation of Atangana-Baleanu (AB) scheme

This section focuses on analyzing the solution of the governed model by employing the fractional definition of AB and utilizing the LT to solve the transformed equation. Here, the AB fractional definition provides a mathematical framework to describe the behavior of the system, particularly in terms of fractional calculus operations. By applying the LT to the transformed equation, we aim to obtain a solution that represents the system's response over time.

4.1. Computations for temperature profile

Implementing the definitions of Atangana-Baleanu fractional operator on Eq. (6) as:

$$\lambda_3\lambda_5 q^{\beta}\frac{\partial^2\overline{T}\left(\zeta,q\right)}{\partial\zeta^2}-\Pr\lambda_4\left(\left(1-\beta\right)q^{\beta+1}+q\beta\right)\overline{T}=0,$$

With

$$\overline{T}(0,q)=\frac{1}{q}-\frac{a}{b+q} \quad and \quad \overline{T}(\infty,q)=0$$

Temperature profile can be solved using AB technique as follows:

$$\overline{T}(\zeta,q)=\left(\frac{1}{q}-\frac{a}{b+q}\right)e^{-\zeta\sqrt{\frac{\Pr\lambda_4}{\lambda_3\lambda_5}q^{1-\beta}\left((1-\beta)q^{\beta}+\beta\right)}}. \tag{11}$$

Where its respective series form

$$\overline{T}(\zeta,q)=\left(\frac{1}{q}-\frac{a}{b+q}\right)+\sum_{n_1=1}^{\infty}\sum_{n_2=0}^{\infty}\frac{(-y)^{n_1}(\Pr)^{n_1/2}(\beta)^{n_2}(\lambda_4)^{n_1/2}}{n_1!(\lambda_3\lambda_5)^{n_2}\Gamma(1-\beta)}\frac{1}{q^{1-(\beta-1)+\beta n_2-\frac{\beta n_1}{2}}}$$

Taking its Laplace inverse

$$\overline{T}(\zeta,q)=\left(1-ae^{-bt}\right)+\sum_{n_1=1}^{\infty}\sum_{n_2=0}^{\infty}\frac{(-y)^{n_1}(\Pr)^{n_1/2}(\beta)^{n_2}(\lambda_4)^{n_1/2}}{n_1!(\lambda_3\lambda_5)^{n_2}\Gamma(1-\beta)\Gamma\left(-3+\beta-\beta n_2+\frac{\beta n_1}{2}\right)}t^{-3+\beta-\beta n_2+\frac{\beta n_1}{2}}$$

4.2. Computations for velocity profile

For the momentum field solution by employing the LT on governed Eq. (4) and using the above solution of thermal field attained by AB-definition

$$\frac{\partial^2\overline{w}(\zeta,q)}{\partial\zeta^2}-\left(MSin(\theta)+q\right)\overline{w}(\zeta,q)+\lambda_2Cos(\delta)\overline{T}(\zeta,q)=0$$

With corresponding conditions

$$\overline{w}(0,q)=\frac{w}{q^2+\omega^2} \quad and \quad \overline{w}(\infty,q)\to 0$$

The solution using AB technique is:

$$\overline{w}(y,q)=\left(\frac{\omega}{q^2+\omega^2}+\frac{\lambda_1\lambda\lambda_2 Cos(\delta)}{\Pr q^{1-\beta}\left((1-\beta)q^{\beta}+\beta\right)-\lambda\left(\lambda_1 q+\lambda_1 MSin(\theta)\right)}\left(\frac{1}{q}-\frac{a}{b+q}\right)\right)e^{-\zeta\sqrt{\lambda_1 q+\lambda_1 MSin(\theta)}}-$$

$$\left(\frac{1}{q}-\frac{a}{b+q}\right)\frac{\lambda_1\lambda\lambda_2 Cos(\delta)}{\Pr q^{1-\beta}\left((1-\beta)q^{\beta}+\beta\right)-\left(\lambda_1 q+\lambda_1 MSin(\theta)\right)}e^{-\zeta\sqrt{\frac{\Pr q^{1-\beta}\left((1-\beta)q^{\beta}+\beta\right)}{\lambda}}}$$

(12)

5. Computations by Caputo-Fabrizio (CF) fractional approach

Now the fractional simulations are further performed by utilizing Caputo-Fabrizio (CF) definitions.

5.1. Computations for temperature profile by CF technique

Now, the fractional outcomes are presented by employing the CF algorithm. Operating the CF definitions on heat equation (6) leads to:

$$\lambda_3\lambda_5 q\frac{\partial^2\overline{T}(\zeta,q)}{\partial\zeta^2}-\Pr\lambda_4\left((1-\alpha)q^2+\alpha q\right)\overline{T}=0, \tag{13}$$

with

$$\overline{T}(0,q)=\frac{1}{q}-\frac{q}{b+q}\quad and\quad \overline{T}(\infty,q)=0$$

Solution via CFD-definition approach for temperature profile is:

$$\overline{T}(\zeta,q)=\left(\frac{1}{q}-\frac{q}{b+q}\right)e^{-\zeta\sqrt{\frac{\Pr\lambda_4}{\lambda_3\lambda_5}((1-\alpha)q+\alpha)}} \tag{14}$$

Writing the series form of exponential function

$$\overline{T}(\zeta,q)=\left(\frac{1}{q}-\frac{a}{b+q}\right)+\sum_{m_1=1}^{\infty}\sum_{m_2=0}^{\infty}\frac{(-y)^{m_1}(\Pr)^{m_1/2}(\lambda_3\lambda_5)^{-m_2}(\lambda_4)^{m_1/2}}{m_1!(\lambda_3\lambda_5)^{m_2}\Gamma(\alpha)}\frac{1}{q^{1-\alpha-\frac{\alpha n_1}{2}+\alpha n_2}}$$

Taking its Laplace inverse

$$\overline{T}(\zeta,q)=\left(1-ae^{-bt}\right)+\sum_{m_1=1}^{\infty}\sum_{m_2=0}^{\infty}\frac{(-y)^{m_1}(\Pr)^{m_1/2}(\beta)^{n_2}(\lambda_4)^{m_1/2}}{m_1!(\lambda_3\lambda_5)^{m_2}\Gamma(\alpha)\Gamma\left(-1+\alpha-\alpha m_2+\frac{\alpha m_1}{2}\right)}t^{-2+\alpha-\alpha m_2+\frac{\alpha m_1}{2}}$$

