Robust Design of Loss-Based Ideal Repetitive Group Sampling Plan under Uncertainty of Input Parameters

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Abstract
Integrating both loss and minimum angle method (MAM) as objective functions in the economic-statistical modeling of variable acceptance sampling plans (VASPs) can yield the most cost-effective plan with the ideal operating characteristic (OC) curve. Nevertheless, occurring crises can disrupt organizational input parameters, causing inefficiencies in providing solutions. This study develops the first robust designs of VASPs, accounting for the uncertainty of input parameters. Unlike previous studies that assume fixed inputs, this research considers deviations from nominal values to address parameter uncertainty. In this way, the challenge of parameter uncertainty's impact on the effectiveness of designs is investigated. We propose a solution procedure based on Particle swarm optimization (PSO). Findings from case studies reveal that (1) a marginal cost increase in the Cost-MAM model significantly reduces overall risks, (2) the repetitive group sampling plan yields lower costs and risks, and (3) tolerating increased costs is imperative to manage potential uncertainty.

Keywords: Robust design, Repetitive group sampling, Economic-statistical design, Loss function, Minimum angle method, Particle swarm optimization

1. Introduction

Businesses can navigate economic challenges by implementing effective policies aimed at minimizing costs and losses while attaining the desired level of quality aligned with consumer expectations. In production systems, specific measures can be employed to regulate the quality of raw materials and finished goods lots. Acceptance Sampling Plans (ASPs) serve to determine the acceptance or rejection of a lot [1]. The quality characteristic (QC) under inspection may manifest as either an attribute or a variable. In situations where both inspection costs and the desired quality level are exceptionally high, choosing variable ASP (VASP) over attribute ASP (AASP) is a more preferable option [2]. Our investigation into past applications of a military standard in the case study has motivated us to delve

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into current research. Given their adverse consequences highlighted by [3], our focus is on selecting appropriate policies for decision-making concerning lots. The overarching objective is to achieve higher quality while minimizing implementation costs, especially in scenarios with uncertain parameters. In this context, we review studies to outline various plans and different approaches to modeling under both certainty and uncertainty conditions.

Various ASPs have been commonly employed in quality control. These include single sampling (SS) [4], double sampling (DS) [5], resubmitted sampling (RS) [6, 7], multiple-dependent state sampling (MDS) [6, 8-10], Skip-lot Sampling (SkS) [11], and Sequential Sampling (SqS) [12] plans. This study focuses on the investigation of the repetitive group sampling (RGS) plan, because of its performance acceptability [8, 13-15]. Additionally, we assume a Normal distribution for the QC. The Normal distribution has been commonly employed in several studies to define a variable QC [5, 13, 14, 16].

Assessing plans requires an Economic-Statistical Design (ESD) to choose the most cost-effective option while ensuring adherence to the statistical criteria, thereby enhancing the Operating Characteristic (OC) curve. ESD models incorporating a sample size objective function (OBF) [8, 9, 11, 15] and a total cost OBF [10, 17] were extended. Taking losses into account, some researchers employed process capability indices (PCIs) in ASPs and optimized the sample size OBF [13, 14, 18, 19]. Another perspective involves including producer and consumer losses in the total cost OBF [6, 20]. The resulting plan becomes more favorable when it simultaneously approaches the ideal OC curve by utilizing the minimum angle method (MAM). Through the integration of loss and MAM OBFs, Fallah Nezhad and Zahmatkesh Saredorahi [21] and Fallahnezhad et al. [22] affirmed the superior performance of the RGS plan compared to other VASPs. They used grid search (GS) to optimize models, in which all feasible solutions are examined. Banihashemi et al. [8] expanded MDS and RGS plans by incorporating PCI and MAM. Under Non-normal distributions, Pawan Teh et al. [23] and Liaqat et al. [24] utilized MAM to develop a group chain sampling plan. So far, all the studies reviewed have focused on designing VASPs under certainty conditions.

However, inaccuracies may arise in the inspection process due to human or tool errors, introducing measurement uncertainty to the results. Various solutions addressing this uncertainty have been proposed, including incorporating inspection error [25, 26] and the utilization of fuzzy and neutrosophic approaches [27, 28]. On the other hand, the design of VASP through modeling necessitates the estimation of input parameters. The presence of uncertainty in estimating the effective input parameters can lead to inefficiencies in achieving the desired results. Addressing the design problem of VASP in the presence of such uncertainty aligns more closely with real-world scenarios. Examining the impact of input parameters on statistical process control models has underscored the critical importance of accurate estimation for achieving desirable results, as highlighted in [29]. In the context of uncertainty, recommended approaches can be found in existing literature [30, 31], although these methods may lack flexibility. Safaei et al. [32] applied a robust design (RD) approach to address the worst-case (wc) scenario for the robust ESD (RESD) of a control chart. Instead of incorporating uncertainty from all inputs, this approach introduces flexibility by incorporating the concept of a budget of uncertainty to counter the overly conservative outcomes associated with wc scenarios. The RD approach ensures that the obtained solution is robust against a wide range of uncertain parameter values. In other words, the solution is determined to perform well even if the actual values of the uncertain parameters deviate from their nominal values. In a similar study on RESD, Jafarian-Namin et al. [33] used interval estimates for influential input parameters and introduced an innovative genetic algorithm (GA)-based approach for optimization. Utilizing the particle swarm optimization (PSO), Salmasnia et al. [34] and Jafarian-Namin et al. [35] have conducted in the RD on models for process monitoring and control. The better performance of PSO over GA in ESD of control charts has already been validated [35-37]. Notably, the RD approach, which has proven effective in various contexts, has not yet been applied to the design of VASPs.

As indicated, numerous studies have investigated the ESD of VASPs under conditions where input parameters are certain. However, to enhance resistance strength and mitigate economic vulnerability during unforeseen crises, it is imperative to research the RD of VASPs is imperative under uncertain conditions. This study introduces novel aspects that set it apart from previous research. Unlike prior fixed-parameter models, this study explicitly accounts for parameter uncertainty by defining interval ranges. Thus, minimax optimization models are developed to minimize the maximum OBF across all potential values of uncertain parameters, ensuring robustness against wc scenarios. The introduction of the budget of uncertainty concept allows a flexible balance between robustness and economic efficiency. Consequently, our main objective is to introduce a novel RD approach that utilizes PSO to optimize minimax models for designing VASPs under uncertainty. This marks the first instance of such an endeavor. Pursuing this research could offer significant opportunities for further exploration by researchers in the field. Typically, our objectives include:

1. Proposing two models by considering loss OBF and integrating loss and MAM OBFs for the RESD of a VASP, i.e. RGS plan, when uncertainty exists in input parameters. By incorporating different shifts and scenarios in uncertain input parameters, the results of both models for designing the RGS plan with normal QC are examined to choose the preferred model.
2. Introducing a novel PSO-based solution procedure in optimizing VASPs. Prior to optimizing models, an orthogonal array design is employed to calibrate PSO factors.
3. Comparing the performance of RGS with other VASPs, including SS, DS, and MDS, in terms of sample size, risk, and loss by optimizing the preferred model to determine the desired VASP.
4. Applying RESD approach to real case studies of a VASP when lower specification limit (LSL) and upper specification limit (USL) are active.

The subsequent sections of this study unfold as follows: In Section 2, we outline the main assumptions. Subsequently, we delve into the RGS plan, loss and MAM functions, and the proposed model under uncertainty. Section 3 introduces a novel solution approach. Moving to Section 4, we scrutinize two real-world case studies to validate the proposed solution approach, while also conducting a comparative analysis of two models and four VASPs. Lastly, the study concludes with a summary of findings and suggestions for future research.

2. Model description

This research aims to design the RGS plan under uncertainty of input parameters by developing a robust model, incorporating the loss and MAM as OBFs. The mathematical equations of the RGS plan are presented in this context. Subsequently, we present the notations and assumptions, Taguchi loss function, MAM method, and the structure of the cost function. Lastly, we propose the first mathematical model under uncertainty.

