A Data-Driven Model for the Energy-Efficient No-Wait Flexible Flow Shop Scheduling Problem with Learning and Deteriorating Effects

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Abstract

This work aims to investigate an energy-efficient no-wait flexible flow shop problem considering deteriorating and learning effects under uncertainty. To do this, a data-driven decision-making framework is developed in this research. At the outset, a multi-objective mathematical model is proposed for the research problem that minimizes the makespan, total tardiness, and total energy consumption. Then, to tackle uncertainty, a data-driven approach based on the fuzzy robust optimization, Seasonal Autoregressive Integrated Moving Average and Support Vector Regression methods is developed. Afterwards, to solve the proposed model, a hybrid approach based on the LP-Metric method and metaheuristic algorithms is proposed. The achieved outputs confirm the appropriate performance of the developed data-driven approach. Based on the obtained results, the developed hybrid metaheuristic algorithm shows an appropriate performance in both computational time and solution quality metrics. Also, the outputs indicate that the objective functions of the proposed model have increased when the due date parameter increases. Additionally, results show that with the increase in the absolute value of the learning coefficient, the first, second, and third objective functions of the model have decreased.

Keywords: No-wait flexible flow shop; Energy-efficient scheduling; Data-driven model; Metaheuristic algorithms; Time series algorithm

1. Introduction

In today's competitive world, effective sequencing and scheduling is a necessity for gaining a competitive advantage and improve the market share. Scheduling is the process of sorting, controlling and optimizing work and workload in a manufacturing or production process, which can have a significant impact on the efficiency of processes [1]. In general, scheduling is one of the critically important tasks of service and production systems that has a significant impact on the productivity of the company [2,3]. One of the most studied scheduling problems that has several real-world applications in different fields is the No-Wait Flexible Flow Shop Problem (NWFFSP). The NWFFSP has some differences from the traditional flow shop problem. For example, in the NWFFSP, no holding up is permitted among two consecutively used machines for any job. Indeed, by starting the process of a job on the first machine, this job should be processed without disruption until the end of its processing on the last machine [3,4]. Since this problem plays an important role in many real-world production operations, such as pharmaceutical processing, steel making, chemical processing, etc., investigating the mentioned problem can help managers to improve the performance of their manufacturing systems [5].

In the traditional scheduling problems, only classic and general objectives have been considered by researchers (e.g., minimizing the completion times and tardiness). However, recently, other indicators and objective have attracted the attention of researchers. For instance, in the nowadays complex and uncertain conditions, energy is a critically important element to support economic and social progress and build a better quality of life. Therefore, responsible consumption of resources (energy) is a critical issue in this time. In this way, in manufacturing systems, the attention of researchers and managers has drastically attracted toward energy-efficient systems for reaching resource-efficient manufacturing, which is considered as a crucial problem [6,7].

In the traditional scheduling problems, the processing time of jobs considered as a constant value. However, researchers indicated that this assumption may be wrong according to the learning and deteriorating effects [8,9]. In this regard, when the deteriorating effect is considered, a delay in processing a job may result in increasing its processing time [10,11]. On the other hand, in scheduling with learning effect, the processing time of a job may be reduced by the repetition of the processing operations [12]. The mentioned concepts are two crucial phenomena in the scheduling literature and there are several examples from their real-world application that the interested readers can see [12,13]. Hence, owing to the importance of these concepts, the current work attempts to incorporate them into the research problem.

Motivated by the real-world cases and owing to the importance of the above-mentioned points, this study attempts to investigate the energy-efficient no-wait flexible flow shop scheduling problem (EENWFFSP). For this purpose, in this research, a data-driven decision-making framework has been proposed. At the outset, a multi-objective programming model (MOPM) is suggested for the research problem. Then, to tackle the uncertainty, an efficient data-driven approach based on the fuzzy robust programming, SARIMA, and SVR methods is developed. Given the intricate nature of the research problem, metaheuristic algorithms are employed to address and solve the proposed model effectively. Overall, in comparison with the previous studies in the relevant literature, the following advantages can be mentioned for this research: (i) this is the first paper that investigates the energy-efficient no-wait flexible flow shop problem considering learning and deteriorating effects under uncertainty, (ii) this work developed an efficient data-driven approach to tackle uncertainty, (iii) this research develops efficient hybrid metaheuristic algorithm is utilized to tackle and resolve the research problem.

2. Literature review

In recent years, numerous studies have addressed workshop flow scheduling. For instance, [14] introduced a stochastic flow scheduling model considering deterioration and learning effects, solved with a mixed integer programming and fireworks algorithm, outperforming other methods. Later, [15] tackled the same issue with a two-population evolutionary algorithm, focusing on minimizing delay. [16] applied a hybrid artificial bee colony algorithm for parallel jobs, considering operator learning, improving performance and diversity in results.

[17] used a fuzzy mathematical model and self-adaptive fish swarm algorithm to address outsourcing in flow scheduling, proving more efficient than CPLEX. [2] developed a time-uncertain scheduling model using interval gray numbers and bee colony algorithms, also showing superior results. Energy consumption was the focus of [18], who presented an improved iterative greedy algorithm.

Further studies include [19], who proposed a water wave optimization algorithm tested on large and small samples, outperforming competitors. [20] applied genetic algorithms and robust models for assembly line scheduling with uncertain times, and [21] offered a fuzzy approach to manage uncertainties in worker learning rates.

Recent advances include a real-time multi-objective evolutionary algorithm [22] for flow scheduling and [23] who developed a repetitive greedy algorithm with resource constraints.

Finally, [24] integrated reinforcement learning and heterogeneous graph neural networks for better accuracy.

Despite advancements, gaps remain, particularly in combining energy efficiency, learning, and deterioration effects in flexible no-wait flow scheduling. This paper addresses these gaps by proposing a data-driven, hybrid metaheuristic solution for this issue. Key contributions include a novel machine learning model and an efficient hybrid algorithm for energy-efficient scheduling. A summary of the research literature is shown in the supplementary materials S.1.

3. Problem definition and uncertainty modelling

3.1. Defining the research problem and proposing the MOPM

Consider a flow shop manufacturing system. Suppose that there are N jobs (indexed by j) that should be processed on M machines (indexed by i) with R positions (indexed by r). For machines, there are L speed levels (indexed by l) that each job can be processed at a level. The parameters of the research problem are as follows. p_{jil} shows the normal processing time of job j in machine i at level l, and d_j is the due date for job j. Moreover, α_{ji} and β_{ji} respectively represent the learning and deteriorating effects for job j on machine i. Also, EN_{il} shows the energy consumption of machine i at level l. On the other hand, the decision variables of the research problem are as follows. p'_{rjil} is the actual processing time of job j that processed on r-th position of machine i at level l, which calculated based on the learning and deteriorating effects. s_{rjil} represents the start time of job j that processed on r-th position of machine i at level l, C_{rji} shows the completion time of job j that processed on r-th position of machine i, CM_{rj} is the completion time of job of j in r-position of the last machine, C_{max} denotes the makespan, and x_{rjil} is a binary variable that equal to 1 if job j is assigned to r-th position of machine i at level l. According to the mentioned points, the research problem (energy-efficient NWFFSP) can be formulated as follows.

$$Min\ Z1 = C_{max} \tag{1}$$

$$Min\ Z2 = \sum_{j} Max\left(CM_{Rj} - d_j, 0\right) \tag{2}$$

$$Min\ Z3 = \sum_r \sum_j \sum_i \sum_l E_{il} . p'_{rjil} . x_{rjil} \tag{3}$$

$S.t:$

$$\sum_r \sum_l x_{rjil} = 1 \qquad \forall j, i \tag{4}$$

$$\sum_j \sum_l x_{rjil} = 1 \qquad \forall r, i \tag{5}$$

$$s_{rjl} + p'_{rijl} \le s_{rj(i+1)l} \qquad \forall r, j, l, i = 1, 2, \ldots, m-1 \tag{6}$$

$$s_{rjil} + p'_{rjil} \le s_{(r+1)jil} \qquad \forall j, i, l, r = 1, 2, \ldots, n-1 \tag{7}$$

$$p'_{rjil} = \left(p_{rjil} + \alpha_{ri} . s_{rjil} \right) . r^{\beta_{ri}} \qquad \forall r, j, i, l \tag{8}$$

$$C_{rji} \ge \sum_l s_{rjil} + p'_{rjil} \qquad \forall r, j, i \tag{9}$$

$$\sum_l s_{rjil} \ge C_{r(j-1)i} \qquad \forall r, i, j = 2, \ldots, n \tag{10}$$

$$\sum_l s_{rjil} \ge C_{(r-1)ji} \qquad \forall j, i, r = 2, \ldots, n \tag{11}$$

$$C_{rji} \le BM . \sum_l x_{rjil} \qquad \forall r, j, i \tag{12}$$

$$CM_{rj} \ge C_{rji} \qquad \forall r, j, i \tag{13}$$

$$s_{rjil} - \left(s_{rjil'} + \left(p'_{rjil} . x_{rjil} \right) \right) = 0 \qquad \forall r, j, i, l \tag{14}$$

$$C_{rji} - C_{rj(i-1)} \ge \sum_l p'_{rjil} * x_{rjil} \qquad \forall r, j, i \tag{15}$$

$$C_{max} \ge CM_{Rj} \qquad \forall j \tag{16}$$

$$CM_{rj}, C_{rji}, C_{max}, p_{rjil}, p'_{rji}, s_{rjil} \ge 0 \qquad \forall r, j, i \tag{17}$$

$$x_{rjil} \in \{0,1\} \qquad \forall r, j, i \tag{18}$$

Relation (1) is the first objective function (OF) that minimizes the makespan. Relation (2) is the second OF that aims at minimizing the total tardiness. Also, the third OF (relation (3)) minimizes the total energy consumption. Constraint (4) indicates that each job only can be processed on one position and one level. Constraint (5) shows that in each position of each machine only one job can be processed. Constraints (6)-(15) are the scheduling constraint that calculate start time and completion time of jobs based on the learning and deteriorating effects. It should be noted that BM is a big positive number. Relation (16) calculates the makespan. Finally, relations (17) and (18) define the range of variables.