5.2.Computations for velocity profile by CF technique

Again employing the LT retaining to Eq. (4), governing the system, while utilizing the CFD definition for solving the thermal field as previously described. By integrating the solution of the thermal field obtained through CFD with the LT technique applied to Equation (14), we attain the transformed ODE of momentum field as follows

$$\frac{\partial^2\overline{w}(\zeta,q)}{\partial\zeta^2}-\lambda_1\left(MSin(\theta)+q\right)\overline{w}(\zeta,q)+\left(\frac{1}{q}-\frac{a}{b+q}\right)\lambda_1\lambda_2Cos(\delta)e^{-\sqrt{\frac{\Pr(1-\alpha)q+\alpha}{\lambda}}}$$

Now by using the transformed conditions and solving the above non-homogeneous ODE, we get

$$\overline{w}(y,q)=\left(\frac{\lambda\lambda_1\lambda_2Cos(\delta)}{\left(\Pr(1-\alpha)q^2+\alpha q\right)-\lambda\lambda_1q^2+\lambda_1qMSin(\theta)}-\frac{a}{b+q}\frac{\lambda_1\lambda_2Cos(\delta)}{\frac{\left(\Pr(1-\alpha)q^2+\alpha q\right)}{\lambda}-\lambda_1q^2+\lambda_1qMSin(\theta)}+\frac{\omega}{q^2+\omega^2}\right)e^{-\zeta\sqrt{\lambda_1q+\lambda_1MSin(\theta)}}-$$

$$\frac{\lambda\lambda_1\lambda_2Cos(\delta)}{\left(\Pr(1-\alpha)q+\alpha\right)-\lambda\lambda_1q+\lambda_1MSin(\theta)}\left(\frac{1}{q}-\frac{a}{b+q}\right)e^{-\zeta\sqrt{\frac{\Pr(1-\alpha)q+\alpha}{\lambda}}}\qquad(15)$$

The momentum field solutions attained by AB and CFD definitions in Eqs. (12 & 15) are so complex to find L-inverse. As many other researchers [32-35] have utilized many different numerical algorithms like Stehfest and Tzou's methods, we have also used the numerical algorithms as mathematically below.

$$w(\xi,t)=\frac{\ln(2)}{t}\sum_{n=1}^{N}v_n\overline{u}\left(\xi,n\frac{\ln(2)}{t}\right),$$

$$v_n=(-1)^{n+\frac{N}{2}}\sum_{l=\left[\frac{p+1}{2}\right]}^{\min\left(p,\frac{N}{2}\right)}\frac{l^{\frac{N}{2}}(2l)!}{\left(\frac{N}{2}-l\right)!l!(l-1)!(p-l)!(2l-p)!}$$

And

$$w(\xi,t)=\frac{e^{4.7}}{t}\left[\frac{1}{2}\bar{w}\left(r,\frac{4.7}{t}\right)+\mathrm{Re}\left\{\sum_{j=1}^{N}(-1)^{k}\frac{1}{2}\bar{w}\left(r,\frac{4.7+k\pi i}{t}\right)\right\}\right]$$

6. Validation of fractional model

The validation of current fractional model is verified by comparing the numerical algorithms used and comparing the results with those obtained by Awan et al. [29], as depicted in Fig. 1(a-b) for both velocity and temperature field. Both figures claim satisfactory agreement with current model.

7. Results and discussion

The physical insight of problem is now addressed with variation of various parameters [37, 38]. A comparative visualization of thermal problem is performed for suspension of titanium dioxide (TiO_2) and molybdenum disulfide (MoS_2) with water and blood base liquids. The results are computed with definitions of Caputo-Fabrizio (CF) and Atangana-Baleanu (AB) fractional operators. The current mathematical model relies on some theoretical flow constraints, with numerical values have been assigned to modeled parameters for performing the computational analysis. The fixed values are taken as $\alpha=\beta=0.8, \lambda=2.5, a=1.4, b=2.5, \mathrm{Pr}=3.5, M=2.4, \omega=0.5, t=1.5$. The analysis relies on comparative simulations for based hybrid nanofluid $(TiO_2-MoS_2)/H_2O$ and blood-based hybrid nanomaterial $(TiO_2-MoS_2)/blood$. Fig. 2(a-b) representing the influence of fractional parameters (α,β) on temperature profile for water based $(TiO_2-MoS_2)/H_2O$ and $(TiO_2-MoS_2)/H_2O$. A decrement in temperature profile is claimed for both hybrid nanofluid suspensions for both fractional parameters. The heat transfer rate is relatively slower for $(TiO_2-MoS_2)/blood$ as compared to $(TiO_2-MoS_2)/H_2O$. Furthermore, more prominent simulations are obtained for AB fractional operators. Fig. 3(a-b) examines the contribution of Prandtl number Pr on fluctuation of temperature field. Decreasing effects of Prandtl number has been incorporated against both decompositions. The reduction in temperature is comparatively larger for blood based hybrid nanofluid $(TiO_2-MoS_2)/blood$. Physically, higher change in Pr conveying low mass diffusivity which leads to decrement to thermal profile. The range of Prandtl number is important in various cooling and heating processes.

Fig. 4(a-b) analyzing the role of fractional parameters α, β on velocity profile. The velocity profile reduces due to both fractional parameters for both hybrid nanofluid models. The outcomes for Hartmann number M on velocity profile have been predicted in Fig. 5(a-b). The velocity profile reduces due to larger values of M. Physical aspects of such trend is associated to applications of Lorentz force which has tendency to slow down the fluid velocity. The resistance in velocity is more convenient for $\left(TiO_2 - MoS_2\right)/blood$. Fig. 6(a-b) proposed the significance of inclination angle θ on velocity profile. The exhibited observations reveal that the velocity profile gets decedent due to different values of θ

Fig. 7(a) predicts the analysis for Nusselt number against Prandtl number for fractional parameters α and β. The Nusselt number reduces due to both fractional parameters. Fig. 7(b) investigated the inspection for wall shear force for α and β. The increasing change in skin friction is exhibited due to both parameters.

Table 3 presents numerical insight of Nusselt number for fractional parameters α and β. The numerical computations are performed with help of both AB and CF schemes. The numerical computations are performed at two different time instants $t = 1.0$ and $t = 1.5$. The Nusselt number increases at $t = 1.0$ while reducing observations are inspected at $t = 1.0$ A fine accuracy of AB and CF operators have been examined. Furthermore, Nusselt number declined for larger ξ.

Table 4 addressing the numerical variation of skin friction with help of AB and CF operators. The analysis is performed at two different time instants $t = 1.0$ and $t = 1.5$ The skin friction increases for both fractional parameters α and β.

