Comprehensive Analysis of Conformable Mathematical Model of Ebola Virus with Effective Control Strategies

Syeda Alishwa Zanib[a], Sehrish Ramzan[b], Nadeem Abbas[c], Aqsa Nazir[d], Wasfi Shatanawi[*c,e,f]

[a] *Department of Mathematics, Riphah International University, Main Satyana Road, Faisalabad 44000, Pakistan.*

[b] *Department of Mathematics, Government College University Faisalabad, 38000, Pakistan.*

[c] *Department of Mathematics and Sciences, College of Humanities and Sciences, Prince Sultan University, Riyadh, 11586, Saudi Arabia.*

[d] *Department of Engineering and Computer Science, National University of Modern Languages, Islamabad 44000, Pakistan.*

[e] *Department of Medical Research, China Medical University Hospital, China Medical University, Taichung, 40402, Taiwan.*

[f] *Department of Mathematics, Faculty of Science, The Hashemite University, P.O Box 330127, Zarqa 13133, Jordan*

*__Correspondence__: +962799724027 *wshatanawi@psu.edu.sa

Abstract The Ebola virus is a highly infectious disease that can propagate throughout a population depending on how people interact in society. his research introduces a modified mathematical model of Ebola Virus Disease (EVD), incorporating effective control strategies such as quarantine, self-isolation, and hospitalization. These compartments have played a key role in understanding the transmission of the Ebola virus disease in the society. By using a conformable derivative, a system of equations has been developed for the Ebola virus disease model. The basic reproduction number $\mathbb{R}_0$ has been determined using the Next-generation matrix method. To understand the impact of parameter variations on Ebola virus disease, sensitivity analysis of $\mathbb{R}_0$ has been observed. Stability analysis has been calculated at both the DFEP and the DPEP to assess the behaviour of virus. The conformable derivative facilitates a smooth transition from fractional order to classical models as the parameter (c) approaches to 1. Additionally, implementation of quarantine, self-isolation, and hospitalization emerges as a highly effective strategy, significantly reduced Ebola virus disease in society. These findings enhance our understanding of Ebola dynamics and offer critical implications for effective outbreak control strategies.

Keyword: Ebola virus, Mathematical modelling, Reproduction number, Sensitivity analysis, Stability Analysis.

1 Introduction

The Ebola virus, which causes Ebola virus disease, is a serious and frequently deadly infection that has created major problems for public health. The virus has intermittently spread since its discovery in 1976, particularly in Central and West African countries, with disastrous results. It is named after the Ebola River in the Democratic Republic of the Congo and is a member of the Filoviridae family (previously known as Zaire) [1–4]. The Ebola virus is spread to people by close contact with infected animals or their body fluids and is thought to have originated in animals, notably fruit bats. Once the virus has infected humans, by coming into contact with fluids from the body, including saliva, blood, organs, or other parts of the body, it can spread from one person to another [5]. In 2014-2015, Sierra Leone, Liberia and Guinea collectively reported 3.9 cases in West Africa [6]. As an entire population, healthcare professionals were responsible for 25 per cent of all contamination during disease transmission, according to Al-Smadi (2020) [7]. Even when they had an infectious condition, Africans continued to shake hands, kiss, and caress their relatives. At funerals, Africans also practised the tradition of bathing and clothing the bereaved family. Cities, villages, and towns are places where vast crowds of people meet and spread the Ebola virus [8]. In general, Ebola virus disease symptoms include fever, lack of appetite, abdominal pain, severe headaches, sore throats, and weariness. Liver and kidney damage, vomiting, rash and other symptoms accompany these symptoms. Disorders characterized by low white blood cell counts, increased platelet counts, and elevated liver enzyme levels can all be associated with uncontrollably occurring internal and external bleeding. Three days on average are followed by a rapid progression toward death for the patient. The interval between the moment of Ebola virus infection and the beginning of symptoms is known as the incubation period, and it usually lasts between two and twenty-one days. Anywhere might have symptoms appear. An infected individual cannot transmit the Ebola virus to the host population unless they exhibit the previously listed symptoms. A mathematical model of the Ebola virus called SEIVR (susceptible, exposed, infected, vaccinated, recovered) is created by Tahir et al. (2019) [9]. Then they found its basic reproduction number R_0, furthermore, a sensitivity analysis of R_0 is also discussed, and all local equilibrium points concerning the disease are derived, conditional on the investigation of all possible equilibria of the model in terms of primary reproductive number. Rafiq et al. (2020) [10] explore the dynamics of the Ebola virus disease spread. They created a coupled nonlinear differential equation SEIR-type model. These equations offer a useful tool for discussing how domestic and wild animals can spread the Ebola virus to human populations. They initially create the suggested model and get the threshold parameter (R_0) value for the model. They establish the disease-free equilibrium (DFE) and endemic equilibrium (EE), as well as discuss the stability of the model. A basic mathematical model that developed in 2014

Ebola spread in Liberia is being studied by Maheshwari et al. (2020) [11]. The resultant mathematical model is then validated using numerical simulations and data made accessible by the WHO. They also create a new mathematical model that takes the inorganization of people into account. They describe different cases of vaccination to observe the effect of vaccination on infected individuals by the passage of time. To investigate how vaccination affects the spread of the Ebola virus, they use the best controls possible. Khan et al. (2020) [12] delved into the development of qualitative theory and approximate solutions for a fractional-order Ebola model, utilizing Atangana-Baleanu-Caputo (ABC) fractional operators. Conditions for the model's existence and stability were established through diverse analytical tools. The Laplace Adomain Decomposition method was employed to obtain approximate solutions, with Matlab-generated graphs illustrating the model's dynamics for different fractional order values of γ. Until an infected person manifests the aforementioned EVD symptoms, they cannot spread the disease to the host population. To comprehend the spread of this pandemic, Farman et al. (2022) [13] provide a nonlinear time-fractional mathematical model of the Ebola virus. The Ebola virus is an extremely infectious disease that, depending on the population's size and social dynamics, can spread throughout it. A set of fractional differential equations is solved using the fractional derivative operators Caputo and Atangana-Baleanu. A qualitative investigation of the fractional order model is conducted. To determine existence and uniqueness, the fixed-point theorem and an iterative strategy are utilized. The Laplace domain decomposition approach is used to determine the time-fractional model's true behaviour. The study of Khan et al. (2022) [14] on the numerical calculations and analytical behaviour of the fractional order Ebola model. With the use of fixed point results, they determined the uniqueness, and stability conditions of the model. Using the two-step fractional Adam Bashforth approach, they computed the numerical solution to the fractional order smoke model. Yadav et al. (2023) [15] utilized the Atangana–Baleanu–Caputo fractional derivative to analyze Ebola virus transmission dynamics, offering novel insights. Numerical techniques were employed, demonstrating the enhanced accuracy of non-integer order derivatives and exploring previously uncharted aspects of the model. Mbah et al. (2023) [16] studies delve into the virus's transmission dynamics, formulating an eighteen-equation system to comprehensively describe its spread. The research explored the local and global stability of disease-free and endemic equilibria, revealing effective strategies for population segments and a forward bifurcation in the system. Nisar et al. (2023) [17] utilized a hybrid genetic algorithm (HGASQP), incorporating sequential quadratic programming and feed-forward neural networks, to optimize the Ebola virus disease model. The approach, minimizing mean squared error, demonstrated efficacy and robustness in comparison to the Adam approach.

After deeply studying the literature review, we found some model gaps. We have added quarantine, self-isolated and hospitalized classes in our model,

which was very important to control the transmission of the Ebola Virus. **Section 2:** The modified Ebola virus disease transmission model with quarantine, self-isolation, and hospitalized class will be discussed. **Section 3** will focus on determining the basic reproduction number using the Next-generation Method, as well as discussing local and global stability at the disease-free and endemic equilibrium point. Additionally, we will provide a sensitivity analysis of the basic reproduction number. **Section 4** will cover the outcomes and observations of the Ebola virus disease model, while **Section 5** will conclude our study.

2 Model description

A compartmental model is used to study the following compartments: Susceptible (S), Exposed (E), Infected (U), Quarantine (Q), Hospitalized (H), Self-isolation (J), and Recovered (R). Individuals in the Susceptible (S) compartment are not infected. Those in the Exposed (E) compartment carry the disease-causing pathogen but do not yet show clinical symptoms. Infected (U) individuals are infectious, can spread the disease, and exhibit symptoms. After their infectious period, these individuals are placed under sanitary care and then classified as hospitalized. Quarantine (Q) contains individuals who are infected but asymptomatic and are isolated to prevent transmission. The Hospitalized (H) compartment holds individuals receiving treatment, who still pose a risk of infection. Individuals in Self-isolation (J) are infected but have mild or no symptoms, remaining at home in isolation. Finally, the Recovered (R) compartment consists of those who have survived the disease, developed natural immunity, and are no longer infectious. The dynamics of the model are illustrated in Figure 1.

The parameters in the model are defined as follows: Δ represents the natural birth rate, α is the propagation rate of susceptible individuals $S(\xi)$ to exposed individuals $E(\xi)$, β_1 is the propagation rate of exposed individuals $E(\xi)$ to quarantined individuals $Q(\xi)$, and β_2 is the propagation rate of exposed individuals $E(\xi)$ to infected individuals $U(\xi)$. The parameter ρ denotes the propagation rate of infected individuals $U(\xi)$ to hospitalized individuals $H(\xi)$, while ω represents the natural death rate. The parameters ε_2 and ε_1 describe the propagation rates from quarantined individuals $Q(\xi)$ to hospitalized individuals $H(\xi)$ and to self-isolated individuals $J(\xi)$, respectively. The parameter η indicates the propagation rate from self-isolated individuals $J(\xi)$ to recovered individuals $R(\xi)$, and γ represents the propagation rate from hospitalized individuals $H(\xi)$ to recovered individuals $R(\xi)$. The model also includes the following death rates: d_E for exposed individuals $E(\xi)$, d_Q for quarantined individuals $Q(\xi)$, d_U for infected individuals $U(\xi)$, d_J for self-isolated individuals $J(\xi)$, and d_H for hospitalized individuals $H(\xi)$, all due to

the virus.

