Reliability analysis of bearing capacity of the foundation resting on rock mass using Subset Simulation method

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Abstract

In this article, a new reliability analysis algorithm is proposed to calculate the probability density function of the bearing capacity of the foundation resting on rock mass. Despite common approaches used by other investigators, four parameters with uncertainties have been adopted in this study as random variables, including *GSI* index, strength of intact rock (σc_i), intact rock constant (m_i), and rock mass disturbance factor (D). In the extended Subset Simulation (SS) proposed in this study, the samples at the first stage are produced using the Monte Carlo Simulation (MCS), while at the next levels, a Markov chain based on the Metropolis-Hastings algorithm is applied to each subset. Finally, statistical parameters of the PDF of bearing capacity are discussed. The results obtained showed that (A) The SS method converges with a much smaller number of samples than those given by the MCS method; (B) Parameters *UCS* and *GSI* have the greatest effect on the bearing capacity; (C) As the coefficient of variation of the input variables increases, the value of the reliability index decreases and therefore the probability of system failure increases.; (D) When the negative coefficient of correlation is used, a decrease in the variation of bearing capacity is observed.

Keywords: Bearing capacity; Rock mass; Reliability Analysis; Monte Carlo simulation; Subset simulation.

1. Introduction

Load-bearing capacity is one of the most significant requirements for the safe and reliable performance of foundations. Safe and reliable design of foundations is the most critical part of structural design. Compared to soils, rocks have generally more compressive strength and, hence, foundations founded on rocks have more bearing capacity. However, the load of large structures, such as dams, skyscrapers, or bridge piers, induced on foundations resting on rock masses may be problematic. As a result, in designing the foundations resting on the rocks, all the structural features of rock the mass and environmental conditions should be attentively taken into account. Usually more conservative design is reached by using safety factors [1]. In general, designing the foundation of structures founded on rock mass is not an easy task and, in real cases, requires preliminary field investigations and detailed office work. These include drilling boreholes, excavation of exploratory tunnels in the rock beneath the foundation, performing laboratory rock strength tests, as well as an accurate analysis of the induced and acting loads on the foundation. Insufficient studies at this stage and inaccuracy in the evaluation of the design parameters for the foundation design may lead to catastrophic consequences. When deterministic approaches are used to design the foundations, the resulting safety factor plays a significant role in the project cost, which may not be in favor of the project owners. In such situations, reliability methods are considered as an alternative to deterministic methods, as their use leads to a more realistic design, especially about the uncertainty of the design parameters.

Different methods have been proposed to calculate ultimate bearing capacity or limit state of rock mass by use of a deterministic approach. Serrano et al and Galindo et al offered the research on the ultimate bearing capacity of the rock masses according to the modified Hoek-Brown (H-B) and the modified Mohr-Coulomb failure criterion [2-4]. Mansouri et al studied the ultimate bearing capacity of rock mass below rectangular and square foundations using three-dimensional finite element analysis [5]. Galindo & Millan offered a method for computing the bearing capacity of shallow foundations on anisotropic rock masses using the H-B failure criterion [6]. In the present study, the situation of a shallow foundation located on the rock masses was investigated by probabilistic methods. The probabilistic methods provide the possibility of considering the uncertainties of input parameters on the system response. Most of the previous probabilistic studies focused on the condition of foundations located on soil layers, including reliability analysis of foundations located on undrained soils [7-8], mechanism of failure of soil-based foundations [9-10], reliability analysis of bearing capacity of piled raft foundations [11-12], and reliability analysis of the ultimate dynamic bearing capacity of foundations [13-14].

A review of previous studies shows that limited research has been done on the analysis of foundation on rock mass by use of probabilistic methods. For example, Millan et al investigated the use of Artificial Neural Networks (ANN) to predict the bearing capacity of foundation located on rock mass [15]. In their study, the bearing capacity was predicted with the general shear failure assumption using the FLAC numerical code of practice based on the H-B criterion. Predictions of the ANN model agreed well with those obtained from numerical analysis. Albitar & Soubra considered the geological strength index and the compressive strength parameters (*GSI*, σc_i) as random [16-17]. Their probabilistic method was based on optimized MCS using the chaos expansion. Their study focused on the correlation distance between parameters, which showed the greater sensitivity of the σ_{ci} at high correlations and the lower sensitivity of the *GSI* of the rock at low correlations. Basha & Moghal studied the allowable bearing capacity of foundation on the jointed rock mass using the probabilistic method [18]. In their study, the Bell equation was used for calculating bearing capacity. In addition, joint orientation, material cohesion, joint spacing, and shear strength (friction angle of joints and rock mass) were selected as random variables. A design algorithm based on the reliability index was proposed. Zawaki et al predicted the σ_{ci} of rock masses using statistical methods [19]. They measured the strength parameters of rock by testing 50 samples of rock taken from 11 different regions in the Czech Republic. They also determined a suitable probability distribution on the frequency histogram of each parameter and proposed a new relationship to appraise the σ_{ci} of rock masses.

As investigated in previous studies, in most of the probabilistic studies performed on rock mass using the H-B failure criterion, two parameters (GSI & σ_{ci}) were chosen as random variables. In the extended method offered in this study, the parameters of disturbance factor (D) and constant of intact rock (m_i) are also considered as random variables to enhance the precision and performance. The MCS method is an ordinary technique and is capable of estimating the failure probability of the problems regardless of their complexity and with reasonable accuracy.

However, this method suffers drawbacks including: a) it is usually used as a basis to evaluate the Probability Density Function (PDF) of the failure probability of the system, however, it may not be efficient in some particular problems and hence, it may lose its generality. Because of this, the MCS method may need to be significantly optimized. b) To achieve a suitable accuracy, the MCS method usually needs large number of simulations which leads to a very time-consuming process. c) The application of the MCS method becomes cumbersome, or even formidable, when the fundamental equations and the system response do not follow linear relations. d) Despite simplicity and applicability, the MCS method has proved to be inefficient in evaluating small probabilities [20-21]. To overcome the inefficiency of the MCS method in calculating small failure probabilities, several advanced simulation methods have been developed, including

Subset Simulation, Spherical Subset Simulation, Line Sampling, Asymptotic Sampling, and other methods.

In this paper, reliability analysis of the bearing capacity of the foundations on the rock masses is presented by use of the Subset Simulation (SS). This method is used when the probability of failure is very small or when subjects are very complex because the computation time is acceptable [22]. SS is well suited for quantitative analysis of systems experiencing functional failures, which are identified based on one or more safety variables. SS requires much fewer samples to reach a given accuracy than does the MCS method. It can efficiently calculate the probabilities of rare events in reliability problems with complex system features and a high number of uncertain or random variables in failure events In this method, the problem is turned into a sequence of problems with conditional failure probabilities. The failure probability of the main and target problems will be equal to the multiplication (product) of these conditional probabilities [23]. On the other hand, the SS method is very efficient and can analyze systems with a large number of random variables or with small failure probabilities. SS, therefore, is a method that is found to have efficiency, stability, and capability in the reliability analysis of complex and nonlinear problems. Hence, the method is adopted here for the base of the analysis while enhancement and optimizations are considered.

This study includes the following sections: First, the idea of MCS and the SS method is explained. This is followed by a presentation of the modified H-B failure criterion, which is applied to calculate the bearing capacity of the rock mass. The reliability analysis algorithm to compute the bearing capacity PDF is offered. Convergence of the bearing capacity results achieved from MCS and proposed SS methods is compared. Statistical parameters related to the bearing capacity PDF are presented. Discussion of the results is presented and continued throughout the paper.

