A Robust Primary Frequency Response Constrained Power Management in Microgrids Considering Distribution Energy Resources Virtual Inertia

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Abstract

A new power management model is presented here to prevent excessive frequency deviations by the more commitment of higher inertia power plants and more contribution of renewable resources or energy storage systems fast inertia response. To have a mixed-integer linear programming model, the primary frequency response constraints are linearized. Using distributionally robust optimization to model uncertainty of renewable sources in the primary frequency response strategy, considering technical limitation of network, and considering suitable case studies to investigate the capability of the proposed scheme are contributions of this paper. Model is examined on a real isolated microgrid. Results show that by activation of distributed energy resources the power management can be done with lower cost. Energy not supplied of microgrid can be reduced when energy storage systems are utilized as energy buffers in network. Comparing robust model with deterministic method shows the more expensive management procedure, however, a more frequency stability is obtained in the contingency condition. The proposed plan with the presence of only frequency control leads to a 361% increase in planning cost. However, if renewable resources are added to this scheme, a 62% reduction in planning cost occurs.

Keywords: Microgrid, Power Management, Mixed-Integer Linear Programming, Primary Frequency Response, Reliability.

NOMENCLATURE

Abbreviations

PFR	Primary Frequency Response
PV	Photovoltaic

Symbol

b, n, g, l	Bus, node, generator, and line indices
fr, to	Origin and destination buses of a line
pv, w	Photovoltaic and wind unit indices
$t, t^{'}, \Delta t$	Time interval indices
B_l^{line}	Susceptance of the line l
c_g^{gen}, c_g^{NL}	generation and no-load cost of unit g

c_g^{inv}, c_l^{inv}	Investment cost if a unit g and line l
c_{pv}^{inv}, c_w^{inv}	Investment cost of PV and wind units
$c_b^{lshd}, \zeta^{\max}$	Cost and maximum allowed load shed
c_g^{pfr}, c_g^{res}	Cost of primary and territory reserve
$c_{pv}^{pvgen}, c_w^{wgen}$	Generation cost of PV and wind units
c_g^{st}, c_g^{sd}	Startup and shutdown cost of unit g
c^{Total}	Total expansion cost
$D_{n,t}^{Fore}$ $D_{n,t}^{Fore}$	Forecasted load of node n at time t
$f_0, \underline{f}_{\min}$	Nominal and minimum frequency
f^{gen}, f^{res}	Total cost of generation and reserve
f^{Ished}, f^{renew}	Load shedding and renewable unit cost
f^{st}, f^{sd}, f^{pfr}	Total startup, shutdown, and Primary Frequency Response (PFR) cost
H_g^G, H_{pv}^{PV}	Inertia of conventional and PV unit
H_t, H_w^w	Total inertia and wind unit inertia
MUT, MDT	Minimum up and down time of unit g
$\tilde{P}_{pv}^{forcast}, \tilde{P}_w^{forcqst}$	Average forecasted PV and wind power
$\overline{p}_{pv}^{forcast}, \underline{p}_{pv}^{forcqst}$	Min and max forecasted power of PV
$\overline{p}_w^{forcast}, \underline{p}_w^{forcqst}$	Min and max forecasted power of wind
$P_{g,t}^{gen}, P_{lost}$	Generation power and total lost power
$\overline{P}_g^{gen}, \underline{P}_g^{gen}$	Maximum and Minimum of Generation power
$P_{l,t}^{Line}, \overline{P}_l^{Line}$	Line power and max capacity of line l
$P_{b,t}^{Load}, \tilde{P}_{b,t}^{Load}$	Mean and forecasted load of bus b
$P_{b,t}^{lshd}, P_{b,t}^{load}\big\|_{y+1}$	The load shed /next year load of bus b
$P_{pv,t}^{pvgen}, p_{w,t}^{wgen}$	Power generation of PV and wind units
R_g^G, R_{pv}^{PV}	Droop value of unit g and PV unit
$R_{g,t}^{pfr}, R_{g,t}^{res}$	Primary and territory reserve of unit g
$\overline{R}_g^{pfr}, \underline{R}_g^{pfr}$	Max and min primary frequency reserve
$\overline{R}_g^{res}, \underline{R}_g^{res}$	Max and min territory frequency reserve
R_t, R_w^W	Total droop and drop of wind unit
$\overline{\Delta P_g^{RD}}, \overline{\Delta P_g^{RU}}$	Maximum ramp-up/down of unit g
$u_{g,t}^G, u_{pv,t}^{PV}, u_{w,t}^W$	PFR activation status of unit/PV/ wind
x_l^{Line}	Line status
$x_{g,t}^{on}, x_{pv,t}^{on}, x_{w,t}^{on}$	On/off status of unit g , PV and wind
$x_{g,t}^{sd}, x_{g,t}^{st}$	Shutdown and startup status of unit g
$\varepsilon_{b,t}^{load}, \varepsilon_0^{load}$	Average value of load in each bus
$\varepsilon_{b,t}^{pv}, \varepsilon_0^{\text{wind}}$	Average value of PV/wind in each bus
$\xi^{PV}, \xi^{wind}, \xi^{gen}$	Radius of PV/wind/Load variations
$\theta_{fr,t}^n, \theta_{to,t}^n$	Bus angle of node fr and to at t

$\Omega^{gen}, \Omega^{line}$ Set of generators and lines
$\Omega^{PV}, \Omega^{wind}$ Set of PV and wind units

1. Introduction

1.1. Motivation

By the expansion of low inertia renewable energy resources in microgrids, control of frequency deviation after occurring a contingency using provision of frequency response becomes a challenge [1]. In the traditional power systems, providing the secondary [2] or territory frequency control [3] was the aim of researchers. While, inertia and primary frequency response (PFR) has gained more importance than before [4]. If the primary frequency deviation cannot be limited in the first moments after an outage, under-frequency protections will be activated that would lead to a partial disconnection or blackout in the system [5]. Therefore, considering the primary frequency parameters such as rate of change of the frequency, frequency nadir and quasi steady state behavior is included in new power management models of microgrids [6]. For frequency response improvement either remedial actions [7] or preventative consideration [8] can be adopted which the latter is the focus of this paper.

