Research on application of optimization installation position for spur gear in gear rack drilling rig transmission unit

Wang Jiangang[1] Feng Ding[2*] Shi Lei[3] Sun Qiaolei[4] Miao Enming[5]

1 Ph.D. student. Institute for Hubei Engineering Research Center for Oil & Gas Drilling and Completion Tools. Dept. of Mechanical Engineering, Yangtze University, Jingzhou, Hubei, 434023, China. Email: 371152725@163.com.

2 Professor. Institute for Hubei Engineering Research Center for Oil & Gas Drilling and Completion Tools. Dept. of Mechanical Engineering, Yangtze University, Jingzhou, Hubei, 434023, China. Email: fengd0861@163.com, Tel: +86 13607219921. * Corresponding author.

3 Associate professor. student. Institute for Hubei Engineering Research Center for Oil & Gas Drilling and Completion Tools. Dept. of Mechanical Engineering, Yangtze University, Jingzhou, Hubei, 434023, China. Email: shilei0909@163.com.

4 Associate professor. Institute for Hubei Engineering Research Center for Oil & Gas Drilling and Completion Tools. Dept. of Mechanical Engineering, Yangtze University, Jingzhou, Hubei, 434023, China. Email: 649647220@qq.com.

5. Professor. College of Mechanical Engineering, Chongqing University of Technology, Chongqing 401135, China. Email: 2695522528@qq.com.

Abstract: The gear and rack transmission unit is a crucial load transmission device in the Gear Rack Drilling Rig (GRDR). Vibration induced by stiffness excitation during meshing between the gear and rack is a significant factor that affects the transmission performance. This paper focuses on the transmission unit of the GRDR and proposes a gear position design method based on time-varying meshing stiffness. A time-varying mesh stiffness model is established, considering tooth profile by the slice-iteration method. With the White Shark Optimizer (WSO) global search optimization algorithm, the gear position conditions are explored based on the fluctuation of stiffness. The load-bearing performance and dynamic characteristics of the mechanism are effectively improved. Dynamic analysis is conducted before and after the optimization of the scheme, and the improvements of the gear position optimization to the displacement of the transmission mechanism are verified through the meshing process of the rack and gear. The results show that the proposed optimization design may reduce the fluctuation by 89.64% and the maximum displacement by 9% compared to before. The proposed design method can effectively improve the motion performance, which is significant for optimizing the transmission unit of the GRDR.

Keywords: Gear Rack Drilling Rig, Transmission unit, Rack and Gear, Time-Varying Meshing Stiffness, Optimization.

0 Introduction

The global oil industry plays a vital role in driving economic development. To enhance the economic efficiency of oil extraction, increasing the efficiency of drilling operations and shortening the drilling cycle is imperative[1]. Traditional drilling rigs apply drilling pressure using the weight of the travel system, which may be insufficient, especially in shallow and horizontal drilling sections, thereby reducing efficiency. The Gear Rack Rig (GRDR), a novel active pressurized drilling rig[2, 3], has been developed to tackle this challenge. The GRDR increases the bit weight during drilling and utilizes a gear and rack mechanism for both bit pressure transmission and drill string lifting. The dynamic performance of the mechanism is a critical factor that affects the structural mechanical response and the safety performance of the drilling rig[4, 5, 6]. Therefore, it is imperative to ensure a reasonable design of the transmission unit. Overall, the development of the GRDR represents a promising avenue for improving the efficiency of oil extraction.

The GRDR transmission unit has the characteristics of low speed and heavy load. Strength calculation based on statics cannot meet the requirements of the gear and rack vibration. Vibration induced by stiffness is an important factor affecting the dynamic performance and service life of gear-rack transmission unit [7, 8]. A large number of scholars have carried out a series of studies. Considering the influence of time-varying meshing stiffness, Zhang[9]established the dynamic model of Epicyclic gear, and an optimal design method based on genetic algorithm was proposed. The results show that the effect of time-varying meshing stiffness on load-sharing performance is obvious. Younes[10] examined how tooth profiles and geometric parameters of gears affect their stiffness. This study utilized the genetic algorithm NSGA-II for multi-objective optimization, focusing on improving transmission efficiency and reducing errors. Offshore platform transmission systems optimize the number of teeth in gear meshes to minimize vibration, shock, and noise during gear rack transmission.[11]. Mounica[12]studied the 7-gear rack lifting system by using finite element method to obtain the optimal fillet radius of gear teeth. On this basis, considering the effect of meshing position on gear stress. Kondaker[13] studied the mechanical response of gear/rack transmission unit and stiffness analysis of gear shaft. However, the stress analysis is only focused on the gear single contact under static load. Gear rack transmission unit is used to lift drill string in land drilling rigs, and the related literature is very limited. Base on the transmission unit of six gears and four racks as the research object, Feng[2]studied the load-carrying capacity of the gear and rack in the meshing process through the finite element method. Lei[14] studied the stability and strength check of the meshing process considering the load. The result shows that the force load of the gear under the asymmetric load is larger than that on the other side. Literature review shows that the work of gear rack transmission unit in GRDR is very limited, especially the research report on reducing stiffness fluctuation through parameter optimization.

This study investigates the optimal gear installation positions within the transmission unit of a gear rack drilling rig to enhance stability and reduce vibration and shock during operation. It develops an advanced mesh stiffness model considering specific gear and rack tooth profiles. The impact of gear placement on meshing tooth count is analyzed while ensuring internal collision space constraints are observed. Optimization focuses on minimizing Time-Varying Meshing Stiffness (TVMS) fluctuations using the Whale Shark algorithm. Validation includes integrating a dynamic transmission model with field tests to ensure optimization reliability.

1 Problem description

The rigid connection between the gear and rack causes more pronounced vibration in the drilling rig compared to the traditional drilling rig with wire ropes. The gear and rack transmission unit is a crucial component of the GRDR and serves as the fundamental equipment for ensuring drilling performance and reliability of the transmission unit. In this paper, the GRDR transmission unit, which features an 8-gear and 4-rack back-to-back style transmission, as the primary research object. The gears are designed with a large module, which offers the advantage of high load-carrying capacity. The transmission unit is formed by installing the gear on the box and fixing the rack to the rig derrick, as shown in Figure 1. The transmission unit and Derrick's center of gravity are located in the same plane, providing several technical benefits, such as uniform forces on each gear, reduced friction of the guide wheel during lifting, and more flexibility in gear design size. Influenced by the working conditions, the transmission unit is characterized by low speed and heavy load. Optimization of gear parameters is used in gear design, but the approach lacks advantages in economy. The vibration in the transmission unit is caused by the dynamic excitation resulting from the periodic changes in the gear contact.

