Optimization of multi-objective reliability redundancy allocation problem with non-homogeneous components using mixed redundancy strategy under uncertainty conditions

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Abstract.

Maximizing system dependability, lowering overall system costs, and accounting for system constraints are critical to preventing the negative outcomes that result from the failure of today's modern industries' whole systems. In order to optimize the reliability-redundancy allocation problem (RRAP) in a series-parallel system and reduce overall cost, we present a novel multi-objective mixed integer non-linear model in this study. The majority of research on redundancy-aware parallel processing (RRAP) assumes homogeneous components, predetermined component dependability, and active or cold-standby redundancy techniques in each subsystem. We will use the NSGA-II strategy to tackle the given multi-objective mixed-integer non-linear problem. Each of the aforementioned assumptions acts as a barrier, preventing the solution regions from increasing. The suggested multi-objective approach is made up of many diverse components, the dependability of which is not immediately apparent. Furthermore, hybrid techniques (active and cold-standby redundancy) can be employed in any subsystem. The trustworthiness of subsystem components is uncertain and considered a decision variable. The proposed optimization problem is NP-hard, requiring accurate algorithms. A multi-objective model with non-homogeneous, cold standby components will be evaluated using a well-known problem-testing approach. The recently provided mathematical model outperforms previous studies in terms of dependability values and total system costs, demonstrating its high efficiency.

Keywords: Reliability optimization, Redundancy, Reliability redundancy allocation problem (RRAP), Cold standby, Warm standby, Hot standby, Perfect switching, Imperfect switching

1. Introduction

Engineers and managers play a crucial role in designing and constructing systems, ensuring high reliability in various industries like aerospace, defense, chemical, and automotive through component reliability, redundant components, and inter-changeable components. "redundancy allocation problem" (RAP) and "reliability-redundancy allocation problem" (RRAP), The RAP aims to maximize system reliability by determining the optimal number of redundant components in each subsystem, considering factors like reliability, cost, and weight. The RRAP is a more complex problem in the reliability optimization area that aims to find the best structure with the highest reliability by calculating component reliability and redundancy levels for each subsystem simultaneously. In other words, in RRAPs, component reliability is not given but treated as a design variable while component cost, weight, volume, etc.

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Active and standby are the two basic strategies for increasing system reliability in RRAP and RAP. In the active strategy, redundant components must work simultaneously from time zero, whether or not they are required to operate at all. The redundant components are only required to back up failed primary components in the standby strategy, also known as the backup strategy. For each standby component, there are three different types of standby strategies:

1. The hot standby: Each associated hot standby component, like active components, runs in parallel with the whole subsystem, ensuring that the subsystem has the same information. As a result, the mathematical formulations for both the hot standby and active techniques are the same.
2. The warm standby: A warm standby component is a backup to a primary component, periodically mirroring information from the primary component.
3. Cold standby: Cold standby systems use redundant components only when needed, resulting in higher downtime compared to hot standby systems. They are often used for non-critical applications or data with minimal changes.

Researchers that study on reliability optimization problems often use an active redundancy strategy and a variety of methods to solve them, the most famous of which are exact and meta-heuristic methods. Exact optimization techniques used in such studies include dynamic programming by Fyffe et al. [1], Lagrange multipliers proposed by Misra [2], integer programming proposed by Gen et al. [3],The branch and bound approach by Way and Hwang [4], the branch and cut approach by Caserta and Kuo [5], implicit enumeration by Prasad and Kuo [6], and dynamic programming by Yeh and Hsieh [7] ,are the most important exact solution approaches. RRAP and RAP are too difficult to solve using exact optimization methods, especially when the problem size is large, since they belong to the NP-hard (non-deterministic polynomial time hard) class of optimization problems proposed by Ha and Kuo [8]. For this reason, numerous meta-heuristic algorithms such as artificial neural networks (ANN) (Habib, Alsieidi, and Youssef [9]; Chamberi et al. [10]), genetic algorithms (GA) (Abouei Ardakan and Rezvan [11]; Abouei Ardakan and Hamadani [12]; Zoulfaghari et al. [13]), artificial immune systems (AIS) (Chen [14]; Hsieh and You [15]), tabu search (TS) (Kulturel-Konak et al. [16]), particle swarm optimization (PSO) (Zhang et al. [17]; Garg et al. [18]) It has been widely studied to solve RRAP and RAP problems.

However, because it is difficult to assess the reliability of the RRAP and RAP cold standby subsystems, very few studies have focused on the development of cold standby algorithms. Abouei Ardakan and Rezvan [11] proposed the RRAP by using a cold standby strategy and solved it with the Non dominate sorting genetic algorithm-II (NSGA-II). Yeh and Hsieh [7] used the ant colony algorithm to solve RRAP problems and compared the results with those obtained from the other meta-heuristic algorithm. Tavakkoli-Moghadam et al. [19] presented a genetic algorithm and a simulated annealing algorithm, respectively, to solve RAP problems Coit [20] considered the allocation problem with using an integer programming solution when the system contains only cold standby redundancy. He assumed imperfect switching and used the K-Erlang distribution for time to failure (TTF) of components. Coit [21] considered the problem of choosing an active or cold standby redundancy strategy for each subsystem component as a decision variable and solved it using the integer programming method. Coit [21] proposed the relationship between the cold standby strategy and the active redundancy strategy in a specific subsystem and showed that for each jth component, the maximum level of reliability of cold standby is greater than or equal to the reliability of the active component. Tavakkoli-Moghaddam and Safari [22] presented a new method with mixed components and a redundancy strategy with a cold-standby component.

For the first time Ardakan and Hamadani [23] developed a modified. genetic algorithm to solve a mixed-integer non-linear optimization model for RRAP to use a cold-standby strategy. They proposed a novel redundancy technique termed "mixed strategy" in another study, in which certain components may well be active and others were on standby. Kim and Kim [24] developed an RRAP with a redundancy strategy selection (active or cold-standby). They also calculated the reliability of a cold standby subsystem using a continuous time Markov chain. Abouei Ardakan and Rezvan [11] developed a bi-objective RRAP with cold-standby strategy that used a multi-objective evolutionary algorithm (NSGA-II). Gholinejad and Zeinal Hamadani [25] introduced a new method for RAP in which the objective function is to maximize reliability by considering a mixed redundancy strategy so that active and cold-standby strategies can occur simultaneously in each component of a subsystem. They used a genetic algorithm to solve it.

To solve RRAP and RAP problems, some researchers used a combination of meta-heuristic algorithms. TS-differential evolution (DE) (Liu and Qin [26]), estimation technique (IT2FLS-PSO) Interval type-2 fuzzy logic set particle swarm optimization (Drik. [27]), (DE-HS) Differential evolution -harmony search (Wang and Li [28]), (SA-

GA) Simulated annealing-genetic algorithm (Dalanezi Mori, Fiori de Castro, and Lucchesi Cavalca [29]), (CS-GA) Cuchoo search-genetic algorithm (Kanagaraj, Ponnambalam, and Jawahar [30]), (Haiying and Yubao [31]).

For solving multi-objective function RRAP and RAP in the form of multi-state, Salazar et al. [32] solved three types of reliability optimization problems: finding the optimal number of redundant components (component reliability problem), finding the reliability of components, and determining both their redundancy and reliability (component reliability and redundancy allocation) by NSGA-II. They showed that the results of their proposed algorithm coincide with the selection optimization redundancy allocation problem (SORAP). Li et al. [33] proposed a new two-phase approach for the multi-objective redundancy allocation problem. They first, using the NSGA algorithm, find the para-optimal set. Then, they applied a self-organizing map (SOM) to classify those para-optimal solutions into several clusters, and finally, by using data envelopment analysis (DEA), they determined the final solution to the problem. s. Attar et al. [34] proposed RRAP, in which they considered system components as active and non-repairable and modeled them in the form of multi-objective functions. Kulturel-Konak et al. [16] solved the problem by using the Tabu Search method with three objective functions: maximize reliability, minimize the cost of the system, and minimize the weight of the system. Safari [35] considered system reliability and cost as objective functions and solved the model presented by Coit [21] using the NSGA-II algorithm. Salmasnia et al. [36] considered RAP with multi-objective functions as maximizing the system reliability estimate, minimizing the overall system cost, minimizing system reliability variance estimate, and minimizing overall system cost variance. Taboada and Coit [37] considered RAP in the form of a multi-objective problem with three objective functions: maximizing system reliability, minimizing system cost, and weight. To solve it, they proposed an algorithm called multi-objective evaluation algorithm (MOEA). Eshraghniaye Jahromi and Feizabadi [38] considered RAP as a multi-objective problem to maximize system reliability and minimize the cost of the system and assumed that all components of a subsystem are non-homogeneous. They used the NSGA-II algorithm to solve it. Garg and Sharma [39] used RRAP as a multi-objective problem with objective functions of reliability and cost. They applied the active strategy to the components and used the particle swarm optimization (PSO) meta-heuristic algorithm to solve the problem. Abouei Ardakan et al. [40] developed a multi-objective model in which system cost and reliability were considered as objective functions. They used the NSGA-II algorithm to solve it. Chambari et al. [41] considered reliability and cost of the system as objective functions and used a meta-heuristic algorithm (NSGA-II) and multi-objective particle swarm optimization (MOPSO) to solve the problem. Liang and Lo [42] considered the RAP problem in order to maximize system reliability while minimizing system cost or weight, and they used a modified variable neighborhood search (VNS) algorithm. Coit and Konak [43] proposed a new model for the RAP to maximize the reliability of each individual subsystem simultaneously. They used a multiple-weighted objective based on a transformation of the problem into a single objective problem. R. Soltani et al. [44] considered interval programming for the redundancy allocation with choices of redundancy strategy and component type under uncertainty Wang et al. [45] considered RAP as a multi-objective optimization problem to maximize system reliability and minimize the cost of the system. They used NSGA-II to solve the model.

