Analyzing Causality in Uncertain Domains using Extended Fuzzy Logic (FLe) Applied to Coronary Heart Disease Diagnosis

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Abstract: To extend FLe application in computing the degree of causality, we employ granular fuzzy causality tactics to determine the direction of causality among related variables when there is only imperfect information. Our approach involves a hierarchy of nested inferences of interval Type-2 fuzzy sets to achieve more approximate reasoning for more workable solutions, in the sense of *f*-valid philosophy. To deal with intrinsic hard uncertainty in the problem architecture, we leverage expert knowledge about the problem structure. Our method involves three key steps: encoding reasons into interval Type-2 fuzzy, utilizing the interaction of concepts in forward reasoning through qualitative descriptions and allowing a certain level of uncertainty, and determining the direction of valid results based on extended fuzzy logic. The simulation results for the application of coronary heart disease demonstrate the reliability of our proposed method when compared with traditional paradigms of precise reasoning. The proposed method accommodates patterns in patient and environmental data, performs well with limited data, and can be adapted for forecasting the types of patient volumes. The approach is validated using the real-world dataset. Overall, this work highlights the potential of FLe in causality problems and provides a basic framework for handling causality in uncertain domains.

Keywords: Causality, Coronary Heart Disease, Extended Fuzzy Logic, *f*-Validity, Uncertainty

1 Introduction

Modeling imprecise and uncertain information is a longstanding and critical topic in the field of information theory. In many challenging problems of computing and decision sciences, the ability to make effective decisions is crucial, particularly in decision support systems. Decision-making can be broadly classified into two areas: forward and backward reasoning. Backward reasoning, which requires less information, plays a significant role in decision support systems. If executed appropriately, backward reasoning can lead to faster and more reliable decisions. In contrast, forward reasoning is data-driven and involves inferring a goal from all available rules and data, followed by determining whether the objective is included within the information that was inferred. Consequently, the final results of forward reasoning may contain extraneous and irrelevant information. In contrast to forward reasoning, backward reasoning is goal-driven and requires only goal-relevant information, making it a more efficient approach.

The increasing uncertainty of knowledge necessitates the development of efficient new approaches, such as fuzzy set theory, to address uncertainty-related problems. Such strategies enable better handling of uncertainty by providing precise solutions despite the presence of imperfect information. Fuzziness-based uncertainty management involves organizing resources from both objective and subjective perspectives, i.e., with respect to data resources and data quality, respectively. Causality is a fundamental concept in science, essential for understanding the relationship between events. However, the analysis of causality is a complex and challenging task, primarily due to the presence of various confounding factors and the uncertainty and imprecision associated with the data. The problem of causality is further compounded by the fact that in many instances, the relationship between events is indirect and dependent on multiple factors, making it challenging to determine the degree to which each of these factors contributes to the observed effect. Moreover, the causality problem is not limited to the natural sciences but also extends to the social sciences and other fields where the complexity and ambiguity of the causal relationships are even more pronounced. In addition, causal relations present significant challenges in dealing with open-world problems that are characterized by existing hard uncertainties, which arise due to a variety of interrelated factors, including incomplete knowledge, measurement errors, and the stochastic nature of the underlying processes. Furthermore, feedback in the context of cause-effect relations can significantly hinder the propagation of causality, adding to the complexity and uncertainty of the analysis. Therefore, the analysis of causality requires a careful consideration of various factors and the use of appropriate methods and techniques to address the inherent uncertainties and complexities of the problem.

Forward reasoning, which involves reasoning from cause to effect, has been approached from various perspectives, including deterministic, probabilistic, and fuzzy solutions. In contrast, backward reasoning, which involves reasoning from effect to cause, has been relatively underexplored. The open-world assumption further complicates this problem. To the best of our knowledge, none of the few works on backward reasoning, i.e., the problem of causality, has addressed open-world problems. Fuzzy backward reasoning has been defined based on the generalized modus ponens and applied to the case of multiple-condition rules in [1]. It uses a set of possible solutions and searches for the answer within that set. Most works on backward reasoning are based on fuzzy Petri nets.

In 2008, Baiyi et al. proposed [2] a backward reasoning algorithm based on a fuzzy Petri net model for expert systems. Additionally, Jie et al.[3] used the structural and behavioral properties of fuzzy Petri nets to approach backward reasoning. They improved causality inferences by using a vector-computational approach to identify middle places. The article [4] reports on an exploratory study that investigated the influence of quadratic functions instruction on students' prior ways of reasoning about linear functions. Using paper-and-pencil assessments, the study found three types of backward transfer influences related to a shift in how students reasoned about functions. The results provide insights into backward transfer, the relationship between prior knowledge and new learning, and instructional approaches to teaching functions. The study [5] tested how risk exposure and strategic ambiguity affect bargaining behavior in a class of pie-sharing games. Results show that higher exposure to strategic ambiguity leads to distorted beliefs and less responsive behavior. Latent change score modeling is a statistical method for measuring change over time, but regressing change on the initial value of the outcome variable may lead to regression to the mean. The study [6] found that such modeling tends to indicate an effect of a predictor on the change even when no change has occurred, and recommends defining the initial value as a covariance instead of regressing change on it. The article [7] proposed a method for solving inverse fuzzy relational equations with max-min composition, which is useful for solving problems related to fuzzy abductive/backward reasoning.

For practical application, the study in [8] proposed methods to improve computational efficiency in diagnosing faults in industrial systems by representing and reasoning about uncertain causalities demonstrating their effectiveness in diagnosing faults in large-scale dynamic systems using data from a nuclear power plant simulator. In [9], an approach proposed based on Gaussian processes for predicting patient volumes by considering causal relationships instead of relying solely on correlations. The method incorporates the Gaussian processes-based convergent cross mapping framework for causal discovery and introduces a novel approach for selecting the appropriate history or look-back length of features. The paper [10] focuses on causality inference in Gene Regulatory Networks (GRNs) and evaluates the effectiveness of seven different inference methods. Time-series expression data from the DREAM challenges is used to assess the methods, and the bestperforming method, Causation Entropy, is found to be time-consuming. The second-best method, Transfer Entropy, is then applied to infer a causal network for Breast Cancer, revealing the importance of genes such as SLC39A5 in cancer progression. The study in [11] focuses on causal inference in the context of infectious diseases, aiming to understand the potential causal relationships between risk factors and diseases. By employing causal decomposition analysis, the researchers investigate the interactions between three infectious diseases and related factors to characterize the nature of disease transmission. In [12], a study investigated the effects of climate policy uncertainty on the economy based on time-varying causality analysis. The research in [13] has examined the relationship between supply chain pressures and China's key resource industries using the time-varying Granger causality method. In addition, the paper [14] introduces a hybrid deep learning model called attention-causality-based graphical gated network to accurately predict the spatio-temporal ground settlement during excavation in building-intensive areas.

The motivation behind our research stems from the limitations of existing approaches in handling imperfect information and intrinsic uncertainty, as well as the need for more flexible and approximate reasoning methods. Despite the aforementioned approaches, the motivation behind of our idea is driven by the challenges associated with analyzing causality in situations where there is imperfect or incomplete information. In various fields such as medical diagnosis, environmental analysis, and complex systems modeling, it is common to encounter scenarios where the relationships between variables are not fully understood or are subject to uncertainty. The motivation is further reinforced by the need for practical and reliable causality analysis in real-world applications.

This paper addresses the problem of causality under open-world assumptions, which includes the convex phenomena of Type-2 fuzzy sets. While fuzzy logic has been used in causality analysis before, the explicit consideration of Type-2 fuzzy sets in FLe system as a framework for handling uncertainties and ambiguities is a distinctive aspect of this work. We present new definitions of causality and propose theorems that demonstrate the validity and convergence of our approach, which leads to a new, more accurate solution. We believe that converting words into interval Type-2 fuzzy sets, along with common-sense forward reasoning, is crucial in reducing the double unknown uncertainties associated with the process of backward decision-making in causal relations. Our proposed approach is based on extended fuzzy logic and utilizes specific rules and fuzzy extended arithmetic operations. The main contribution of this paper is the development of new definitions of causality and the proposal of theorems that establish the validity and convergence of the approach. These contributions provide a foundation for addressing the problem of causality under open-world assumptions, specifically focusing on the complexities handled by Type-2 fuzzy sets and FLe philosophy. By offering a more accurate solution, we aim to enhance the understanding and modeling of causality in uncertain domains.

This paper is divided into six sections, as follows. In Section 2, we introduce FLe. Causality is discussed in Section 3, and the proposed approach is presented in Section 4. Section 5 provides a coronary heart diseases application to verify the approach. Finally, Section 6 presents the conclusions.