Let $K : (0, \infty) \to R$ the conformable fractional derivative of K (of order c) provided by Khalil et al. [18], can be define as,

$$G_c(K)(\xi) = \lim_{\sigma \to 0} \frac{K(\xi + \sigma \xi^{1-c}) - K(\xi)}{\sigma}, \tag{1}$$

if K is differentiable, then,

$$G_c(K)(\xi) = t^{1-c} \frac{dK}{d\xi}, \;\; c \in (0, 1]. \tag{2}$$

For $G_c(K)(\xi)$, we can sometimes utilize the notation $K(c)(\xi)$ to represent the conformable fractional derivatives of K of order c. Furthermore, we may state that K is c-differentiable if the conformable fractional derivative of K of order c exists. This involves taking the limit as σ approaches 0 of the difference quotient, which captures the change in K concerning ξ over a small interval $\sigma \xi^{1-c}$. Now we discuss the mathematical behaviour of the ebola transmission in figure 2. Where the non-linear system of equations we have,

$$\frac{d^c}{d\xi^c} S = \Delta - (\alpha \, E + \omega) \, S, \tag{3}$$

$$\frac{d^c}{d\xi^c} E = \alpha \, SE - (\beta_2 U + \beta_1 Q + \omega + d_E) \, E, \tag{4}$$

$$\frac{d^c}{d\xi^c} Q = \beta_1 EQ - (\varepsilon_1 + \varepsilon_2 + \omega + d_Q) \, Q, \tag{5}$$

$$\frac{d^c}{d\xi^c} U = \beta_2 EU - (\rho + \omega + d_U) \, U, \tag{6}$$

$$\frac{d^c}{d\xi^c} J = \varepsilon_1 Q - (\eta + \omega + d_J) \, J, \tag{7}$$

$$\frac{d^c}{d\xi^c} H = \rho U + \varepsilon_2 Q - (\gamma + \omega + d_H) \, H, \tag{8}$$

$$\frac{d^c}{d\xi^c} R = \eta J + \gamma \, H - (\omega) \, R. \tag{9}$$

With initial conditions,

$$S(0) \geq 0, E(0) \geq 0, Q(0) \geq 0, U(0) \geq 0, J(0) \geq 0, H(0) \geq 0, R(0) \geq 0. \tag{10}$$

3 Model Analysis

3.1 Invariant Region

To find the invariant region of system of equations (3-9) with non-negative initial conditions (10) solution is bounded, taking total population $\mathbb{N}(S, E, Q, U, J, H, R) = S(\xi) + E(\xi) + Q(\xi) + U(\xi) + J(\xi) + H(\xi) + R(\xi)$. When there is no disease, take the derivative of N with respect to ξ.

We obtain:

$$\xi^{1-c}\mathbb{N}' = \Delta - \omega\mathbb{N}, \tag{11}$$

after solving (11) and $\xi \to \infty$, then,

$$\Omega = \{(S, E, Q, U, J, H, R) \in \mathbb{R} : \mathbb{N}(t) \leq \frac{\Delta}{\omega}\}. \tag{12}$$

which is the feasible solution set of a system of equations that are bounded.

3.2 Positivity of Solution

Theorem 3.1. *If* $S(0) > 0, E(0) > 0, Q(0) > 0, U(0) > 0, J(0) > 0, H(0) > 0, R(0) > 0$ *are positive in the feasible set* Ω*, then the solution set* $(S(\xi), E(\xi), Q(\xi), U(\xi), J(\xi), H(\xi), R(\xi))$ *of system of equations is positive* $\forall\ \xi \geq 0$

Proof. Taking the first equation from the system of equations,

$$\frac{d^c}{d\xi^c}S = \Lambda - (\alpha\, E + \omega)\, S, \tag{13}$$

after simplification,

$$S \geq S(0)e^{-\xi^{c-1}(\alpha\, E+\omega)\xi}, \tag{14}$$

similar to another system of equations. Therefore, we can say the solution set of all system of equations are positive for $\xi \geq 0$ [19]. □

Disease-Free Equilibrium Point (DFEP)
When population has no infectious individuals of Ebola virus disease, then

$$E = Q = U = J = H = R = 0.$$

Then we have DFEP,

$$\mathbb{E} = (\mathbb{S}, 0, 0, 0, 0, 0, 0) = \left(\frac{\Delta}{\omega}, 0, 0, 0, 0, 0, 0\right). \tag{15}$$

Disease-Present Equilibrium Point (DPEP)
When the population has an infectious person with Ebola virus disease, then DPEP is,

$$\begin{aligned}
E^* &= \frac{\varepsilon_1 + \varepsilon_2 + \omega + d_Q}{\beta_1}, \\
H^* &= \frac{\varepsilon_2(\alpha S^* - \omega - d_E)}{\beta_1(\gamma + \omega + d_H)}, \\
U^* &= 0, \\
J^* &= \frac{\varepsilon_1(\alpha S^* - \omega - d_E)}{\beta_1(\eta + \omega + d_J)}, \\
Q^* &= \frac{\alpha S^* - \omega - d_E}{\beta_1}, \\
R^* &= \frac{(\alpha S^* - \omega - d_E)(\eta\gamma\varepsilon_1 + \eta\gamma\varepsilon_2 + \eta\omega\varepsilon_1 + \eta d_H\varepsilon_1 + \gamma\omega\varepsilon_2 + \gamma d_J\varepsilon_2)}{\beta_1(\eta + \omega + d_J)(\gamma + \omega + d_H)\omega}.
\end{aligned} \tag{16}$$

3.3 Basic Reproduction Number

The next-generation matrix method applies a system of equations to calculate the basic reproduction number $\mathbb{R}_0$ [20]. Taking Transmission matrix $\mathbb{F}$ and transition matrix $\mathbb{V}$ from system of equations at DFEP,

$$\mathbb{F}\mathbb{V}^{-1} = \begin{bmatrix} \frac{\alpha\,\Delta}{\omega\,(\omega + d_E)} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{17}$$

Therefore, we find the basic reproduction number,

$$\mathbb{R}_0 = \frac{\alpha\,\Delta}{\omega\,(\omega + d_E)}. \tag{18}$$

3.4 Local Stability of DFEP

Theorem 3.2. *The DFEP point is locally stable if* $\mathbb{R}_0$ *is less than unity and unstable if* $\mathbb{R}_0$ *is greater than unity [21].*

Proof. The Jacobian matrix of system of equations (3-9) at DFEP, we get,

$$\mathbb{J}_0 = \begin{bmatrix} -\omega & -\frac{\alpha\Delta}{\omega} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\alpha\Delta}{\omega} - \omega - d_E & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\omega - d_Q - \varepsilon_1 - \varepsilon_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\omega - \rho - d_U & 0 & 0 & 0 \\ 0 & 0 & \varepsilon_1 & 0 & 0 & -\eta - \omega - d_J & 0 \\ 0 & 0 & \varepsilon_2 & \rho & -\gamma - \omega - d_H & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma & \eta & -\omega \end{bmatrix}, \tag{19}$$

The characteristic equations are,

$$\begin{aligned} \mathbb{J}_0 = (-\omega - \delta)\left(\frac{\alpha\,\Lambda}{\omega} - \omega - d_E - \delta\right)(-\omega - d_Q - \varepsilon_1 - \varepsilon_2 - \delta)(-\omega - \rho - d_U - \delta) \\ (\tau + \delta + \omega)\left(\omega^2 + (\eta + \gamma + d_J + d_H)\,\omega - \delta^2 + (\gamma + d_H)(\eta + d_J)\right). \end{aligned} \tag{20}$$

After solving (20), we have the root of δ are,

$$\begin{aligned} \delta_1 = -\omega, \delta_2 = (d_E + \omega)(\mathbb{R}_0 - 1), \delta_3 = -\omega - d_Q - \varepsilon_1 - \varepsilon_2, \delta_4 = -\omega - \rho - d_U, \\ \delta_5 = -\omega, \delta_6 = -\sqrt{(\gamma + \omega + d_J)(\eta + \omega + d_H)}, \delta_7 = -\sqrt{(-\gamma - \omega - d_J)(-\eta - \omega - d_H)}, \end{aligned} \tag{21}$$

where δ_1, δ_3, δ_4, δ_5, δ_6 and $\delta_7 < \mathbb{R}_0$ and $\delta_2 < 1$ if $\mathbb{R}_0 < 1$ in equation (21). □

3.5 Global Stability of DFEP

Lemma 1. *If* $\mathbb{R}_0 < 1$*, then, the system of equation is globally asymptotic stable at DFEP if condition* ***(A1)*** *and* ***(A2)*** *are satisfied [22].*

$$\textbf{\textit{A1:}}\ \frac{dF_H}{d\xi} = \mathbb{G}(F_H, 0)$$

$$\textbf{\textit{A2:}}\ H(F_H, V_H) = P_H V_N - \hat{H}(F_H, V_H)$$

where P_H *shows the matrix of parameters of infectious stages.* F_H *is individuals without Ebola virus and* V_H *individuals with Ebola virus disease.*

Theorem 3.3. *If R_0 is less than unity, the DFEP is globally stable; if $\mathbb{R}_0$ is greater than unity, it is unstable.*

Proof. Firstly, we satisfy condition **(A1)**, so rewrite the system of equations $F_H = (S)$ and, $V_H = (E, Q, U, J, H, R)$. Then, the DFEP is given by the fixed point, $\mathbb{E}_0 = (\mathbb{S}^0) = \left(\frac{\Delta}{\omega}\right)$, the system $\frac{dF_H}{d\xi} = \mathbb{G}(F_H)$ becomes,

$$\frac{dS^*}{d\xi} = -\omega\, S + \Delta, \tag{22}$$

By solving Eq. (22), the equation has a unique equilibrium point,

$$S^* = \frac{\Delta}{\omega}, \tag{23}$$

$\mathbb{S}^0$ is therefore globally asymptotically stable. The criterion **(A1)** is therefore satisfied. The second condition **(A2)** now has to be confirmed.

$$H(F_H, V_H) = P_H V_N - \hat{H}(F_H, V_H),$$

and $\hat{H}(F_H, V_H) \geq 0$, For that, system of equations (3-9). We have,

$$H(F_H, V_H) = \begin{bmatrix} (S\alpha - \beta_2 U - \beta_1 Q - \omega - d_E)\, E - \beta_1 QE - \beta_2 EU \\ \beta_1 QE + (\beta_1 E - \omega - d_U - \varepsilon_1 - \varepsilon_2)\, Q \\ \beta_2 EU + (\beta_2 E - \omega - \rho - d_U)\, U \\ \varepsilon_1 Q + (-\eta - \omega - d_J)\, J \\ \rho\, U + \varepsilon_2 Q + (-\gamma - \omega - d_H)\, H \\ \eta\, J + \gamma\, H + (-\omega)\, R \end{bmatrix}, \tag{24}$$

$$\hat{H}(F_H, V_H) = P_H V_N - H(F_H, V_H) = \begin{bmatrix} \alpha\, E\, (S^* - S) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{25}$$

This demonstrates that $\hat{H}(F_H, V_H) \geq 0$, with V_N denoting an M-matrix that has non-negative off-diagonal elements. Lemma 1 states that since the DFEP $\mathbb{E}_0$ is globally asymptotically stable when $\mathbb{R}_0 < 1$, this implies that the criteria **(A1)** and **(A2)** are met. This completes the proof.