A more recent study considered different model and algorithm for RRAP as Ramezani Dobani et al. [46] proposed for the first time CM in a RRAP with the active redundancy strategy considered as the predetermined one for all the subsystems. Mahdavi-Nasab et al. [47] used water cycle algorithm for solving RRAP. Yeh et al. [48] General RRAP, is proposed to extend the series-parallel structure or bridge network to a more general network structure a new algorithm called the binary addition tree algorithm small sampling tri-objective orthogonal array (BAT-SS3OA). They used the simplified swarm optimization (SSO) to update solution, the small-sampling tri-objective orthogonal array (SS30A). Peykany et al. [49] considered a novel mathematical approach for the fuzzy multi-period, multi-objective portfolio optimization problem under uncertainty. Mekawy [50] proposed a novel method for solving multi-objective linear fractional programming problems under uncertainty. Nath and Muhuri [51] formulated the many objective RRAP (many objective reliability redundancy allocation problem (MaORRAP)) and for solve the used non-dominated sorting genetic algorithm-III (NSGA-III). Mellal et al. [52] considered RRAP with cold-standby strategy and used enhanced nest cuckoo optimization algorithm (ENCOA) for solve them. Yeh et al. [53] used a hybrid algorithm (SEB) (simplified swarm optimization (SSO), elite selection (ES) and boundary search (BS) called the SEB) solving cold-standby RRAP. Najmi et al. [54] used a parallel stochastic fractal search algorithm for mathematical model of RRAP with heterogeneous components. Karamasa et al. [55] considered a single-valued neutrosophic analytic hierarchy process (AHP) based on multi-objective optimization on the basis of ratio analysis plus a full multiplicative form (multi-objective optimization on the basis of ratio analysis plus a full multiplicative form (MULTIMOORA)) to rank the training aircraft as the alternatives. Rasoulzadeh et al. [56] considered a multi-objective approach based on Markowitz. Wang et al. [57] used Bare-bones multi-objective particle swarm (BB-MOPS). Ouyang et al. [58] proposed an improved particle swarm optimization algorithm (PSO) for RRAP with CM and mixed redundancy strategy. Wei et al. [59] considered RRAP as a bi-objective and used a new simplified swarm optimization (SSO) with a penalty function. Mahdavi-Nasab et al. [60] Considered RRAP with mixed redundancy strategy and is developed new model based on a continuous-time Markov chain model and used water cycle algorithm for solve them.

The problem's limitations include homogeneity in components, cold redundancy, weight, cost, volume, and arrangement. Removing homogeneity could improve system reliability and mathematical model. As a result, three new redundancy strategies are introduced in this article:

Strategy 1: Non-homogenous of subsystem components.

Strategy 2: The problem model is considered to be a multi-objective problem by considering the reliability and cost of the system as objective functions and solving the model by using the NSGA-II algorithm.

Strategy 3: The reliability of the system is considered to be uncertain.

Previous research has assumed homogeneous, predefined system components, limiting problem model expansion and optimal solution improvement. However, none of the presented papers have considered system components with uncertain reliability, non-homogeneous, and mixed strategies simultaneously.

Among the research gaps in previous articles is that none of the articles presented so far have simultaneously mentioned the following: 1- Heterogeneous components 2- Uncertainty of component reliability 3- Mixed strategies 4- Multiple key objective functions 5 - Series-parallel components 6- Use of Gamma distribution 7- redundancy strategies in each subsystem are considered cold-standby or active, which are considered in this article as a combination of all of them, and a new mathematical model is presented. In most research for system design, the components were considered homogenous and the reliability of the system was determined, which is a bigger limitation, prevents reaching a high level of system reliability, and increases the cost of system design. For the first time in this study the RRAP is reformulated, a new redundancy strategy known as the "mixed strategy" is developed to improve system reliability. In this strategy, cold-standby and active components are simultaneously used in each subsystem, and the reliability of the components is uncertain. The cold-standby and active determine the number and reliability of components in each subsystem in order to optimize the reliability of each subsystem and the whole system.

The research utilizes meta-heuristic algorithms to optimize reliability and cost in non-homogeneous redundancy components, outperforming previous literature and demonstrating superiority and innovation in real systems.

This article focuses on maximizing system reliability using heterogeneous components, mixed series strategies, and weight and cost limitations using a new method, offering the best answer to previous research. It turns out that the results are clearly shown in Table 1 compared to other previous studies, and the reason for this concentration is the best answer so far, and what it brings to managers and decision makers is that by using this mathematical model presented in the system optimization mentioned and with the help of metaheuristic algorithms to solve it and then get the optimal solution.

The rest of this paper is structured as follows: in Section 2, mixed strategies are explained along with the concepts. Section 3 deals with the research questions and mathematical modeling of multi-objective functions, symbols, decision variables, constraints, assumptions, and optimal solutions. In Section 4, well-known tests are used to test the validity of the model, and the optimal solutions are compared to those obtained from other models in the literature. The NSGA-II Algorithm is explained in Section 5. Chapter 6 presents conclusions and suggestions for further studies.

2. Reliability (mixed strategies)

Engineers and managers plan, design, and build systems based on probability, satisfactory performance, time, and specific operation conditions. Reliability assessment is crucial, estimating indicators based on real tests and analytical methods. The reliability of a system can be improved in four ways:

1. Increasing the reliability of components
2. Use redundant components in parallel.
3. The use of redundancy components in parallel systems in conjunction with increasing component reliability.
4. Displacement of components in the subsystem

Reliability optimization is crucial for various industries, including communication, transportation, and manufacturing. Researchers are exploring methods to improve system reliability, such as using starting operation redundancy components in parallel series systems. The paper explores RRAP, a new method focusing on reliability maximization, cost minimization, non-homogeneous components, and mixed redundancy strategies, aiming to maximize system reliability in various systems. The types of issues studied and researched are:

- The problem of allocating continuous reliability
- Allocation of continuous-discrete reliability
- Redundancy allocation problems
- Reliability and redundancy allocation problems

The methods used for optimization purposes include:

- The exact method (linear programming, integer programming, and dynamic programming)
- Approximate methods (geometric programming, Lagrange coefficient method, random search, and lexicographic method)
- Heuristic and meta-heuristic methods

3. Problem modeling

The paper explores RRAP, focusing on maximizing reliability and cost optimization of series components in mixed strategies under uncertainty. It considers continuous monitoring of failure detection and switching performance, and switches dependent on component failure. As Table 1 shows, none of the researchers have simultaneously investigated the afore-mentioned scenarios in any single study. the study considers non-homogeneous redundancy components in parallel series systems with mixed, active, and cold-standby components under uncertainty conditions. Reliability maximization and cost minimization are objective functions, solved using the NSGA-II algorithm, and are defined as shown in Figure 1.

3.1. Assumptions

- The state of components has only two options (good or bad).
- The cost and weight of components are known and deterministic.
- Three redundancy strategies (namely, active, cold standby, and mixed)
- There are no repairable components or preventive maintenance.
- Component failure is considered an independent event.
- Failures of components do not cause the entire system to crash.
- Components within a similar subsystem can be of different types, in other words. The use of complex components is allowed.
- There is imperfect switching for the cold standby redundancy strategy.