8. Closing remarks

Thermal dynamical of hybrid nanofluid due to inclined surface has been inspected by using the fractional computations, leveraging the Atangana-Baleanu (AB) and Caputo-Fabrizio (CF) models. A comparative assessment of thermal profile due to water based hybrid $\left(TiO_2 - MoS_2\right)/H_2O$ and blood based hybrid nanomaterial $\left(TiO_2 - MoS_2\right)/blood$ has been performed. Both fractional techniques are validated for ensuring the accuracy, and confining the reliability of claimed results. The analysis unveils a critical insight into the fractional operators, inclined angle, Prandtl and Hartmann number for heat transfer, velocity profile, skin friction and Nusselt number. The major results have been summarized as:

- ❖ The temperature profile declined due to fractional parameters for both blood and water based hybrid nanofluids. However, the decrement in thermal profile is slower for water based hybrid nanofluid.
- ❖ Higher values of Prandtl number leads to decrement of temperature profile. The control of heat transfer is more impressive for blood based hybrid nanofluid.
- ❖ The variation of inclined angle leads to decrement of fluid velocity.
- ❖ The Nusselt number reduces for fractional parameters.
- ❖ Increasing profile of skin friction have been observed when fractional parameters enhance gradually.
- ❖ The future investigations could explore the potential of hybrid nanomaterials with applications of entropy generation, slip effects, joule heating and external heating effects. The results may further have enhanced for tri-hybrid nanomaterials by utilizing various nanoparticles.
- ❖ Current findings conveying applications in the heat transfer devices, optimizing the thermal managements systems, energy storage applications, drug delivery, cooling processes etc.

Conflict of Interests:

Authors declared no conflict of Interests.

Acknowldmegent:

Dr. Ahmed S. Sowayan would like to special thanks and acknowledgment to Imam Mohammad Ibn Saud Islamic University (IMSIU) Riyadh, Saudi Arabia.

This study is supported by funding from Prince Sattam bin Abdulaziz University's project number (PSAU/2024/R/1445).

Reference

1. Sheikholeslami, M., Ellahi, R., “Three dimensional mesoscopic simulation of magnetic field effect on natural convection of nanofluid”. *Int. J. Heat Mass Transf.* **89**. 799-808 (2015). DOI: https://doi.org/10.1016/j.ijheatmasstransfer.2015.05.110.

2. Khan, S. U., Alabdan, R., Al-Qawasmi, A. R., et al. “Bioconvection applications for double stratification 3-D flow of Burgers nanofluid over a bidirectional stretched surface: Enhancing energy system performance”, *Case Stud. Therm. Eng.*, **26**, pp. 101073 (2021). DOI: https://doi.org/10.1016/j.csite.2021.101073.
3. Sheikholeslami, M., “Magnetic source impact on nanofluid heat transfer using CVFEM”, *Neural. Comput. Applic.* **30**, pp. 1055–1064 (2018). DOI: https://doi.org/10.1007/s00521-016-2740-7.
4. Hejazi, H. A., Khan, M. I., Raza, A., et al. “Inclined surface slip flow of nanoparticles with subject to mixed convection phenomenon: Fractional calculus applications”, *J. Ind. Chem. Soc.*, **99**, p. 100564 (2022). DOI: https://doi.org/10.1016/j.jics.2022.100564.
5. Gangadhar, K., Victoria, E. M., Chamkha, A.J. “Effect of entropy minimization and melting heat on gold-magnesium oxide hybrid nanofluid in squeezing channel with magnetic field with radiation”, *Int. J. Modern Phys. B.* pp. 2450172 (2023). DOI: https://doi.org/10.1142/S0217979224501728.
6. Sheikholeslami, M., “Numerical approach for MHD Al_2O_3 water nanofluid transportation inside a permeable medium using innovative computer method”, *Comput. Methods Appl. Mech. Eng.* **344,** pp. 306-318 (2019). DOI: https://doi.org/10.1016/j.cma.2018.09.042.
7. Li, S., Faizan, M., Ali, F., et al. “Modelling and analysis of heat transfer in MHD stagnation point flow of Maxwell nanofluid over a porous rotating disk”, *Alex. Eng. J.* **91**, pp. 237-248 (2024). DOI: https://doi.org/10.1016/j.aej.2024.02.002.
8. Basha, H., “Significance of Variability in Liquid Properties on 3D MHD Maxwell Nanofluid Flows Over a Stretching Surface with Heat Generation/Absorption and Chemical Reaction”, *J. Nanofluids,* **13(1)**, pp. 15-27 (2024). DOI: https://doi.org/10.1166/jon.2024.2124.
9. Murugan, R. D., Sivakumar N., Tarakaramu N., et al., “Entropy and energy transfer analysis of a Maxwell thin-film fluid over an inclined surface with viscous dissipation effect”, *Z Angew. Math. Mech.* **104(6)**, pp. e202300381 (2024). DOI: https://doi.org/10.1002/zamm.202300381.
10. Sheikholeslami, M. “Numerical investigation for concentrated photovoltaic solar system in existence of paraffin equipped with MWCNT nanoparticles”, *Sustain. Cities Soc.* **99**, pp. 104901 (2023). DOI: https://doi.org/10.1016/j.scs.2023.104901.

11. Adnan; Nadeem, A.; Said, N. M., et al. “LSM Analysis of Thermal Enhancement in KKL Model-Based Unsteady Nanofluid Problem Using CCM and Slanted Magnetic Field Effects”, *J. Therm. Anal. Calorim.*, **149(2)**, pp. 839–851 2024. DOI: https://doi.org/10.1007/s10973-023-12801-1.
12. Aich, W., Adnan, Abbas, W., et al., “Impacts of nanoscaled metallic particles on the dynamics of ternary Newtonian nanofluid laminar flow through convectively heated and radiated surface”, *Case Stud. Therm. Eng.*, **53,** pp. 103969 (2024). DOI: https://doi.org/10.1016/j.csite.2023.103969.
13. Sheikholeslami, M.; Khalili, Z., “Simulation for Impact of Nanofluid Spectral Splitter on Efficiency of Concentrated Solar Photovoltaic Thermal System”, *Sustain. Cities Soc.*, **101**, pp. 105139 (2024). DOI: https://doi.org/10.1016/j.scs.2023.105139.
14. Ikram, M. D., Asjad, M. I., Akgül, A., et al. “Effects of hybrid nanofluid on novel fractional model of heat transfer flow between two parallel plates”, *Alex. Eng. J.*,**60(4)**, 3593-3604 (2021). DOI: https://doi.org/10.1016/j.aej.2021.01.054.
15. Shatnawi, T. A., Abbas, N., Shatanawi, W., “Comparative study of Casson hybrid nanofluid models with induced magnetic radiative flow over a vertical permeable exponentially stretching sheet”, *AIMS Math*. **7(12)**, pp. 20545-20564 (2022). DOI: 10.3934/math.20221126
16. Ali, K., Reddy, Y. R., Shekar, B. C., “Thermo-fluidic transport process in magnetohydrodynamic couette channel containing hybrid nanofluid”, *Partial Diff. Equ. Appl. Math.*, **7**, pp. 100468 (2023). DOI: https://doi.org/10.1016/j.padiff.2022.100468.
17. Kumar, A., Hassan, M. A. “Heat transfer in flat tube car radiator with CuO-MgO-TiO2 ternary hybrid nanofluid”, *Powder Technol*.: pp. 119275 (2023). DOI: https://doi.org/10.1016/j.powtec.2023.119275.
18. Sharma, B. K., Sharma, P., Mishra, N. K. et al., “Bayesian regularization networks for micropolar ternary hybrid nanofluid flow of blood with homogeneous and heterogeneous reactions: Entropy generation optimization”, *Alex. Eng. J.*, **77,** pp. 127-148 (2023). DOI: https://doi.org/10.1016/j.aej.2023.06.080.
19. Yasir, M., Khan, M., Alqahtani, A. S., “Mass transpiration effect on rotating flow of radiative hybrid nanofluid due to shrinking surface with irregular heat source/sink”, *Case*

