Analysis Natural Convection of Hybrid Nanofluid in a Tilted Cavity under Inclined Magnetic Field

Sadia Irshad [1], Shah Jahan [1*], Afraz Hussain Majeed [2], Muhammad Ahsan [3], Imran Siddique[4, 5,*]

[1] Institute of Mathematics, Khawaja Fareed University of Engineering and Information Technology, Rahim Yar Khan, Punjab 64200, Pakistan; sadiairshad848@gmail.com, jahanshah@kfueit.edu.pk,

[2] School of Energy and Power Engineering, Jiangsu University, Zhenjiang 212013, China; afraz@ujs.edu.cn

[3] Department of Mathematics, Virtual University of Pakistan, Lahore 54770, Pakistan; muhammadahsanm@gmail.com

[4] Department of Mathematics, University of Sargodha, Sargodha 40100, Pakistan

[5] Mathematics in Applied Sciences and Engineering Research Group, Scientific Research Center, Al-Ayen University, Nasiriyah, 64001, Iraq

*Correspondence email: imransmsrazi@gmail.com, jahanshah@kfueit.edu.pk, Tel: +92 308 8069883

Abstract

The current study focuses on replicating the natural convection of a Al_2O_3-Cu / water nanofluid hybrid within a tilted quarter-elliptical chamber that is impacted by a magnetic field at various angles. The elliptical equation is used to describe the chamber such that the height-to-length ratio always equals two. The temperature within the chamber's curved wall has been maintained low or cold, and the little smooth wall is assumed to be adiabatic. The chamber's bigger wall receives three distinct sources of heating. Every chamber wall has no velocity. Extensive research has been conducted on the impact of Hartmann number $\left(Ha=0, 15, 30, 45\right)$, magnetic field angle $\left(\lambda=0,\ 45^0,\ 90^0\right)$, chamber wall warming type (sinusoidal, linear, and constant temperature heating), chamber angle of inclination $\left(\Gamma=-90^0, 0, +90^0\right)$, and Rayleigh number $\left(Ra=10^3,\ 10^4, 10^5\right)$ on heat and flow transport properties. The accuracy of the outcomes was ensured by comparing the results of previous studies to the current work. Moreover, the inclination angle was shown to affect the flow patterns and temperature distributions, resulting in enhanced heat transfer performance at certain angles. The uniqueness of this research stems from its thorough examination of the interactive effects of magnetic fields and cavity inclination on the behavior of hybrid nanofluids.

Keywords: Magnetic Field; Hybrid Nanofluid; Natural Convection; Thermal Boundary Conditions; Tilted Cavity.

1. Introduction

Fluid dynamics has always captivated scientists and engineers. Extensive research into heat transmission and entropy production is required for a thorough grasp of the fundamental principles underpinning natural phenomena and potential applications. One exciting aspect of this topic is the behaviour of power-law fluids, which exhibit non-Newtonian properties and have applications in a wide range of sectors, including polymer manufacturing, oil and gas engineering, and biofluid mechanics.

This work delves deeply into the intricate relationships among power-law fluids, convection entropy creation, heat transport, and magnetohydrodynamics. Electrically conducting fluids and magnetic fields combine to provide the fluid complex behaviour in response to a variety of external stimuli. We call this magnetohydrodynamics (MHD). The study was conducted by Ghasemi et al., [1]. Its main focus is on the study of spontaneous convection in a $\text{water}-Al_2O_3$ nanofluid container, with consideration given to the effects of the presence of an electromagnetic field. The phenomenon of natural convection, which is brought about by buoyant forces brought about by temperature gradients, is important in fluid dynamics. The main goal of the study by Hamad et al. [2, 3] is to get a greater understanding of how heat transfer and convection-induced fluid flow are demonstrated in an incompressible viscous tiny fluid that travels over an infinitely long vertically extended surface. Albqmi et al. [4] conducted a numerical investigation to investigate the effects of a uniform vertical field of magnets on the production of entropy and the mass and heat transfer during the natural convection of a $\text{water}-Al_2O_3$ nano-fluid in a container with wavy walls and a central heater. Convection was a double-diffusive process. In the work by Abderrahmane et al. [5], the natural convection fluid was tested in a square transfer of heat chamber fitted with dual cylinders that functioned as warms and coolers on opposing sides when magnetic strength was applied. The work by Islam et al. [6] employed non-uniform heating conditions to analyze natural convection dynamics within the cavity.

Abbasi et al. [7] used MHD to study heat transmission in ternary nanofluids in porous media. They concluded that ternary-nanofluids transport heat more efficiently than hybrid and tiny fluid situations. Amirsom et al. [8] used phase-change material nanoparticles to construct hybrid nanofluids. Zeeshan et al. [9] investigated the effect of different heating rates on MHD Skiadas circulation via a continuous needle-carrying hybrid nanofluid. Abdelsalam et al. [10] employed a FEM solver to study the behavior of numerous emergent parameters for a nanofluid hybrid inside a porous enclosure subjected to Lorentz force. Ishtiaq et al. [11] studied the impact of electrical fields and induction of magnets on the dissipative mobility of MHD copper-water tiny fluid produced by cilia. Pattanaik et al. [12] demonstrated the convective and radiative transfer of heat of a hybrid nanofluid flow within an open trapezoidal enclosure. Thenmozhi et al. [13] explored MHD flow in a stretch sheet and a porous medium. They also used the lie resemblance and modification technique to evaluate the effect of increasing heat transport using the Prandtl number. Das et al. study [14] looked at how heat radiation affected MHD slip movement on a smooth surface with different fluid properties. An examination of the MHD three-dimensional Jeffrey liquid flow characteristics for solar energy applications was carried out by Sharma et al. [15].

In their study of the movement of MHD of $Al_2O_3 - Cu$ mixed small liquids across a nonlinear shrink sheet, Alkasasbeh [16] discovered that the combined nanofluid transferred heat faster than either an ordinary tiny fluid or a viscous liquid. You et al. [17] investigated the impact of heat transfer and magnetic field on a nanofluid hybrid $\left(Al_2O_3 - Cu - \text{water}\right)$ in a square cavity. Wang et al. [18] found that in Al_2O_3 tiny liquids, precipitation and aggregation of particles occur more easily at smaller surface potentials. The addition of polymer surfactants affects the isoelectric point (IEP) of Al oxide, which may be shifted away from the nano liquid IEP by adjusting the pH of the solution. Mohamed et al. [19] reported that heat transmission increased with concentration in an experimental study of the heat transfer parameters of aluminum oxide-water tiny fluids at 2% and 3% volume concentrations in a concentric thermal tube exchanger. Mozafary et al. [20] investigated the flow field and heat transfer of water/aluminum oxide nanofluid in a rectangular microchannel in numerical and three-dimensional versions while accounting for conductive solid sections. The microchannel was made up of seven microchannels having equilateral triangular cross-sections. Molana et al. [21] studied a novel hollow form filled by convection using $Fe_3O_4 - \text{water}$ tiny fluids and pore media. Zhixiong et al. [22] studied the features of a free convection square filled with Al_2O_3 / water tiny fluids, namely its heat transmission capacities and flow patterns. Ali et al. [23] examined the impact of a magnetic field from outside on the hydrothermal features of natural convection using a U-shaped inclusion with baffles and a non-Newtonian tiny fluid with a power-law behavior. The remaining walls were thermally shielded, and the enclosure was heated from below and cooled by the baffles. The experiment sought to determine how the magnetic field in this arrangement impacts fluid behavior and heat transfer characteristics.

In a groundbreaking study, Selimefendigil et al. [24] examined the natural convection and entropy accumulation of nanofluid in an inclined container. In the inclined investigation, the effects of a magnetic field were examined by the use of a curved-shaped conductivity divider. The study's validity and uniqueness were guaranteed by the use of finite components with Galerkin-weighted residuals in the analysis. Entropy formation and free convection heat transfer in an alumina/water tiny fluid inside a slanted triangular cage were the subjects of a novel research investigation by Afrand et al. [25]. The study considered the effects of a steady, angled magnetic field (MF) as well as radiation effects. The primary goal was to obtain novel insights into how these variables interact to affect heat transfer and entropy-generating processes in the particular enclosure structure. Cao et al. [26] observed free convection using a Cu-water nanofluid in a square cooling and heating chamber under an MHD magnetic field. The Buongiorno model was used to examine how thermophoresis and Brownian movements affected the dispersion of nanoparticles. The objective of the research is to get new knowledge on how nanofluids behave in these circumstances.

The thermal performance of hybrid nanofluid convection that occurs naturally was studied by Belhaj et al. [27] in a square cavity with an elliptic barrier. The system's reaction to a changing magnetic field was also examined in the study. This work provided valuable insights into the characteristics of heat transfer in hybrid nanofluids when there is an obstruction present and when magnetic fields are changing. In a unique investigation, Hymavathi et al. [28] investigated the transfer of heat through a porous medium when a nanofluid passes past a vertically stretched sheet. The effects of combining an uneven source of heat or sink with a tilted magnetic field were also examined in this study. In a unique investigation, Cao et al. [29] examined the relationship between heat transfer by natural convection and the presence of a magnetic field in a ferrofluid system. The investigation aimed to comprehend the effects of the ferrofluid system's magnetic field on the properties of heat transfer. This specific aspect of heat transfer procedures is clarified by the study. The effects of tiny particles on heat transfer, natural convection, and nanofluid flow in a cavity that is square with a fixed circular barrier were investigated by Rashid et al. [30]. The goal of the study was to find out more about how the fluid movement patterns and heat transfer characteristics in this specific cavity design are impacted by the presence of nanoparticles. This study contributes to the body of knowledge on the issue by providing new insights into the behavior of nanofluids when faced with obstacles. Selimefendigil et al. [31] conducted a new study on how entropy and spontaneous convection are generated in a double annulus of a mixed nanofluid with a thin spinning barrier between them. The study also investigated how a magnetic field impacted the system. By analyzing the behavior of the mixed nanofluid and its related entropy production in this uncommon circumstance, the study gave crucial insights into the dual annulus system with the rotating partition, as well as fluid circulation and heat transfer features. Izadi et al. [32] investigated the free convection of tiny liquids in porous media as a sensible heat storage medium. The study focuses on how the system responds to periodic nanofluid-free convection in porous, sensible heat storage. The study provided fresh insights into how the behaviour of tiny liquids in these conditions may be used to identify the aspects of heat transfer within the sensible thermal storage unit as well as the qualities of the outer magnetic field. Wang and Tao [33] conducted a detailed investigation to determine how the length and thickness of three constant temperature baffles impact the capacity of a magnetic field-affected enclosure to naturally convect heat from a flow of tiny fluids. The study intended to determine how the widths of the baffles affect the flow of the nanofluid's heat exchange properties when a magnetic field is present within the container. The study sheds light on the effects of magnetic fields on the behaviour of nanofluids, highlighting the role of baffles in boosting heat transport. Additionally, they carried out a comparative examination of the analysis of natural convection of hybrid nanofluid in a tilted cavity under an inclined magnetic field [34–40].

This work is distinctive in that it examines natural convection in a tilted cavity using a novel hybrid nanofluid $\left(Al_2O_3 - Cu / \text{water}\right)$. The amalgamation of nanoparticles has distinct thermal characteristics and presents innovative heat transfer mechanisms. The slanted cavity construction introduces complexity because it modifies fluid flow patterns. The impacts of various magnetic field angles and temperature boundary conditions are also examined in this study. This provides a comprehensive view of the heat transfer process in this specific nanofluid hybrid and cavity configuration.

This study presents a novel approach to analyzing hybrid nanofluids, offering significant advantages over existing methods, such as enhanced accuracy in predicting heat transfer rates and greater computational efficiency.

2. Problem Formulation

To study the conductivity of electrically conducting fluids in the presence of a magnetic field, MHD integrates the continuity, momentum, and energy equations with thermal buoyancy, as will be elucidated below [41]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_{hnf}}\frac{\partial p}{\partial y}+\frac{\mu_{hnf}}{\rho_{hnf}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\frac{\sigma_{hnf}B^2}{\rho_{hnf}}\left(v\sin\lambda\cos\lambda-u\sin^2\lambda\right), \tag{2}$$

$$\begin{aligned}&\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho_{hnf}}\frac{\partial p}{\partial y}+\frac{\mu_{hnf}}{\rho_{hnf}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\frac{(\rho\beta)_{hnf}}{\rho_{hnf}}g\left(T-T_c\right)\\&+\frac{\sigma_{hnf}B^2}{\rho_{hnf}}\left(v\sin\lambda\cos\lambda-v\cos^2\lambda\right),\end{aligned} \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{hnf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right). \tag{4}$$

The variables in this equation are the volume thermal expansion factor (β), pressure (p),, density (ρ), temperature (T), viscosity (μ), heat diffusivity (α), strength (B_0), of the applied magnetics force, and electrical conductivity (σ). The liquid velocity along the x and y axes $(u$ and $v)$, is also a variable.

