Finite Element Analysis and Aspects of Solar Energy for Prandtl Nanofluid with Heat Transfer

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Abstract: Recently the work on nanofluids are aimed by ultrafine elements (≤100 nm) that are adjourned in a diversity of conventional liquids, for instance, lubricate, ethylene glycol, remedial liquefied emollient, etc. The straightforward liquids heat transport possessions improved by consuming nano-particles. Often exploited nano-particles, for instance non-metallic and metallic significantly progress the transporter liquids' thermal features. Furthermore, nanofluids can be developed in security structures, control fusion plants, reheating astral, chilling, etc. The objective of the current effort is to explore the thermal aspects of radiation in mixed convection flow of magneto Prandtl nanofluids in a stagnation point frame. The concept of boundary layer approaches a mathematical formulation is performed. Additionally, the similarity variables approach has been exploited to attain ODEs and solved via finite element strategy. Furthermore, researchers are observing small nano-particles as they have remarkable possessions, for instance outstanding thermal transfer, which is required in progressive nanotechnology; materials work, heat exchangers, and machineries. The numerous features of the aggregation factors on nanofluid velocity, temperature and concentration fields, skin friction, local Nusselt and Sherwood number are examined graphically and in table form. The current outcomes specify that the mixed

convection and Prandtl fluid factors decay the velocity component.

Keywords: Finite Element Method; Nanofluid; Prandtl fluid model; Mixed convection flow; Stagnation point flow; Thermal radiation.

1. Introduction

Recently, the wide-ranging uses of nanofluids in the arena of engineering, heat industries and technological progressions, the study of nanofluids have become a fascinating area of research. Nanofluids have received research attention due to their enormous uses such as fuel depletion, drug transport, nuclear vessels, chilling of chips, food protection, etc. Furthermore, some dynamic uses of this material comprise control engines, creations of polymers, gas makers, paper forming, cut-glass material, wire design etc. Nanofluids are introduced as most energy bases when these transported nanoparticles, specially transition metals as well as metal oxides are added because they meaningfully raise the thermal performance of conductivity and provide extra heat transfer. Refining the thermal necessities of the fluid supports a developed amount of flexibility in the thermal unit construction. The word nanoliquid first studied by Choi et al. [1]. Sheikholeslam el al. [2] reviewed the CuO nanofluid flow and heat transfer considering the Lorentz forces features. The heat sink-source aspects for 3D Oldroyd-B nanofluid were considered by Khan el al. [3]. The effects of $CO-H_2O$ nanoparticles on an reversed perpendicular cone were reported by Ellahi et al. [4] . Khan et al. [5] examined the aspect of Burgers nanofluid considering new mass flux conditions. The fluid flow due to non-linearly stretched plates with convective conditions was discussed by Waqas et al. [6]. Khan et al. [7] scrutinized Burgers fluid characteristics with nanomaterials. Hayat et al. [8] scrutinized the MHD influence on nanofluid flow via exponential expansion plate with impact of radiation. Sheikholeslami and Shehzad [9] studied nanofluid properties by taking into account the properties of the Lorentz force. The numerous studies that have been made in the field of nanofluids are discussed in Refs. [10-20].

Recent advances report that thermal radiation has significant importance in technical strategies and powered manufacturing. Heat transport progression and thermal radiation has various uses in industrial processes such as, astronomical vehicles, satellites, gas turbines, fissile plants, arsenals, airplanes, materials processing, transparent ecological processes, cubicle heating, rocket impulsion, decoration, and other solar accumulators. The wide-ranging uses in engineering and physics mostly in planning the equipment’s vapour turbines and astronomical equipment has developed very significantly. Furthermore, the radiation effect on blood flow is an essential study focus in bio-medical work and in many therapies, specifically in thermo-therapeutic measures. Mahanthesh et al. [21] scrutinized the characteristics of radiation developments and heat sink-source in nanofluid flow in a rotating disk. Hayat et al. [22] inspected the features of entropy generation and radiation in nanofluid flow. Jamshed et al.[23] analyzed the Joule heating and thermal radiation outcomes on viscous second grade nanofluid over a horizontal surface. Danish et al.[24] deliberated the effects of chemical reaction and thermal radiation on Williamson nanofluid with combined MHD electrical and activation energy. Kumar et al.[25] considered the consequences of thermal radiation on the MHD flow of a connecting nanofluid passing through a permeable vertical flat plate. The relevant studies deal with radiation was presented in [26-31].