3.2. Notation

Parameters	
s	The number of subsystems
A	Set of all subsystems using active redundancy
S_1	Set of all subsystems using standby redundancy
NR	Set of all subsystems with no redundancy
M	Set of all subsystems using mixed redundancy
T	Mission time
$R(t)$	System reliability at a time (t)

n_i	The number of available component choices for sub s
n_{maxi}	Upper bound for (n_i)
g_i	Index for active components of type j in sub-system i
d_i	Index for standby component of type j in subsystem i
c_{ij}, c'_{ij}	The cost for the jth available active and standby component for the subsystem i
w_{ij}, w'_{ij}	Weight for the jth available active and standby component for subsystem i
$P_i(t)$	Failure – detection / switching reliability at time t for case 1
P_i	Failure – detection / switching reliability at time t for case 2
$(c_{\text{switch},i}, w_{\text{switch},i})$	Cost and weight of switch used in subsystem i
w	System – level constraint limit for weight
c	System – level constraint limit for cost
$r_{ij}(t)$	Reliability component of type of type j in subsystem i at time t
$f_{ij}(t)$	pdf of the failure time for type j components for subsystem i at time t
k_{ij}	An index for the number of failures of standby components of type j in subsystem i $(k_{ij} = 1, \ldots, y_{ij}, i = 1, \ldots, s, j = 1, 2, \ldots, d_i)$
$F_{ij}(t)$	is the CDF(cumulative distribution function) for the failure time of the jth component type in subsystem i
$F_{ik}(t)$	is the CDF for the failure time of the kth component type in the subsystem i
$f_{ij}^{k_{ij}}(t)$	represents the pdf for the k_{ij} failure time of component type j in the sun system i
	Indices
l	Index for standby component types that have been allocated.
Z_i	Set of Z_{il} , $\{Z_{i1}, Z_{i2}, \ldots, Z_{il}\}$
Z_{il}	Index of standby component choices used for subsystem i, $\{1, 2, \ldots, d_i\}$
	Decision variables
x_{ij}	The number of active redundancies used in the subsystem
y_{ij}	The number of cold standby redundancies used in the subsystem
n_{ij}	The number of redundancies used in the sub-system
$r_{ij}(t)$	The reliability of the redundancies in the sub-system $(0 \le r_{ij}(t) \le 1)$

3.3. Selection of strategies

Strategies applied to each subsystem can be classified under four scenarios:

1. The subsystem components have an active component and no standby component.
2. The subsystem components have several active components.
3. The subsystem components have one active component and several cold-standby components.
4. The subsystem components have several active and several cold standby components.

3.4. Mathematical model

In the first scenario, the reliability of the system can be calculated in the following way:

$$\prod_{i \in NR} r_{ij}(t) \tag{1}$$

where $r_{ij}(t)$ denotes the probability that only the *j*th active component in the subsystem will operate normally until time *t*.

The second scenario assumes active components in parallel configuration and subsystems in series, resulting in at least one system operating until time *t*, indicating system reliability.

$$\prod_{i\in active}\left(1-\prod_{j=1}^{g_i}\left(1-r_{ij}\left(t\right)\right)^{x_{ij}}\right) \tag{2}$$

If the system has a strategy of three, considering that in each system we have one active component and several inactive components and a cold standby, then when the active component breaks down, the switch is constantly running with reliability $P_i(t)$ and the cold standby is activated. After the breakdown, the first component of cold standby, the second component of cold standby, starts operating until the end.

In this case, the system reliability can be calculated as follows:

$$\prod_{i\in \text{standby}}\left[r_{ij}\left(t\right)+\sum_{k_{ij}=0}^{y_{ij}-1}\int_0^t P_i\left(t\right)r_{ij}\left(t-u\right)\right. \\ \left.\times f_{ij}^{k_{ij}}\left(u\right)du\right] \tag{3}$$

where $f_{ij}^{k_{ij}}$ denotes the probability density function for the k_{ij} time of failure in the component *j* used for the subsystem *i*, when the component continues operating until time *t* and no standby component is needed. Here, the summation is equal to the probability that a k_{ij} component of type *j* in a subsystem *i* at a time *u* is broken and the next component of the same type survives until *t*.

When the fourth scenario applies, the system reliability can be calculated as follows:

$$\prod_{i\in mixed}\left[\left(1-\prod_{j=1}^{g_i}\left(1-r_{ij}\left(t\right)\right)^{x_{ij}}\right)\right. \\ +\int_0^t P_i\left(t\right)r_{ij}\left(t-u\right)f_{ij}^{\max}\left(u\right)du \\ +\sum_{j\in Z_i}\sum_{k_{ij}=0}^{y_{ij}-1}\int_0^t\int_{t_0}^t P_i\left(t\right)r_{ij}\left(t-u\right)f_{ij}^{\left(k_{ij}\right)}\left(u-t_0\right) \\ \left.\times f_i^{\max}\left(t_0\right)du\,dt_0\right] \tag{4}$$

The first part includes the probability that the active components in the subsystem will operate until time t without any need for a standby component. The second part covers the probability that all active components fail before time *u* (and that the last active component fail exactly at time *u*) and that the first standby component starts operating at time u and remains in operation until time *t*. The third part of the equation shows the probability that the last active component fails at time t_1 just when the first standby component starts operating, and that k_{ij} is the time of failure of the type *j* component used for ith subsystem failing at time u and the standby component survives until *t*. In this equation, $f_{ij}^{k_{ij}}$ denotes the probability density function for k_{ij} time of failure of the component *j* used for the subsystem *i*, and represents the probability density function of the time of failure in the last active component.

If $P_i(t)$ is a nonincreasing function ($P_i(u)\geq P_i(t)$ for all $t\geq u$), $0\leq P_i(t)\leq 1$, if switching is perfect, then $P_i(t)=1$ (Gholinezhad [25]).

As a result, the objective function is as follows:
The mathematical model for maximizing the reliability of the problem has the following two modes.

1. In the case of failure detection and switching to be done continuously, and this is displayed with the possibility of $P_i(t)$ in the issue.
2. After the failure of the active member, the switch is displayed as a $P_i^{k_{ij}}$ in the problem.

Case 1: Detection and switching on a continuous basis

Because the subsystem is sequential, the reliability function of maximizing the reliability of the whole system is modeled as a product.

$$\begin{aligned}
&\max R = \prod\nolimits_{i \in NR} \left(r_{ij}(t) \right) \\
&\times \prod\nolimits_{i \in \text{active}} \left(1 - \prod\nolimits_{j=1}^{g_i} \left(1 - r_{ij}(t) \right)^{x_{ij}} \right) \\
&\times \prod\nolimits_{i \in \text{standby}} \left[r_{ij}(t) + \sum_{k_{ij}=0}^{y_{ij}-1} \int_0^t P_i(t) r_{ij}(t-u) f_{ij}^{k_{ij}}(u) du \right] \\
&\times \prod_{i \in \text{mixed}} \Bigg[\left(1 - \prod\nolimits_{j=1}^{g_i} \left(1 - r_{ij}(t) \right)^{x_{ij}} \right) \\
&+ \int_0^t P_i(t) r_{ij}(t-u) f_{ij}^{\max}(u) du \\
&+ \sum\nolimits_{j \in Z_i} \sum\nolimits_{k_{ij}=0}^{y_{ij}-1} \int_0^t \int_{t_0}^t P_i(t) r_{ij}(t-u) f_{ij}^{(k_{ij})}(u - t_0) \\
&\times f_i^{\max}(t_0) du \, dt_0 \Bigg]
\end{aligned} \tag{5}$$

$$\begin{aligned}
\min c = \sum\nolimits_{i=1}^{s} \sum\nolimits_{j=1}^{g_i} c_{ij} x_{ij} + \sum\nolimits_{i=1}^{s} \sum\nolimits_{j=1}^{d_i} c_{ij} y_{ij} \\
+ \sum\nolimits_{i \in \{s,x\}} c_{\text{switch},i}
\end{aligned}$$

$$\text{s.t.} \quad \sum\nolimits_{i=1}^{s} \sum\nolimits_{j=1}^{g_i} w_{ij} x_{ij} + \sum\nolimits_{i=1}^{s} \sum\nolimits_{j=1}^{d_i} w_{ij} y_{ij} \le w,$$

$$\sum\nolimits_{i=1}^{s} \sum\nolimits_{j=1}^{g_i} c_{ij} x_{ij} + \sum\nolimits_{i=1}^{s} \sum\nolimits_{j=1}^{d_i} c_{ij} y_{ij} \le c,$$

$$\lambda_{ij}, \beta_{ij}, x_{ij}, y_{ij} \ge 0,$$

where

$$f_i^{\max}(t) = \sum\nolimits_{j=1}^{g_i} x_{ij} \prod\nolimits_{\substack{k=1 \\ k \ne 1}}^{g_i} \left(F_{ik}(t) \right)^{x_{ij}} . \left(F_{ij}(t) \right)^{x_{ij=1}} f_{ij}(t), \tag{6}$$

$$F_{ik}(t) = \int_{-\infty}^{t} f_{ik}(u) du \Rightarrow F_{ik}(t) = \int_0^t f_{ik}(u) du, \tag{7}$$

$$F_{ij}(t) = \int_{-\infty}^{t} f_{ij}(u) du = \int_0^t f_{ij}(u) du,$$

$$r_{ij}(t) = e^{-\lambda_{ij} t} \sum\nolimits_{l=0}^{\beta_{ij=1}} \frac{\left(\lambda_{ij} t \right)^l}{l_i}, \tag{8}$$

and

$$f_{ij}(t) = \lambda_{ij}^{\beta_{ij}} t^{\beta_{ij-1}} e^{-\lambda_{ij} t} / \Gamma\left(\beta_{ij} \right) \tag{9}$$

is the probability density function of failure with a gamma distribution.