Stud. Therm. Eng. **44**, pp. 102870, (2023). DOI: https://doi.org/10.1016/j.csite.2023.102870.

20. Ahmad, B., Abbas, T., Fatima, K., et al., "Nonlinear flow of hybrid nanofluid with thermal radiation: A numerical investigation", *Z Angew. Math. Mech*. **104(1)**, pp. e202200170 (2023). DOI: https://doi.org/10.1002/zamm.202200170.
21. Gangadhar, K., Victoria, E. M., Chamkha, A. J., "Effect of entropy minimization and melting heat on gold-magnesium oxide hybrid nanofluid in squeezing channel with magnetic field with radiation", *Int. J. Modern Phys. B.*, **38(14)**, pp. 2450172 (2023). DOI: https://doi.org/10.1142/S0217979224501728.
22. Ige, E. O., Falodun, B. O., Adebiyi, D. O. O., et al., " Computational Analysis of Mixed Convection in a Blood-Based Hybrid Nanoliquid under Boussinesq Approximation in a Transient Regime", *J. Comput. Biophys. Chem.* , **22(03),** pp. 347-359 (2023). DOI: 10.1142/S2737416523400094
23. Li, X., Abbasi, A., Al-Khaled, K., et al., "Thermal performance of iron oxide and copper (Fe_3O_4, Cu) in hybrid nanofluid flow of Casson material with Hall current via complex wavy channel", *Mater. Sci. Eng.* , **289**, 116250, (2023). DOI: https://doi.org/10.1016/j.mseb.2022.116250.
24. Turkyilmazoglu, M., "Hyperthermia therapy of cancerous tumor sitting in breast via analytical fractional model", *Comput. Biology Medic.,* **164**, pp. 107271, (2023). DOI: https://doi.org/10.1016/j.compbiomed.2023.107271.
25. Turkyilmazoglu, M., Altanji, M., "Fractional models of falling object with linear and quadratic frictional forces considering Caputo derivative", *Chaos, Solit. Frac.,* **166**, pp. 112980 (2023). DOI: https://doi.org/10.1016/j.chaos.2022.112980
26. Turkyilmazoglu, M., "Transient and passage to steady state in fluid flow and heat transfer within fractional models", *Int. J. Numer. Methods Heat Fluid Flow*, **33(2)**, pp. 728-750 (2023). DOI: https://doi.org/10.1108/HFF-04-2022-0262.
27. Sheikh, N. A., Ali, F., Saqib, M., et al., "A comparative study of Atangana-Baleanu and Caputo-Fabrizio fractional derivatives to the convective flow of a generalized Casson fluid", *Eur. Phys. J. Plus*, **132**, pp. 1-14 (2017). DOI: https://doi.org/10.1140/epjp/i2017-11326-y

28. Raza, A., Khan, S. U., Farid, S., et al., "Thermal activity of conventional Casson nanoparticles with ramped temperature due to an infinite vertical plate via fractional derivative approach", *Case Stud. Therm. Eng.*, **27**, pp. 101191 (2021). DOI: https://doi.org/10.1016/j.csite.2021.101191.
29. Awan, A. U., Shah, N. A., Ahmed, N., et al., "Analysis of free convection flow of viscous fluid with damped thermal and mass fluxes", *Chin. J. Phys.* **60**, pp. 98-106 (2019). DOI: https://doi.org/10.1016/j.cjph.2019.05.006.
30. Lisha, N., Vijayakumar, A., "Analytical investigation of the heat transfer effects of non-Newtonian hybrid Nanofluid in MHD flow past an upright plate using the Caputo fractional order derivative", *Symmetry*, **15**, p. 399 (2023). DOI: https://doi.org/10.3390/sym15020399.
31. Hanif, A., Iqbal, A., Butt, K., et al., "Numerical approach to solve Caputo-Fabrizio-fractional model of corona pandemic with optimal control design and analysis", *Math. Methods Appl. Sci.,* **46(8),** pp. 9751-9782 (2023). DOI: https://doi.org/10.1002/mma.9085.
32. Adel, W., Amer, Y. A., Youssef, E. S. M., et al., "Mathematical analysis and simulations for a Caputo-Fabrizio fractional COVID-19 model", *Partial Diff. Equ. Appl. Math.,* **8**, pp. 100558 (2023). DOI: https://doi.org/10.1016/j.padiff.2023.100558.
33. Alzahrani, A. B. M., Saadeh, R., Abdoon, M. A., et al., "Effective methods for numerical analysis of the simplest chaotic circuit model with atangana–baleanu caputo fractional derivative", *J. Eng. Math.* **144(1)**, 9 (2024). DOI: https://doi.org/10.1007/s10665-023-10319-x.
34. Le, Q. H., Ali, Q., Al-Khaled, K., et al., Study of hybrid nanofluid containing graphene oxide and molybdenum disulfide nanoparticles with engine oil base fluid: A non-singular fractional approach", *Ain Shams Eng. J.* **15(1),** pp. 102317 (2024). DOI: https://doi.org/10.1016/j.asej.2023.102317.
35. Mahitha, O., Golla, V. K. A., Öztop, H.F., et al., "Application of Caputo fractional approach to MHD Casson nanofluid with alumina nanoparticles of various shape factors on an inclined quadratic translated plate", *Hybrid Adv.* **6**, 100183 (2024). DOI: https://doi.org/10.1016/j.hybadv.2024.100183.

36. Turkyilmazoglu, M., Alofi, A. S., Liquid Vortex Formation in a Swirling Container Considering Fractional Time Derivative of Caputo, *Fract. Fractional*, **8(4)**, pp. 231 (2024). DOI: https://doi.org/10.3390/fractalfract8040231.