2 Extended Fuzzy Logic

According to Zadeh [15], Extended Fuzzy Logic (FLe) can be seen as an exploration of uncharted territory, where analysis is based on a quasi-mathematical interpretation rather than a strictly mathematical one. FLe is classified as a type of logic that aims to determine what is happening and how to influence it with less precise available data. The development of FLe involves creating a type of fuzzy logic that is not precisiated, meaning that membership functions and generalized constraints are determined simply from perception rather than measurement and are not necessarily specified beforehand. The goal is to construct a more flexible fuzzy logic that can handle imprecise and uncertain information. In this framework, the main concepts include membership functions, linguistic variables, fuzzy sets, and fuzzy rules. For a more detailed explanation, readers may refer to [16].

A *f*-Transformation

The concept of *f*-transformation can be demonstrated by considering examples of a map that is one-to-many and preserves the originality of 'one,' such that the 'many' retains the defining characteristics of the 'one.' These examples were demonstrated by Zadeh [15] using geometric shapes such as circles, lines, and triangles, which are depicted in Fig. 1. These examples illustrate that *f*-transformation can be applied to any mapping of any frame, but in order to obtain a result with a high validity index, the 'one' property must be maintained throughout the *f*-transformation. It is important to note that the concept of *f*-transformation is used in accordance with Zadeh's definition in [15].

B Validation Principle

In FLe, solutions must have sufficient validity degree. Zadeh [15] introduced a validation principle that specifies the permissible validity value that FLe's result should have. The principle states that " Let p be a p -valid conclusion drawn from a chain of premises $p_1,\ldots,p_n$. Then f - p is an f -valid conclusion drawn from f - $p_1,\ldots,p_n$, and f - p has a high validity index."

C Cointensive Assumption

The validation principle is based on the cointensive assumption, which means that unless stated otherwise, *f*-transforms are assumed to be cointensive, *i.e.*, *f*-C is close-fitting to its true prototype, C. Zadeh [15] introduced this assumption, and we refer to it as the cointensive assumption.

D P/I Principle

Despite the high flexibility provided by FLe, the computation of the *f*-transform function is a significant concern. Zadeh [15] discusses the challenges of computing h when it is considered as a function, functional, or operator, specifically when *f*- transform f_C is an argument of h, denoted as $h(f\_C)$. In this case, computing h becomes nontrivial. One possible approach to address this issue is to utilize an *f*-valid approximation, which is a straightforward method to compute h through the Precisiation/Imprecisiation principle or P/I principle, *i.e.*, $h(f\_C)\, f\_= f\_h(C)$, where $f\_=$ represents approximately equal (read as *f*-equal).

E S-answer

Within the framework of Fuzzy Logic Extension (FLe), two types of answers can be obtained: S-answer and f-answer [16]. The S-answer can be likened to the centroid of a spray pen with adjustability chosen according to validity index, while the *f*-answer can be represented by Z-mouse, a novel conceptual visual data entry and retrieval tool introduced by Zadeh. The *f*-answer is composed of multiple answers, whereas the S-answer is the centroid (precisiated) of the f-answer, which is selected as the final applicable result of the given problem. These two degrees are assumed to be unknown a priori and are structurally distinct from one another. The validity degree of the S-answer relates to the framework's validity in handling the given problem, whereas the other degree relates to the method's ability to handle incomplete information.

F A-Granule

Granulation helps us explain boundaries of perceived classes and values of perceived attributes. This type of perception-based granular probability distribution can be further computed in arrival times, and then the problem is reduced to the ranking of granular probability distributions [17]. In this work, we use causal schemes where findings are represented as reason granules, which is a processing module at the level of collected data in the causal-effect relation. The output we focus on in this work accepts some results of processing offered with the underlying representation topology of the incoming evidence when considering reason granules. In fact, the use of this process can be viewed as translating evidence to a specific reason with the help of reason granules. This translation is perception based, and by applying the IT2FS method, we also take advantage of measurement-based method.

Let us consider Q be a granule in $f \ \_C$, we now define a new concept—A-granule.

<u>Definition</u>1: A-granule Q is the smallest granule that contains *f*- transform of the problem P such that $f \ \_P \quad f \ \_=Q$, where P within Q is valid at least to a degree [18].

We assume a granule large enough to include complete information about the *f*-transform, but small enough to exclude irrelevant information about the *f*-transform, which we call an A-granule. In other words, we can attach as a tracer to the *f*-transform so that we can manipulate the given problem with scalar measures such as distance. However, when we have incomplete information, we cannot define distance measures.

3 Causality

As has been noted at least since Hume, causality is an unobservable entity. Neither qualitative descriptions of processes nor statistical correlations can determine causal mechanisms with certainty.

Forward reasoning, from cause to effect, has been studied from various perspectives, including deterministic and probabilistic, as well as through fuzzy solutions [19]. However, in comparison, backward reasoning, which involves arguing from effect to cause, has been studied by relatively few in comparison to forward reasoning when it comes to the problem of causality.

These terminologies are commonly used in analysis and are crucial for understanding the concept of causality in the context of time series data. Suppose X_t represents the stationary time series with mean zero. For better clarity and completeness, a few terminologies are reviewed/defined here.

1. Causality: A causal relationship between two variables X and Y exists if a change in X causes a change in Y.

2. Granger causality: A statistical concept that determines whether one time series is useful in forecasting another time series. If the inclusion of past values of one time series improves the forecast of another time series, then it is said to Granger-cause the other series.

3. Autoregressive (AR) model: A statistical model that predicts future values of a time series based on past values of the same series.

4. Moving average (MA) model: A statistical model that predicts future values of a time series based on past forecast errors.

5. Autoregressive moving average (ARMA) model: A combination of the AR and MA models that predicts future values of a time series based on both past values of the same series and past forecast errors.

6. Autoregressive integrated moving average (ARIMA) model: A variation of the ARMA model that can handle non-stationary time series by differencing the series to make it stationary before applying the ARMA model.

7. Vector autoregression (VAR) model: A statistical model that predicts future values of multiple time series based on past values of all these series.

Let U_t denote the series of the universe of all information at time $t-1$, $U_t - Y_t$ denote all of the information except Y_t the series, $\overline{Y_t}$ represent the set of past values $Y_{t-j}, j = 1, \ldots, \infty$, and σ^2 denote the variance parameter. These definitions are based on the assumption of a stationary series. With these notations, we have the following definitions:

Definition 2 [20]:"*Causality*. If $\sigma^2(X \mid U) < \sigma^2(X \mid (\overline{U - Y}))$ we say that *Y* is causing *X*, denoted by $Y_t \to X_t$. We say that is causing if we are better able to predict X_t using all available information than if the information apart from had been used. "

Definition 3 [20]: "*Feedback*. If $\sigma^2(X \mid \bar{U}) < \sigma^2(X \mid (\overline{U - Y}))$ and $\sigma^2(Y \mid \bar{U}) < \sigma^2(Y \mid (\overline{U - X}))$, we say that feedback is occurring, which is denoted by $Y_t \leftrightarrow X_t$, *i.e.*, feedback is said to occur when is causing *and* also is causing."

Furthermore, when reviewing uncertainty in causal relations, an important question arises regarding the type of uncertainty that we face in cause-and-effect relationships. Answering this question proves to be challenging due to the hidden nature of the causality system. It is common to monitor only certain principal factors of the problem that are known a priori. However, the debate remains as to what constitutes a cause of an event.

Understanding these factors highlights the overall uncertainty in causal relations and emphasizes the complexity of comprehending causality in real-world systems (see Fig. 2). It is imperative to recognize that different causes relating to an effect are often vague, and it is impossible to pinpoint any specific one as the sole cause. Consequently, it is unrealistic to define precise bounds, be it crisp or fuzzy. However, it is crucial to focus on rational factors that are genuinely causal while also addressing the key issues mentioned earlier. In other words, narrowing down a factor in a cause-and-effect relationship may lead to an understanding of the actual cause. Considering both sides, effects may have various known and unknown causes with different interactions, impacting both themselves and the effects. Causality appears to be a matter of degree in terms of the strength, sign, necessity, and sufficiency of factors.

4 Proposed Approach

A. Uncertainty in Causal Relation

We commence this section through the fact that the observation for *A* is a starting point to determine intervals, *i.e.*, the study of several (*n*) people opinions must be performed to estimate the degree of truth *d*, which made us take $d = \frac{m}{n}$, where *m* is the number of people who believe that A is true. This concept is known as interval-valued fuzzy techniques [21].

As a basis for consideration of two statements *A* and *B*, we need to estimate the degrees of truth $d(A), d(B)$, and $d(A \& B)$ for *A, B*, and $A \& B$. Due to the fact that, as with any possible Boolean combinations, it is not possible to ask human experts about the truth values of all, so it is obvious that $d(A \& B)$ must be estimated.