□

3.6 Local Stability of DPEP

Theorem 3.4. *The DPEP (16) state that, system of equations (3-9) is locally stable if $\mathbb{R}_0$ is greater than unity and unstable if $\mathbb{R}_0$ is less then unity.*

Proof. The jacobian matrix of (3-9) at DPEP,

$$J_e = \begin{bmatrix} j_{11} & j_{12} & 0 & 0 & 0 & 0 & 0 \\ j_{21} & j_{22} & j_{23} & j_{24} & 0 & 0 & 0 \\ 0 & j_{32} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & j_{44} & 0 & 0 & 0 \\ 0 & 0 & j_{53} & 0 & 0 & j_{56} & 0 \\ 0 & 0 & j_{63} & j_{64} & j_{65} & 0 & 0 \\ 0 & 0 & 0 & 0 & j_{76} & j_{76} & j_{77} \end{bmatrix}, \tag{26}$$

where $j_{11} = -\frac{\alpha\left(\varepsilon_1+\varepsilon_2+\omega+d_Q\right)}{\beta_1}-\omega$, $j_{12} = -\frac{\alpha\,\Delta\,\beta_1}{\alpha\,\omega+\alpha\,d_Q+\alpha\,\varepsilon_1+\alpha\,\varepsilon_2+\omega\,\beta_1}$, $j_{21} = \frac{\alpha\left(\varepsilon_1+\varepsilon_2+\omega+d_Q\right)}{\beta_1}$, $j_{22} = \frac{\alpha\,\Delta\,\beta_1}{\alpha\,\omega+\alpha\,d_Q+\alpha\,\varepsilon_1+\alpha\,\varepsilon_2+\omega\,\beta_1}-\frac{\alpha\,\Delta\,\beta_1-\alpha\,\omega^2-\alpha\,\omega\,d_E-\alpha\,\omega\,d_Q-\alpha\,\omega\,\varepsilon_1-\alpha\,\omega\,\varepsilon_2-\alpha\,d_E d_Q-\alpha\,d_E\varepsilon_1-\alpha\,d_E\varepsilon_2-\omega^2\beta_1-\omega\,\beta_1 d_E}{\alpha\,\omega+\alpha\,d_Q+\alpha\,\varepsilon_1+\alpha\,\varepsilon_2+\omega\,\beta_1}-\omega-d_E$, $j_{23} = -\varepsilon_1-\varepsilon_2-\omega-d_Q$, $j_{24} = -\frac{\beta_2\left(\varepsilon_1+\varepsilon_2+\omega+d_Q\right)}{\beta_1}$, $j_{32} = \frac{\alpha\,\Delta\,\beta_1-\alpha\,\omega^2-\alpha\,\omega\,d_E-\alpha\,\omega\,d_Q-\alpha\,\omega\,\varepsilon_1-\alpha\,\omega\,\varepsilon_2-\alpha\,d_E d_Q-\alpha\,d_E\varepsilon_1-\alpha\,d_E\varepsilon_2-\omega^2\beta_1-\omega\,\beta_1 d_E}{\alpha\,\omega+\alpha\,d_Q+\alpha\,\varepsilon_1+\alpha\,\varepsilon_2+\omega\,\beta_1}$, $j_{44} = \frac{\beta_2\left(\varepsilon_1+\varepsilon_2+\omega+d_Q\right)}{\beta_1}-\omega-\rho-d_U$, $j_{53} = \varepsilon_1$, $j_{56} = -\eta-\omega-d_J$, $j_{63} = \rho$, $j_{64} = \rho$, $j_{65} = -\gamma-\omega-d_H$, $j_{75} = \gamma$, $j_{76} = \eta$, $j_{77} = -\omega$. The characteristics polynomial of J_e is given by,

$$\Omega^3 + (-j_{11}-j_{22})\,\Omega^2 + (j_{11}\,j_{22}-j_{12}\,j_{21}-j_{23}\,j_{32})\,\Omega + j_{11}\,j_{23}\,j_{32}, \tag{27}$$

We rename the coefficients of Ω in the above Eq (27),

$$P_1 = -j_{11}-j_{22},$$

$$P_2 = j_{22}-j_{12}\,j_{21}-j_{23}\,j_{32},$$

$$P_3 = j_{11}\,j_{23}\,j_{32}.$$

For (n = 1,2,3), the eigenvalue of the Jacobian matrix contains the negative real component if and only if $P_n > 0$, as per the Routh-Herwitz criteria [23].

$$S_1 = P_1 > 0,$$

$$S_2 = P_1P_2 - P_3 > 0,$$

$$S_3 = P_1P_2P_3 - P_1^2P_4 - P_3^2P_4 > 0.$$

For DPEP (16), when the basic reproduction number of the model is greater than one, and all the Routh Herwitz matrix determinants ($S_1, S_2, S_3 > 0$) are positive. Negative real components are present in each of the Jacobian matrix's eigenvalues. Then the model (3-9) is locally asymptotically stable. □

3.7 Global Stability at DPEP

Using the geometric technique devised by Li et al. [24], the global stability analysis at DPEP has been examined.

Lemma 2. *Let us assume the following system of equations:*

$$\bar{\xi} = f(\xi), \tag{28}$$

where D_1 is an open set and connected, and $f : D_1 \to R^n$ and $f \in C_1(D_1)$.If the following circumstances hold true, then $L \subset D_1$, such that a compact absorbing set L exists, can be considered.There exists a unique equilibrium point for Eq. (28) if $\xi^* \in D_1$. For Eq. (28), the equilibrium point (ξ^*) is globally stable if,

$$q = \lim_{t\to\infty} \sup_{\xi\in L} \frac{1}{t} \int_0^t \Gamma(\mathbb{N}(x(s, \xi_0)))ds). \tag{29}$$

Theorem 3.5. *If $\mathbb{R}_0 > 1$ then Ebola virus disease model (3-9) is said to be globally asymptotically stable at DPEP (16).*

Proof. The first three stages of the Ebola virus disease model (3-9) are selected, using the Jacobian system of equations, for the DPEPs, to generate

global stability for DPEP (16).

$$J_1 = \begin{bmatrix} -\alpha\, E^* - \omega & -\alpha\, S^* & 0 \\ \alpha\, E^* & \alpha\, S^* - U^*\beta_2 - Q^*\beta_1 - \omega - d_E & -\beta_1 E^* \\ 0 & Q^*\beta_1 & \beta_1 E^* - \omega - d_Q - \varepsilon_1 - \varepsilon_2 \end{bmatrix}, \tag{30}$$

The second additive compound matrix is defined as follows:

$$Z = \begin{bmatrix} k_{11} + k_{22} & k_{23} & -k_{13} \\ k_{32} & k_{11} + k_{33} & k_{12} \\ -k_{31} & k_{21} & k_{22} + k_{33} \end{bmatrix}. \tag{31}$$

Where k_{ij} are entries of J_1, shown as, $K_{11} = -\alpha\, E^* - \omega, K_{12} = -\alpha\,^* S, K_{13} = 0, K_{21} = -\alpha\, E^*, K_{22} = \alpha\, S^* - U^*\beta_2 - Q^*\beta_1 - \omega - d_E, K_{23} = -\beta_1 E^*, K_{31} = 0, K_{32} = Q^*\beta_1, K_{33} = \beta_1 E^* - \omega - d_Q - \varepsilon_1 - \varepsilon_2,$

dividing H_3 from a diagonal matrix before the matrix, the differential matrix H_3 multiplied to get the matrices shown below.

$$H_3 H_1^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{\left(\left(\frac{\mathrm{d}}{\mathrm{d}\xi}E^*\right)Q^* - \left(\frac{\mathrm{d}}{\mathrm{d}\xi}Q^*\right)E^*\right)R^*}{Q^{*2}E^*} & 0 \\ 0 & 0 & \frac{\left(\left(\frac{\mathrm{d}}{\mathrm{d}\xi}Q^*\right)E^* - \left(\frac{\mathrm{d}}{\mathrm{d}\xi}E^*\right)Q^*\right)}{Q^{*2}E^*} \end{bmatrix}, \tag{32}$$

now, $H_2 H_3 H_2^{-1}$ add $H_2 H_1 H_2^{-1}$ and, then got H_4 matrix in the block matrix. Now, matrix R in block matrix form of H_4, the block matrix $R = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}$.

Let $\hat{\mathbb{E}}(R)$ be a norm-related measure. Next, let us examine Lozinski's function about ω_1 norms, which may be expressed as follows:

$$\hat{\mathbb{E}}(R^*) \leq sup(\eta_1, \eta_2), \tag{33}$$

$$\eta_1 = \hat{\mathbb{E}}(R_{11}) + |R_{12}|), \tag{34}$$

$$\eta_2 = \hat{\mathbb{E}}(R_{21}) + |R_{22}|), \tag{35}$$

$$\eta_1 \leq \frac{Q^{*'}}{Q^*} - 2\omega, \tag{36}$$

$$\eta_2 \leq \frac{Q^{*'}}{Q^*} - 2\omega. \tag{37}$$

By using (36) and (37) in equation (33),

$$\hat{\mathbb{E}}(R^*) \leq sup(\eta_1, \eta_2) \leq -\omega, \tag{38}$$

this implies that, $q \leq -\omega \leq 0$.

Hence proved the global stability at DPEP. Positive equilibrium (S^*, E^*, Q^*) is globally asymptotically stable. Moreover, consider a sub-system of system of equation (3-9) using lemma 2 by Martian [25],

$$U(\xi) = \frac{\beta_2 E^* U^*}{-(\rho + \omega + d_U)}, \tag{39}$$

$$J(\xi) = \frac{\varepsilon_1 Q^*}{-(\eta + \omega + d_J)}, \tag{40}$$

$$H(\xi) = \frac{\rho U^* + \varepsilon_2 Q^*}{-(\gamma + \omega + d_H)}, \tag{41}$$

$$R(\xi) = \frac{\eta J^* + \gamma H^*}{-\omega}. \tag{42}$$

Hence proved, that the Ebola virus disease model is globally stable at the DPEP. □

3.8 Sensitivity Analysis

Sensitivity analysis [26] is an important analysis that shows how each parameter affects disease transmission. To establish how relevant each parameter is to disease transmission, the sensitivity index of factors with relation to the basic reproduction number was produced; intervention control strategies that target such parameters should be employed in Ebola disease control/prevention.