37. Bedram, A., "A novel method (a tube with successive increase and reduction in diameter) to increase nanofluid heat transfer in a tube", *Sci. Iran.* **31(13),** pp. 1030-1042 (2024). DOI: 10.24200/SCI.2023.60676.6935.

38. Sarfraz, M., Khan, M., "Energy optimization of water-based hybrid nanomaterials over a wedge-shaped channel", *Sci. Iran.* **31(1)**, pp. 71-82 (2024). DOI: 10.24200/sci.2023.60254.6689.

Figure and table captions

Fig. 1. Comparison of fractional model with results of Awan et al. [29].

Fig. 2(a-b): Effects of α, β on temperature field.

Fig. 3(a-b). Effects of $\Pr$ on temperature field.

Fig. 4. Impact of α, β on velocity field.

Fig. 5: Impact of M on velocity field.

Fig. 6. Impact of θ on velocity field.

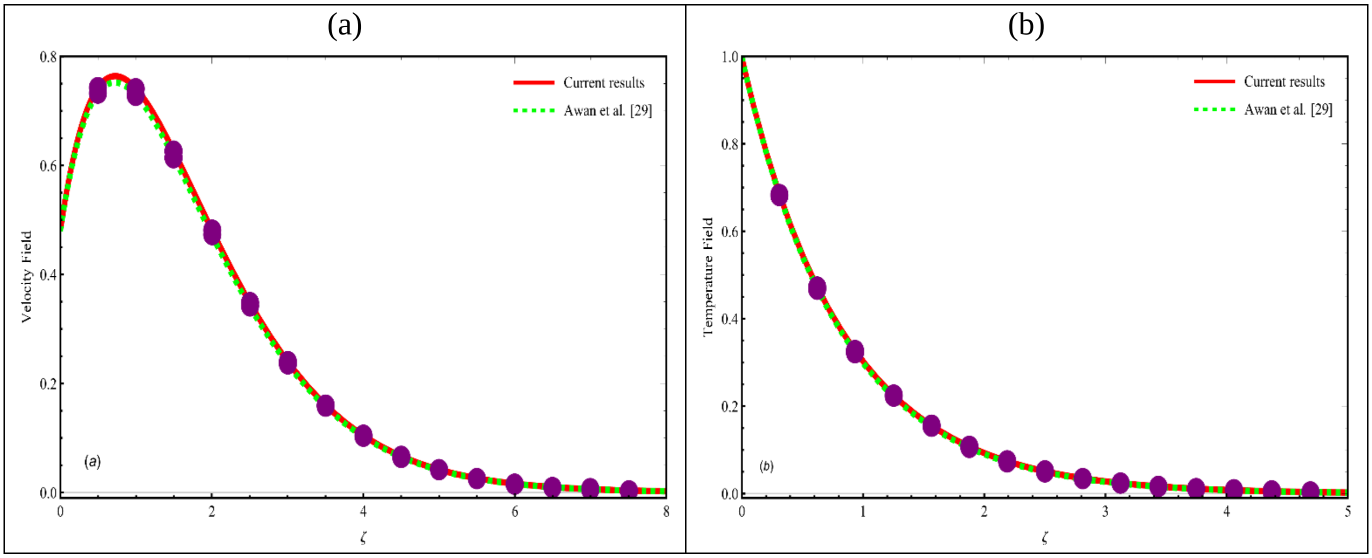
Fig. 7(a-b): (a)Variation of Nusselt number with Prandtl number for various values of α and β , (b)Variation of skin friction with Prandtl number for various values of α and β .

Table 1: Mathematical relations for various properties of nanofluids.

Table 2: Thermal properties of nanoparticles and base fluids.

Table 3. Numerical values of Nusselt number Nu.

Table 4. Numerical variation of skin friction.



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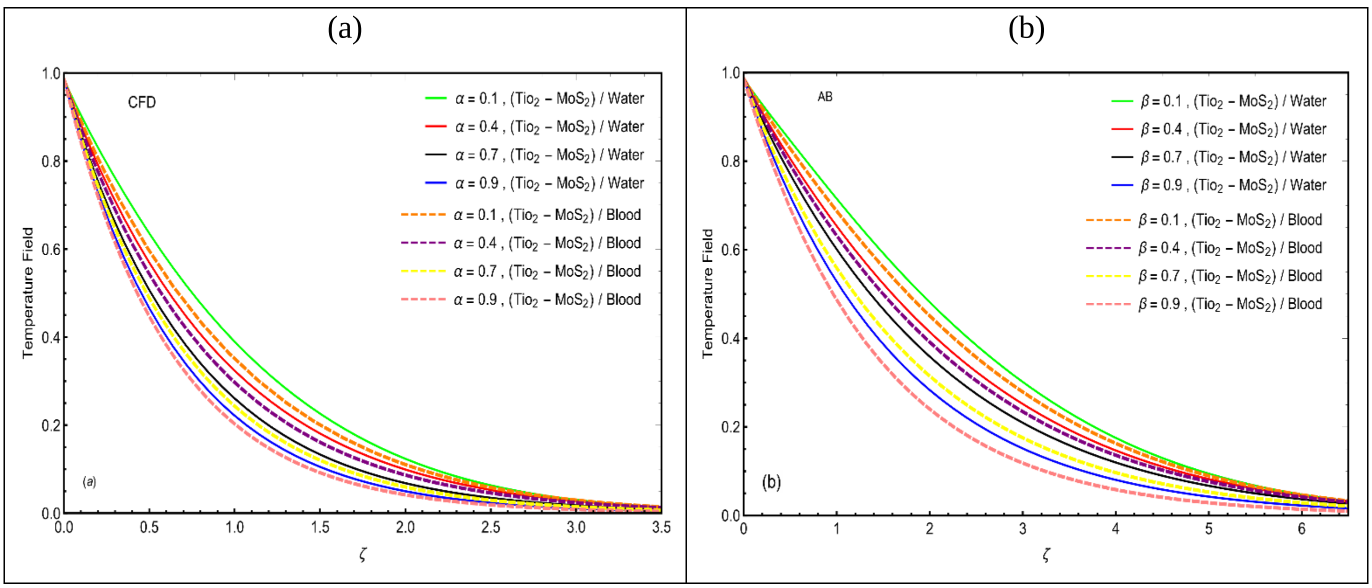


Fig. 2(a-b): Effects of α, β on temperature field.