2.1. Notations and assumptions

Table 1 provides an overview of the applied notations and abbreviations. To better characterize the general framewok of mathematical modeling, some assumptions are outlined and should be adhered to:

1. The QC follows a Normal distribution, expressed as $x \sim N(\mu,\sigma)$,
2. The standard deviation is historically known (note that the unknown standard deviation must be estimated beforehand),
3. Only either *LSL* or *USL* is active, and the k method is employed,
4. Items produced sequentially are chosen for inspection,
5. The proportion of nonconforming items (PNI) remains constant in submitted lots,
6. Each sampling plan is designed to assess the PNI,
7. Rectifying inspection is applied to any rejected lot,
8. No lot exhibits lower quality than preceding lots.

{Please insert Table 1 about here.}

2.2. RGS plan

When the measured value of a chosen item falls below the LSL, it is considered nonconforming. The calculation of the PNI for a lot is as follows:

$$p = P\left(x < LSL\right) = 1 - \Phi\left(\frac{\mu - LSL}{\sigma}\right) = 1 - \Phi\left(v\right), \tag{1}$$

where, according to Normal distribution, we have:

$$\Phi\left(v\right) = \int_{-\infty}^{v} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-z^2}{2}\right) dz\,. \tag{2}$$

Thus, the mean value of the QC can be calculated as follows:

$$\mu = \sigma\left[\Phi^{-1}\left(1-p\right)\right] + LSL. \tag{3}$$

If USL is active, the PNI of a lot is calculated as follows:

$$p = P\left(x > USL\right) = 1 - \Phi\left(\frac{USL - \mu}{\sigma}\right) = 1 - \Phi\left(v\right). \tag{4}$$

The probabilities of acceptance and rejection, along with the probability of choosing a new sample, are determined as follows, respectively:

$$\begin{aligned} P_a &= P\left(v \geq k_a\right) = 1 - \Phi\left(w_1\right), \\ P_r &= P\left(v < k_r\right) = \Phi\left(w_2\right), \\ P_{new} &= P\left(k_r \leq v < k_a\right) = \Phi\left(w_1\right) - \Phi\left(w_2\right), \end{aligned} \tag{5}$$

where:

$$w_1 = (k_a - v)\sqrt{n},$$
$$w_2 = (k_r - v)\sqrt{n}. \tag{6}$$

Calculating the operating characteristic (OC) curve at a specified quality level, denoted as p, is as follows:

$$P_a(p) = \frac{P_a}{P_a + P_r} = \frac{P_a}{1 - P_{new}} = \frac{1 - \Phi(w_1)}{1 - \Phi(w_1) + \Phi(w_2)}. \tag{7}$$

When p matches the acceptable quality level (AQL), we establish $w_1=w_{11}$, and $w_2=w_{21}$. At this juncture, we have:

$$P_a(p_1 = AQL) = \frac{1 - \Phi(w_{11})}{1 - \Phi(w_{11}) + \Phi(w_{21})}. \tag{8}$$

When p matches the limiting quality level (LQL), we establish $w_1=w_{12}$, and $w_2=w_{22}$. At this juncture, we have:

$$P_a(p_2 = LQL) = \frac{1 - \Phi(w_{12})}{1 - \Phi(w_{12}) + \Phi(w_{22})}. \tag{9}$$

Under active USL, the Equations (5-9) are respectively rewritten as follows:

$$P_a = P(v \geq k_a) = \Phi(w_2),$$
$$P_r = P(v < k_r) = 1 - \Phi(w_1), \tag{10}$$
$$P_{new} = 1 - P_a - P_r = \Phi(w_1) - \Phi(w_2),$$

$$w_2 = (v - k_a)\sqrt{n},$$
$$w_1 = (v - k_r)\sqrt{n}. \tag{11}$$

$$P_a(p) = \frac{P_a}{P_a + P_r} = \frac{\Phi(w_2)}{1 - \Phi(w_1) + \Phi(w_2)}. \tag{12}$$

$$P_a(p_1 = AQL) = \frac{\Phi(w_{21})}{1 - \Phi(w_{11}) + \Phi(w_{21})}. \tag{13}$$

$$P_a(p_2 = LQL) = \frac{\Phi(w_{22})}{1 - \Phi(w_{12}) + \Phi(w_{22})}. \tag{14}$$

Average sample number (ASN) is determined by multiplying the sample size by the expected frequency of sampling repetitions over the long term:

$$ASN = \frac{n}{1 - P_{new}}. \tag{15}$$

The execution of the RGS plan requires the predefinition of its three decision variables (DVs) as ($z_1=n$, $z_2=k_a$, $z_3=k_r$). The steps outlining its process are briefly summarized as follows:

1. A random sample with size n is taken as (x_1, x_2, ..., x_n) from the lot,
2. Following the computation of the sample mean $\left(\bar{x} = \sum_{i=1}^{n} x_i / n\right)$, the subsequent statistic is determined:

$$v = \frac{\bar{x} - LSL}{\sigma}. \tag{16}$$

When USL is active, v equals $\left(USL-\bar{x}\right)/\sigma$.

3. Decision-making. If $v \geq k_a$, the lot is accepted. If $v<k_r$, the lot is rejected. If $k_r \leq v<k_a$, go to the first step.

2.3. Taguchi loss function

Against the traditional notion of adhering strictly to specifications, Taguchi introduced a loss function designed to minimize deviations from the target value [38]. When LSL is active, a more-better loss function is formulated as follows:

$$L\left(x\right)=C_{cs}\left(x\right)=\frac{K}{x^2}, \tag{17}$$

where x is the QC value, $K=A\Delta^2$ is the coefficient of the loss function, A is the average quality loss, and Δ is the consumer's tolerance. When USL is active, $L(x)$ equals Kx^2 where $K=A/\Delta^2$. If both LSL and USL are active, $L(x)$ equals $K\left(x-T\right)^2$. The relationship among the target value of T, tolerance, and specification limits is as $T\pm\Delta=\left[LSL,USL\right]$. Note that the inspection cost per item (C_{ins}) and the producer's cost associated with repairing or replacing an item ($C_{pr}=B$) are regarded as constant, remaining unaffected by the QC value.

2.4. Structure of cost function

The components contributing to the expected loss cost, utilized in the OBF of the proposed model, are delineated as follows:

$$E_L=L_1+L_2+L_3. \tag{18}$$

where (note that $C_{pr}=B$ and $P_r(p)=1-P_a(p)$):

$$\begin{aligned}
L_1 &= ASN\left(C_{ins}+\int_{\delta}^{LSL} C_{pr}(x)f(x)dx+\int_{LSL}^{\infty} C_{cs}(x)f(x)dx\right),\\
L_2 &= \left(N-ASN\right)\left(\int_{\delta}^{\infty} C_{cs}(x)f(x)dx\right)P_a(p),\\
L_3 &= \left(N-ASN\right)\left(C_{ins}+\int_{\delta}^{LSL} C_{pr}(x)f(x)dx+\int_{LSL}^{\infty} C_{cs}(x)f(x)dx\right)P_r(p).
\end{aligned} \tag{19}$$

L_1 determines the expected cost of inspected items, encompassing the expenses associated with (1) inspecting sample items, (2) non-conforming items with QC values below the LSL, sent back to the producer, and (3) losses incurred due to QC deviations from the target value for items with QC values surpassing LSL, dispatched to the consumer. L_2 addresses the expected cost related to the remaining (uninspected) items within the lot when the decision involves accepting that lot. Notably, the remaining items are exempt from inspection costs, placing the burden of cost solely on the consumer. L_3 tackles the expected cost tied to the remaining (uninspected) items within the lot when the decision leads to rejecting that lot. It encompasses inspection costs, and producer and consumer expenses. Initially, all remaining items undergo inspection to categorize them as conforming or nonconforming. The producer is accountable for the repair or replacement costs of nonconforming items, and the consumer assumes the potential costs associated with the conforming items. When USL is active, we have:

$$
\begin{aligned}
&L_1 = ASN\left(C_{ins} + \int_{USL}^{\delta} C_{pr}(x)f(x)dx + \int_{-\infty}^{USL} C_{cs}(x)f(x)dx\right),\\
&L_2 = \left(N - ASN\right)\left(\int_{-\infty}^{\delta} C_{cs}(x)f(x)dx\right)P_a(p),\\
&L_3 = \left(N - ASN\right)\left(C_{ins} + \int_{USL}^{\delta} C_{pr}(x)f(x)dx + \int_{-\infty}^{USL} C_{cs}(x)f(x)dx\right)P_r(p).
\end{aligned}
\tag{20}
$$