The purpose of this study is to examine the effects of thermal radiation and MHD [32-37] in a stagnation point flow of Prandtl nanofluid with mixed convection. The novelty includes the magnetic field, thermal radiation; stagnation point, mixed convection and the theory of nanofluid (Brownian diffusion and thermophoresis) are a part of the recent study. The Brownian and thermophoresis nano-particles in fluids which are important in transport of heat rapidly sustained and continuous temperature variant. It is essential from energy and momentum equations that nano-particle diffusion is extra and the Brownian strength is influential when the base/local fluid temperature is higher. The nanoparticle interruption, insignificant nano-particles dissolve quicker in warmer areas and slower in cooler fields when there is temperature difference in the flow area. The Thermal aspects of radiation in flow have worth in various progressive energy adaptation schemes functioning at extraordinary temperature. Such thought inside the structure is usually the outcome of radiation by the operational fluid and warm walls. Lastly, the numerical solutions via FEM of the ODEs are obtained and discussed.

2. Formulation

The mixed convection and stagnation point flow of magnetite Prandtl nanofluid with thermal features of radiation are examined. The flow is due to the linear expansion of a linearly stretching sheet over a distance x, i.e. $u_w = cx$. Furthermore, a magnetic field of intensity B_0 is applied in the transverse direction to the flow. The considered assumptions give the following ODEs:

$$u_x + v_y = 0, \tag{1}$$

$$uu_x + vu_y = U\left(U_y\right) + \frac{\nu A}{2C^3}\left(u_y\right)^2\left(u_{yy}\right) + \nu\frac{A}{C}\left(u_{yy}\right) + g\beta_1\left(T - T_\infty\right) + g\beta_2\left(C - C_\infty\right) + \frac{\sigma B_0^2}{\rho}\left(U - u\right), \qquad (2)$$

$$uT_x + vT_y = \alpha_m\left(T_{yy}\right) - \frac{1}{\rho c_p}\left(q_r\right)_y + \tau\left(D_B\left(C_y\right)\left(T_y\right) + \frac{D_T}{T_\infty}\left(T_y\right)^2\right), \qquad (3)$$

$$uC_x + vC_y = D_B\left(C_{yy}\right) + \frac{D_T}{T_\infty}\left(T_{yy}\right), \qquad (4)$$

$$u = u_w\left(x\right) = cx, \;\; v = 0, \;\; T = T_w, \;\; C = C_w, \quad \text{at} \quad y = 0, \qquad (5)$$

$$u = U\left(x\right) = ax, \;\; v = -ay, \;\; T = T_\infty, \;\; C = C_\infty \quad \text{as} \quad y \to \infty. \qquad (6)$$

Where $x-$ and $y-$velocity components are u and v. $\left(T,\ C\right)$ the temperature and concentration, respectively, ρ fluid density, α_m thermal diffusivity, D_B Brownian diffusion coefficient, B_0 magnetic field, D_T thermophoretic diffusion coefficient, β_1 and β_2 thermal and concentrations expansion factors, respectively, g gravitational acceleration.

The radiative heat flux is clarified as:

$$q_r = -\frac{4\sigma_1}{3k}\frac{\partial T^4}{\partial y}; \quad T^4 = 4T_\infty^4 - 3T_\infty^4, \qquad (7)$$

here k and σ_1 are the mean absorption coefficient and the Stefan-Boltzmann constant, respectively.

Utilizing eqs (7) to obtain

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma_1 T_\infty^3}{3k}\frac{\partial^2 T}{\partial y^2}. \qquad (8)$$

2.1. Similarity transformations:

$$u = cxf'\left(\eta\right), \quad v = -\sqrt{c\nu}\, f\left(\eta\right), \quad \eta = \left(\frac{c}{\nu}\right)^{\frac{1}{2}} y, \quad \theta\left(\eta\right) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi\left(\eta\right) = \frac{C - C_\infty}{C_w - C_\infty}. \qquad (9)$$

Eq. (9), report Eqs (10)-(14):

$$\left(\alpha + \beta f''^2\right) f''' - f'^2 + ff'' - Mf' + M\frac{a}{c} + \frac{a^2}{c^2} + \Lambda\left(\theta + N\phi\right) = 0 \qquad (10)$$

$$\left(1 + R\right)\theta'' + \Pr N_b \theta'\phi' + \Pr N_t \theta'^2 + \Pr f\theta' = 0 \qquad (11)$$

$$\phi'' + \frac{N_t}{N_b}\theta'' + Le \Pr f\phi' = 0, \qquad (12)$$

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1, \tag{13}$$

$$f(\infty) \to \frac{a}{c}, \; \theta(\infty) \to 0, \; \phi(\infty) \to 0. \tag{14}$$