Definition: The normalized forward sensitivity index of a variable ϕ that depends differentiable on a parameter $\mathbb{P}$ is defined,

$$L_{\mathbb{P}}^{\phi} = \frac{\partial \phi}{\partial \mathbb{P}} \times \frac{\mathbb{P}}{\phi}.$$

As we have explicit formula for $\mathbb{R}_0$, analytical expression has been derived for the sensitivity of $\mathbb{R}_0$ as,

$$L_{\mathbb{P}}^{\mathbb{R}_0} = \frac{\partial \mathbb{R}_0}{\partial \mathbb{P}} \times \frac{\mathbb{P}}{\mathbb{R}_0}.$$

Parameters for sensitivity analysis of $\mathbb{R}_0$,

$$L_{\alpha}^{\phi} = \frac{\partial \mathbb{R}_0}{\partial \alpha} \times \frac{\alpha}{\mathbb{R}_0} = 1 > 0,$$

$$L_{\omega}^{\phi} = \frac{\partial \mathbb{R}_0}{\partial \omega} \times \frac{\omega}{\mathbb{R}_0} = \frac{-2\,\omega - d_E}{\omega + d_E} < 0,$$

$$L_{d_E}^{\phi} = \frac{\partial \mathbb{R}_0}{\partial d_E} \times \frac{d_E}{\mathbb{R}_0} = -\frac{d_E}{\omega + d_E} < 0.$$

Those parameters, which have positive indices, show that if their values increase, they impact the rate of infection in the community. Those parameters, which have negative indices, show that if their values are increasing, then they minimize the disease in the community.

3.9 Existence and Uniqueness of Solution

In this find the existence of the non-linear systems of the equation of the Ebola virus disease model (3-9) by using fixed point theory [27].

The system of non-linear is,

$$S(\xi) - S(0) = \int_{\xi}^{c} [\Delta - (\alpha\, E + \omega)\, S] d\xi, \tag{43}$$

$$E(\xi) - E(0) = \int_{\xi}^{c} [\alpha\, SE - (\beta_2 U + \beta_1 Q + \omega + d_E)\, E] d\xi, \tag{44}$$

$$Q(\xi) - Q(0) = \int_{\xi}^{c} [\beta_1 EQ - (\varepsilon_1 + \varepsilon_2 + \omega + d_Q)\, Q] d\xi, \tag{45}$$

$$U(\xi) - U(0) = \int_{\xi}^{c} [\beta_2 EU - (\rho + \omega + d_U)\, U] d\xi, \tag{46}$$

$$J(\xi) - J(0) = \int_{\xi}^{c} [\varepsilon_1 Q - (\eta + \omega + d_J)\, J] d\xi, \tag{47}$$

$$H(\xi) - H(0) = \int_{\xi}^{c} [\rho U + \varepsilon_2 Q - (\gamma + \omega + d_H)\, H] d\xi, \tag{48}$$

$$R(\xi) - R(0) = \int_{\xi}^{c} [\eta J + \gamma\, H - (\omega)\, R] d\xi. \tag{49}$$

Now let's start the procedure,

$$S(\xi) - S(0) = \int_{0}^{\xi} \sigma^{c-1} [\Delta - (\alpha\, E + \omega)\, S] d\sigma, \tag{50}$$

$$E(\xi) - E(0) = \int_{0}^{\xi} \sigma^{c-1} [\alpha\, SE - (\beta_2 U + \beta_1 Q + \omega + d_E)\, E] d\sigma, \tag{51}$$

$$Q(\xi) - Q(0) = \int_{0}^{\xi} \sigma^{c-1} [\beta_1 EQ - (\varepsilon_1 + \varepsilon_2 + \omega + d_Q)\, Q] d\sigma, \tag{52}$$

$$U(\xi) - U(0) = \int_{0}^{\xi} \sigma^{c-1} [\beta_2 EU - (\rho + \omega + d_U)\, U] d\sigma, \tag{53}$$

$$J(\xi) - J(0) = \int_{0}^{\xi} \sigma^{c-1} [\varepsilon_1 Q - (\eta + \omega + d_J)\, J] d\sigma, \tag{54}$$

$$H(\xi) - H(0) = \int_0^{\xi} \sigma^{c-1}[\rho U + \varepsilon_2 Q - (\gamma + \omega + d_H)\, H]d\sigma, \tag{55}$$

$$R(\xi) - R(0) = \int_0^{\xi} \sigma^{c-1}[\eta J + \gamma\, H - (\omega)\, R]d\sigma. \tag{56}$$

now we define the kernels,

$$\lambda_1(\xi, S) = \Delta - (\alpha\, E(\xi) + \omega)\, S(\xi), \tag{57}$$

$$\lambda_2(\xi, E) = \alpha\, S(\xi)E(\xi) - (\beta_2 U(\xi) + \beta_1 Q(\xi) + \omega + d_E)\, E(\xi), \tag{58}$$

$$\lambda_3(\xi, Q) = \beta_1 E(\xi)Q(\xi) - (\varepsilon_1 + \varepsilon_2 + \omega + d_Q)\, Q(\xi), \tag{59}$$

$$\lambda_4(\xi, U) = \beta_2 E(\xi)U(\xi) - (\rho + \omega + d_U)\, U(\xi), \tag{60}$$

$$\lambda_5(\xi, J) = \varepsilon_1 Q(\xi) - (\eta + \omega + d_J)\, J(\xi), \tag{61}$$

$$\lambda_6(\xi, H) = \rho U(\xi) + \varepsilon_2 Q(\xi) - (\gamma + \omega + d_H)\, H(\xi), \tag{62}$$

$$\lambda_7(\xi, R) = \eta J(\xi) + \gamma\, H(\xi) - (\omega)\, R(\xi). \tag{63}$$

Theorem 3.6. *If the following inequality is proven:*

$$0 < r_1, r_2, r_3, r_4, r_5, r_6, r_7 \leq 1, \tag{64}$$

where $\|S\| \leq b_1, \|E\| \leq b_2, \|Q\| \leq b_3, \|U\| \leq b_4, \|J\| \leq b_5, \|H\| \leq b_6, \|R\| \leq b_7$, *and* $\alpha b_1 = r_1, \beta_1 b_3 + \beta_2 b_4 + \omega + d_E = r_2, \varepsilon_1 + \varepsilon_2 + \omega + d_Q = r_3, \rho + \omega + d_U = r_4, \eta + \omega + d_J = r_5, \gamma + \omega + d_H = r_6, \omega = r_7$.

Proof. Consider S_1 and S_2 as two functions for the kernel λ_1. Then,

$$\begin{aligned} &\|\lambda_1(\xi, S_1) - \lambda_1(\xi, S_2)\| \\ &\leq r_1 \|S_1(\xi) - S_2(\xi)\|. \end{aligned} \tag{65}$$

Since $\|S\|$ is a bounded function of r_1, we have

$$\|\lambda_1(\xi, S_1) - \lambda_1(\xi, S_2)\| \leq r_1 \|S_1(\xi) - S_2(\xi)\|. \tag{66}$$

Similarly, for each kernel $\lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7$, the Lipschitz conditions are satisfied. If $0 < r_1, r_2, r_3, r_4, r_5, r_6, r_7 \leq 1$, then $r_1, r_2, r_3, r_4, r_5, r_6, r_7$ also satisfy the contraction conditions for $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7$ respectively. This concludes the proof of the theorem. □

Now consider the kernels $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7$ and rewrite the system of equations,

$$S(\xi) = S(0) + \int_0^{\xi} \lambda_1(\sigma, S)d\sigma, \tag{67}$$

$$E(\xi) = E(0) + \int_0^{\xi} \lambda_2(\sigma, E)d\sigma, \tag{68}$$

$$Q(\xi) = Q(0) + \int_0^{\xi} \lambda_3(\sigma, Q)d\sigma, \tag{69}$$

$$U(\xi) = U(0) + \int_0^\xi \lambda_4(\sigma, U)d\sigma, \tag{70}$$

$$J(\xi) = J(0) + \int_0^\xi \lambda_5(\sigma, J)d\sigma, \tag{71}$$

$$H(\xi) = H(0) + \int_0^\xi \lambda_6(\sigma, H)d\sigma, \tag{72}$$

$$R(\xi) = R(0) + \int_0^\xi \lambda_7(\sigma, R)d\sigma, \tag{73}$$

now proceed with the recursive formula which is as follows,

$$S_r(\xi) = S(0) + \int_0^\xi \lambda_1(\sigma, S_{r-1})d\sigma, \tag{74}$$

$$E_r(\xi) = E(0) + \int_0^\xi \lambda_2(\sigma, E_{r-1})d\sigma, \tag{75}$$

$$Q_r(\xi) = Q(0) + \int_0^\xi \lambda_3(\sigma, Q_{r-1})d\sigma, \tag{76}$$

$$U_r(\xi) = U(0) + \int_0^\xi \lambda_4(\sigma, U_{r-1})d\sigma, \tag{77}$$

$$J_r(\xi) = J(0) + \int_0^\xi \lambda_5(\sigma, J_{r-1})d\sigma, \tag{78}$$

$$H_r(\xi) = H(0) + \int_0^\xi \lambda_6(\sigma, H_{r-1})d\sigma, \tag{79}$$

$$R_r(\xi) = R(0) + \int_0^\xi \lambda_7(\sigma, R_{r-1})d\sigma, \tag{80}$$

where,

$$S(0) \geq 0, E(0) \geq 0, Q(0) \geq 0, U(0) \geq 0, J(0) \geq 0, H(0) \geq 0, R(0) \geq 0. \tag{81}$$