2. Bearing capacity of rock mass

Probabilistic analysis of engineering problems, especially reliability analysis, appears rational when compared with conventional deterministic approaches. In deterministic analysis, parameters are considered certain without scattering and error. Then, the design parameters are calculated, followed by applying a safety factor. A large safety factor may not necessarily imply the safety of a structure, especially when the input parameters are indeterminate and scattered in their distribution. In these cases, reliability analysis is preferred to reach a rational engineering design value. Predicting the possibility of common fractures in rock provides a better understanding of the overall long-term state of the rock mass. Due to the complexities and limitations, theoretical criteria are not preferred to predict rock mass behavior and strength. Instead, experimental failure criteria are generally applied in rock engineering practices. These criteria are expressed in both linear and non-linear equations relating the principal stresses while failure is expressed based on some experimental or regressed constants.

2.1. Hoek-Brown failure criterion

The strength behavior of the rocks is commonly indicated by a failure criterion. The Hoek–Brown criterion is utilized by engineers in practice to estimate the strength of rock masses, being one of the limited non-linear criteria. Hoek-Brown failure criterion was first presented in 1980 and has been extended to many versions, all of which are non-linear. Initially, Hoek and Brown suggested a relation between principal stresses at failure in rock as follows [24]:

$$\sigma_1 - \sigma_3 = (m * \sigma_3 * \sigma_{ci} + \sigma_{ci}^2)^{0.5} \qquad (1)$$

In the above relationship, σ_1 and σ_3 represent the principal stress at failure, respectively, while σ_{ci} represents the uniaxial compressive strength of intact rock. The parameter *m* is a material constant of the rock and is obtained by statistical analysis, especially the regression approach, on the available uniaxial and triaxial test results performed on a variety of rock types. Hoek and Brown recommend that at least 5 pairs of σ_1 and σ_3 values of triaxial test results should be used to achieve a reliable regression analysis.

As mentioned previously, the H-B failure criterion has been improved since its initial introduction in 1980, and several updated and expanded versions of the criterion have been introduced. For example, the Hoek-Brown (1992) relationship is as follows:

$$\sigma_1 - \sigma_3 = (m * \sigma_3 * \sigma_{ci} + s * \sigma_{ci}^2)^{0.5} \qquad (2)$$

$$m = m_i * \exp\left(\frac{RMR - 100}{a}\right) \qquad (3)$$

$$s = \exp\left(\frac{RMR - 100}{b}\right) \qquad (4)$$

Parameter *s*, in equation (2), has been introduced to account for the structural characteristics of the rock masses, especially concerning the extent and pattern of jointing in a rock mass. Parameter *m* in equation (1), also has been modified and reintroduced based on the type and general class of the rock mass which relies on the *RMR* (Rock Mass Rating) classification offered by Bieniawski [1]. This classification system evaluates the rock mass quality based on a summation of 6 parameters with a maximum of 100 (full mark) for an ideal rock to a minimum value of 0 for extremely weak or crushed rock masses. In equations (2-4), the maximum value of

s=1.0 is assigned for an ideally intact rock (no jointing) while the minimum value of s=0.0 is set for completely a crushed rock. Practically s=1.0 is equivalent to *RMR*=100 in the Bieniawski classification. Further, the theoretical value of *RMR*=0 leads minimum value for m and s in equations (3-4). Parameter m_i is an experimentally determined constant for various types of rock and *RMR*=100, the parameter m reaches its maximum value of m_i (i.e. m=m_i if *RMR*=100). For intact rock, parameters "a" and "b" in equations (3-4) are introduced for adjustment of the strength in various rocks such that for intact rock a=28 and b=9 while for jointed rock a=14 and b=6 is best adapted.

2.2. Modified Hoek-Brown Failure Criterion

The most applicable version of the failure criterion suggested by Hoek and Brown (2002) is defined as follows [25]:

$$\sigma_1 = \sigma_3 + \sigma_{ci}\left(m_b \frac{\sigma_3}{\sigma_{ci}} + s\right)^a \qquad (5)$$

$$m_b = m_i * \exp\left(\frac{GSI - 100}{28 - 14D}\right) \qquad (6)$$

$$s = \exp\left(\frac{GSI - 100}{9 - 3D}\right) \qquad (7)$$

$$a = 0.5 + \frac{1}{6}\left(\exp\left(\frac{-GSI}{15}\right) - \exp\left(\frac{-20}{3}\right)\right) \qquad (8)$$

In the above equations, σ_{ci} generally is obtained by uniaxial loading of cylindrical samples (cores) taken from the intact part of the rock mass under study.

The *GSI* value determines the rock mass quality. It is dependent on the structure of rock mass and the surface condition of joints. The use of *GSI* needs a good comprehension of the engineering aspects and the geological features of rock mass. As shown in the above relations, values of m_b and s also depend on the *GSI* value of rock masses.

The *D* (rock mass disturbance coefficient) in equations (6-7) also has values from 0 to 1 and is determined experimentally. A value of zero is used for intact and undisturbed rock masses while a limit value of one is applied for completely disturbed rock masses. The *D* coefficient depends on the weathering and the damage caused by disturbing explosions close to or in the rock mass. The parameter (*a*) in equations (5-8) ranges practically from 0.5 to 0.65.

Among the versions of the H-B failure criterion, the modified version of Hoek-Brown (2002) seems to be more extensible and complete than the previous versions because it applies to all types of rocks. Therefore, the modified version (2002) is used in this study.

3. Subset Simulation method

3.1.The Monte Carlo Simulation Method

Simulation methods refer to any numerical method for creating system conditions in a real and natural state. The most common simulation technique is the MCS method, an effective method for statistical analysis of uncertainties in engineering problems [26]. The results of this method are very similar to the real solutions. Implementation of this method includes the following steps:

Step 1: Choose an appropriate deterministic analysis solution method;

Step 2: Choose the input parameters for the probabilistic model and quantify their variations;

Step 3: Generate random samples for each parameter selected from the PDF or data related to those parameters;

Step 4: Solve the problem using the deterministic analysis methods by the parameters selected to calculate the performance function;

Step 5: Continue the operation and repeat the last two steps till a sufficient number of simulations are reached, then, using the output values, the PDF and the failure probability are determined.

In the MCS method, n values are first produced for each random parameter in the response equation. The response equation is then solved for each generated random number and, finally, n values for the system response equation are obtained, which may be applied to obtain statistical information about the response of the system.

A system failure probability can also be calculated by use of the MCS method. For this target, failure limits must be specified in advance. Then, the MCS method is carried out for each data sample and it is checked whether failure occurs or not. The probability of failure is estimated by dividing the number of samples with failure by the total number of samples.

Using the concept of the MCS method, the probability of failure is easily obtained from the following equation:

$$p_f = \frac{1}{N_t}\sum_{i=1}^{N_1} I\left(X\right) \qquad (9)$$

The total number of limit conditions analyzed is represented by N_t. The function $I(X)$ indicates whether a simulated point is in the region of failure or not and is determined according to the following relationship:

$$I(X) = \begin{cases} 1 & \text{if } g(X) \le 0 \\ 0 & \text{if } g(X) > 0 \end{cases} \qquad (10)$$

According to Equation (9), the number of N_t sets of independent design variables is obtained based on their distribution function. Then the failure function or the limit function is calculated for them. Eventually, the estimated failure probability is computed as follows:

$$P_f = \frac{N_f}{N_t} \qquad (11)$$

where N_f indicates the number of failures in the system.