Case 2: Switch activation only in response to a failure (P_i)

$$\max R = \prod_{i\in NR}\left(r_{ij}\left(t\right)\right)\times\prod_{i\in active}\left(1-\prod_{j=1}^{g_i}\left(1-r_{ij}\left(t\right)\right)^{x_{ij}}\right)$$
$$\times\prod_{i\in \text{standby}}\left[r_{ij}\left(t\right)+\sum_{j\in Z_i}\sum_{k_{ij}=0}^{y_{ij}-1}P_i^{k_{ij}}\int_0^t r_{ij}\left(t-u\right)f_{ij}^{k_{ij}}\left(u\right)du\right]$$
$$\times\prod_{i\in \text{mixed}}\left[1-\prod_{j=1}^{g_i}\left(1-r_{ij}\left(t\right)\right)^{x_{ij}}+P_i\int_0^t r_{ij}\left(t-u\right)f_{ij}^{\max}\left(u\right)du\right.$$
$$+\sum_{j\in Z_i}\sum_{k_{ij}=0}^{y_{ij}-1}P_i^{k_{ij}}\times\int_0^t\int_{t_0}^t r_{ij}\left(t-u\right)f_{ij}^{k_{ij}}\left(u-t_0\right).f_i^{\max}\left(t_0\right)$$
$$\left.du\,dt_0\right] \tag{10}$$
$$\min c=\sum_{i=1}^{s}\sum_{j=1}^{g_i}c_{ij}x_{ij}+\sum_{i=1}^{s}\sum_{j=1}^{d_i}c_{ij}y_{ij}+\sum_{i\in\{s,x\}}c_{switch,i}$$
$$\text{s.t.}\ \sum_{i=1}^{s}\sum_{j=1}^{g_i}w_{ij}x_{ij}+\sum_{i=1}^{s}\sum_{j=1}^{d_i}w_{ij}y_{ij}\le w,$$
$$\sum_{i=1}^{s}\sum_{j=1}^{g_i}c_{ij}x_{ij}+\sum_{i=1}^{s}\sum_{j=1}^{d_i}c_{ij}y_{ij}\le c,$$
$$\lambda_{ij},\beta_{ij},x_{ij},y_{ij}\ge 0,$$

where

$$F_{ik}\left(t\right)=\int_{-\infty}^{t}f_{ik}\left(t\right)dt,$$
$$F_{ij}\left(t\right)=\int_0^t f_{ij}\left(u\right)du,$$
$$r_{ij}\left(t\right)=e^{-\lambda_{ij}t}\sum_{l=0}^{\beta_{ij=1}}\frac{\left(\lambda_{ij}t\right)^l}{l_i},$$
$$f_{ij}\left(t\right)=\frac{\lambda_{ij}^{\beta_{ij}}t^{\beta_{ij-1}}e^{-\lambda_{ij}t}}{\Gamma\left(\beta_{ij}\right)}$$
$$\Gamma\left(\beta_{ij}\right)=\int_0^{\infty}t^{\beta_{ij-1}}e^{-t}dt, \tag{11}$$
$$f_{ij}^{\left(k_{ij}\right)}=\int_{u_0}^t\int_{u_1}^t\cdots\int_{uk_{ij}-1}^t\int_{uk_{ij}}^t f_{ij}\left(u_{k_{ij}+1}-u_{k_{ij}}\right).f_{ij}\left(u_{k_{ij}}-u_{k_{ij}-1}\right)\ldots f_{ij}\left(u_2-u_1\right).f_{ij}\left(u_1-u_0\right))\,du_{k_{ij}+1}.du_{k_{ij}}\ldots.du_2du_1, \tag{12}$$
$$f_i^{\max}\left(t\right)=\sum_{j=1}^{g_i}\prod_{\substack{k=1\\k\neq j}}^{g_i}\left(F_{ik}\left(t\right)\right)^{x_{ij}}.\left(F_{ij}\left(t\right)\right)^{x_{ij}=1}f_{ij}\left(t\right). \tag{13}$$

and $\Gamma\left(\beta_{ij}\right)$ is Gamma function, λ_{ij} is rate parameters of Gamma function, and β_{ij-1} is shape parameters of Gamma function.

By instead $r_{ij}(t)=e^{-\lambda_{ij}t}\sum_{l=0}^{\beta_{ij}-1}\left(\left(\lambda_{ij}t\right)^{l}\big/l_i\right)$, in the equations of the case 2, the mathematical model of the problem is in the following form:

$$
\begin{aligned}
&\max R=\prod_{i\in NR}\left(e^{-\lambda_{ij}t}\sum_{l=0}^{\beta_{ij}-1}\frac{\left(\lambda_{ij}t\right)^{l}}{l_i}\right)\\
&\times\prod_{i\in active}\left(1-\prod_{j=1}^{g_i}\left(1-e^{-\lambda_{ij}t}\sum_{l=0}^{\beta_{ij}-1}\frac{\left(\lambda_{ij}t\right)^{l}}{l_i}\right)^{x_{ij}}\right)\\
&\times\prod_{i\in\text{mixed}}\Bigg[\left(1-\prod_{j=1}^{g_i}\left(1-r_{ij}(t)\right)^{x_{ij}}\right)\\
&+P_i\int_0^t r_{ij}(t-u)\,f_{ij}^{\max}(u)\,du+\\
&\sum_{j\in Z_i}\sum_{k_{ij}=0}^{y_{ij}-1}P_i^{k_{ij}}\times.du_{k_{ij}}.\\
&\int_0^t\int_{u_0}^t\int_{u_1}^t\cdots\int_{u_{k_{ij}}}^t r_{ij}\left(t-u_{k_{ij+1}}\right)\\
&f_{ij}\left(u_{k_{ij+1}}-u_{k_{ij}}\right).f_{ij}\left(u_1-u_0\right)).f_i^{\max}\left(t_0\right)du_{k_{ij+1}}\\
&\ldots.du_3du_2du_1\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
\min c=&\sum_{i\in NR}\sum_{j=1}^{g_i}c_{ij}\\
&+\sum_{i\in\text{standby}}\sum_{j=1}^{g_i}\left(c_{ij}+c'_{ij}y_{ij}\right)\\
&+\sum_{i\in\text{active}}\sum_{j=1}^{g_i}c_{ij}x_{ij}\\
&+\sum_{i\in\text{mixed}}\sum_{j=1}^{d_i}\left(c_{ij}x_{ij}+c'_{ij}y_{ij}\right)\\
&+\sum_{i\in\{\text{standby},\text{mixed}\}}c_{\text{switch},i}
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
\text{s.t. }&\sum_{i\in\{NR,\text{active},\text{standby},\text{mixed}\}}i\le S,\\
&\sum_{i\in NR}\sum_{j=1}^{g_i}w_{ij}\\
&+\sum_{i\in\text{standby}}\sum_{j=1}^{g_i}\left(w_{ij}+w'_{ij}y_{ij}\right)\\
&+\sum_{i\in\text{active}}\sum_{j=1}^{g_i}w_{ij}x_{ij}\\
&+\sum_{i\in\text{mixed}}\sum_{j=1}^{d_i}\left(w_{ij}x_{ij}+w'_{ij}y_{ij}\right)\\
&+\sum_{i\in\{\text{standby},\text{mixed}\}}w_{\text{switch},i}\le W,\\
&\sum_{i\in NR}\sum_{j=1}^{g_i}c_{ij}+\\
&+\sum_{i\in\text{standby}}\sum_{j=1}^{g_i}\left(c_{ij}+c'_{ij}y_{ij}\right)\\
&+\sum_{i\in\text{active}}\sum_{j=1}^{g_i}c_{ij}x_{ij}+\\
&+\sum_{i\in\text{mixed}}\sum_{j=1}^{d_i}\left(c_{ij}x_{ij}+c'_{ij}y_{ij}\right)\\
&+\sum_{i\in\{\text{standby},\text{mixed}\}}c_{\text{switch},i}\le C,\\
&\sum_{j=1}^{g_i}x_{ij}+\sum_{j=1}^{d_i}y_{ij}\le n_i,\\
&\lambda_{ij},\beta_{ij},x_{ij},y_{ij}\ge 0.
\end{aligned}
$$

3.5. Solving a multi-objective mathematical model using the NSGAI meta-heuristic algorithm

Evolutionary algorithm implementation stages are as follows:

1. Generating a set of random solutions
2. Comparing the solutions, rating them, and choosing the best one.
3. Combining the obtained solutions by simulation of natural processes such as reproduction and integration of new answers with old answers
4. Return to stage 2 (end)

The genetic algorithm implementation steps are as follows:

1. Generating a random population and evaluating it
2. Selecting parents and combining them to create offspring population
3. Selecting the most fitted members of the population to perform mutations and create mutant populations
4. Combining the parent, offspring, and mutant populations and generating a parent population
5. Repeat steps 2 through 6 until termination criteria are satisfied.
6. End

Different end conditions are as follows:

1. Achieve an acceptable level of response.
2. Iteration time out/definition repetition.
3. The chance of achieving significant changes in the next generation is excessively low.

The coding solution in the proposed GA consists of a $(2m)*s$ matrix. Although $m=\max\{m_i\}$ and m_i denote the number of variants available in the subsystems, m in the initial rows represents the active components and m in the last row represents the standby components.

The first row shows the number of type I active components that have been allocated, and the second row shows the number of allocated type II active components. Moreover, (m+1)th row shows the allocated type I standby components, and (m+2)th row shows the allocated type II standby components. The components that are in an idle state are represented by the number zero.