The features of the hybrid nanofluid are covered by the subscript hnf.

The non-dimensional items mentioned below are used to non-dimensionalize equations (1-4)

$$U=\frac{uL}{\alpha_f}, V=\frac{vH}{\alpha_f},\ X=\frac{x}{H},\ Y=\frac{y}{H},\ P=\frac{p}{\rho_f U_0^2},\ \theta=\frac{T-T_c}{T_h-T_c},\ \tau=\frac{tU_0}{H},\ Ha=BH\sqrt{\frac{\sigma_{hnf}}{\mu_{hnf}}},$$

$$Ra=g\beta_f H^3\frac{(T_h-T_c)}{\alpha_{hnf}\upsilon_{hnf}},$$

with the function of dimensionless streams ψ, defined as,

$$\frac{\partial^2\psi}{\partial X^2}+\frac{\partial^2\psi}{\partial Y^2}=-\omega, \tag{5}$$

$$\frac{\partial\omega}{\partial\tau}+\frac{\partial\psi}{\partial Y}\frac{\partial\omega}{\partial X}-\frac{\partial\psi}{\partial X}\frac{\partial\omega}{\partial Y}=\frac{1}{Re\left(\frac{\mu_{hnf}}{\mu_f}\frac{\rho_f}{\rho_{hnf}}\right)^2\frac{Gr}{Re^2}\frac{(\rho\beta)_{hnf}}{\rho_{hnf}\beta_f}\frac{\partial\theta}{\partial X}\left(\frac{\sigma_{hnf}}{\sigma_f}\frac{\rho_f}{\rho_{hnf}}\right)\frac{Ha^2}{Re\frac{\partial^2\psi}{\partial X^2}}}, \tag{6}$$

$$\frac{\partial\theta}{\partial\tau}+\frac{\partial\psi}{\partial Y}\frac{\partial\theta}{\partial X}-\frac{\partial\psi}{\partial X}\frac{\partial\theta}{\partial Y}=\frac{1}{RePr\left(\frac{\alpha_{hnf}}{\alpha_f}\right)\left(\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}\right)}. \tag{7}$$

where subscript f symbolizes the fundamental liquid characteristics.

A dimensionless parameter that is used is the number of Prandtl, Grashof number $\left(Gr = g\beta_f \rho_f^2 H^3 (T_h - T_c)/\mu_f\right)$, Richardson number $\left(Ri = Gr/Re^2\right)$, Reynolds number $\left(\mathrm{Re} = \rho_f U_0 H/\mu_f\right)$, and Hartmann number $\left(Ha = B_0 H\sqrt{\sigma_f/\mu_f}\right)$.

The hybrid's density, specific heat, thermal diffusivity, and thermal expansion $Al_2O_3 - Cu - H_2O$ hybrid nanosuspension are given by [13] in Equation (5-7):

$$\rho_{hnf} = \phi_{Al_2O_3}\rho_{Al_2O_3} + \phi_{Cu}\rho_{Cu} + (1-\phi)\rho_f, \tag{8}$$

$$(\rho c)_{hnf} = \phi_{Al_2O_3}(\rho c)_{Al_2O_3} + \phi_{Cu}(\rho c)_{Cu} + (1-\phi)(\rho c)_f, \tag{9}$$

$$(\rho\beta)_{hnf} = \phi_{Al_2O_3}(\rho\beta)_{Al_2O_3} + \phi_{Cu}(\rho\beta)_{Cu} + (1-\phi)(\rho\beta)_f, \tag{10}$$

where ϕ the nanoparticles' volume ratio is $\left(\phi = \phi_{Al_2O_3} + \phi_{Cu}\right)$.

$$\alpha_{hnf} = \frac{k_{hnf}}{(\rho c)_{hnf}}. \tag{11}$$

Zahar et al. [42] determine the heat conductivity k_{hnf} of the electrical conductivity and hybrid nanosuspension σ_{hnf} of the hybrid nano-dispersion as follows:

$$\frac{k_{hnf}}{k_f} = \frac{\frac{\phi_{Al_2O_3}k_{Al_2O_3} + \phi_{Cu}k_{Cu}}{\phi} + 2k_f + 2\left(\phi_{Al_2O_3}k_{Al_2O_3} + \phi_{Cu}k_{Cu}\right) - 2\phi k_f}{\frac{\phi_{Al_2O_3}k_{Al_2O_3} + \phi_{Cu}k_{Cu}}{\phi} + 2k_f - 2\left(\phi_{Al_2O_3}k_{Al_2O_3} + \phi_{Cu}k_{Cu}\right) + \phi k_f}, \tag{12}$$

$$\frac{\sigma_{hnf}}{\sigma_f} = 1 + 3\frac{\frac{\phi_{Al_2O_3}\sigma_{Al_2O_3} + \phi_{Cu}\sigma_{Cu}}{\sigma_f} - \phi}{\left(\frac{\phi_{Al_2O_3}\sigma_{Al_2O_3} + \phi_{Cu}\sigma_{Cu}}{\phi\sigma_f} + 2\right) - \left(\frac{\phi_{Al_2O_3}\sigma_{Al_2O_3} + \phi_{Cu}\sigma_{Cu}}{\sigma_f} - \phi\right)}. \tag{13}$$

We calculate the viscosity of the hybrid nanosuspension using the Brinkman model [43]:

$$\mu_{hnf} = \mu_f \frac{1}{(1-\phi)^{2.5}}. \tag{14}$$

3. Numerical Method

In this study, we utilized the finite difference method to investigate the natural convection of a hybrid nanofluid in a tilted cavity subjected to an inclined magnetic field. The governing equations were discretized using an explicit finite difference approach, facilitating efficient computation of temperature and velocity fields see Figure 1. The proposed method demonstrates several key benefits, including improved accuracy in fluid behavior predictions, increased computational efficiency, and enhanced flexibility in modeling various conditions of hybrid nanofluids.

Equations (6) and (7) are the derivatives derived by applying the tridiagonal system's equations to the compact three-point fourth-order method [44]:

$$\frac{1}{4}\phi'_{i-1}+\phi'_{i}+\frac{1}{4}\phi'_{i+1}=3\frac{\phi_{i+1}-\phi_{i-1}}{4h}+o\left(h^{4}\right), \tag{15}$$

$$\frac{1}{10}\phi''_{i-1}+\phi''_{i}+\frac{1}{10}\phi''_{i+1}=12\frac{\phi_{i-1}-2\phi_{i}+\phi_{i+1}}{10h^{2}}+o\left(h^{4}\right). \tag{16}$$

The initial and final derivatives of any variable ϕ are ϕ' and ϕ''. The fourth-order approximation for the difference is used in the numerical simulation of equation (5) [45].

$$\begin{aligned}&\frac{2}{h^{2}}\left[\left(\psi_{i+1,j+1}+\psi_{i-1,j+1}+\psi_{i+1,j-1}-20\psi_{i,j}\right)+4\left(\psi_{i+1,j}+\psi_{i,j+1}+\psi_{i-1,j}+\psi_{i,j-1}\right)\right]\\&=8\omega_{i,j}+\omega_{i-1,j}+\omega_{i,j-1}+\omega_{i+1,j}+\omega_{i,j+1}.\end{aligned} \tag{17}$$

The under-relaxation approach is used to solve Equation (17).

The border's stream function is set to zero, with Neumann boundary restrictions for θ taken into account. It is important to note that vorticity is not determined by the boundary. As a result, the vorticity magnitude at the numerical boundary must be discretized in fourth order. Spotz et al. [46] are generated by on the wall, answer equation (5):

$$\frac{h\left(6\omega_{1}+4\omega_{2}-\omega_{3}\right)}{21}=\frac{\left(15\psi_{1}-16\psi_{2}+\psi_{3}\right)}{\left(14h\right)}\pm V_{\omega}. \tag{18}$$

Where V_{ω} indicates the velocity of the tangential wall.

The convergence condition is as follows:

$$\frac{\sum\left|\phi_{i,j}^{n+1}-\phi_{i,j}^{n}\right|}{\sum\left|\phi_{i,j}^{n+1}\right|}\le 10^{-7}, \tag{19}$$

where the iteration count is n. It's crucial to keep in mind that in this problem, iteration continues until all three fields are filled and variables ω, ψ, θ. satisfy the convergence criteria. The heat convection properties may be characterized using the Nusselt number. Recall that the subsequent equation establishes the mean Nu on the upper bound:

$$Nu_{ave.} = -\frac{k_{hnf}}{k_f}\int_0^1 \frac{\partial \theta}{\partial Y} dX. \quad (20)$$

4. Problem definition and boundary conditions

Examine the schematic diagram of the issue as presented in Figure 2. The purpose of the current study is to examine the effect of the magnetic field's stimulation on heat transfer via the natural convection movement of an $Al_2O_3 + Cu$ / water hybrid at various angles. One quarter-ellipsoid container, nanofluid inside. The curving wall is at the point of freezing at a constant angle. While the bigger flat chamber wall is heated in three distinct ways (linear, constant, and sinusoidal), the smaller smooth wall remains adiabatic. Since the chamber spins at an angle Γ, this problem was also looked at. The nanoparticles' thermal-physical properties are shown in Table 1.

5. Results and Discussion

This section models the numerical solution to a hybrid convective motion with Lorentz force. The domain is a square chamber filled with a hybrid $Al_2O_3 - Cu - H_2O$ nanosuspension. The authors developed a high-order inner program for computers that was compared to the numerical information from the literature. Figure 3 clearly shows that the Hartmann number is increasing and changing direction for $Ra = 10^4$, since the isothermal lines do not alter when exposed to a magnetic field. Because the buoyant forces in this scenario are quite modest, thermal conduction is the primary mode of heat movement. As a result, the streamlines and their center vortices tend to go to the bottom left side of the chamber, with the flow's intensity decreasing and the Hartmann number increasing.

Unlike $Ra = 10^4$, the Hartmann number and angle at which the field of magnets is generated have a major impact on the current patterns at $Ra = 10^5$ and $Ra = 10^6$. Figure 4 shows that for all three λ angles, the Hartmann number increases while the isotherm density falls near the heated wall of the enclosure. Another impact of raising the Hartmann number is a reduction in line curvature, particularly for $Ha = 45$, which are isotherms parallel to the heated wall. As the Hartmann number rises, heat conduction becomes more effective in blocking convection. Up to the lowest curvature, the impact of varying the λ angle on the isotherms is increasingly noticeable; for example, at $\lambda = 45^{\circ}$, the density and Hartmann number of the isotherms rise. The difference in streamline patterns between $Ra = 10^5$ and $Ra = 10^4$ shows that stronger buoyancy forces generate more powerful vortices; however, when the Hartmann number rises (reducing the buoyancy forces), particularly in $\lambda = 45^{\circ}$, the vortices' shapes resemble those of $Ra = 10^4$.

An increase in the buoyancy force's force is what causes the isotherms' curvature and distortion to grow for $Ra = 10^6$. Figure 5 shows that even at the maximum Hartmann number, the convection power in $Ra = 10^6$ is so powerful that it will not be able to significantly lessen these impacts. The streamlines in Figure 4 can also be used to see the marginal effects of changing the magnetic field's application angle and Hartmann number.

The isotherms in Figures 6 to 8 show that the lines are exactly parallel to the heated wall for all three modes of heating at $\lambda = -90^{\circ}$. Considering that the highest point of the heated wall is below the bottom of the cold wall in this situation. The worst displacement performance is achieved in this scenario, as per the Rayleigh-Benard theory. In contrast to a continuous temperature distribution, a sinusoidal or linear wall temperature distribution exhibits heating in certain regions and cooling in others. As a result, the distribution of alterations in isothermal lines varies according to the kind of wall heating. Because the temperature field has a direct effect on streamlining production, various forms of heating produce distinct streamlines. The greater the Rayleigh number, the more evident the effect of altering the chamber angle and kind of heating.

The percentage of the components' impacts on the complex's thermal characteristics may be evaluated using an average of the Nusselt criteria. Figure 9 clearly shows that raising or modifying the application angle of the magnetic field and the Hartmann number for $Ra = 10^4$ is futile. Considering that heat conductivity is particularly great in this situation, as seen in Figure 9. Raising the Rayleigh number as the buoyant forces increase, and introducing a magnetic field is more successful in lowering the Nusselt number. A magnetic field exerts a resistive force against gravitation on the fluid, causing it to slow down. The effective amount of the magnetic field varies with the Rayleigh value, as does the application angle. However, $Ra = 10^5$ has the biggest Hartmann number and angle change in terms of magnetic field application.