3.2. Uncertainty modeling

3.2.1. FRO

One of the common approaches to investigate an optimization problem under uncertainty is to consider fuzzy parameters that were widely used in the literature. Studying optimization problems under fuzzy environment uncertainty is crucial due to the pervasive nature of uncertainty in real-world decision-making. There are different methods to deal with the fuzzy uncertain environment that one of the efficient and widely employed of them is the chance-constrained fuzzy programming model ($CCFP$). The $CCFP$ is a popular method of posibilistic programming that aims at dealing with uncertainty. This approach is based on mathematical concepts, particularly the utilization of the expected value of fuzzy numbers, necessity (Nec), and possibility (Pos) [25,26]. To better understand, see the following compact model (model (19)) where f is the deterministic parameter of objective function, $\tilde{c}$ shows the fuzzy parameter of the objective function, A, B, and S are the deterministic coefficient of constraints, and $\tilde{d}$, $\tilde{L}$, and $\tilde{N}$ are the fuzzy coefficient of constraints. Also, y and x are the decision variables. It should be noted that the fuzzy parameters have trapezoidal fuzzy distribution with four critical points (i.e., $\left(\tilde{\theta} = \theta_{(1)}, \theta_{(2)}, \theta_{(3)}, \theta_{(4)}\right)$).

$$Min\, Z = f\,.y + \tilde{c}.x$$

$$A.x \geq \tilde{d} \qquad (19)$$

$$B.x = \tilde{L}$$

$$S.x \le \tilde{N}.y$$

Now, by considering σ_j as the satisfaction level of j-th uncertain constraint, the CCFP counterpart for Model (19) can be formulated as Model (20) [25,27].

$$Min\, Z = f.y + \frac{c_1 + c_2 + c_3 + c_4}{4}.x$$

$$A.x \ge \left(1 - \sigma_j\right).d_2 + \sigma_j.d_1$$

$$B.x \ge \left(1 - \frac{\sigma_j}{2}\right).\left(\frac{L_1 + L_2}{2}\right) + \left(\frac{\sigma_j}{2}\right).\left(\frac{L_3 + L_4}{2}\right) \tag{20}$$

$$B.x \le \left(1 - \frac{\sigma_j}{2}\right).\left(\frac{L_3 + L_4}{2}\right) + \left(\frac{\sigma_j}{2}\right).\left(\frac{L_1 + L_2}{2}\right)$$

$$S.x \le \left(\left(1 - \sigma_j\right).N_2 + \sigma_j.N_1\right).y$$

Moreover, according to the literature, the robust counterpart for Model (20) can be written as Model (21) where $E[Z]$ is the objective function of Model (20), η shows the penalty coefficient for the optimality robustness, π_i represent the penalty coefficients for the feasibility robustness, and $Zmax$ is the worst value of the objective function shown ($Zmax = f.y + c_4.x$). See [25,27,28] to read more details.

$$\begin{aligned} Min\, Z = E[Z] + \eta.(Zmax - E[Z]) + \pi_1.\left(d_4 - \left(1 - \alpha_j\right).d_3 + \alpha_j.d_4\right) \\ + \pi_2.\left(L_4 - \left(1 - \frac{\alpha_j}{2}\right).\left(\frac{L_1 + L_2}{2}\right) + \left(\frac{\alpha_j}{2}\right).(\frac{L_3 + L_4}{2})\right) \\ + \pi_3.\left(\left(1 - \frac{\alpha_j}{2}\right).\left(\frac{L_3 + L_4}{2}\right) + \left(\frac{\alpha_j}{2}\right).\left(\frac{L_1 + L_2}{2}\right) - L_1\right) \\ + \pi_4.\left(\left(1 - \alpha_j\right).N_2 + \alpha_j.N_1 - N_1\right) \end{aligned} \tag{21}$$

$$A.x \ge \left(1 - \alpha_j\right).d_3 + \alpha_j.d_4$$

$$B.x \ge \left(1 - \frac{\alpha_j}{2}\right).\left(\frac{L_1 + L_2}{2}\right) + \left(\frac{\alpha_j}{2}\right).\left(\frac{L_3 + L_4}{2}\right)$$

$$B.x \le \left(1-\frac{\alpha_j}{2}\right).\left(\frac{L_3+L_4}{2}\right)+\left(\frac{\alpha_j}{2}\right).\left(\frac{L_1+L_2}{2}\right)$$

$$S.x \le \left(\left(1-\alpha_j\right).N_2+\alpha_j.N_1\right).y$$

3.2.2. Seasonal Auto-Regressive Integrated Moving Average algorithm (SARIMA)

The deterioration rate parameter plays a crucial role in the current model and is subject to uncertainty. To handle this, time series algorithms can be employed to predict and estimate the parameter. In this study, the SARIMA was selected due to the presence of seasonality in the data. Using data-driven models to estimate parameters by considering various influencing factors is more efficient than heuristic approaches. This leads to increased accuracy in decision-making derived from scheduling models [29,30]. SARIMA is a combination of the Auto-Regressive Integrated Moving Average (ARIMA) model and the Seasonal Autoregressive (SAR) model[31,32]. The main steps of the SARIMA algorithm are presented in the Supplementary Materials S2.

3.2.3. SVR

One of the important parameters in the designed model is the processing time. The mentioned parameter, which is considered as fuzzy, depends on different features. In order to consider different features in estimating the amount of processing time, regression methods are used, in this article, Support Vector Regression (SVR) algorithm is used. SVR algorithm is a machine learning algorithm used for regression and prediction problems. This algorithm is based on the SVM (Support Vector Machine) method for classification, with the difference that instead of separating data into different categories, it tries to find a function to predict continuous values [33,34]. Using the data-driven SVR algorithm increases the accuracy in estimating the processing time parameter, which in turn enhances the accuracy of the scheduling problem [35]. The main steps of the SVR algorithm are presented in the Supplementary Materials S3.

4. Solution method

4.1. LP-Metric

The LP-Metric method is a well-known MODM approach that is commonly employed for solving multi-objective models. The main advantage of this approach is its easy concept and implementation which makes it understandable for practical managers. In the following, we briefly define this approach. Suppose w_i is the importance of i -th OF, Z_i is the mathematical relation of i -th objective function, Z_i^- represents the negative ideal solution for i -th OF, and Z_i^+ denotes the positive ideal solution for i -th OF. According to the above-mentioned definitions, the formulation of the LP-Metric method can be written as Relation (22) (for the minimization objective functions).

$$Z_{LP} = \sum_i w_i \cdot \frac{(Z_i - Z_i^+)}{Z_i^- - Z_i^+} \tag{22}$$

4.2. Simulated Annealing (SA)

SA is a metaheuristic algorithm introduced by Kirkpatrick et al. [36] , renowned for its effectiveness in addressing optimization problems, draws inspiration from the physical process of solid annealing. the SA algorithm initiates by generating an initial solution. Subsequent to this, the algorithm evaluates this initial solution using an objective function. To enhance the initial solution, it employs various operators that generate neighboring solutions. For each of these neighboring solutions, their respective objective function values are calculated. The algorithm then computes the difference in the objective function values between the initial solution and its neighbors. If the objective function value of a neighboring solution shows improvement over the initial one, this new solution is adopted in place of the previous one. However, if the objective function of the old solution is better than the new solution, the new solution can be accepted with a positive probability that is calculated based on the Boltzmann function ($e^{\Delta T}$) where T represents the corresponding temperature and Δ shows the difference between the objective function of the initial solution and neighborhood solution [37].

