Path Overlapping Problem in Estimating Origin-Destination Matrix Using Stochastic User Equilibrium

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Abstract

Estimating the origin-destination (OD) matrix based on traffic counts in a transportation network has received substantial attention in recent decades. Several studies have attempted to incorporate a stochastic user equilibrium (SUE) constraint into the OD estimation framework to deal with uncertainties. These studies have mainly adopted a multinomial logit (MNL) route choice model which has a restrictive assumption that does not allow for overlapping paths. This paper addresses the path overlapping problem by employing corrected logit route choice models, namely C-logit and path-size logit (PS-logit), that partially capture similarity/correlation among paths by a correction term in the MNL structure. A gradient algorithm (developed by Spiess) is also utilized to solve the SUE-based OD matrix estimation problem. Numerical experiments on the well-known Winnipeg network show that considering correlated/overlapping paths in the OD estimation process using C-logit or PS-logit route choice models results in more accurate OD matrices than the MNL-based procedure.

Keywords: origin-destination matrix; stochastic user equilibrium; path overlapping; corrected logit route choice; gradient algorithm; C-logit; path-size logit.

1. Introduction

Origin-destination (OD) matrix is crucial information for transportation planning and has a basic role in transportation network analysis. The principal method to obtain this demand matrix is a household travel survey, which is very costly. Thus, estimating techniques using readily available data (especially traffic counts) have been widely investigated over the past decades [1-4]. Earlier models were developed for uncongested networks based on proportional assignments in which route choice proportions were flow-independent. The most common estimation methods in this approach are entropy maximization or information minimization [5], maximum likelihood [6, 7], and generalized least squares [8, 9]. Although these models are relatively easy to solve, the assumption of uncongested conditions is usually violated in urban networks.

In congested networks, the assumption of a predetermined route choice matrix is no longer valid. Researchers have attempted to simultaneously estimate link choice proportions and trip matrix since they are interdependent in general networks [10]. Most studies have addressed this problem by incorporating user equilibrium (UE) conditions in a bi-level optimization problem [11]. The advantage of the bi-level approach is that the resulting OD matrix will be consistent with the assumed route choice behavior [12]. Crucial elements of the bi-level OD estimation problem are prior OD, upper-level (OD estimation) formulation, lower-level (traffic assignment), solution algorithm, and extra constraints to limit the search space [13]. The main focus of the present study is the traffic assignment component, which implies the travelers' route choice behavior. In addition, an upper-level formulation and a solution algorithm suitable for large-scale networks are selected.

Trip makers' hypothesized route choice behavior is a key factor affecting the quality of the OD demand matrix derived from link counts [14]. Deterministic UE-based methods assume that all motorists have complete information on network conditions and choose the best (shortest) route from an origin to a destination [15]. These presumptions are not necessarily correct and could be relaxed. According to the stochastic user equilibrium (SUE), travelers perceive travel costs differently and thus may choose a route different from the shortest one [15]. Gholi et al. have compared SUE-based OD estimation with UE-based estimation based on the different variances of users' perceptions in the Tehran network [16].

Researchers have incorporated the SUE condition into the OD estimation problem to model route choice behavior more accurately. At first, Shihsien and Fricker [17] developed a two-stage heuristic method to estimate the OD trip table and the θ parameter in a logit-based stochastic assignment. However, the model ignores the congestion effect and requires traffic counts and travel costs of all links to calibrate the θ parameter. Maher et al. [18] presented a solution algorithm for the bi-level problem of trip matrix estimation with SUE assignment on congested networks. Unlike Liu and Flicker, this study addressed congestion, but the dispersion parameter (θ) was introduced to the model exogenously. Yang et al. [12] suggested a nonconvex optimization problem for estimating an OD matrix and the travel cost coefficient (θ) simultaneously for a congested network under the logit-based SUE condition. A successive quadratic programming algorithm employing the analytical derivatives of the SUE constraints was applied to solve the simultaneous estimation model. Lo and Chan [19] proposed a maximum likelihood estimator in a two-stage iterative procedure to simultaneously estimate an OD matrix and dispersion parameter (θ) from traffic counts and OD surveys. This procedure uses SUE traffic assignment to consider congestion effects, resulting in maximum likelihood estimates with established statistical properties. Wang et al. [20] used a generalized least square (GLS) estimator and an SUE assignment in a two-stage algorithm for simultaneous estimation of the OD matrix and link choice probabilities by incorporating a dynamic dispersion parameter (according to traffic flow profile by time of day) into the logit route choice model. Ma and

Qian [11] developed a generalized single-level formulation to estimate an OD matrix under an SUE constraint since the bi-level programming approach could be computationally intensive when applied to large networks. They showed that the single-level approach is much more computationally efficient for large-scale networks, and the model's accuracy is similar to the bi-level formulation.

The primary issue that is focused on in the present study is that almost all OD estimation models with SUE constraints have used the popular logit route choice model because of its straightforward structure. The widely known weakness of multinomial logit (MNL) is the inability to properly account for correlated or overlapping routes. This drawback is due to the property of independence from irrelevant alternatives (IIA) and may lead to unrealistic results [15]. The multinomial probit (MNP) model does not suffer from this weakness. However, the MNP-based model is computationally intensive. Some route choice models have been established in the stochastic assignment context to address the overlapping path problem while preserving closed-form expression for choice probability. Two major types of such models are: 1) corrected logit route choice models, e.g., C-logit and path-size logit, and 2) generalized extreme value (GEV) models, e.g., cross nested logit (CNL), generalized nested logit (GNL), and paired combinatorial logit (PCL). A more detailed review of these types of route choice models can be found in [21] and [22]. Based on our best knowledge, many studies have acknowledged the inadequacy of MNL for route choice modeling. Still, SUE models that take overlapping routes into account have not been applied to estimate the OD demand matrix. This paper aims to apply well-known route choice models, i.e., C-logit and path-size logit [23], that consider correlated routes into the OD estimation framework and compare the results with a logit-based model. Furthermore, sensitivity analyses are conducted to evaluate the robustness of models.