In the bearing capacity analysis using the MCS, the best probability density function for each input random variable is obtained. In the current study, a log-normal probability distribution function is considered for the random variables, since such a distribution provides only positive values. The PDF related to the lognormal probability distribution is obtained from the following equation:

$$f(x) = \frac{\exp\left(-0.5\left(\frac{Ln(x) - \mu}{\sigma}\right)^2\right)}{x * \sigma\sqrt{2\pi}} \qquad (12)$$

That x is a random variable and μ and σ represent the mean and standard deviation of random variables. Initially, the desired number of data points is generated for random values from the PDF of each parameter. A similar process is repeated several times for each random variable at each level, based on the obtained probability densities, the result values are obtained [27].

3.2. Subset Simulation method

As pointed out in previous sections, one of the disadvantages of the MCS method is that it is relatively time-consuming. To resolve the issue, the subset simulation (SS) is adopted in the present study.

The advantages and disadvantages of the SS method need to be carefully studied to come up with a better approach. One of the simulation methods that provides acceptable results in terms of computational time is the SS method suggested by Au and Beck [22]. This method is highly effective for high-dimensional problems and issues with a very small probability of failure. The SS method has been used in recent years to analyze various structural and geotechnical problems [28-35]. Therefore, this method is utilized in this research to assess the reliability of the bearing capacity of the foundation on rock mass. The main aim of the SS method is to convert the subject into a series of smaller problems with conditional failure probabilities so that the failure probability of the main problem is equal to the product of these conditional probabilities [22].

3.2.1 Fundamental of subset simulation

Based on the event of failure $F=\{X : g(x)<0\}, g(x)$ is the function of the random variables $X=(x_1, x_2, ..., x_n)$. The PDF of X is determined by $f_x(X)$. It is assumed that $b_1>b_2>...>b_m=0$ as a decreasing series of values of the threshold of a failure event $F_k=\{X : g(x)<b_k\}\ (k=1,2,...,m)$ has been given as depicted in figure (1). Then, the following relationship is established between the failure limits of the thresholds [23]:

$$F_1 \supset F_2 \supset ... \supset F_m \qquad (13)$$

$$F_k = \bigcap_{i=1}^{k} F_i \qquad (14)$$

The failure probability may be noted as follows, according to the description of conditional probability in the probability theory:

$$P_f = P(F) = P(F_m | F_{m-1}) P(F_{m-1}) = \ldots = P(F_1) \prod_{i=1}^{m-1} P(F_{i+1} | F_i) \qquad (15)$$

Equation (15) presents that the probability of failure is the result of multiplying the conditional probability series $P(F_i | F_{i-1})(i = 2,3,...,m)$ and $P(F_1)$. The main purpose of SS is to calculate the probability of failure by measuring the conditional probabilities. By determining $P(F_1) = P_1$ and $P_i = P(F_i | F_{i-1})\ (i = 2,3,...,m)$, the failure probability in equation (15) is written as follows:

$$P_f = \prod_{i=1}^{m} P_i \qquad (16)$$

By choosing the correct value of the intermediate failure events, the conditional probability in equation (16) will be large enough to be estimated by the simulation. Thus, the subject of calculating the probability of a small failure in the main problem is changed by a series of conditional probabilities with higher frequencies in the conditional probability space. In equation (16), *P1* can be estimated by the MCS method [23]:

$$P_1 = \frac{1}{N_1} \sum_{k=1}^{N_1} I_{F_1}\left[X_k^{(1)}\right] \qquad (17)$$

Where $x_k^{(1)}(k = 1,2,...,N)$ are the independent samples with the same distribution obtained from the PDF of $f_x(X)$. The term $I_{F_1}\left[x_k^{(1)}\right]$ is also an indicator function such that $x_k^{(1)} \in F_1$ then $I_{F_1}\left[x_k^{(1)}\right] = 1$ and otherwise $I_{F_1}\left[x_k^{(1)}\right] = 0$. In the same way, the conditional probability

$P_i \left(i = 2,3,...,m\right)$ in equation (15) may be measured by generating a sample of the conditional probability density function $\left[x\right] f_x\left(x\right)/P\left(F_{i-1}\right) q\left(x\middle|F_{i-1}\right) = I_{F_{i-1}}$:

$$P_i = \frac{1}{N_i}\sum_{k=1}^{N_i} I_{F_i}\left[x_k^{(i)}\right]\left(i = 2,3,...,m\right) \qquad (18)$$

As $x_k^{(i)}\left(k = 1,2,...,N_i; i = 2,3,...,m\right)$ are independent conditional samples with the same distribution, taken from the probability density function $q\left(x\middle|F_{i-1}\right)$. Also, $I_{F_i}\left[x_k^{(i)}\right]$ is a function that indicates such that when $x_k^{(i)} \in F_i$ then $I_{F_i}\left[x_k^{(i)}\right] = 1$, otherwise $I_{F_i}\left[x_k^{(i)}\right] = 0$.

In the SS method, the first-stage samples are produced using the MCS method, while in the next levels, a Markov chain based on the Metropolis-Hastings algorithm will be applied to each subset. The Markov Chain Monte Carlo algorithm shown in Figure (2) generates samples with a distribution $q\left(x\middle|F_{i-1}\right)\left(i = 2,3,...,m\right)$, which is very convenient for calculating conditional probabilities. Therefore, the SS simulation is performed according to the following steps:

1) Generate N_1 independent samples with the same distribution $x_k^{(1)}\left(k = 1,2,...,N_1\right)$ from the PDF $f_x\left(X\right)$ using the MCS method for $i = 1$.

2) Determine values of response $g\left(x_k^{(1)}\right)\left(k = 1,2,...,N_1\right)$. The $\left(p_0 N_1\right)th$ value from the descending list N_1 is selected as the first value of the intermediate threshold (b_1). Also, p_0 is considered a predefined value for conditional probability values, such as $p_0=0.1$, where p_0N_1 must be an integer value. Then $F_1 = \left\{X : g\left(X\right) < b_1\right\}$ defines the first intermediate failure event. Therefore, the failure probability $P_1 = P\left(F_1\right)$ is estimated as $P_1 = p_0$.

3) At this stage, starting from these p_0N_{i-1} conditional samples that sit in F_{i-1} for the ith level $(i = 2,3,...,m)$, the Markov chain is performed to produce $(N_i - p_0N_{i-1})$ the remaining samples obey the PDF $q\left(x \mid F_{i-1}\right)$.

4) Estimate the values of corresponding response $g\left(x_k^{(i)}\right)(k = 1,2,...,N_i)$. The intermediate threshold value b_i is selected as the value of (p_0N_i)th in the descending list of N_i response values. Afterward, the next intermediate failure event is determined as $F_i = \{X : g(X) < b_i\}$. The conditional failure probability $P_i = P\left(F_i \mid F_{i-1}\right)$ may be calculated by $P_i = p_0$ and the probability of failure $P(F_{i-1})$ is evaluated as $P(F_{i-1}) = \prod_{j=1}^{i-1} P_j$.