4.2.1. Solution structure

One of the most important parts of implementing the metaheuristic algorithms is to design a appropriate solution structure. In this regard, in this section, the structure of the solution is explained. The structure that is considered to display the solution in this research includes two main parts involving the jobs and speed levels. For example, suppose there are 5 jobs to process and 3 speed levels for machines. The structure of the solution is as described in Figure 1. In this figure, the first row shows the jobs and the second row demonstrates the speed levels. For example, in Figure 1, job 3 is processed first on a machine at speed level 1, then job 2 is processed on machine at speed level 3.

4.2.2. Neighborhood solution

In this section, the method of generating the neighborhood solution is described. In this research, in order to design the neighborhood solution, two operators have been used as described below. To better understand, Figure 2 illustrates these operators.

(i) Swap: Two elements of the solution are chosen randomly and their places are changed with each other.

(ii) Inversion: Two points are chosen randomly within a solution, and then the positions of the elements between these two points are reversed.

4.3. Genetic algorithm (GA)

GA is one of the widely used meta-heuristic algorithms in solving optimization problems [38,39]. Initially, a population of chromosomes is generated by the algorithm, and the fitness function of each chromosome is evaluated. Subsequently, parents are selected using a strategy such as a roulette wheel. Next, new chromosomes are created through the application of operators like mutation and crossover. This methodology has been extensively employed in research literature and has demonstrated favorable performance. Below, we outline the primary configuration of this algorithm for solving the proposed model.

4.3.1. chromosome

In this study, the structure of chromosome for the GA is completely like to the structure of the SA presented in Section 4.2.1.

4.3.2. Crossover

Crossover is one of the most important parts of the GA algorithm that plays a crucial role in improving the fitness functions of the chromosomes. In this research, in order to implement the crossover operator, the single point method, as described in Figure 3, has been used. In this approach, first, two parents are selected randomly and then one cut point is randomly selected in each of the parents. Afterwards, the elements of cut points of parents are substituted with each other. In this operator, maybe some repeated elements are generated in the offspring that can be revised by identifying the repeated element and substituting it with the correct element (see Figure 3).

4.3.3. Mutation

Another crucial operator of the GA is the mutation, which aims at improving the initial population of chromosomes. In the current paper, the mutation operator for the GA algorithm is similar to the neighbor solution creation operator in the simulated annealing method explained in Section 4.2.2.

4.4. Particle Swarm Optimization (PSO)

The PSO algorithm was introduced by Eberhart & Kennedy [40] and is one of the popular algorithms to solve complex problems. The PSO is a swarm intelligence-based algorithm that is inspired by the social behavior of birds, bees, and fish [41–43]. Within this approach, every particle within the group is characterized by its position and velocity. The position of a particle represents a potential solution to the optimization problem, while its velocity indicates both the direction and magnitude of movement. By leveraging the collective knowledge of the particles and their neighbors, each particle continuously adjusts its movement to search for the optimal solution. This process allows for the identification of the best solution through iterative refinement. In each iteration, the velocity and position of the particles are updated using relations (23) and (24) where Vel_i and X_i respectively show the velocity and position of particle i, $pbest_i$ represents the best position of particle i and $gbest$ represents the best position obtained so far. r_1 and r_2 are random numbers between zero and one, c_1 and c_2 demonstrate the acceleration coefficients, respectively. Also, w is the inertia coefficient.

$$Vel_i(k+1) = w.Vel_i(k) + c_1.r_1.\left(pbest_i - X_i(k)\right) + c_2.r_2.\left(gbest - X_i(k)\right) \tag{23}$$

$$X_i(k+1) = X_i(k) + Vel_i(k+1) \tag{24}$$

4.4.1. Solution structure

In this research, the solution structure of the PSO algorithm is similar to the simulated annealing method. However, since the PSO technique is an algorithm developed in the continuous solution space, the Random Key (RK) method is used to generate the solution. In this approach, first, a sequence of real numbers is created, where each generated number has a position in the vector. Then the numbers are sorted in ascending order, the position number of the numbers in the sorted vector indicates the job number. For an example, see Figure 4.

4.5. Hybrid GA-SA

In this research, to improve the performance of the GA, we combine it with the SA algorithm. In this regard, at the outset, the population of chromosomes is formed and the fitness function is computed. Afterwards, by employing the Boltzman function of the SA, the new solutions are chosen (the flowchart of the proposed GA-SA is provided in the Supplementary Materials S4).

4.6. Hybrid PSO-SA

In the current work, to concurrently benefit from the advantages of the PSO and SA algorithms, we have combined them using the following procedure. In the first step, the initial swarm is generated and the OF is computed to determine $Pbest$ and $Gbest$. In the developed algorithm, the Boltzman operator of the SA is employed to update $Pbest$. The flowchart of the proposed PSO-SA is provided in the Supplementary Materials S4.

5. numerical results

5.1. Input data

In this section, the input data of the research problem is stated. It should be noted that some parameters of the model such as the learning coefficient and deteriorating coefficient are estimated in the next section using data mining techniques. In this section, other parameters of the model will be estimated according to the research literature and experts' opinions. The current work considers three different speed level for machines namely (1) slow, (2) normal, and (3) fast. In this regard, we define a conversion factor for processing speed level (denoted by s_i) that considered equal to $s_1 = 0.8$, $s_2 = 1$, and $s_3 = 1.2$. Also, suppose that PP_{ji} shows the expected processing time for job j on machine i, that estimated using the data mining approaches in the next section. Now, the value of p_{jil} can be calculated using Relation (25). Also, the value of EN_{il} is considered equal to $E_{i1} = 0.6$, $E_{i2} = 1$ and. $E_{i3} = 1.5$

$$p_{jil} = \frac{PP_{ji}}{s_l} \qquad \forall j, i, l \tag{25}$$

5.2. Estimating data using SARIMA and SVR

5.2.1. Estimating data using SARIMA

In order to estimate the rate of deterioration in the devices and facilities of the studied factory, since the data related to the rate of deterioration of the machinery has been recorded on a monthly basis for 8 years, this parameter is estimated using time series algorithms that are based on the data available in past years. Therefore, since the deterioration rate is recorded monthly and 8 years of data are available, there are currently 96 data records available. Figure 5 shows the estimation value of the deterioration rate for the current model.

It can be seen that the value of deterioration rate is also 1.05. In Table 1, SARIMA algorithm with other time series algorithms is compared, and it can be seen that the error rate of SARIMA algorithm is lower than other algorithms.

5.2.2. Estimating data using SVR

One key parameter in the current research model is processing time, the duration an operator takes to complete a task. Various factors impact this, including:

- Machine depreciation: Refers to the machine's age and usage.

- Operator's experience: Duration of the operator's specialized work.

- Tool quality: Rated from 1 to 10 based on quality control.

- Work shift: The factory runs three shifts (morning, evening, night), each divided into two, totaling six parts.

- Season: Seasonal effects (cold/heat) influence operator efficiency.

- Operator's age and gender: Both factors affect the operation speed.

Due to different machines and tasks, processing time is treated as a fuzzy number. A regression model estimates this, and Pearson correlation identifies key factors before applying the SVR algorithm. Figure 6 shows the heatmap diagram of components and processing time.

According to the outputs of Figure 6, it can be seen that the user experience is more related to the activity time. After that, things like work shift, gender, operator's age, depreciation rate of machines, quality of tools, and season affect the time of doing the activity is ordered respectively.

By following the steps mentioned in Section 3, the step of building the regression model to estimate the amount of activity time in the form of fuzzy numbers should be implemented. To implement the model, the training and testing data were separated, and, 70% of the data are selected for training and 30% for testing. By separating these data and running the model, the models should be evaluated according to performance and accuracy evaluation indicators. The evaluation metrics of the regression model to estimate the activity and work time data are shown in Table 2.

It can be seen that the desired parameters are estimated according to the designed model with 91% accuracy. In Table 3, you can see the model parameters estimation in this section.

5.3. Parameters setting

The parameter setting section is provided in the Supplementary Materials S5.

5.4. The outputs of the algorithms

This section details the outcomes of applying the Multi-Objective Optimization Model (MOPM) using the methods introduced earlier. It's important to note that the meta-heuristic algorithms employed here are integrated with the LP-Metric method, leading to the prefix "LP" being added to each algorithm's name. The research involved solving the model for two different sets of problems: small-scale and large-scale problems. To this end, 20 large-sized and 20 small-sized instances were created. Each problem was solved ten times, and the best result achieved by each algorithm, along with the computational time, was recorded. Table 4 presents the results for the small-sized instances. For evaluating the effectiveness of the algorithms, two key metrics were used: the quality of the solutions and the computational time. In the context of small-sized test problems in which obtaining the global optimal solution is feasible, the PRE (Percentage Relative Error) metric was utilized for evaluating the solution quality. This metric is calculated as per Relation (26), where A_{sol} represents the solution derived from the algorithm and E_{sol} is the solution achieved from the exact approach.

$$PRE = \frac{A_{sol} - E_{sol}}{E_{sol}} \times 100 \tag{26}$$

Figure 7 compares the algorithms in terms of the computational time criterion and Figure 8 compares the performance of the algorithms based on the PRE criterion. As can be seen in achieved results, the algorithms developed in this study achieved optimal and near-optimal solutions within a more feasible computational timeframe when compared to the exact method. The findings indicate that the LP-SA algorithm excels in terms of time efficiency, while the LP-GA-SA algorithm stands out for the quality of the solutions it produces.