Rather than considering assigning a single value to $d(A \& B)$, when only the degrees of $d(A), d(B)$ are known, assigning the interval $[\max(d(A)+d(B)-1,0), \min(d(A),d(B))]$,seems reasonable. Because if *d (A) < d (B)* and people who believe in *A*, believe in B too, then $d(A \& B) = d(A) = \min(d(A), d(B))$. Hence it arises that, if people who believe in *A* do not to believe in B, then $d(A \& B) = \max(d(A)+d(B)-1,0)$

After the appropriate (and approximate) interval 1s found then footprint of uncertainty (FOU) can be calculated, and using the similarity factor from the codebook, the related granules can be chosen. Also, in order to determine the associated interval Type-2 fuzzy sets (IT2FS), FOU has to be first calculated due to the fact that an IT2FS is completely known by its FOU. Here the interval approach (IA) [22]is used, which is composed of the data part and the fuzzy set (FS) part. In the data part, collected data from a group of subjects are preprocessed, and data statistics are computed. In the FS part, uncertainty is measured based on a pre-specified Type-1 membership function (T1MF). Parameters of the T1MF are calculated based on information received from the first part, and then the union of T1MFs leads to a mathematical model for the FOU.

When discussing uncertainty in causal relations, an important question to consider is: "What types of uncertainty do we face in cause-and-effect relationships?" Due to the hidden nature of the causality system, it is common to monitor only certain principal effects that are known beforehand. Furthermore, the definition of a "cause" in an event is often debated. To address this phenomenon, several significant issues need to be considered. The first significant issue is that some reasons may be rational but not causal. Rational principles allow for causal reasoning by analyzing contingency information and arriving at an outcome based on the rational principle. Rational analysis enables an assessment of the fit between the effect and a particular causal relationship. However, a paradox arises as some reasons may be rational because they are causal in most similar situations but may not be causal in a particular situation. The degree of uncertainty depends on our understanding of the events. Such reasons, whether fundamental or rational, are closely linked to human reasoning and may lead to decision making through another transition. The degree of correctness of this transition determines the extent to which the predictor remains highly accurate, moderately accurate, and so on. The effectiveness of these reasons in the cause-and-effect relationship is not always known. By establishing a basis for these reasons, which are characterized by modes of behavior that are not precisely defined but cannot be ignored, we can develop a framework for dealing with these unusual situations. Although this may lead to imprecise results, using approximating reasoning with linguistic words that have been developed during a recursive process can minimize the degree of imprecision, as discussed later in the following section.

The second significant issue is that sometimes evidence contradicts rationality. This is the opposite of the first issue, where some reasons are rational but not causal. In this case, the aspects of causal relationships may not have a rational justification. Based on counter-evidence, it can be concluded that the probability of an effect differs as the number of observations increases. This suggests that these differences are less likely to be due to chance. However, research has shown that participants are sensitive to sample size when inferring causal relations, which indicates a level of ignorance [23].

The third important issue is the presence of unknown reasons and the need to measure the degree of causality of an unknown number of unknown reasons.

The fourth issue is that we often interpret the vagueness of symptoms as the degree of uncertainty in our understanding of events. Therefore, we need to consider this in our analysis. It is also worth noting that some effects, although statistically believed to be near zero, are considered here to increase the reliability of our results.

By factoring in these significant issues into the second-order uncertainty, we can increase the accuracy of our results, even in complex situations. This discussion has motivated the incorporation of a tractable character, such as stochastic property, into our analysis. In the absence of certainty, data on the subject must be collected from various sources to obtain a meaningful uncertainty model for reasoning. The changes in the effect of a cause are usually vague and not focused on any particular effect, at least not to a specific amount. Therefore, it is not practical to define crisp bounds or even Type-1 fuzzy bounds. However, assuming a Type-2 fuzzy logic system as a "perturbed Type-1 fuzzy logic system," we can think of the type-reduced set as the uncertainty in the perturbed crisp output. Measuring variation in this is done by the spread of the type-reduced set, similar to the use of intervals indicating the amount of confidence in a stochastic-uncertainty situation [24].

In many practical settings, the encoding method named interval Type-2 fuzzy seems to be the best solution to handle this uncertainty model. In fact, a similarity metric for interval Type-2 fuzzy sets (IT2 FS) can be used to form a third dimension known as uncertainty[2], making it possible to have a majority of different sets of effect categorizations while also giving us the ability to handle a wide range of open-end inputs.

It is noteworthy that, in addition to focusing on the pervious key issues that motivated us to use IT2 FS models for reasoning about casual relationships, we also need to focus on real rational reasons that are causal. If there is understanding about a reason across a relationship, it is the next that really causes it. We cannot miss linearity because of the inherent nonlinearity of the discussed problem. This means that we need to consider only overlapping granules, which called reasonable granules, like the intervals that named after similar words for different people.

The crisp case of backward reasoning can be defined as:

If a rule denoted by $A \to B$ would be true, A would be true with sufficient condition B, which is true again if $A \to B$ would be true. Solutions returned from backward reasoning are, proving A is true with considering B as its output.

In specifying characteristic of a crisp backward reasoning; we come to a concept called backtracking [1]. To define it, let consider y *is* B' as a true proved goal, one rule as:

$$\text{If } x \text{ is } A\text{, then } y \text{ is } B \tag{1}$$

and several sub goals are as x *is* $A1'?\ldots.x$ is Ap'?. In this case, however it is sufficient to prove that if one of those sub goals will be true, we can conclude that the goal y *is* B' is true. Therefore, we need to search sub goals one by one until we find a true one. However, this means we need all the possible solutions. In this case our search may be face with redundant solutions. Accessing to this set which contains all possible sub goals is impossible with our open world assumption.

We can express causality relations as:

$$XoR = Y \tag{2}$$

Where *X* is a set of possible effects, and *Y* is a set of outputs. R is defined as the relation, here causality matrix, which in that r_{ij} defined as:

r_{ij} = amount of the *jth* effect that is known to cause the *ith* symptom.

Syntactically speaking, causal knowledge base is similar to IF-THEN rules but semantically speaking they are different. The set of causal relationships among concepts can be represented as:

If x is $G \rightarrow y$ is F with causality e_{xy}, then if x is $G_i \rightarrow y$ is F_i with causality e_{xy}

If x is $G \rightarrow y$ is F with causality e_{xy}, then if x is not $G_i \rightarrow y$ is not F_i with causality e_{xy}

The simplest form of backward causality can be driven as:

$$R^{-1} \circ Y = X \tag{3}$$

One approach by which we can compute the inverse relation is reported in [25]. That paper computes the inverse relation by a heuristic function based on the inherent constraints of the problem.

B Proposed Algorithm

The applied methodology consists of the following steps:

First (Statistical): We collect related data about the problem and create a precise and rational causal diagram to track cause-and-effect relationships.

Second (Knowledge): We study patterns of causes from different human perspectives to gain insight into the granulation/classification of factors. We also study cause-and-effect relationships to estimate their stability, strength, importance, and signs.

Third (Granule Definition): We define granules for causes based on the information from the previous two steps. We also include two additional granules: one for unknown factors and another for unexpected events that can tolerate the potential non-stationary behavior of causality. These two extra parts allow us to handle the continuous behavior of causes due to slight variations in their characteristics.

Due to the complexity of causality (as is in the above settings), it is impossible to determine all causes; therefore, by introducing a granule for unknown factors, we have an open set of factors that can give results that are more reasonable. The additional granule second is for unexpected events that can tolerate the potential non-stationary behavior of causality. The addition of these two extra parts allows us to handle the continuous behavior of causes, which are due to slight variations of the causes' characteristics. The discontinuous behavior of causes changes the state of a given causality problem. Moreover, these two parts have expressed an analogy of using the concept of the spray pen. Consequently, we insert into our rational causal diagram a little irrationality due to the imprecision that exists in natural language, granular translation, and unknown-factor and unexpected-event parts.

Fourth (Uncertainty Adjustment): We use perception to analyze the lack of knowledge for the unknown-factor and unexpected-event parts and their effects on outputs. This process is accomplished by using Step 2 and other resources to reach a bound for their levels of uncertainty.

Fifth (Estimating): We estimate data fuzzistics [22], necessity measure, and possibility measure for each granule to model f-transformed parts.

Sixth (Mapping): We map/translate components of the problem into *f*-C space based on the models defined in the previous step.

Seventh (Computing and Aggregating Potential Output Granules): We compute potential outputs, which can be taken as evidence, and combine multiple pieces of evidence through aggregation.

Finally (Computing Centroid): The centroid, *s*-answer, is computed, which may be subject to pre-assigned validity.

By following these steps, we can effectively handle the complexity of causality and incorporate unknown factors and unexpected events into our analysis. The methodology also allows for the use of perception and uncertainty adjustment to account for imprecision and lack of knowledge.