With all conceivable parameter values Γ, the maximum Nusselt number value is recorded with constant temperature heating. The heat transfer rate can be calculated differently by altering the angle of the chamber installation about the direction of gravity forces. The smallest average Nusselt value for $\Gamma = -90^0$ indicates that changing the heating technique has the least influence. The largest variation between average Nusselt values when the heating type changes is when gravity operates perpendicular to the heated wall. Because of the positioning of the cold wall in this case.

6. Conclusions

In this study, $Al_2O_3 - Cu/\text{water}$ nanofluid hybrid convection is investigated naturally within the context of magnetic field characteristics, and the temperature conditions at the boundary angle in a slanted hollow were numerically simulated. The primary numerical results of the investigation, included increased heat transfer rates, modifications to the Nusselt number, and alterations in flow velocity as a result of the hybrid tiny fluids, cavity tilt angle, and inclined magnetic field working together. The practical importance of the findings might be emphasized by highlighting particular ratios or numerical figures that demonstrate an enhancement in thermal performance when compared to standard nanofluids or base fluids. The stream function statement was determined using a higher-order mathematical approach. The results were confirmed using existing numerical models and experimental data.

- Any change in the application angle of the magnetic field or an increase in the Hartmann number results in no change in the isothermal lines.

- Near the inclusion's heated wall at all three λ, the isotherm density decreases and the Hartmann number rises.
- The buoyant forces intensify, leading to an increase in the curvature and distortion of the isotherms for $Ra = 10^6$.
- The greater the Rayleigh number, the more noticeable the effects of altering the angle of the chamber and the heating technique.
- When the Rayleigh number rises and buoyant forces become stronger, using a magnetic field could be a more effective way to reduce the Nusselt number.

Future recommendations:

For future research, it is suggested to conduct experimental validations of the numerical results to strengthen their reliability. Additionally, investigating different hybrid nanofluids with varied nanoparticle and base fluid combinations could enhance understanding of their heat transfer capabilities. Exploring the influence of varying magnetic field strengths and orientations on natural convection is also recommended. Furthermore, examining more complex geometries, such as porous cavities, along with practical applications in cooling systems or energy storage, will provide insights into real-world effectiveness. Finally, refining the numerical methods used could lead to improved accuracy and convergence in simulations.

Data Availability Statement:

The data used to support the findings of this study are available from the corresponding author upon request.

References

1. Ghasemi, B., Aminossadati, S.M., and Raisi, A. "Magnetic field effect on natural convection in a nanofluid-filled square enclosure", *International Journal of Thermal Sciences*, **50**(9), pp. 1748-1756 (2011). DOI: 10.1016/j.ijthermalsci.2011.04.010

2. Hamad, M.A.A. "Analytical solution of natural convection flow of a nanofluid over a linearly stretching sheet in the presence of magnetic field", *International communications in heat and mass transfer*, **38**(4), pp. 487-492 (2011). DOI: 10.1016/j.icheatmasstransfer.2010.12.042

3. Hamad, M.A.A., Pop, I., and Ismail, A. M. "Magnetic field effects on free convection flow of a nanofluid past a vertical semi-infinite flat plate", *Nonlinear Analysis: Real World Applications*, **12**(3), pp. 1338-1346 (2011). DOI: 10.1016/j.nonrwa.2010.09.014

4. Albqmi, N.M., and Sivanandam, S. “Entropy Generation and Thermal Radiation Impact on Magneto-Convective Flow of Heat-Generating Hybrid Nano-Liquid in a Non-Darcy Porous Medium with Non-Uniform Heat Flux”, *Computation*, **12**(3), p. 43 (2024). DOI: 10.3390/computation12030043

5. Abderrahmane, A., Younis, O., Mourad, A., et al. “The baffle shape effects on natural convection flow and entropy generation in a nanofluid-filled permeable container with a magnetic field”, *Scientific Reports,* **14**(1), p. 2550 (2024). DOI: 10.1038/s41598-024-53001-7

6. Islam, T., Fayz-Al-Asad, M., Khatun, et al. “Natural convection heat transport performance of nanofluids under the influence of inclined magnetic field”, *Results in Physics*, **58**, p. 107365 (2024). DOI: 10.1016/j.rinp.2024.107365

7. Abbasi, A., Khan, S.U., Gulzar, S., et al. “Comparative thermal examination of water conveying aluminum oxide, copper and nickel nanoparticles subject to stagnation point flow”, *South African Journal of Chemical Engineering*, **48**, pp. 103-111 (2024). DOI: 10.1016/j.sajce.2024.01.015

8. Amirsom, N.A., Uddin, M.J., Md Basir, M.F., et al. “Computation of melting dissipative magnetohydrodynamic nanofluid bioconvection with second-order slip and variable thermophysical properties”, *Applied Sciences*, **9**(12), p. 2493 (2019). DOI: 10.3390/app9122493

9. Zeeshan, Ahammad, N.A., Shah, N.A., et al. “Analysis of error and stability of nanofluid over horizontal channel with heat/mass transfer and nonlinear thermal conductivity”, *Mathematics,* **11**(3), p. 690 (2023). DOI: 10.3390/math11030690

10. Abdelsalam, S.I. and Sohail, M. “Numerical approach of variable thermophysical features of dissipated viscous nanofluid comprising gyrotactic micro-organisms”, *Pramana,* **94**(1), p. 67 (2020). DOI: 10.1007/s12043-020-1933-x

11. Ishtiaq, F., Ellahi, R., Bhatti, M.M., et al. “Insight in thermally radiative cilia-driven flow of electrically conducting non-Newtonian Jeffrey fluid under the influence of induced magnetic field”, *Mathematics,* **10**(12), p. 2007 (2022). DOI: 10.3390/math10122007

12. Pattanaik, P.C., Jena, S., Mishra, S.R., et al. “Illustration of convective boundary conditions on the Darcy–Forchheimer flow of nanofluid with the impact of chemical reaction”, *Symmetry*, **15**(9), p. 1728 (2023). DOI: 10.3390/sym15091728

13. Thenmozhi, D., Rao, M.E., Devi, R.R., et al. “Analysis of Jeffrey fluid on MHD flow with stretching–porous sheets of heat transfer system”, *Forces in Mechanics*, **11**, p. 100180 (2023). DOI: 10.1016/j.finmec.2023.100180

14. Das, K., Acharya, N., & Kundu, P.K. “Radiative flow of MHD Jeffrey fluid past a stretching sheet with surface slip and melting heat transfer”, *Alexandria Engineering Journal*, **54**(4), pp. 815-821 (2015). DOI: 10.1016/j.aej.2015.06.008

15. Sharma, B.K., Kumar, A., Gandhi, R., et al. “Entropy generation and thermal radiation analysis of EMHD Jeffrey nanofluid flow: Applications in solar energy”, *Nanomaterials,* **13**(3), p. 544 (2023). DOI: 10.3390/nano13030544

16. Alkasasbeh, H.T. “Exploring Three-Dimensional MHD Maxwell Hybrid Nanofluid Flow: A Computational Study on a Stretching sheet”, *Journal of Computational Applied Mechanics*, **55**(1), pp. 77-91. DOI: 10.22059/jcamech.2024.369138.916

17. You, X. and Wang, Y. “Series Solutions of Three-Dimensional Magnetohydrodynamic Hybrid Nanofluid Flow and Heat Transfer”, *Nanomaterials*, **14**(3), p. 316 (2024). DOI: 10.3390/nano14030316

18. Wang, R.T. and Wang, J.C. “Properties Related to the HLB Value of Hybrid Thermoelectric Nanofluids at Different Temperatures”, *Polymers,* **16**(4), p. 509 (2024). DOI: 10.3390/polym16040509

19. Mohamed, M.A.E.M., Meana-Fernández, A., González-Caballín, J. M., et al. “Numerical Study of a Heat Exchanger with a Rotating Tube Using Nanofluids under Transitional Flow”, *Processes,* **12**(1), p. 222 (2024). DOI: 10.3390/pr12010222

20. Mozafary, B., Akbar, A., Arani, A., Nooshabadi, G.A.S., et al. “Brownian motion models effect on the nanofluid fluid flow and heat transfer in the natural, mixed, and forced convection”, *Journal of Thermal Engineering,* **10**(1), pp. 88-100 (2024). DOI: 10.18186/thermal.1429382

21. Molana, M., Dogonchi, A.S., Armaghani, T., et al. “Investigation of hydrothermal behaviour of Fe3O4-H2O nanofluid natural convection in a novel shape of porous cavity subjected to magnetic field dependent (MFD) viscosity”, *Journal of Energy Storage,* **30**, p. 101395 (2020). DOI: 10.1016/j.est.2020.101395

22. Li, Z., Hussein, A.K., Younis, O., et al. “Natural convection and entropy generation of a nanofluid around a circular baffle inside an inclined square cavity under thermal radiation and magnetic field effects”, *International Communications in Heat and Mass Transfer*, **116**, p. 104650 (2020). DOI: 10.1016/j.icheatmasstransfer.2020.104650

23. Ali, F.H., Hamzah, H.K., Egab, K., et al. “Non-Newtonian nanofluid natural convection in a U-shaped cavity under magnetic field”, *International Journal of Mechanical Sciences*, **186**, p. 105887. DOI: 10.1016/j.ijmecsci.2020.105887

24. Selimefendigil, F. and Öztop, H.F. “Effects of conductive curved partition and magnetic field on natural convection and entropy generation in an inclined cavity filled with nanofluid”, *Physica A: Statistical Mechanics and its Applications*, **540**, p. 123004 (2020). DOI: 10.1016/j.physa.2019.123004

25. Afrand, M., Pordanjani, A.H., Aghakhani, S., et al. “Free convection and entropy generation of a nanofluid in a tilted triangular cavity exposed to a magnetic field with sinusoidal wall temperature distribution considering radiation effects”, *International Communications in Heat and Mass Transfer*, **112**, p. 104507 (2020). DOI: 10.1016/j.icheatmasstransfer.2020.104507

26. Cao, Y., Ayed, H., Jarad, F., et al. “MHD natural convection nanofluid flow in a heat exchanger: Effects of Brownian motion and thermophoresis for nanoparticles distribution”, *Case Studies in Thermal Engineering*, **28**, p. 101394 (2021). DOI: 10.1016/j.csite.2021.101394

27. Belhaj, S., & Ben-Beya, B. “Thermal performance analysis of hybrid nanofluid natural convection in a square cavity containing an elliptical obstacle under variable magnetic field”, *International Journal of Numerical Methods for Heat & Fluid Flow*, **32**(6), pp. 825-1860 (2022). DOI: 10.1108/HFF-04-2021-0300

28. Hymavathi, T., Mathews, J., and Kiran Kumar, R.V.M.S.S. “Heat transfer and inclined magnetic field effects on unsteady free convection flow of MoS2 water-based nanofluids over a porous stretching sheet”, *International Journal of Ambient Energy*, **43**(1), pp. 5855-5863(2022). DOI: 10.1080/01430750.2021.1995491

29. Cao, Y., Mansir, I.B., Singh, P.K., et al. “Heat transfer analysis on ferrofluid natural convection system with magnetic field”, *Ain Shams Engineering Journal*, **14**(9), p. 102122 (2023). DOI: 10.1016/j.asej.2023.102122

30. Rashid, U., Lu, D., and Iqbal, Q. “Nanoparticles impact natural convection nanofluid flow and heat transfer inside a square cavity with a fixed circular obstacle”, *Case Studies in Thermal Engineering*, **44**, p. 102829 (2023). DOI: 10.1016/j.csite.2023.102829

31. Selimefendigil, F., Chouikhi, H., and Oztop, H.F. “Natural convection and entropy generation of hybrid nanofluid in double annulus separated by a thin rotating partition under magnetic field”, *Journal of Magnetism and Magnetic Materials*, **582,** p. 170974 (2023). DOI: 10.1016/j.jmmm.2023.170974

32. Izadi, M., Alshuraiaan, B., Hajjar, A., et al. “Free convection of nanofluids in a porous sensible heat storage unit: Combined effect of time-periodic heating and external magnetic field”, *International Journal of Thermal Sciences*, **192**, p. 108404 (2023). DOI: 10.1016/j.ijthermalsci.2023.108404

33. Wang, D. and Hai, T. “Effect of the length and thickness of three constant temperature baffles on the natural convection heat transfer of nanofluid flow inside an enclosure affected by a magnetic field”, *Engineering Analysis with Boundary Elements*, **150**, pp. 70-83 (2023). DOI: 10.1016/j.enganabound.2023.01.038