Here Prandtl fluid factor α, elastic factor β, mixed convection factor Λ, buoyancy ratio factor N, magnetic factor M, Prandtl number $\Pr$, thermophoresis parameter N_t, Brownian motion factor N_b, radiation factor R and Lewis factor Le

$$\alpha = \frac{1}{\mu AC}, \; \beta = \frac{c^3 x^2}{2C^2 \nu}, \; \Lambda = \frac{g\beta_1 (T_w - T_\infty) x^3 / \nu^2}{u_w^2 x^2 / \nu^2}, \; N = \frac{g\beta_2 (C_w - C_\infty)}{g\beta_1 (T_w - T_\infty)}, M = \frac{\sigma B_0^2}{\rho c},$$
$$\Pr = \frac{\nu}{\alpha_m}, \; N_t = \frac{\tau D_T (T_w - T_\infty)}{\nu}, \; N_b = \frac{\tau D_B (C_w - C_\infty)}{\nu}, R = \frac{16\sigma_1 T_\infty^3}{3k}, \; Le = \frac{\nu}{D_B}. \tag{15}$$

The coefficient of skin-friction C_{fx}, Nusstle number Nu_x and Sherwood number Sh_x are:

$$C_{fx} = \frac{\tau_w}{\rho u_w^2}, \quad Nu_x = \frac{-xq_w}{k(T_w - T_\infty)}, \quad Sh_x = \frac{-xq_m}{D_B (T_w - T_\infty)}, \tag{16}$$

with

$$\tau_w = \left[\frac{A}{C}\frac{\partial u}{\partial y} + \frac{A}{6C^3}\left(\frac{\partial u}{\partial y}\right)^3 \right]_{y=0}, \; q_w = \left[\left(-k \frac{\partial T}{\partial y} \right) \right]_{y=0}, \; q_m = \left[\left(-D_B \frac{\partial C}{\partial y} \right) \right]_{y=0}. \tag{17}$$

Dimensionless representations

$$C_{fx} \operatorname{Re}_x^{\frac{1}{2}} = \left(\alpha f''(0) + \beta f'''(0)^3 \right), \; Nu_x \operatorname{Re}_x^{-\frac{1}{2}} = -(1+R)\theta'(0), \; Sh_x \operatorname{Re}_x^{-\frac{1}{2}} = -\varphi(0). \tag{18}$$

3. Finite Element Analysis

The finite element method has been utilized to solve an Eqs. (10)-(14). For solving ordinary or partial differential equations we exploited an excellent technique called the finite element method. The steps related to finite element analysis are:

1) Domain discretization into elements
2) Element equations derivation
3) Element equations assembly
4) Boundary conditions Imposition
5) Assembled equations Solution.

For solving we use

$$f' = h \qquad (19)$$

Equations (10)-(14) are

$$\left(\alpha+\beta\left(h'\right)^{2}\right)h''-h^{2}+fh'-Mh+M\frac{a}{c}+\frac{a^{2}}{c^{2}}+\Lambda\left(\theta+N\phi\right)=0, \qquad (20)$$

$$\left(1+R\right)\theta''+\Pr f\theta'+\Pr N_{b}\theta'\phi'+\Pr N_{t}\theta'^{2}=0, \qquad (21)$$

$$\phi''+Le\Pr f\phi'+\frac{N_{t}}{N_{b}}\theta''=0, \qquad (22)$$

$$f\left(0\right)=0,\ h\left(0\right)=1,\ \theta\left(0\right)=1,\ \phi\left(0\right)=1, \qquad (23)$$

$$h\left(\infty\right)\to\frac{a}{c},\theta\left(0\right)\to0,\phi\left(\infty\right)\to0. \qquad (24)$$

3.1. **Variational design**

Eqs., (25)-(28) in variable form with (η_{e},η_{e+1}) linear elements are

$$\int_{\eta_{e}}^{\eta_{e+1}}w_{1}\left(f'-h\right)d\eta=0, \qquad (25)$$

$$\int_{\eta_{e}}^{\eta_{e+1}}w_{2}\left(\left(\alpha+\beta\left(h'\right)^{2}\right)h''-h^{2}+fh'-Mh+M\frac{a}{c}+\frac{a^{2}}{c^{2}}+\Lambda\left(\theta+N\phi\right)\right)d\eta=0, \qquad (26)$$

$$\int_{\eta_{e}}^{\eta_{e+1}}w_{3}\left(\left(1+R\right)\theta''+\Pr f\theta'+\Pr N_{b}\theta'\phi'+\Pr N_{t}\theta'^{2}\right)d\eta=0, \qquad (27)$$

$$\int_{\eta_{e}}^{\eta_{e+1}}w_{4}\left(\phi''+\Pr Lef\phi'+\frac{N_{t}}{N_{b}}\theta''\right)d\eta=0, \qquad (28)$$

where w_{i} $(i=1-4)$ are weight functions and can be thought of as the variance of f, h, θ and ϕ respectively.