Figure 2, for instance, shows the coding solutions for s = 14 and m = 4 in this matrix. The results show that there is one solution in subsystem 1 with a mixed strategy consisting of two type I and four type IV active components, as well as one type I and three type II standby components. Subsystem 2 has a mixed strategy consisting of two type I and two type II active components and two types II and one type IV standby component. The fourteenth subsystem has an active strategy with three type II active components.

3.6. Non-dominant Sorting Genetic Algorithm (NSGA-II)

NSGA-II and Controlled NSGA II are popular multi-objective optimization algorithms, with genetic algorithms emerging as a new approach. These multidimensional, universal search algorithms can solve complex problems without relying on the problem's mathematical structure.

1. Fitness distribution to population members based on non-dominated sorting
2. preserving diversity in solutions in the non-dominant border

Although multi-objective evolutionary algorithms provide efficient solutions to such problems, Deb et al. [16] proposed an algorithm called NSGA-II, which could partially eliminate the shortcomings of the previous algorithm (NSGA). These shortcomings include:

1. The computational complexity of the previous algorithm was reduced from
2. Lack of efficient elitism
3. The obviation of the need to determine parameters in the splitting process.

The NSGA-II algorithm selects solutions from each generation using a binary tournament selection method. It primarily considers solution rank and crowding distance. Crossover is applied to some selected population members, mutation is applied to others, and a new population is generated. Members are sorted by rank and crowding distance.

3.6.1. Generating the initial population

Literature typically introduces two methods for generating an initial population: random selection and heuristic algorithms.

3.6.2. Evaluation of chromosomes

The NSGA-II algorithm evaluates each member's potential for passage to the next generation using non-dominated ranking and scattering diversity. The optimal front solution algorithms are placed in F1, while non-dominated solutions are placed in F2 and F3. The crowding distance approach maintains diversity at each front.

3.6.3. Selection operator

This paper presents a selection operator based on the roulette wheel selection operator, a crucial element in evolutionary algorithms for random selection of better members for reproduction.

3.6.4. Crossover operators

This study uses spin crossover to demonstrate the permutation of elements in complementation, a genetic algorithm technique used to generate better offspring with positive traits inherited from parents.

3.6.4.1 Spin crossover operator in permutation presentation

The spin recombination operator is a genetics method that displays permutation by shifting genes between parents, repeating for the second parent, and then back to the first parent, determining remaining genes until the first offspring is complete. (Figure 3).

3.6.5. Mutation operator

The proposed algorithm uses a mutation operator to prevent premature convergence by altering offspring chromosomes, dislocating genes from each family, and employing inversion mutation operator permutation presentation.

3.6.5.1 Inversion mutation operator in permutation presentation

The inversion mutation operator, similar to the hashing mutation operator, copies gene values between two cut points, resembling a mirror between them. (Figure 4).

3.6.6. Validation

Model accuracy measurement ensures numerical solutions' accuracy, while validation introduces verified models as alternatives to experimental methods, eliminating challenges from tests and comparing components and models to literature solutions. This article uses a numerical method as an alternative to experimental tests, ensuring system implementation aligns with intended results and conforms to description, using problem test and MATLAB.

4. Numerical example

A well-known example adopted by Fyffe et al. will be used to compare the proposed model with models presented in the literature. This example has been used in many articles. The system designed in this example includes 14 subsystems, and there are three or four component choices for each subsystem. The cost and weight of components such as k-erlang distribution parameters are presented in Table 2. The goal is to maximize system reliability in 100 units of time while the system cost and system weight limits are Each subsystem may have an active, cold-standby, or mixed redundancy strategy, or none of them at all. Also, different types of components can be simultaneously chosen in each subsystem. The maximum number of components within each subsystem is 6. Once the proposed model is applied to the example mentioned above, optimal solutions are obtained. These solutions are presented in Table 3. Maximum reliability is achieved by the proposed NSGAII algorithm. The data used for this purpose is presented in Table 2. As Table 3 shows, the system reliability obtained using the new model is equal to 0.9939. Moreover, the optimal values for system cost and weight were found to be 98 and 170, respectively.

Figure 5 shows the performance of the proposed model compared to Gholinejad and Zeinal Hamedani's models [25] in obtaining the reliability of subsystems. Comparison of the proposed method to the models used by Fyffe et al. [1], Tavakkoli-Moghadam and Safari [19], Soltani et al. [44], Abouei Ardakan and Zeinal Hamedani [23] and Gholinejad and Zeinal Hamedani [25], showed that the system obtained from the proposed model is more reliable and incurs lower costs (Table 4 and Figure 6). Therefore, it can be argued that the proposed model outperforms the previous models.

According to Table 3, Section 4 shows the optimal reliability values of all subsystem components and the optimal number of active and standby members for cold-standby and what type they are. In Table 4, Section 4, the optimal reliability values, the optimal cost values, and the optimal total system weight values, related to previous studies and the present article, show the superiority of the proposed method in this article in terms of the optimal system reliability value and the optimal cost value compared to other methods. It shows the methods in the previous studies, and it is noteworthy that the optimal cost of the whole system compared to the optimal cost of the whole system of the studies mentioned in Table 4, Section 4, shows a very noticeable superiority with a large difference. In this proposed mathematical model, due to the limitations of cost, weight, and heterogeneity of components, uncertainty of component reliability, and considering mixed strategies, the best capability of the whole system is 0.9939, with the lowest consumption cost value of 98, which indicates the tangible superiority and cost-effectiveness of the proposed method in this article.

5. Conclusion

In the present paper, a new model for RRAP is presented. In this system, the components in the subsystems are non-homogeneous and have uncertain reliability, providing design engineers with an opportunity to use the proposed model to increase the reliability of individual components and, consequently, the whole system. Since the homogeneity constraint has been crossed out in the component selection, this may lead to improvements in the reliability of the whole system, and the mixed redundancies and strategies applied to each subsystem may provide the ground for access to more realistic solutions. In this case, the subsystem will be much more realistic and can be used as a useful tool by designers to simultaneously choose standby and active redundancies for subsystems. The choice and reliability of strategies can be assumed as decision variables, and the number of each active and cold-standby component can also be considered as decision variables. The proposed model is a non-linear programming model of the NP-hard type that cannot be solved by exact algorithms and is usually solved using the NSGA-II algorithm.
The performance of the proposed mathematical model was evaluated by a well-known test problem. The optimization results show the effectiveness of the proposed model, and according to the results in Table 4, it shows that it has led to more reliability at a very low cost, which indicates that the mathematical model works better than the previous methods. Further studies can consider system components independently, considering multiple failure modes and integrating multiple parallel, repairable switches. The RRAP mathematical model can be integrated with game theory to address attacker-defender problems and artificial goals, leading to improved articles for defending sensitive military and civilian areas.

References

[1] Fyffe, D. E., Hines , W. W., and Lee, N. K. "System reliability allocation and a computational algorithm", IEEE Transactions on Reliability, 2, pp. 64–69 (1968).
Doi.org/10.1109/TR.1968.5217517

[2] Misra,K.B. "Reliability optimization of a series-parallel system", IEEE Transactions on Reliability, 21(4), pp. 230–238 (1972).
Doi.org/10.1109/TR.1972.5216000

[3] Gen ,M., Ida , K., and Lee, J. U. "A computational algorithm for solving 0–1 goal programming with GUB structures and its application for optimization problems in system reliability", Electronics and Communications in Japan, (Part III: Fundamental Electronic Science), 73(12), pp. 88–96 (1990) .
Doi.org/10.1002/ecjc.4430731210

[4] Hwang C-L , K. F. "Optimal reliability design: fundamentals and applications". Cam- bridge, UK: Cambridge University Press, 2001.

[5] Caserta, M., and Voß, S. "An exact algorithm for the reliability redundancy allocation problem", European Journal of Operational Research, 244(1) ,pp. 110–116 (2015).