It can also be written in sequential term differences which are as follows,

$$\Omega 1_r = S(\xi) - S(0) = \int_0^\xi \sigma^{c-1}(\lambda_1(\sigma, S_{r-1}) - \lambda_1(\sigma, S_{r-2}))d\sigma, \tag{82}$$

$$\Omega 2_r = E(\xi) - E(0) = \int_0^\xi \sigma^{c-1}(\lambda_2(\sigma, E_{r-1}) - \lambda_2(\sigma, E_{r-2}))d\sigma, \tag{83}$$

$$\Omega 3_r = Q(\xi) - Q(0) = \int_0^\xi \sigma^{c-1}(\lambda_3(\sigma, Q_{r-1}) - \lambda_3(\sigma, Q_{r-2}))d\sigma, \tag{84}$$

$$\Omega 4_r = U(\xi) - U(0) = \int_0^\xi \sigma^{c-1}(\lambda_4(\sigma, U_{r-1}) - \lambda_4(\sigma, U_{r-2}))d\sigma, \tag{85}$$

$$\Omega 5_r = J(\xi) - J(0) = \int_0^\xi \sigma^{c-1}(\lambda_5(\sigma, J_{r-1}) - \lambda_5(\sigma, J_{r-2}))d\sigma, \tag{86}$$

$$\Omega 6_r = H(\xi) - H(0) = \int_0^\xi \sigma^{c-1}(\lambda_6(\sigma, H_{r-1}) - \lambda_6(\sigma, H_{r-2}))d\sigma, \tag{87}$$

$$\Omega 7_r = R(\xi) - R(0) = \int_0^\xi \sigma^{c-1}(\lambda_7(\sigma, R_{r-1}) - \lambda_7(\sigma, R_{r-2}))d\sigma, \tag{88}$$

the system of equations implies that,

$$\begin{aligned} S_r(t) &= \sum_{j=1}^{r} \Omega 1_r(\xi), E_r(t) = \sum_{j=1}^{r} \Omega 2_r(\xi), \\ Q_r(t) &= \sum_{j=1}^{r} \Omega 3_r(\xi), U_r(t) = \sum_{j=1}^{r} \Omega 4_r(\xi), \\ J_r(t) &= \sum_{j=1}^{r} \Omega 5_r(\xi), H_r(t) = \sum_{j=1}^{r} \Omega 6_r(\xi), R_r(t) = \sum_{j=1}^{r} \Omega 7_r(\xi). \end{aligned} \tag{89}$$

Now, we take norm both sides of the system of equations, then kernels satisfy the Lipschitz condition. Now triangle inequality applies to a system of equations, then we have,

$$\| S_r(\xi) - S_{r-1}(\xi) \| \leq r_1 \int_0^{\xi} \sigma^{c-1} \| (S_{r-1} - S_{r-2}) \| d\sigma, \tag{90}$$

$$\| E_r(\xi) - E_{r-1}(\xi) \| \leq r_2 \int_0^{\xi} \sigma^{c-1} \| (E_{r-1} - E_{r-2}) \| d\sigma, \tag{91}$$

$$\| Q_r(\xi) - Q_{r-1}(\xi) \| \leq r_3 \int_0^{\xi} \sigma^{c-1} \| (Q_{r-1} - Q_{r-2}) \| d\sigma, \tag{92}$$

$$\| U_r(\xi) - U_{r-1}(\xi) \| \leq r_4 \int_0^{\xi} \sigma^{c-1} \| (U_{r-1} - U_{r-2}) \| d\sigma, \tag{93}$$

$$\| J_r(\xi) - J_{r-1}(\xi) \| \leq r_5 \int_0^{\xi} \sigma^{c-1} \| (J_{r-1} - J_{r-2}) \| d\sigma, \tag{94}$$

$$\| H_r(\xi) - H_{r-1}(\xi) \| \leq r_6 \int_0^{\xi} \sigma^{c-1} \| (H_{r-1} - H_{r-2}) \| d\sigma, \tag{95}$$

$$\| R_r(\xi) - R_{r-1}(\xi) \| \leq r_7 \int_0^{\xi} \sigma^{c-1} \| (R_{r-1} - R_{r-2}) \| d\sigma, \tag{96}$$

we have,

$$\begin{aligned} &\| \Omega 1_r \| \leq r_1 \int_0^{\xi} \| \Omega 1_{r-1} \| d\sigma, \| \Omega 2_r \| \leq r_2 \int_0^{\xi} \| \Omega 2_{r-1} \| d\sigma, \\ &\| \Omega 3_r \| \leq r_3 \int_0^{\xi} \| \Omega 3_{r-1} \| d\sigma, \| \Omega 4_r \| \leq r_4 \int_0^{\xi} \| \Omega 4_{r-1} \| d\sigma, \\ &\| \Omega 5_r \| \leq r_5 \int_0^{\xi} \| \Omega 5_{r-1} \| d\sigma, \| \Omega 6_r \| \leq r_6 \int_0^{\xi} \| \Omega 6_{r-1} \| d\sigma, \\ &\| \Omega 7_r \| \leq r_7 \int_0^{\xi} \| \Omega 7_{r-1} \| d\sigma. \end{aligned} \tag{97}$$

The following theorem may be derived from these findings.

Theorem 3.7. *The modified Ebola virus disease model offers a solution under the condition that can be formed τ_{max} property,*

$$r_i \tau_{max} \leq 1, i = i, 2, \ldots, 7. \tag{98}$$

Proof. Consider the function $S(\xi), E(\xi), Q(\xi), U(\xi), J(\xi), H(\xi)$, and $R(\xi)$ are the bounded and having the kernels $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7$ satisfied the Lipschitz condition. We apply the recursive method to a system of equations,

$$\begin{aligned}
&\| \Omega 1_r \| \leq S(0) \| \{r_1\tau_{max}\}^r, \| \Omega 2_r \| \leq E(0) \| \{r_2\tau_{max}\}^r, \\
&\| \Omega 3_r \| \leq Q(0) \| \{r_3\tau_{max}\}^r, \| \Omega 4_r \| \leq U(0) \| \{r_4\tau_{max}\}^r, \\
&\| \Omega 5_r \| \leq J(0) \| \{r_5\tau_{max}\}^r, \| \Omega 6_r \| \leq H(0) \| \{r_6\tau_{max}\}^r, \\
&\| \Omega 7_r \| \leq R(0) \| \{r_7\tau_{max}\}^r.
\end{aligned} \tag{99}$$

Thus, we assume that this is the model for the Ebola virus disease solution.

$$\begin{aligned}
&S(\xi) - S(0) = S_r(\xi) - B1_n(\xi), E(\xi) - E(0) = E_r(\xi) - B2_n(\xi), \\
&Q(\xi) - Q(0) = Q_r(\xi) - B3_n(\xi), U(\xi) - U(0) = U_r(\xi) - B4_n(\xi), \\
&J(\xi) - J(0) = J_r(\xi) - B5_n(\xi), H(\xi) - H(0) = H_r(\xi) - B6_n(\xi), \\
&R(\xi) - R(0) = R_r(\xi) - B7_n(\xi).
\end{aligned} \tag{100}$$

it is shown that the term in equation (100) hold $\| B1_n(\xi) \| \to 0, \| B2_n(\xi) \| \to 0, \| B3_n(\xi) \| \to 0, \| B4_n(\xi) \| \to 0, \| B5_n(\xi) \| \to 0, \| B6_n(\xi) \| \to 0$, and $\| B7_n(\xi) \| \to 0$, so we have,

$$\begin{aligned}
\| B1_n(\xi) \| &\leq \| \int_0^\xi \sigma^{c-1}[\lambda_1(\sigma, S) - \lambda_1(\sigma, S_{r-1})]d\sigma \|, \\
&\leq \int_0^\xi \| \sigma^{c-1}[\lambda_1(\sigma, S) - \lambda_1(\sigma, S_{r-1})] \| d\sigma, \\
&\leq \xi r_1 \| S - S_{r-1} \| .
\end{aligned} \tag{101}$$

Similarly for others,

$$\| B2_n(\xi) \| \leq \xi r_2 \| E - E_{r-1} \|, \tag{102}$$

$$\| B3_n(\xi) \| \leq \xi r_3 \| Q - Q_{r-1} \|, \tag{103}$$

$$\| B4_n(\xi) \| \leq \xi r_4 \| U - U_{r-1} \|, \tag{104}$$

$$\| B5_n(\xi) \| \leq \xi r_5 \| J - J_{r-1} \|, \tag{105}$$

$$\| B6_n(\xi) \| \leq \xi r_6 \| H - H_{r-1} \|, \tag{106}$$

and,

$$\| B7_n(\xi) \| \leq \xi r_7 \| R - R_{r-1} \|, \tag{107}$$

apply recursive relation, then obtain,

$$\| B1_n(\xi) \| \leq \xi^{r-1} r_1^r \Phi, \tag{108}$$

$$\| B2_n(\xi) \| \leq \xi^{r-1} r_2^r \Phi, \tag{109}$$

$$\| B3_n(\xi) \| \leq \xi^{r-1} r_3^r \Phi, \tag{110}$$

$$\| B4_n(\xi) \| \leq \xi^{r-1} r_4^r \Phi, \tag{111}$$

$$\| B5_n(\xi) \| \leq \xi^{r-1} r_5^r \Phi, \tag{112}$$
$$\| B6_n(\xi) \| \leq \xi^{r-1} r_6^r \Phi, \tag{113}$$
$$\| B7_n(\xi) \| \leq \xi^{r-1} r_7^r \Phi, \tag{114}$$

taking at τ_{max} point, we get

$$\| B1_n(\xi) \| \leq \{\tau_{max}\}^{r-1} r_1^r \Phi, \tag{115}$$

$$\| B2_n(\xi) \| \leq \{\tau_{max}\}^{r-1} r_2^r \Phi, \tag{116}$$
$$\| B3_n(\xi) \| \leq \{\tau_{max}\}^{r-1} r_3^r \Phi, \tag{117}$$
$$\| B4_n(\xi) \| \leq \{\tau_{max}\}^{r-1} r_4^r \Phi, \tag{118}$$
$$\| B5_n(\xi) \| \leq \{\tau_{max}\}^{r-1} r_5^r \Phi, \tag{119}$$
$$\| B6_n(\xi) \| \leq \{\tau_{max}\}^{r-1} r_6^r \Phi, \tag{120}$$
$$\| B7_n(\xi) \| \leq \{\tau_{max}\}^{r-1} r_7^r \Phi, \tag{121}$$

$r \to \infty$ apply both sides, then using the result of theorem 3.6, then we get, $\| B1_n(\xi) \| \to 0$, $\| B2_n(\xi) \| \to 0$, $\| B3_n(\xi) \| \to 0$, $\| B4_n(\xi) \| \to 0$, $\| B5_n(\xi) \| \to 0$, $\| B6_n(\xi) \| \to 0$ and $\| B7_n(\xi) \| \to 0$. □

Theorem 3.8. *if*

$$(1 - r_i \xi) \geq 0, \ i = 1, 2, \ldots, 7. \tag{122}$$

then modified Ebola virus disease model has a unique system of solutions.