34. Jafaripournimchahi, A., Shateri, A., Jalili, B., et al. “The effects of magnetic field and thermal radiation on the mixed convection of Al2O3-Cu/water hybrid nanofluid over a permeable vertical flat plate”, *Modern Physics Letters B*, **38**(25), p. *2450242* (2024). DOI: 10.1142/S0217984924502427

35. Jalili, B., Emad, M., Jalili, P., et al. “Thermal analysis of boundary layer nanofluid flow over the movable plate with internal heat generation, radiation, and viscous dissipation”, *Case Studies in Thermal Engineering*, **49**, p. 103203 (2023). DOI: 10.1016/j.csite.2023.103203

36. Jalili, B., Ganji, A.M., Shateri, A., et al. “Thermal analysis of Non-Newtonian visco-inelastic fluid MHD flow between rotating disks”, *Case Studies in Thermal Engineering,* **49** p. 10333 (2023). DOI: 10.1016/j.csite.2023.103333

37. Hussain, M., Ali, A., Ghaffar, A., et al. “Squeezing flow of Casson fluid between two circular plates under the impact of solar radiation”, *J. Therm. Anal. Calorim*, **147**, pp. 6959-6969 (2023). DOI: 10.1002/zamm.202200455

38. Jalili, P., Sharif Mousavi, S.M., Jalili, B., et al. “Thermal evaluation of MHD Jeffrey fluid flow in the presence of a heat source and chemical reaction”, *International Journal of Modern Physics B*, **38**(8), p. 2450113 (2024). DOI: 10.1142/S0217979224501133

39. Ahmadi Azar, A., Jalili, B., Jalili, P., et al. “Investigating the effect of structural changes of two stretching disks on the dynamics of the MHD model”, *Scientific Reports*, **13**(1), pp. 21833 (2023). DOI: 10.1038/s41598-023-48988-4

40. Jalili, B., Azar, A.A., Esmaeili, K., et al. “A novel approach to micropolar fluid flow between a non-porous disk and a porous disk with slip”, *Chinese Journal of Physics*, **87**, pp. 118-137 (2023). DOI: 10.1016/j.cjph.2023.11.023

41. Talebi, F., Mahmoudi, A.H., and Shahi, M. “Numerical study of mixed convection flows in a square lid-driven cavity utilising nanofluid”, *International Communications in Heat and Mass Transfer*, **37**(1), pp. 79-90 (2010). DOI: 10.1016/j.icheatmasstransfer.2009.08.013

42. El-Zahar, E.R., Rashad, A.M., Saad, W., et al. “Magneto-hybrid nanofluids flow via mixed convection past a radiative circular cylinder”. *Scientific Reports*, **10**(1), p. 10494 (2020). DOI: 10.1038/s41598-020-66918-6

43. Brinkman, H.C. “The viscosity of concentrated suspensions and solutions”, *The Journal of Chemical Physics*, **20**(4), pp. 571-571 (1952). DOI: 10.1063/1.1700493

44. Lele, S.K. “Compact finite difference schemes with spectral-like resolution”, *Journal of computational physics*, **103**(1), pp. 16-42 (1992). DOI: 10.1016/0021-9991(92)90324-R

45. Tian, Z.F. and Dai, S.Q. “High-order compact exponential finite difference methods for convection–diffusion type problems”, *Journal of Computational Physics*, **220**(2), pp. 952-974 (2007). DOI: 10.1016/j.jcp.2006.06.001

46. Spotz, W.F. “Accuracy and performance of numerical wall boundary conditions for steady, 2D, incompressible stream function vorticity”, *International Journal for Numerical Methods in Fluids*, **28**(4), pp. 737-757 (1998). DOI: 10.1002/(SICI)1097-0363

Sadia Irshad, Mathematics PhD Scholar at KFUEIT, researches Fluid Dynamics, focusing on computational modeling and simulation of fluid flow phenomena.

Dr. Shah Jahan is an Assistant Professor of Mathematics at Khwaja Fareed University of Engineering and Information Technology (KFUEIT), where he teaches and conducts research in Fluid Mechanics, Computational Fluid Dynamics, and related areas.

Dr Afraz Hussain Majeed is currently a postdoctoral research fellow at the School of Energy and Power Engineering, Jiangsu University, Zhenjiang, China. His research interests span various topics, including computational techniques, Newtonian and non-Newtonian fluid dynamics, fluid forces, heat and mass transfer, and cavity flows. He specializes in solving highly nonlinear differential equations using numerical methods such as the finite difference Method (FDM), Finite Element Method (FEM), and Finite Volume Method (FVM). His primary focus is on applied mathematics, emphasizing real-world engineering problems and solutions.

Dr. Muhammad Ahsan is an accomplished academic professional currently serving as an Assistant Professor in the Mathematics Department at the Virtual University of Pakistan, where he has been a faculty member since February 2005. He earned his Ph.D. in Mathematics from the University of Management and Technology, Lahore, in December 2021, with a focus on computational mathematics. His research interests include graph theory, particularly the edge metric dimension of planar graphs, and he has also contributed to research in multicriteria decision-making techniques and applied mathematics.

Dr. Imran Siddique is currently working as a Full Professor at the University of Sargodha, Sargodha, Pakistan. He is a reviewer of several well-known SCI and ESCI journals. His research interests include artificial intelligence, fuzzy algebra, fuzzy fluid dynamics, fluid mechanics, lubrication theory, soliton theory, and fractional Calculus.

Figure 1. Flowchart of the Finite Difference Method Algorithm Steps for Analyzing Hybrid Nanofluid Behavior.

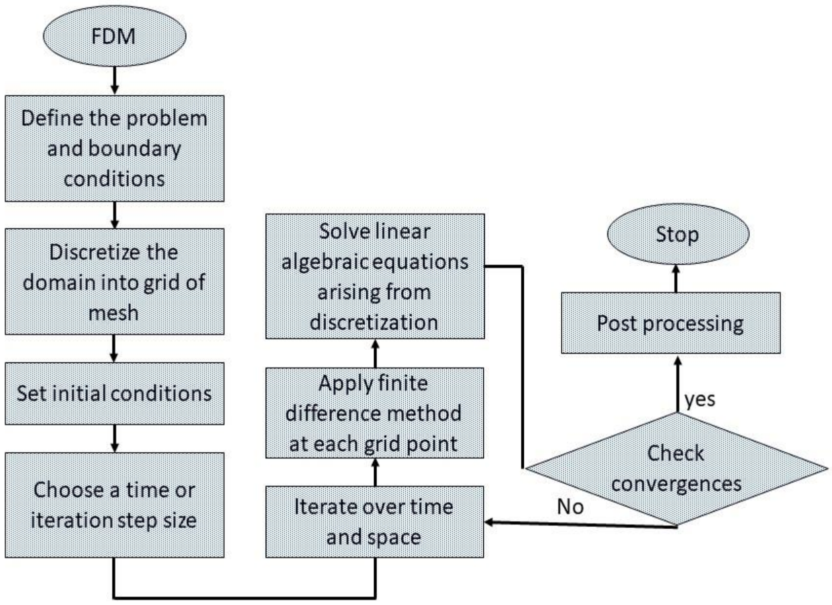


Figure 2. Flow configuration of the problem.

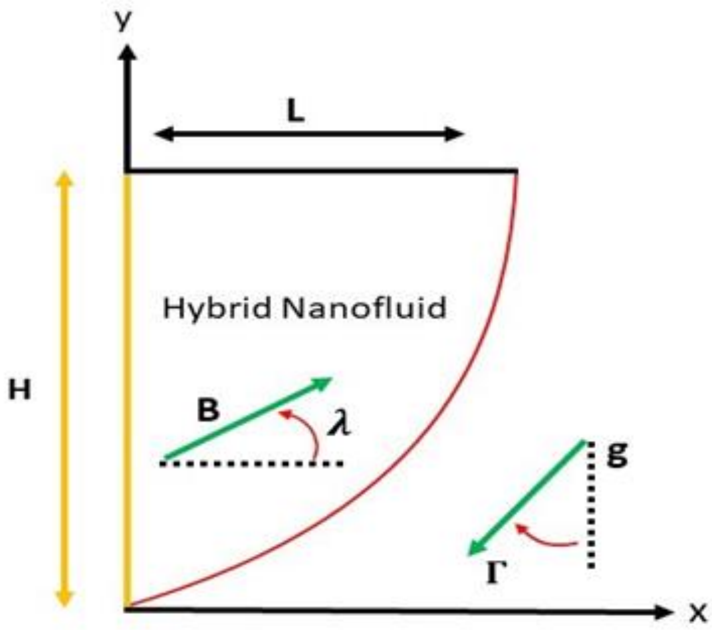


Figure 3 (a): Streamlines for different Hartmann values at Ra=10^4 and magnetic field inclinations.

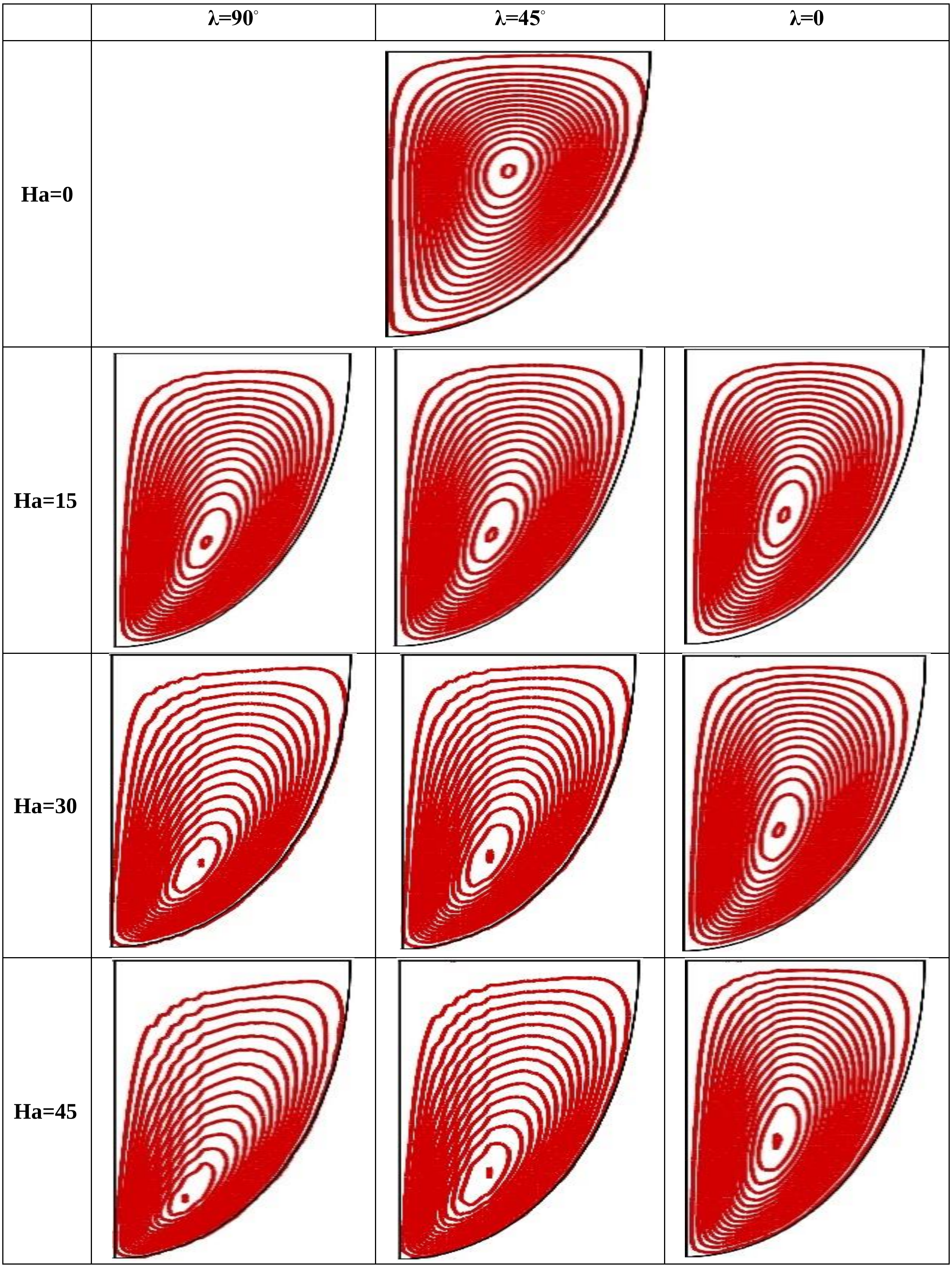


Figure 3 (b): Isotherms for different Hartmann values in Ra=10^4 and magnetic field inclinations.