2.5. MAM method

The ideal OC curve is achieved when all conforming lot are accepted, and all nonconforming lots are rejected. Soundararajan and Christina [39] introduced the MAM method to attain this ideal OC curve. The approach involves minimizing the angle θ illustrated in Figure 1, bringing the OC curve closer to its optimal configuration. The tangent of θ is determined as follows:

$$
\tan(\theta) = \frac{LQL - AQL}{P_a(AQL) - P_a(LQL)}. \tag{21}
$$

Given that AQL and LQL remain constant, and the goal is to minimize $\tan(\theta)$, it is imperative to maximize the denominator of the aforementioned fraction. In situations where an additional OBF, denoted as $g(S) = E_L$, needs minimization, an integrated function can be formulated as $g(S) / \left(P_a(AQL) - P_a(LQL)\right)$.

{Please insert Figure 1 about here.}

2.6. Proposed Models under Uncertainty

Assume that the values of the input parameters, which are unknown but bounded, are defined within a set denoted as UB. Utilizing a consistent percentage of positive and negative deviations from the nominal value enables the derivation of bounds for each parameter. Considering the uncertainty for all parameters in UB may lead to quite conservative results. To discern the trade-off between flexibility and conservatism, we define the budget of uncertainty $\left(\Gamma = 0, 1, \ldots, m_{UB}\right)$ where m_{UB} indicates the maximum number of uncertain parameters. Under $\Gamma = 0$, the outcome is exposed to uncertainty conditions. In contrast, when $\Gamma = m_{UB}$, the resulting conservative outcome is secure against uncertainty entirely. Indicating all possible scenarios by S', the vector of input parameters is defined as $I^{s'} = \left\{I_1^{s'}, I_2^{s'}, \ldots\right\}$ for each specific scenario $s' \in S'$. After generating various scenarios, the OBF value for each scenario s', denoted as $E_L^{s'}$, is computed. With a non-linear OBF, the economic-statistical model under uncertainty (M1) is expressed as follows:

$$
\begin{aligned}
&\min\left(\max_{s'\in S'} E_L{}^{s'}\right)\\
&\textit{Subject to}\\
&\qquad P_a(AQL)^{s'} \geq 1-\alpha^{Up} \quad \forall s'\in S'\\
&\qquad P_a(LQL)^{s'} \leq \beta^{Up} \quad \forall s'\in S'\\
&\qquad k_a{}^{s'} > k_r{}^{s'} \quad \forall s'\in S'\\
&\qquad n^{s'} \in N^+;\, k_a{}^{s'}, k_r{}^{s'} > 0
\end{aligned}
\tag{22}
$$

where the OBF indicates minimizing the wc scenario among all scenarios. The initial constraint reflects the producer's inclination to accept conforming items by the consumer under scenario s', with a higher probability. Therefore, the upper value α^{Up} is set for the producer's risk or Type-I error to determine a lower bound for $P_a(AQL)^{s'}$. In contrast, the second constraint accounts for the consumer's inclination to accept nonconforming products, albeit with a lower probability. Thus, $P_a(LQL)^{s'}$ is limited by the upper bound β^{Up} for consumer's risk or Type-II error. According to the third constraint, the acceptance number ($k_a{}^{s'}$) must surpass the rejection number ($k_r{}^{s'}$) under scenario s'. It is noteworthy that $n^{s'}$ is a positive discrete variable, while $k_a{}^{s'}$ and $k_r{}^{s'}$ are continuous variables.

Searching for ideal designs needs simultaneously optimizing the E_L and MAM functions. Thus, the economic-statistical model with integrated OBFs under uncertainty (M2) is expressed as follows:

$$
\begin{aligned}
&\min\left(\max_{s'\in S'} \frac{E_L{}^{s'}}{P_a(AQL)^{s'} - P_a(LQL)^{s'}}\right)\\
&\textit{Subject to}\\
&\qquad P_a(AQL)^{s'} \geq 1-\alpha^{Up} \quad \forall s'\in S'\\
&\qquad P_a(LQL)^{s'} \leq \beta^{Up} \quad \forall s'\in S'\\
&\qquad k_a{}^{s'} > k_r{}^{s'} \quad \forall s'\in S'\\
&\qquad n^{s'} \in N^+;\, k_a{}^{s'}, k_r{}^{s'} > 0
\end{aligned}
\tag{23}
$$

where the integrated OBF under scenario s' is formulated as stated in subsection 2.5. The constraints in Equation (23) is the same as those defined in Equation (22).

To transform the models into unconstrained forms and explore the optimal solution within the feasible region, the penalized OBF for a given solution (S) is expressed as follows:

$$
fp^{s'}(S) = OBF^{s'}(S) \times \left(1 + viol_1^{s'}(S) + viol_2^{s'}(S) + viol_3^{s'}(S)\right), \tag{24}
$$

where the violation terms are characterized as follows:

$$
\begin{aligned}
viol_1^{s'}(S) &= \max\left\{0, 1-\left(P_a(AQL)^{s'} / 1-\alpha^{Up}\right)\right\},\\
viol_2^{s'}(S) &= \max\left\{0, \left(P_a(LQL)^{s'} / \beta^{Up}\right)-1\right\},\\
viol_3^{s'}(S) &= \max\left\{0, 1-\left(k_a{}^{s'} / k_r{}^{s'}\right)\right\}.
\end{aligned}
\tag{25}
$$

The following section introduces an innovative PSO-based approach to solve the unconstrained models.

3. Solution Procedure

The aim is to determine optimal DVs through the optimization of the proposed models, specifically Mixed Integer Nonlinear Programming (MINLP). Obtaining optimal DVs by explicitly solving these MINLPs is nearly impossible due to several complexities. These include: (1) the nonlinear nature of OBFs and constraints, (2) the coexistence of discrete and continuous DVs, and (3) the uncertainty associated with estimating input parameters. Given the development of metaheuristics for obtaining near-optimal solutions within a reasonable timeframe, a novel PSO-based approach is introduced to provide the RD of the RGS plan. PSO has been widely used in solving similar problems [34-37]. It benefits from both global and local search capabilities to find solutions. To provide the RD of the RGS plan, this proposed PSO-based approach minimizes the wc scenario among all generated scenarios, as follows:

$$S' = N_R \binom{m_U}{\Gamma} + N_E \binom{m_U}{\Gamma}, \tag{26}$$

where in each possible combination, random scenarios involve the repetition of N_R times, while extreme scenarios involve the repetition of $N_E = 2^{\Gamma}$ times, both chosen randomly. Within random scenarios, Γ parameters get values within their bounds, selected randomly. Meanwhile, in extreme scenarios, Γ parameters obtain extreme values at the limits of their bounds. The remaining $m_{UB} - \Gamma$ parameters maintain nominal values.

The proposed solution procedure aims to minimize the maximum OBF across various scenarios while ensuring the feasibility of the solution, even when input parameters deviate from their nominal values. Figure 2 illustrates the steps of the RD optimization. Once the inputs are determined, the desired number of scenarios is generated. Then, the PSO-based solution procedure is implemented with an initial population, calculating the OBF across all scenarios. The wc scenario, which has the maximum OBF among the generated scenarios, is identified and then optimized to find the minimax solution. Throughout the optimization process, each particle tries to reach the best position by learning from its own movements and those of the entire population. Each particle may follow: (1) its current velocity, (2) its personal best, which is the best value experienced by the particle, or (3) the global best, which is the best solution found by the entire population up to a specific iteration. The procedure concludes when a predefined number of iterations is reached.