The secondary issue that is addressed is that most reviewed studies have developed models that could not be efficiently applied to large networks [11]. Traffic assignment is the most time-consuming part of the OD estimation problem [24]. The computation of derivatives is another challenging part of the OD estimation. Spiess [25] developed a gradient-based algorithm for solving a UE-based problem that is scalable to large-scale networks because of the simplicity of gradient computation. This gradient approach is general enough to accommodate different assignment models [25]. As far as we know, this model has not been implemented with SUE assignment models. In order to exploit both the Spiess method and the SUE approach, we propose an SUE-based gradient approach to estimate the OD matrix from traffic counts in a real-size network.

Therefore, the contribution of the paper to the SUE-based OD matrix estimation problem (ODMEP) exploiting traffic counts is twofold:

(1) Applying corrected logit route choice models with the ability to deal with overlapping paths in the SUE assignment (lower level),

(2) Taking advantage of the Spiess gradient method to solve this problem in large-scale networks (upper level).

The remainder of the paper is organized as follows. The next section presents the gradient approach for OD estimation, an SUE assignment model, and two route choice models (other than MNL). Next, the results of the proposed OD estimation formulation with three different route choice models are examined. Finally, conclusions and suggestions are summarized.

2. Methodology

In this section, the Spiess gradient approach to the OD matrix estimation problem (ODMEP) is described. Next, an algorithm used in this paper to solve the SUE assignment is presented. Then, two multinomial logit (MNL) modifications employed to model route choice behavior are explained. Figure 1 shows the steps of solving SUE-based ODMEP in this research.

<Figure 1>

2.1 Spiess's gradient approach for solving ODMEP

Let $G = (N, A)$ be a graph representing a road network composed of node set N and link set A. Each link $a \in A$ has a flow-dependent, non-decreasing cost function $t_a(v_a)$. Assume that $\hat{A} \subseteq A$ denotes a subset of links on which traffic flow ($\hat{v}$) is observed. Spiess [25] employed the convex minimization problem (1) for ODMEP:

$$\begin{aligned} \min \quad & Z(g) = \frac{1}{2}\sum_{a\in\hat{A}}(v_a - \hat{v}_a)^2 \\ \text{s.t.} \quad & v = \text{assign}(g), \\ & g \geq 0, \end{aligned} \tag{1}$$

where g is the estimated OD matrix, assign(.) indicates the traffic assignment model, and v is the vector of assigned volumes resulting from the assign(g).

Since optimization problem (1) is highly underdetermined, extra information is required to find the best demand matrix among optimal solutions. This information is usually an outdated matrix that is called the target matrix, which contains crucial structural information on origin-destination demands [25]. Most studies utilize a distance measure between the estimated and target matrix in the objective function to choose the best matrix. However, this method increases the complexity of the problem and makes it challenging to apply to large-scale networks. Spiess [25] suggested the gradient method, also referred to as the steepest descent method, which follows the direction of the largest yield in terms of minimizing the value of the objective function. Consequently, this method inherently finds a solution that will not deviate more than necessary from the initial solution (or target matrix). Despite this, since the distance between estimated and target matrices is not explicitly considered, deviation may grow as the number of iterations increases.

The basic formula of the gradient method for estimating the OD demand matrix from traffic flows is written as follows:

$$g_i^{l+1} = \begin{cases} \hat{g}_i & \text{for} \quad l = 0, \\ g_i^l \left(1 - \lambda^l \left[\dfrac{\partial Z(g^l)}{\partial g_i} \right]_{g_i^l} \right) & \text{for} \quad l = 1,2,3,..., \end{cases} \tag{2}$$

in which $\hat{g}_i$ is the prior (or target) demand for OD pair *i*, g_i^l is the estimated demand for OD pair *i* at iteration *l*, and λ^l is the step length for iteration *l*. The following equations are used to calculate the gradient matrix $\dfrac{\partial Z(g)}{\partial g_i}$ and the step length λ^l in equation (2). See Spiess's original paper for details on how these expressions are derived [25].

$$\frac{\partial Z(g)}{\partial g_i} = \sum_{k \in K_i} p_k \sum_{a \in \hat{A}} \delta_{ak} (v_a - \hat{v}_a), \quad \forall i \in I, \tag{3}$$

$$p_k = \frac{h_k}{g_i}, \quad \forall k \in K_i, \quad \forall i \in I, \tag{4}$$

$$\lambda^l = \frac{\sum_{a \in \hat{A}} v'_a (\hat{v}_a - v_a)}{\sum_{a \in \hat{A}} v'^2_a}, \tag{5}$$

$$v'_a = -\sum_{i \in I} g_i \left(\frac{\partial Z(g)}{\partial g_i}\right)\left(\sum_{k \in K_i} \delta_{ak} p_k\right), \tag{6}$$

where *I* denotes the set of OD pairs, K_i is the set of working paths for OD pair *i*, h_k and p_k represent the flow and the probability of path *k*, respectively. Whenever link *a* is contained within path *k*, δ_{ak} equals one; otherwise, it equals zero. Selecting large values for the step length can increase the objective function value, and the algorithm's convergence would be lost. Thus, in order to be feasible, the optimal step length needs to be bounded by the following inequality:

$$\lambda \frac{\partial Z(g)}{\partial g_i} \leq 1, \quad \forall i \in I \quad \text{with} \quad g_i > 0. \tag{7}$$

The steps of the gradient algorithm for solving the ODMEP (relation 1) can be outlined as follows [26]:

Step 1: Set $g_i = \hat{g}_i$ for all $i \in I$, and the iteration counter $l = 0$.

Step 2: Assign matrix g to the network to find link flows (v), as well as path flows (h) and path sets (K) for each origin-destination pair.