C Inference Approach

To specify numbers, we define each number based on the granule's domain definition. With this definition, each number can be usually specified in terms of its assigned granule, and we can separate same numbers corresponding to different granules; for example, 15% unknown factor is not represented as 15% unimportant factor (see Fig. 3). In fact, all numbers that belong to [0,100] are mapped onto the particular granule's domain definition and modeled based on that granule model. An uncertainty bound is hence considered as $[\text{mapped number} - \delta \pm e_1, \text{mapped number} + \delta \pm e_2]$ where δ, e_1, e_2 belong to $[0,100]$ and depend on granule's domain definition. These parameters give sufficient freedom for interior trapezoidal modeling. Different values for δ , e_1, and e_2 in determining model of different granules are allowed due to the FLu part of FLe and can be handled by *f*-validity index. Moreover, the height of LMF depends on the granule model. By this conversion, we can also take the importance of various factors into account.

For inference part, to avoid the major computing difficulty of the inversion part, we solve the problem with modus ponens and attain potential outputs that considered as evidence in the rest of the procedure. Then, by aggregation them, we obtain final output to a reasonable degree of validity. The output appears to be more reliable since we have tried to capture more uncertainty by validity metric.

Our inference approach is similar to Perceptual Reasoning [26, 27]. Because the achieved output in the approach is in the form of IT2 FS, the aggregated output can be obtained simply using Type 1 Fuzzy Set aggregation technique. The method is performed by aggregating the lower membership functions (LMFs) and the upper membership functions (UMFs) of all potential outputs, respectively, that constitute FOU. The uncertainty of IT2 FS is determined by FOU's size. The following theorem addresses this issue.

Theorem 1. (Aggregation): Let $\tilde{P}_1,\tilde{P}_2,\ldots,\tilde{P}_m$ be IT2 FS. Then the aggregation operator denoted by $A(\tilde{P})=\text{Aggregate}(\tilde{P}_1,\tilde{P}_2,\ldots,\tilde{P}_m)$ is defined as $A(\tilde{P})=\dfrac{1}{[A(\underline{P}),A(\bar{P})]}$ where $A(\underline{P})$ is the aggregation of LMFs of all $\tilde{P}_i$ and $A(\bar{P})$ the aggregation of UMFs of all $\tilde{P}_i$, $i=1,\ldots,m$.

Proof: First, it is obvious that the aggregation of IT2 FSs leads to another IT2 FS, and an IT2 FS can be determined by its FOU. According to wavy slice representation [20], the union of the embedded TI FSs gives the FOU. Thus, for computing FOU of aggregated IT2 FS, we can use wavy slice. FOU is equal to $[\text{LMF},\text{UMF}]$. By defining $A(P_I^e)=\text{Aggregate}(P_{I_1},P_{I_2},\ldots,P_{I_m})$ where P_I^e is the embedded TI FS of aggregated IT2 FS $\tilde{P}$, and $P_{I_1},P_{I_2},\ldots,P_{I_m}$ are the embedded TI FSs of $\tilde{P}_1,\tilde{P}_2,\ldots,\tilde{P}_m$ with considering $I=1,\ldots,\prod_{i=1}^{N}M_i$ where N and M_i are the number of primary variables and primary membership functions, respectively, the aggregated membership function in wavy slice analysis would be as $A(\tilde{P})=\bigcup_{j=1}^{I}A(P_j^e)=[\text{A}(\underline{P}),\text{A}(\bar{P})]$. We know $A(\underline{P})=\text{Aggregate}(\text{LMF}_1,\text{LMF}_2,\ldots,\text{LMF}_m)$ and $A(\bar{P})=\text{Aggregate}(\text{UMF}_1,\text{UMF}_2,\ldots,\text{UMF}_m)$ that leads to $\text{FOU}(A(\tilde{P}))=[A(\underline{P}),A(\bar{P})]$. As earlier mentioned, this special form is as due to embedded T1 FSs representation $A(\tilde{P})=\dfrac{1}{\text{FOU}(A(\tilde{P}))}$; hence, $A(\tilde{P})=\dfrac{1}{[A(\underline{P}),A(\bar{P})]}$. Q.E.D.

To perform aggregation[28], we write down Type 1 fuzzy sets in terms of intervals of confidence and sum them up. Now, we have the overall intervals of confidence. Then we compare it with the standard formula of representing a fuzzy number based on intervals of confidence, the corresponding aggregation can hence be found.

An interesting question arises regarding the degree to which the input causes the output. To evaluate this degree, we rely on the complementarity of probability2 and FLe [16, 29]. In this study, we utilize the possibility-probability distribution (PPD) [30] as a tool to account for this complementarity.

We believe that in order to cope for handling of uncertain piece of knowledge, computing the degree of effective effects in terms of a possibility distribution over [0, 1], transforms to calculate the α-cut of its membership function. We use PPD in the following form. Here, we have LMF and UMF as shown in Fig. 4.

We have $\alpha_{\min}$-cut and $\alpha_{\max}$-cut respect to LMF and UMF membership functions. Thus,

$$\pi_{i_{\min}}=1-\int p\,dx\ ,\ \ \pi_{i_{\max}}=1-\int p\,dx \tag{4}$$

Where p is posterior distribution and π_i is associated possibility distribution. Now, consider Figure 5. The α-plane of the possibility distribution can be equal to the shaded domain of probability distribution. So, we have probability in terms of interval possibility distribution.

$$\tilde{p}(x\in A)=[\pi_L,\pi_U] \tag{5}$$

Through the use of FLe, it is possible to assume the distribution of the aforementioned interval as a uniform distribution. In this way, the expected value of the above interval can be considered as the f-transform of the real effect with a standard deviation. The expected value and standard deviation are given as follows:

$$\bar{\pi} = \frac{\pi_L + \pi_U}{2} \tag{6}$$

$$\bar{\sigma} = \frac{-\pi_L + \pi_U}{\sqrt{12}}$$

Thus, the *f*-degree of effect is $\bar{\pi}$ with s.t.d $\bar{\sigma}$. Now, we have *f*-transformation as:

$$\text{Degree of causality} \rightarrow \bar{\pi} \text{ with } \bar{\sigma} \tag{7}$$

It is important to note that in order to minimize the imprecision of the final result, the use of fuzzy Type 2 and the precise determination of FOU are critical. By utilizing fuzzy Type 2 and carefully selecting FOU, we can enhance the accuracy and reliability of the analysis, leading to more robust and valid conclusions.

D Proving Accuracy

In analysis of the validation of the method, it must be mentioned that there is no guarantee, even in the presence of IT2FS that there is an actual casual-effect relation.

However, IT2FS should be used because of its technique to handle such uncertainty. To discuss its convergence rate, we explore a theorem to prove stationary state of a result representing by fuzzy number of Type 2.

Theorem 2. (Accuracy) Consider the causal-effect relation with using interval Type 2 fuzzy to estimate the posterior distribution of effect. Then, the result converges, that is, $|E(\pi_i(\alpha_j)) - E(\pi_i(\alpha_{j-1}))| < \varepsilon$.

Proof: without loss of generality, we can assume that events have composed of normal probability events. Consider p_i as occurred event, q_i *denotes* p_i's degree , p_j as other events, *e* as evidence and H_i the final result. Thus, we can write:

$$\text{prob}(p_i \rightarrow H_i) = \text{prob}(p_i \mid p_j, e) \tag{8}$$

$$\text{prob}(p_i \rightarrow H_i) = E(\pi_i)$$

According to this fact that Karnik-Mendel Algorithm converges monotonically [31] .*i.e.*

$$(c_l(\alpha_1) - c_l) \le c_0 - c_l \tag{9}$$

Where cl is the centroid of the IT2FS. We have:

$$|c_l(\alpha_j) - c_l(\alpha_{j-1})| \le \varepsilon \tag{10}$$

Furthermore, we can represent and calculate c_l using the α-plane, which also converges [18]:

$$|\alpha_{\text{Plane t}}\text{represen}(\alpha_j) - \alpha_{\text{Plane}}\text{ represent}(\alpha_{j-1})| \le \varepsilon \tag{11}$$

Thus,

$$|\operatorname{proj}(\alpha_{\text{Plane t}}\text{represen}(\alpha_j)) - \operatorname{proj}(\alpha_{\text{Plane t}}\text{represen}(\alpha_{j-1}))| \leq \epsilon \tag{12}$$

Since α-plane is two dimensional of α-cut, we have:

$$\operatorname{proj}(\alpha_{\text{Plane}}(\alpha_j)) < \alpha_{\text{cut-max}} \Rightarrow k \cdot \operatorname{proj}(\alpha_{\text{Plane}}) = \alpha_{\text{cut-max}} \tag{13}$$

We can conclude:

$$E(\pi_i) < \alpha_{\text{cut-max}} \Rightarrow k' \cdot E(\pi_i) = \alpha_{\text{cut-max}} \tag{14}$$

Thus, with substituting (15) in (13) for both $c_l(\alpha_j)$ and $c_l(\alpha_{j-1})$,we obtain:

$$\left|\left(\frac{\alpha_{\text{cut-min}}(\alpha_j)}{k} - \frac{\alpha_{\text{cut-max}}(\alpha_{j-1})}{k'}\right)\right| < \epsilon \tag{15}$$

Subsequently,

$$\left|\left(\frac{k' E(\pi_i)_j}{k} - \frac{k' E(\pi_i)_{j-1}}{k}\right)\right| < \epsilon \tag{16}$$

Thus,

$$|E(\pi_i)_j - E(\pi_i)_{j-1}| < \frac{k\epsilon}{k'} \tag{17}$$

By defining,

$$\frac{k\epsilon}{k'} = \epsilon_1 \tag{18}$$

We see since $\epsilon \to 0$, we can conclude that $\epsilon_1 \to 0$ and this means that the algorithm converges. Q.E.D.