Proof. Suppose different system of solution such as $\hat{S}, \hat{E}, \hat{Q}, \hat{U}, \hat{J}, \hat{H}, \hat{R}$, then it may write,

$$S(\xi) - \hat{S}(\xi) = \int_0^\xi \sigma^{c-1} [\lambda_1(\sigma, S) - \lambda_1(\sigma, \hat{S})] d\sigma, \tag{123}$$

$$E(\xi) - \hat{E}(\xi) = \int_0^\xi \sigma^{c-1} [\lambda_2(\sigma, E) - \lambda_2(\sigma, \hat{E})] d\sigma, \tag{124}$$
$$Q(\xi) - \hat{Q}(\xi) = \int_0^\xi \sigma^{c-1} [\lambda_3(\sigma, Q) - \lambda_3(\sigma, \hat{Q})] d\sigma, \tag{125}$$
$$U(\xi) - \hat{U}(\xi) = \int_0^\xi \sigma^{c-1} [\lambda_4(\sigma, U) - \lambda_4(\sigma, \hat{U})] d\sigma, \tag{126}$$
$$J(\xi) - \hat{J}(\xi) = \int_0^\xi \sigma^{c-1} [\lambda_5(\sigma, J) - \lambda_5(\sigma, \hat{J})] d\sigma, \tag{127}$$
$$H(\xi) - \hat{H}(\xi) = \int_0^\xi \sigma^{c-1} [\lambda_6(\sigma, H) - \lambda_6(\sigma, \hat{H})] d\sigma, \tag{128}$$
$$R(\xi) - \hat{R}(\xi) = \int_0^\xi \sigma^{c-1} [\lambda_7(\sigma, R) - \lambda_7(\sigma, \hat{R})] d\sigma, \tag{129}$$

apply norm on both sides (123-129) and results of kernels which fulfil the Lipschitz condition. We can write it as,

$$\| S(\xi) - \hat{S}(\xi) \| \leq r_1 \xi \| S(\xi) - \hat{S}(\xi) \|, \tag{130}$$

$$\| E(\xi) - \hat{E}(\xi) \| \leq r_2\xi \| E(\xi) - \hat{E}(\xi) \|, \tag{131}$$
$$\| Q(\xi) - \hat{Q}(\xi) \| \leq r_3\xi \| Q(\xi) - \hat{Q}(\xi) \|, \tag{132}$$
$$\| U(\xi) - \hat{U}(\xi) \| \leq r_4\xi \| U(\xi) - \hat{U}(\xi) \|, \tag{133}$$
$$\| J(\xi) - \hat{J}(\xi) \| \leq r_5\xi \| J(\xi) - \hat{J}(\xi) \|, \tag{134}$$
$$\| H(\xi) - \hat{H}(\xi) \| \leq r_6\xi \| H(\xi) - \hat{H}(\xi) \|, \tag{135}$$
$$\| R(\xi) - \hat{R}(\xi) \| \leq r_7\xi \| R(\xi) - \hat{R}(\xi) \|, \tag{136}$$

then,

$$\| S(\xi) - \hat{S}(\xi) \| (1 - r_1\xi) \leq 0, \tag{137}$$
$$\| E(\xi) - \hat{E}(\xi) \| (1 - r_2\xi) \leq 0, \tag{138}$$
$$\| Q(\xi) - \hat{Q}(\xi) \| (1 - r_3\xi) \leq 0, \tag{139}$$
$$\| U(\xi) - \hat{U}(\xi) \| (1 - r_4\xi) \leq 0, \tag{140}$$
$$\| J(\xi) - \hat{J}(\xi) \| (1 - r_5\xi) \leq 0, \tag{141}$$
$$\| H(\xi) - \hat{H}(\xi) \| (1 - r_6\xi) \leq 0, \tag{142}$$
$$\| R(\xi) - \hat{R}(\xi) \| (1 - r_7\xi) \leq 0, \tag{143}$$

consequently,

$$\| S(\xi) - \hat{S}(\xi) \| = 0, \tag{144}$$
$$\| E(\xi) - \hat{E}(\xi) \| = 0, \tag{145}$$
$$\| Q(\xi) - \hat{Q}(\xi) \| = 0, \tag{146}$$
$$\| U(\xi) - \hat{U}(\xi) \| = 0, \tag{147}$$
$$\| J(\xi) - \hat{J}(\xi) \| = 0, \tag{148}$$
$$\| H(\xi) - \hat{H}(\xi) \| = 0, \tag{149}$$

and,

$$\| R(\xi) - \hat{R}(\xi) \| = 0. \tag{150}$$

This shows that the model has a unique solution. which is the complete proof of the theorem. □

4 Numerical Simulations

A numerical solution of the non-linear fractional-order system of odes has been obtained using RK4 method [28–30] on Maple 2019. Using the initial condition $S(0) = 5, E(0) = 1, U(0) = 2, Q(0) = 1, H(0) = 7, J(0) = 1, R(0) = 3$, and parameter of Table 1, behavior of individuals have been display in figures for $c = 0.4, 0.6, 0.8, 1$. Fractional order ODEs help us to understand, how population effect under observation changes. The advantage of this method is that, when $c = 1$ the fractional model goes to the classical model. In Figure 3, the dynamic behaviour of the susceptible class can be studied. Over 30 days, the number of susceptible classes is getting very increase. Dynamic behaviour of Exposed, Quarantine, Infected, Self-Isolated, Hospitalized and Recovered

shown in Figure (4-9). As the results show when people are self-isolated, quarantined and hospitalized, the number of infected persons is decreased, So we can say the rate of Ebola virus disease going to decrease.

5 Conclusion

In conclusion, we have included self-isolation, hospitalized classes, and quarantine to a mathematical model for studying the dynamics of the Ebola virus disease, based on research done by Khan et al. (2022) [14]. The problem has been governed by a system of conformable differential equations, and its invariant region has been ensured by using a well-known theorem. We calculated basic reproduction using the next-generation matrix method and checked its sensitivity analysis. For fractional values of derivatives between 0 and 1, graphs have been shown to demonstrate how the solutions of all classes behave. The advantage of the fractional order is that, as ξ approaches 1, the solution of fractional models (3-9) tends to the solution of classical models. In the graphical representation, we have observed that when people are quarantined, self-isolated and hospitalized, the infection rate decreases. Based on these findings, it can be concluded that the addition of above mentioned three compartments has a significant role in controlling the transmission of Ebola virus disease in society. Future research could focus on finding the best ways to control strategies and vaccine using our model and incorporating real-world data to make our predictions even more accurate and also using a different type of vaccination. By exploring these areas, we can make our model more effective and relevant in addressing the challenges of disease modelling and control.

Acknowledgement
Authors (Dr. Nadeem Abbas and Prof. Dr. Wasfi Shatanawi) would like to thank Prince Sultan University for their support through the TAS research lab.

References

[1] Jacob, S.T., Crozier, I., Fischer II, W.A., et al. 'Ebola virus disease (Primer).' *Nature Reviews: Disease Primers* **6**(1) (2020).

[2] Feldmann, H., and Thomas W. G. 'Ebola haemorrhagic fever.' *The Lancet* **377**(9768), pp. 849-862 (2011).

[3] Mohamadzadeh, M., Lieping C., and Alan L. S. 'How Ebola and Marburg viruses battle the immune system.' *Nature Reviews Immunology*, **7**(7), pp. 556-567 (2007).

[4] Rugarabamu, S., Mboera, L., Rweyemamu, M., et al. 'Forty-two years of responding to Ebola virus outbreaks in Sub-Saharan Africa: a review.' *BMJ Global Health* **5**(3) (2020).

[5] Leroy, E. M., Baize, S., Volchkov, V. E., et al. 'Human asymptomatic Ebola infection and strong inflammatory response.' *The Lancet* **355**(9222), pp. 2210-2215 (2000).

[6] Guinea, L. 'Assessing the socio-economic impacts of Ebola virus disease in guinea, liberia and sierra leone.' pp.1-72 (2014)

[7] Al-Smadi, M., Omar A., and Shaher M. 'Numerical computations of coupled fractional resonant Schrödinger equations arising in quantum mechanics under conformable fractional derivative sense.' *Physica Scripta*, **95**(7), Article ID 075218 (2020).

[8] Berge, T., Lubuma, J. S., Moremedi, G. M., et al. 'A simple mathematical model for Ebola in Africa.' *Journal of biological dynamics*, **11**(1), pp. 42-74 (2017).

[9] Tahir, M., Zaman, G., Shah, S. I. A., et al. 'The stability analysis and control transmission of mathematical model for Ebola Virus.' *Open Journal of Mathematical Analysis*, **3**(2), pp. 91-102 (2019).

[10] Rafiq, M., Ahmad, W., Abbas, M., et al. 'A reliable and competitive mathematical analysis of Ebola epidemic model.' *Advances in Difference Equations*, **2020**, pp. 1-24 (2020).

[11] Maheshwari, H., Shetty, S., Bannur, N., et al. 'CoSIR: Managing an Epidemic via Optimal Adaptive Control of Transmission Policy.' *medRxiv*, **2020**(11) (2020).

[12] Khan, Z. U., Ali, A., Khan, F. M., et al. 'Existence theory and numerical simulation of ebola model corresponding to nonlocal fractional order derivative.' *Arab Journal of Basic and Applied Sciences*, **27**(1) pp. 375-388 (2020).

[13] Farman, M., Akgül, A., Abdeljawad, T., et al. 'Modeling and analysis of fractional order Ebola virus model with Mittag-Leffler kernel.' *Alexandria Engineering Journal*, **61**(3), pp. 2062-2073 (2022).

[14] Khan, F. M., Ali, A., Bonyah, E., et al. 'The mathematical analysis of the new fractional order Ebola model.' *Journal of Nanomaterials*, **2022**(1-12) (2022).

[15] Yadav, P., Shah J., and Kottakkaran S. N. 'Fractional order mathematical model of Ebola virus under Atangana–Baleanu–Caputo operator.' *Results in Control and Optimization* **13**, Article ID 100332 (2023).

[16] Mbah, G. C. E., Onah, I. S., Ahman, Q. O., et al. 'Mathematical modelling approach of the study of Ebola virus disease transmission dynamics in a developing country.' *African Journal of Infectious Diseases,* **17**(1), pp. 10-26 (2023).

[17] Nisar, K. S., Shoaib, M., Raja, M. A. Z., et al. 'A novel design of evolutionally computing to study the quarantine effects on transmission model of Ebola virus disease.' *Results in Physics*, **48**, Article ID 106408 (2023).

[18] Khalil, R., Al Horani, M., Yousef, A., et al. 'A new definition of fractional derivative.' *Journal of computational and applied mathematics* **264**, pp. 65-70 (2014).

[19] Zanib, S. A., Ramzan, S., Abbas, N., et al. enquoteA mathematical approach of drug addiction and rehabilitation control dynamic. *Modeling Earth Systems and Environment*, **5**, pp. 1-8 (2024).

[20] Van den Driessche, P., and Watmough, J. 'Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission.' *Mathematical biosciences,* **180**(1-2), pp. 29-48 (2002).