We set the PSO parameters according to Appendix A (in Supplementary Data) as inertia weight w=0.8, recognition and social learning factors c_1=1.5 and c_2=2.5, population size N_{Pop}=20, and iteration number N_{Itr}=100. For l=1,2,…,N_{Pop} and Itr=1,…,N_{Itr}, PSO characterizes each particle (Prt_l^{Itr}) through its position $X_l^{Itr} \sim U(b_{low}, b_{up})$ and velocity $V_l^{Itr} \sim U(-|b_{up} - b_{low}|, |b_{up} - b_{low}|)$, where b_{low} and b_{up} respectively represent the boundaries defining the lower and upper limits within the search space. For example, in designing the RGS, a position is established by uniformly selecting values {z_1=n, z_2=k_a, z_3=k_r} from their boundaries. Primary value of a discrete DV, such as sample size, is determined by ($Rv_i \sim U(0,1)$ is a random number):

$$z_i = \min\left\{ z_{i\,min} + \left[Rv_i \left(z_{i\,max} - z_{i\,min} + 1 \right) \right], z_{i\,max} \right\}. \tag{27}$$

{Please insert Figure 2 about here.}

4. Experimental results

To explore the influence on OBF in situations where precise estimates of model parameters are unavailable, and to consider diverse levels of uncertainty in these parameters, two minimax models were developed for various scenarios under uncertainty. This section compares the performance of the proposed models and four VASPs. The superior VASP optimized by the effective model is implemted in practice. Valuable insights are derived from a practical example and a case study, contributing to a comprehensive review.

4.1. Practical example

In an industrial example, the wall thickness of 2-inch plastic pipes is evaluated in batches of 2500 pieces per week (refer to the "example of creating a single sampling plan" on "support.minitab.com"). Nominal values for the input parameters are outlined in Table 2. First, PSO-based algorithm was compared with GS [21] in optimizing M1 and M2 models for RGS under certainty (Γ=0). Optimizing M1 using PSO and GS resulted in E_L=17830 in 7.52 seconds and E_L=19416 in 4569.16 seconds, respectively. Similarly, optimizing M2 using PSO and GS resulted in E_L=18084 in 7.67 seconds and E_L=19991 in 4566 seconds, respectively. Therefore, utilizing PSO is preferred due to its ability to produce better quality solutions in a shorter computational time.

In Appendix B (in Supplementary Data), a set of model input parameters was considered, as shown in Table S.3 (in Supplementary Data). Based on the results of the sensitivity analysis shown in Figure S.1 (in Supplementary Data), five input parameters were identified with significant effects on OBF. Consequently, UB was set as {α^{Up}, β^{Up}, K, N, AQL} where m_{UB}=5. For parameters within that set subject to uncertainty, deviations of 10% and 20% from the nominal value are taken into account. This implies m_{UB}=5 and a range of Γ=0,1,...,5. Additionally, we establish N_R=4 in our analysis. The outcomes for the M1 and M2 models are indicated in Table 3 and Table 4, respectively (MATLAB codes can be obtained upon request from the authors). In Table 3, the smallest E_L is observed when Γ=0. Conversely, the most conservative scenario with Γ=5 yields the highest E_L. The gradual rise in E_L becomes apparent as Γ increases. This upward trend reveals the following points:

- The rate of cost increase in 20% deviation is steeper than that of 10%. For instance, $E_{L(\Gamma=1)}$-$E_{L(\Gamma=0)}$ for 20% and 10% deviations results in 3506.41 and 1753.42, respectively.
- Initially, the E_L trend experiences an increase ($E_{L(\Gamma=1)}$-$E_{L(\Gamma=0)}$=3506.41 under a 20% shift). Conversely, subsequent results, such as 17.06, 1.21, 1.27, and 3.11, display a decreasing trend.

Our analysis demonstrates the superiority of RESD over standard ESD in handling parameter uncertainty. For instance, under a 10% shift, as shown in Table 3, standard ESD yields E_L=17830.52, whereas RESD shows varied E_L values as Γ increases, reflecting its adaptability to uncertainty. These results provide strong evidence for RESD's applicability in real-world scenarios where parameter uncertainty is a concern. After Γ=$m_{UB}^{0.5}$=$5^{0.5}$≈2, the oscillations in the trend appear to diminish [40]. The consistency of the outcomes for the 10% deviation becomes more prominent. Hence, opting for Γ=2 appears to be a suitable choice for addressing the present example. Typically, decision-makers should embrace some risks to handle potential uncertainties. Comparable findings can be derived from the M2 results in Table 4. Figure 3 compares the optimal costs and risks associated with RD for the proposed models under both 10% and 20% deviations. The visualization illustrates that a slight increase in the cost of M2 in comparison to M1 leads to a noteworthy reduction in the overall risks of the RGS plan. Consequently, the effective performance of M2 for RD is validated.

Utilizing M2 under Γ=2, we conducted a comparison of four sampling plans. Table 5 reveals that the RGS plan yields lower cost, ASN, and risk values (the DVs for each plan were equated to z_i values). Additionally, the OC curve of the RGS plan closely aligns with the ideal situation (see Figure 4). The comprehensive findings demonstrate the superior performance of the RGS plan. Consequently, its application is recommended for further evaluation. Table 6 indicates 104 samples taken from one received lot. A P-value=0.704>0.05 confirms the normality assumption based on the Anderson-darling index. Considering the optimal DVs by M2 under 20% shift and Γ=2, including (n, k_a, k_r)=(61, 2.25, 1.89), the inspection procedure is summarized in the following steps:

1. A random sample with size 61 is taken from the lot as (x_1, x_2, ..., x_{61})= (0.1735, 0.1810,...,0.1710),
2. After obtaining the sample mean $\bar{x} = 0.200043$ from the first step, the following statistic is calculated:
   $$v = \frac{\bar{x} - LSL}{\sigma} = \frac{0.200043 - 0.09000}{0.02500} \approx 4.40,$$
3. Decision-making. Since v=4.40≥k_a=2.25, the lot is accepted.

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4.2. Case study

A company manufactures a diverse range of spare parts for automobiles. On a daily basis, it receives batches containing 350 plano-convex glass lenses, which are integral components for producing a specific model of fog light. Its external diameter is a critical QC for assembly, with a targeted value of 57 mm. To assess it, the company has transitioned to utilizing RGS, departing from its previous approach based on MIL-STD-105E. Table 7 indicates the nominal values of the input parameters. The key distinction in this study, unlike previous examples, is the activation of USL. Following the execution of the solution algorithm, Table 8 and Table 9 indicate the outcomes of models M1 and M2, respectively. Figure 5 depicts the optimal costs and risks associated with both models under uncertainty conditions. Accordingly:

- The minimum (maximum) cost is attained in the most risky (most conservative) scenario with Γ=0 (Γ=5),
- The rate of cost escalation in the 20% shift surpasses that of the 10% shift,
- Initially, there is an upward trend in the cost changes, succeeded by a subsequent decline,
- The discernible shifts in the cost trend diminish notably after $\Gamma=5^{0.5}\approx 2$,
- After Γ=2, the uniformity of outcomes for the 10% shift becomes more evident,
- The superior efficacy of M2 is validated in the design of RGS.

- The superiority of RESD is confirmed over standard ESD in handling parameter uncertainty.

The results of the comparison among four VASPs are presented in Table 10 and Figure 6. Given that the RGS plan exhibits the most closed OC curve with the lowest associated cost, it has been chosen for the evaluation of the lot in this particular case study. Considering the DVs (n, k_a, k_r)=(65, 1.87, 1.48) obtained by optimizing M2 under 20% shift and Γ=2, a sample of 65 items was taken from one lot (see Table 11). The inspection procedure is summarized in the following steps:

1. A random sample with size 65 is taken from the lot as (x_1, x_2, ..., x_{65})= (57.05, 57.10,..., 57.05),
2. After obtaining the sample mean $\bar{x} = 57.05354$ from the first step, the following statistic is calculated:
$$v = \frac{USL - \bar{x}}{\sigma} = \frac{57.10 - 57.05354}{0.0222} \approx 2.09,$$
3. Decision-making. Since v=2.09≥k_a=1.87, the lot is accepted. If we used M1, the decision criteria would be closer to the threshold according to v=2.08≥k_a=1.88. In other words, decision making in M2 is associated with less risk.