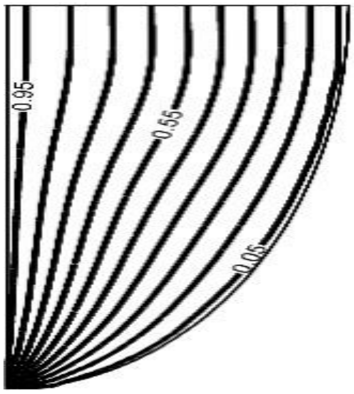
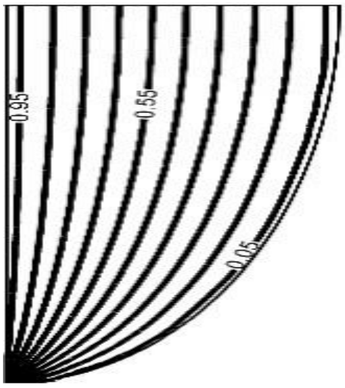
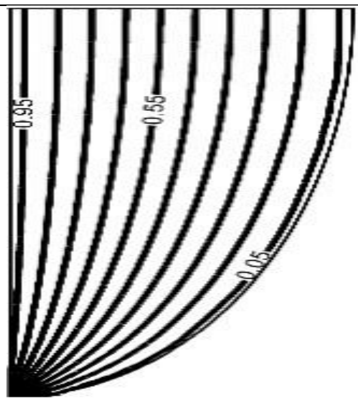
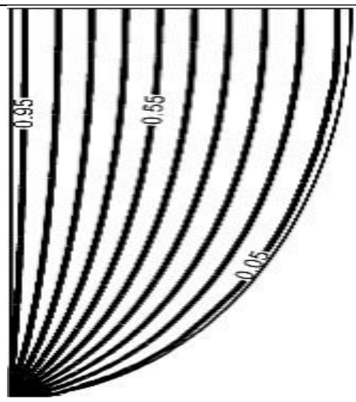
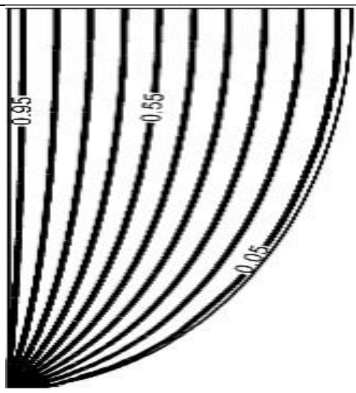
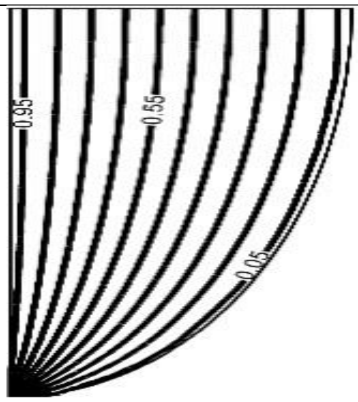
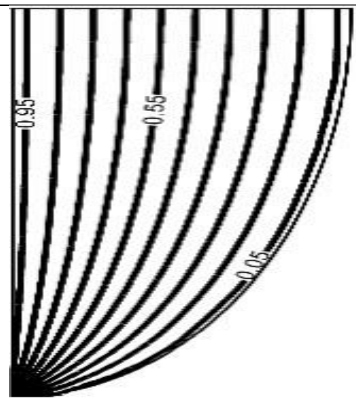
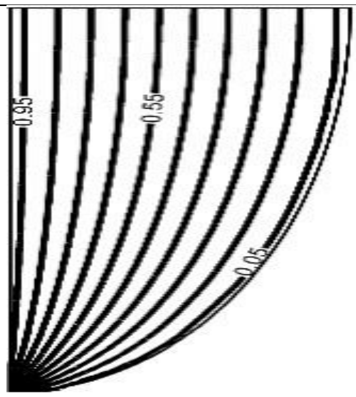
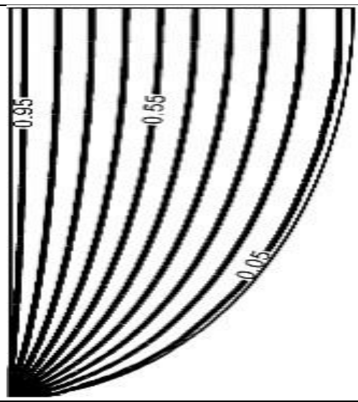
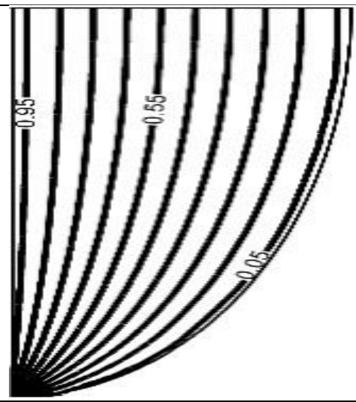
	λ=90°	**λ=45°**	**λ=0**
Ha=0			
Ha=15			
Ha=30			
Ha=45			

Figure 4 (a): Streamlines for different Hartmann values in Ra=10^5 and magnetic field inclinations.

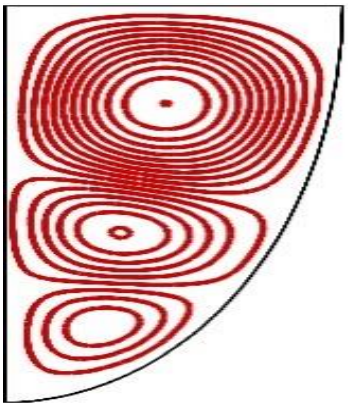
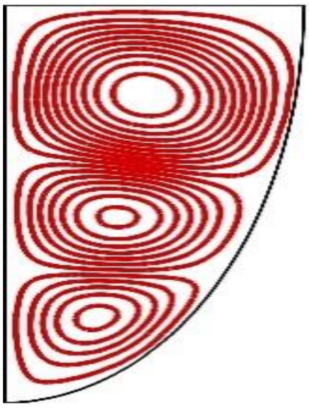
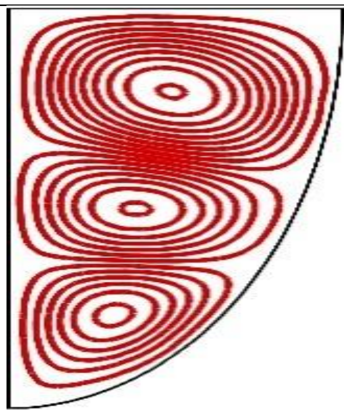
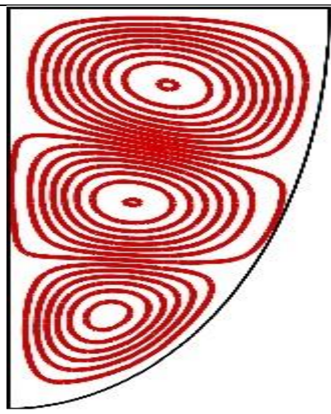
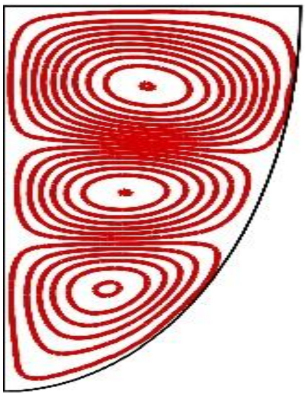
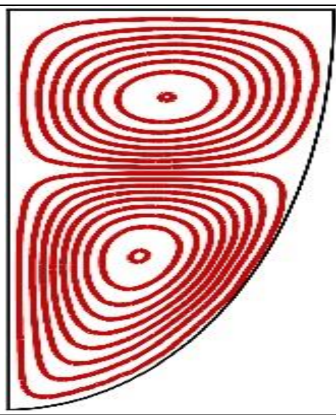
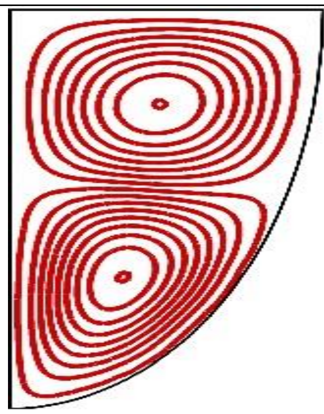
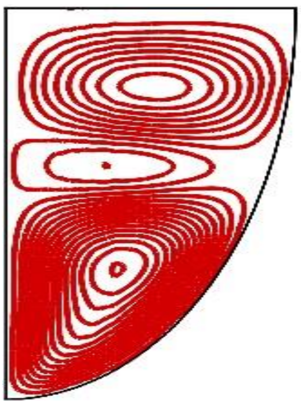
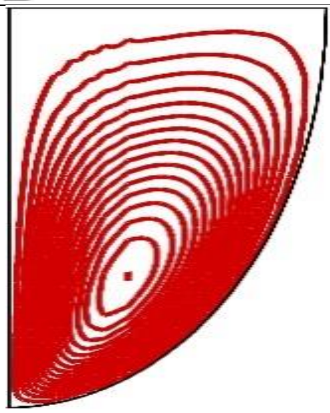
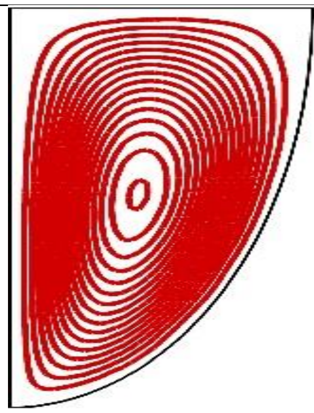
	λ=90°	**λ=45°**	**λ=0**
Ha=0			
Ha=15			
Ha=30			
Ha=45			

Figure 4 (b): Isotherms for different Hartmann values in Ra=10^5 and magnetic field inclinations**.**

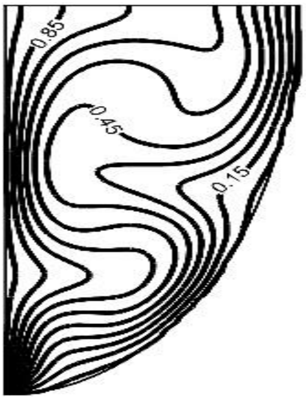
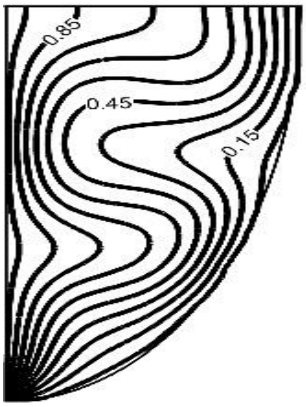
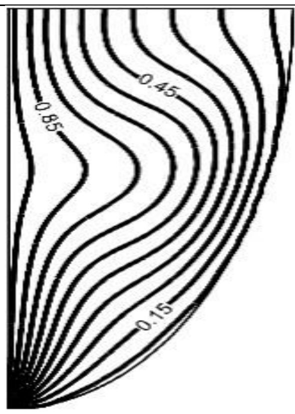
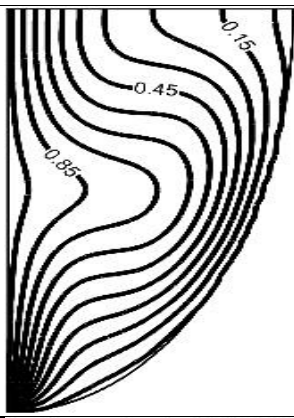
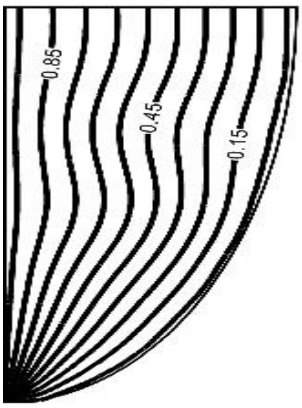
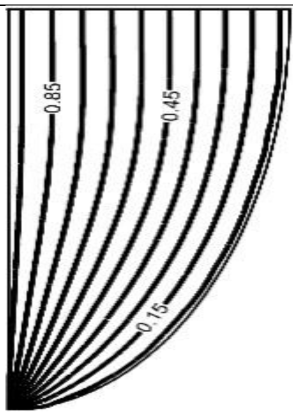
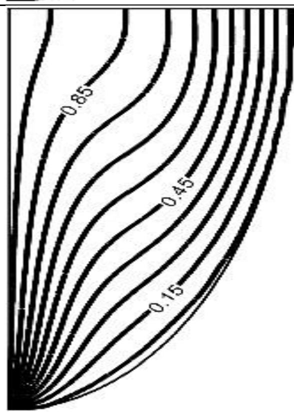
	λ=90°	**λ=45°**	**λ=0**
Ha=0			
Ha=15			
Ha=30			
Ha=45			

Figure 5 (a): Streamlines for different Hartmann values in Ra=10^6 and magnetic field inclinations.