1.2. Literature review

Most of frequency control preventative actions is based on unit-commitment or power management of generation units or loads during the day. So, frequency constrained unit commitments models have been developed [9]. Contribution of load shedding [10] or integration of renewable energy resources [11] or energy storage systems [12] are also included in these models. The review of frequency constrained unit commitment models are presented in [13].

Primary frequency response is a concept that analyze the frequency dynamics immediately after an event that leads to a power imbalance followed by frequency oscillations in the network [14]. After a generation unit outage, the frequency decreases by a fast rate called rate of change of frequency. The absolute value of this ramp must not exceed from 1Hz/s (for less than 500 ms) [15]. According to the single-machine equivalent model of system and swing equation, rate of change of frequency can be limited merely by inertia of the system [16]. Frequency decreases until the sum of inertia responses of other online units overcomes the lost power [17]. Then the frequency reach to its minimum value called nadir which is vital not to exceed its limits (about 500 mHz) [18]. According to [11, 19] the frequency nadir has a non-linear relation with inertia, governor droop and power fraction of high pressure turbine. In this regard, an analytical model for minimum frequency prediction is obtained in [20] using the polynomial fitting of governor PFR characteristic. Furthermore, in [21] a model predictive approach is introduced that calculate the time and frequency of nadir by solving a set of non-linear equation in isolated low inertia networks.

Frequency stability in isolated grids or small microgrids is somehow more complex than large scale power system. A small outage in an microgrid can lead to a large frequency deviation [10]. So, due to this sensitivity, more attention must be paid prevent measures. Utilizing any apparatus that improves the frequency stability is of great importance in microgrids [22]. Accordingly, the use of any virtual inertia of renewable energy resources or energy storage systems for frequency support is on the agenda of system operators [23]. Hence, in this paper a new microgrid power management model is presented that contribute the inertia response of renewable energy resources and energy storage systems for primary frequency support. The more details about the main distinction of this work with other models are mentioned below.

In [24], several models of frequency-constrained unit commitment problem are compared from frequency improvement. In [25] using the general order frequency support model and swing equation, time domain dynamics of frequency obtained considering the same governor time constant. After calculation of frequency nadir, a nonlinear function representing the nadir constrain is piecewise linearized using multi variable repression method. The same approach is also stated in [11] while the primary frequency response of renewable energy resources such as photovoltaic and wind turbine are added to its model. A more complete frequency dynamics model is introduced in [26] that consider the details of converter control and generators dynamics. The piecewise linearization with curve fitting is done in it and a new bound extraction method is applied to reduce the computational burden.

A different nadir calculation approach is investigated in [27] which consider a predefined function for response of generation units and calculates the nadir point using time domain analysis in the presence of battery energy storage systems. The [28] assumes a predefined primary frequency response of each unit without any direct reference to nadir point. Instead, the provision of total lost power or energy is assumed as a linear constraint for frequency support. In [22] considering the droop behavior, headroom of generation units, and linear approximation of frequency changes from an outage to nadir time, several linear boundary conditions are obtained to support the lost power and energy of the network in the first moments. In [29-32] a logarithmic equation is formulated for nadir frequency constraints using a predefined function for generation unit primary frequency response. Their nonlinear nadir related terms are linearized using big M method. Some forms of nonlinear nadir constraints are extracted in [33, 34]. Extraction of nadir constraints in multi-area networks is performed in [12, 35] an linearized using Pseudo-Boolean functions.

In [36], it proposes a power instruction correction-based frequency response strategy for Grid forming inverter in islanded microgrids. Firstly, the fundamental principle of the power instruction correction strategy that originates the basic droop control is theoretically analyzed. Ref. [37] proposes two control loops as secondary frequency control, first is an improved optimized delay-dependent frequency control to ensure robust frequency control under daily changes in system parameters that may occur in prospective, renewable-rich future AC microgrids. In [38], it proposes a PFR ancillary service market mechanism to ensure the security of frequency nadir and quasi-steady-state frequency while providing equitable settlements to multiple PFR providers. In [39], a model predictive control scheme integrated with two-layer moving-horizon estimation observer is proposed and applied to the secondary frequency control of a photovoltaic high-penetration microgrid. Ref. [40] presents a method of using a distributed control architecture to support primary frequency response in networked microgrids operations. The support of primary frequency response is accomplished using the Open Field Message Bus reference architecture and Grid Friendly Appliance controllers. Ref. [41] proposes a dynamic demand response load control strategy for primary frequency regulation in microgrids that utilizes Electric water heaters as responsive loads.