The transmission unit is a parallel symmetrical distribution of eight gears, which the front gears are used as the research object. The phase difference between the initial gears is considered to be 0°. As shown in Figure 2, the number of gear meshing teeth before optimization varied greatly at different moments, which resulted in the TVMS large fluctuation of TVMS. Installation positions of the gears are optimized to enhance the dynamics. Amplitude of change in the gear mesh tooth number is reduced, which in result reduces the TVMS fluctuation.

2 Description of gear mounting position design method

The investigation of gear installation position design proceeds through three sequential steps, outlined in Figure 3: establishing the "gear and rack TVMS model," reducing transmission unit TVMS fluctuation, and verifying dynamic performance. Each step aims to achieve specific research objectives. The gear and rack TVMS model is developed by integrating structural parameters of the gear and rack with theories such as Hamiltonian, energy, and Muskhelishvili fillet theories. Optimization of gear installation positions utilizes the white shark optimization algorithm, taking into account its impact on TVMS fluctuation. The reliability of this optimization approach is subsequently validated through dynamic performance calculations and testing.

Stiffness is a key factor in determining the vibration displacement correlation function in the dynamic formula, and therefore has a significant impact on the overall dynamic performance of transmission unit. To enhance the stability of the transmission unit, a novel design optimization method is proposed that can effectively accommodate internal collision constraints. In this method the installation position of the gears is optimized by which the fluctuation of the TVMS of the gears is reduced. The phase difference between the gears is modified and the magnitude of variation in the number of meshed teeth at different moments is decreased.

2.1 Objective functions and constraints

The optimization model for multi-gear TVMS is characterized as a nonlinear programming problem in which the design variables need to be re-selected in each iteration. An objective function for the optimization problem is proposed, which aims to minimize the variance of multi-gear TVMS in a cycle. The objective function is composed of the four gears TVMS and is influenced by the gear installation distance. Specifically, Equation (1) is used to calculate the stiffness fluctuation value during the gear transmission process and optimize the installation distance of the multi-gears system.

$$\begin{gathered} \min f = \delta\left(G_{x_1} + G_{x_2} + G_{x_3} + G_{x_4}\right) \\ G_{x_i} = [G_o(x_i:), G_o(1:x_i)] \end{gathered} \tag{1}$$

Where δ is calculated variance, x_i is relative to the pre-design changes in the location of each gear,

G_{x_i} is the gear at the installation distance of the TVMS.

When the installation distance of gears is designed, the space constraints of hydraulic motors, reducers, and other devices are considered. To satisfy the practical engineering requirements, the design parameters are set to integer values. The constraint function is presented in Equation (2), where all the above factors are considered and the constraints on the gear mounting position are satisfied.

$$\begin{gathered} 0 \le x_i \le 62 \\ x_i \in z \end{gathered} \tag{2}$$

2.2 Time-varying meshing stiffness

The TVMS is composed of bending, shear, axial compressive, contact and fillet-foundation. The gear and rack TVMS is calculated as shown in Equation (3):

$$K_e = \frac{1}{\frac{1}{K_{a1}} + \frac{1}{K_{b1}} + \frac{1}{K_{s1}} + \frac{1}{K_{f1}} + \frac{1}{K_h} + \frac{1}{K_{a2}} + \frac{1}{K_{b2}} + \frac{1}{K_{s2}} + \frac{1}{K_{f2}}} \tag{3}$$

The stiffness of single teeth mesh by calculating axial(k_a) Shear(k_s), bending(k_b), fillet-foundation stiffness(K_f), subscripts 1 and 2 denote gear and rack, respectively.

2.2.1 Function of tooth profile

The profile of the gear tooth is constituted by the gear tooth structure and the pressure angle, which is influenced by the TVMS by the tooth profile. The tooth profile curve comprises the straight-line and curve types, as depicted in Figure 4. The tooth profile of the research object is composed of straight line and curve. The gear tooth profile curve is composed of involute tooth profile, linear profile, and tooth root arc. The rack consists of straight line and tooth root circular profiles, where the involute tooth profile, root circular tooth profile, and linear tooth profile are represented in Equation (4)-(6), respectively:

Involute tooth profile：

$$\left.\begin{aligned}x &= r_i \sin(\pi / 2N - (\mathrm{inv}\alpha_i - \mathrm{inv}\alpha_0)) \\ y &= r_i \cos(\pi / 2N - (\mathrm{inv}\alpha_i - \mathrm{inv}\alpha_0))\end{aligned}\right\} \tag{4}$$

Where r_i is the distance from the engaging point to the center of the gear; N is the number of teeth; $\mathrm{inv}\alpha_i$ is the involute Angle on the circle of radius r_i, $\mathrm{inv}\alpha_0$ is the involute Angle of the pitch diameter.

Root circular tooth profile：

$$\left.\begin{aligned}x &= r \times \sin\phi - \left(a_1 / \sin\gamma + r_\rho\right) \times \cos(\gamma - \phi) \\ y &= r \times \cos\phi - \left(a_1 / \sin\gamma + r_\rho\right) \times \sin(\gamma - \phi)\end{aligned}\right\} \tag{5}$$

Where r is reference radius. r_ρ is the distance between the center of the Fillet and the center line, a_1 is the radius of the fille.

Straight Line of tooth profile：

$$y = y_L - \tan(\alpha_\mathrm{r})x \tag{6}$$

Where y_L is the distance from the intersection point of meshing line and the root circle to the center, a_r is the Pressure Angle of rack.

2.22 Function of single tooth stiffness

The property of meshing stiffness is affected by structure and material parameters, such as module, number of teeth, pressure angle, tooth width, and material, which directly influence the value of meshing stiffness. The stiffness of a single tooth is determined through the calculation of the bending, shearing, and axial directions of each tooth, as depicted in Equation (7):

$$\begin{aligned}\frac{1}{k_b} &= \int_0^d \frac{\left((d-x)\cos(a_m) - h\sin(a_m)\right)^2}{EI_x}dx \\ \frac{1}{k_s} &= \int_0^d \frac{1.2\cos^2(a_m)}{GA_x}dx \\ \frac{1}{k_a} &= \int_0^d \frac{\sin^2(a_m)}{EA_x}dx\end{aligned} \tag{7}$$

Where h, a_m, x, dx and d are defined in Fig. 4, E, G, A_x are Young's modulus, shear modulus and the tooth section area at meshing point.