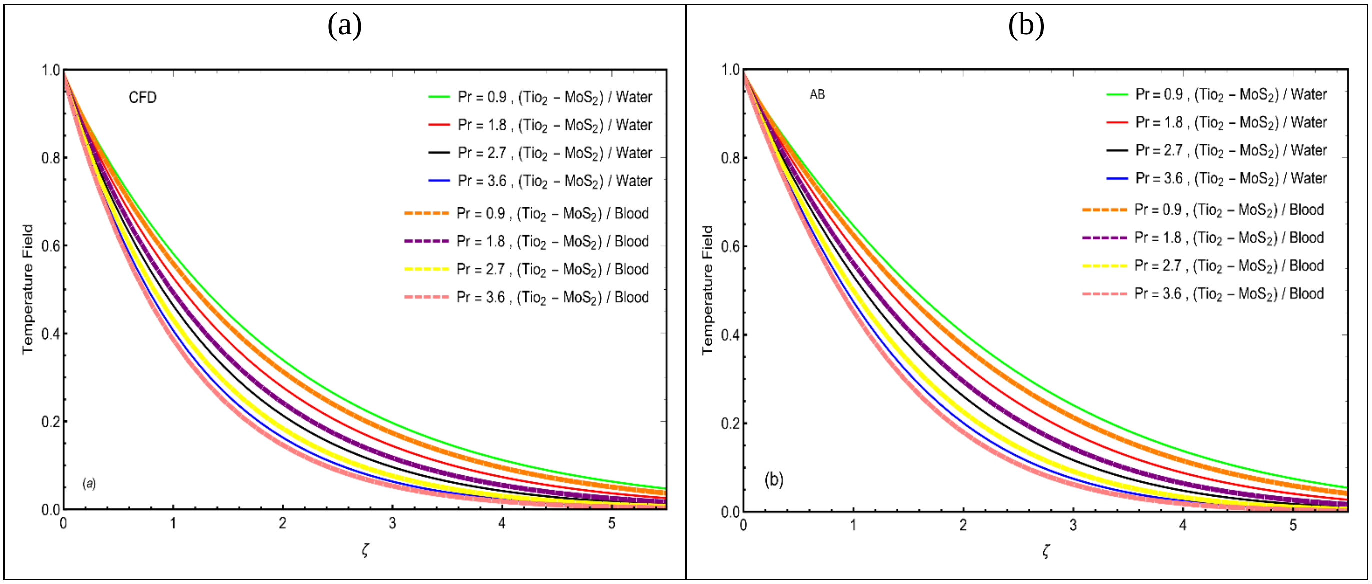


Fig. 3(a-b). Effects of Pr on temperature field.

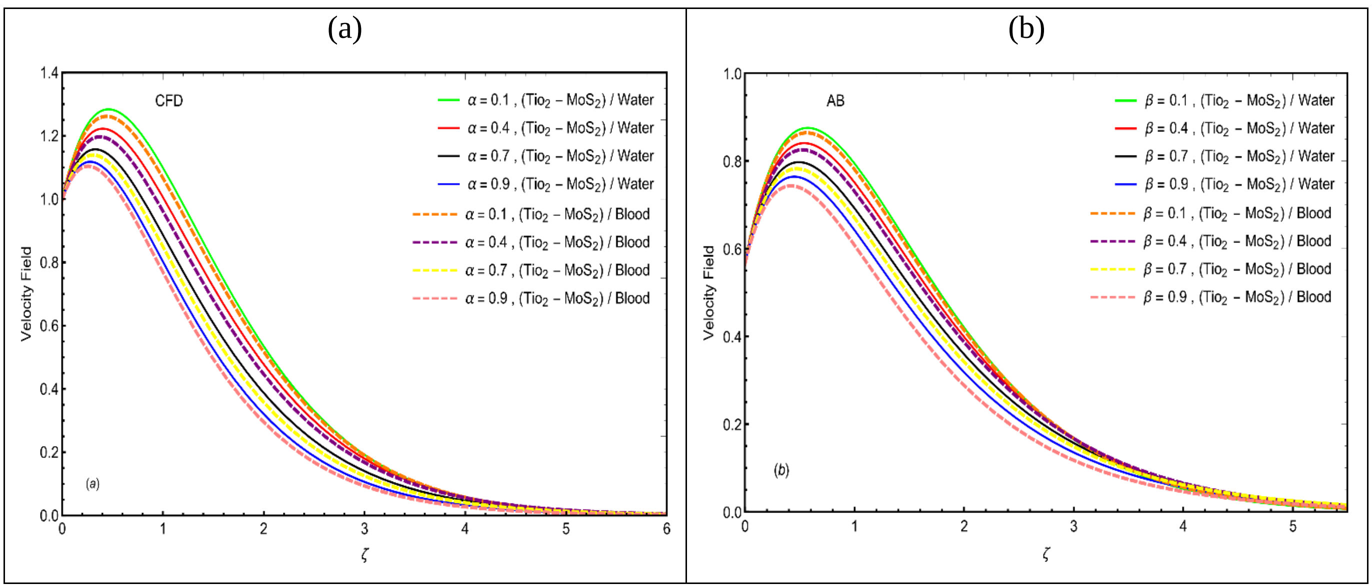


Fig. 4. Impact of α, β on velocity field.