Step 3: Compute $\frac{\partial Z(g)}{\partial g_i}$ by equations (3) and (4) and λ^l by equations (5) and (6). If possibility condition (7) does not hold, adjust λ^l using the following equation [27]:

$$\lambda^l = \min\left(\max\left(\frac{\sum_{a\in\hat{A}} v'_a(\hat{v}_a - v_a)}{\sum_{a\in\hat{A}} {v'_a}^2}, \underline{\lambda^l} \right), \overline{\lambda^l} \right), \tag{8}$$

where $\underline{\lambda^l}$ and $\overline{\lambda^l}$ are the lower and upper bounds defined as:

$$\underline{\lambda^l} = \begin{cases} \dfrac{1}{\max\limits_{i\in I}(\frac{\partial Z(g)}{\partial g_i})} & \text{if} \quad \max\limits_{i\in I}(\frac{\partial Z(g)}{\partial g_i}) > 0 \\ +\infty & \text{else} \end{cases}$$

$$\overline{\lambda^l} = \begin{cases} \dfrac{1}{\min\limits_{i\in I}(\frac{\partial Z(g)}{\partial g_i})} & \text{if} \quad \min\limits_{i\in I}(\frac{\partial Z(g)}{\partial g_i}) < 0 \\ -\infty & \text{else} \end{cases}$$

Step 4: For each OD pair i, set g_i^{l+1} using equation (2) and increment l by 1 ($l = l$+1).

Step 5: If the stopping criterion is met, terminate the procedure; otherwise, go to step 2.

The stopping criterion in step 5 is defined as the similarity between assigned and observed flows as below:

$$\frac{\sqrt{\sum_{a\in\hat{A}} (v_a - \hat{v}_a)^2}}{\sum_{a\in\hat{A}} \hat{v}_a} \le k\,. \tag{9}$$

In step 2, a stochastic user equilibrium (SUE) model is used to solve the lower level of the problem, as explained in the subsequent section.

2.2 Stochastic user equilibrium assignment

A stochastic user equilibrium model considers perceived rather than measured travel cost. A random error term is included in perceived travel cost functions to represent the variations in the motorists' perception. As a result, paths with a higher actual cost than the shortest path may also be used. The perceived travel cost on route *k*, which connects origin *r* and destination *s*, can be expressed as [15]:

$$C_k^{rs} = c_k^{rs} + \xi_k^{rs}, \quad \forall k,r,s, \tag{10}$$

where c_k^{rs} is the actual travel cost at a designated flow, and ξ_k^{rs} is the random error term. The choice probability of route *k* in OD pair (*r*, *s*) is [15]:

$$P_k^{rs} = \Pr(C_k^{rs} \le C_l^{rs}, \forall l \in K_{rs}). \tag{11}$$

Different discrete choice models have been developed based on the probability distribution selected for the random components (ξ_k^{rs}) to calculate route choice probabilities and, thus, route flows. The well-known multinomial logit model assumes that error components have independent and identical distributions of extreme value type I (Gumbel) [28]. Given the measured travel costs (c_k^{rs}), the closed-form of logit route choice probability is written as [15]:

$$P_k^{rs} = \frac{e^{-\theta c_k^{rs}}}{\sum_l e^{-\theta c_l^{rs}}}, \quad \forall k,r,s, \tag{12}$$

in which θ is a positive dispersion parameter inversely proportional to the variance of perceived travel cost ($\mathrm{var}(C_k^{rs}) = \frac{\pi^2}{6\theta^2}$) [28]. Traffic flows on each route are computed by the following equation, which characterizes stochastic user equilibrium conditions:

$$f_k^{rs} = q_{rs} P_k^{rs}, \quad \forall k,r,s, \tag{13}$$

where q_{rs} is the travel demand between origin-destination *r* and *s*. This paper employs a path-based algorithm developed by Damberg et al. [29] to perform an SUE assignment. This algorithm extends the disaggregate simplicial decomposition (DSD) algorithm introduced by Larsson and Patriksson [30] for solving the UE-based traffic assignment problem. Briefly, the DSD algorithm is as follows [29]. A restricted master problem based on a subset of all routes is solved. Given its solution, link flows and link costs are updated. Subsequently, shortest paths are found for all OD pairs using a column (or route) generation strategy. The route set for each OD pair is augmented with the new shortest path if it does not already exist in the corresponding route set. Next, a new restricted master problem is solved using the new route sets. The procedure is then repeated until convergence is achieved. The steps of the DSD algorithm for solving the SUE assignment can be summarized as follows [29]:

Step 1 (initialization): Form the initial route set for all OD pairs and compute the initial route flows (f_k^0). Put m = 1.

Step 2 (restricted master problem): Put n = 0 and repeat the following until convergence:

(2.1) Update route cost for all routes in the route sets.

(2.2) Compute auxiliary route flow (h_k^n) by a stochastic network loading model.

(2.3) Set the new route flow: $f_k^{n+1} = f_k^n + (\frac{1}{n})(h_k^n - f_k^n)$

(2.4) If the convergence criterion is met, go to step 3; otherwise, go to 2.1.

Step 3 (column or route generation): For each OD pair, generate new routes and add them to the working route sets if they are not already contained. If no new routes are found or a predetermined number of iterations (m) is reached, stop; otherwise, put m = m+1 and go to step 2.

Finally, route flows are employed to compute link flows as below:

$$x_a = \sum_{rs} \sum_{k} f_k^{rs} \delta_{a,k}^{rs}, \quad \forall a . \tag{14}$$

In this paper, the algorithm applied to solve the restricted master problem (step 2) is the widely known method of successive averages (MSA) that employs a predetermined step length. However, other step-size schemes, such as self-regulated averaging (SRA) and self-adaptive Armijo (SAA), are used in the literature to accelerate the convergence of the algorithm [31]. The convergence criterion in step 2 is defined as similarity in link flows (x_a) in successive iterations:

$$\frac{\sqrt{\sum_a (x_a^{n+1} - x_a^n)^2}}{\sum_a x_a^n} \le k . \tag{15}$$

Initial path flows in step 1 and auxiliary path flows in step 2.2 are computed by the stochastic network loading model. In this model, travel costs are assumed to be independent of flow. Given a route set for an OD pair, route choice probabilities are obtained (e.g., by equation (12) for the MNL route choice model), and then route flows are computed using equation (13).