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5. Conclusions

To enhance resistance strength and mitigate economic vulnerability in the face of unforeseen crises, it was imperative to conduct RD for VASPs under uncertain conditions. The proposed models involved some influential input parameters, challenging accurate estimation in reality. Introducing the concept of RD for the first time in VASP modeling aimed to provide an optimal policy considering the potential effects of uncertainty in input parameters. Evaluation of the RD for cost-based and cost-MAM models, utilizing practical examples and case studies, resulted in the presentation of optimal policies for the RGS plan. The findings demonstrated that the cost-MAM model exhibited superior performance in the RGS plan, resulting in the lowest risk under various budgets of uncertainty. Additionally, the RGS plan, when compared to SS, DS, and MDS plans, incurred lower costs and risks.

As indicated, some input parameters have a major impact on optimal solutions and can undermine the effectiveness of the models. When these parameters deviate from their nominal values, deterministic-based models become ineffective in the face of such uncertainty. Disregarding this can lead to additional costs. The proposed models, while upholding statistical conditions and minimizing costs, afford the decision-maker the flexibility to apply the desired level of protection against uncertainty. Indeed, for various levels of uncertainty in the parameters, there are observable changes in the total cost, with these incremental changes initially being noticeable. However, as the uncertainty budget further increases after specific points, the changes in the OBF become imperceptible. In essence, dealing with potential uncertainty necessitates taking some level of risk and paying for it. Recognizing the risks and associated costs, utilizing robust optimization in modeling VASPs provides decision-makers with a robust solution for managing potential uncertainties in real-world scenarios.

Pursuing the current research could offer opportunities for further exploration by researchers. In this context, additional aspects to expand include evaluating novel VASPs, extending new models, and proposing efficient solution approaches. Studies on RDs in experimental design [41] and system reliability [42] already exist, and applying the PSO-based RD in these contexts could be expanded. Investigating the RD approach in larger manufacturing systems could enhance the model's applicability to more complex real-world scenarios. However, this would also present challenges, such as increased parameter dimensionality and higher computational requirements. Moreover, the current approach has several limitations. These include dependence on accurate estimation of uncertain parameter ranges, the assumption of a uniform distribution for defining intervals, disregard for potential dependencies among uncertain parameters, and challenges in accounting for uncertainty in the model structure. Our upcoming investigation will focus on the RD of VASPs for life-testing under both certainty and uncertain conditions.

Supplementary Data

Supplementary data is available at:
file:///C:/Users/pc/Downloads/Supplementary%20data-SCI-2403-8879.pdf

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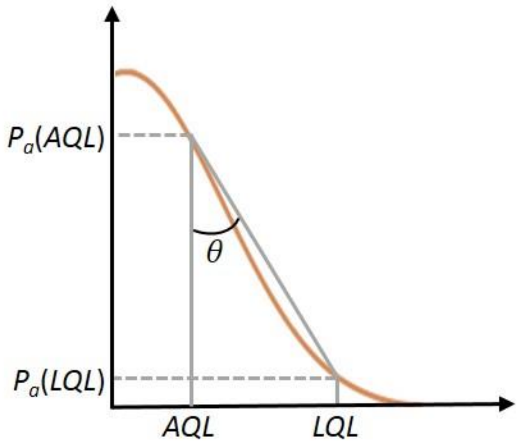