1.3. Research gaps and Contributions

Based on the research background in the field of PFR in the power system, there are various research gaps. As a research gap, the generation power of renewable resources is based on natural phenomena. For example, the production power in a wind turbine is dependent on the wind speed, or in photovoltaic systems, it is dependent on the solar radiation. Since the prediction of the amount of these natural phenomena is erroneous, their values are uncertain. Therefore, the power generation of renewable resources is accompanied by uncertainty. This has been addressed in less research in the field of PFR, and the power of renewable resources is generally assumed to be constant. One of the uncertainty modeling trends is robust optimization. This method uses only one scenario. In this scenario, the worst value of uncertainty is selected in terms of the objective function. Therefore, the optimal solution obtained in the worst-case scenario represents the most resistant solution against the prediction error of the uncertainty parameters. This topic has been considered in less research in the field of PFR in the power system. As another research gap, there are various technical limitations in the PFR problem, such as the technical limitations of the power system. However, in few studies, these limitations were considered, or only one or two limitations were considered.

To address the research gaps, this paper presents a novel power management model to control frequency by committing power generation units that have large inertia. Another solution adopted in this research is the utilization of renewables and storage devices with fast inertia response. Linearization of the constraints related to the primary frequency response has also been done to provide a mixed-integer linear programming model. To take the uncertainty of renewables into account when dealing with power frequency control, the paper uses a distributionally robust model, which is solved with the help of MOSEK and YALMIP.

Therefore, the contribution of this paper can be stated as follows:

A new distributionally robust optimization is presented that considers the uncertainty of renewable energy resources contribution in primary frequency response.

Some limitation or boundary of technical parameters is considered.

Several case studies are done for more understanding of the proposed model.

The rest of the paper is organized as follows. In section 2 the description of the proposed model is presented. Then, the solution method of the model is introduced in 3 as a flowchart. The simulation and analysis are expressed in 4. Finally, the conclusion and future work is included in the part 5.

2. Description and modeling

2.1. Unit commitment basic model

Objective function of the basic model of power management of microgrid is written as equation (1):

$$\min f^{Total} = \sum_{t}\sum_{g}\left(c_g^{gen}P_{g,t}^{gen}+c_g^{NL}x_{g,t}^{on}+c_g^{res}R_{g,t}^{res}+c_g^{st}x_{g,t}^{st}+c_g^{sd}x_{g,t}^{sd}\right) +\sum_{t}\sum_{re}\left(c_{re}^{gen}P_{re,t}^{RE}+c_{re}^{res}R_{re,t}^{res}\right)+\sum_{t}\sum_{b}\left(c_b^{lshd}P_{b,t}^{lshd}\Delta t\right) \quad (1)$$

Full description of all symbols is given in the nomenclature. According to equation (1), total cost of power management can be calculated by the summation of operating cost of conventional power plants which includes power generation, no-load, reserve, startup, shutdown, and load shedding cost which are stated by $c_g^{gen} P_{g,t}^{gen}$, $c_g^{NL} x_{g,t}^{on}$, $c_g^{res} R_{g,t}^{res}$, $c_g^{st} x_{g,t}^{st}$, $c_g^{sd} x_{g,t}^{sd}$, and $c_b^{lshd} P_{b,t}^{lshd} \Delta t$ symbols, respectively. Furthermore, renewable energy resources costs are added to the objective function that includes generation and reserve cost shown $c_{re}^{gen} P_{re,t}^{RE}$ and $c_{re}^{res} R_{re,t}^{res}$, respectively. The boundary constraints of the basic unit commitment model are described as the equations (2)-(18).

$$x_{g,t}^{on}, x_{g,t}^{st}, x_{g,t}^{sd} \in \{0,1\}, \quad \forall g \in \Omega^{gen,RE}, \forall t \tag{2}$$

$$x_{g,t}^{on}, -x_{g,t-1}^{on} = x_{g,t}^{st} - x_{g,t}^{sd}, \quad \forall t, g \in \Omega^{gen,RE} \tag{3}$$

$$x_{g,t}^{st} + x_{g,t}^{sd} \le 1, \quad \forall g \in \Omega^{gen,RE}, \forall t \tag{4}$$

$$x_{g,t'}^{on} \ge x_{g,t}^{st}, \ \mathrm{t}' = [t, t + MUT - 1], \forall t, g \in \Omega^{gen,RE} \tag{5}$$

$$x_{g,t'}^{on} \le \left(1 - x_{g,t}^{sd}\right), \ \mathrm{t}' = [t, t + MDT - 1], \forall t, g \in \Omega^{gen,RE} \tag{6}$$

$$\underline{P}_g^{gen} x_{g,t}^{on} \le P_{g,t}^{gen} \le \overline{P}_g^{gen} x_{g,t}^{on}, \quad \forall g \in \Omega^{gen,RE}, \forall t \tag{7}$$

$$\underline{R}_g^{res} x_{g,t}^{on} \le R_{g,t}^{res} \le \overline{R}_g^{res} x_{g,t}^{on}, \quad \forall t, g \in \Omega^{gen,RE} \tag{8}$$

$$\underline{P}_g^{gen} x_{g,t}^{on} \le P_{g,t}^{gen} + R_{g,t}^{res} \le \overline{P}_g^{gen} x_{g,t}^{on}, \forall g \in \Omega^{gen,RE}, \forall t \tag{9}$$

$$P_{g,t}^{gen} - P_{g,t-1}^{gen} \le \overline{\Delta P_g^{RU}}, \quad \forall t, g \in \Omega^{gen,RE} \tag{10}$$

$$P_{g,t-1}^{gen} - P_{g,t}^{gen} \le \overline{\Delta P_g^{RD}}, \quad , \forall t, g \in \Omega^{gen,RE} \tag{11}$$