2.23 Function of contact stiffness

The stiffness of a single tooth is calculated through the aforementioned equations. However, the gear-rack contact is not comprised of a single tooth, and the process of gear-rack meshing transmission involves at least a pair of teeth. Therefore, it is necessary to consider the contact stiffness of a pair of teeth. In this study, to simulate nonlinear Hertz contact, Hamilton's approximate Hertz contact is employed to represent the contact stiffness of a single pair of meshes[15], as depicted in Equation (8):

$$k_H = \frac{\pi L}{[4+\pi]}\left[\left(\frac{E_1}{1-v_1^2}\right)+\left(\frac{E_2}{1-v_2^2}\right)\right] \tag{8}$$

Where v_i (i=1,2) is the Poisson's ratio of gear and rack respectively.

2.14 Function of fillet-foundation stiffness

The fillet-foundation stiffness plays a crucial role in the calculation of TVMS, as the influence of gear body deflection is considered. It is important to include the fillet-foundation stiffness in the TVMS calculation. Sainsot's fillet-foundation stiffness theory[16], as presented in Equation (9), is utilized in this study. The fillet-foundation stiffness is determined by the ratio of the radius of the tooth root circle to the center hole and the angle between the center line of the tooth and the joint of the tooth root circle.

$$y_{\mathrm{f}} = \frac{\cos^2\beta}{EL}\left\{L^*\left(\frac{u_{\mathrm{f}}}{S_{\mathrm{f}}}\right)^2 + M^*\left(\frac{u_{\mathrm{f}}}{S_{\mathrm{f}}}\right) + P^*\left(1+Q^*\tan^2\beta\right)\right\} \tag{9}$$

Where u_{f} $S_{\mathrm{f}}, L^*, M^*, P^*$ 和 Q^* Cited in the literature[9].

2.3 white shark optimizer

The White Shark Optimizer (WSO) is proposed by Malik Braik[17], drawing inspiration from the hunting strategies of white sharks, one of the most perilous predators of the oceans. The group-based WSO algorithm is modeled after the sharks' innate hunting abilities, which rely on their superior sense of hearing, smell, and prey orientation. Due to the ever-changing positioning of the biological traits of white sharks, the WSO exhibits exceptional efficiency in avoiding local optima and accelerating the attainment of the global optimum solution for complex objective functions, as shown in Figure 5.

In the initial phase, the WSO algorithm begins with the white shark moving towards the optimal prey position based on its natural hunting behavior, while the initial position of the white shark is randomly generated. During the search iteration process, the vector method is utilized to explore the solution space, as presented in Equation (10):

$$w_j^i = l_j + r \times \left(u_j - l_j\right) \tag{10}$$

Where w_j^i represents the vector of the i white shark on the J dimension, u_j and l_j represent the lower and upper bounds of the white shark on the J dimension respectively, and r is a random number created in the interval [0,1].

The auditory, visual and olfactory senses are used by white sharks to search for prey. Based on this perception, the white shark moves towards its prey in a wave-like motion. By comparing the optimal prey with the known prey location, the white shark can adjust its speed during the pursuit, as presented in Equation (11).

$$v_{k+1}^i = \mu\left[v_k^i + p_1\left(w_{\text{gbest}_k} - w_k^i\right) \times c_1 + p_2\left(w_{\text{best}}^{v_k^i} - w_k^i\right) \times c_2\right] \tag{11}$$

v_{k+1}^i, v_k^i represents the speed at which the white shark moves to the new prey, w_{gbest_k} is the optimal prey in the current k iterations, w_k^i is position vector of the white shark i in the K step, and c_1 and c_2 are two random numbers uniformly generated within the range of [0,1].

2.4 Multi-gear dynamics model

The gear meshing process is assumed to be uniform, and torsional deformation at the gearing shaft is neglected. Figure 6 illustrates gears in various meshing positions. Based on Newton's second law, combined with the time-varying meshing stiffness in Section 2, the multi-rack and gear dynamic model is developed. This model is represented by Equation (12):

$$\begin{cases} m_i \ddot{x}_i + c_{i(t)} \dot{x}_i + k_{i(t)} x_i = F_i R_i \\ \sum_{i=1}^{8} F_i \cos\alpha = F_{\mathrm{load}} + Ma \\ F_1 l_1 + F_2 l_2 + F_3 l_3 + \cdots + F_8 l_8 = 0 \\ \sum_{i=1}^{8} F_i = \sum_{i=1}^{8}\left(k_l \delta_l + c_t \dot{\delta}_l\right) = \sum_{i=1}^{8}\left[k_i\left(r\theta - e_i\right) + c_i(r\dot{\theta} - \dot{e}_i)\right] \end{cases} \tag{12}$$

Where: M is the total mass of the lifting system, F_{load} is the load of the lifting device, F_i is the meshing force of the gears (i=1-8), a is the acceleration of the lifting device, l_i is the distance from the gears to the center of gravity of the lifting device, m_i is the mass of the gears, R_i is the indexing circle of the gears, $k_{i(t)}$ is the stiffness of the gears, which is calculated according to the subsection 2.2, and $c_{i(t)}$ is the damping of the gears. Damping, related to the stiffness, is calculated as shown in Equation (13):

$$c_{(t)} = 2\zeta\sqrt{k_{(t)}\frac{r_1^2 r_2^2 I_1 I_2}{r_1^2 I_1 + r_2^2 I_2}} \tag{13}$$

3 Case study

The GRDR has specific features, including the handling of large lifting weights (up to 250 tons) and exposure to different types of loads, creating complex working conditions. Parameters from the GRDR transmission unit, as shown in Table 1.

3.1 TVMS Analysis for rack and gear in GRDR

TVMS calculation methods include analytical method, finite element method, and hybrid method.

Finite element methods are applied for theoretical validation due to accuracy and efficiency[18, 19].

3.1.1 Finite element analysis setup

Based on the geometric and material parameters provided in Table 1, the gear and rack is modeled using the finite element method. To enhance the calculation accuracy and efficiency, a multi-scale meshing technique is employed for the gear and rack. The mesh cell type is selected as C3D8 cell, the number is 1334540, and the Jacobian of the generated mesh is greater than 0.7. The details of the meshing of the gear and rack are illustrated in Figure 7.