Figure 1. θ angle in OC curve

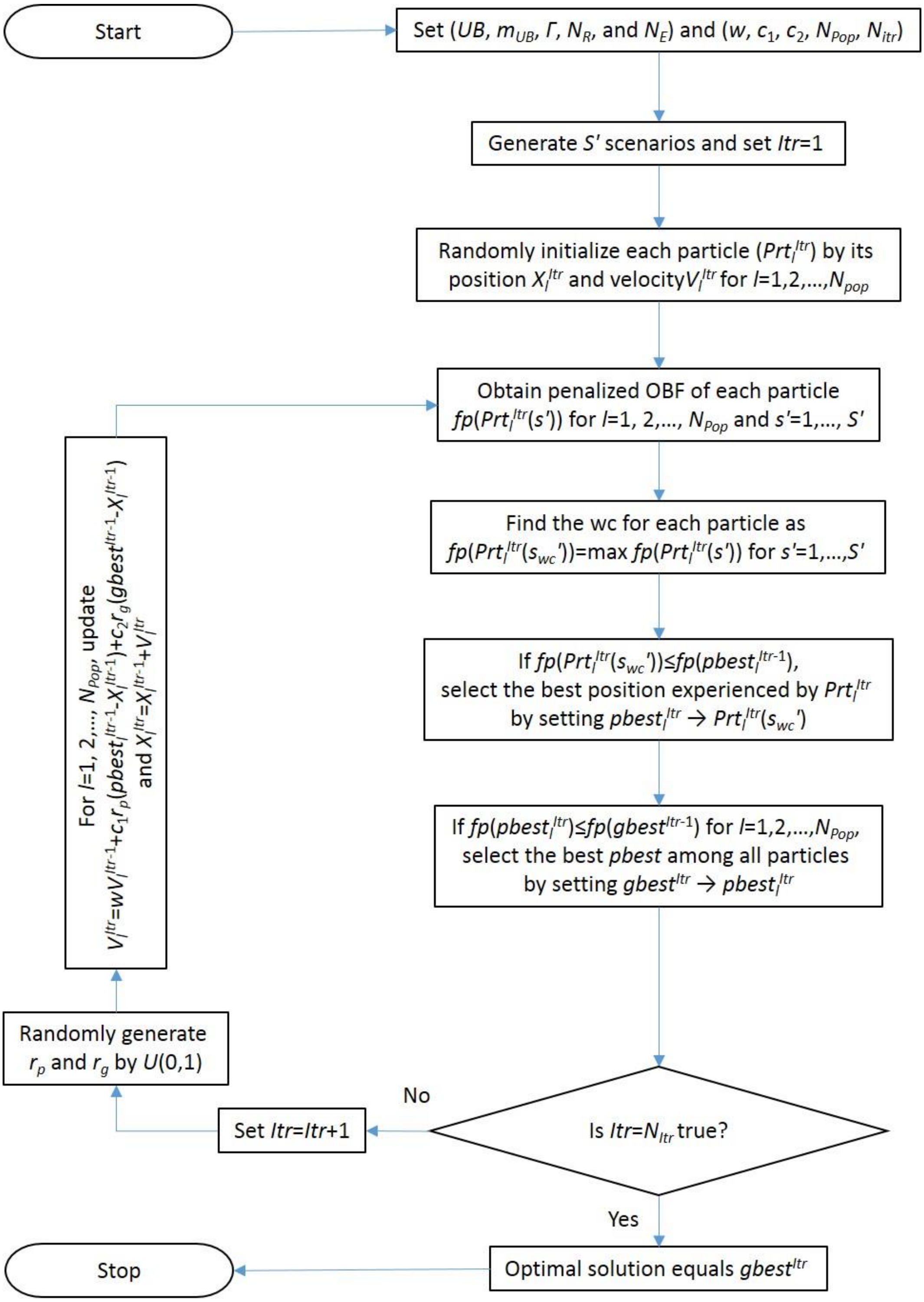

Figure 2. PSO-based solution procedure

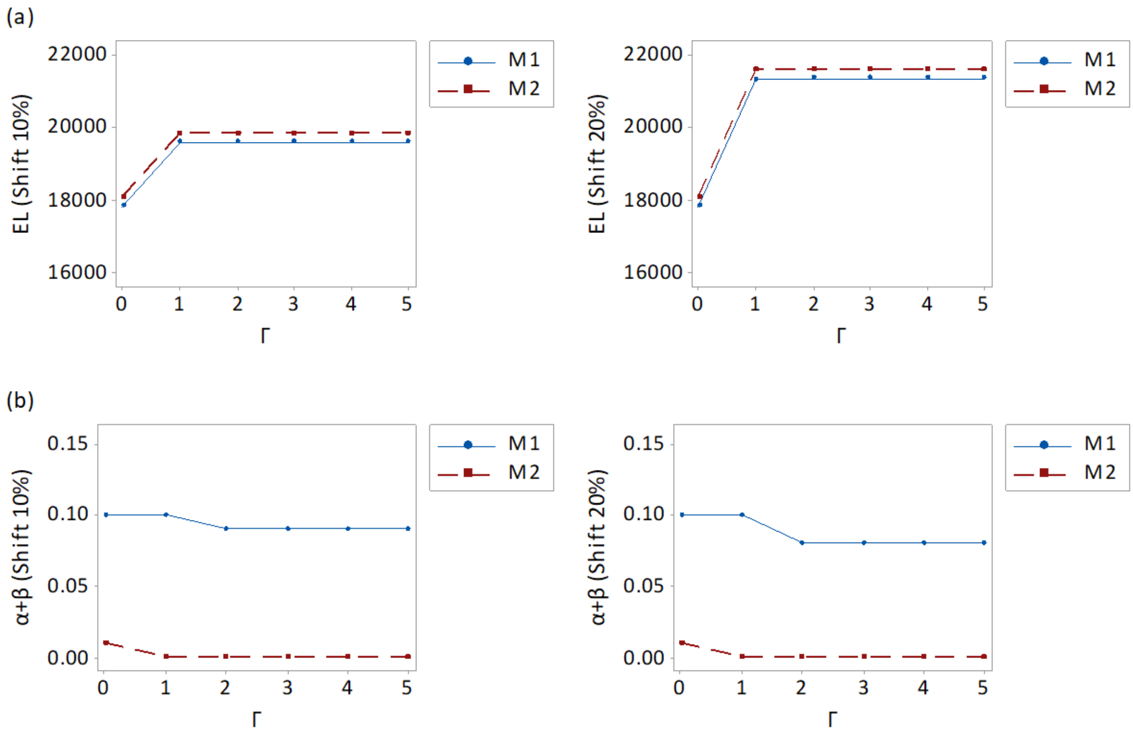

Figure 3. Robust design of RGS plan in the practical example using M1 and M2 to examine the changes in: (a) cost under 10% and 20% shift scenarios, (b) total risk under 10% and 20% shift scenarios for different uncertainty budgets

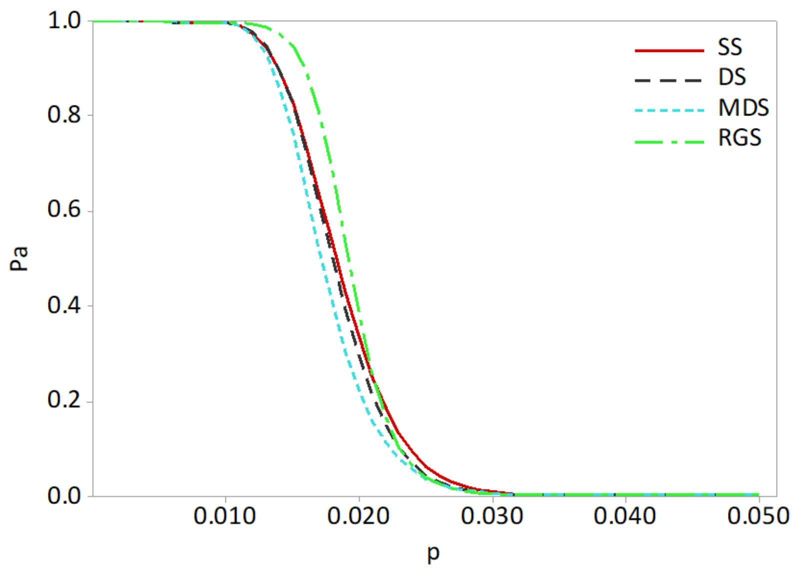

Figure 4. OC curves of four VASPs with M2 model in the practical example under 20% shift

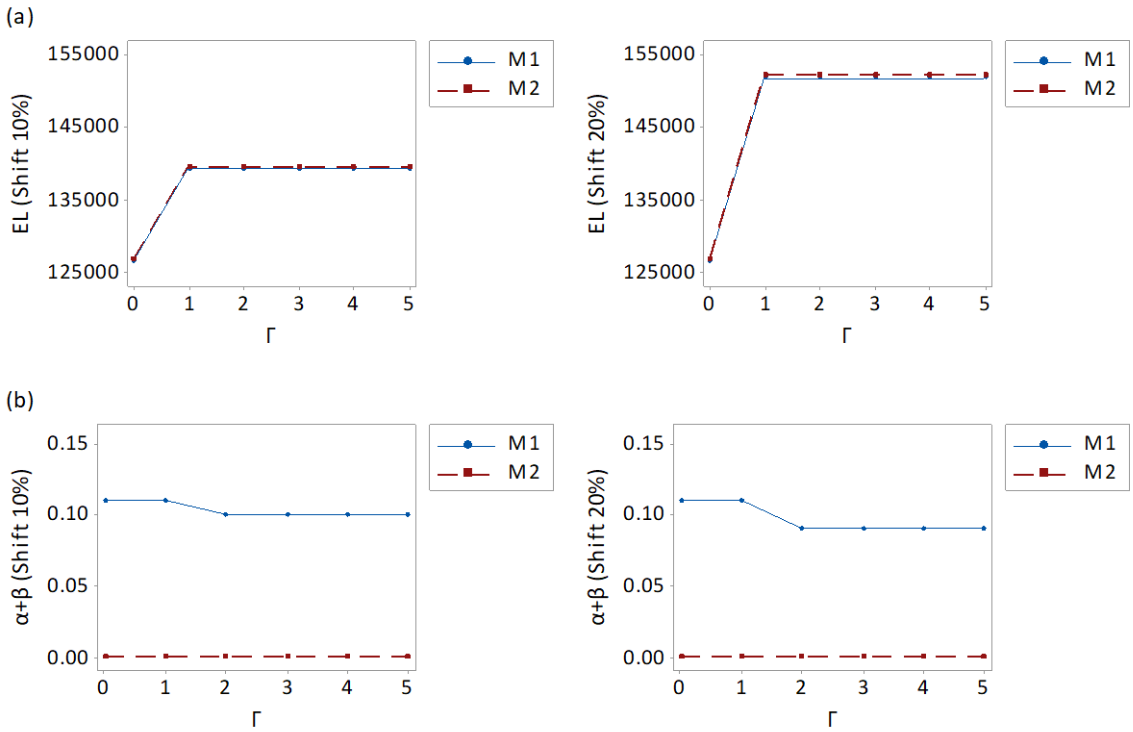

Figure 5. Robust design of RGS plan in the case study using M1 and M2 to examine the changes in: (a) cost under 10% and 20% shift scenarios, (b) total risk under 10% and 20% shift scenarios for different uncertainty budgets

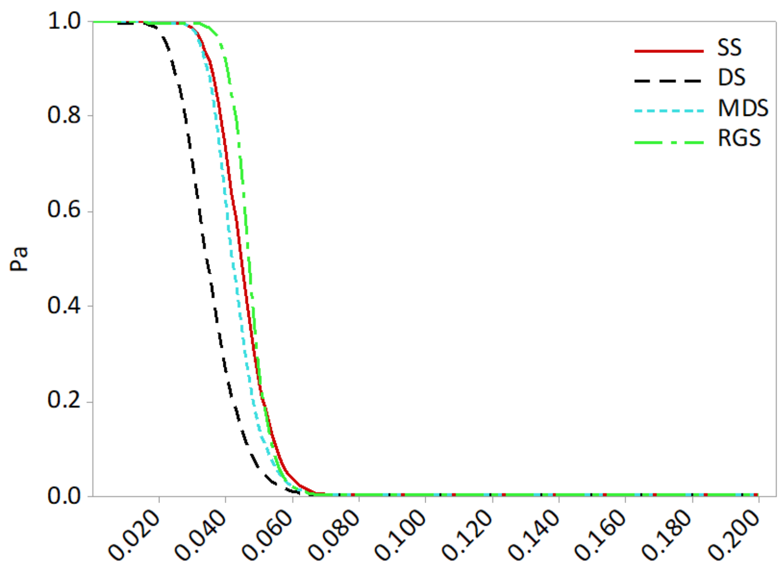

Figure 6. OC curves of four VASPs with M2 model in the case study under 20% shift