$$\underline{P}_g^{forcast} x_{g,t}^{on} \le P_{g,t}^{gen} \le \overline{P}_g^{forcast} x_{g,t}^{on}, \forall g \in \Omega^{RE}, \forall t \tag{12}$$

$$\sum\nolimits_g P_{g,t}^{gen} \ge \sum\nolimits_b \left(P_{b,t}^{load} + P_{b,t}^{lshd}\right), \quad \forall t \tag{13}$$

$$\sum_{g \in \Omega_b^{gen,RE}} P_{g,t}^{gen} = P_{b,t}^{load} + P_{b,t}^{lshd} + \sum_{l \in \Omega_b^{line}} P_{l,t}^{Line}, \quad \forall t, b \tag{14}$$

$$0 \le P_{b,t}^{lshd} \le \zeta^{\max} P_{b,t}^{load}, \quad \forall b, t \tag{15}$$

$$P_{l,t}^{Line} = B_l^{Line} \left(\theta_{fr,t}^l - \theta_{to,t}^l\right), \quad \forall \{fr, to\} \in l, t \tag{16}$$

$$-\overline{P}_l^{Line} \le P_{l,t}^{Line} \le \overline{P}_l^{Line}, \quad \forall l, t \tag{17}$$

$$-\pi \le \theta_{b,t} \le \pi, \quad \forall b, t \tag{18}$$

Definition of binary variables is given in the equation (2) which includes on/off, startup, and shutdown status of power plants shown by $x_{g,t}^{on}$, $x_{g,t}^{st}$, and $x_{g,t}^{sd}$, respectively. Furthermore, the relation between these binary variables is stated in the equation (3). Also, equation (4) prevents each powerplant to be started or shutdown simultaneously [42]. The minimum up/down time constraints are applied by equations (5), (6) [42]. Equations (7)-(9) limits the power generation and reserve of each unit [43]. The ramp-up/down limitation is formulated as equations (10), (11) [42]. The power generation of each renewable energy resource is bounded by its forecasted power as equation (12). Power flow relations are given by equations (13)-(15) [44-47]. According to equation (13) it would be necessary for units to provide power more than load consumption. While, total input power to each bus must be equal to total output power of it as claimed in the equation (14), considering the limitation of loadshedding according to equation (15). Line power limits are included in the model by equations (16)-(18) [48-51]. Other technical limits will be extracted and described as follows.

2.2. Primary Frequency Response Constraints

The frequency dynamics of the power system follows the swing equation as follows:

$$2H_t \frac{d\Delta f}{dt} + D\Delta f = \sum_g \Delta P_{g,t}^{gen} + \sum_{re} \Delta P_{re,t}^{\mathrm{Re}} - \sum_b \Delta P_{b,t}^{load} \tag{19}$$

In addition, in equations (13) and (14), the load is in the opposite direction of the producer. That is, the load is the power consumer, and the producer has the generation power. Therefore, as a positive coefficient is used for the producer in equation (19), a negative coefficient is used for the load. For the load, disturbances are also considered.

Each of loads and generation units power my variate and change the frequency. So far, some assumptions have been considered for simplicity as follows:

1- The frequency was stable in its nominal value before the contingency. So, the right side of the equation (19) is zero at this circumstance.

2- A unit outage is called contingency, here.

3- During the contingency the load variation is assumed neglectable in comparison to generation unit outage.

4- After an outage each generation unit or renewable energy resource can participate in the primary frequency response support using multi-machine system frequency response as shown in the Figure 1.

5- Although aggregating a multi-machine system frequency response model is a complicated task, an aggregated system frequency model of [16] can be applied.

6- Renewable energy resource follows the same dynamics of thermal unit primary frequency response but their technical parameters are different.

In the Figure 1 a schematic of system frequency model is depicted that describe the frequency swing equation Laplace transform. According to this schematic and swing equation and assuming ΔP^{Load} » 0, we have [16]:

$$\begin{aligned}
&(2H_t s + D)\Delta f = \\
&\sum_g \left(-\frac{K_{mg}^G}{R_g^G} \times \frac{1 + sT_g^R F_g^G}{1 + sT_g^R} \right) \Delta f + \\
&\sum_{re} \left(-\frac{1}{R_{re}^{RE}} \times \frac{1}{1 + sT_{re}^{RE}} \right) \Delta f - \Delta P^{lost}
\end{aligned} \tag{20}$$

So, for simplifying the equation, an aggregated model of [16] is used. So, it can be written:

$$\begin{aligned}
&\sum_g \left(-\frac{K_{mg}^G}{R_g^G} \times \frac{1 + sF_g^G}{1 + sT_g^R} \right) + \sum_{re} \left(-\frac{1}{R_{re}^{RE}} \times \frac{1}{1 + sT_{re}^{RE}} \right) \\
&\quad \approx -\frac{1}{R_t} \times \frac{1 + sT_R F_t}{1 + sT_R}
\end{aligned} \tag{21}$$

where,

$$H_t = \sum_g u_{g,t}^{\mathrm{Res}} H_g \bar{P}_g^{gen} \Big/ \sum_n D_{n,t}^{Fore}, \quad \forall t \tag{22}$$

$$F_t = \sum_g u_{g,t}^{\mathrm{Res}} K_{mg} F_g^G \bar{P}_g^{gen} \Big/ \sum_n D_{n,t}^{Fore}, \quad \forall t \tag{23}$$

$$\frac{1}{R_t} = \sum_g u_{g,t}^G \frac{K_{mg}}{R_g^G} \bar{P}_g^{gen} \Big/ \sum_n D_{n,t}^{Fore}, \quad \forall t, \tag{24}$$