Step 2 solves a restricted problem based on a given subset of the complete set of routes. Next, the route choice set needs to be updated based on the new costs to add the most likely utilized routes to the existing choice set. The route generation procedure in step 3 is performed through the shortest path problems based on perceived travel costs. In order to calculate perceived costs, randomized link costs are drawn from a random distribution. The number of random draws obviously affects the outcome of this strategy. Based on Damberg et al.'s recommendation [29], perceived costs are derived by a single drawing from a truncated normal distribution with actual travel costs as the average.

This paper uses two route choice models besides the MNL model in the SUE assignment. These models are extensions to basic MNL that make them capable of considering the issue of path overlapping while keeping the logit model's closed-form function. In the next section, these two models, namely C-logit and PS-logit models, are discussed.

2.3 Corrected logit route choice models

These models somewhat relax the logit's assumption by applying a correction factor to the cost function. The following are two well-known models from this family.

2.3.1 C-logit model

Cascetta et al. [32] developed the C-logit model, which has the following functional form:

$$P_k^{rs} = \frac{e^{(-\theta c_k^{rs} - cf_k)}}{\sum_l e^{(-\theta c_l^{rs} - cf_k)}}, \tag{16}$$

in which cf_k is the commonality factor of route k and represents the part of route k that is common with other routes in the route set of OD pair r and s. Different ways have been proposed to specify the commonality factor [21]. According to Cascetta et al. [32], the formula is as follows:

$$cf_k = \beta \ln \sum_l \left(\frac{L_{kl}}{\sqrt{L_k L_l}} \right)^{\gamma}, \tag{17}$$

where L_{kl} is the common length of routes k and l. L_k and L_l are the total length of routes k and l, respectively. β and γ are positive parameters that require calibration.

2.3.2 Path-size logit model

Ben-Akiva and Bierlaire [33] developed the PS-logit model with the following functional form:

$$P_k^{rs} = \frac{e^{(-\theta c_k^{rs} + \ln S_k)}}{\sum_l e^{(-\theta c_l^{rs} + \ln S_k)}}, \tag{18}$$

where S_k is the size of route k. The route size equals one and does not need utility adjustment if no overlapping links exist. Different forms have been adopted to specify the route size. Ben-Akiva and Bierlaire [33] suggested the following formula:

$$S_k = \sum_{a \in \Gamma_k} \frac{l_a}{L_k} \frac{1}{\sum_j \delta_{aj}}, \tag{19}$$

where L_k is the overall length of route *k*, l_a is the length of link *a*, Γ_k is the set of links included in route *k*, and δ_{aj} is the link-path incidence variable that is one if route *j* contains link *a*, and zero otherwise. C-logit and PS-logit have similar functional forms but with different interpretations regarding the correction factor to the utility function of the MNL model. The commonality factor in the C-logit formula is greater than or equal to one and thus decreases the route utility if any similarity (overlapping) is found. The path size factor in the PS-logit formula represents the fraction of a route that could be a complete alternative [21].

3. Results and Discussion

This section reports and discusses the results of the proposed SUE-based OD estimation model. Three variations of the stochastic user equilibrium (i.e., with MNL, C-logit, and PS-logit route choice models) are employed as the lower level of the Spiess OD estimation algorithm. These models are coded in Python, run on a PC with a Dual Core 2.5 GHz CPU and 8 GB RAM, and implemented on the well-known Winnipeg network (Manitoba, Canada).

The Winnipeg network used for the analysis includes 1,052 nodes, of which 147 are centroids, 2,836 links, and 4,345 OD pairs with non-zero travel demand. The total demand is 64,775 trips. The volume-delay function for each link is according to the typical formula established by the Bureau of Public Roads (BPR) with specific parameters for each link. The information is obtained from https://github.com/bstabler/Transportation Networks.

The existing OD matrix is considered the network's true matrix (i.e., the matrix the model seeks to approximate). Since no outdated matrix is available, the target (or primary) matrix is acquired by manipulating the true matrix [34]. Assuming that demand has grown over time, the true matrix has been disturbed to reduce total demand in the prior matrix. Two scenarios are applied to generate the target (prior) matrix. In the first scenario, the total demand is reduced by 10% by randomly decreasing most matrix elements (and increasing some others). In the second one, the total demand is reduced by 30% by randomly reducing all matrix elements. Similar scenarios could be created, but these seem sufficient for this study. Scenarios are:

(1) Disturbing the true OD matrix elements by multiplying them with the randomly generated uniform numbers in (0.7, 1.1). Thus, the total demand is reduced by 10%,

(2) Disturbing the true OD matrix elements by multiplying them with the randomly generated uniform numbers in (0.5, 0.9). Thus, the total demand is reduced by 30%.

Since there are no link flow observations, flows resulting from the assignment of the true matrix to the network are regarded as traffic counts. In order to equitably compare three different route choice models (i.e., MNL, C-logit, and PS-logit) at the lower level of ODMEP, an assignment method other than them is utilized to produce link counts. The MNL and Multinomial probit (MNP) models were applied in the earlier developments of stochastic assignments [15]. Unlike MNL, MNP lacks a closed-form expression and thus requires considerable computational effort in large-scale networks. However, it does not suffer from the IIA assumption of MNL and hence can deal with the issue of overlapping (or correlated) paths. Therefore, the MNP-SUE assignment is employed to obtain observed link flows. Also, a link-based algorithm presented by Sheffi [15] is used for the MNP-based stochastic assignment.