5) Continue Repeating step(3) and step (4) till the value of m_{th} threshold b_m is equal to or less than 0. Then, it is assumed that $b_m=0$ and the failure probability level of the target (final) $P(F) = P(F_m)$ is achieved. The probability of conditional failure $P_m = P\left(F_m \mid F_{m-1}\right)$ may be calculated as $P_m = N_f / N_m$ where N_f is equal to the number of samples located in the final failure zone $F=F_m$. The probability of final failure $P_f = P(F) = P(F_m)$ may be calculated as follows:

$$P_f = P_0^{(m-1)} \times \frac{N_f}{N_m} \tag{19}$$

4. Ultimate Bearing Capacity

A literature study shows that investigations on the ultimate bearing capacity of rock mass are few. Serrano et al proposed a method for predicting the ultimate bearing capacity of a strip footing on a weightless rock mass with or without a surface surcharge. The ultimate bearing capacity q_u, as proposed by Serrano et al. using the Hoek–Brown criterion is defined as [2] :

$$q_u = \beta_n \left(N_\beta - \zeta_n \right) \tag{20}$$

where ζ_n and β_n are constants for the rock mass which depend on m_b, a, s and σ_{ci} according to

$$\beta_n = A_n \sigma_{ci} \quad , \quad A_n = \left(\frac{m_b \left(1-a\right)}{2^{\frac{1}{a}}} \right)^{\frac{a}{(1-a)}} \quad , \quad \zeta_n = \frac{s}{m_b A_n}$$

ζ_n is known as the toughness of the rock mass while β_n is known as the strength modulus. N_β is a function of the normalized external load on the boundary adjacent to the footing. If there is no surcharge on the surface boundary, then N_β can be determined using the method outlined by Serrano and Olalla [2]. The parameters s, a, and m_b are commonly obtained from equations (6-8). The ultimate bearing capacity obtained from deterministic equation (20) using parameters in the table (1) is (1.4944 MPa). This is used as a limit state value (failure mode) for calculating the failure probability. If the results of the reliability analysis of bearing capacity are less than this value, it will cause failure otherwise safety is considered.

5. Simulation results

As mentioned earlier simulation methods refer to any numerical approach for creating system conditions in a real and natural state. The results of these methods are very similar to the real solutions. In the previous sections, the adoption of the reliability method and failure criterion were described. In the modified H-B failure criterion (2002) used in this study, all the affective parameters are assumed random variables. The determination of the σ_{ci} of rocks can be affected by systematic or human error and therefore uncertainty will govern the values resulting from tests performed on rock samples. The parameters (*GSI, D,* m_i, σ_{ci}) of the modified H-B failure criterion are assumed as random variables. The factors (s), (m_b), and (a) also are dependent on the *GSI,* m_i, and *D* values, as shown in equations (5 to 8). Therefore, they are neither random nor constant, but for each sample, when the values of *GSI*, m_i, and *D* change, the values of (s), (m_b), and (a) also change. It is also assumed that the foundations are placed on the surface of the rock masses without overloading. For investigating the dependence between the parameters σ_{ci} and *GSI*, the coefficient of correlation is determined between them. The statistical parameters related to random variables introduced above were adopted from ref. [36] and presented in table (1).

5.1. Effect of coefficient of variation of variables on bearing capacity

The Coefficient Of Variation (COV) represents the scattering rate per unit mean value. The lower the coefficient of variation, the less scattered the data. This value is dimensionless, making it appropriate for comparing statistical data with different units. In this section, an investigation is applied to check the effect of the COV of input variables on bearing capacity. Keeping the COV of the three random variables constant, the coefficient of variation of the fourth variable is achieved by gradually a 25% and 50% decrease and increase, respectively. The effect of such variation is then investigated using SS on the system's probability density function. Finally, a random variable with the greatest effect on the response system is reached. In this paper, to study the behavior of a system, in addition to the PDF format of the bearing capacity, the parameters of reliability index (β), system failure probability (P_f), factor of safety (F_s), and statistical parameters related to system output, including standard deviation (σ), mean (μ), kurtosis (κ) and skewness (δ) were used. The output results are shown in tables (2) to (5). In addition, Figures (3a) to (3d) present the PDF diagrams of bearing capacity resulting from the change in the COV of random variables.

As displayed in diagrams (3a) to (3d), it can be seen that the parameter (σ_{ci}) has the greatest effect on system response (bearing capacity). This is because the diagram (3a) is more scattered and more spread out than other diagrams. Indeed, in equation (20) the bearing capacity value has a direct relationship with σ_{ci} and any change in σ_{ci} directly affects the bearing capacity, Since σ_{ci} is a prime factor of the compressive strength of rock mass, the higher the σ_{ci}, the higher is bearing capacity for rock mass, provided that all other parameters are assumed to remain unchanged. According to the PDF diagram resulting from the changes in the COV of σ_{ci}, the scatter of the response of bearing capacity is larger than that of other variables. Another effective variable is the *GSI* (Geological Strength Index), with a great effect on the scatter of the system response. This is because the parameter *GSI* describes the structural quality of the rock mass and depends on its structure and its joint surface condition. The other two variables, m_i, and D, have less effect on the system response.

These results help practicing and engineering professionals make decisions based on the importance of the impact of variables on design features. As a result, a safe structural design can be achieved considering uncertainty in design parameters and engineering experience. It is also observed that as the coefficient of variation decreases, the PDF diagram becomes more compact, which means the bearing capacity output is less variable and can provide high reliability.

The statistical parameters related to the output results of bearing capacity presented in Tables (2) to (5) are explained. The results clearly show that by increasing the COV of the input variables, the standard deviation of the bearing capacity also increases, indicating a greater scatter of the outputs. Skewness also indicates the degree of asymmetry of the probability distribution. As can be seen in the diagrams from Figures (3a) to (3d), the skewness of the outputs is positive and the skewness also increases as the COV of the input variables increases, meaning that the PDF becomes more asymmetrical. This is more evident in the diagrams of figures (3a) and (3b) related to σ_{ci} and *GSI*. Kurtosis describes the degree to which a probability distribution is peaked or flat. In this study, by increasing the COV of the input variables, the kurtosis of the output decreases, meaning that the bearing capacity PDF becomes wider and moves away from the normal state. furthermore, the coefficient of variation represents the rate of scattering per unit of average and is a dimensionless value. In the current research, increasing the coefficient of variation of the input variables leads to an increment in the COV of bearing capacity. This implies a direct relation between these two coefficients of variation.

The reliability index classically refers to the ratio between the mean of a performance function and its standard deviation. Evaluation of such an index is straightforward if the density function of the bearing capacity probability is predefined by any method. However, due to the complexity of the performance function, it is often challenging to calculate the statistical properties of the PDF, such as the standard deviation and the mean. The purpose of the simulation method is to calculate these parameters using numerical analysis techniques. The larger the reliability index, the greater the safety of the design. The reliability index is described as the inverse of the COV and may be used to assess the probability of failure. In this study, it was found that increasing the COV of the input variables leads to a decrease in the reliability index value, indicating a decrease in the safety of the system. There are always errors and uncertainties in implementation that design engineers must take into account in designing structures. For this reason, the allowable load values for the design must be such as to prevent unforeseen failures due to uncertainties, thus using safety factors. In this study, it was found that as the COV of input variables increases, the safety factor required for design also increases, which indicates a decrease in structural safety. Failure probability shows the risk of system failure throughout its life cycle. Usually, this parameter is used in probabilistic methods to test the stability or system failure. The findings reveal that as the COV of input variables increases, the probability of failure also increases.