Doi.org/10.1016/j.ejor.2015.01.008
[6] Prasad ,V. R., and Kuo, W. “Reliability optimization of coherent systems”, IEEE Transactions on Reliability, 49(3) ,pp. 323–330 (2000). Doi: 10.1109/24.914551
[7] Yeh , W.-C. and Hsieh ,T. J. “Solving reliability redundancy allocation problems using an artificial bee colony algorithm”, Computers and Operation Research, 38 ,pp. 1465–1473 (2011).
Doi.org/10.1016/j.cor.2010.10.028
[8] Ha , C., and Kuo, W. “Reliability redundancy allocation: an improved realization for nonconvex nonlinear programming problems”, European Journal of Operational Research, 171(1), pp. 24–38 (2006).
Doi.org/10.1016/j.ejor.2004.06.006
[9] Habib ,A., Aleida ,R., and Youssef , G. “Reliability analysis of a consecutive r-out-of-n: F system based on neural networks”, Chaos, Solitons & Fractals ,39 (2),pp.610–24 (2009).
Doi.org/10.1016/j.chaos.2007.01.151
[10] Chambari, A., Najafi, A. A., Rahmati, S. H. A., et al “An efficient simulated annealing algorithm for the redundancy allocation problem with a choice of redundancy strategies”, Reliability Engineering and System Safety, 119 , 158-164 (2013) .
Doi.org/10.1016/j.ress.2013.05.016
[11] Abouei Ardakan, M., and Rezvan, M.T. “Multi-objective optimization of reliability-redundancy allocation problem with cold-standby strategy using NSGA-II”, Reliability Engineering & System Safety ,172, pp. 225–238 (2018). Doi.org/10.1016/j.ress.2017.12.019
[12] Abouei Ardakan, M., and Zeinal Hamadani, A. “Reliability–redundancy allocation problem with cold standby redundancy strategy”, Simulation Modelling Practice and Theory, 42, pp.107–118 (2014).
Doi.org/10.1016/j.simpat.2013.12.013
[13] Zoulfaghari , H., Zeinal Hamadani, A., and Abouei Ardakan, M. “Bi-objective redundancy allocation problem for a system with mixed repairable and non-repairable components”, ISA Transactions ,53(1) ,pp. 17–24 (2014).
Doi.org/10.1016/j.isatra.2013.08.002
[14] Chen, T. C. “IAs based approach for reliability redundancy allocation problems”, Applied Mathematics and Computation, 182 (2), pp. 1556–1567 (2006).
Doi.org/10.1016/j.amc.2006.05.044
[15] Hsieh , Y. C., and You, P. S. “An effective immune based two-phase approach for the optimal reliability–redundancy allocation problem”, Applied Mathematics and Computation ,218 (4) ,pp. 1297–1307 (2011).
Doi.org/10.1016/j.amc.2011.06.012
[16] Kulturel-Konak, S., Smith, A., and Coit, D. W. “Efficiently solving the redundancy allocation problem using tabu search”, IIE Transaction, 35(6) pp. 515–526 (2003).
Doi.org/10.1080/07408170304422
[17] Zhang ,E., Wu ,Y., and Chen, Q. “A practical approach for solving multi-objective reliability redundancy allocation problems using extended bare-bones particle swarm optimization”, Reliability Engineering and System Safety, 127(C) , pp. 65–76 (2014).
Doi.org/10.1016/j.ress.2014.03.006
[18] Garg , H., Rani, M., Sharma, S. P., et al, “Bi-objective optimization of the reliability-redundancy allocation problem for series-parallel system”, Journal of Manufacturing Systems, 33(3), pp. 335–347 (2014).
Doi.org/10.1016/j.jmsy.2014.02.008
[19] Tavakkoli-Moghaddam , R., Safari, J., and Sassani, F. “Reliability optimization of series-parallel systems with a choice of redundancy strategies using a genetic algorithm”, Reliability Engineering & System Safety, 93(4) ,pp.550–556 (2008).
Doi.org/10.1016/j.ress.2007.02.009
[20] Coit, D. W. “Cold-standby redundancy optimization for nonrepairable systems”, IIE Transactions ,33(6) ,pp. 471–478 (2001).
doi.org/10.1023/A:1007689912305
[21] Coit, D. W. “Maximization of system reliability with a choice of redundancy strategies”, IIE Transaction, 35(6) ,pp. 535–543 (2003).
Doi.org/10.1080/07408170304420
[22] Tavakkoli-Moghaddam , R., and Safari, J. “A new mathematical model for a redundancy-allocation problem with mixing components redundant and choice of redundancy strategies”, Applied Mathematical Sciences, 45(1) ,pp. 2221–2230 (2007).
[23] Abouei Ardakan, M., and Zeinal Hamadani, A. “Reliability optimization of series–parallel systems with mixed redundancy strategy in subsystems”, Reliability Engineering & System Safety ,130 ,pp. 132–139(2014).
Doi.org/10.1016/j.ress.2014.06.001
[24] Kim , H., and Kim, P. “Reliability–redundancy allocation problem considering optimal redundancy strategy using parallel genetic algorithm”, Reliability Engineering & System Safety ,159, pp. 153–160 (2017).
Doi.org/10.1016/j.ress.2016.10.033
[25] Gholinezhad ,H., and Zeinal Hamadani, A. “A new model for the redundancy allocation problem with component mixing and mixed redundancy strategy”, Reliability Engineering & System Safety, 164 ,pp. 66–73 (2017).
Doi.org/10.1016/j.ress.2017.03.009
[26] Liu ,Y., and Qin, G. “A modified particle swarm optimization algorithm for reliability redundancy optimization problem”, Journal of Computers ,9(9) ,pp. 2124–2131 (2014).
Doi:10.4304/jcp.9.9.2124-2131
[27] Drik, M. “Type-2 fuzzy logic controller design optimization using the PSO approach for ECG prediction”, journal of fuzzy extension and application, 3 , pp. 158–168 (2022).
Doi.org/10.22105/jfea.2022.333786.1207
[28] Wang , L., and Li , L. P. “A coevolutionary differential evolution with harmony search for reliability–redundancy optimization”, Expert Systems with Applications, 39(5), pp. 5271–5278 (2012).
Doi.org/10.1016/j.eswa.2011.11.012

[29] Dalanezi Mori , B., Fiori de Castro, H., and Lucchesi Cavalca, K. “Development of hybrid algorithm based on simulated annealing and genetic algorithm to reliability redundancy optimization”, International Journal of Quality & Reliability Management, 24(9) pp. 972–987 (2007).
Doi.org/10.1108/02656710710826225
[30] Kanagaraj , G., Ponnambalam, S. G., and Jawahar, N. “A hybrid cuckoo search and genetic algorithm for reliability–redundancy allocation problems”, Computers & Industrial Engineering, 66(4), pp. 1115–1124 (2013).
Doi.org/10.1016/j.cie.2013.08.003
[31] Haiying ,Z., and Yubao , L. “A mixed TS-ISA algorithm for reliability redundancy optimization problem”, International Journal of Security and Its Applications, 10(4) ,pp. 71–78 (2016).
Doi.org/10.14257/ijsia.2016.10.4.08
[32] Salazar , D., Rocco , C. M., and Galvan, B. J. “Optimization of constrained multiple-objective reliability problems using evolutionary algorithms”, Reliability Engineering & System Safety 2006, 91, pp. 1057–1070 (2006).
Doi.org/10.1016/j.ress.2005.11.040
[33] Li ,Z., Liao ,H., and Coit, D.W. “A two-stage approach for multi-objective decision making with applications to system reliability optimization”, Reliability Engineering & System Safety ,94(10) ,pp. 1585–1592 (2009).
Doi.org/10.1016/j.ress.2009.02.022
[34] Attar ,A., Raissi, S., and Khalili-Damghani, K. “A simulation-based optimization approach for free distributed repairable multi-state availability-redundancy allocation problems”, Reliability Engineering & System Safety, 157 ,pp. 177–191(2017).
Doi.org/10.1016/j.ress.2016.09.006
[35] Safari, J. “Multi-objective reliability optimization of series-parallel systems with a choice of redundancy strategies”, Reliability Engineering & System Safety, 108, pp. 10–20 (2012).
Doi.org/10.1016/j.ress.2012.06.001
[36] Salmasnia , A., Ameri , E,. and Akhavan Niaki, S. T. “A robust loss function approach for a multi-objective redundancy allocation problem”, Applied Mathematical Modeling, 40(1) ,pp. 635–645 (2015).
Doi.org/10.1016/j.apm.2015.06.007
[37] Taboada , H. A., and Coit, D. W. “Development of a multiple objective genetic algorithm for solving reliability design allocation problems”, In: Proceedings of Industrial Engineering Research Conference, (2008).
[38] Eshraghniaye Jahromi , A., and Feizabadi, M. “Optimization of multi-objective redundancy allocation problem with non-homogeneous components”, Computers & Industrial Engineering, 108, pp. 111–123 (2017).
Doi.org/10.1016/j.cie.2017.04.009
[39] Garg, H., and Sharma, S. P. “Multi-objective reliability-redundancy allocation problem using particle swarm optimization”, Computers & Industrial Engineering, 64 ,pp. 247–255 (2013) .
Doi.org/10.1016/j.cie.2012.09.015
[40] Abouei Ardakan, M., and Zeinal Hamadani, A., and Alinaghian, M. “Optimizing bi-objective redundancy allocation problem with a mixed redundancy strategy”, ISA Transactions ,55 ,pp. 116–128 (2015).
Doi.org/10.1016/j.isatra.2014.10.002
[41] Chambari, A., Rahmati, S. H. A., Najafi, A., et al, “A bi-objective model to optimize reliability and cost of system with a choice of redundancy strategies”, Computers & Industrial Engineering ,63(1) , pp.109–119. (2012)
Doi.org/10.1016/j.cie.2012.02.004
[42] Liang ,Y. C., and Lo, M. H. “Multi-objective redundancy allocation optimization using a variable neighborhood search algorithm”, Journal of Heuristics, 16 ,pp. 511–535 (2010) .
Doi.org/10.1007/s10732-009-9108-4
[43] Coit, D. W., and Konak, A. “Multiple weighted objectives heuristic for the redundancy allocation problem”, IEEE Transactions on Reliability, 55(3), pp. 551–558(2006) .
Doi.org/10.1109/TR.2006.879654
[44] Soltani , R., Sadjadi, S. J., and Tavakkoli-Moghaddam, R. “Interval programming for the redundancy allocation with choices of redundancy strategy and component type under uncertainty: erlang time to failure distribution”, Applied Mathematics and Computation, 244 ,pp. 413–421 (2014).
Doi.org/10.1016/j.amc.2014.06.113
[45] Wang, Z., Chen ,T., Tang, K., et al. “A multi-objective approach to redundancy allocation problem in parallel-series systems”, IEEE Congress on Evolutionary Computation ,pp. 582–589 (2009).
Doi. 10.1109/CEC.2009.4982998
[46] Ramezani Dobani, E., Abouei Ardakan , M., Davari-Ardakani ,H., et al. “RRAP-CM: a new reliability- redundancy allocation problem with heterogeneous components”, Reliability Engineering & System Safety, 191, 106563 (2019).
Doi.org/10.1016/j.ress.2019.106563
[47] Mahdavi-Nasab , N., Abouei Ardakan , M., and Mohammadi, M. “Water cycle algorithm for solving the reliability-redundancy allocation problem with a choice of redundancy strategies”, Communications in Statistics - Theory and Methods, 49(11) ,pp. 2728–2748 (2019).
Doi.org/10.1080/03610926.2019.1580741
[48] Yeh , W.-C., Zhu , W., Tan , S.-Y., et al. “Novel general active reliability redundancy allocation problem and algorithm (GRRAP)”, Reliability Engineering and System Safety, 218 , 108167 (2022).
Doi.org/10.1016/j.ress.2021.108167
[49] Peykani , P., Nouri, M., Eshghi , F., et al. “A novel mathematical approach for fuzzy multi-period multi-objective portfolio optimization problem under uncertain environment and practical constraints”, Journal of Fuzzy Extension and Application, 2 ,pp. 191–203 (2021).
Doi.org/10.22105/jfea.2021.287429.1150
[50] Mekawy, I. M. “A novel method for solving multi- objective linear fractional programming problem under uncertainty”, journal of fuzzy extension and application, 3 ,pp. 169–176 (2022).