The value of the dispersion parameter θ (in equations (12), (16), and (18)) is supposed to be known. For each route choice model, the θ value is determined using a line search method to minimize the root mean square error (RMSE) between the resulting link flows and traffic counts. Moreover, the RMSE measure, besides the coefficient of determination (R^2), is used to evaluate the quality of estimations. For this purpose, these evaluation criteria are exploited: RMSE and R-squared between the elements of true and estimated OD matrix, and RMSE and

R-squared between observed and estimated link flows. The RMSE is calculated as follows [10]:

$$RMSE = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(y_{est}^{n} - y_{obs}^{n})^2}, \tag{20}$$

where N is the number of observations, and y_{obs} and y_{est} are observed/ true and estimated flows, respectively. Besides metrics based on discrepancies between individual OD flows, metrics developed for structural comparison of OD matrices (such as MSSIM, GSSI, etc.) can also be used [35]. The convergence tolerance k in equation (9) is set to 0.001, and the gradient algorithm for ODMEP (section 2.1) is stopped when the convergence criterion becomes less than k or iteration counter (l) hits 20. MSA algorithm in step 2 of the SUE assignment (section 2.2) is stopped as soon as the stopping criterion becomes less than 0.00001 ($k = 10^{-5}$ in equation (15)). Furthermore, the maximum number of iterations (m in step 3 of the SUE) is fixed at 5.

3.1 SUE-based ODMEP using the complete set of link flows

The results of ODMEP under an SUE constraint (with each MNL, C-logit, and PS-logit route choice models) and based on the complete network link flows as traffic counts and for the two scenarios mentioned above are presented in Tables 1 and 2. All reported outputs are acquired after 20 iterations of the Spiess method (section 2.1). As explained above, the value of the dispersion parameter of the route choice models is determined exogenously. In this example, the estimated values of θ are 0.30, 0.33, and 0.35 for MNL, C-logit, and PS-logit, respectively. Based on the closeness of estimated and observed link flows, all three models have performed almost similarly in terms of R^2 (LF). It is likely because all models explicitly consider link flow deviation at the upper level of ODMEP. Nonetheless, in terms of RMSE (LF), the model with MNL has performed somewhat better (up to 5%) than models with C-logit and PS-logit.

<Table 1>

<Table 2>

A comparison regarding the fitness between estimated and true OD matrices indicates that OD estimation based on C-logit and PS-logit is more accurate than OD estimation based on MNL in terms of both R^2 (OD) and RMSE (OD) (Tables 1 and 2). Nevertheless, the MNL-based model estimates are slightly more accurate than the other two models regarding the sum of the OD matrix elements. It can be seen that R^2 (OD) for models with C-logit and PS-logit that take the overlapping paths problem into account are almost similar but superior to the model with MNL that is not able to address this problem. Besides, using the C-logit and PS-logit models in the OD estimation reduces RMSE (OD) by about 9.5 to 13.5 percent compared to the MNL-based model (Table 2). These results show that employing route choice models that account for the issue of path overlapping could significantly affect the OD demand estimates. In other words, incorporating this issue through route choice models like C-logit and PS-logit could result in a more accurate estimate of the OD matrix.

Figure 2 (a-b) displays the logarithm of the objective function value in each iteration for both scenarios. Three models with MNL, C-logit, and PS-logit at the lower level of ODMEP perform practically similarly in minimizing the objective function. The reason could be that all models use the same algorithm to minimize the objective function. However, the MNL-based model is slightly better than models with C-logit and PS-logit.

<Figure 2>

As noted in section 2.1, the objective function does not explicitly consider the discrepancy between estimated and true demand. Figure 3 (a-b) shows that all models worsen as iterations proceed, based on RMSE (OD). All models function almost similarly in the early stages. However, as iterations move forward, the MNL-based model deteriorates further than the other two models, resulting in a less precise OD matrix. As a result, there is no guarantee that the MNL-based model will produce an accurate OD matrix, even though it minimizes the objective function. Note that the Spiess algorithm explicitly uses path probabilities to correct the OD matrix at each step. Consequently, different OD demand estimates can be derived from the

SUE-based ODMEP with different underlying route choice models.

<Figure 3>

Table 3 shows the average time required to solve the SUE-based ODMEP with the Spiess gradient algorithm according to the specifications mentioned above. Thus, the proposed method could be applied to real networks in a reasonable time. A Clogit or PS-logit-based model requires more time to calculate than an MNL-based model because of the need to calculate an extra parameter (commonality factor in Clogit and route size in PS-logit).

<Table 3>

3.2 An analysis of counting locations coverage

In section 3.1, all network links were supposed to be counted. This part analyzes the impact of observing an incomplete set of links. In this regard, traffic volumes on 50, 25, and 10 percent of links intercepting with most OD pairs are considered the observed link flows. In other words, following an SUE-MNL assignment with true demand, links are sorted according to how many ODs are intercepted. Then, the first 50, 25, or 10 percent of links are selected. Intercepting with more OD pairs brings about more OD demands to be adjusted since only those OD pairs with counting locations between them are updated in the Spiess algorithm. In addition, path overlapping is more likely to occur on such links.

Table 4 presents the results of OD estimation based on each of the three incomplete sets of network link flows. The predominance of models considering path correlation over the MNL is preserved when the number of counted links changes. Utilizing C-logit or PS-logit improves RMSE(OD) compared to the MNL-based model by about 5 to 15 percent (Table 4). Therefore, the results are stable with respect to the location of observed links. According to total demand, no significant difference between models in different situations is observable. However, as the number of counting locations decreases, the total demand becomes more distant from the true value because fewer OD pairs are adjusted when the number of observed links decreases.

<Table 4>

3.3 An analysis of the choice set size

An important factor affecting SUE results is the size of the path choice set. This factor is controlled by the number of iterations in the column generation step of the SUE assignment. The results presented above are derived through five iterations of column generation. In this part, models are also implemented with 10 and 15 iterations. According to figure 4 (a-b), in both scenarios, $\Delta RMSE^{MNL}(OD)$ increases when iterations of column generation are fixed at 10 (except C-logit in scenario 2) but decreases at 15. Indeed, as column generation iterates more, the size of the path choice set increases as a consequence. The difference between path flows diminishes as the number of active paths increases beyond a limit since all active paths should receive flow for calculating auxiliary path flows in the SUE procedure. Hence, OD flow breaks between many paths, resulting in similar auxiliary path flows. Figure 4 shows that the PS-logit provides more robust results than the C-logit for the evaluated range of iterations.