Additionally, Table 5 displays the results produced by the algorithms for large-sized instances. In these large-sized test cases, where the global optimal solution isn't accessible, the Relative Percentage Deviation (RPD) metric is used to evaluate solution quality. This metric is determined according to Relation (27), with B_{sol} representing the best solution found across all algorithms. Table 5 lists the RPD values for each objective function. Moreover, Figure 8 illustrates a comparison of the algorithms' performance based on CPU time for the large-sized instances. Moreover, for the statistical validation, the Least Significant Deviation (LSD) chart is depicted for algorithms in terms of the solutions' quality (RPD criterion) in Figure 9. Based on this figure, the developed L-GA-SA has significantly better performance in comparison with the other algorithms.

$$RPD = \frac{A_{sol} - B_{sol}}{B_{sol}} \times 100 \qquad (27)$$

As can be seen in Tables 4, 5, and Figure 10, the LP-GA-SA algorithm has the best performance in terms of the quality of the obtained solutions, and the LP-SA algorithm showed the best performance in terms of CPU time. The comparisons are shown in Supplementary Materials S6.

5.5. Sensitivity analysis

5.5.1. Learning coefficient

Since this research has included the learning effect, in order to investigate the role of this parameter in the research problem, in this section, the sensitivity analysis of this parameter is discussed. In this regard, Figure 11 shows the behavior of the objective functions with respect to the change in this parameter. As can be seen in this figure, the learning coefficient parameter has a positive role in the research problem, and with the increase in the absolute value of the learning coefficient, the first, second, and third objective functions of the model have decreased.

5.5.2. Deteriorating effect

This section focuses on examining the role of the deterioration factor in the research problem. To do so, the problem has been solved with different values assigned to this parameter, and the results have been reported. By analyzing the outcomes obtained for varying deterioration factor values, valuable insights are gained regarding its influence on the problem and any discernible trends or patterns. The reported findings shed light on the significance of the deterioration factor and its implications within the research problem. Figure 12 shows the behavior of the research problem based on the changes in this parameter. As can be seen in this figure, by increasing this parameter, the values of all objective functions become worse.

5.6. Managerial Insight

This article aims to design a flow shop scheduling model considering machine decay, the learning effect, and energy consumption. A key innovation is using data-driven methods to estimate parameters with a future-oriented approach, helping managers plan realistically based on potential future events. Traditional models often rely on intuition and past data, neglecting future impacts. With growing data and machine learning algorithms, these can now predict parameters more accurately, aiding decision-making.

For instance, processing time in flow shops is influenced by factors like machine depreciation, tool quality, and operator characteristics. Machine learning can analyze these factors and predict future activity times, improving decision-making. Managers are advised to design data-driven Decision Support Systems for better factory planning.

6. Conclusions

This research addresses the no-wait flexible flow shop problem, incorporating learning, deteriorating effects, and uncertainty. A multi-objective mathematical model was proposed to minimize makespan, tardiness, and energy consumption, using data-driven methods like FRO, SARIMA, and SVR. Hybrid methods combining LP-Metric and metaheuristics were used, with the LP-GA-SA algorithm excelling in solution quality and LP-SA in CPU efficiency. Sensitivity analyses showed that increasing the learning coefficient improved results, while the deteriorating effect worsened them. Limitations include focusing only on fuzzy environments, suggesting future studies explore hybrid uncertainty, social impacts, agility, and maintenance.

The supplementary data is available at:

https://scientiairanica.sharif.edu/article_23722.html

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Job	3	2	1	4	5
Speed level	1	3	2	1	2