[21] Rahman, S. A. 'Study of Infectious Diseases by Mathematical Models: Predictions and Controls'. *Electronic Thesis and Dissertation Repository,* **3**(487) (2016).

[22] Castillo-Chavez C., Blower S., Driessche P., et al. *Mathematical approaches for emerging and reemerging infectious diseases: models, methods, and theory.* Vol.**126**. Springer Science & Business Media, 2002.

[23] Kot, M. 'Elements of mathematical ecology'. *Cambridge University Press* (2001).

[24] Liu, X., and Lijuan Y. 'Stability analysis of an SEIQV epidemic model with saturated incidence rate.' *Nonlinear analysis: real world applications*, **13**(6), pp. 2671-2679 (2012).

[25] Martin Jr, R. H. 'Logarithmic norms and projections applied to linear differential systems'. *Journal of Mathematical Analysis and Applications,* **45**(2), pp. 432-454 (1974).

[26] Blower, S. M., and Dowlatabadi, H. 'Sensitivity and uncertainty analysis of complex models of disease transmission: an HIV model, as an example'. *International Statistical Review/Revue Internationale de Statistique,* **4**, pp. 229-243 (1994).

[27] Koca, I. 'Modelling the spread of Ebola virus with Atangana-Baleanu fractional operators'. *The European Physical Journal Plus,* **133**(1-11) (2018).

[28] Arif, M. S., Abodayeh, K., and Nawaz, Y. 'A Computational Scheme for Stochastic Non-Newtonian Mixed Convection Nanofluid Flow over Oscillatory Sheet'. *Energies,* **16**(5), pp. 2287-2298 (2023).

[29] Nawaz, Y., Arif, M. S., and Abodayeh, K. 'Predictor–Corrector Scheme for Electrical Magnetohydrodynamic (MHD) Casson Nanofluid Flow: A Computational Study'. *Applied Sciences,* **13**(2), pp. 1209-1216 (2023).

[30] Nawaz, Y., Arif, M. S., Abodayeh, K., et al. 'A new explicit numerical scheme for enhancement of heat transfer in Sakiadis flow of micro polar fluid using electric field.' *Heliyon*, **9**(10) (2023).

Author Biography

Syeda Alishwa Zanib: Syeda Alishwa Zanib is a highly dedicated mathematician, with an Master of Philosophy in Mathematics from Riphah International University Faisalabad Campus, Pakistan and a bachelor's degree from Government College University Faisalabad, Pakistan. She is working on the mathematical modeling of infectious diseases, computational neurobiology, numerical methods, and machine learning. Her research expertise demonstrates a multidisciplinary approach, addressing complex challenges at the intersection of mathematics and diverse scientific fields.

Sehrish Ramzan: Sehrish Ramzan holds a Bachelor's degree in Mathematics from Government College University Faisalabad and a Master of Philosophy from Riphah International University Faisalabad. Her research focuses on mathematical modeling, spanning topics such as differential equations, optimization techniques, data-driven approaches, and interdisciplinary applications.

Dr. Nadeem Abbas: Dr. Nadeem Abbas completed his PhD at the Quaid I Azam University in Islamabad, Pakistan. He was working as an Assistant Professor at Riphah International University Faisalabad Campus, Faisalabad, Pakistan. He is highly cited and has published many articles in ISI highly impact factor journals. Now, he is working as a researcher at Prince Sultan University Saudi Arabia.

Dr. Aqsa Nazir: Dr. Aqsa Nazir completed her PhD from HITEC University Taxila, Pakistan, in 2020. She worked as an Assistant professor at Riphah International University Faisalabad campus, Faisalabad, Pakistan. Currently, she is working as a lecturer at National University of Modern Languages University, Faisalabad Campus, Pakistan.

Prof. Dr. Wasfi Shatanawi: He was working as a full professor in the De-

partment of Mathematics at Hashemite University, Jordon. Now, he is working as a full professor in the Department of Mathematics at Prince Sultan University, Riyadh, Saudi Arabia. According to clarivate analytic, Prof. Shatanawi was highly cited researcher for the years 2015, 2016, 2017 and 2018. Moreover, Prof. Shatanawi has been ranked as one of the top 2 of the highly cited scientist list created by Stanford University for the years 2019, 2020, 2021, 2022. He also published many papers and achieved the Best Researcher award from Prince Sultan University.

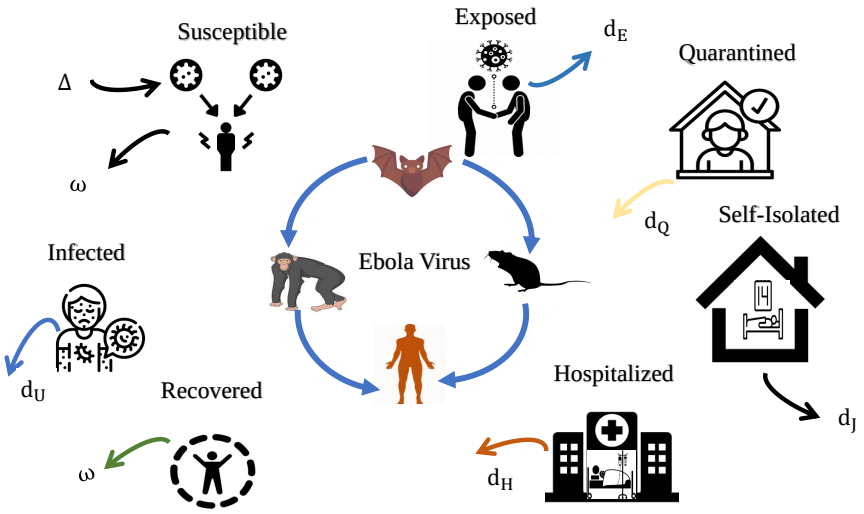

Figure 1. Transmission rate of Ebola virus in the society by discussing the Susceptible, Exposed, Infected, Quarantine, Hospitalized, Self-isolation and Recovered individuals.

Parameters	**Values**	**Sources**	**Parameters**	**Values**	**Sources**
ε_1	0.048	[14]	d_E	0.075	[14]
ε_2	0.35	[14]	β_1	0.2	Assumed
β_2	0.13	Assumed	d_Q	0.35	[14]
ρ	0.0379	Assumed	d_U	0.24	Assumed
γ	0.08	[14]	ω	0.7	[14]
d_J	0.23	Assumed	α	0.14280	[14]
d_H	0.22	Assumed	η	0.09	Assumed
Δ	10	Assumed			

Table 1
Values of Parameters

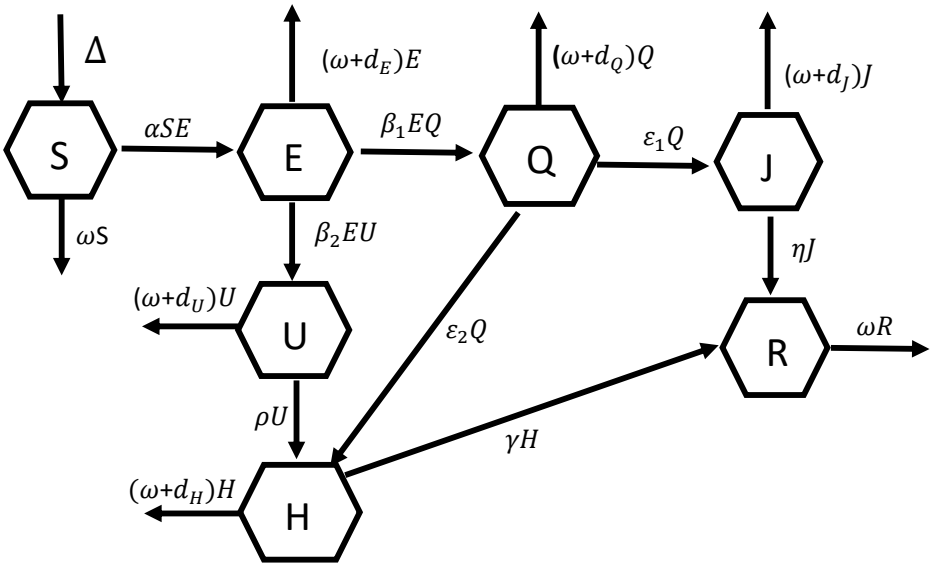

Figure 2. Flow Chart of Ebola Virus Transmission

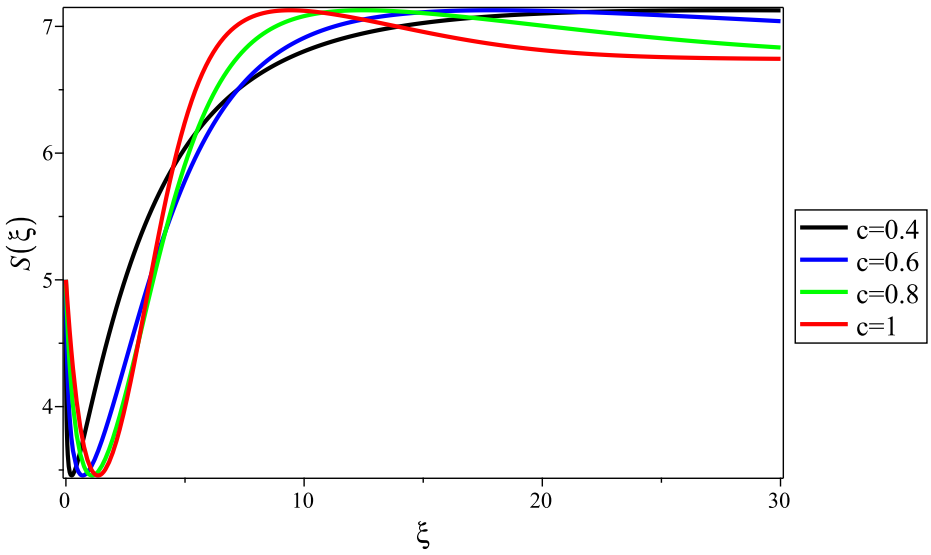

Figure 3. Susceptible Individual $S(\xi)$ after the implementation of self-isolated, quarantined and hospitalized of Ebola virus individuals for $c = 0.4, 0.6, 0.8, 1$.