The gear with 17 teeth rotates 21.177° in a single tooth meshing cycle. The meshing surface is segmented into 21 parts and a load of 53125 N/mm is applied to the nodes (node 1 ~ node 21). Rack bottom of the fixed frame is used as a constraint in the simulation.

3.1.2 Verification of calculation results

Figure 8 depicts the TVMS is calculated by the ISO standard Ishikawa method, the method of reference [20], the method proposed in this paper, and the finite element method. Table 2 shows the results of the comparison between the single tooth meshing stiffness, average meshing stiffness and obtained by the four methods. Three theoretical calculations are compared with the finite element method based on the results of the finite element method. The proposed method in this paper is shown to be close in the maximum meshing stiffness of a single tooth and the average value of TVMS.

The results of bending stiffness, axial compression stiffness and meshing stiffness of single tooth calculated by the reference [20] and propose model are compared.

The stiffness of the proposed model is calculated and compared to that of the reference 12, yielding a consistently lower value, as illustrated in Figure 9. Although the shear stiffness law is akin to the meshing stiffness of a single tooth, the difference in bending stiffness is accentuated during the meshing process. Notably, this study accounts for the pressure angle at each meshing point on the gear and rack in the developed model.

3.2 Optimization on gear mounting position

The global optimum for gear installation position optimization is employed as WSO algorithm. The minimum multi-gear TVMS variance is optimized as a single objective. The optimal installation position of the multi-gear system is searched efficiently while ensuring that the TVMS satisfies the stiffness variance search. The parameters of the algorithm are set to a population size of 30, a maximum of 100 iterations, and maximum wave and minimum current values of 0.75 and 0.07, respectively. The initial and secondary velocities were set to 0.5 and 1.5. During the optimization process, the TVMS variance iterations are monitored and visualized as shown in Figure 10. The algorithm and visualization provide valuable insights into the optimization process and facilitate the identification of optimal design solutions.

Table 3 presents the parameters of the gear installation before and after optimization. The results demonstrate that all optimization variables meet the necessary constraints, and the stiffness variance is reduced by 89.64%. This significant reduction indicates that the smoothness of the transmission process is greatly improved by optimizing the structural stiffness. The reduction in stiffness variance confirms the effectiveness of the proposed optimization approach.

3.3 Verification of dynamic performance

The feasibility of the optimization results is verified by validating the dynamic model of the transmission unit before and after the optimization of the gear installation position parameters. Dynamic differential equations are used in the Lunger-Kutta method to obtain the dynamic response of the multi-gear system. The dynamic behavior of the transmission unit is investigated to get a better understanding of the effect of the optimized gear mounting distance parameters on the operation of the system.

Figure 11(a) depicts that the velocity curves of the transmission unit match, before and after the gear installation position optimization. It shows that the gear position optimization design ensures the performance of good synchronization of the lifting device. The amplitude fluctuation of the optimized transmission unit in the start-up phase is alleviated, and the overall view of the speed fluctuation amplitude is gentler.

Figure 11(b) shows that the optimized gear meshing forces are all smaller than the pre-optimization gear meshing forces, with the most significant effect in Gear 1, which is reduced by about 6.58%. The gear installation position is conducive to improving the dynamic characteristics of the transmission unit and its reduction of gear force. The optimized gear installation position ensures stable movement of the transmission unit and the service life of the gears is extended.

3.4 Stability test verification of transmission unit

This test is conducted under specific conditions, in which the GRDR load of 36 tons. The vibration of the lifting box is observed during its downward speed, using the attitude detection of the lifting box.

3.4.1 Test setting

The sensor is fixed at the designated point, as shown in Figure 12. Non-gravitational acceleration effects during rotation are considered. Smooth friction is made between the installation surface of the sensor and the measuring surface of the transmission unit. The transmission unit is parallel to the carrier plane, reducing the effect of errors on the measured data.

3.4.2 Test results analysis

In this test, real-time monitoring of the space attitude of the GRDR transmission unit was conducted during its operation at speed is set as 0.28 m/s. To measure the system stability, the rotation angle of the attitude sensor around the x and y axes is observed, and the changes in the angle of these axes were recorded. These changes are graphically represented in Figure 13(a) (b), which demonstrates the fluctuations in the rotation angles of the x and y axes, respectively.

Figure 13 depicts the angular fluctuation of the transmission unit around the x and y axes during operation. With a velocity of 0.28 m/s, the maximum angular change around the x-axis is 0.1121° and the maximum angular change around the y-axis is 0.0326°. These results show that the vibration response of the GRDR transmission unit is small, and the feasibility of the design method proposed in this paper and the stability of the system are verified.

4 Conclusion

In this paper, the optimization of gear mounting position is investigated for the vibration problem of GRDR transmission unit, which the TVMS is considered to be affected by the gear installation position. Combined with the gear and rack TVMS is developed and the global search of the White Shark algorithm being utilized, the dynamic performance of the GRDR transmission unit is optimized. The feasibility of the design methodology is verified based on the dynamic model developed and field tests. These findings have important implications for future design and engineering efforts aimed at reducing vibration issues and improving the performance of the GRDR transmission unit.

1. Based on the 8-gear-4-rack transmission unit in the GRDR, the TVMS model is developed by combining the tooth profile function with the energy method. Other theoretical computational methods are compared that proposed model is closer to the finite element method in maximum single tooth meshing

stiffness and average TVMS values.

2. The gear installation position is an effective measure to improve the dynamic performance of the GRDR by reducing the fluctuations in the TVMS stiffness. The fluctuation of TVMS in the optimized transmission unit is reduced by 89.64%. The magnitude of meshing force in gearing is reduced by 6.58%.

3. Combining the TVMS model with the WSO global optimal search algorithm, the optimal design method considering the gear installation position is proposed. The effectiveness of the method is verified by test results, which validate the feasibility of the method in applications.

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Data availability

The authors confirm that the data supporting the findings of this study are available within the article.

Code availability

The authors confirm that the code supporting the findings of this study are available within the article.

Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Author Contributions

All authors contributed to the study's conception and design. Jiangang Wang, Lei Shi performed material preparation, data collection, and analysis. Jiangang Wang wrote the first draft of the manuscript, and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

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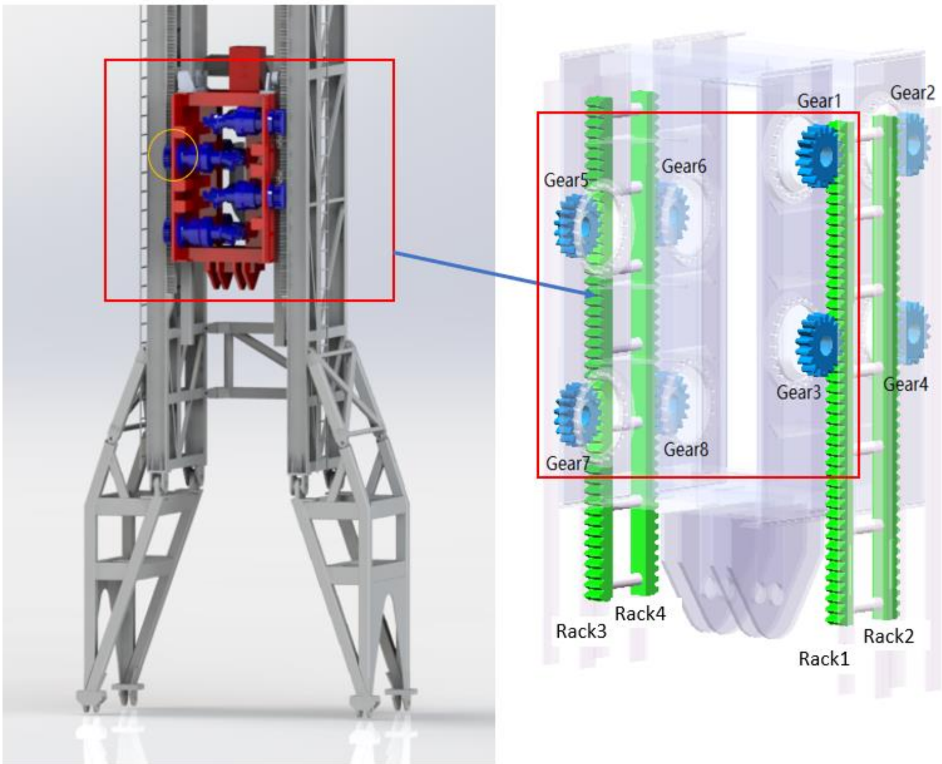