5.2.Effect of the correlation coefficient on bearing capacity

The correlation coefficient, used to determine the relationship between two quantitative variables, is a number between +1 and -1 and is 0 if there is no relationship between the two variables. The (+1) value expresses a complete direct relationship, and a (-1) value expresses a perfect inverse relationship between the two variables. In the present research, an investigation is performed on the impact of changing the correlation coefficient between the two variables, including *GSI* and σ_{ci} of the rock mass, on a PDF diagram of bearing capacity, as displayed in Figure (4). As shown, when the negative coefficient of correlation is used (when decreasing one parameter will increase the other parameter), the PDF is less spread out, and the kurtosis index of the diagram increases, which indicates a decrease in bearing capacity scatter. According to the results displayed in Table (6), in this case, the standard deviation, mean value, and skewness of the PDF diagram of bearing capacity decrease, leading to an increment in the reliability index of bearing capacity. In a positive correlation case, when both parameters increase or decrease together, there is a significant variation in the ultimate bearing capacity.

5.3. Comparison with Monte Carlo simulation

In the present paper, the log-normal probability distribution is considered for random variables because this distribution provides only positive values. Using MATLAB programming, the calculation of the ultimate bearing capacity value generated from equation (20) is carried in the order of 10^3, 10^4, 10^5, and 10^6 iterations, finally leading to the failure probability calculated from equation (11).

The PDF diagrams of ultimate bearing capacity at 10^5 and 10^6 iterations are shown in Figure (5). Table (7) also presents a comparison between SS and MCS methods. One of the most important factors in the MCS is determining the number of iterations required to perform the calculations using the deterministic equation. Comparing the results of the bearing capacity value achieved from the MCS with various iterations, it is found that the bearing capacity value increases as the number of iterations increases. Standard deviation values resulting from 10^3 up to 10^6 repetitions have an increasing trend, which is reasonable due to the increase in the number of repetitions. A review of the probability of failures achieved from the MCS shows that the higher the number of repetitions, the lower the failure probability. Additionally, as the number of repetitions increases, the rate of reduction in the probability of failure decreases. Therefore, it can be inferred that as the number of repetitions increases, the results of the failure probability converge, and hence more repetitions do not cause further reduction in the failure probability.

To show the effectiveness of the SS, the obtained results are compared with those given by the classical MCS. For this purpose, the results of 10^6 MCS repetitions are calculated. These orders of repetition are sufficient to achieve convergence to the bearing capacity results obtained from the SS. In Figures (6a) to (6d), comparative diagrams of statistical parameters of standard deviation, mean, kurtosis and skewness related to the mean of ultimate bearing capacity versus the number of MCS and SS methods are shown. The SS method is a suitable alternative to the MCS because the SS method can reach the same results as the MCS with less computational effort. In summary, the SS method discussed here is advantageous since it is faster (fewer repetitions required) and demands less computational time when compared with the MCS method.

5.4. Comparison with other studies

Table (8) shows the comparison between the results achieved by the suggested method and the findings of previous studies. The suggested method was validated by comparison of its results with those presented by Al-Bittar and Soubra, Mao et al and Merifield et al for different values of the rock parameters [17,36,37]. The chosen disturbance factor value in this paper is ($D=0$). It is important to note that the findings reported by Merifield et al represent the mean values between the upper and lower bound solutions of the limit analysis theory [37]. Meanwhile, Mao et al only offer an upper bound solution of the ultimate bearing capacity [36]. Table 8 shows that the results of the ultimate bearing capacity obtained from the proposed model have suitable agreement with those of other researchers.

6. Conclusions

In the common deterministic analysis of the bearing capacity of the foundation on rock mass, the uncertainties in the rocks are neglected and therefore no idea of the probability of failure is reached. Therefore, practical values are obtained using safety factors which often leads to overdesigned structures. This indicates the need to use reliable methods based on statistical analysis of variations on affecting parameters and probabilistic analysis of the structure failures. In this paper, the main purpose of the analysis performed is to quantify the uncertainty associated with the bearing capacity of strip foundations placed on rock mass and determine the PDF of bearing capacity. Despite the common approaches used by other investigators, four parameters of the H-B failure criterion with uncertainties were adopted in this study as random variables, including *GSI* index, strength of intact rock (σ_{ci}), constant of intact rock (m_i), and disturbance factor (D) to increase precision and performance of the method. A new reliability analysis algorithm was proposed to calculate the PDF of bearing capacity. In the SS method extended and suggested in this study, the samples of the first stage were calculated using the MCS method, while at later levels, the Markov chain based on the Metropolis-Hastings algorithm was applied to each subset. Moreover, statistical parameters related to the PDF of bearing capacity were presented and discussed. Therefore, both MCS and the proposed SS methods were adopted for the probabilistic analysis. Discussing the results of statistical values of PDF of the bearing capacity obtained from these simulations, the following conclusions were proposed:

1) A diagram of the PDF of bearing capacity resulting from changing the coefficient of variation of variables shows that the uniaxial compressive strength parameter σ_{ci} has the greatest impact on the system response (bearing capacity). Another important affective variable was the geological strength index *GSI*, which affects the scattering of system response. The other two variables, m_i, and D, have less effect on the system response.
2) With a reduction in the COV of variables, the PDF diagram of bearing capacity becomes more compressed, meaning that the output results of the bearing capacity change less,

and more reliability can be applied to them. It is also observed that with increasing COV of input variables, the standard deviation of bearing capacity also increases, which indicates more scattered outputs.

3) As can be seen in the output diagrams of figures (3a) to (3d), the skewness of the results is positive which increases with increasing the COV of the input variables. This means that PDF becomes more asymmetric, which is more evident in the σ_{ci} and *GSI* diagrams figures (3a) and (3b). Also, with an increment in the COV of the input variables, the kurtosis of results decreases, meaning that the PDF diagram of bearing capacity becomes wider and moves away from the normal state.

4) It should also be mentioned that by increasing the COV of the input variables, the reliability index value decreases, and as a result, the failure probability of the system increases. Also, the safety factor that is used for the design increases accordingly. This further implies the safety reduction for structure.

5) By investigating the coefficient of correlation between the two variables, uniaxial compressive strength and geological strength index of the rock mass, it was observed that when a negative coefficient of correlation was used (when decreasing one parameter will increase the other parameter), the kurtosis of the diagram increases, indicating a decrease in variation of bearing capacity.

6) Comparing the results of current SS and MCS methods shows that the SS method presents almost similar and precise results with a much smaller number of data samples than those of MCS. Also, in the MCS method, if the number of repetitions reaches 10^5 and 10^6 times, results converge to those given by the SS method.