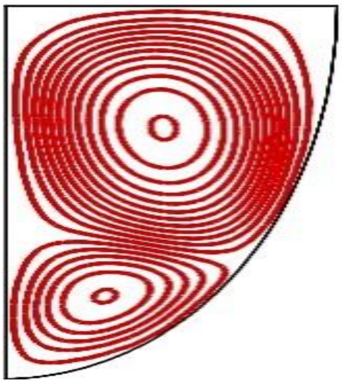
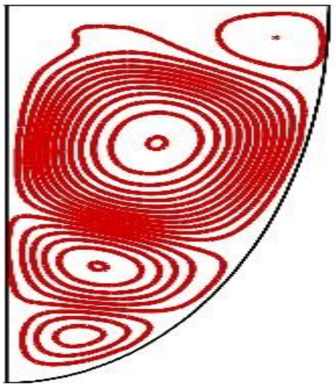
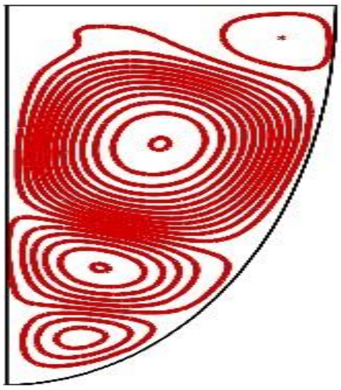
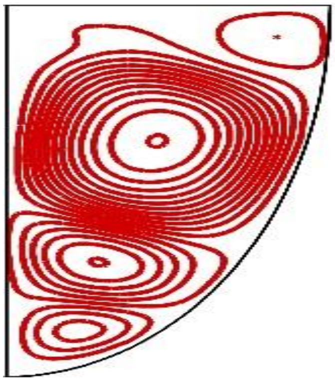
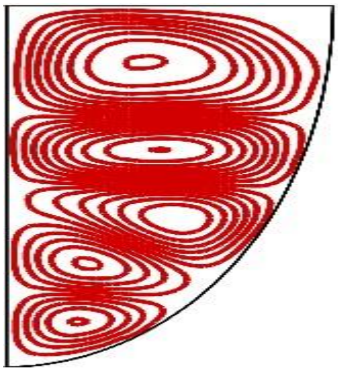
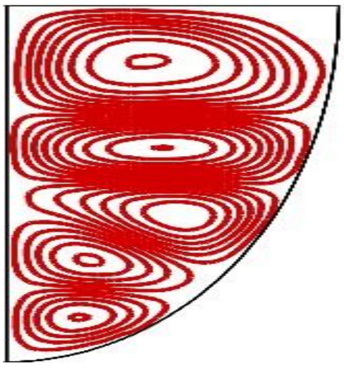
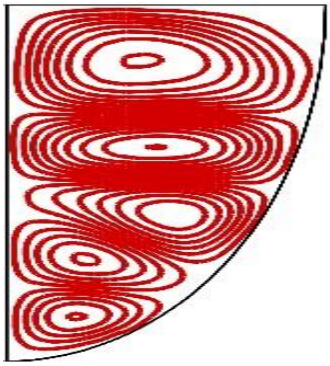
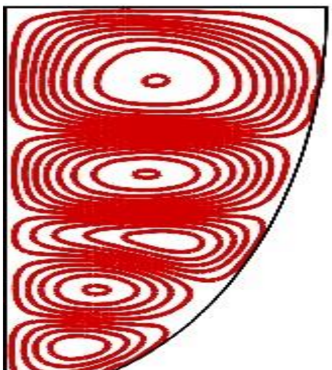
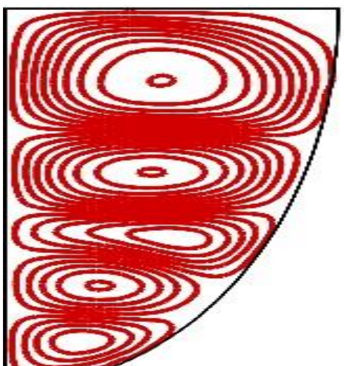
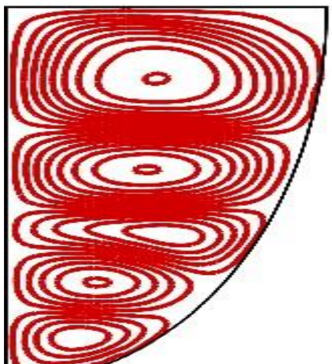
	λ=90°	**λ=45°**	**λ=0**
Ha=0			
Ha=15			
Ha=30			
Ha=45			

Figure 5 (b): Isotherms for different Hartmann values in Ra=10^6 and magnetic field inclinations.

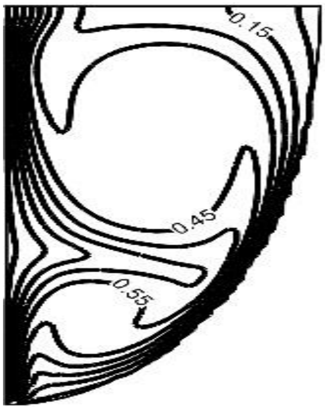
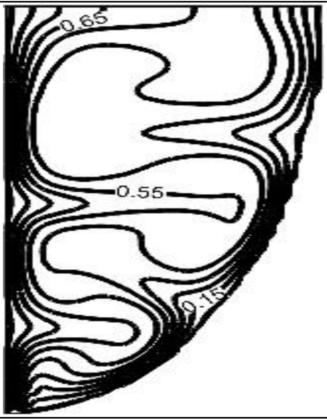
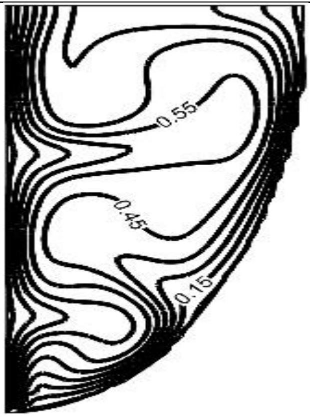
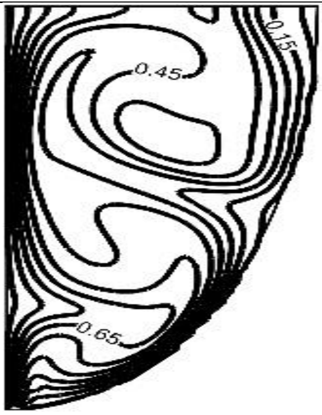
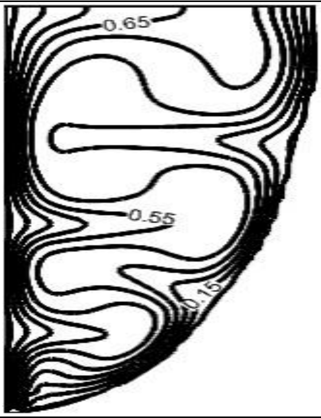
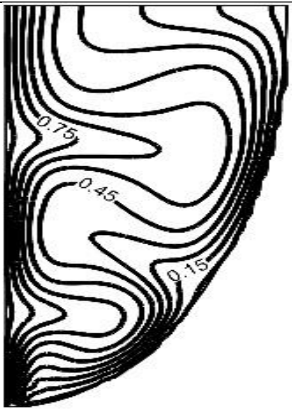
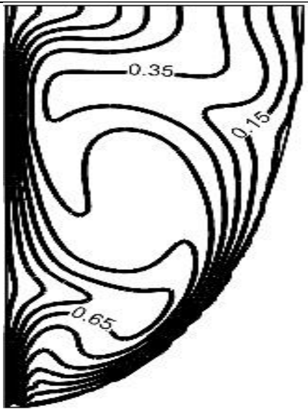
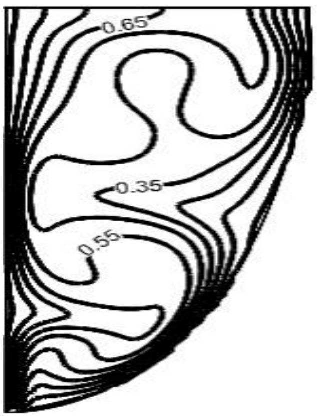
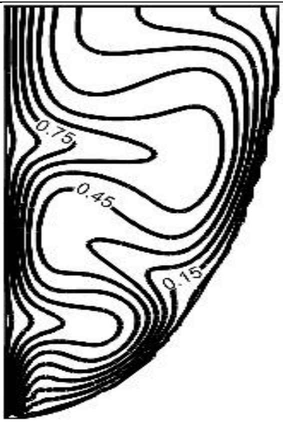
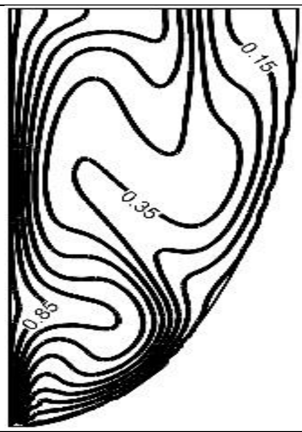
	λ=90°	**λ=45°**	**λ=0**
Ha=0			
Ha=15			
Ha=30			
Ha=45			

Figure 6(a): Streamlines for different cavity angles and Rayleigh values in a linear wall distribution of temperatures.

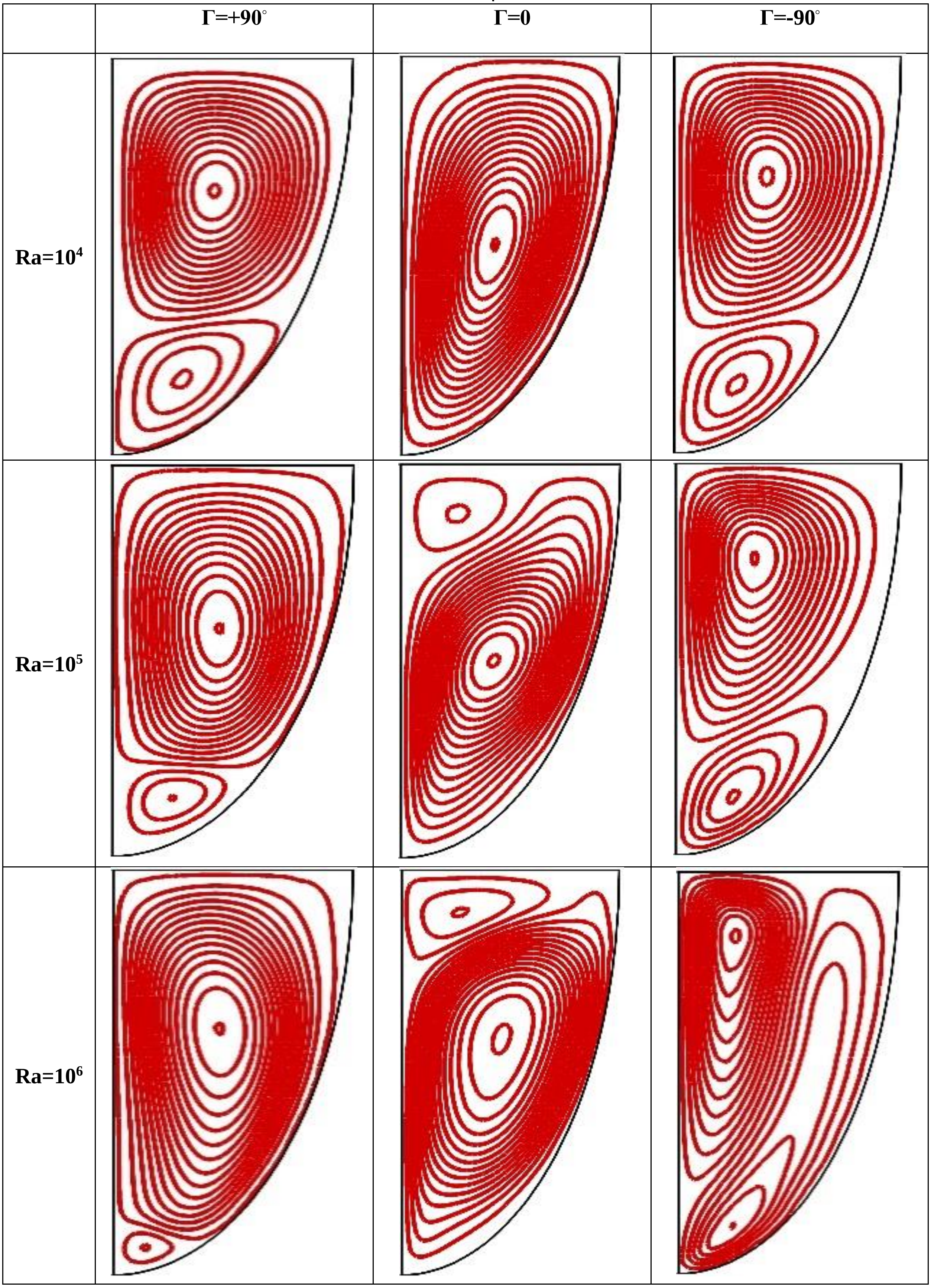


Figure 6 (b): Isotherms for different cavity angles and Rayleigh values in a linear wall distribution of temperatures.