Figure 1. The solution structure for the SA algorithm

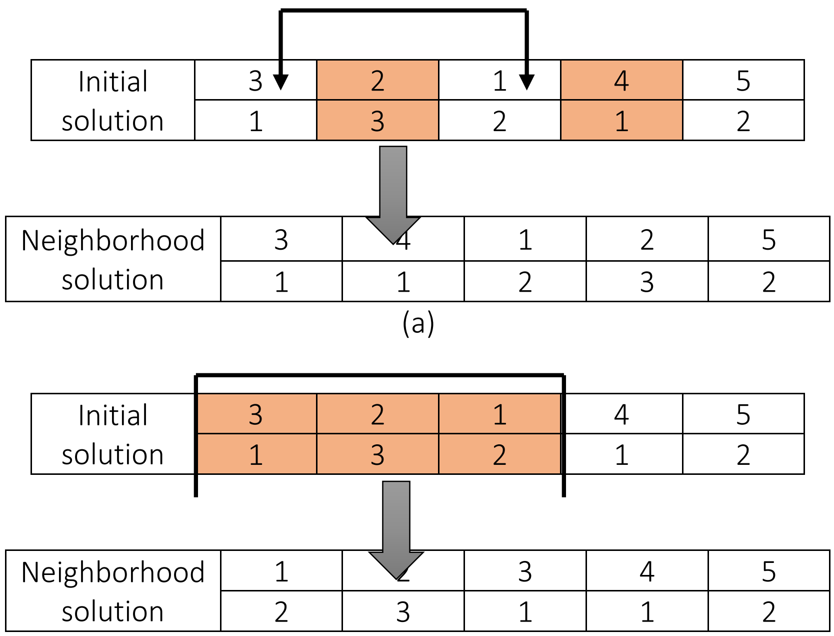


Figure 2. Operators of the SA algorithm: (a) swap, (b) inversion

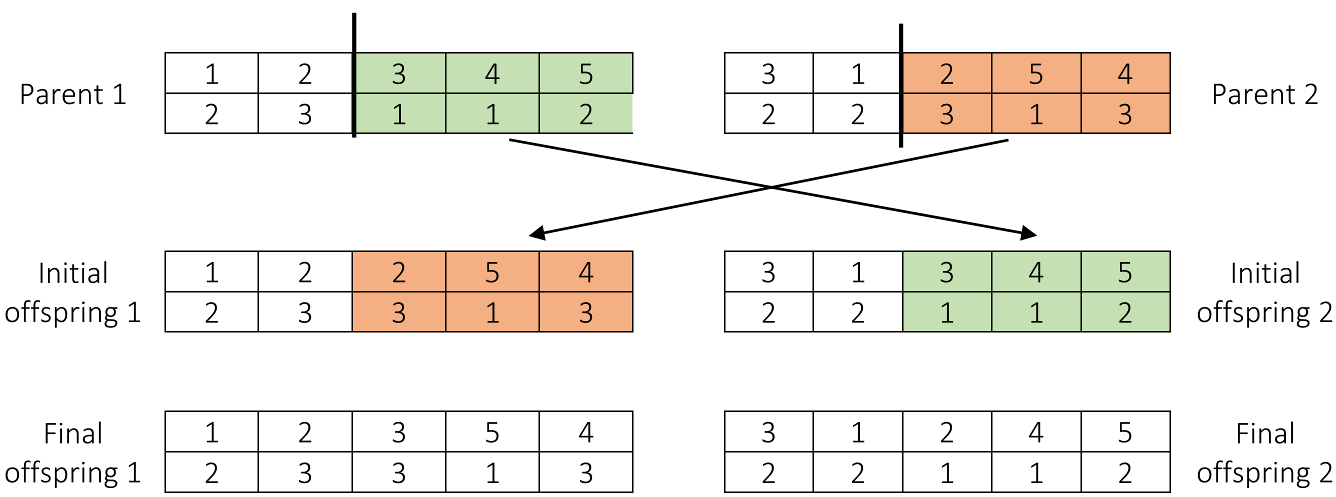


Figure 3. The crossover operators of the GA algorithm

0.3	0.75	0.42	0.1	0.68	0.05	0.55

Random numbers $\in (0,1)$

1	2	3	4	5	6	7

The position of numbers

0.05	0.1	0.3	0.42	0.55	0.68	0.75

The sorted numbers

6	4	1	3	7	5	2

The position of the sorted numbers

Figure 4. An example of the RK approach

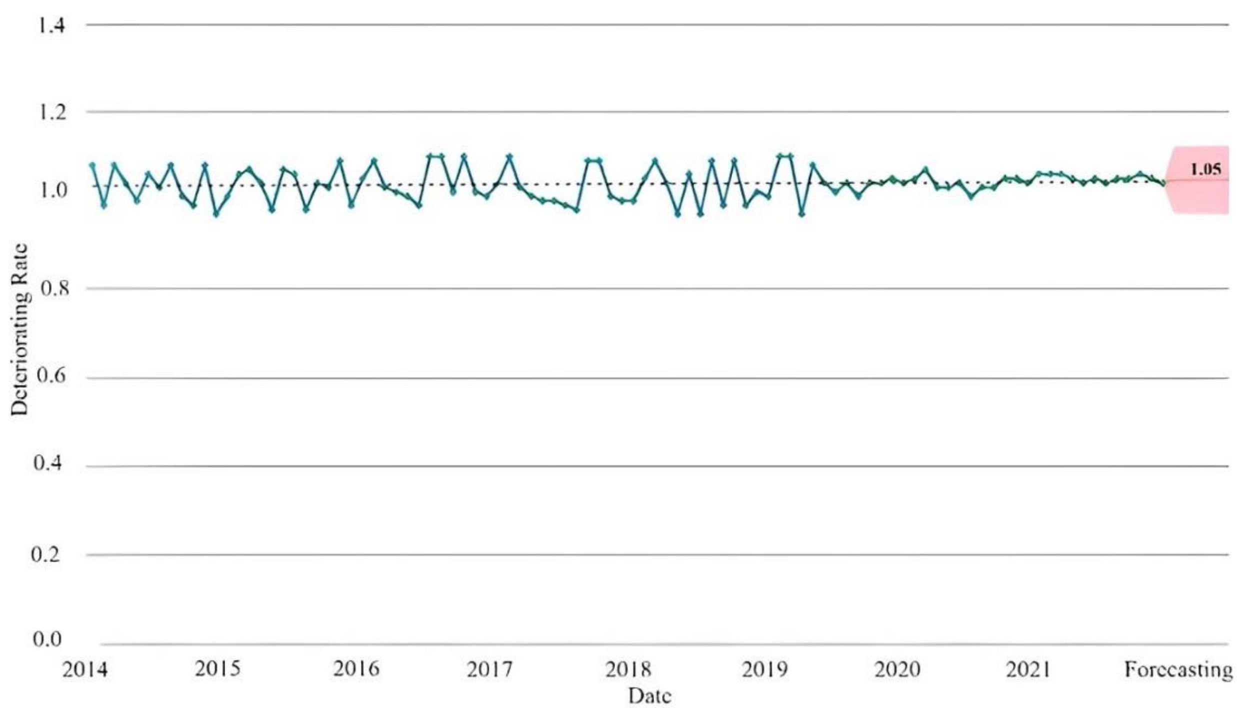


Figure 5. Deterioration rate of machinery

Table 1. SARIMA model error compared to other models

	SARIMA	ARIMA	Exponential smoothing
RMSE	4.236	16.251	19.521

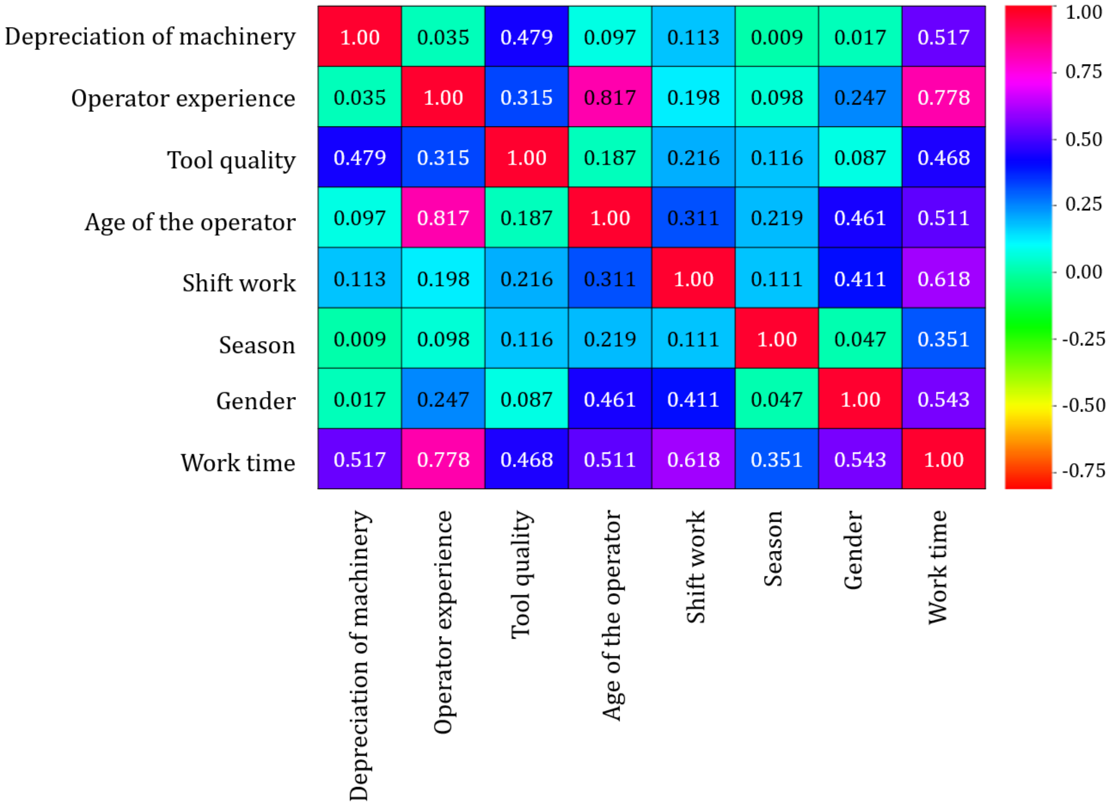


Figure 6. Correlation of all features

Table 2. Evaluation indicators in SVR model to estimate operator processing time

Regressor	**Training set accuracy**	**Testing set accuracy**
SVR	91%	89%

Table 3. Evaluation indicators for the activity time parameter

Processing time estimation		
$\boldsymbol{\theta_3}$	$\boldsymbol{\theta_2}$	$\boldsymbol{\theta_1}$
40	25	10