By inserting equation (21) in equation (22), a simplified form is obtained as follows:

$$\left(2H_t s + D\right)\Delta f = -\frac{1}{R_t} \times \frac{1+sT_R F_t}{1+sT_R}\Delta f - \frac{\Delta P^{lost}}{s} \tag{25}$$

By some mathematical manipulation, we have:

$$\Delta f = \frac{-\Delta P^{lost}\left(1+sT_R\right)}{s^2 + 2\zeta\omega_n + \omega_n^2}\frac{1}{s} \tag{26}$$

where,

$$\omega_n^2 = \frac{DR_t + 1}{2H_t R_t T_R} \tag{27}$$

$$\zeta = \frac{DR_t T_R + 2H_t R_t + F_t T_R}{2\left(DR_t + 1\right)}\omega_n \tag{28}$$

$$\omega_r = \omega_n\sqrt{1-\zeta^2} \tag{29}$$

Using inverse Laplace transform, the frequency dynamic is obtained as follows:

$$\Delta f\left(t\right) = \frac{R_t \Delta P^{lost}}{DR_t + 1}\left[1 + \alpha e^{-\xi\omega_n t}\sin\left(\omega_r t + \varphi\right)\right] \tag{30}$$

where,

$$\alpha = \sqrt{\frac{1-2T_R\zeta\omega_n + T_R^{\,2}\omega_R^2}{1-\zeta^2}} \tag{31}$$

$$\varphi = \arctan\left(\frac{\omega_r T_R}{1-\xi\omega_n T_R}\right) - \arctan\left(\frac{\sqrt{1-\xi^2}}{-\xi}\right) \tag{32}$$

According to the frequency dynamics of equation (31), the nadir frequency in which the derivative of $\Delta f\left(t\right)$ is zero, is obtained as follows [16]:

$$t_{nadir} = \frac{1}{\omega_r}\tan^{-1}\left(\frac{\omega_r T_R}{\zeta\omega_r T_R - 1}\right) \tag{33}$$

$$\Delta f_{nadir} = \frac{R_t \Delta P^{lost}}{DR_t + 1}\left[1 + \sqrt{1-\xi^2}\,\alpha e^{-\xi\omega_n t_{nadir}}\right] \tag{34}$$

So, we can write that:

$$\Delta P^{lost} = \frac{R_t \Delta f_{nadir}}{DR_t + 1}\left[1 + \sqrt{1-\xi^2}\,\alpha e^{-\xi\omega_n t_{nadir}}\right] \tag{35}$$

Using linear approximation, we have:

$$\Delta P^{lost} = \Delta f_{nadir}\left(\beta_0 + \beta_H H_t + \beta_F F_t / R + \beta_{1/R}/R\right) \tag{36}$$

Frequency deviation in nadir frequency must be limited to maximum allowable frequency deviation $\Delta f_{nadir} \le \overline{\Delta f}$. So, it can be written that:

$$\Delta P^{lost} \le \overline{\Delta P} \le \overline{\Delta f}\left(\beta_0 + \beta_H H_t + \beta_F F_t / R + \beta_{1/R}/R\right) \tag{37}$$

Piecewise linearization methos is used for calculation of beta parameters that can be time consuming when the number of subspaces is increased. Other constraints are given as following:

$$\underline{H}_t \le H_t \le \overline{H}_t \tag{38}$$

$$\underline{F}_t \le F_t \le \bar{F}_t, \quad \forall t \tag{39}$$

$$\frac{1}{\bar{R}_t} \le \frac{1}{R_t} \le \frac{1}{\underline{R}_t}, \quad \forall t, \tag{40}$$

$$\bar{P}_g^{gen} - P_{g,t}^{gen} \ge u_{g,t}^G \frac{K_{mg}}{R_g^G} \bar{P}_g^{gen} \kappa\left(f_0 - \underline{f}_{\min}\right), \ \forall g, t \tag{41}$$

Equations (38)-(40) shows the limitation of total inertia, governor factor, and droop coefficients. The headroom, limitation of generation units for provision of the primary frequency response is given in equation (41) [16].

2.3. Distributionally Robust Optimization model

In this paper, we supposed that the contribution of online units in the primary frequency response is uncertain. It means that the real contribution status of a unit is lower that its predicted condition $u_{g,t}^G \le \tilde{u}_{g,t}^G$, so the probability constraints can be given as follows:

$$\Pr\left\{u_{g,t}^G \ge \tilde{u}_{g,t}^G\right\} \ge 1 - \varepsilon^{PFR}, \quad \forall t, g \tag{42}$$

the probability distribution function of $\tilde{u}_{g,t}^G$ variable is not known. So, only the average and variance of the probability function can be available. equation (42) can be expressed as follows:

$$\Pr\left\{u_{g,t}^G \le \tilde{u}_{g,t}^G\right\} \ge \varepsilon^{PFR}, \quad \forall t, g \tag{43}$$

Considering $\alpha = u_{g,t}^G, \beta = \tilde{u}_{g,t}^G$, equation (43) is presented as follows:

$$F_\alpha(\beta) = \Pr\left\{\alpha \le \beta\right\} \ge \varepsilon^{PFR},$$

$$F_\alpha(\beta) = \begin{cases} 0 & \beta \le \beta^{\min} \\ \dfrac{\beta - \beta^{\min}}{\beta^{\max} - \beta^{\min}} & \beta^{\min} \le \beta \le \beta^{\max} \\ 1 & \beta \ge \beta^{\max} \end{cases} \tag{44}$$

F is Probability density function. It includes linear format. Upper and lower values of β determinate by the average and variance the probability function.