Note that, if we denote $p_{(\mu,\upsilon)}$ as a cause with degree μ and validity υ, q as other causes, and H_μ as the corresponding effect with degree μ, then by considering the concepts of validity and degree of causality, we can infer that: $\text{probability}(p_{(\mu,\upsilon)} \to H_\mu) = \text{probability}(p_{(\mu,\upsilon)} \cap q \to H_\mu)$. In fact, $p_{(\mu,\upsilon)}$ and q are independent from each other while p and q are not in the classic sense.

5 Illustrative Example and Simulation: Coronary Heart Disease

Coronary Heart Disease (CHD) refers to a medical condition where the inner walls of the coronary arteries that are responsible for delivering oxygen-rich blood to the heart muscle develop a buildup of plaque, which is a waxy substance [32]. The number of people affected by CHD is on the rise, and there is mounting evidence to support this claim. To aid in the prediction and diagnosis of this condition, various computational intelligence algorithms have been utilized. These include neural networks [33, 34], machine learning [35], fuzzy systems [36-39], decision support system [40, 41], Dempster-Shafer evidence theory [42], evolutionary algorithms [43, 44]. Although the papers mentioned have made valuable contributions to the assessment of CHD, determining all the influential factors and accurately measuring their impact can be challenging due to incomplete or inaccurate data.

This study draws upon 152 documents from Iranian hospitals, which encompass a variety of causes related to coronary heart disease. To construct our granule classes, we partition our reasons for the increase in disease into several parts, each an *f*-set representing one of the possible factors described earlier, based on the methodology. These parts, based on a various case report, we found that many factors influence coronary heart disease. Following the initial list of factors, they were adjusted based on the interviews with experts. Then, with a systematic analysis, we advance our understanding of the risk factors and studied relevant hypotheses. If many of the cases agreed with the hypothesis, then we conclude of higher possibility of the factor's risk., These Factors are in Table 1.

We will now consider an example as a test bed to verify our approach. To provide an example, consider the attributes of the user is as: Female, Diastolic blood pressure =$78\ mmHg$, Systolic blood pressure=$110\ mmHg$, Blood cholesterol= $35\ kg/m^2$, Diabetes mellitus =89, No Smoking, less than 5 minutes of physical activity, high stress, age$=60$, sometimes unhealthy diet, Triglycerides$=150\ mg/dL$ family history= Second-degree relatives; this risk factor has increased the uncertainty of the final results.

The above example demonstrates a causality problem where the main objective is to determine the cause of the disease and to quantify the extent to which the increase in this factor contributed to the disease. However, the diseases are not straightforward, as we cannot attribute a specific cause, and we are not dealing with a closed world problem. Furthermore, due to imperfect information, no theory of causality can provide a precise answer. To analyze causality, sufficient information is required, and without such information, the conclusions reached may not be provably valid. Zadeh [15] highlights that FLe is employed when a provably valid (p-valid) solution is either not feasible, has a high implementation cost, or when a precise solution is not necessary. In such cases, an *f*-valid solution may suffice, which indicates the degree of result validity in a fuzzy way. Given the lack of a p-valid solution for this problem, we aim to obtain an *f*-valid solution.

Using "FH" as an expression of "family history," in the example, we now face the following knowledge: Input-FH is set at 30% then Output becomes 20%. The questions now are these: whether Input-FH is a cause? And to which degree is Input-FH effective if it is a cause? The first part of above statement can be rewritten as:

If the more Input- FH is 30%, Then the more Output is 20% can be Valid.

Or more concretely,

*The more (If the more Input- F*H *is 30%, Then the more Output is 20%) is True, The higher is the Validity of Input- FH being the cause.*

Based on generalist accounts, there is not a simple correlation between input and output; however, we observe the inner layer, alternatively, that can be written in a familiar form as:

IF *Input- FH is 30% and Other known/unknown factors are unknown%* THEN *Output is 20%*

It is obvious that the information is missing here regarding connections between other known/unknown factors and the output. There is not any measurement-based information about them, but there may be perception-based information that is granular. In fact, the only information we have is about the relation between input and output is our granules and apparently correlated facts:

Input- FH is thirty percent, and output is twenty percent.

Potential outputs and the *f*-set of output after aggregation are shown in Fig. 6 and Fig. 7, respectively. In these figures, x-axis represents the percentage of output and y-axis is its primary degree. As indicated in Fig. 7, with respect to the output of twenty percent there is a non-zero degree in every set, so the thirty percent family history seems to be the cause of twenty-percent output but to a degree. As shown in Fig. 7, the degree of increasing in degree of family history is an interval [0, 0.575]. As is often the case with causality problems, the obtained results cannot be proven. However, we can provide indications that support the cause-and-effect relation and evaluate the performance of the method. To accomplish this, we calculate the output in the absence of family history (input), and compare it with the output of thirty-percent due to family history. We find the output in this case results in a sixteen-percent increase. Therefore, it can be conjectured that increasing family history degree can be an effective cause for the twenty- percent increases in it along with other factors, to a certain degree.

It is worth noting that since the range in Fig. 7 pertains to the primary membership degree with the secondary membership degree equal to one, we can think of the latter as the probability of the former. This presupposition is not unreasonable since we propagate uncertainty, and we view this range as a more probable degree. By utilizing t-norm, we can get the information of both data, i.e., the above range (Fig. 7) and the range obtained by PPD. With assuming uniform distribution in calculating PPD, we apply operator min (the largest t-norm) to acquire the maximal information of both ranges that we have for two cases of thirty and zero percent FH. As a result, we achieve Fig. 8 and Fig. 9 with respect to each case. The degree of causality of thirty-percent FH is found to be at 0.7841. From these figures, we observe that even though we may reach a non-zero degree of twenty percent in the case of zero family history, it is obtained with a smaller possibility as well as a smaller probability as shown in Fig. 10. Furthermore, both of these figures show that in the case of several unknown and unexpected factors, the start of the support in the case of thirty-percent family history is less than that of the zero case. These results indicate dependence linking of the increase in disease and the thirty-percent increase in family history; therefore, increase in family history by thirty percent ends up to being a cause to a degree.

The results of examining these two cases are detailed in Table 2. In that, the degree of causality, probability related to it, and validity index of obtained result for default values $m = 2$ and $\tau = 0.5$ are provided. The table indicates that the validity index for 0.7841 degree for the thirty-percent family history is 0.8702. To examine the reliability and robustness of decisions made using the approach, we apply it to 52 cases of testing set. The twenty cases of results are included in Table 3, where the percentages express the rate of being affected and not being affected as actual results. For comparison with existing approaches, we apply the same database to neural network, FLe system [29], Type 2 and Type 1 fuzzy system, and alternative approaches of [9, 29, 37] in Table 3. These approaches are selected due to using data set, fuzzy rules, *f*-rules, and causality to provide a fair comparison. The neural system has three layers including input layer and twenty neurons in the hidden layer. It has sixteen inputs representing the sixteen factors. Additionally, the neural system has two outputs: "Affected" and "Not Affected," indicating that it is designed to predict whether the factor is affected or not. The backpropagation learning algorithm is mentioned as the chosen method for training the neural system. For fair comparison, training and testing sets are composed 100 and 52 cases, respectively and the system has been run ten times. The fuzzy system employed in this study utilizes the Mamdani inference system and multiplication for t-norm. As observed, the results obtained from the proposed approach are superior in comparison. This is due to the incorporation of *f*-rules and the consideration of causality nature based on the validity degree, which enhances the accuracy and effectiveness of the system.