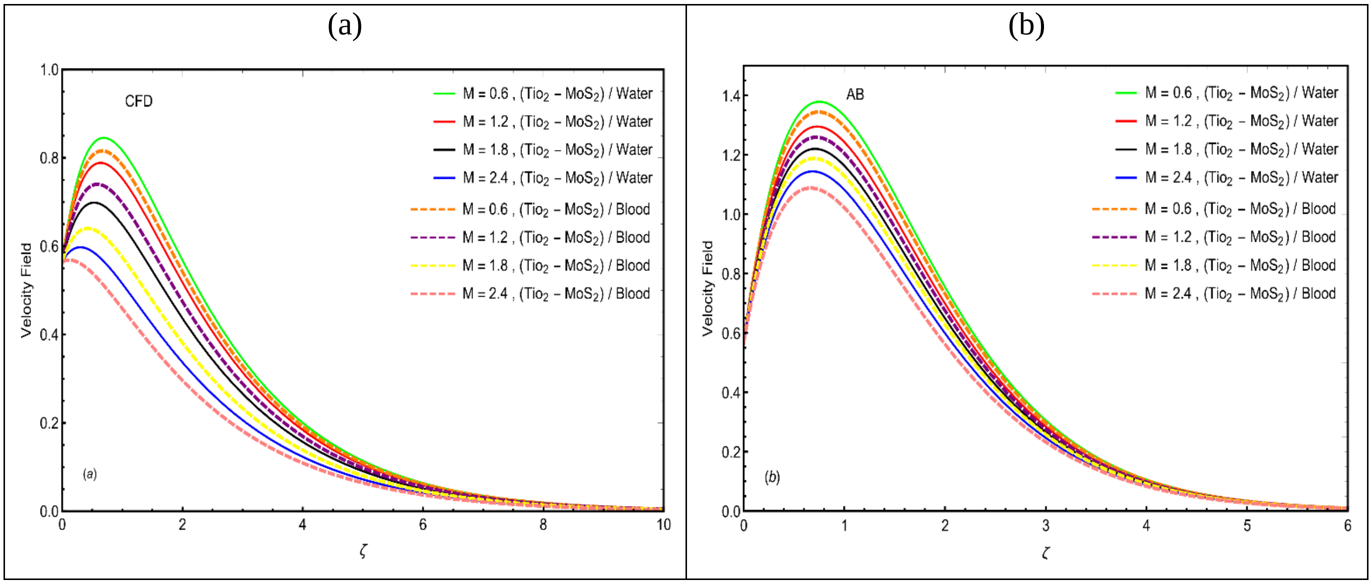
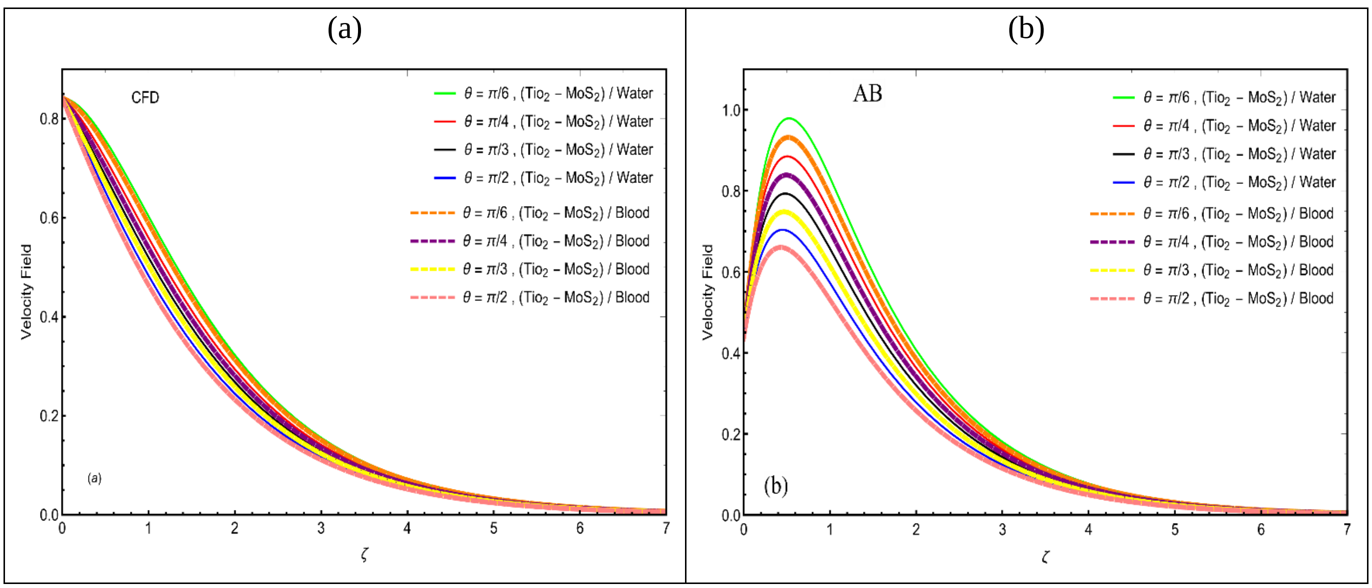


Fig. 5: Impact of M on velocity field.



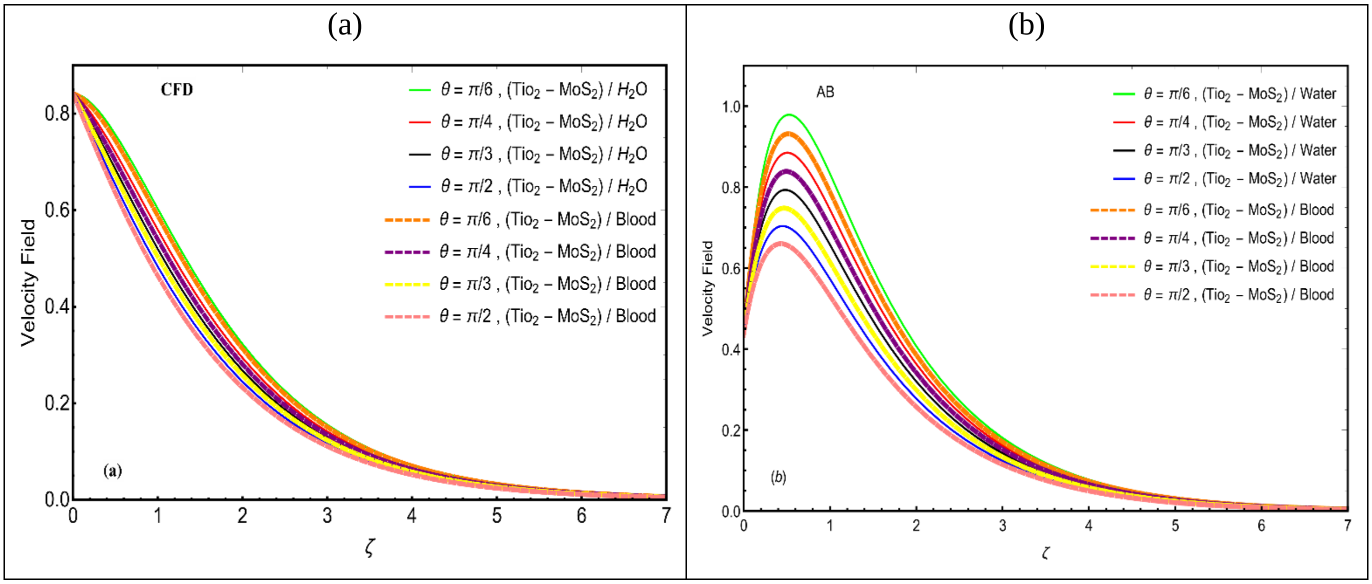


Fig. 6. Impact of θ on velocity field.