3. Solution Procedure

The flowchart of optimization implementation is shown in Figure 2. First, all the necessary data required for the simulation will be called and will be in the desired format. These data are related to power network information, loads per bus, capacity and location of lines, technical specifications of power plants, information related to weather conditions, network load fluctuations, economic information (interest rate, initial investment cost, types of coefficients Cost of power plants), information related to the frequency response of each network component, etc. After receiving the information, once for the first year, the problem is implemented in the installation circuits of power plants constraints to the frequency response and virtual inertia of renewable resources, and the status of power plants and the capacity of its transmission lines (amount of congestion) are monitored to Determine if the network needs development or not. If development is required, expansion planning is implemented for the first year only and the results are obtained. Then the network specifications are updated for the second year and the amount of network load is increased for the new year. The planning is carried out again in the location of the power plants. After observing the new condition of the lines, the network is developed. This trend continues until the last year. The reason for separating the years of network development, and not considering all the planning years in the model, is to save time. As the problem variables increase from one year to a longer period of time, the number of boundary conditions increases exponentially, which in turn increases the execution time of the program exponentially. Therefore, it is preferred that the network development process be initially "development based solely on current needs." Finally, after identifying all the equipment in need of development, it can be decided based on the inflation rate that the development will take place in the first year or in the same year. Finally, the results of the simulations and network development will be shown.

The growth of energy consumption in the coming years may lead to an imbalance of generation and consumption in the power system, if a new generation unit is not installed in the network. This imbalance leads to frequency deviations from the normal value. So, if the amount of load is more than the output, the frequency will drop. Therefore, there is a need for power system expansion plans so that a high frequency drop that leads to the shutdown of the power system is prevented. PFR was also considered as an index in expansion planning until the capacity of the resources installed in the power system to maintain the frequency of the system at the desired value is calculated.

4. Simulation results

4.1. Solver

The proposed optimization model is MILP that can be solved with a variety of commercial software. In this paper, the combination of YALMIP and MOSEK toolboxes is used to solve the problem. Initially, using a variable called semi-definite programming (SDP variable of YALMIP software), all boundary constraints, variables, and the objective function are modeled in MATLAB (2021b) software [52]. Then, by setting the solver to MOSEK 9.2, a branch and bound algorithm based optimization is started [53]. We use the academic version. Implementation is done by a DELL latitude laptop with a sixth-generation corei5 CPU, 8 GB of RAM and a 256 GB SSD hard drive.

4.2. Case Study

The 24-bus IEEE RTS system is recognized for the optimization. The base information about the branch data, and generation units such as location, capacity, operation costs of generation are extracted from MATPOWER software (case24_IEEE_rts.m) [54]. The primary frequency response data for each thermal and renewable energy units is adopted from [11] and show in the table 1.

It is considered that in each location the planner can install up to 4 units with the same sized that installed before. Also, up to four parallel lines can be installed in the network if it is required. We supposed that the length of each line is 100 km and the investment cost of each line can be 10 *$/km/MW*. Furthermore, a 20% (percentage of load) outage is considered in the problem. The frequency nadir must not exceed from 49.5 *Hz*. The nominal frequency is 50 *Hz*. The gap error is set to 0.05 and the maximum iteration is set to 3000. For renewable energy resources we consider wind power (in bus 1,16, and 21) and photovoltaic power plants (in bus 3,21,22) with 500 MW capacity. Three case studies are considered in the problem as follows:

Case 1: This is a base case with no PFR constraints and no contribution of renewable energy virtual inertia.

Case 2: This is a PFR-based expansion planning without renewable energy contribution in PFR

Case 3: This is the complete model with PFR constraints and renewable energy contribution

for linearizing the nadir constraints, we use 1000 samples to calculate $\beta_0, \beta_H, \beta_F, \beta_R$. According to these samples the value of these coefficients are obtained 0.5621, 0.2319, -27.17, and 13.64, respectively that their computation takes less than 3s. Finally, it should be noted that the proposed scheme has no restrictions for implementation on different data of the power system, PFR, and renewable resources.

4.3. Results

The difference between the linearized estimation of g function and its actual value is represented in the Figure 3 that show a small error for $F_t = 0.02$. Furthermore, the results of implementing case 1 to case 3 is demonstrated in the table 2 and table 3, respectively. As it is clear from these results, the lowest cost is obtained in the case1 (base case), without any PFR consideration. By considering the primary frequency response constraints the total cost of expansion is increased and the number of installation units is more than the other cases. By contribution of renewable energy resources in the system the value of inertia is increased and the total cost is decreased. So, it can be concluded that by activating a primary frequency response ability of renewable energy resources, a techno-economic merit is achieved. The flexibility of the network against the power outage and frequency stability issues is increased. Moreover, the cost of installation is decreased.

the comparison of inertia variation of the network at the end of the planning is illustrated in the Figure 4 for case 2 and three. As it is clear from the picture, total inertia of the network in case 3 is somehow more than case 2 in each hour (by the same load). So, the more robust planning is obtained in case 3. Furthermore, the number of online units in case 3 is more than case 2 that leads to more capacity in the network for any event.