6 Conclusions

In this study, we have analyzed the properties of extended fuzzy logic and discussed its rationality, as well as investigated an example of causality based on it. Our analysis has shed light on the execution of problem solutions from the perspective of the philosophy of extended fuzzy logic and the role of *f*-validity in such solutions. By utilizing the extended fuzzy logic approach, we were able to derive new results that are particularly useful for systems with large states and/or unknown dimensions. The PPD was used as a tool to evaluate the degree of causality, which provided a more comprehensive understanding of the problem. The results of our study fall within the class of *f*-answer. FLe has allowed the derivation of numerous new results that are beneficial for systems with vast states or unknown dimensions. Integrating both empirical and logical facets of a problem can enhance the capture of uncertainty, which is done on family history factor for decision-making in practical scenarios. The last note that is implied throughout in this paper is the assumption that we do not loose certain general properties of the given problem during the *f*-transformation; specifically, the mapped system as well as its prototype is convex. In our future work, we aim to address the analysis of concave systems. Additionally, we plan to investigate real-time causality analysis methods that have the ability to adapt and update causality models dynamically in response to changing data or system conditions. This research direction holds great promise and will contribute to advancing our understanding of causality analysis in dynamic systems.

Conflict of interest statement

The author declares that there is no conflict of interest.

REFERENCES

[1] Arnould, T. and Tano, S. "Interval-valued fuzzy backward reasoning," *IEEE Transactions on Fuzzy Systems*, **3**(4), pp. 425-437, (1995), DOI: 10.1109/91.481951.

[2] Baiyi, Z. and Shangsen, C. "A Parallel Backward Reasoning Study Using Fuzzy Petri Net," in *2008 International Conference on Computer Science and Software Engineering*, vol. 3, pp. 315-319, (2008), DOI: 10.1109/CSSE.2008.825.

[3] Jie, Y., Hai-bo, S., Chang, L., and Wen-li, S. "Backward concurrent reasoning based on fuzzy Petri nets," in *2008 IEEE International Conference on Fuzzy Systems (IEEE World Congress on Computational Intelligence)*, pp. 832-837, (2008), DOI: 10.1109/FUZZY.2008.4630467.

[4] Hohensee, C., Willoughby, L., and Gartland, S. "Backward transfer, the relationship between new learning and prior ways of reasoning, and action versus process views of linear functions," *Mathematical Thinking and Learning*, (2022), DOI: 10.1080/10986065.2022.2037043.

[5] Conte, A., Güth, W., and Pezanis-Christou, P. "Strategic ambiguity and risk in alternating pie-sharing experiments," *Journal of Risk and Uncertainty*, **66**(3), pp. 233-260, (2023), DOI: 10.1007/s11166-022-09401-z.

[6] Sorjonen, K., Ingre, M., Nilsonne, G., and Melin, B. "Dangers of including outcome at baseline as a covariate in latent change score models: Results from simulations and empirical re-analyses," *Heliyon*, **9**(5), (2023), Art no. e15746, DOI: 10.1016/j.heliyon.2023.e15746.

[7] Wen, C. F., Wu, Y. K., and Li, Z. "Algebraic formulae for solving systems of max-min inverse fuzzy relational equations," *Information Sciences*, **622**(1), pp. 1162-1183, (2023), DOI: 10.1016/j.ins.2022.11.123.

[8] Dong, C. and Zhou, J., "A New Algorithm of Cubic Dynamic Uncertain Causality Graph for Speeding Up Temporal Causality Inference in Fault Diagnosis," *IEEE Transactions on Reliability,* **72**(2), pp. 662-677, (2023), DOI: 10.1109/TR.2022.3170063.

[9] Feng, G., Yu, K., Wang, Y., Yuan, Y., and Djurić, P. M. "Exploiting Causality for Improved Prediction of Patient Volumes by Gaussian Processes," *IEEE Journal of Biomedical and Health Informatics,* **25**(7), pp. 2487-2496, (2021), DOI: 10.1109/JBHI.2021.3089459.

[10] Ahmed, S. Roy, S. , S., and Kalita, J. "Assessing the Effectiveness of Causality Inference Methods for Gene Regulatory Networks," *IEEE/ACM Transactions on Computational Biology and Bioinformatics,* **17**(1), pp. 56-70, (2020), DOI: 10.1109/TCBB.2018.2853728.

[11] Sun, J., Yuan, K., Chen, C., Xu, H., Wang, H., Zhi, Y., Peng, S., Peng, C.K., Huang, N., Huang, G., Yang, A. "Causality Network of Infectious Disease Revealed With Causal Decomposition," *IEEE Journal of Biomedical and Health Informatics,* **27**(7), pp. 3657-3665, (2023), DOI: 10.1109/JBHI.2023.3268081.

[12] Uğurlu-Yıldırım, E. and Dinç-Cavlak, Ö. "Climate policy uncertainty, media coverage of climate change, and energy markets: New evidence from time-varying causality analysis," *Energy and Climate Change,* **5** (1)**,** p. 100134, 2024/12/01/ (2024), DOI:10.1016/j.egycc.2024.100134.

[13] Ren, X., Fu, C., Jin, C., and Li, Y. "Dynamic causality between global supply chain pressures and China's resource industries: A time-varying Granger analysis," *International Review of Financial Analysis,* **95** (1)**,** p. 103377, 2024/10/01/ (2024), DOI: 10.1016/j.irfa.2024.103377.

[14] Zhou, X., Pan, Y., Qin, J., J., Chen, J., and Gardoni, P. "Spatio-temporal prediction of deep excavation-induced ground settlement: A hybrid graphical network approach considering causality," *Tunnelling and Underground Space Technology,* **146** (1), pp. 105605, (2024), DOI: 10.1016/j.tust.2024.105605.

[15] Zadeh,L. A. "Toward extended fuzzy logic—a first step," *Fuzzy Sets and Systems,* **160**(21), pp. 3175-3181, 2009/11/01/ (2009), DOI: 10.1016/j.fss.2009.04.009.

[16] Sabahi, F. and Akbarzadeh-T, M. R. "A qualified description of extended fuzzy logic," *Information Sciences,* **244,** pp. 60-74, 2013/09/20/ (2013), DOI: 10.1016/j.ins.2013.03.020.

[17] Decò, A. and Frangopol, D. M. "Risk assessment of highway bridges under multiple hazards," *Journal of Risk Research,* **14**(9), pp. 1057-1089, (2011), DOI: 10.1080/13669877.2011.571789.

[18] Sabahi, F. and Akbarzadeh-T, M. R. "A framework for analysis of extended fuzzy logic," *Journal of Zhejiang University SCIENCE C,* **15**(7), pp. 584-591, (2014), DOI: 10.1631/jzus.C1300217.

[19] Cheng, J., Nara, S., and Goto, Y. "FreeEnCal: A Forward Reasoning Engine with General-Purpose," in *Knowledge-Based Intelligent Information and Engineering Systems*, Berlin, Heidelberg, B. Apolloni, R. J. Howlett, and L. Jain, Eds.: Springer Berlin Heidelberg, pp. 444-452, (2007), DOI : 10.1007/978-3-540-74827-4_56.

[20] Granger, C. W. J. "Investigating Causal Relations by Econometric Models and Cross-spectral Methods," *Econometrica, JSTOR,* **37**(3), pp. 424-438, (1969), DOI : 10.2307/1912791.

[21] Yew, K. M. and Kohout, L. J. "Interval-based reasoning in medical diagnosis," in *Proceedings Intelligent Information Systems. IIS'97*, pp. 32-36, (1997), DOI: 10.1109/IIS.1997.645174.

[22] Liu, F. and Mendel, J. M. "Encoding Words Into Interval Type-2 Fuzzy Sets Using an Interval Approach," *IEEE Transactions on Fuzzy Systems,* **16**(6), pp. 1503-1521, (2008), DOI:10.1109/TFUZZ.2008.2005002.

[23] Sloman, S. and Fernbach, P. M. "The value of rational analysis: an assessment of causal reasoning and learning," pp. 481- 496, (2007), DOI: 10.1093/acprof:oso/9780199216093.003.0021.

[24] Li, Y. and Liu, W., "A method of backward probabilistic logic reasoning of decision problems," in *2008 27th Chinese Control Conference*, pp. 657-662,(2008), DOI: 10.1109/CHICC.2008.4605124.

[25] Chakraborty, S., Konar, A. and Jain, L. C. "An Efficient Algorithm to Computing Max–Min Inverse Fuzzy Relation for Abductive Reasoning," *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans,* **40**(1), pp. 158-169, (2010), DOI: 10.1109/TSMCA.2009.2030784.

[26] Wu, D. and Mendel, J. M. "Corrections to “Aggregation Using the Linguistic Weighted Average and Interval Type-2 Fuzzy Sets”," *IEEE Transactions on Fuzzy Systems,* **16**(6), pp. 1664-1666, (2008), DOI: 10.1109/TFUZZ.2008.2005941.