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Figure captions :

Figure 1. A series of failure events in the SS method

Figure 2. Flowchart showing the implementation of Subset simulation-based reliability analysis for bearing capacity

Figure 3. Effect of the COV of the input random variables on the PDF diagram of the ultimate bearing capacity : (a) effect of cov(σ_{ci}); (b) effect of cov(*GSI*); (c) effect of cov(m_i); (d) effect of cov(*D*)

Figure 4. Effect of the correlation coefficient on PDF variation of the ultimate bearing capacity

Figure 5. PDF of ultimate bearing capacity at 10^5& 10^6 iterations

Figure 6. Effect of the number of simulations on statistic parameters of ultimate bearing capacity : (a) mean; (b) standard deviation; (c) skewness; (d) kurtosis

Table captions :

Table 1. Statistical parameters values of random variables [36]

Tables 2. Effect of the COV of σ_{ci} on statistical parameters of bearing capacity

Tables 3. Effect of the COV of *GSI* on statistical parameters of bearing capacity

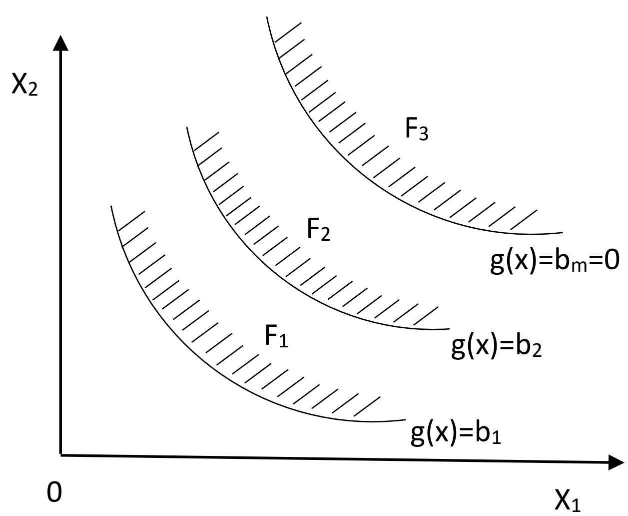
Tables 4. Effect of the COV of *D* on statistical parameters of bearing capacity

Tables 5. Effect of the COV of m_i on statistical parameters of bearing capacity

Table 6. Results of variation of the coefficient of correlations between σ_{ci} and *GSI*

Table 7. Comparative results of Monte Carlo and Subsets Simulations

Table 8. Comparative results of q_u (MPa) as given by the proposed method and previous studies (*D*=0)



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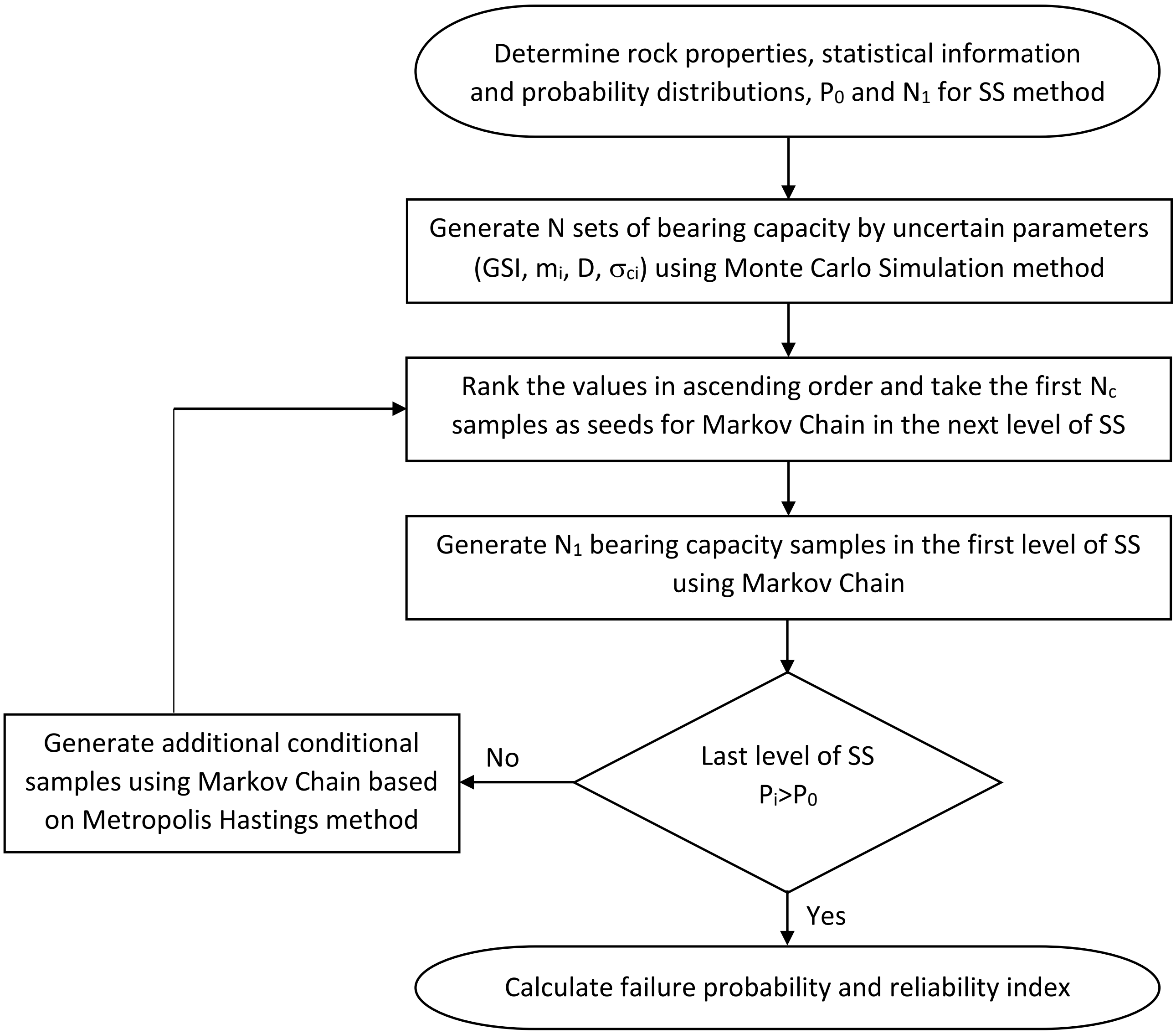