Fig. 1 Diagram of the gear distribution in GRDR transmission unit

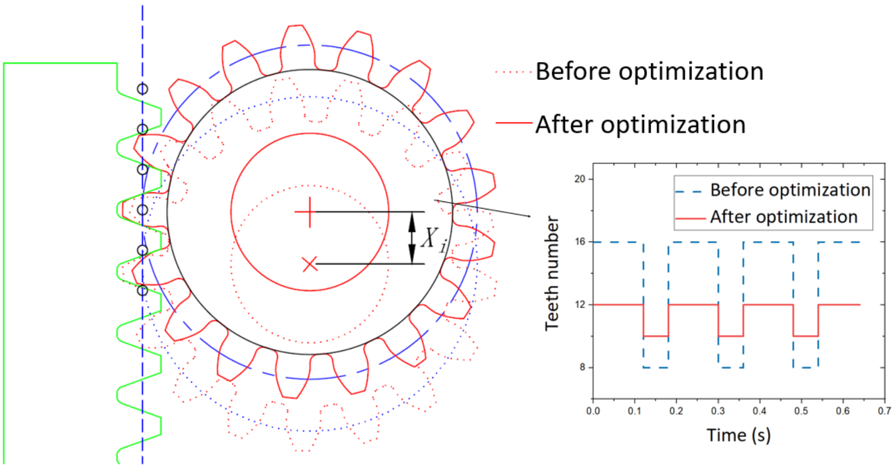


Fig. 2 Effect of installtion position on phase difference

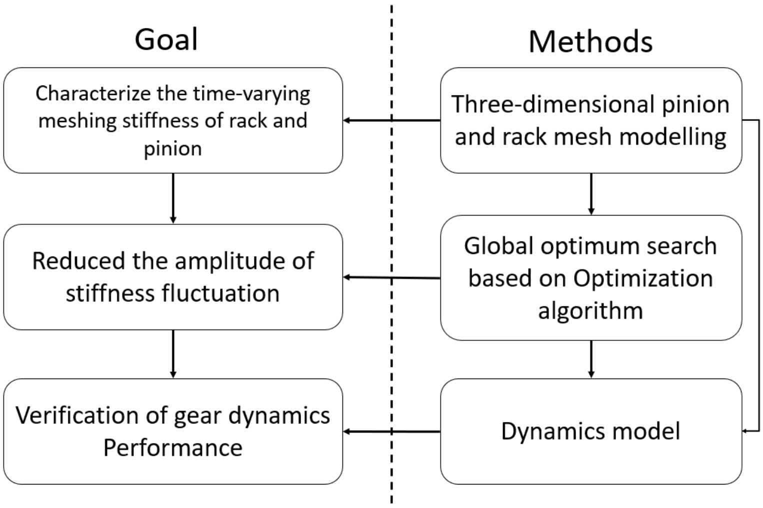


Fig. 3 Methodological framework of gear installation position design

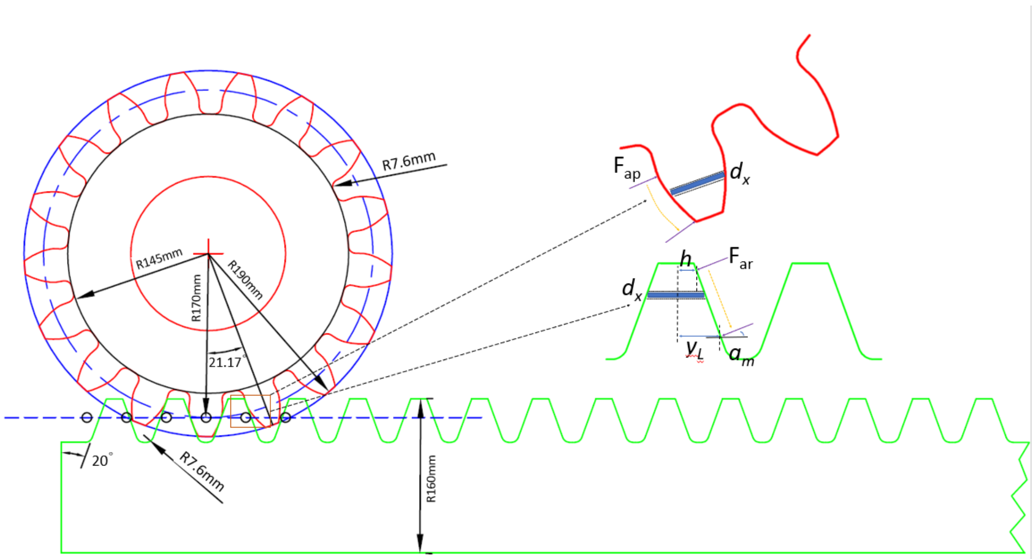


Fig. 4 Schematic of the load of tooth

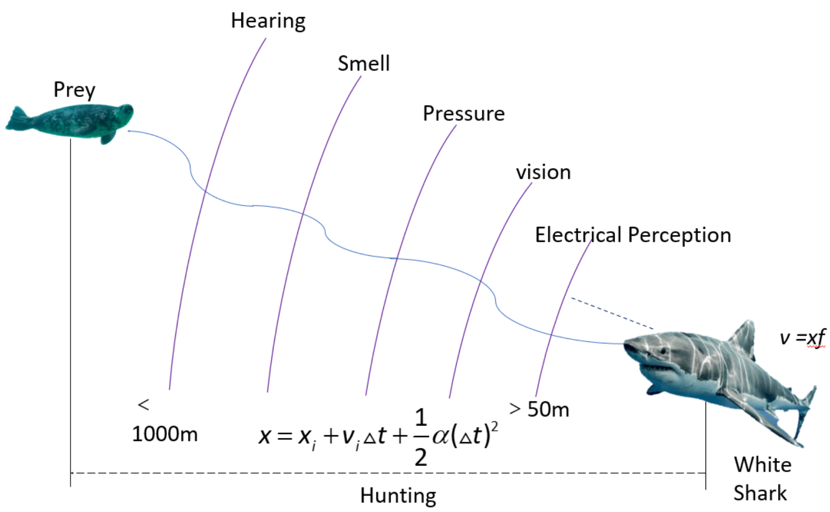