Table 1. Notations and abbreviations

Notation	Description
Decision variables	
$n^{s'}$	The sample size under scenario s'
$k_a^{s'}$, $k_r^{s'}$	The acceptance and rejection numbers under scenario s', respectively
z_i	Representing the decision variables of a RGS plan (i=1,2,3)
Cost parameters	
$E_L^{s'}$	The expected total cost (loss) for scenario s'
L_1	The expected cost of inspected items
L_2	The expected cost of items remaining in the lot if it is accepted
L_3	The expected cost of items remaining in the lot if it is rejected
C_{ins}	The cost of inspecting an item
C_{pr}, C_{cs}	The producer's and consumer's loss functions, respectively
K	The coefficient of consumer's loss function
A	The average quality loss
B	The producer's cost to repair or replace an item
Process parameters	
α, β	The producer's and consumer's risks or Type-I and Type-II errors, respectively
α^{Up}, β^{Up}	The upper bounds on Type-I and Type-II errors in constraints, respectively
USL, LSL	Upper and lower specification limits, respectively
x	A specified value of the QC
v	A standardized normal value
δ	The minimum possible value of the QC
Δ	The consumer's tolerance
T	The target value
μ, σ	The mean and standard deviation of the QC, respectively
N	The lot size
$f(x)$	The probability density function (pdf)
$\Phi(v)$	The cumulative distribution function (cdf)
AQL, LQL	The acceptable and limiting (rejectable) quality levels, respectively
p	The nonconforming proportion of the lot
$P_a(p)$, $P_r(p)$	The probability of accepting and rejecting a lot at the quality level p, respectively
P_{new}	The probability of selecting a new sample when implementing the RGS plan
ASN	The average sample number
S	A set of DVs as a solution
$fp^{s'}(S)$	A Penalized OBF$^{s'}$ for a given solution S under s'
$viol^{s'}(S)$	A violation term associated with a constraint for a given solution S under s'
θ	The angle in MAM method
Solution parameters	
UB	The set of bounded input parameters for uncertainty study
S'	All possible scenarios
s'	A specific scenario (s'=1,…,S')
s_{wc}'	The worst case (wc) scenario among S'
m_{UB}	The maximum number of uncertain parameters
Γ	The budget of uncertainty
$I^{s'}$	The vector of input parameters under s'
N_E	The repetitions of extreme scenarios among S'
N_R	The repetitions of random scenarios among S'
w	The inertia weight
c_1, c_2	The recognition and social learning factors, respectively

N_{Pop}	The population size
N_{Itr}	The iteration number
Prt_l^{Itr}	The l^{th} particle in the Itr^{th} iteration, where l=1,2,…,N_{Pop} and Itr=1,…,N_{Itr}
X_l^{Itr}, V_l^{Itr}	The position and velocity of Prt_l^{Itr}, respectively
U	Uniform distribution
b_{low}, b_{up}	The lower and upper limits of DVs within the search space, respectively
Rv_i, r_p, r_g	The generated random numbers according to Uniform distribution between (0,1)
$pbest_l^{Itr}$, $gbest_l^{Itr}$	The particle best and global best of Prt_l^{Itr}, respectively
Abbreviations	
ASP	Acceptance sampling plan
AASP	Attribute ASP
AQL	Acceptable quality level
DS	Double sampling
DV	Decision variable
ESD	Economic-statistical design
GA	Genetic algorithm
GS	Grid search
LQL	Limiting quality level
M1	The economic-statistical model with loss-based OBF under uncertainty
M2	The economic-statistical model with integrated OBFs under uncertainty
MAM	Minimum angle method
MDS	Multiple-dependent state sampling
MINLP	Mixed integer nonlinear programming
OBF	Objective function
OC	Operating characteristic
PCI	Process capability index
PNI	Proportion of nonconforming items
PSO	Particle swarm optimization
QC	Quality characteristic
RD	Robust design
RESD	Robust ESD
RGS	Repetitive group sampling
RS	Resubmitted sampling
SkS	Skip-lot sampling
SqS	Sequential sampling
SS	Single sampling
VASP	Variable ASP
wc	Worst-case

Table 2. Nominal values of the input parameters in the practical example

μ	σ	USL	LSL	Δ	δ	A	K
0.2025"	0.025"	0.315"	0.09"	0.1125"	0.065"	11.1	0.14
N	AQL	LQL	C_p	α^{Up}	β^{Up}	C_{ins}	C_{pr}
2500	0.01	0.03	1.50	0.05	0.10	7	25

Table 3. Optimal results of M1 with RGS plan under different shift scenarios and uncertainty budgets

Shift	Γ	z_1	z_2	z_3	E_L	$P_a(AQL)$	$P_a(LQL)$	ASN	tan(θ)	$\alpha+\beta$
	0	29	2.23	1.76	17830.52	1.00	0.10	41.99	0.02	0.10
	1	31	2.22	1.76	19583.94	1.00	0.10	43.24	0.02	0.10

10%	2	31	2.23	1.78	19592.03	1.00	0.09	43.39	0.02	0.09
	3	30	2.24	1.76	19591.85	1.00	0.09	43.41	0.02	0.09
	4	31	2.22	1.78	19592.55	1.00	0.09	43.11	0.02	0.09
	5	28	2.27	1.73	19596.44	1.00	0.09	44.87	0.02	0.09
20%	0	30	2.23	1.76	17830.40	1.00	0.10	42.52	0.02	0.10
	1	31	2.23	1.76	21336.81	1.00	0.10	43.46	0.02	0.10
	2	33	2.22	1.78	21353.87	1.00	0.08	45.48	0.02	0.08
	3	34	2.21	1.79	21355.08	1.00	0.08	45.53	0.02	0.08
	4	33	2.22	1.78	21353.81	1.00	0.08	45.67	0.02	0.08
	5	31	2.25	1.74	21356.92	1.00	0.08	46.99	0.02	0.08

Table 4. Optimal results of M2 with RGS plan under different shift scenarios and uncertainty budgets

Shift	Γ	z_1	z_2	z_3	E_L	$P_a(AQL)$	$P_a(LQL)$	ASN	tan(θ)	$\alpha+\beta$
10%	0	59	2.25	1.89	18084.41	1.00	0.00	80.78	0.02	0.01
	1	60	2.25	1.89	19844.75	1.00	0.00	82.15	0.02	0.00
	2	60	2.25	1.89	19844.45	1.00	0.00	82.11	0.02	0.00
	3	60	2.25	1.89	19844.55	1.00	0.00	82.13	0.02	0.00
	4	60	2.25	1.89	19844.43	1.00	0.00	82.11	0.02	0.00
	5	60	2.25	1.89	19844.40	1.00	0.00	82.11	0.02	0.00
20%	0	59	2.25	1.89	18084.58	1.00	0.00	80.81	0.02	0.01
	1	61	2.25	1.89	21603.83	1.00	0.00	83.35	0.02	0.00
	2	61	2.25	1.89	21603.85	1.00	0.00	83.36	0.02	0.00
	3	61	2.25	1.89	21603.78	1.00	0.00	83.35	0.02	0.00
	4	61	2.25	1.89	21603.94	1.00	0.00	83.37	0.02	0.00
	5	61	2.25	1.89	21603.87	1.00	0.00	83.36	0.02	0.00

Table 5. Optimal results of four VASPs using M2 in the practical example under Γ=2