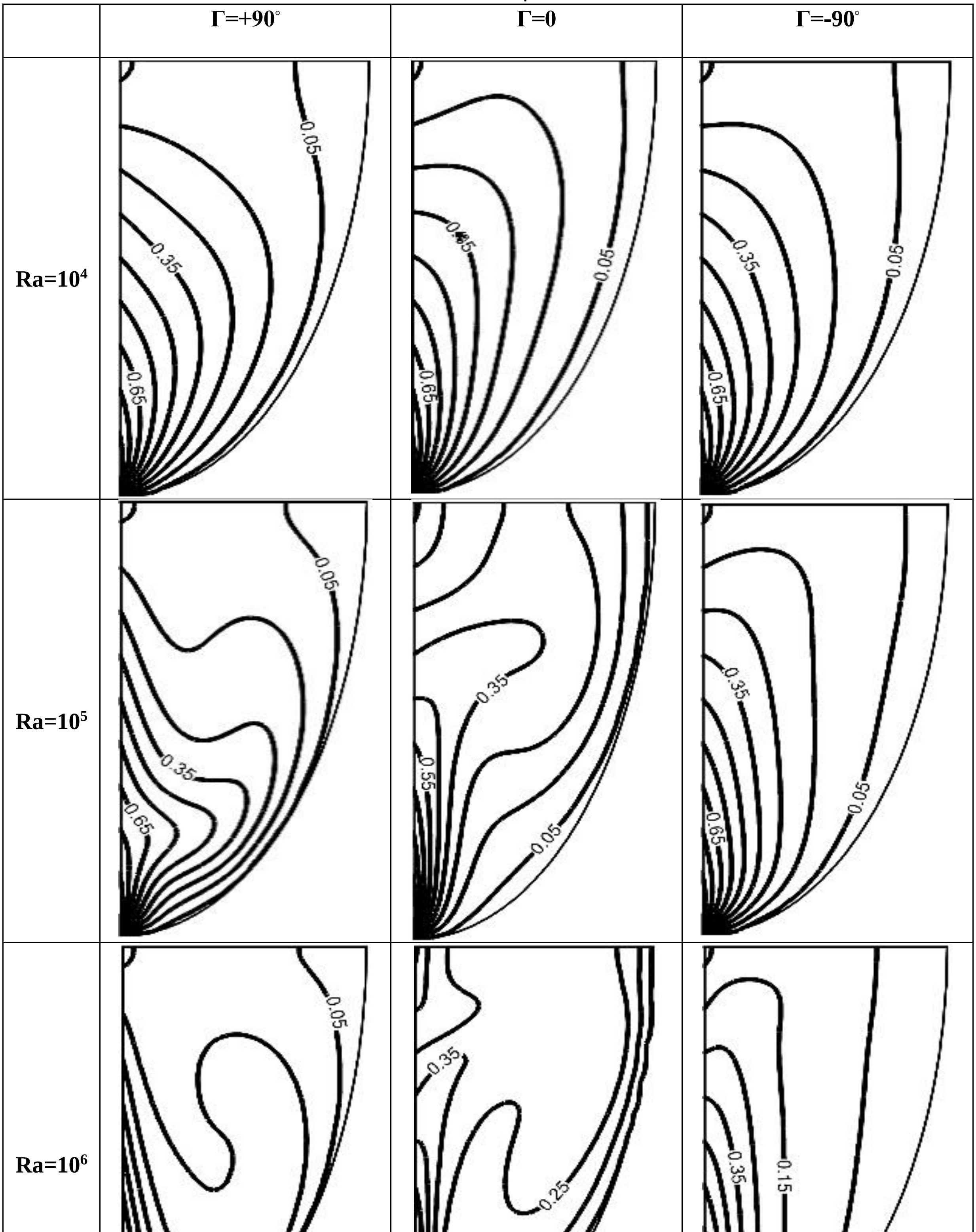


Figure 7 (a): Streamlines for different cavity angles and Rayleigh values in a sinusoidal wall distribution of temperatures.

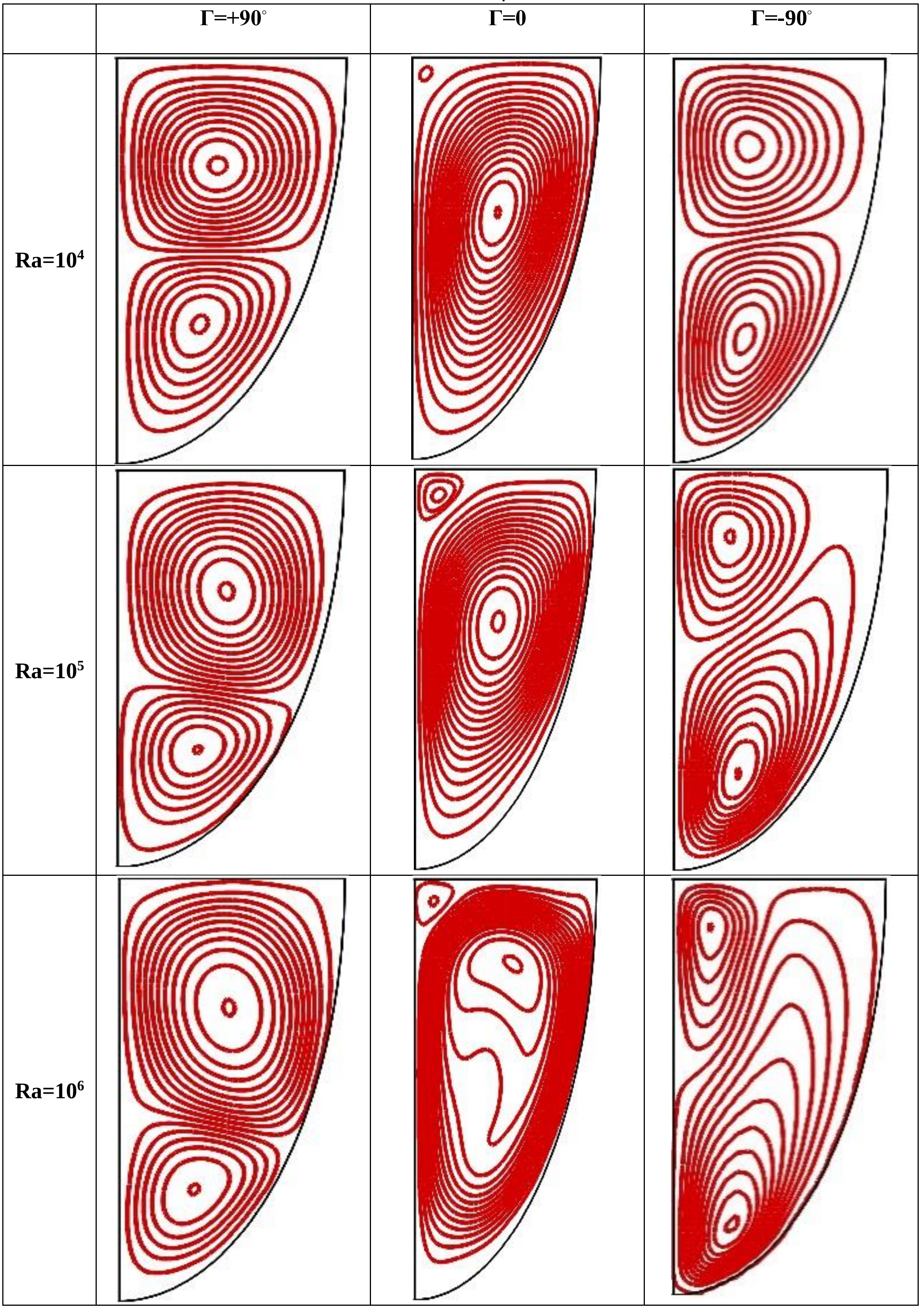


Figure 7 (b): Isotherms for different cavity angles and Rayleigh values in a sinusoidal wall distribution of temperatures.

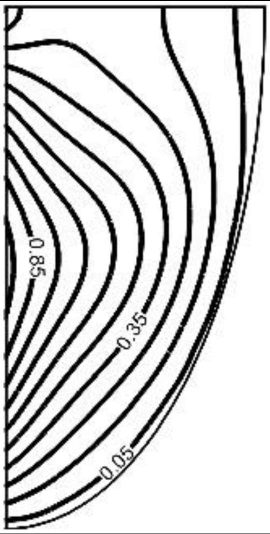
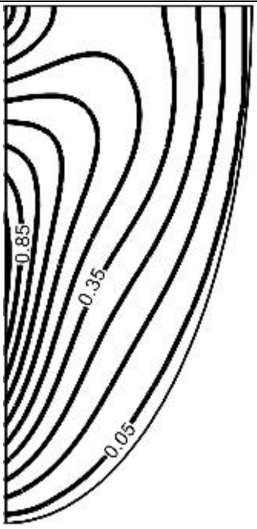
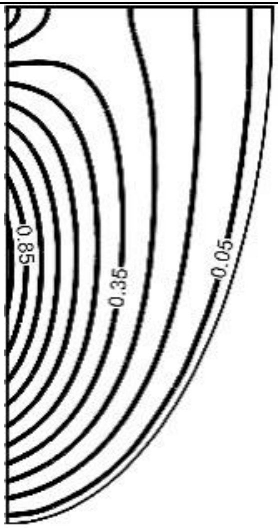
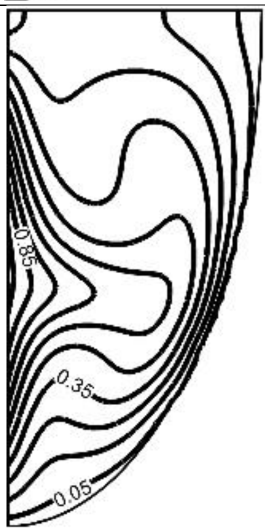
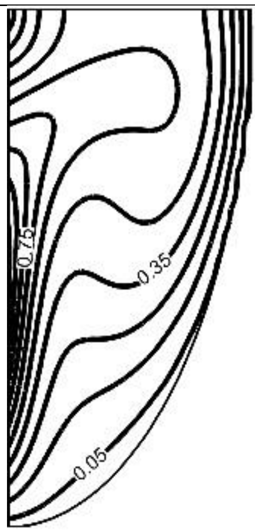
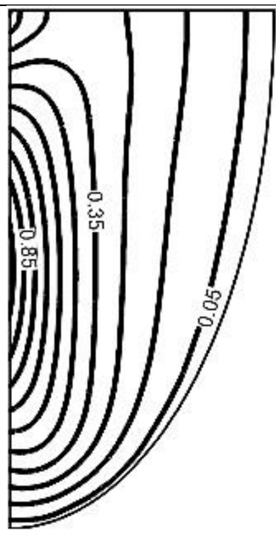
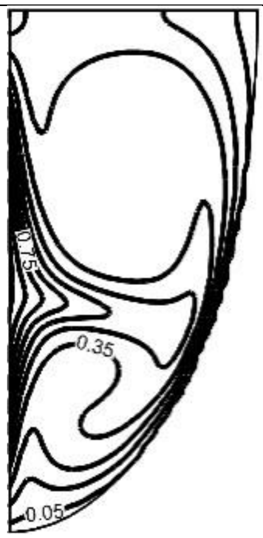
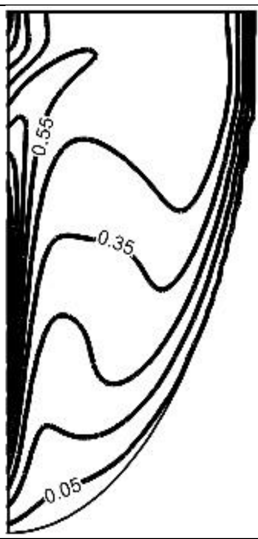
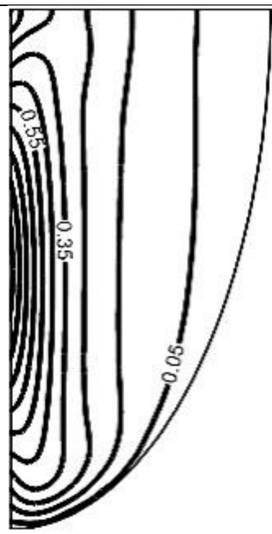
	Γ=+90°	**Γ=0**	**Γ=-90°**
Ra=10^4			
Ra=10^5			
Ra=10^6			

Figure 8 (a): Streamlines for different cavity angles and Rayleigh values in a constant wall distribution of temperatures.