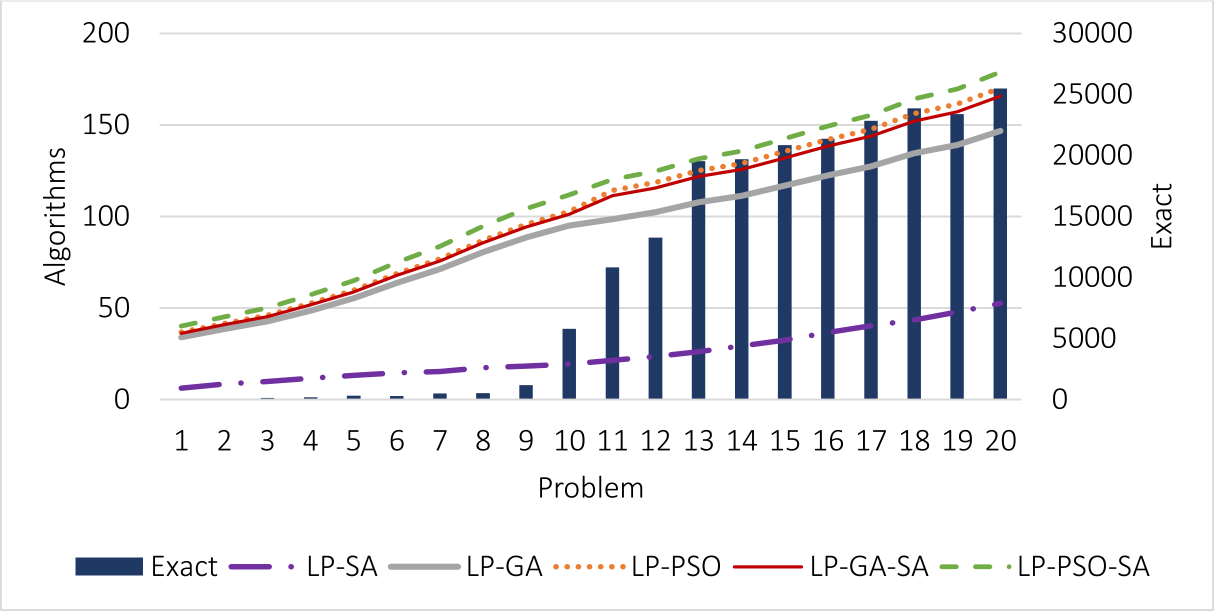
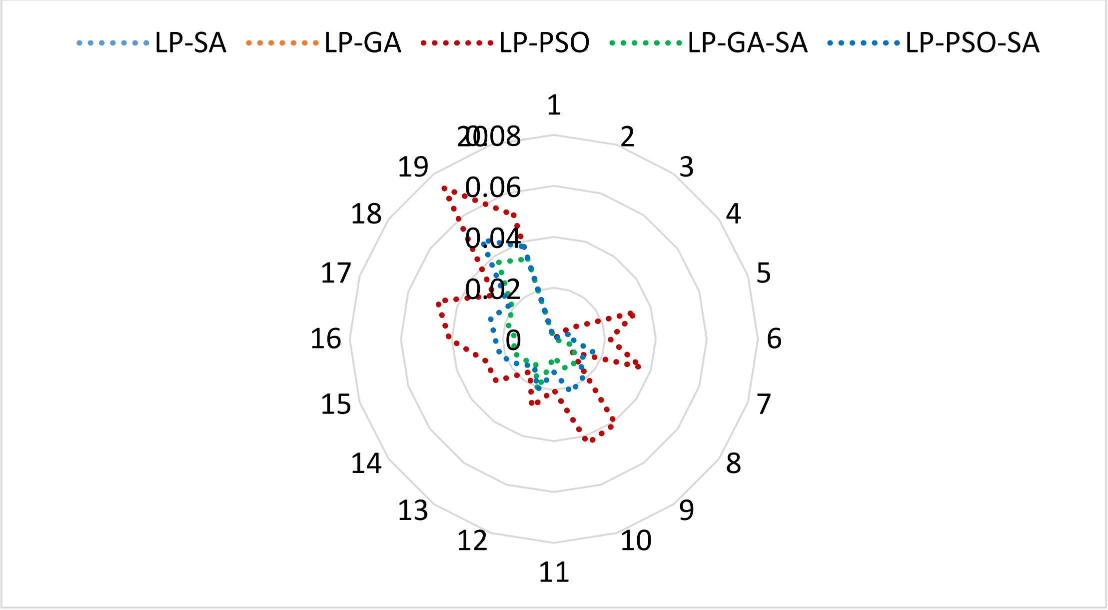
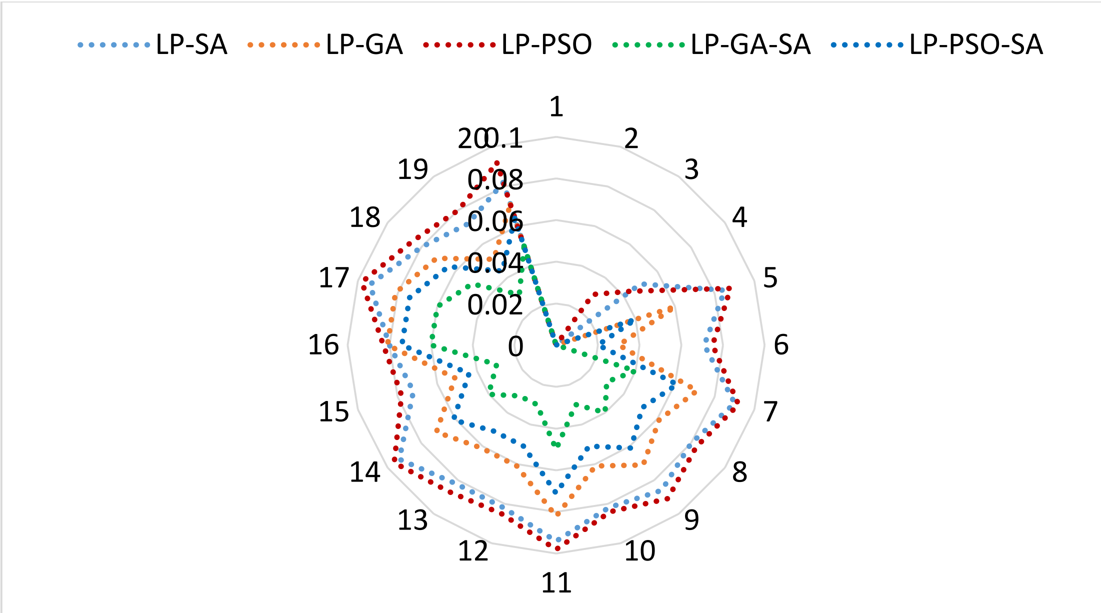


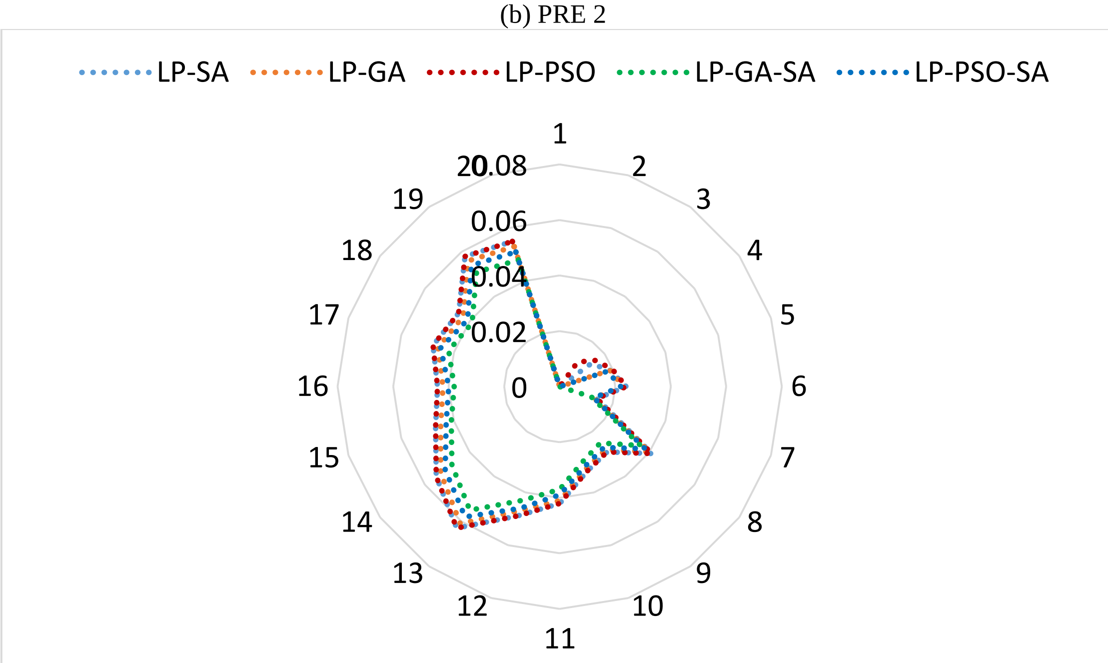
Figure 7. The evaluation of algorithms based on CPU time for small-sized test scenarios



(a) PRE 1



(b) PRE 2



(c) PRE 3

Figure 8. The comparison of algorithms in terms of the quality of solutions

Table 4. The outputs of the algorithms applied to small-scale instances

Problem	Exact			LP-SA			LP-GA			LP-PSO			LP-GA-SA			LP-PSO-SA		
	Z1	**Z2**	**Z3**	**Z1**	**Z2**	**Z3**	**Z1**	**Z2**	**Z3**	**Z1**	**Z2**	**Z3**	**Z1**	**Z2**	**Z3**	**Z1**	**Z2**	**Z3**
1	179.8	8.5	557.38	179.8	8.5	557.38	179.8	8.5	557.38	179.8	8.5	557.38	179.8	8.5	557.38	179.8	8.5	557.38
2	283.7	28.37	562.88	283.7	28.37	562.88	283.7	28.37	562.88	283.7	28.37	562.88	283.7	28.37	562.88	283.7	28.37	562.88
3	226.4	14.7	569.26	226.4	14.7	569.26	226.4	14.7	569.26	235.2	15.14	575.52	226.4	14.7	569.26	226.4	14.7	569.26
4	388.1	15.5	575.46	390.7	16.28	583.51	388.1	15.5	575.46	396.3	16.18	584.66	388.1	15.5	575.46	388.1	15.5	575.46
5	287.4	17	580.42	297.3	18.45	591.72	291.7	18.02	591.28	301.4	18.5	591.81	287.4	17	580.42	289.55	17	590.87
6	459.1	19.6	587.33	469.1	21	601.31	465.9	20.19	600.77	472.4	21.07	601.28	459.1	19.6	587.33	462.5	19.6	600.25
7	337.6	22.6	592.29	349.6	24.64	600.44	345.6	24.21	600.13	351.5	24.68	600.43	340.2	23.5	599.4	342.9	23.5	599.82
8	391.8	26.1	599.02	395.5	28.19	623.75	397.7	27.69	622.8	405.3	28.27	623.71	395.8	26.88	620.59	396.75	26.88	621.88
9	321.5	30.5	606.38	334.8	33.13	623.76	329.5	32.67	623.09	340.2	33.28	623.73	325.6	31.72	621.54	327.55	31.72	622.45
10	344.9	33.4	612.85	359.8	36.14	632.52	356.2	35.43	631.76	361.9	36.21	632.49	349	34.4	630.01	352.6	34.4	631.04
11	527.9	37.5	619.93	538.6	41.04	646.23	537.7	40.56	645.22	544.4	41.18	646.19	531.7	39.38	642.87	534.7	39.38	644.25
12	419.4	42.3	626.31	431.3	45.77	657.19	428.4	44.88	656	439.3	45.9	657.14	428.1	43.57	653.24	428.25	43.57	654.86
13	476.2	46.6	634.11	483.9	50.43	674.1	483.2	49.44	672.56	491.8	50.65	674.04	481.7	48	668.99	482.45	48	671.08
14	408.8	50.4	641.38	420.2	55.07	676.67	416.9	53.99	675.31	433	55.24	676.62	414.7	52.42	672.16	415.8	52.42	674.01
15	550.1	54.8	650.77	565.3	58.72	681.36	564.7	57.57	680.18	572.4	59.07	681.31	559.3	56.44	677.45	562	56.44	679.05
16	456.3	59.1	719.75	475.2	64.67	751.11	470.2	63.9	749.9	483.7	64.01	751.06	463.2	62.65	747.1	466.7	62.65	748.74
17	403.6	65.3	796.04	423.1	71.45	834.37	416.6	70.6	832.9	433	71.7	834.32	411.4	69.22	829.48	414	69.22	831.48
18	525.1	69.5	880.42	540.4	75.92	919.79	538.4	74.43	918.27	543.4	75.41	919.73	535.4	72.98	914.76	536.9	72.98	916.82
19	472.3	75.1	973.75	506.9	80.48	1030.68	500.5	78.9	1028.49	510.9	81.11	1030.6	489.9	77.35	1023.41	495.2	77.35	1026.39
20	523.6	79.3	1076.96	550.1	85.78	1136.23	546	84.93	1133.95	568.1	86.6	1136.14	541.1	83.27	1128.66	543.55	83.27	1131.76