<Figure 4>

4. Conclusions and suggestions

In this paper, an origin-destination matrix estimation problem (ODMEP) with stochastic user equilibrium (SUE) constraint and using link traffic counts is developed employing three underlying route choice models: multinomial logit (MNL), C-logit, and path-size logit (PS-logit). The proposed approach has two main characteristics:

- We apply corrected logit route choice models, namely C-logit and PS-logit, to address the path overlapping issue in the SUE framework.
- We utilize the Spiess gradient algorithm (which can be adapted to large-scale networks)

in which SUE assignment is used at the lower level to account for different user perceptions.

Three OD estimation models (i.e., ODMEP with each SUE-MNL, SUE-C-logit, and SUE-PS-logit as a constraint) are applied to the Winnipeg network under two scenarios for generating a prior (target) matrix. The MNP assignment is used to obtain flows on observed links and compare the results of these models. In addition to the complete set of network links, three incomplete sets of links are considered: 50, 25, and 10 percent of network links intercepted by most OD pairs, and thus, more correlated paths are expected on them. Also, models are executed for various column generation iterations that show the impact of path choice set size. The study reveals the following relevant findings:

- Although the three models perform almost identical in reproducing link counts and total OD demands, the estimated OD matrix estimated by the model with SUE-MNL is significantly less accurate (up to about 15%) than those estimated by models with SUE-C-logit and SUE-PS-logit. In other words, employing route choice models that address overlapping paths can provide an OD matrix closer to the true demand matrix than the model that does not. However, these models require more time to execute because of the need to compute further parameters to relax the IID assumption partially.
- When traffic counts are limited to the subset of links intercepted by most OD pairs, the superiority of models dealing with path overlapping (C-logit and PS-logit in this study) over the MNL-based model is almost preserved. Consequently, the results are robust in relation to the observed links.
- As the number of active paths exceeds a certain limit, the estimated matrices of the three models approach each other. Because increasing the path choice set's size reduces the difference between path flows produced by different route choice models.

- The adapted approach (i.e., Spiess's gradient algorithm) to solve SUE-based ODMEP is successfully applied to the Winnipeg network, which shows that it can be used in real-size networks.

According to the findings of this study on a medium-sized network with moderate congestion, SUE-based OD estimation models would be more accurate if correlated routes were taken into account. The methodology is general enough to be applied to any other network. Thus, more tests on large-scale networks should be conducted to draw more generalized conclusions. Although the Spiess gradient method is suitable to apply to large-scale networks, it may get trapped in local optima because of the non-convexity of the problem. Recently, Shahbandi and Babazadeh [36] have proposed a novel hybrid approach, named GPSO, which integrates particle swarm optimization (PSO) and gradient methods to combine the effective global search ability of PSO and good local convergence properties of the gradient. Utilizing this hybrid method in the SUE-based ODMEP could provide more accurate solutions than the gradient approach. Also, to enhance the quality of OD estimates, other detailed information that may be easily available from emerging data sources such as GPS, Bluetooth, and mobile phones can be used [37].

In the present study, the rule to select a subset of links for traffic counts was the intersection of links with most OD pairs. Another study could examine the role of counting locations by investigating other rules such as links with the heaviest traffic, heaviest traffic on different roads, and random selection (for instance, see [38] on estimating freight tour flows). Furthermore, we have used simulated link counts. It is suggested that real link counts be used to determine whether the above findings are valid in real-life conditions. Moreover, dispersion parameters of the route choice models have been estimated exogenously and considered fixed during the estimation process. For future studies, it is recommended that this parameter be estimated simultaneously with the OD matrix in an integrated framework.

In this study, we have employed two modifications of the MNL model that handle the problem of path overlapping through the deterministic component of the utility function. However, models of the generalized extreme value (GEV) family [28] that apply tree structure can also be used to capture the similarity among paths through the random component of the utility function. A recursive logit model proposed by Knies et al. [39] can also overcome the correlation between paths. Further, a traffic equilibrium assignment based on the network generalized extreme value (NGEV) model with the ability to capture path correlation without explicit path enumeration has been recently proposed [40].

The identical perception variance is another disputable limitation of the MNL model stemming from IIA. Multinomial Weibit (MNW), which was introduced by Castillo et al. [41], relaxes the assumption of the same variance among choice alternatives. In addition, Xu et al. [42] developed a multiplicative hybrid (MH) model that releases both MNL and MNW assumptions. Future works may focus on integrating these variants of route choice models into the OD estimation system.

Correction terms used for C-logit and PS-logit are based upon length/free-flow travel time, which may cause inaccuracy since a short path can have a large flow-dependent travel cost and vice versa. Zhou et al. [43] and Xu et al. [44] presented an SUE-C-logit formulation with commonality factors that capture the route similarity based on flow-dependent costs. Duncan et al. developed this extension for PS-logit [45] and GEV models [23]. Incorporating these SUE formulations based on generalized, flow-dependent costs into the ODMEP would be beneficial.

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Figure and Table Captions

Figure 1 Flowchart of solving SUE-based ODMEP
(SUE: stochastic user equilibrium; ODMEP: OD matrix estimation problem)

Figure 2 Convergence of the proposed algorithm using the complete set of link counts on the Winnipeg network (Log: logarithm; MNL: multinomial logit)

Figure 3 Performance comparison between three models regarding RMSE (OD)
(RMSE: root mean square error; OD: origin-destination; MNL: multinomial logit)

Figure 4 Variations in ΔRMSEMNL(OD) versus iterations of column generation
(ΔRMSEMNL(OD): root mean square error of OD demand matrix compared with that of MNL)