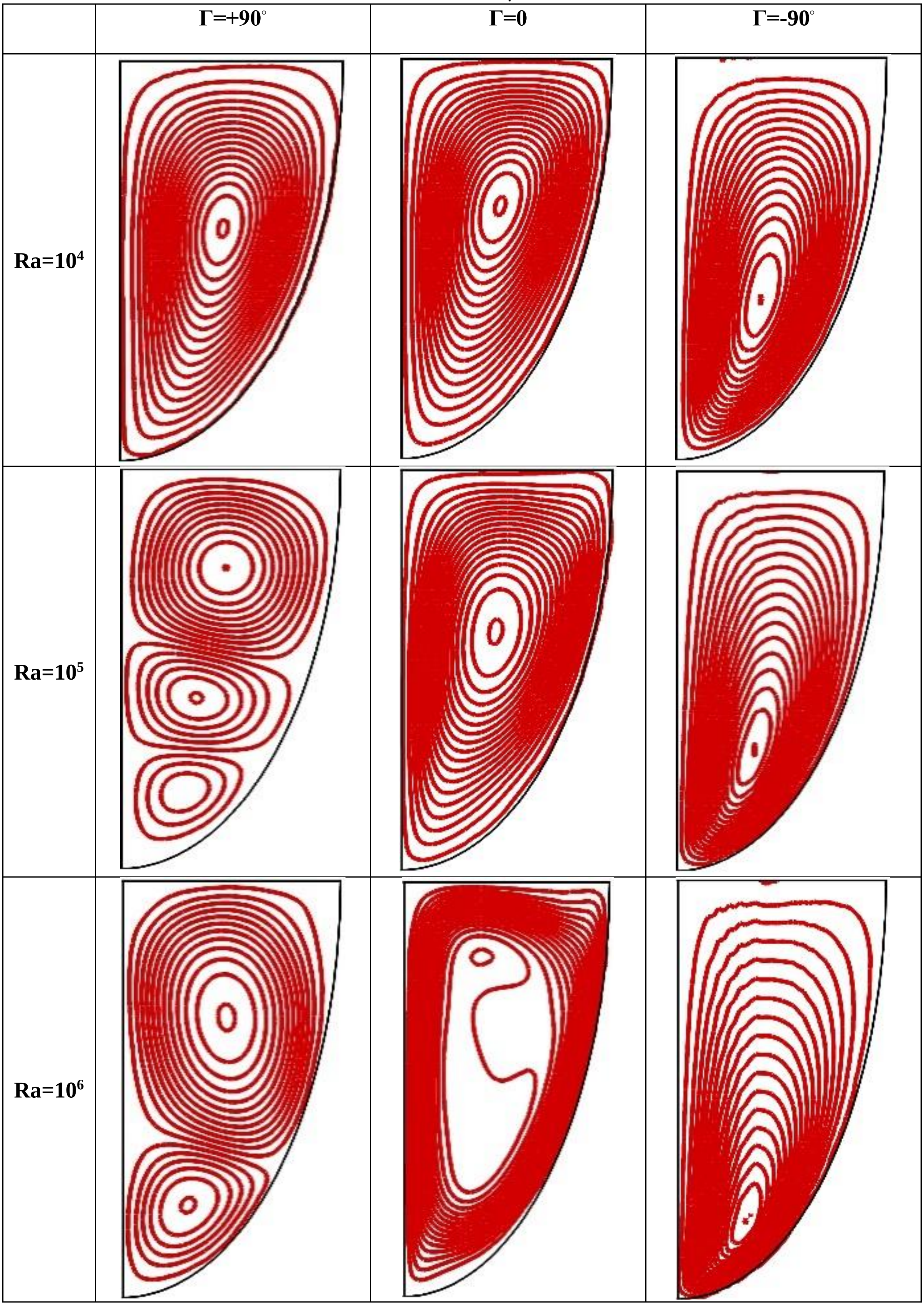


Figure 8 (b): Isotherms for different cavity angles and Rayleigh values in a constant wall distribution of temperatures.

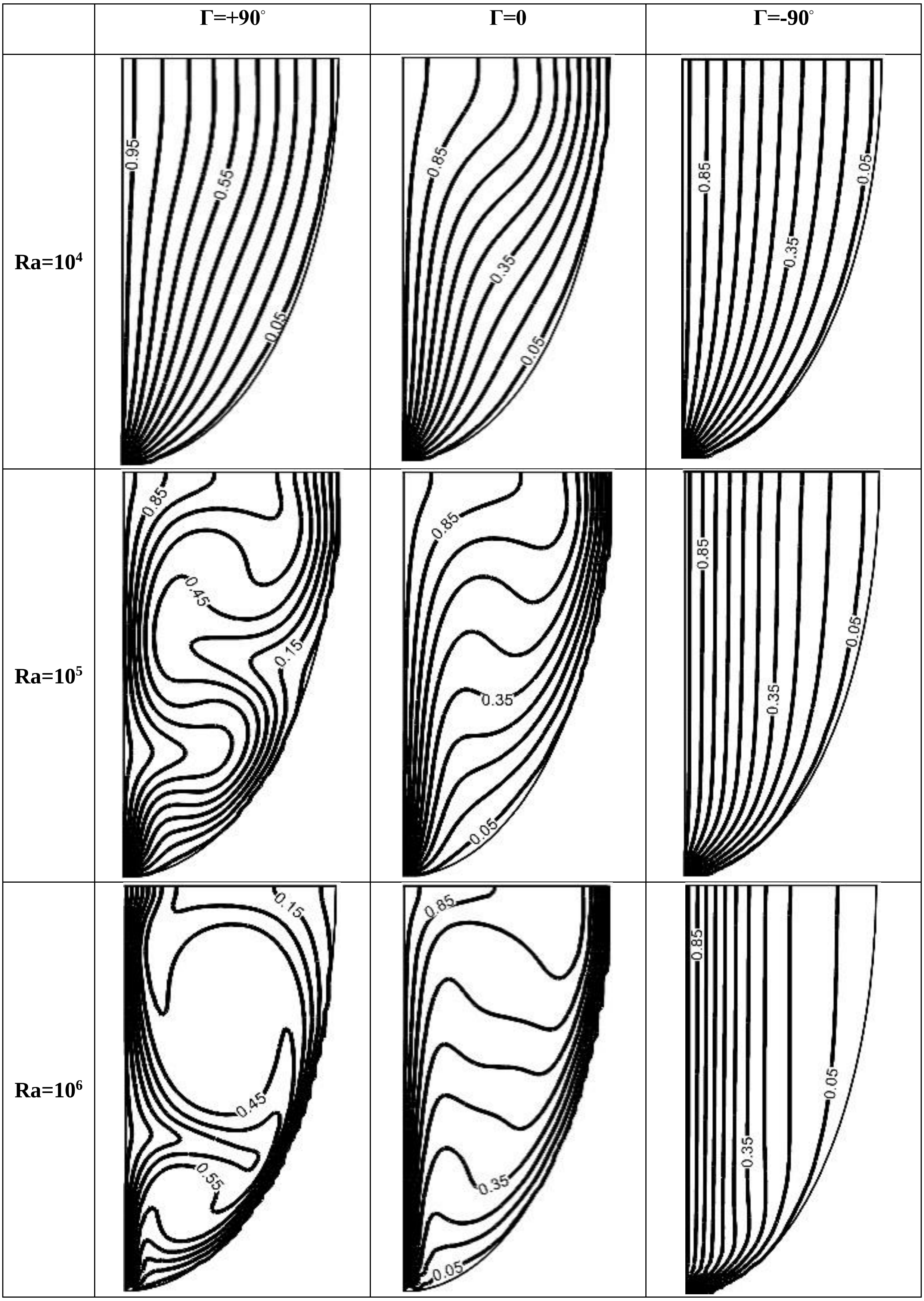


Figure 9: The average Nusselt number, Rayleigh values, and Hartmann values calculated for Γ=+90◦, ϕ=0.04, and constant heating.

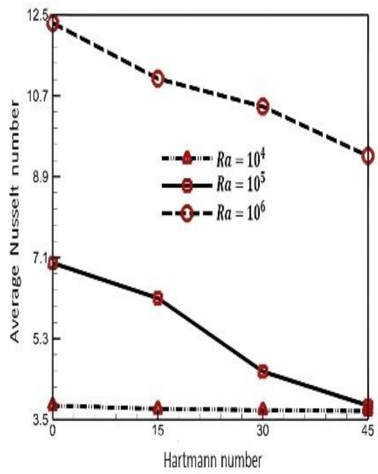
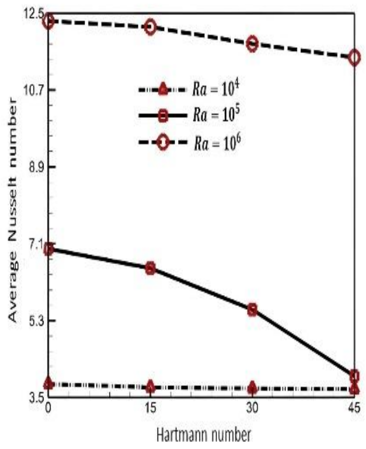
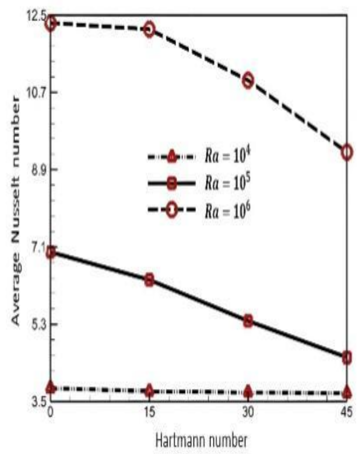
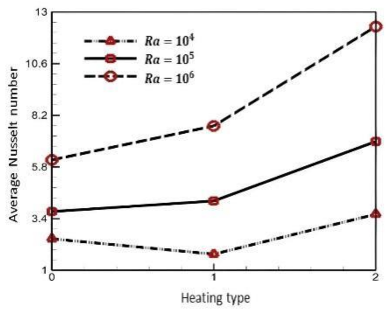
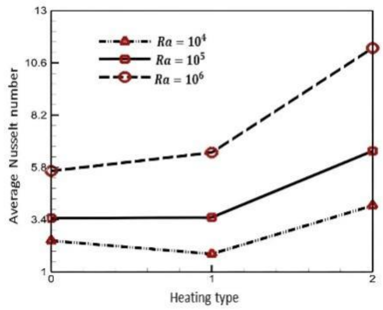
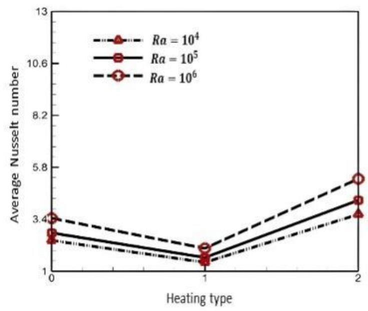
λ=90°	**λ=45°**	**λ=0**
		

Figure 10: The average Nusselt number is calculated for different cavity angles, heating types, and Rayleigh values at ϕ=0.04, Ha=0.

Γ=+90°	**Γ=0**	**Γ=-90°**
		

0: linear heating, 1: sinusoidal heating, 2: constant heating

Table 1. Thermophysical properties of the Cu-Al_2O_3/water (see [47]).

Physical characteristics	Water	Cu	Al_2O_3
$C_p(J/kgK)$	4179	385	765
$k(W/mK)$	0.613	401	40.0
$\alpha \times 10^{-7}(m^2/s)$	1.47	1163.1	131.7
$\beta \times 10^{-5}(K^{-1})$	21	1.67	0.85
$\rho(kg/m^3)$	997.1	8933	3970
$\mu \times 10^{-4}(kg/ms)$	8.9	-	-