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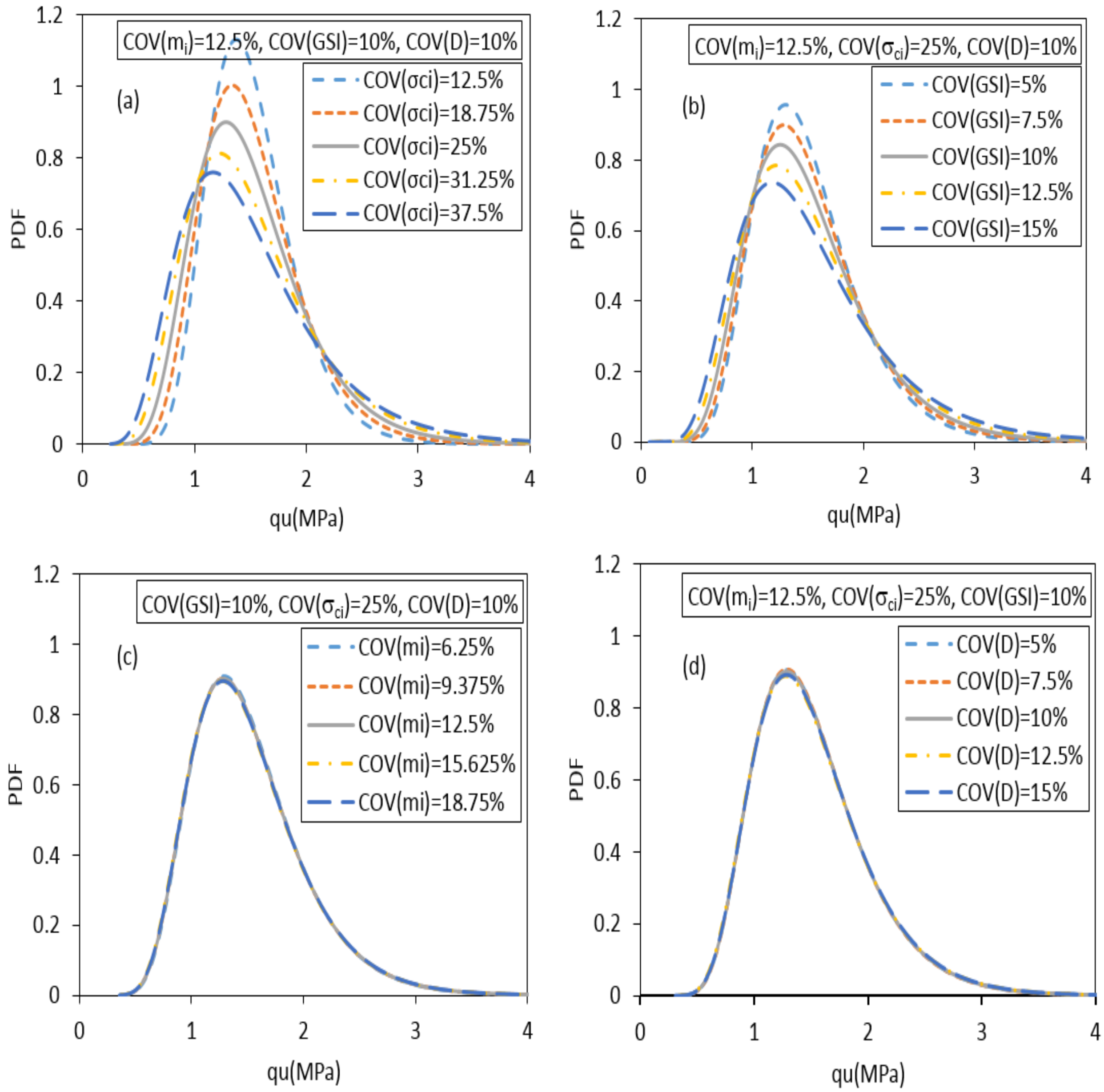

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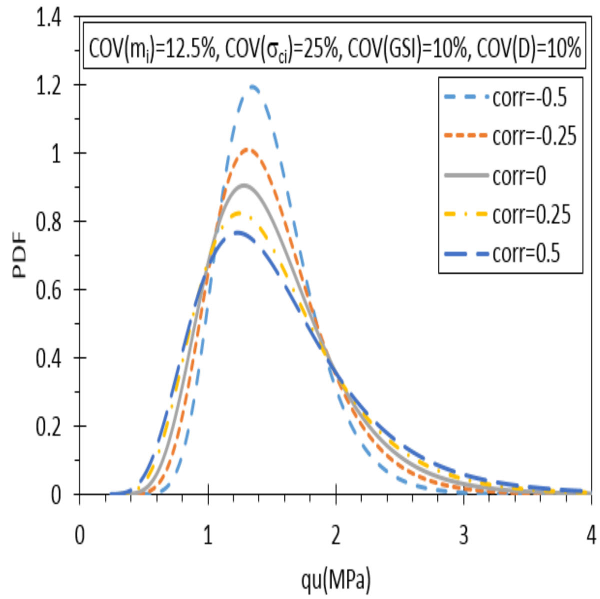

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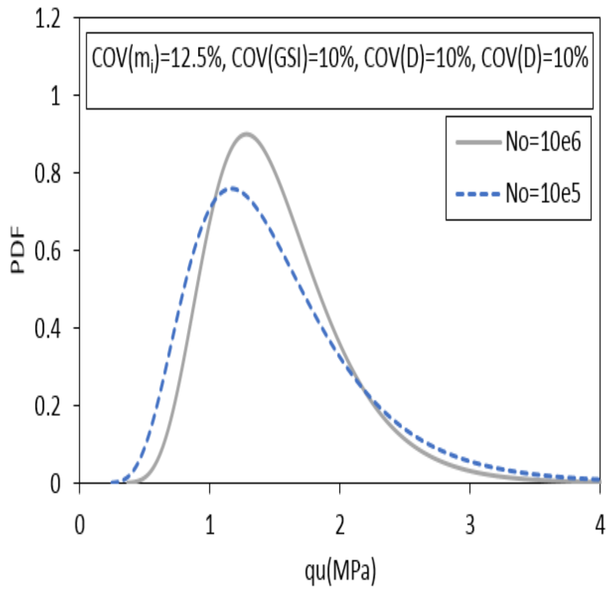

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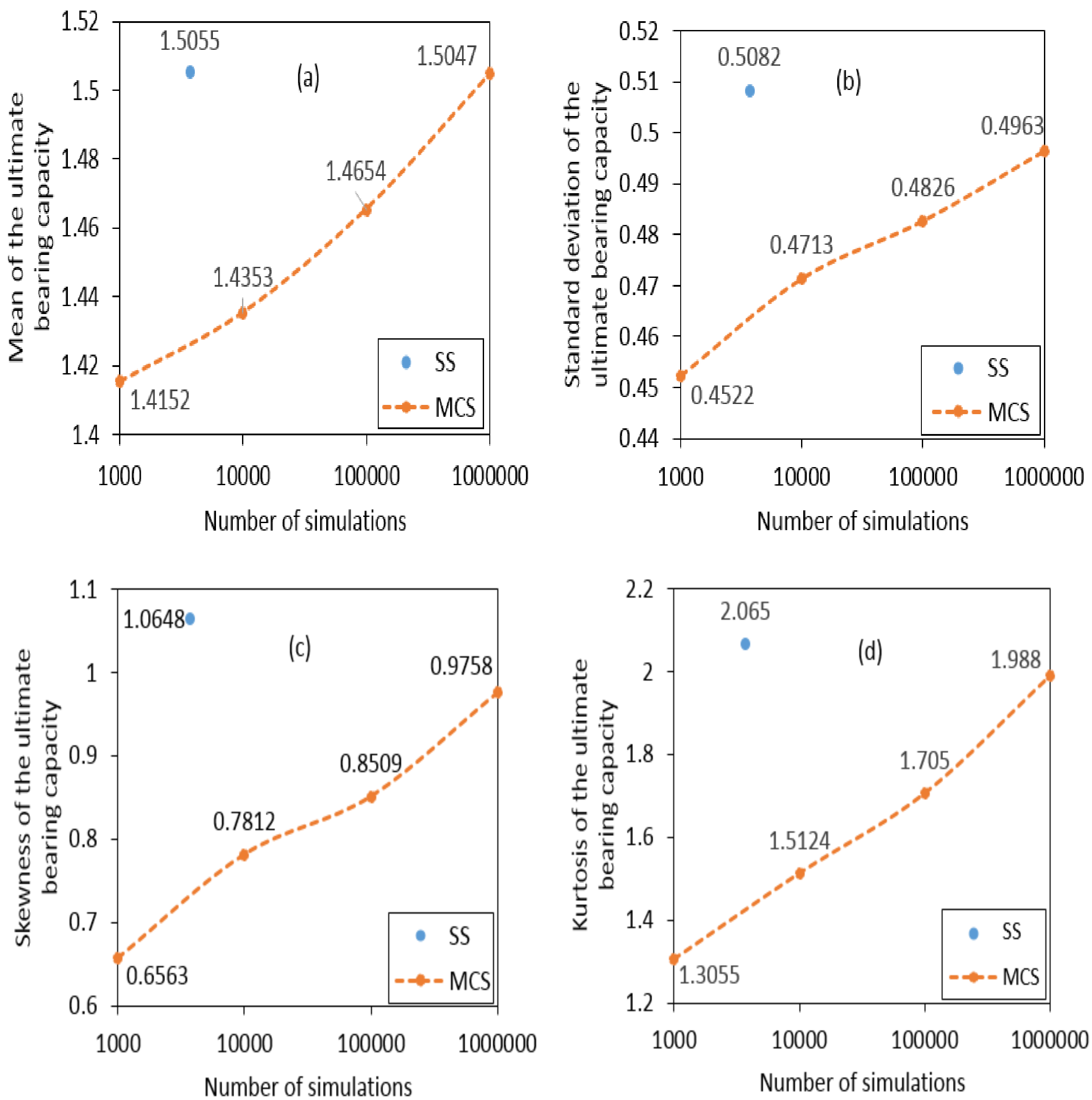