According to Table 3, case 1 considers the network without PFR, but in case 2, the presence of PFR in the network is considered. Based on the comparison of the results of cases 1 and 2 in Table 3, it can be seen that the minimum, average and maximum value of H_t in case 2 compared to case 1 is more reduced about 53.8%, 19.4% and 17.8%, respectively. These values for F_t in case 2 compared to case 1 have the highest increase of 0%, 25% and 100%, respectively. For R_t, they decrease by 52%, 19.6% and 12.3%, respectively.

By comparing the planning results in Table 2, it can be seen that case 1 has the lowest planning cost. The presence of PFR in planning (case 2) leads to a 361% increase in the cost of planning compared to case 1. But the presence of renewable resources along with PFR leads to a reduction of 61.8% in planning cost compared to case 2.

Generally, power system planning is done for a long planning horizon of more than 5 years. In many works, a 5-year horizon is also considered. Therefore, a 5-year planning horizon was considered in this study. In addition, this article has dynamic development planning. In other words, they determine what equipment is built in the network in different years. In addition, one of the important goals of power system planning in optimal conditions is to determinate the location, size and time of installing an equipment in the power system. Therefore, a one-year planning period was chosen until the installation time of the network equipment is also calculated.

5. Conclusions

In this paper a PFR-based expansion planning model is presented that considered the renewable energy contribution in inertia response to limit the frequency nadir when as outage is occurred in the system. The analysis of this paper shows that for expansion planning in the presence of renewable energy resources, it is preferred to provide the frequency control to keep the frequency nadir in its standard range and prevent from blackout in the system or under frequency relay protection. Furthermore, regardless of the initial frequency response (traditional model), network operation costs will be lower because there is no need to consider any reserve for unit. So, power load can be supplied with less power plants, however, there is no guarantee for frequency stability in the unexpected outage. From computational burden, adding frequency response constraints may lead to more complexity in the model but it is required for flexibility. This paper also illustrates that considering the primary frequency response will increase the total inertia of the system by providing more participation of fast response power plants that leads to greater robustness against power imbalances between generation and demand. Furthermore, more online power plants must be existed in the network. It can be concluded that linearization of frequency nadir constraints leads to easier implementation with commercial software.

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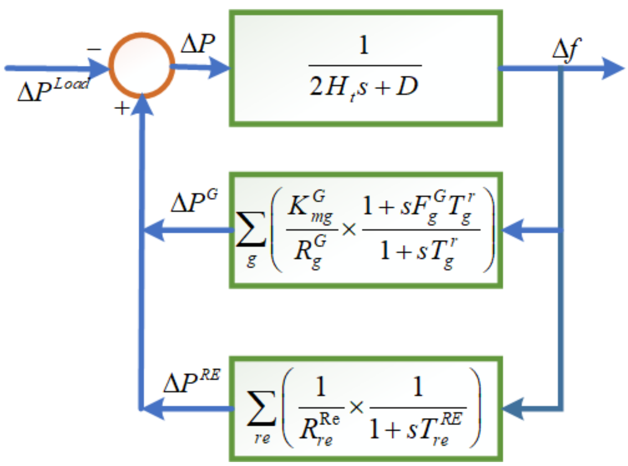


Figure. 1. System frequency model

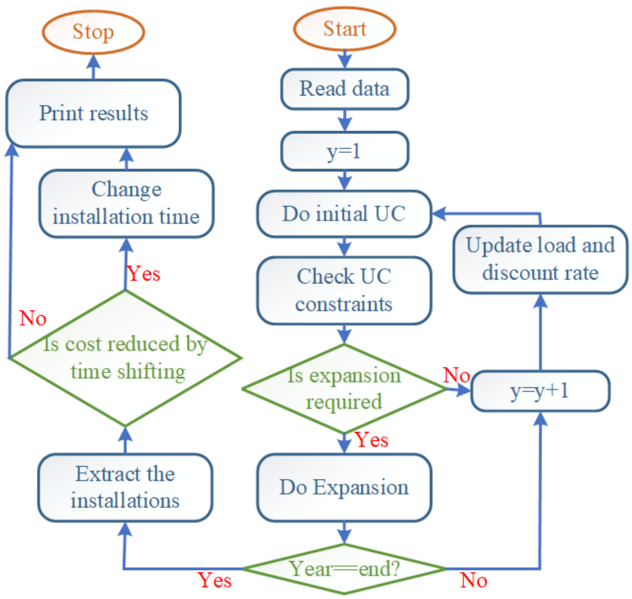


Figure. 2. Flowchart of the proposed algorithm

TABLE. 1. Generation unit parameters

Unit	*H*	*F*	*R*	ST/SD	Invest	OPR	NL
	(s)			$	$/kW	S/kWh	$
U12	4	0.25	0.033	1500	0.680	130	400
U20	4	0.25	0.033	1500	0.667	16.08	212
U50	4	0.25	0.033	1500	0.667	0.001	0.001
U76	4	0.25	0.033	1500	0.667	16.08	212.3
U100	6	0.30	0.05	1500	0.667	43.66	781.5
U155	6	0.30	0.05	1500	0.630	12.38	382.2
U197	6	0.30	0.033	1500	0.630	43.66	781.5
U350	8	0.35	0.05	1500	0.608	4.42	395.37
U400	8	0.35	0.05	1500	0.608	4.42	395.37