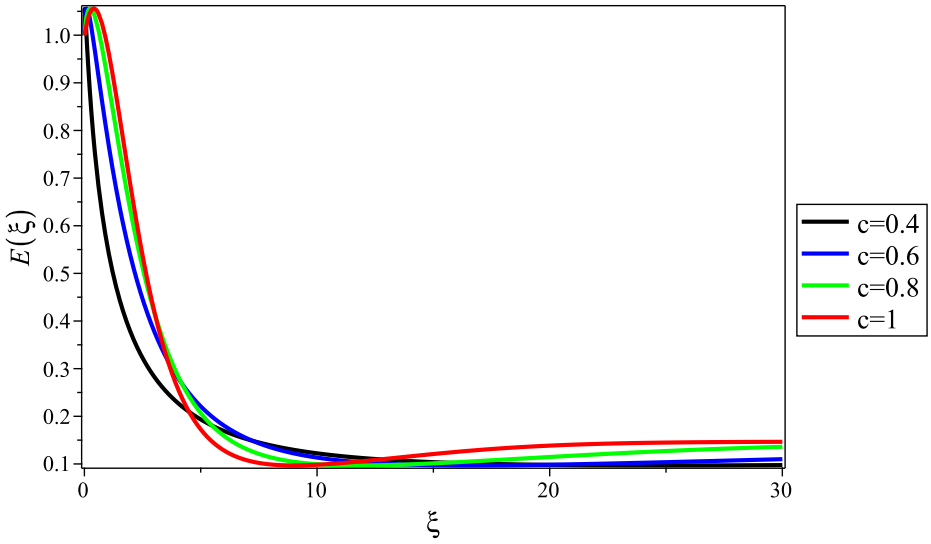

Figure 4. Exposed Individual $E(\xi)$ after the implementation of self-isolated, quarantined and hospitalized of Ebola virus individuals for $c = 0.4, 0.6, 0.8, 1$.

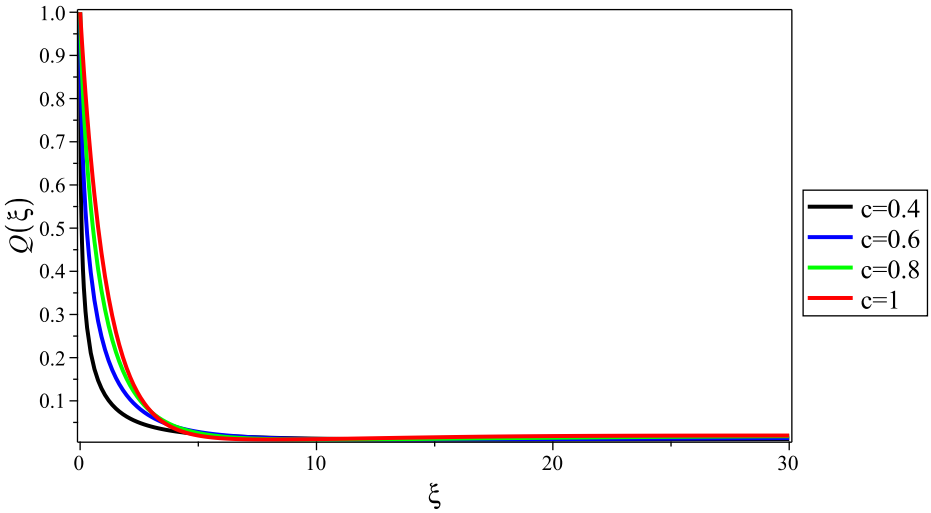

Figure 5. Quarantined Individual $Q(\xi)$ after the strict implementation of self-isolated, quarantined and hospitalized of Ebola virus individuals for $c = 0.4, 0.6, 0.8, 1$.

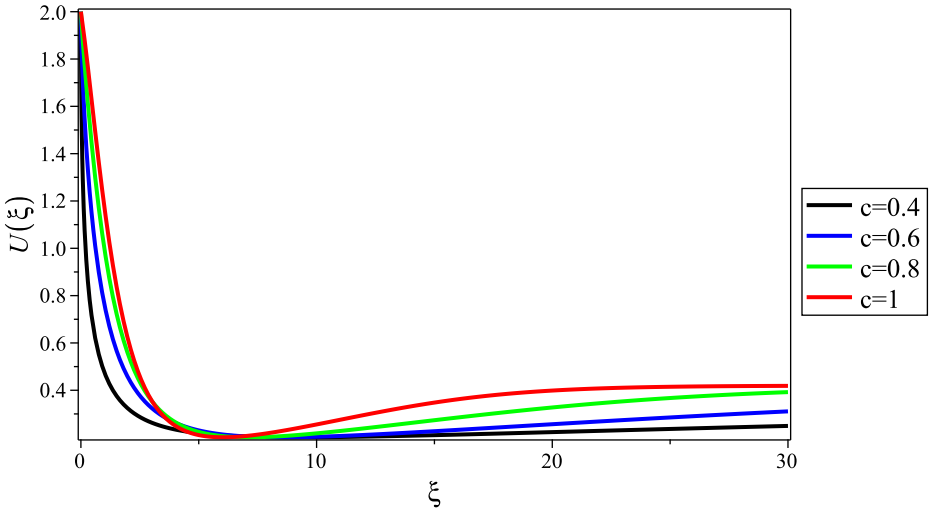

Figure 6. Infected Individual $U(\xi)$ after the strict implementation of self-isolated, quarantined and hospitalized of Ebola virus individuals for $c = 0.4, 0.6, 0.8, 1$.

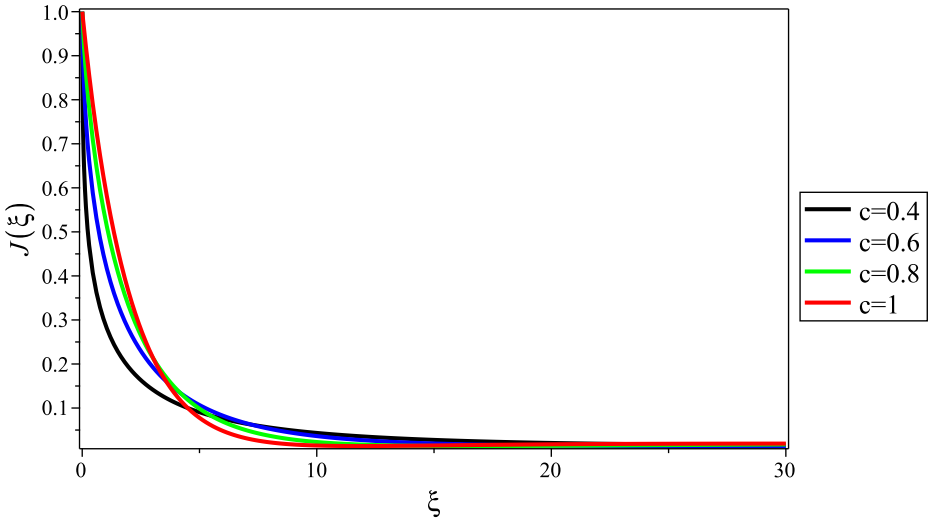

Figure 7. Self-Isolated Individual $J(\xi)$ after the strict implementation of self-isolated, quarantined and hospitalized Ebola virus individuals for $c = 0.4, 0.6, 0.8, 1$.

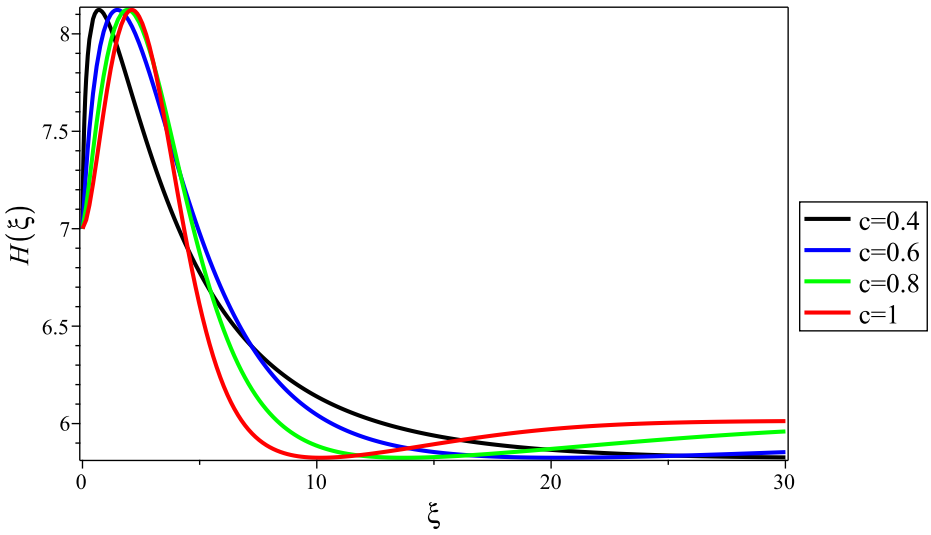

Figure 8. Hospitalized Individual $H(\xi)$ after the strict implementation of self-isolated, quarantined and hospitalized of Ebola virus individuals for $c = 0.4, 0.6, 0.8, 1$.

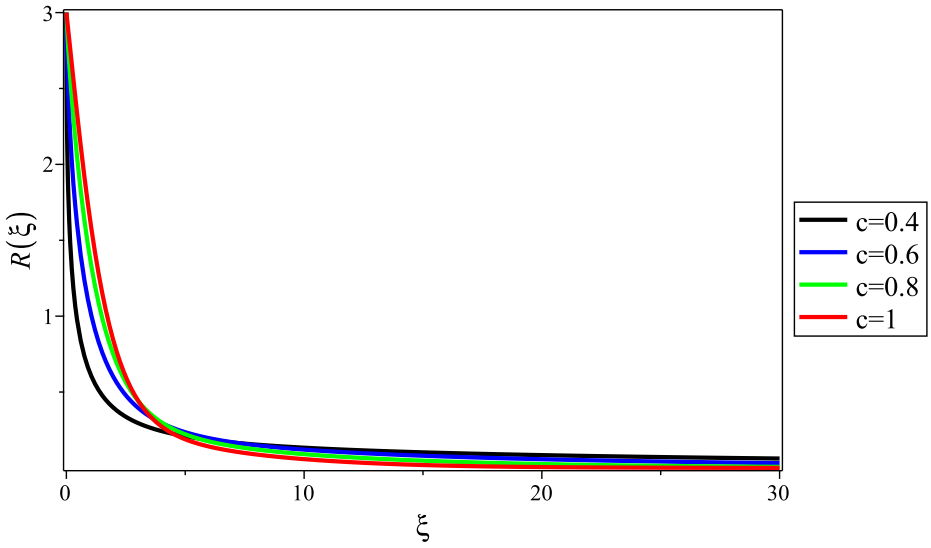

Figure 9. Recovered Individual $R(\xi)$ after the strict implementation of self-isolated, quarantined and hospitalized of Ebola virus individuals for $c = 0.4, 0.6, 0.8, 1$.