Figure 6. Effect of the number of simulations on statistic parameters of ultimate bearing capacity : (a) mean; (b) standard deviation; (c) skewness; (d) kurtosis

Table 1. Statistical parameters values of random variables [36]

Variables	Mean	COV	PDF
σ_{ci} [MPa]	10	25	Log-normal
GSI	25	10	Log-normal
m_i	8	12.5	Log-normal
D	0.3	10	Log-normal

Tables 2. Effect of the COV of σ_{ci} on statistical parameters of bearing capacity

parameters	**value of the COV of σ_{ci}**				
	12.5%	**18.75%**	**25%**	**31.25%**	**37.5%**
mean	1.50741	1.51014	1.50550	1.50763	1.50377
Stdev	0.38293	0.44201	0.50825	0.58920	0.66083
Skew	0.82210	0.92925	1.06488	1.24571	1.40944
Kurt	1.2887	1.56995	2.06502	2.84794	3.70555
COV	25.4034	29.2698	33.7594	39.081	43.9448
β	3.93649	3.41649	2.96214	2.55879	2.27558
FS	1.0128	1.0147	1.0116	1.0130	1.0104
P_f(SS)	0.00532	0.00538	0.00555	0.00561	0.00575

Tables 3. Effect of the COV of *GSI* on statistical parameters of bearing capacity

parameters	**value of the COV of *GSI***				
	5%	**7.5%**	**10%**	**12.5%**	**15%**
mean	1.49056	1.50199	1.50547	1.51899	1.52959
Stdev	0.42207	0.45978	0.50806	0.56469	0.62025
Skew	0.30013	0.46227	0.62467	0.76540	0.87777
Kurt	0.23620	0.45756	0.71196	0.99006	1.31516
COV	28.3164	30.6114	33.7475	37.1752	40.5498
β	3.53152	3.26675	2.96319	2.68997	2.46610
FS	1.0015	1.0092	1.0128	1.0206	1.0277
P_f(SS)	0.00517	0.00518	0.00524	0.00528	0.00530

Tables 4. Effect of the COV of *D* on statistical parameters of bearing capacity

parameters	value of the COV of *D*				
	5%	**7.5%**	**10%**	**12.5%**	**15%**
mean	1.50781	1.50661	1.50773	1.50951	1.51013
Stdev	0.50487	0.50394	0.50919	0.51698	0.51826
Skew	1.08669	1.06335	1.06843	1.09791	1.04747
Kurt	2.29784	2.01102	2.06196	2.23812	1.94653
COV	33.4835	33.4489	33.7717	34.2484	34.3188
β	2.98655	2.98964	2.96106	2.91984	2.91386
FS	1.0131	1.0123	1.0131	1.0143	1.0147
P_f(SS)	0.00550	0.00551	0.00552	0.00551	0.00548

Tables 5. Effect of the COV of m_i on statistical parameters of bearing capacity

parameters	value of the COV of m_i				
	6.25%	**9.375%**	**12.5%**	**15.625%**	**18.75%**
mean	1.50898	1.50676	1.50740	1.50383	1.50530
Stdev	0.50068	0.50708	0.50943	0.51148	0.51842
Skew	1.04331	1.07208	1.07935	1.05642	1.08485
Kurt	1.97126	2.04864	2.10298	1.94907	2.15942
COV	33.1802	33.6534	33.7955	34.0120	34.4400
β	3.01384	2.97147	2.95897	2.94014	2.90360
FS	1.0139	1.0124	1.0128	1.0105	1.0114
P_f(SS)	0.00547	0.00553	0.00552	0.00555	0.00555

Table 6. Results of variation of the coefficient of correlations between σ_{ci} and *GSI*

parameters	value of correlation coefficient between σ_{ci} and *GSI*								
	-1	**-0.75**	**-0.5**	**-0.25**	**0.00**	**0.25**	**0.5**	**0.75**	**1**
mean	1.43472	1.45123	1.46932	1.48880	1.50407	1.52250	1.54795	1.56397	1.58021
Stdev	0.13225	0.26622	0.35890	0.43799	0.50631	0.57281	0.64291	0.70616	0.76435
Skew	0.32085	0.54489	0.73754	0.91968	1.05718	1.20324	1.36173	1.51782	1.58095
Kurt	0.24592	0.53912	0.95678	1.60825	1.99090	2.70541	3.43864	4.55541	4.45567
COV	9.21758	18.3446	24.426	29.419	33.6625	37.6227	41.533	45.1515	48.3702
β	10.8488	5.4512	4.0940	3.39917	2.97066	2.65797	2.40773	2.21476	2.06739
FS	0.964	0.9751	0.9872	1.0003	1.0106	1.023	1.0401	1.0508	1.0618
P_f(SS)	0.00675	0.0059	0.00569	0.00558	0.00552	0.00549	0.0054	0.00541	0.00541

Table 7. Comparative results of Monte Carlo and Subsets Simulations

statistical parameters	No. of Repetitions				
	MCS method				SS method
	10^3	10^4	10^5	10^6	3700
mean	1.4152	1.4353	1.4654	1.5047	1.5055
Stdev	0.4522	0.4713	0.4826	0.4963	0.5082
Skew	0.6563	0.7812	0.8509	0.9758	1.0648
Kurt	1.3055	1.5124	1.7050	1.9880	2.0650
COV	31.953	32.609	32.710	32.983	33.759
β	3.1296	3.0666	3.0572	3.0318	2.9621
FS	3.1161	2.6353	2.2741	1.0588	1.0161
P_f(SS)	0.0510	0.0221	0.0106	0.0075	0.0055

Table 8. Comparative results of q_u (MPa) as given by the proposed method and previous studies (D=0)

GSI	σ_{ci} (MPa)	m_i (MPa)	FLAC^{3d} [17]	Mao, Al-Bittar, and Soubra [36]	Merifield, Lyamin, and Sloan [37]	Proposed method
20	7.5	10	1.460	1.600	1.568	1.585
20	10	10	1.960	2.130	2.090	2.109
20	12.5	10	2.450	2.670	2.613	2.642
20	15	10	2.930	3.200	3.135	3.167
20	20	10	3.920	4.270	4.180	4.225
30	7.5	10	2.784	3.040	2.978	3.009
30	10	10	3.710	4.060	3.970	4.015
30	12.5	10	4.660	5.070	4.963	5.016
30	15	10	5.605	6.120	5.955	6.037
30	20	10	7.498	8.080	7.940	8.010

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