4. Analysis of Results

Our aim in this section is to focus to investigate the Prandtl fluid along with the effects of thermal diffusion by studying the effects of flow control under parameters on velocity $f'(\eta)$, temperature $\theta(\eta)$, concentration $\phi(\eta)$ of nanomaterials with the help of graphs.

Fig. 1 designed to describe the performance of the fluid Prandtl module parameter α as a function of f'. The enhancing behaviour of f' is observed in the case of large α. Here for greater α increases the viscosity of the fluid, we observe intense production values which lead to large drag forces. So, f' increases. Fig. 2 plotted the curve of f' for the value of Λ. We noticed that an improvement in f' occurs for larger value of Λ. The larger Λ impacts the buoyancy force raises and flow of velocity also upturns which escalates f'. The picture of M on f' is displayed in figure 3. By increasing M the velocity profiles f' decreases. As the deviation of M effects the deviation of the Lorentz forces. The Lorentz creates more resistance to transport phenomena that results in a decrease in fluid velocity f'. The impact of N on f' is shown in Fig. 4. It is noted that with increasing values of N; there is an increase in fluid velocity f'.

Fig. 5 shows the trajectories of the nanofluid temperature field with increasing values of Brownian motion parameter N_b. This figure reveals that the temperature distribution of the Prandtl liquid is enriched with a large (Brownian movement). In fact, collusion between the base fluid (Prantl fluid) and nanoparticles beyond the parameter (Brownian movement) increases which results temperature field higher. Fig. 6 shows the aspects of the N_t on the temperature $\theta(\eta)$ field. This figure shows that $\theta(\eta)$ increases for large N_t. The larger values of N_t; the temperature difference between the heated liquid and the surrounding liquid increases. Therefore, $\theta(\eta)$ is enhanced. Fig. 7 displays the behaviour of $\Pr$ on $\theta(\eta)$ with increasing values of $\Pr$. Physically, by increasing $\Pr$ leads to a decrease in the thermal diffusion and a result decrease in $\theta(\eta)$. Fig. 8 reveals the temperature value $\theta(\eta)$ for R. Graphical analysis shows that $\theta(\eta)$ decreases with increasing values of R. Physically, R has a direct effect on ocean temperature which raises the temperature field.

Fig. 9 demonstrates the Lewis number Le effects on $\phi(\eta)$. A larger Lewis number Le reduces the concentration profile $\phi(\eta)$. A larger Brownian diffusion coefficient is responsible for a lower fluid flow concentration. Also the Brownian diffusion coefficient is inversely proportional to the Lewis number Le which is the reason to decline $\phi(\eta)$. Fig. 10 shows the influence of the N_b on $\phi(\eta)$. From this figure, $\phi(\eta)$ decreases with larger N_b. Physically random movement of the nanoparticles is enhanced for larger N_b which increase $\phi(\eta)$. Fig. 11 reports the impact of N_t for concentration $\phi(\eta)$ field. The fluid concentration intensifies for higher N_t.

Table 1 presents the numerical values of $C_f \mathrm{Re}_x^{\frac{1}{2}}$, $Nu_x \mathrm{Re}_x^{\frac{1}{2}}$ and $Sh_x \mathrm{Re}_x^{\frac{1}{2}}$. The $C_f \mathrm{Re}_x^{\frac{1}{2}}$ increases for α, β and R; $Nu_x \mathrm{Re}_x^{\frac{1}{2}}$ and $Sh_x \mathrm{Re}_x^{\frac{1}{2}}$ decays for α and R, whereas, enhance for β.

5. Final Remarks

Here the phenomena of radiation in Prantel magneto nanofluid have been studied via Finite Element Method (FEM). The mixed convection concept stagnation point frame has been also analyzed. The main facts drawn from this analysis are detailed below.

- The influence of α and Λ enhanced f'.
- The temperature field decreased for higher Prandtl values Pr.
- The influence of thermophoresis parameter N_t increased the temperature and concentration fields.
- The effect of Brownian motion N_b is to increases the temperature profiles and decreases the concentration field.
- The thermophoresis parameter N_t enhanced $\theta(\eta)$ and $\phi(\eta)$.