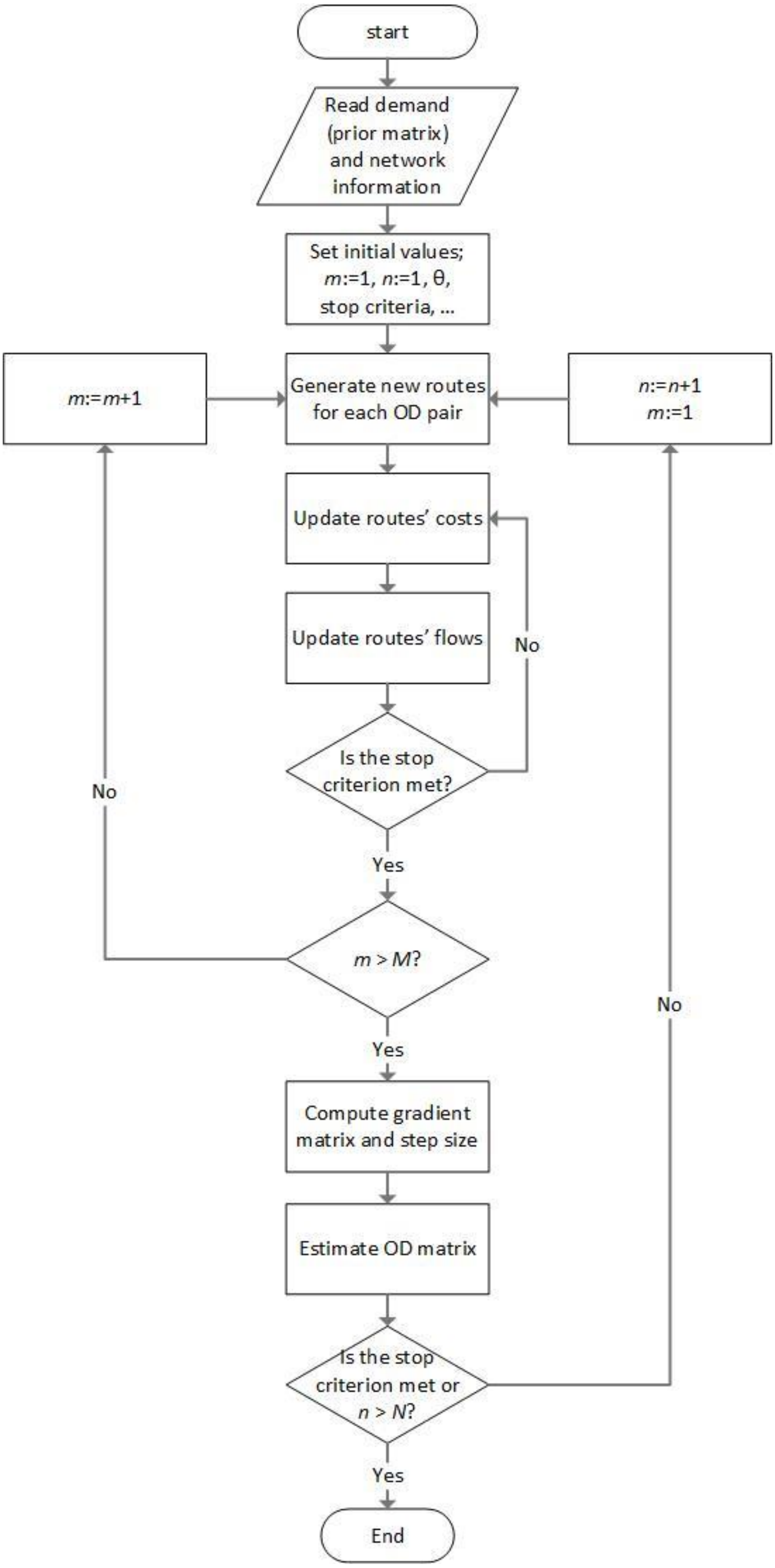
Table 1: Results of ODMEP using SUE with three route choice models

Table 2: Models output compared to MNL-based model

Table 3: Average running time of the Spiess algorithm on the Winnipeg network

Table 4: Results of SUE-based ODMEP based on different sets of counted links

Figures and Tables



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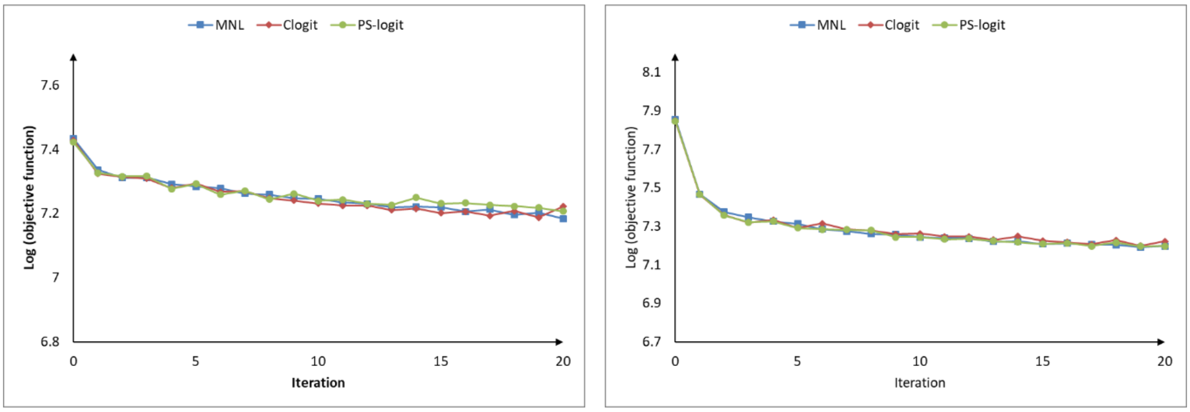
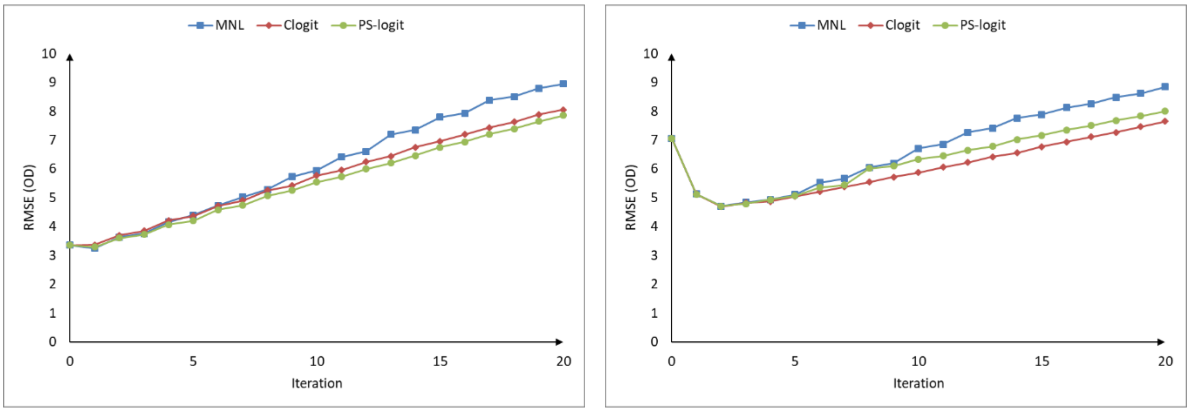