Shift	Plan	z_1	z_2	z_3	z_4	z_5	E_L	$P_a(AQL)$	$P_a(LQL)$	ASN	tan(θ)	$\alpha+\beta$
10%	SS	137	2.09	-	-	-	20280.15	1.00	0.01	137.00	0.02	0.01
	DS	76	214	2.17	1.87	2.07	19921.13	1.00	0.01	94.16	0.02	0.01
	MDS	99	2.14	0.42	2	-	19987.77	1.00	0.01	99.00	0.02	0.01
	RGS	60	2.25	1.89	-	-	19844.45	1.00	0.00	82.11	0.02	0.00
20%	SS	140	2.09	-	-	-	22052.30	1.00	0.01	140.00	0.02	0.01
	DS	141	51	2.11	2.08	2.11	22045.90	1.00	0.00	141.13	0.02	0.01
	MDS	101	2.14	1.85	2	-	21752.80	1.00	0.00	101.00	0.02	0.01
	RGS	61	2.25	1.89	-	-	21603.85	1.00	0.00	83.36	0.02	0.00

Table 6. Obtained Data from a shipment in the practical example (starting from the left side in order from top to bottom)

0.1735	0.1926	0.2175	0.1892	0.1786	0.1942	0.2097	0.1694	0.1994	0.2267	0.1675	0.2083
0	0	0	0	0	0	0	0	0	0	0	0
0.1810	0.2086	0.2046	0.1903	0.2112	0.2296	0.1750	0.1800	0.1708	0.2046	0.2309	0.2075
0	0	0	0	0	0	0	0	0	0	0	0
0.2543	0.2560	0.2331	0.2366	0.1984	0.1640	0.1800	0.2311	0.1867	0.2423	0.2238	0.2277
0	0	0	0	0	0	0	0	0	0	0	0

0.1803	0.1861	0.2135	0.1898	0.1518	0.2195	0.1636	0.2061	0.1593	0.1507	0.1890	0.1463
0	0	0	0	0	0	0	0	0	0	0	0
0.2028	0.2261	0.1894	0.2070	0.2039	0.2159	0.2260	0.1804	0.2095	0.2167	0.1805	0.2471
0	0	0	0	0	0	0	0	0	0	0	0
0.2001	0.1821	0.2160	0.2135	0.2034	0.2028	0.2107	0.2189	0.1917	0.2073	0.1627	-
0	0	0	0	0	0	0	0	0	0	0	
0.1931	0.2124	0.2143	0.2146	0.1882	0.2303	0.1710	0.1414	0.1807	0.2087	0.2135	-
0	0	0	0	0	0	0	0	0	0	0	
0.1585	0.2489	0.1593	0.2357	0.2054	0.1997	0.2238	0.2064	0.1797	0.1973	0.2033	-
0	0	0	0	0	0	0	0	0	0	0	
0.1974	0.1612	0.1564	0.1817	0.1968	0.1689	0.2187	0.2405	0.2466	0.2201	0.1732	-
0	0	0	0	0	0	0	0	0	0	0	

Table 7. Nominal values of the input parameters in the case study

μ	σ	USL	LSL	Δ	δ	A	K
57	0.0222	57.10	56.90	0.10	57.12	11.1	0.11
N	AQL	LQL	C_p	α^{Up}	β^{Up}	C_{ins}	C_{pr}
350	0.025	0.075	1.50	0.05	0.10	7	25

Table 8. Optimal results of M1 with RGS plan under different shift scenarios and uncertainty budgets

Shift	Γ	z_1	z_2	z_3	E_L	$P_a(AQL)$	$P_a(LQL)$	ASN	tan(θ)	$\alpha+\beta$
10%	0	17	1.87	1.34	126539.04	0.99	0.10	26.16	0.06	0.11
	1	17	1.87	1.35	139174.77	0.99	0.10	25.83	0.06	0.11
	2	18	1.87	1.35	139180.97	0.99	0.09	27.35	0.06	0.10
	3	18	1.86	1.37	139181.01	0.99	0.09	26.70	0.06	0.10
	4	18	1.86	1.36	139180.81	0.99	0.09	26.99	0.06	0.10
	5	18	1.86	1.36	139180.81	0.99	0.09	27.04	0.06	0.10
20%	0	18	1.85	1.37	126539.21	0.99	0.10	26.04	0.06	0.11
	1	18	1.86	1.35	151810.45	0.99	0.10	26.61	0.06	0.11
	2	18	1.88	1.34	151823.77	0.99	0.08	28.52	0.06	0.09
	3	19	1.86	1.37	151823.32	0.99	0.08	28.40	0.05	0.09
	4	20	1.85	1.38	151823.78	0.99	0.08	28.69	0.05	0.09
	5	19	1.86	1.38	151823.54	0.99	0.08	28.01	0.06	0.09

Table 9. Optimal results of M2 with RGS plan under different shift scenarios and uncertainty budgets

Shift	Γ	z_1	z_2	z_3	E_L	$P_a(AQL)$	$P_a(LQL)$	ASN	tan(θ)	$\alpha+\beta$
10%	0	63	1.87	1.48	126916.42	1.00	0.00	82.71	0.05	0.00
	1	64	1.87	1.48	139557.60	1.00	0.00	83.78	0.05	0.00
	2	64	1.87	1.48	139557.60	1.00	0.00	83.78	0.05	0.00
	3	64	1.87	1.48	139557.58	1.00	0.00	83.77	0.05	0.00
	4	64	1.87	1.48	139557.61	1.00	0.00	83.78	0.05	0.00
	5	64	1.87	1.48	139557.56	1.00	0.00	83.77	0.05	0.00
	0	63	1.87	1.48	126916.43	1.00	0.00	82.71	0.05	0.00
	1	65	1.87	1.48	152198.35	1.00	0.00	84.78	0.05	0.00
20%	2	65	1.87	1.48	152198.35	1.00	0.00	84.78	0.06	0.00

3	65	1.87	1.48	152198.34	1.00	0.00	84.78	0.06	0.00
4	65	1.87	1.48	152198.33	1.00	0.00	84.77	0.06	0.00
5	65	1.87	1.48	152198.35	1.00	0.00	84.78	0.06	0.00

Table 10. Optimal results of four VASPs using M2 in the case study under Γ=2

Shift	Plan	z_1	z_2	z_3	z_4	z_5	E_L	$P_a(AQL)$	$P_a(LQL)$	ASN	tan(θ)	$\alpha+\beta$
10%	SS	152	1.70	-	-	-	140036.03	1.00	0.00	152.00	0.05	0.00
	DS	78	171	1.82	1.49	1.71	139638.53	1.00	0.00	95.34	0.05	0.00
	MDS	106	1.76	1.56	1.00	-	139713.80	1.00	0.00	106.00	0.05	0.00
	RGS	64	1.87	1.48	-	-	139557.60	1.00	0.00	83.78	0.05	0.00
20%	SS	154	1.70	-	-	-	152683.83	1.00	0.00	154.00	0.06	0.00
	DS	79	170	1.82	1.49	1.71	152277.51	1.00	0.00	96.08	0.06	0.00
	MDS	111	1.75	1.44	2.00	-	152382.52	1.00	0.00	111.00	0.06	0.00
	RGS	65	1.87	1.48	-	-	152198.35	1.00	0.00	84.78	0.06	0.00

Table 11. Obtained Data from a shipment in the case study (starting from the left side in order from top to bottom)

57.05	57.05	57.05	57.04	57.10	57.00	57.08	57.08	57.07	57.04
57.10	57.08	57.05	57.05	57.07	57.02	57.08	57.02	57.00	57.05
57.06	57.03	57.05	57.05	57.05	57.04	57.10	57.00	57.08	-
57.08	57.07	57.10	57.08	57.05	57.05	57.07	57.02	57.08	-
57.02	57.00	57.06	57.03	57.05	57.05	57.05	57.04	57.10	-
57.00	57.08	57.08	57.07	57.10	57.08	57.05	57.05	57.07	-
57.02	57.08	57.02	57.00	57.06	57.03	57.05	57.05	57.05	-

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