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List of Table

Table 1 : Numerical values of $C_f \mathrm{Re}_x^{\frac{1}{2}}$, $Nu_x \mathrm{Re}_x^{\frac{1}{2}}$ and $Sh_x \mathrm{Re}_x^{\frac{1}{2}}$ when $M = 0.3$, $\Lambda = 0.1$ and $\mathrm{Pr} = 1.0$.

α	β	R	$C_f \mathrm{Re}_x^{\frac{1}{2}}$	$Nu_x \mathrm{Re}_x^{\frac{1}{2}}$	$Sh_x \mathrm{Re}_x^{\frac{1}{2}}$
0.0	0.2	0.1	0.6889	0.3692	0.6538
0.2			0.7168	0.4532	0.7157
0.4			0.7451	0.9113	0.7449
0.1	0.0		1.2118	1.8954	1.3125
	0.2		0.7053	3.3532	0.7032
	0.4		0.6508	3.5694	0.6517
	0.1	0.0	0.7964	0.5538	0.7955
		0.2	0.7477	0.6126	0.7468
		0.4	0.6496	0.7444	0.6487

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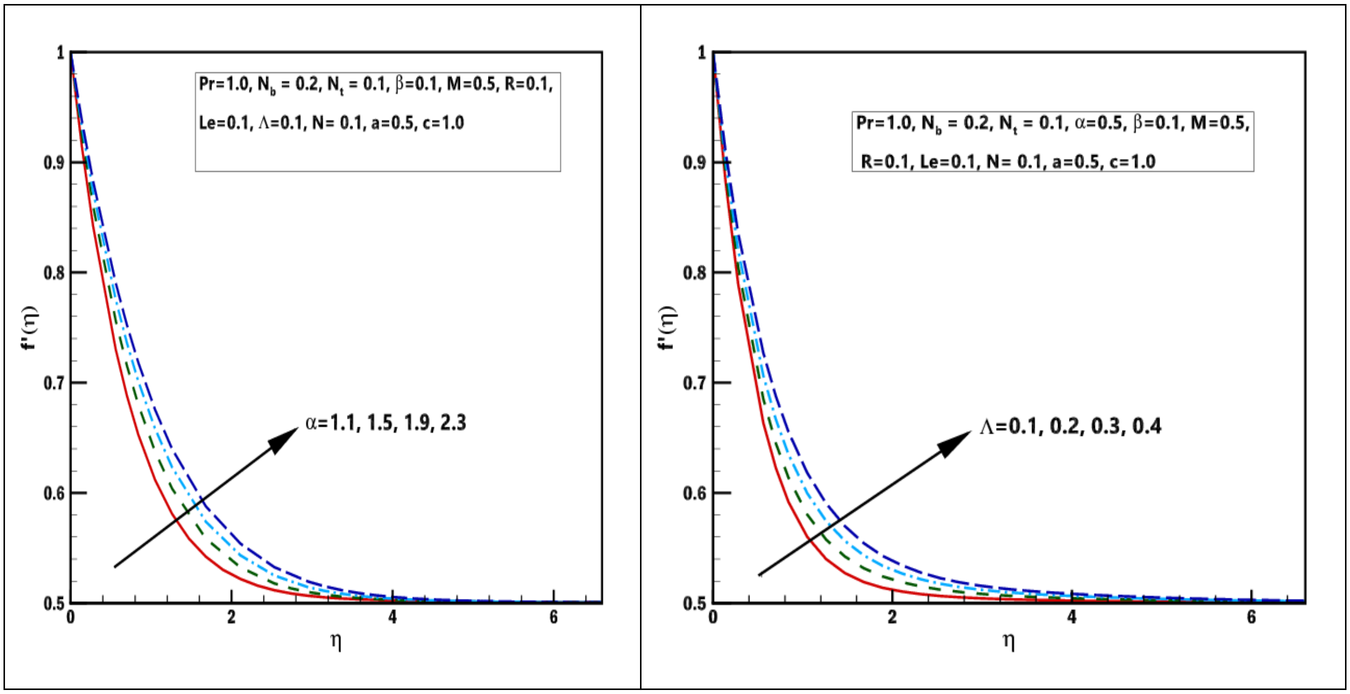
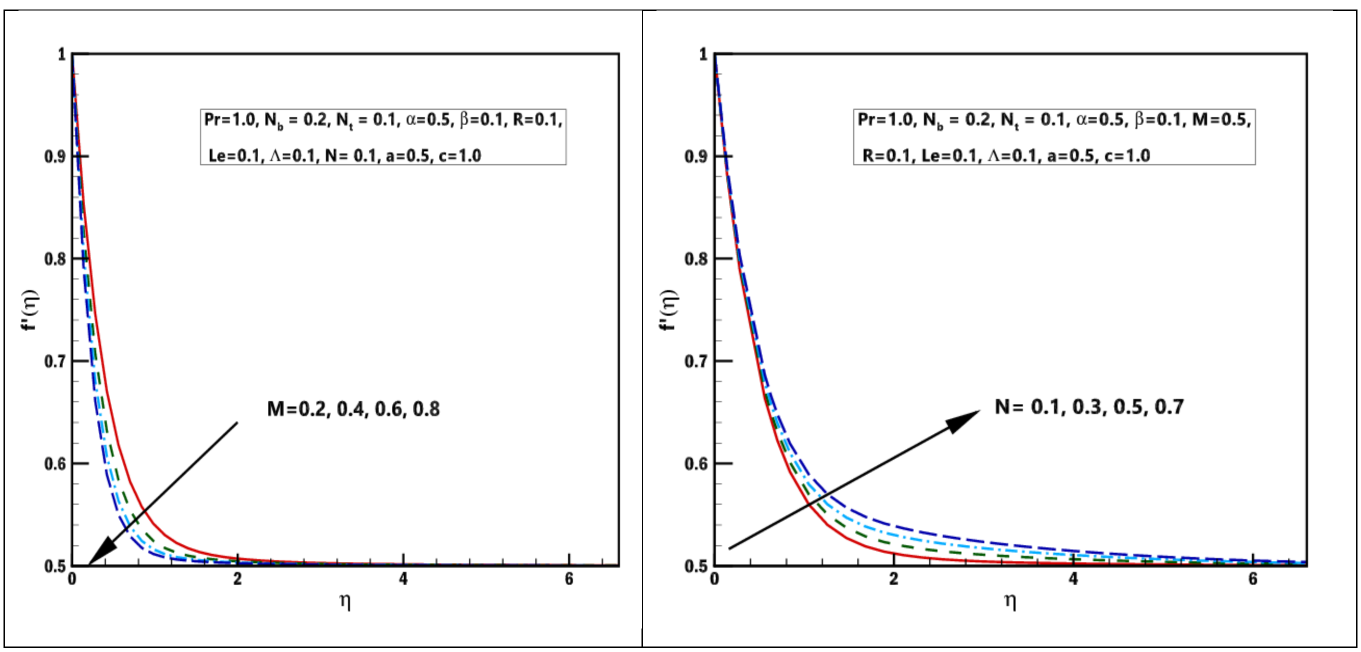
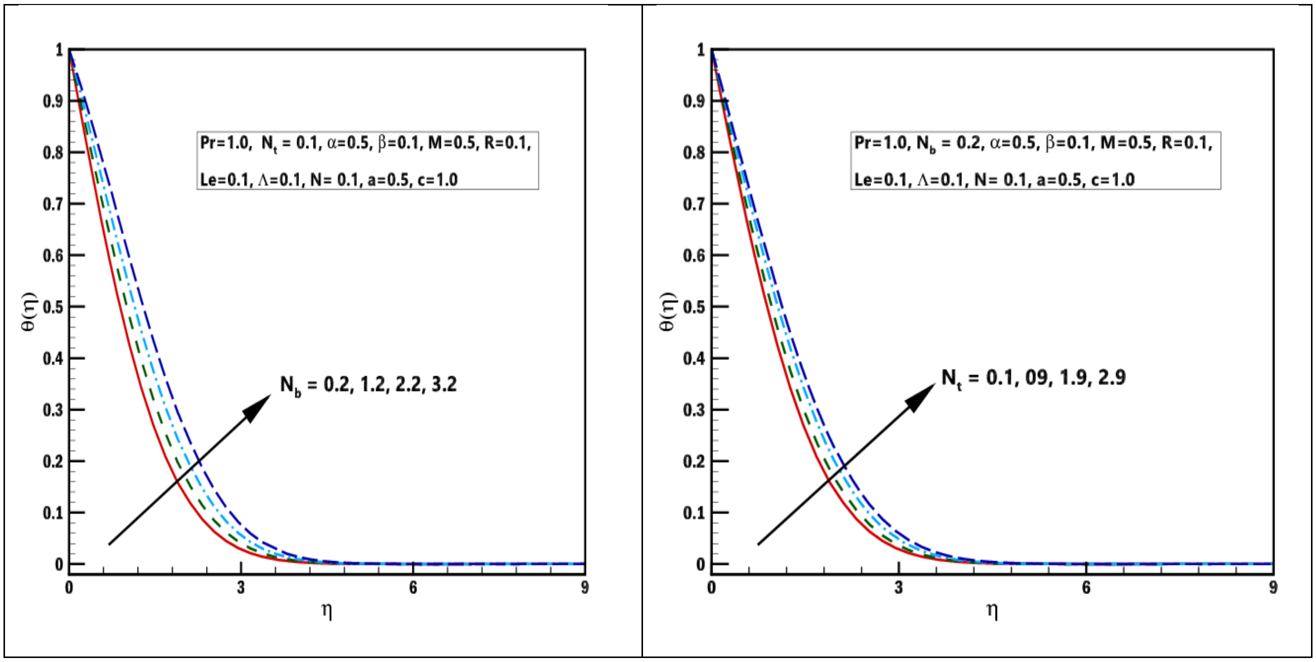
Fig. 11: ϕ impact for N_t