Doi.org/10.22105/jfea.2022.331180.1206
[51] Nath, R., and Muhuri, P.K. “Evolutionary optimization based solution approaches for many objective reliability redundancy allocation problem” , Reliability Engineering & System Safety 220, 108190 (2022).
Doi.org/10.1016/j.ress.2021.108190
[52] Mellal, M. A., and Zio, E. “System reliability-redundancy optimization with cold-standby strategy by an enhanced nest cuckoo optimization algorithm”, Reliability Engineering & System Safety ,201 , 106973 (2022).
Doi.org/10.1016/j.ress.2020.106973
[53] Yeh, W.-C. “solving cold-standby reliability redundancy allocation problems using a new swarm intelligence algorithm”, Applied Soft Computing, 83 , 105582 (2019).
Doi.org/10.1016/j.asoc.2019.105582
[54] Najmi ,A., Abouei Ardakan ,M., and Javid, Y. “Optimization of reliability redundancy allocation problem with component mixing and strategy selection for subsystems”, Journal of Statistical Computation and Simulation, 91(10), pp. 1935–1959 (2021).
Doi.org/10.1080/00949655.2021.1879080
[55] Karamaşa , Ç., Karabasevic, D., Stanujkic , D., et al. “An extended single-valued neutrosophic AHP and MULTIMOORA method to evaluate the optimal training aircraft for flight training organizations”, Facta Universitatis-Series Mechanical Engineering ,19(3), pp. 555–578 (2021).
Doi.org/10.22190/FUME210521059K
[56] Rasoulzadeh , M., Edalatpanah , S. A., Fallah, M., et al. “A multi-objective approach based on Markowitz and DEA cross-efficiency models for the intuitionistic fuzzy portfolio selection problem”, Decision Making: Applications in Management and Engineering ,5 ,pp. 241–259 (2022).
Doi.org/10.31181/dmame0324062022e
[57] Wang ,Z., Yang, G., Sun, Y., et al. “An improved bare-bones particle SWARM algorithm for multi-objective optimization with application to the engineering structures”, Facta Universitatis-Series Mechanical Engineering, (2023).
Doi: 10.22190/FUME220829004W.
[58] Ouyang , Z., Liu ,Y., Ruan, S.-J., et al. “An improved particle swarm optimization algorithm for reliability-redundancy allocation problem with mixed redundancy strategy and heterogeneous components”, Reliability Engineering & System Safety, 181 , pp.62–74 (2019).
Doi.org/10.1016/j.ress.2018.09.005
[59] Yeh , W.-C., Su , Y.-Z., Gao , X.-Z., et al, “Simplified swarm optimization for bi-objective active reliability redundancy allocative problems”, Applied Soft Computing, 106, 107321 (2021).
Doi.org/10.1016/j.asoc.2021.107321
[60] Mahdavi-Nasab , N., Abouei Ardakan, M., and Peiravi, A. “A New model for the reliability redundancy allocation problem with the mixed redundancy strategy”, Journal of Statistical Computation and Simulation, 92 (14), pp. 2956–2979 (2022).
Doi.org/10.1080/00949655.2022.2053126

Table 1. Comparison between previous research and the pro-posed model.

	Reliability	Cost	Other	RAP	RRAP	Non-homogeneous	Uncertain	Mixed Strategy	Algorithm
Kulturel-Konak et al. [16]	*	*	*	*					Tabu Search
Coit and Konak [43]	*			*					GA
Salazar et al. [32]	*			*	*				NSGA-II
Wang et al. [57]	*	*		*					NSGA-II
Tavakkoli-Moghaddam et al. [19]	*			*					GA
Chambari et al. [10]	*			*					Simulated Annealing
Liang and lo [42]		*		*					VNS
Abouei Ardakan et al. [40]	*	*		*				*	NSGA-II
Yeh and Hsieh [7]					*				Ant Colony
Garg and Sharma [39]	*	*			*				PSO
Safari [35]	*	*		*					NSGA-II
Eshraghniaye Jahromi and Feizabadi [38]	*	*		*		*		*	NSGA-II
Abouei Ardakan and Rezvan [11]	*	*			*			*	NSGA-II
Gholinezhad and Zeinal Ham-adani [25]	*			*				*	GA
Salmasnia et al. [36]	*	*	*	*			*		DFS
Proposed Method	*	*	*		*	*	*	*	NSGA-II

Table 2. Component data for the given example.

	Choice 1 ($j=1$)				Choice 2 ($j=2$)				Choice 3 ($j=3$)				Choice 4 ($j=4$)			
i	λ_{ij}	k_{ij}	c_{ij}	w_{ij}	λ_{ij}	k_{ij}	c_{ij}	w_{ij}	λ_{ij}	k_{ij}	c_{ij}	w_{ij}	λ_{ij}	k_{ij}	c_{ij}	w_{ij}
1	0.00532	2	1	3	0.000726	1	1	4	0.00499	2	2	2	0.00818	3	2	5
2	0.00818	3	2	8	0.000619	1	1	10	0.00431	2	1	9	–	–	–	–
3	0.0133	3	2	7	0.011	3	3	5	0.0124	3	1	6	0.00466	2	4	4
4	0.00741	2	3	5	0.0124	3	4	6	0.00683	2	5	4	–	–	–	–
5	0.00619	1	2	4	0.00431	2	2	3	0.00818	3	3	5	–	–	–	–

6	0.00436	3	3	5	0.00567	3	3	4	0.00268	2	2	5	0.000408	1	2	4
7	0.0105	3	4	7	0.00466	2	4	8	0.00394	2	5	9	–	–	–	–
8	0.015	3	3	4	0.00105	1	5	7	0.0105	3	6	6	–	–	–	–
9	0.00268	2	2	8	0.000101	1	3	9	0.000408	1	4	7	0.000943	1	3	8
10	0.0141	3	4	6	0.00683	2	4	5	0.00105	1	5	6	–	–	–	–
11	0.00394	2	3	5	0.00355	2	4	6	0.00314	2	5	6	–	–	–	–
12	0.00236	1	2	4	0.00769	2	3	5	0.0133	3	4	6	0.011	3	5	7
13	0.00215	2	2	5	0.00436	3	3	5	0.00665	3	4	6	–	–	–	–
14	0.011	3	4	6	0.00834	1	4	7	0.00355	2	2	6	0.00436	3	6	9

Table 3. Optimal solution and its comparison with those obtained from other models

		Active components				Standby components				
Subsystem	Redundancy strategy	Type1	Type2	Type3	Type4	Type1	Type2	Type3	Type4	Reliability
1	Mixed	2	0	0	0	1	0	0	0	0.9993
2	Standby	1	0	0	0	1	0	0	0	0.9998
3	Mixed	0	2	0	0	0	1	0	0	0.9998
4	Mixed	1	0	0	0	0	0	1	0	0.9988
5	Mixed	0	0	0	2	0	0	0	1	0.9999
6	Standby	1	0	0	0	0	1	0	0	1
7	Standby	1	0	0	0	0	1	0	0	0.9992
8	Mixed	2	0	0	0	0	0	1	0	0.9991
9	Standby	0	0	1	0	0	0	1	0	0.9992
10	Mixed	2	0	0	0	0	1	0	0	0.9988
11	Standby	0	1	0	0	0	0	1	0	0.9993
12	Mixed	2	0	0	0	0	1	0	0	1
13	Active	0	2	0	0	1	0	0	0	0.9999
14	Standby	0	0	1	0	0	0	1	0	0.9995
	System reliability									0.9939
	System weight									170
	System cost									98

Table 4. A comparison between the results of the system reliability, system weight and system cost.