[27] *Matlab Codes, Copyright by the University of Southern California*. (2009). sipi.usc.edu/~mendel/software/. [Online]. Available:

[28] Kumar, A. and Karmakar, G. "Aggregation of opinions using fuzzy numbers," *International Journal of Systems Science,* **32**(12), pp. 1399-1411, (2001), DOI: 10.1080/00207720110057323.

[29] Sabahi, F. and Akbarzadeh-T, M. R. "Extended Fuzzy Logic: Sets and Systems," *IEEE Transactions on Fuzzy Systems,* **24**(3), pp. 530-543, (2016), DOI: 10.1109/TFUZZ.2015.2453994.

[30] Karimi, I. and Hüllermeier, E. "Risk assessment system of natural hazards: A new approach based on fuzzy probability," *Fuzzy Sets and Systems,* **158**(9), pp. 987-999, 2007/05/01/ (2007), DOI: 10.1016/j.fss.2006.12.013.

[31] Wu, D. and Mendel, J. M. "Enhanced karnik--mendel algorithms," *IEEE Transactions on Fuzzy Systems,* **17**(4), pp. 923-934, (2009), DOI: 10.1109/TFUZZ.2008.924329.

[32] Vosbergen, S., Mulder-Wiggers, J. M., Lacroix J. P., Kemps, H. M., Kraaijenhagen, R. A., Jaspers, M. W., Peek, N. "Using personas to tailor educational messages to the preferences of coronary heart disease patients," *Journal of Biomedical Informatics,* **53** (1), pp. 100-112, (2015), DOI: 10.1016/j.jbi.2014.09.004.

[33] Rao, B. S., Rao, K. N., and Setty, S. P. "An approach for heart disease detection by enhancing training phase of neural network using hybrid algorithm," in *2014 IEEE International Advance Computing Conference (IACC)*, pp. 1211-1220,(2014), DOI: 10.1109/IAdCC.2014.6779500.

[34] Su, Y. S., Ding, T. J., and Chen, M. Y. "Deep learning methods in internet of medical things for valvular heart disease screening system," *IEEE Internet of Things Journal,* **8**(23), pp. 16921-16932, (2021), DOI: 10.1109/JIOT.2021.3053420.

[35] Qadri, A. M., Raza, A., Munir, K., and Almutairi, M. S. "Effective feature engineering technique for heart disease prediction with machine learning," *IEEE Access,* **11**(1)**,** pp. 56214-56224, (2023), DOI: 10.1109/ACCESS.2023.3281484.

[36] Kim, J.-K., Lee, J., and Lee, Y. "Data-mining-based coronary heart disease risk prediction model using fuzzy logic and decision tree," *Healthcare informatics research,* **21**(1), pp. 167-74, 07/01 (2015), DOI: 10.4258/hir.2015.21.3.167.

[37] Lahsasna, A., Ainon, R. N., Zainuddin, R., and Bulgiba, A. "Design of a fuzzy-based decision support system for coronary heart disease diagnosis," *Journal of Medical Systems,* **36**(5), pp. 3293-3306, (2012), DOI: 10.1007/s10916-012-9821-7.

[38] Yuan, X., Chen, J., Zhang, K., Wu, Y., and Yang, T. "A stable ai-based binary and multiple class heart disease prediction model for iomt," *IEEE Transactions on Industrial Informatics,* **18**(3), pp. 2032-2040, (2022), DOI: 10.1109/TII.2021.3098306.

[39] Fuente-Cortes, G. D. L., Diaz-Mendez, A., Flores-Verdad, G. E., and Gonzalez-Diaz,V. R. "On-chip fuzzy logic synthesis of a new ischemic and non-ischemic heartbeat classifier," *IEEE Transactions on Circuits and Systems II: Express Briefs,* **68**(1), pp. 476-480, (2021), DOI: 10.1109/TCSII.2020.3007464.

[40] Fitriyani, N. L., Syafrudin, M., Alfian, G., and Rhee, J. "HDPM: an effective heart disease prediction model for a clinical decision support system," *IEEE Access,* **8**(1)**,** pp. 133034-133050, (2020), DOI: 10.1109/ACCESS.2020.3010511.

[41] Najafi, A., Nemati, A., Ashrafzadeh, M., and Hashemkhani Zolfani, S. "Multiple-criteria decision making, feature selection, and deep learning: a golden triangle for heart disease identification," *Engineering Applications of Artificial Intelligence,* **125**p. 106662, (2023), DOI: 10.1016/j.engappai.2023.106662.

[42] Khatibi, V. and Montazer, G. A. "A fuzzy-evidential hybrid inference engine for coronary heart disease risk assessment," *Expert Systems with Applications,* **37**(12), pp. 8536-8542, (2010), DOI: dx.doi.org/10.1016/j.eswa.2010.05.022.

[43] Muthukaruppan, S. and Er, M. J. "A hybrid particle swarm optimization based fuzzy expert system for the diagnosis of coronary artery disease," *Expert Systems with Applications,* **39**(14), pp. 11657-11665, (2012), DOI: 10.1016/j.eswa.2012.04.036.

[44] Turabieh, H. "A hybrid ANN-GWO algorithm for prediction of heart disease " *American Journal of Operations Research,* **6**(1)**,** pp. 136-146, (2016), DOI: 10.4236/ajor.2016.62016

[45] Sabahi, F. "Bimodal fuzzy analytic hierarchy process (BFAHP) for coronary heart disease risk assessment," (in eng), *J Biomed Inform,* **83**(1)**,** pp. 204-216, 2018/07// (2018), DOI: 10.1016/j.jbi.2018.03.016.

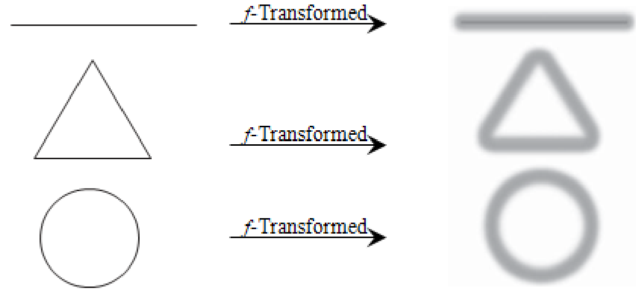

Figure 1. Examples of *f*- transformation.

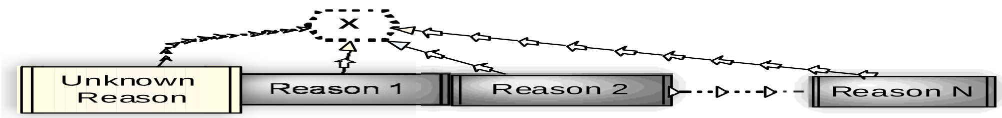

Figure 2. Reasons

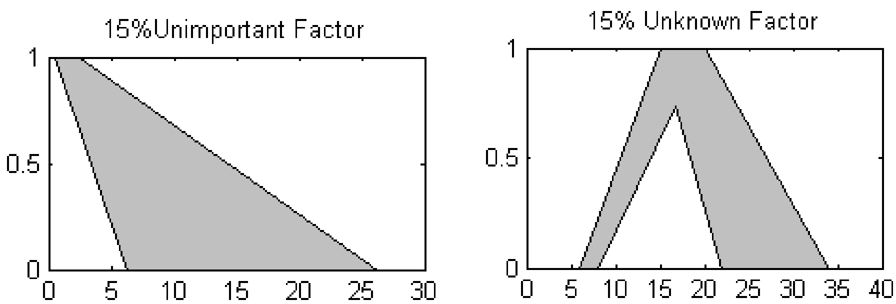

Figure 3. Same numbers belong to different sets. Since an uncertainty class may have more effects, its specific membership function would efficiently reflect its effects. Therefore, depending on the uncertainty class, the effect of the fifteen percent is different.