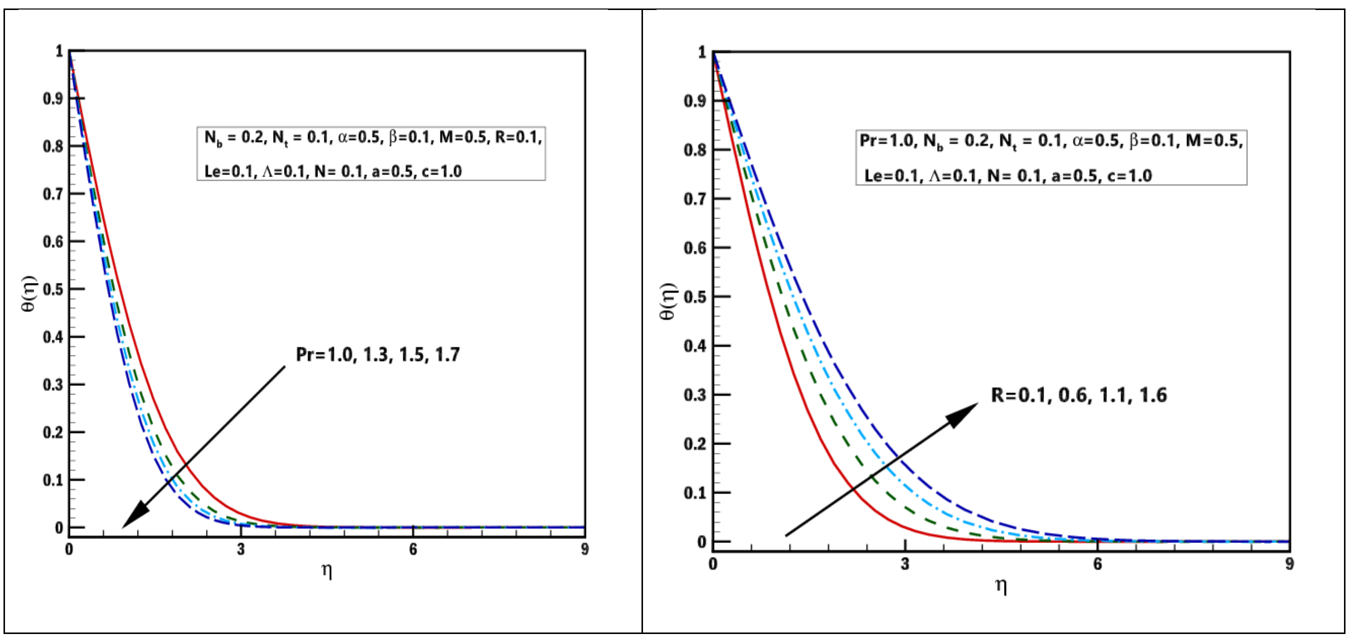
Figs 1 and 2



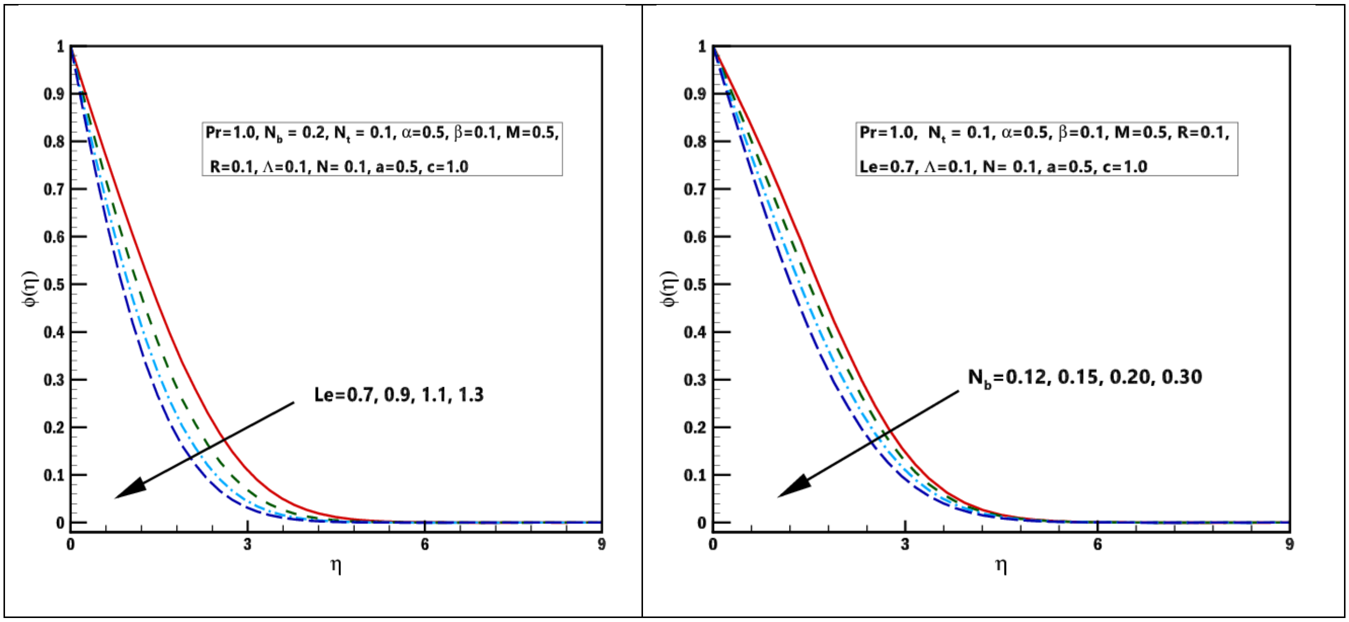