Fig. 5 White sharks track prey

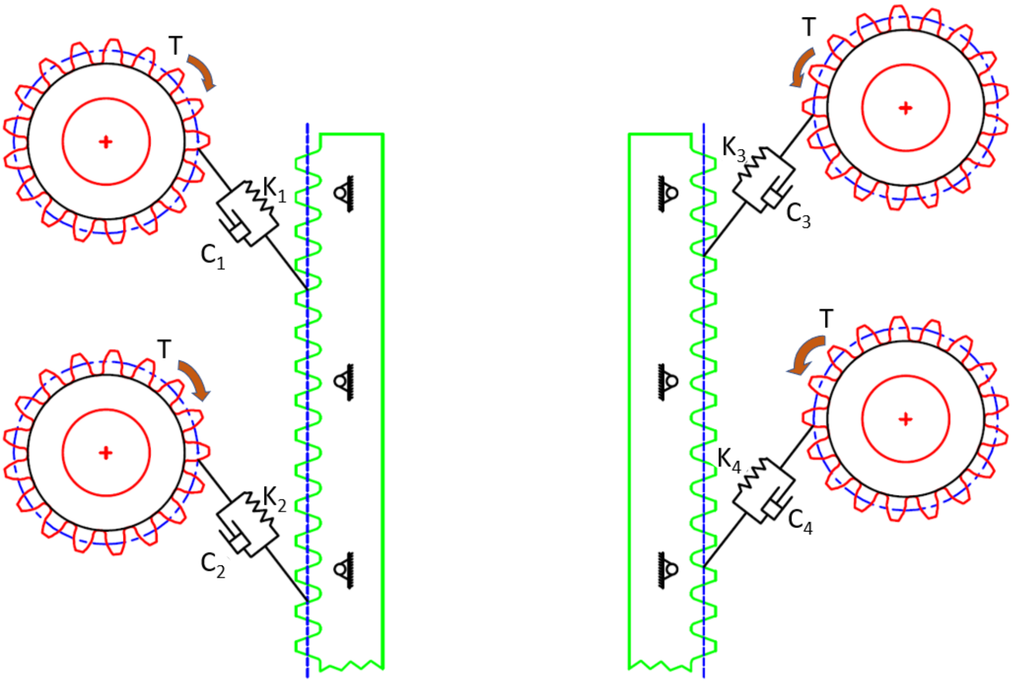


Fig. 6 Dynamics model of gear and rack

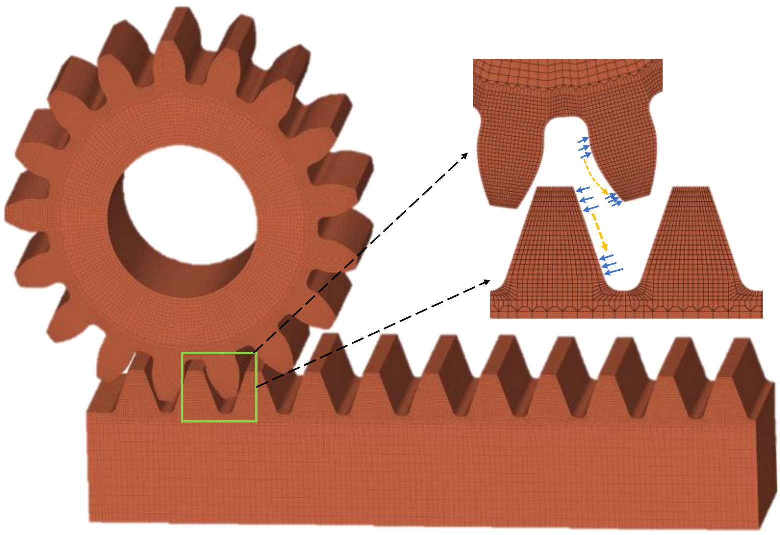


Fig. 7 Finite element model considering meshing Angle

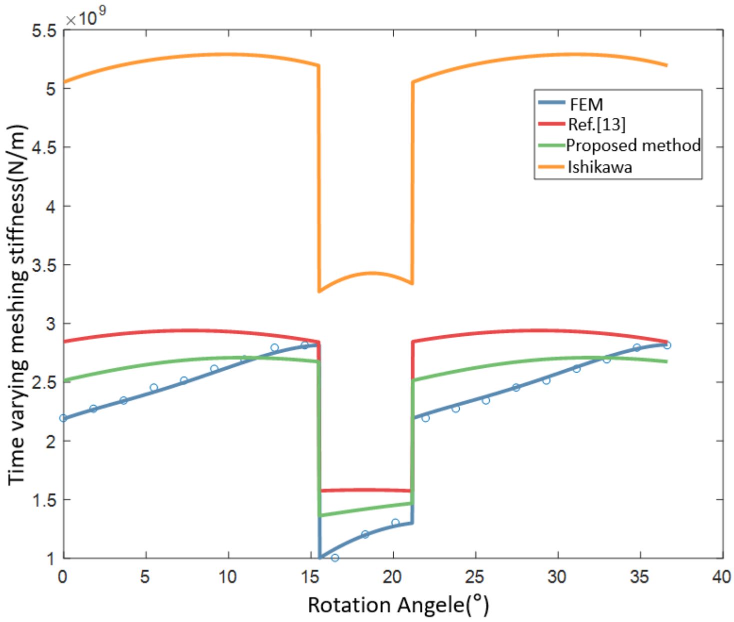


Fig. 8 TVMS based on different methods

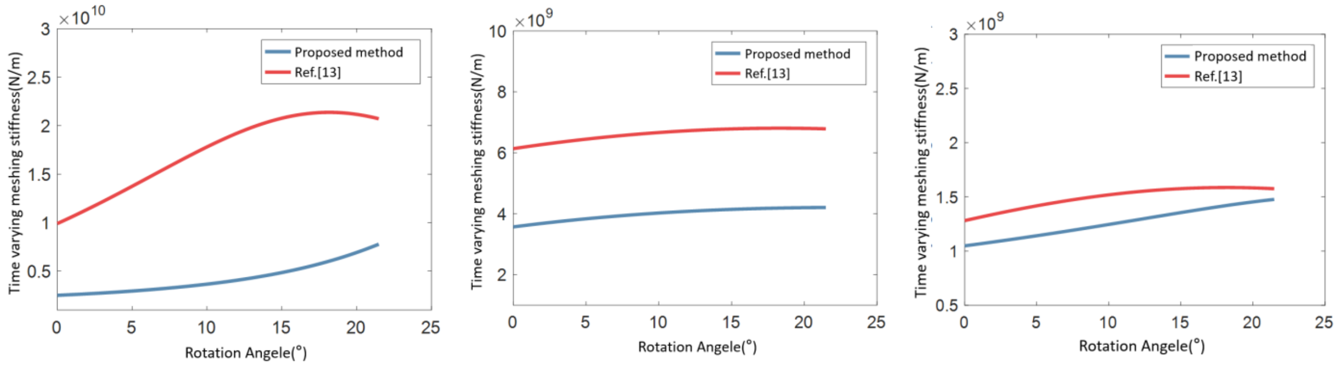


Fig. 9 Comparison of single tooth meshing stiffness curves based on different methods