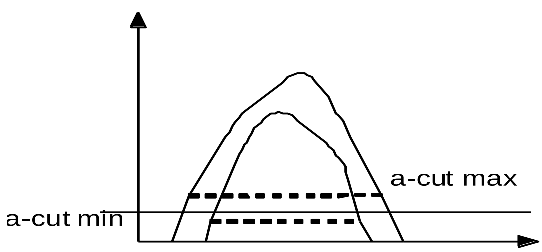

Figure 4. Alpha-cut

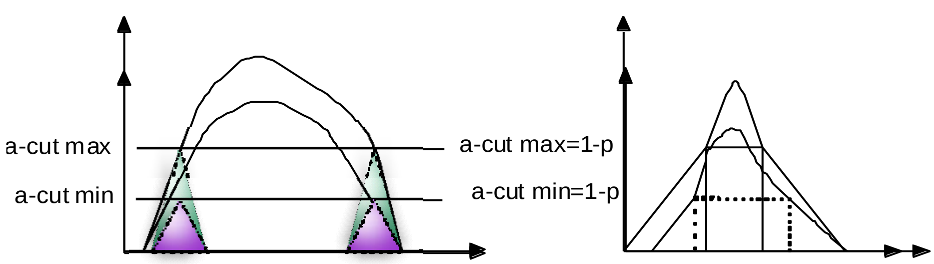

Figure 5. Possibility Distribution

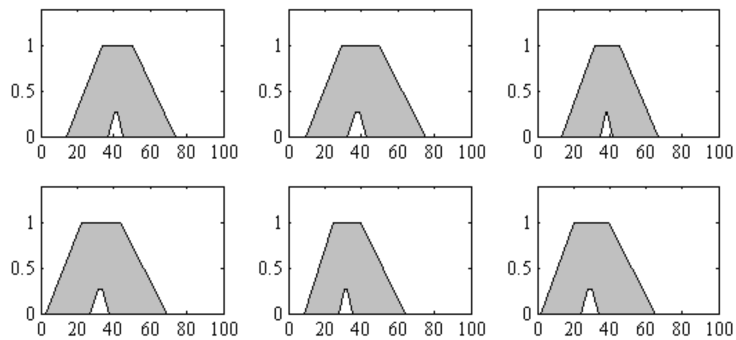

Figure 6. Potential outputs. They are treated as evidence.

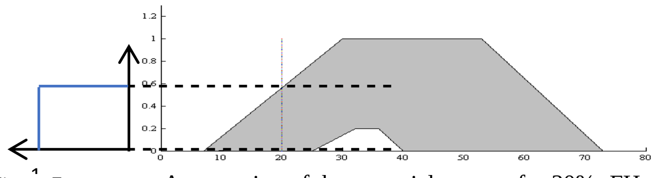

Figure 7. Aggregation of the potential outputs for 30% -FH.

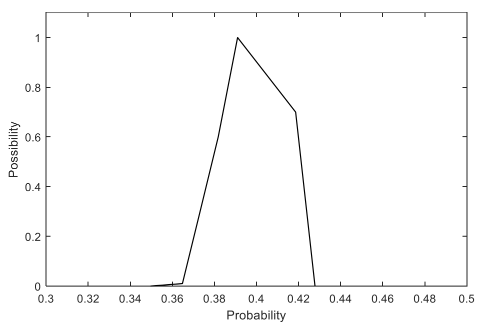

Figure 8. Degree of causality at 0% -FH.

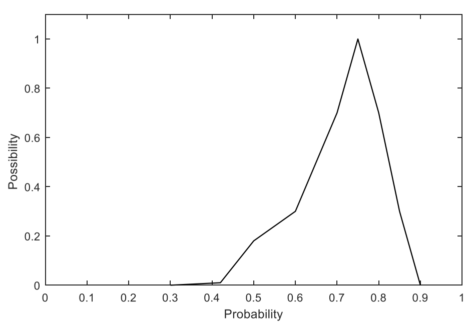

Figure 9. Degree of causality at 30% -FH

Table 1: Factor of Coronary Heart Disease [45]

Factor	**High**	**Medium**	**Low**
Sex	Male	Female middle-aged or higher	Young Female
Diastolic blood pressure (mmHg)	> 85	$80 - 84$	$80 >$
Systolic blood pressure (mmHg)	$131 <$	$121 - 130$	$120 >$
Family history of early heart disease	First-degree relatives	Second-degree relatives	No relatives
Blood cholesterol mg/dL	200>	201-239	240<
Diabetes mellitus	$126 <$	100-125	$70 - 99$
Overweight and obesity	$30\ kg/m^2 <$ BMI	$20\ kg/m^2 <$ BMI $< 29\ kg/m^2$	BMI $< 20\ kg/m^2$
Smoking	Everyday	Three in a Week	Never
Physical Activity	Sometimes and $<$ $10\ min$	$20\ min\ - 30\ min$ low-intensity	Regular and 30 min30 min <rate-intensity
low-density lipoprotein cholesterol LDL-C (mg/dL)	$161 <$	110-160	$100 >$
High-density lipoprotein cholesterol HDL-C (mg/dL)	Female: < 50 Male: < 40	Female: $50 - 60$ Male: $40 - 50$	Female: > 60 Male: > 50
Mental stress and depression	High	Low	No
Age	Female: $55 <$ Male: $50 <$	Female: $45 - 55$ Male: $45 - 50$	Female: $45 >$ Male: $45 >$
Unhealthy Diet	Always	Sometimes	Seldom
Race	-	Asian, African	East European, European
Triglycerides (mg/dL)	$200 <$	$151 - 199$	< 150

Table 2: Examination of Family History 30% and 100%.

Input-Ad	**30%**	**0%**
Degree	0.7841	0.4108
Probability related to the degree	0.9500	0.9606
Validity (for m=2, τ=0.5)	0.8702	0.8031

Table 3: Applying alternative approaches to the same database and comparing of accuracy rate and diagnosis of the approaches: I: Incorrect, C: Correct, U: Unknown.

No	Actual Results	Neural Network		FLe System [29]		Type 1 Fuzzy [37]		Interval Type 2 Fuzzy		Gaussian processes [9]		**Proposed Approach**	
1	Affected	0%	**I**	84.28%	**C**	71.00%	**C**	75.83%	**C**	81.14%	**C**	87.44%	**C**
2	Not Affected	100%	**C**	76.57%	**C**	73.02%	**C**	78.89%	**C**	77.82%	**C**	85.61%	**C**
3	Affected	100%	**C**	77.33%	**C**	74.10%	**C**	76.99%	**C**	81.49%	**C**	87.71%	**C**
4	Not Affected	0%	**I**	79.14%	**C**	72.38%	**C**	80.02%	**C**	82.41%	**C**	89.21%	**C**
5	Affected	0%	**I**	58.91%	**C**	52.45%	**U**	61.22%	**C**	54.24%	**C**	78.91%	**C**
6	Affected	100%	**C**	81.72%	**C**	79.21%	**C**	83.60%	**C**	83.29%	**C**	84.39%	**C**
7	Affected	0%	**I**	79.01%	**C**	73.21%	**C**	80.11%	**C**	80.21%	**C**	87.71%	**C**
8	Not Affected	0%	**I**	71.61%	**C**	61.21%	**C**	67.71%	**C**	73.26%	**C**	82.51%	**C**
9	Affected	0%	**I**	72.14%	**C**	71.38%	**C**	76.30%	**C**	73.09%	**C**	79.30%	**C**
10	Affected	0%	**I**	67.81%	**C**	74.29%	**C**	81.29%	**C**	38.01%	**I**	89.03%	**C**
11	Affected	100%	**C**	50.27%	**U**	35.21%	**I**	45.89%	**I**	74.95%	**C**	91.04%	**C**
12	Affected	0%	**I**	75.84%	**C**	71.23%	**C**	77.00%	**C**	79.31%	**C**	80.32%	**C**
13	Affected	0%	**I**	51.00%	**U**	49.73%	**U**	51.41%	**U**	57.31%	**C**	80.00%	**C**
14	Not Affected	100%	**C**	68.22%	**C**	45.75%	**I**	55.15%	**C**	76.71%	**C**	78.81%	**C**
15	Not Affected	0%	**I**	61.78%	**C**	63.12%	**C**	70.82%	**C**	71.52%	**C**	80.63%	**C**
16	Not Affected	0%	**I**	72.77%	**C**	71.25%	**C**	79.20%	**C**	78.39%	**C**	82.81%	**C**
17	Affected	0%	**I**	75.28%	**C**	68.71%	**C**	76.281%	**C**	81.49%	**C**	85.00%	**C**
18	Not Affected	100%	**C**	77.12%	**C**	67.59%	**C**	71.57%	**C**	80.28%	**C**	81.55%	**C**
19	Affected	100%	**C**	79.01%	**C**	51.10%	**U**	64.00%	**C**	72.51%	**C**	82.32%	**C**
20	Not Affected	100%	**C**	50.77%	**U**	41.20%	**I**	44.32%	**I**	44.75%	**I**	78.13%	**C**
	Mean of Success	40.00%		70.5290		63.3570%		69.8800		72.1090%		**83.6215%**	
	Standard Deviation	50.2625		10.5745		12.7826		12.1779		13.0316		**4.0066**	

Biography

Farnaz Sabahi received the Ph.D. degree in control engineering from the Department of Electrical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran. She is currently an Associate Professor and the Director of S-Answer Laboratory with the Department of Electrical Engineering, Urmia University, Urmia, Iran. Her current research interests include fuzzy logic, machine learning, deep learning, control theories, artificial intelligence, cognitive sciences, and robotics. She has published different peer-reviewed articles in these research fields.