Figs 3 and 4



Figs 5 and 6



Figs 7 and 8



Figs 9 and 10

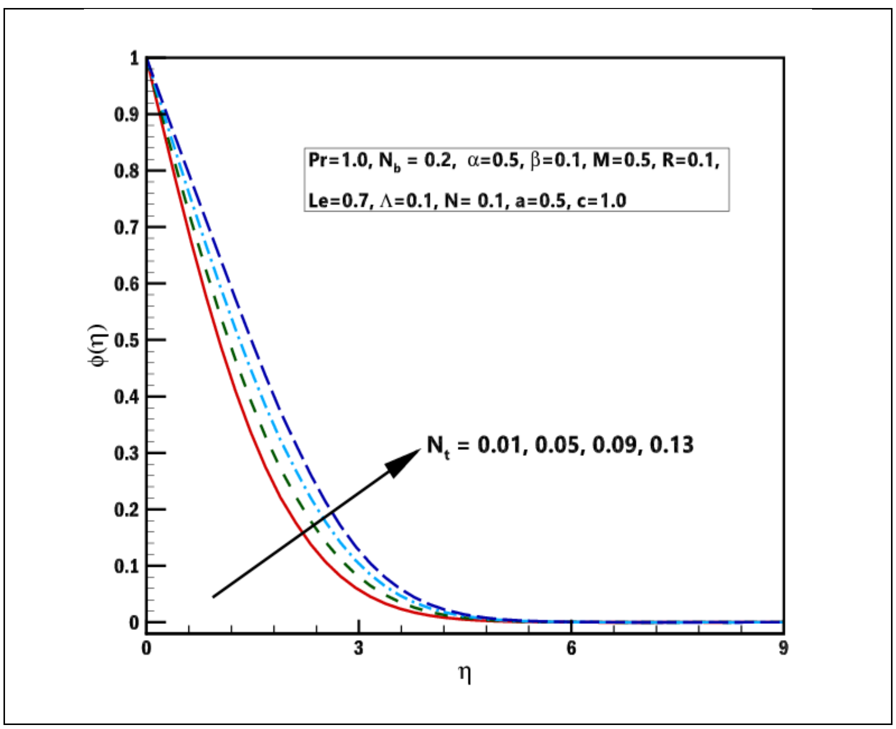


Fig 11