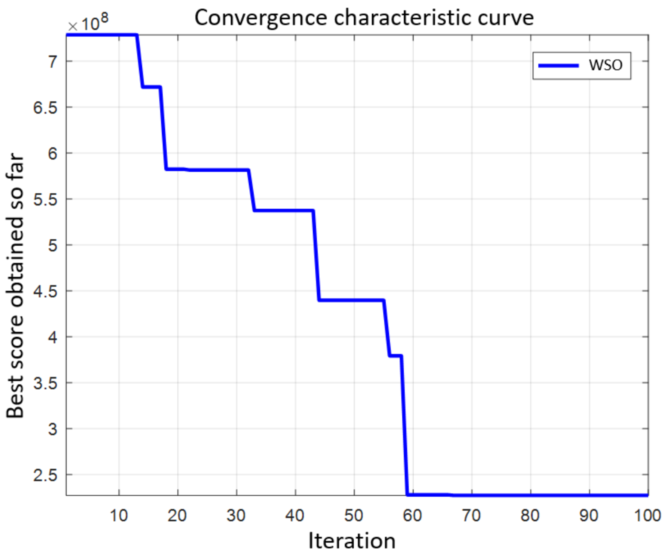


Fig. 10 Convergence Curves

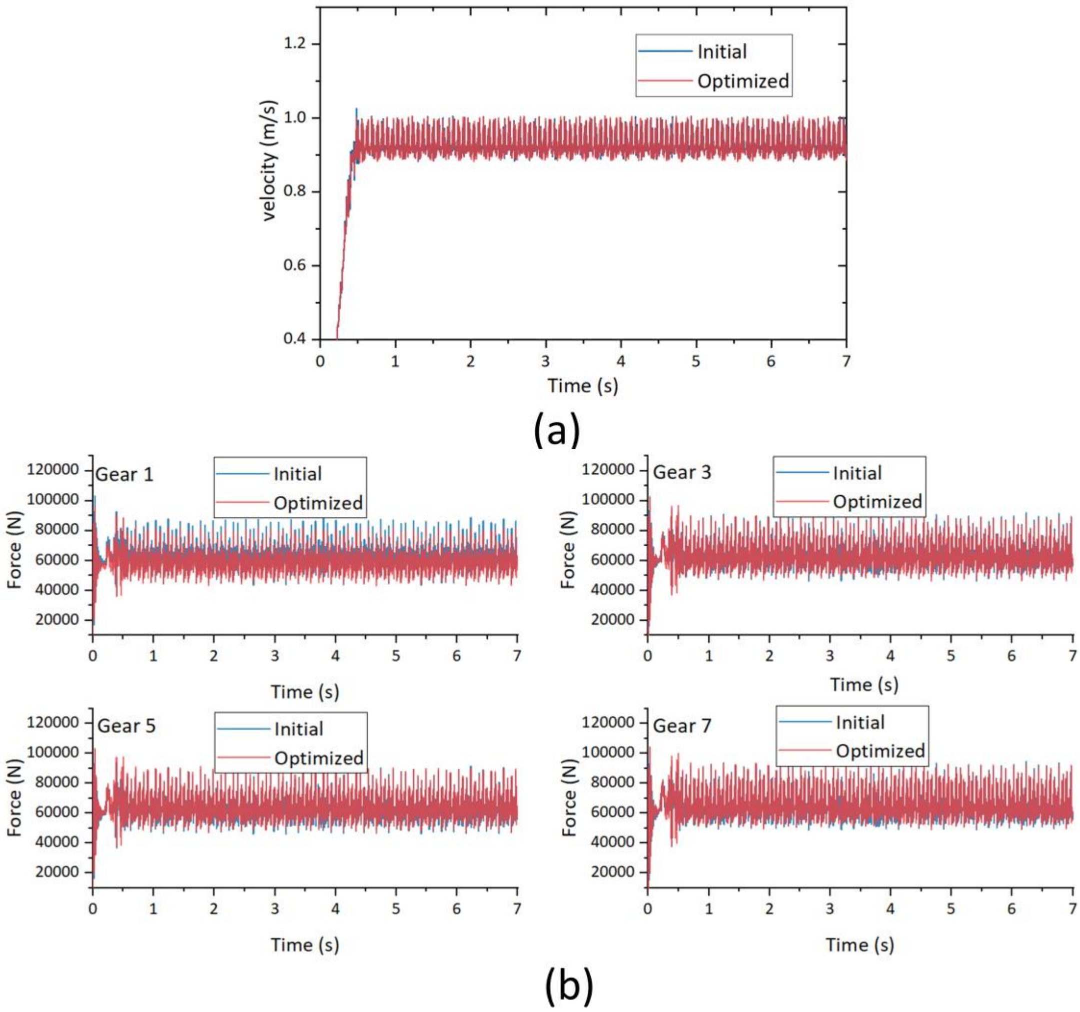
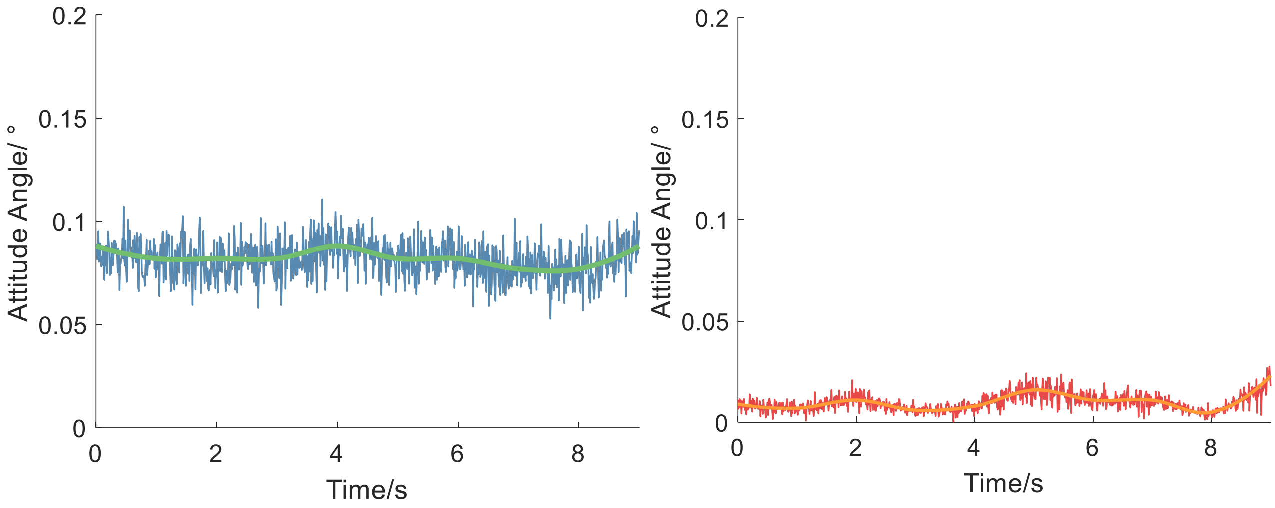


Fig. 11 Velocity and gear meshing force before and after optimization

Fig. 12 Orientation of attitude sensor installation



(a) (b)

Fig. 13 x/y axis attitude angle detection in 0.28m/s velocity

Table 1 Parameters of gear and rack

Parameters	Value
Modulus m(mm)	20
Pressure angle α (°)	20
Number of teeth Z	17
Pitch diameter (mm)	340
Tooth width L(mm)	120
Gear elastic modulus E_1 (GPa) / Poisson ratio v_1	207 / 0.254
Rack elastic modulus E_2 (GPa) / Poisson ratio v_2	212 / 0.28

Table 2 Comparison of TVMS calculations

Method	Maximum meshing stiffness of a single tooth (N/m)	Comparison with finite element(%)	The average value of TVMS /(N/m)	Comparison with finite element(%)
Ishikawa	3.43e+09	62	4.74E+09	51
Reference	1.58e+09	17	2.55E+09	9
Proposed	1.46e+09	10	2.33E+09	0.4
Finite element	1.30E+09	-	2.32e+09	-

Table 3 Optimize before and after parameters

	Gear 1	Gear 3	Gear 5	Gear 7	Variance of meshing stiffness
Before optimization	0	0	0	0	2.19e+09
After optimization	+10	+56	+25	+39	2.27e8

Biographies

Jiangang Wang is a current Ph.D. student at Yangtze University. As a major participant, he has participated in 3 national provincial and ministerial projects and Published 8 related papers. His research interests: theoretical and technical application research in the design, diagnosis and dynamic simulation of petroleum machinery.

Lei Shi, Associate Professor, his major in oil field equipment, published several papers and participated in the translation of a series of books on foreign oil and gas exploration and development progress. He has participated in 6 projects at national and provincial levels published 8 first-author academic papers in relevant academic journals.

Ding Feng is a Professor of Mechanical Engineering at Yangtze University, China. He received his Ph.D. at the China University of Petroleum (Beijing). His main research areas are design, testing and diagnostics of oil and gas drilling and completion tools and equipment and offshore engineering equipment and tubular column mechanics and dynamic simulation technology. He has participated in nearly 90 projects at national and provincial levels and published 242 relevant academic papers.

Sun Qiaolei, Associate professor, has presided over 8 vertical and horizontal projects of Hubei Natural Science Foundation, participated in more than 10 projects of National Natural Science Foundation, youth projects and national major special projects, and published more than 40 relevant journal papers;.

Enming Miao is a professor and Ph.D. supervisor in the School of Instrument Science and Optoelectronic Engineering at the Hefei University of Technology. His main research interests are precision mechanical engineering, accuracy theory, thermal error compensation of CNC machine tools, and structural design theory and application technology of mechanical thermal robustness. Published more than 50 papers.