TABLE .2. Expansion costs in all cases

Case	Year	Investment (M$)	Operation (M$)	NPV (M$)
1	1	1047.537	431.698	1232.696
	2	671.380	475.079	796.152
	3	668.109	611.610	740.578
	4	965.104	775.467	839.395
	5	990.392	1182.05	873.054
Total				**4481.875**
2	1	9179.452	457.032	9179.45
	2	310.959	449.404	310.959
	3	1874.640	743.560	1874.640
	4	8445.938	535.306	8445.938
	5	865.07	765.421	865.070
Total				**20676.482**
3	1	5643.146	357.479	5000.52
	2	816.833	294.636	771.853
	3	810.295	389.822	694.512
	4	842.275	528.719	661.1662
	5	1107.322	791.343	762.0308
Total				**7890.082**

TABLE .3. The frequency response characteristics of all cases

	H_t			F_t			R_t		
Case 1									
Y	min	Avg	max	min	avg	max	min	avg	max
1	10.5	12.2	14.3	0.02	0.025	0.03	0.46	0.54	0.63
2	10.8	12.4	14.6	0.02	0.025	0.03	0.48	0.56	0.65
3	11.2	12.9	15.2	0.02	0.025	0.03	0.49	0.57	0.65
4	11.7	12.7	14.9	0.02	0.024	0.03	0.52	0.57	0.66
5	11.6	12.7	14.3	0.02	0.024	0.03	0.52	0.57	0.64
Case 2									
Y	min	Avg	max	min	avg	max	min	avg	max
1	5.7	11.7	13.9	0.02	0.03	0.06	0.24	0.53	0.62
2	4.9	10.0	12.5	0.02	0.03	0.06	0.23	0.45	0.57
3	7.0	10.6	12.5	0.02	0.03	0.04	0.32	0.49	0.58
4	5.4	11.0	13.5	0.02	0.03	0.05	0.25	0.50	0.61
5	7.2	10.7	12.9	0.02	0.03	0.04	0.32	0.49	0.58
Case 3									
Y	min	Avg	max	min	avg	max	min	avg	max
1	9.6	13.0	15.7	0.02	0.02	0.02	0.13	0.46	0.62
2	7.1	12.2	14.8	0.02	0.02	0.02	0.09	0.24	0.59
3	10.4	13.1	15.5	0.02	0.02	0.02	0.26	0.48	0.59
4	9.4	12.5	14.8	0.02	0.02	0.02	0.25	0.48	0.58
5	11.1	12.7	15.0	0.02	0.02	0.02	0.36	0.50	0.59

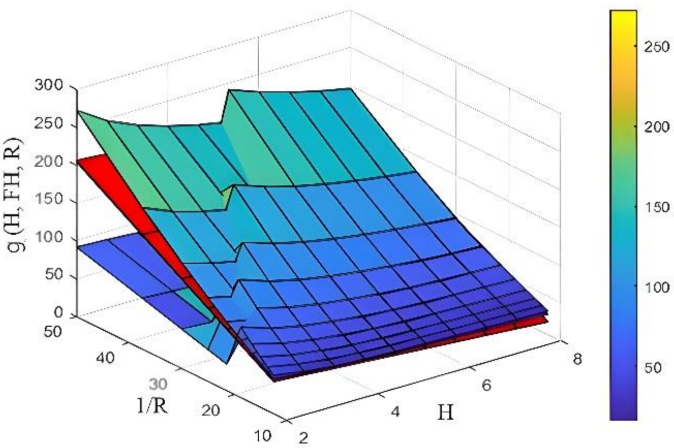
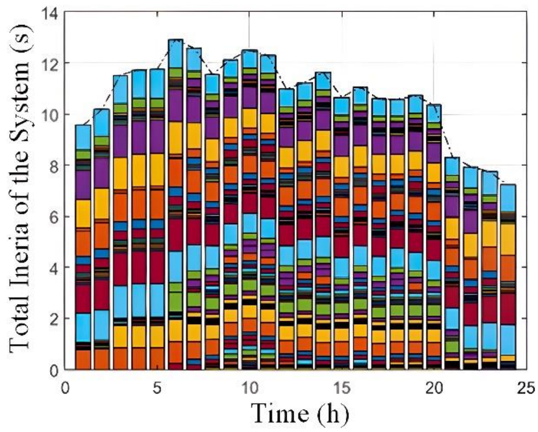
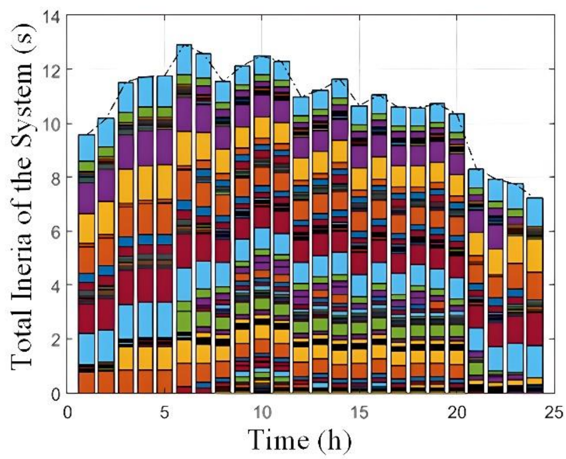


Figure. 3. The difference between the linearized estimation of g function



(a)



(b)

Figure. 4. The variation of equivalent inertia of the network and the contribution of each generating units in inertia at the final year in (a) case 2 without wind units' contribution and (b) case 3 with contribution of wind generation in primary frequency response