	Fyffe et al. [16]	Tavakkoli-Moghaddam and safari [50]	Soltani et al. [47]	Abouei Ardakan and Zeinal Hamadani [2]	Gholinezhad and Zeinal Hamadani [20]	Proposed Model
System reliability	0.97	0.9875	0.9863	0.99233	0.99266	0.9939
System weight	170	170	170	170	170	170
System cost	119	123	123	116	118	98

Figure 1. System structure.

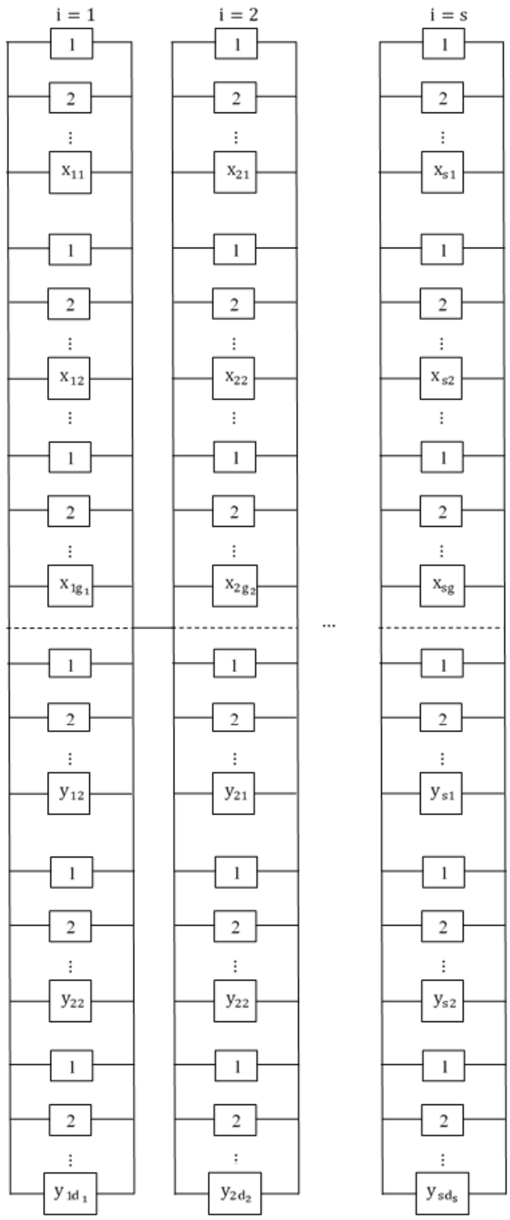


Figure 2. Encoding solution as a chromosome representation.

Type 1	2	2	0	0	0	0	3	0	1	0	1	0	3	0
Type 2	0	0	0	4	0	0	0	0	0	2	0	3	0	3

Type 3	0	2	0	1	3	0	0	0	1	0	1	0	2	0
Type 4	4	0	0	0	0	0	0	0	4	1	0	0	0	0
Type 1	1	0	0	1	0	0	3	0	0	0	1	1	0	0
Type 2	3	2	0	0	2	0	0	0	0	1	0	0	1	0
Type 3	0	0	0	0	0	0	0	0	1	0	1	0	0	0
Type 4	0	1	0	0	1	0	1	0	1	2	1	1	0	0

Figure 3. (a) Step one: Determine the steps. (b) Step second: Copy steps in offspring. Spin crossover operator in permutation presentation.

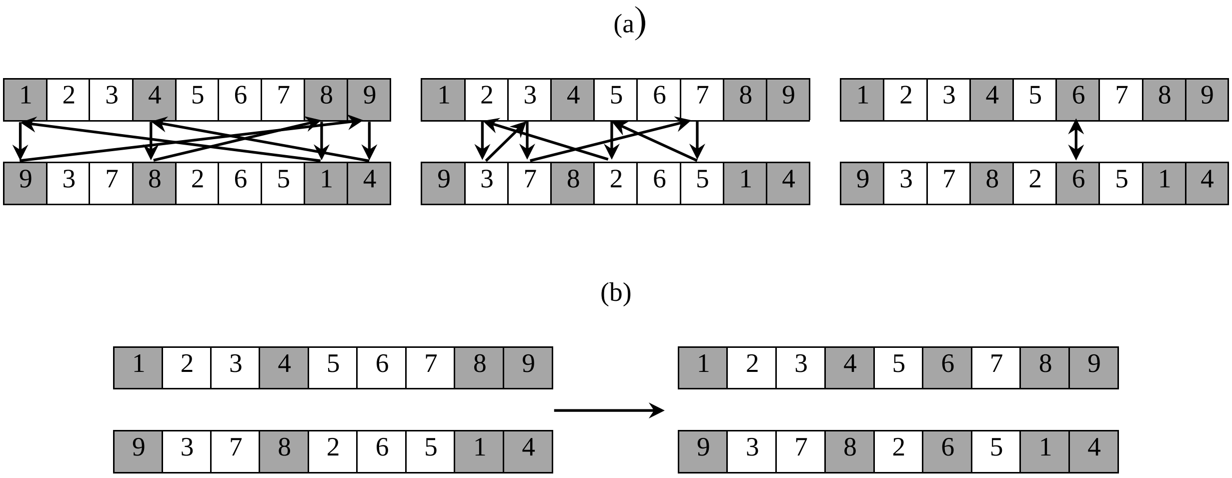


Figure 4. Inversion mutation operator in permutation presentation.

.1	.2	.3	.4	.5	.6	.7	.8	.9		.1	.5	.4	.3	.2	.6	.7	.8	.9

Figure 5. A comparison between the results of the subsystems reliability.

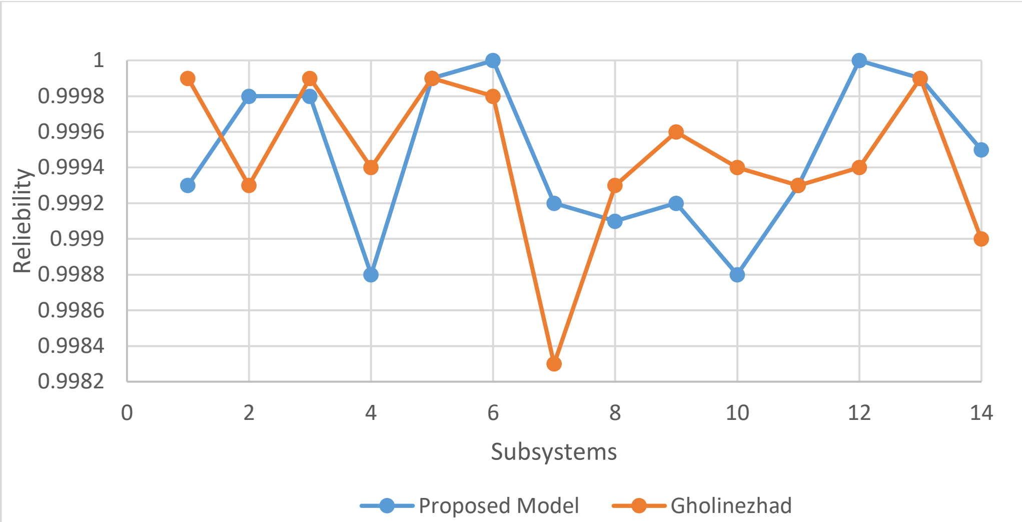
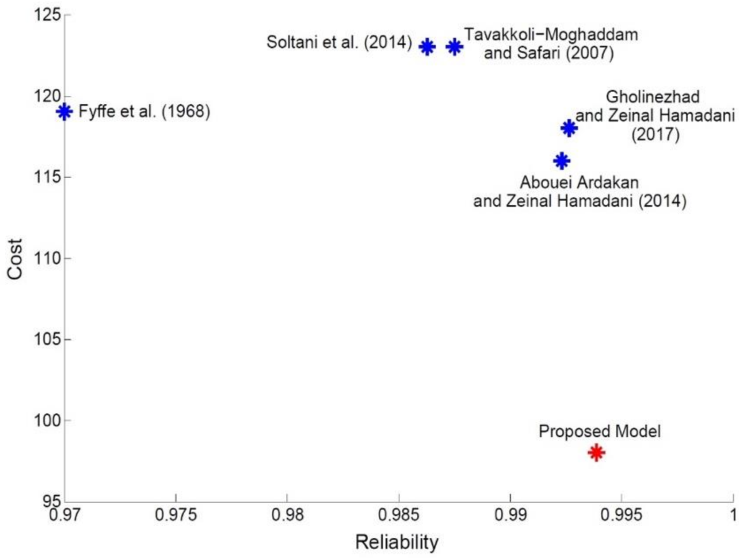


Figure 6. Comparison between the solutions based on system reliability and system cost.



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