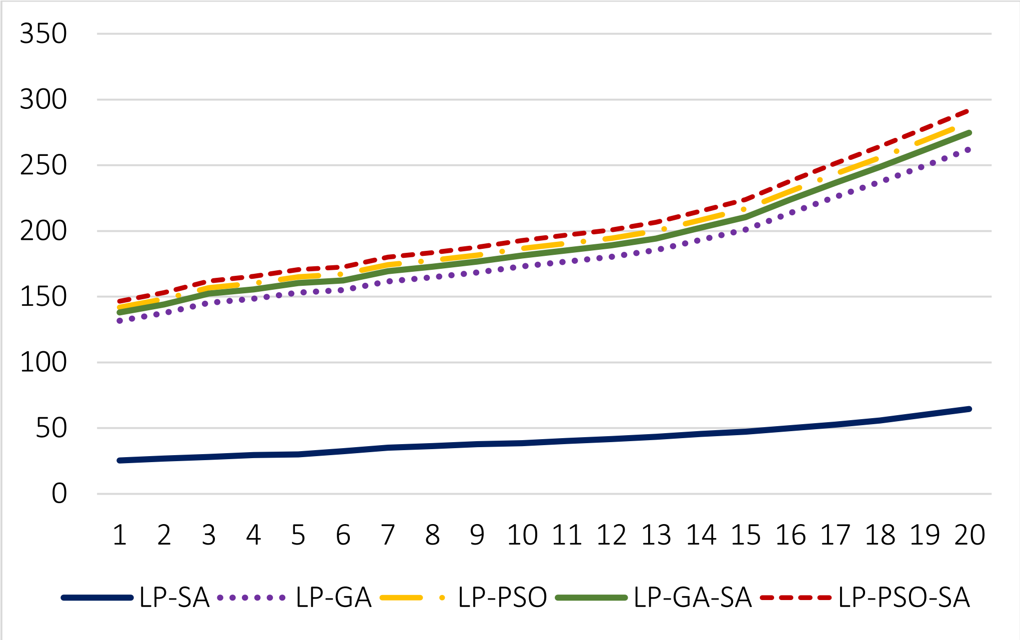
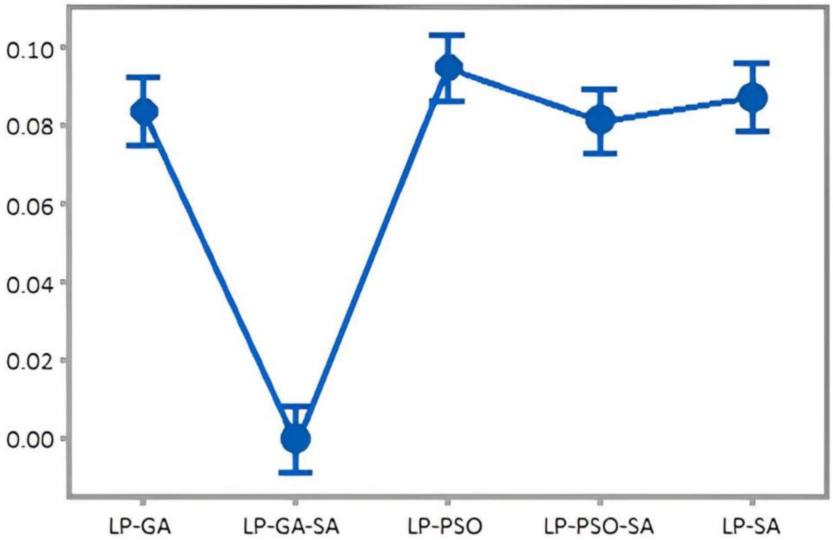
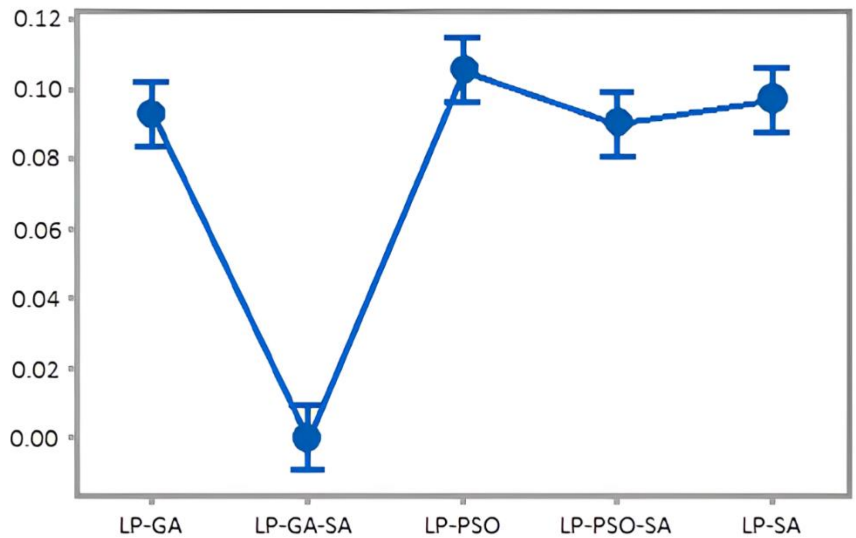


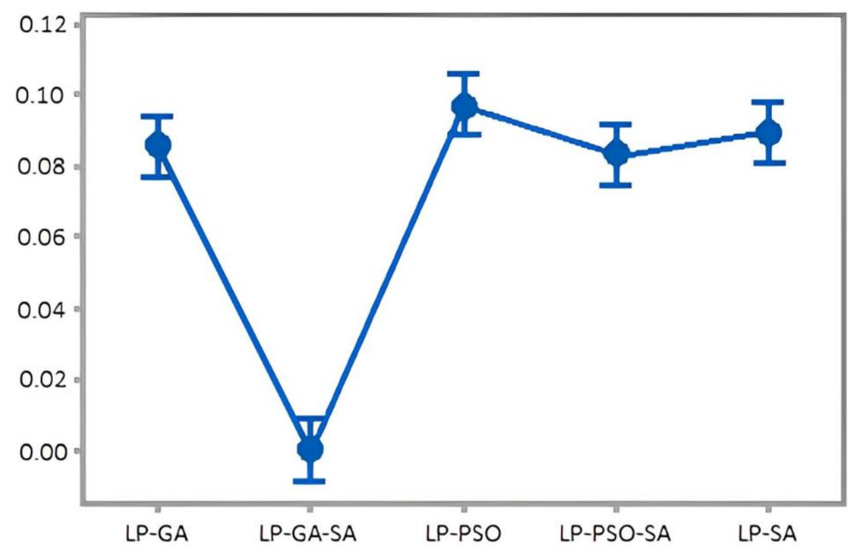
Figure 9. Evaluating the algorithms' performance according to CPU time for small-sized test cases