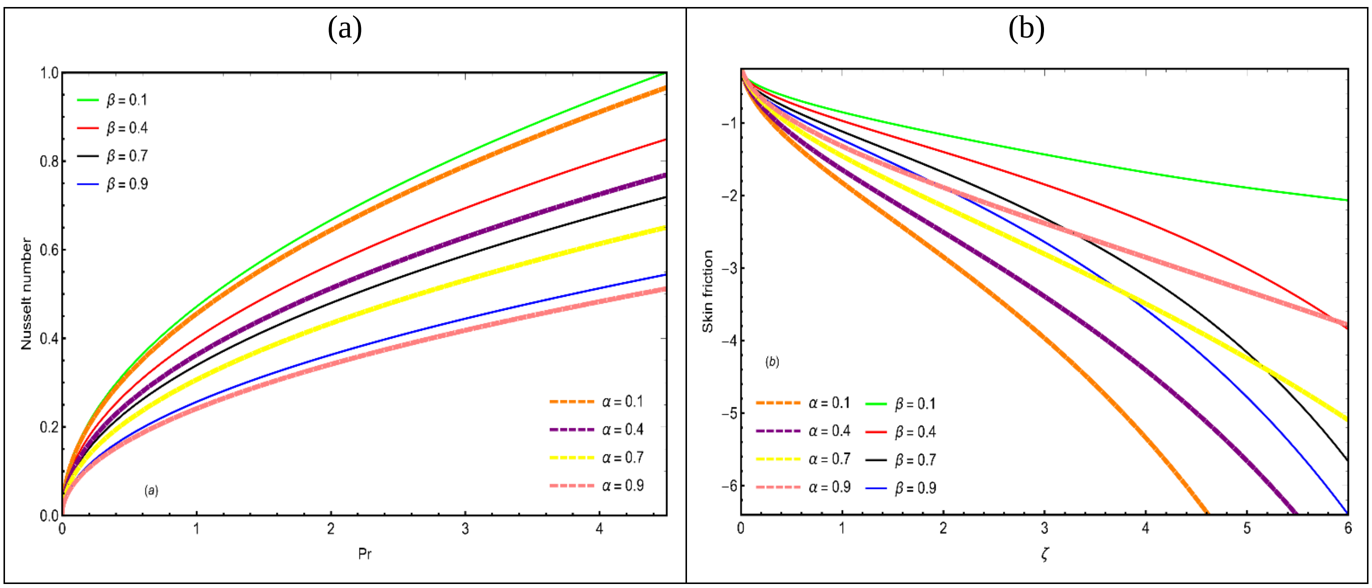


Fig. 7(a-b): (a)Variation of Nusselt number with Prandtl number for various values of α and β, (b)Variation of skin friction with Prandtl number for various values of α and β.

Table 1: Mathematical relations for various properties of nanofluids.

Thermal quantities	Nanofluid

Solidity	$\rho_f = \dfrac{\rho_{nf}}{(1-\varphi)+\varphi\dfrac{\rho_s}{\rho_f}}$
Dynamic Thickness	$\mu_f = \mu_{nf}(1-\varphi)^{2.5}$
Electrical Conduction	$\sigma_f = \dfrac{\sigma_{nf}}{\left(1+\dfrac{3\left(\dfrac{\sigma_s}{\sigma_f}-1\right)\varphi}{\left(\dfrac{\sigma_s}{\sigma_f}+2\right)-\left(\dfrac{\sigma_s}{\sigma_f}-1\right)\varphi}\right)}$
Thermal Conductivity	$k_f = \dfrac{k_{nf}}{\dfrac{k_s+(n-1)k_f-(n-1)(k_f-k_s)\varphi}{k_s+(n-1)k_f+(k_f-k_s)\varphi}}$
Heat Capacitance	$(\rho C_p)_f = \dfrac{(\rho C_p)_{nf}}{(1-\varphi)+\varphi\dfrac{(\rho C_p)_s}{(\rho C_p)_f}}$
Thermal Extension	$(\rho\beta)_f = \dfrac{(\rho\beta)_{nf}}{(1-\varphi)+\varphi\dfrac{(\rho\beta)_s}{(\rho\beta)_f}}$
Absorption Development	$(\rho\beta_C)_f = \dfrac{(\rho\beta_C)_{nf}}{(1-\varphi)+\varphi\dfrac{(\rho\beta_C)_s}{(\rho\beta_C)_f}}$

Table 2: Thermal properties of nanoparticles and base fluids.

Corporeal stuff	H_2O	Blood	MoS_2	TiO_2
$\rho(kgm^{-3})$	997.1	1063	5060	4250
$Cp\ (Jk^{-1}Kg^{-1})$	4179	3594	397.7	686.4
$k(Wm^{-1}K^{-1})$	0.613	0.492	904.5	8.9538
$\beta_T \times 10^5 (1/K)$	21	0.18	2.842	0.90

Table 3. Numerical values of Nusselt number (Nu).

Parameters	AB-Simulations		CF-Simulations	
α,β	$t=1.0$	$t=1.5$	$t=1.0$	$t=1.5$
0.1	0.5585	0.4932	0.3089	0.2677
0.15	0.5547	0.4978	0.3327	0.2916
0.2	0.5509	0.5026	0.3548	0.3150
0.25	0.5473	0.5078	0.3754	0.3380
0.3	0.5438	0.5134	0.3948	0.3608
0.35	0.5404	0.5193	0.4132	0.3833
0.4	0.5373	0.5256	0.4306	0.4056
0.45	0.5344	0.5323	0.4471	0.4278
0.5	0.5317	0.5394	0.4628	0.4498
0.55	0.529346	0.5469	0.4776	0.4716
0.6	0.5274	0.5549	0.4916	0.4931
0.65	0.5258	0.5633	0.5048	0.5144
0.7	0.5247	0.5721	0.5172	0.5354
0.75	0.5241	0.5819	0.5287	0.5560
0.8	0.5241	0.5904	0.5395	0.5763
0.85	0.5244	0.5995	0.5494	0.5962
0.9	0.5249	0.6082	0.5585	0.6157

Table 4. Numerical variation of skin friction.

Parameters	AB-Simulations		CF-Simulations	
α,β	$t=1.0$	$t=1.5$	$t=1.0$	$t=1.5$
0.1	-1.2787	-2.1328	-1.2631	-2.1786
0.15	-1.2704	-2.1293	-1.2311	-2.1474
0.2	-1.2587	-2.1236	-1.1997	-2.1163
0.25	-1.2436	-2.1152	-1.1689	-2.0851
0.3	-1.2252	-2.1037	-1.1386	-2.0540
0.35	-1.2036	-2.0889	-1.1089	-2.0231
0.4	-1.1791	-2.0704	-1.0798	-1.992
0.45	-1.1518	-2.0481	-1.0512	-1.9619
0.5	-1.1221	-2.0220	-1.0231	-1.9317
0.55	-1.0903	-1.9923	-0.9956	-1.9019
0.6	-1.0567	-1.9590	-0.9685	-1.8725
0.65	-1.0215	-1.9225	-0.9420	-1.8435
0.7	-0.9851	-1.8829	-0.9160	-1.8151
0.75	-0.9478	-1.8409	-0.8906	-1.7872
0.8	-0.9097	-1.7966	-0.8658	-1.7599
0.85	-0.8713	-1.7506	-0.8416	-1.7334
0.9	-0.8326	-1.7033	-0.8180	-1.7074

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