(a) scenario 1 (b) scenario 2

Figure 2 Convergence of the proposed algorithm using the complete set of link counts on the Winnipeg network (Log: logarithm; MNL: multinomial logit)



(a) scenario 1 (b) scenario 2

Figure 3 Performance comparison between three models regarding RMSE (OD)
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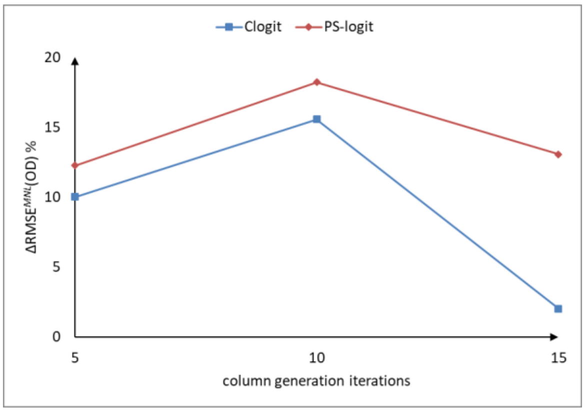
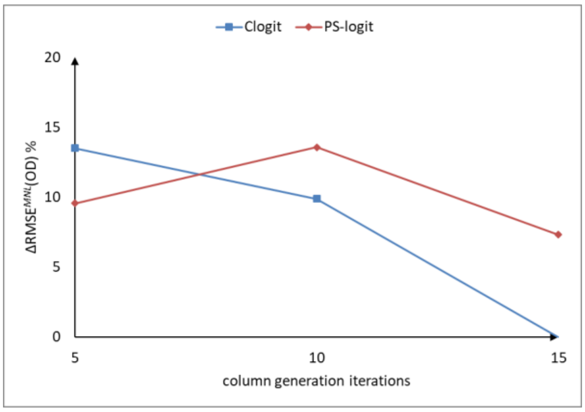


(a) scenario 1



(b) scenario 2

Figure 4 Variations in $\Delta RMSE^{MNL}(OD)$ versus iterations of column generation ($\Delta RMSE^{MNL}(OD)$: root mean square error of OD demand matrix compared with that of MNL)

Table 1: Results of ODMEP using SUE with three route choice models

Performance measures		scenario 1			scenario 2		
		MNL	C-logit	PS-logit	MNL	C-logit	PS-logit
OD demand	R^2 (OD)	0.761	0.795	0.804	0.759	0.805	0.792
	RMSE (OD)	8.945	8.050	7.849	8.838	7.647	7.995
	Total demand	64,043	63,596	63,345	62,714	61,835	61,914
Link flows	R^2 (LF)	0.965	0.962	0.964	0.965	0.963	0.965
	RMSE (LF)	103.752	108.464	106.539	105.389	108.430	105.396

Total true demand = 64,775

(Note: RMSE: root mean square error; OD: origin-destination (demand); LF: link flow; MNL: multinomial logit; SUE: stochastic user equilibrium; ODMEP: OD matrix estimation problem)

Table 2: Models output compared to MNL-based model

Performance measures		scenario 1		scenario 2	
		C-logit	PS-logit	C-logit	PS-logit
OD demand	ΔR^2 (OD)	0.034	0.043	0.046	0.033
	ΔRMSE (OD) (%)	-10.00	-12.25	-13.48	-9.54
	ΔTotal demand (%)	-0.7	-1.1	-1.4	-1.3
Link flows	ΔR^2 (LF)	-0.003	-0.001	-0.002	0.000
	ΔRMSE (LF) (%)	4.54	2.69	2.89	0.01

Total true demand = 64,775

(Note: RMSE: root mean square error; Δ: variation; OD: origin-destination (demand); LF: link flow; MNL: multinomial logit)

Table 3: Average running time of the Spiess algorithm on the Winnipeg network

Route choice model	MNL	Clogit	PS-logit
Running time (minutes)	12.1	13.5	13.5

Table 4: Results of SUE-based ODMEP based on different sets of counted links

		Scenario 1			Scenario 2		
Counted links (%)	SUE model	RMSE (OD)	$\Delta RMSE^{MNL}$ (OD) (%)*	Total demand**	RMSE (OD)	$\Delta RMSE^{MNL}$ (OD) (%)	Total demand
50	MNL	8.853	-	63,139	8.813	-	60,811
	C-logit	7.747	-12.49	62,512	7.891	-10.46	60,368
	PS-logit	7.705	-12.97	62,371	7.703	-12.59	60,410
25	MNL	7.926	-	61,221	8.428	-	58,101
	C-logit	7.561	-4.61	61,160	7.558	-10.33	57,631
	PS-logit	6.892	-13.05	61,006	7.200	-14.57	57,635
10	MNL	7.224	-	58,957	8.418	-	54,256
	C-logit	6.347	-12.15	58,847	7.892	-6.26	54,137
	PS-logit	6.157	-14.78	58,606	7.504	-10.86	54,299

* RMSE(OD) compared with that of MNL

** Total true demand = 64,775

(Note: RMSE: root mean square error; Δ: variation; OD: origin-destination (demand); MNL: multinomial logit; SUE: stochastic user equilibrium; ODMEP: OD matrix estimation problem)

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