(a) RPD 1



(b) RPD 2



(c) RPD 3

Figure 10. The LSD chart for comparing the algorithms based on the RPD criterio

Table 5. The outputs of the algorithms in large-scale instances

Problem	LP-SA			LP-GA			LP-PSO			LP-GA-SA			LP-PSO-SA		
	Z1	Z2	Z3	Z1	Z2	Z3	Z1	Z2	Z3	Z1	Z2	Z3	Z1	Z2	Z3
1	607.38	146.02	1347.3	605.35	145.5	1342.89	611.80	147.16	1356.89	556.54	132.89	1237.01	603.88	145.12	1339.71
2	734.51	151.35	1225.19	731.68	150.73	1220.58	740.65	152.71	1235.21	663.91	135.74	1109.99	729.65	150.28	1217.27
3	940.00	154.82	1358.61	936.42	154.19	1353.55	947.78	156.2	1369.62	850.51	139.00	1232.09	933.84	153.73	1349.91
4	866.41	158.66	1475.63	863.82	158.15	1471.31	872.05	159.77	1485.02	801.56	145.89	1367.62	861.95	157.78	1468.2
5	1283.39	170.81	1692.81	1277.98	170.04	1685.82	1295.17	172.49	1708	1147.99	151.48	1518.06	1274.08	169.48	1680.79
6	1146.09	179.96	1856.78	1141.75	179.22	1849.91	1155.51	181.54	1871.71	1037.72	161.69	1685.05	1138.63	178.7	1844.96
7	1100.56	195.31	2126.07	1095.15	194.29	2115.84	1112.32	197.55	2148.3	965.36	169.61	1870.40	1091.26	193.55	2108.48
8	1393.30	188.53	2224.72	1389.49	187.97	2218.78	1401.58	189.73	2237.64	1298.12	174.67	2076.15	1386.75	187.57	2214.5
9	1285.64	205.18	2509.9	1281.33	204.44	2501.68	1294.99	206.78	2527.76	1178.07	186.73	2304.52	1278.23	203.91	2495.77
10	1149.29	212.54	2843.33	1144.58	211.61	2831.92	1159.54	214.58	2868.14	1031.42	189.16	2558.02	1141.18	210.93	2823.7
11	1819.39	211.78	3069.87	1813.81	211.08	3060.66	1831.54	213.3	3089.92	1679.70	194.30	2839.40	1809.78	210.58	3054.02
12	1671.94	212.87	3315.2	1668.56	212.4	3308.67	1679.27	213.87	3329.42	1587.57	201.29	3151.73	1666.13	212.07	3303.96
13	1590.13	229.94	3815.98	1584.72	229.1	3803.28	1601.90	231.77	3843.59	1454.83	208.92	3498.43	1580.82	228.49	3794.13
14	2103.30	231.39	4143.08	2097.91	230.75	4132.68	2115.04	232.78	4165.67	1968.36	215.41	3883.25	2094.02	230.29	4125.2
15	1947.47	240.88	4610.85	1942.27	240.19	4598.83	1958.75	242.38	4636.97	1817.66	223.60	4310.41	1938.53	239.69	4590.18
16	1884.55	248.73	5120.91	1879.49	248.01	5107.46	1895.56	250.3	5150.16	1757.93	230.75	4784.55	1875.84	247.5	5097.77
17	2400.38	261.75	5798.26	2392.13	260.79	5778.77	2418.32	263.85	5840.65	2194.07	237.58	5310.86	2386.18	260.09	5764.73
18	2227.55	266.08	6227.9	2222.68	265.46	6214.59	2238.15	267.45	6256.85	2105.73	250.41	5895.05	2219.17	265.01	6205
19	2674.26	279.31	6987.99	2667.3	278.53	6970.21	2689.39	281.01	7026.64	2500.25	259.75	6543.51	2662.29	277.96	6957.41
20	2603.52	301.75	7961.66	2594.18	300.58	7933.73	2623.82	304.27	8022.39	2370.04	272.68	7263.29	2587.46	299.75	7913.61

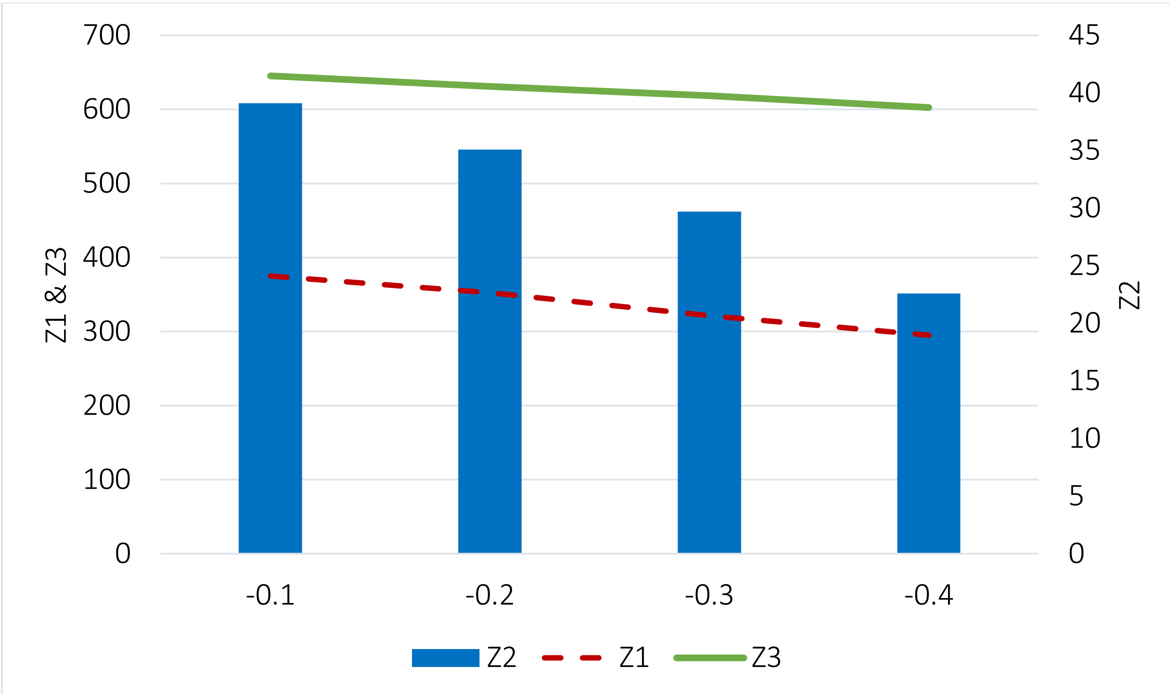


Figure 11. The behavior of the model according to changing the learning coefficient parameter

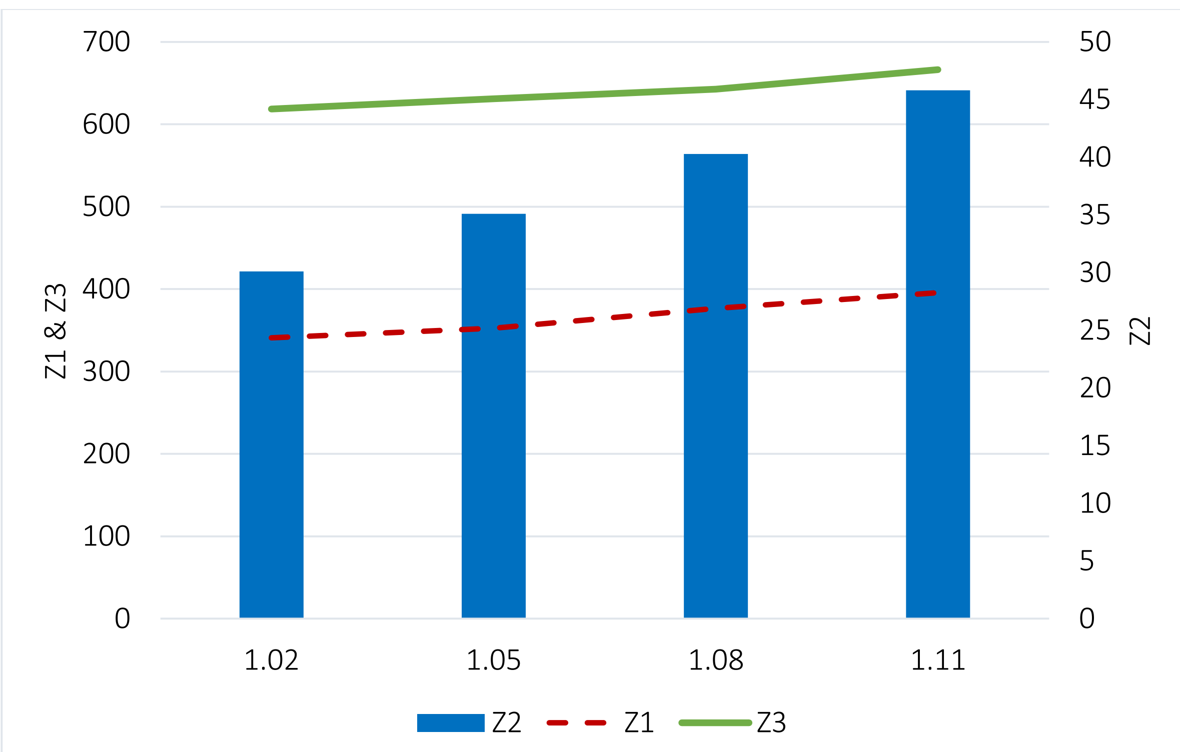


Figure 12. The behavior of the model according to changing the deteriorating coefficient parameter

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