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Brief Note

Corrections to "On the global practical stabilization of discrete-time switched affine systems: Application to switching power converters" [Scientia Iranica (2021) 28(3), 1621-1642] and "Global practical stabilization of discrete-time switched affine systems via switched Lyapunov functions and state-dependent switching functions", [Scientia Iranica (2021) 28(3), 1606-1620]

M. Hejri*

Department of Electrical Engineering, Sahand University of Technology, P.O. Box 51335-1996, Sahand New Town, Tabriz, Iran.

Abstract. This note corrects the proof of Theorem 1 in [1], the statement and proof of Lemma 1, and a part of proofs in Theorems 1 and 2 of [2].

1. Corrections to [1]

A step is incorrect in the proof of Theorem 1 of [1] when showing a positive constant γ exists to satisfy item (b) of Lemma 1. The error lies in the definition of function $\phi(s)$ in Eq. (35) where the norm function $\|e(k)\|$ must be replaced by the distance function $\|e(k)\|_{\mathcal{E}(P)} = \inf_{y \in \mathcal{E}} \|e(k) - y\|$ denoting the distance from point $e(k)$ to the ellipsoid $\mathcal{E}(P)$. Therefore, using Eq. (32) in [1] and Lyapunov set asymptotic stability arguments [3], we can only conclude that

$\phi(\|e(k)\|_{\mathcal{E}(P)}) \to 0$ as $k \to \infty$, and the existence of a positive constant γ is not proved.

1.1. Correction to the proof of Theorem 1
In [1], pages 1627–1628, Relation (25) in this note is added, minor modifications are made in Eqs. (25)–(32) of [1] by changing "< 0" to "$\leq -\gamma$" in these relations, and arguments between Relations (32) and (37) of [1] are omitted.

Remark 1. *According to the preceding modifications in the proof of Theorem 1 in [1], the relation numbers (25)–(32) in [1] must be increased by 1. For instance, the equation number (32) in [1] must be changed to*

*. *Corresponding author. Tel.: +98 41 33459363*
 E-mail address: hejri@sut.ac.ir (M. Hejri)

Eq. (33). Furthermore, the equation numbers (38)–(62) in [1] must be reduced by 4. For example, the Relation (62) must be changed to Relation (58) in the modified version.

In what follows, all equation numbers are set based on the comment in Remark 1 of this note. The rest of correct proof of Theorem 1 in the paragraph after Relation (24), page 1627 of [1], is given in below.

In the sequel, we plan to prove the attractiveness of the set $\mathcal{E}(P)$ via Relations (7) and (8) and $v(e(k)) = e(k)^T P e(k)$. From Relation (7), one can conclude $\exists \gamma > 0$ such that:

$$\begin{bmatrix} \tau_2 P + \tilde{M}_1 - \sum_{i \in \mathbb{K}} \lambda_{2i} P & * \\ \tilde{M}_2 & \tilde{M}_3 - \tau_2 \end{bmatrix} \preceq -\gamma I_{n+1}. \quad (25)$$

This comes from the fact that for any real symmetric matrix $M = M^T \in \mathbb{R}^{n \times n}$ one can write $\lambda_{\min}(M) I_n \preceq M \preceq \lambda_{\max}(M) I_n$. This inequality can be derived via Rayleigh's inequality in the context of the Matrix Analysis. Now, if M is a negative definite matrix as well, i.e., $M = M^T \prec 0$, then its all eigenvalues are negative, including $\lambda_{\max}(M)$. Therefore, one can choose $-\gamma = \lambda_{\max}(M)$ which always exists. Pre-multiplying Relation (25) by $[e(k)^T 1]$ and post-multiplying by $[e(k)^T 1]^T$ one can obtain:

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tau_2 P + \tilde{M}_1 - \sum_{i \in \mathbb{K}} \lambda_{2i} P & * \\ \tilde{M}_2 & \tilde{M}_3 - \tau_2 \end{bmatrix}$$

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq -\gamma \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq -\gamma,$$

$$\forall e(k) \in \mathbb{R}^n. \quad (26)$$

Relation (26) can be rewritten as Relation (27):

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tilde{M}_1 - \sum_{i \in \mathbb{K}} \lambda_{2i} P & * \\ \tilde{M}_2 & \tilde{M}_3 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix}$$

$$- \tau_2 \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} -P & * \\ 0_{1 \times n} & 1 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq -\gamma,$$

$$\forall e(k) \in \mathbb{R}^n. \quad (27)$$

By using S-procedure, Relation (27) implies Relation (28):

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} -P & * \\ 0_{1 \times n} & 1 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} < 0 \Rightarrow$$

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tilde{M}_1 - \sum_{i \in \mathbb{K}} \lambda_{2i} P & * \\ \tilde{M}_2 & \tilde{M}_3 \end{bmatrix}$$

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq -\gamma. \quad (28)$$

By substituting $\tilde{M}_1$, $\tilde{M}_2$, and $\tilde{M}_3$ from Eqs. (13)–(15)

into Relation (28) and after some algebra, one can reach:

$$e(k)^T P e(k) > 1 \Rightarrow \sum_{i \in \mathbb{K}} \lambda_{2i} [(A_i e(k) + l_i)^T$$

$$P(A_i e(k) + l_i) - e(k)^T P e(k)] \leq -\gamma. \quad (29)$$

According to Relation (9), since $\sum_{i \in \mathbb{K}} \lambda_{2i} > 0$, $\lambda_{2i} \geq 0, i \in \mathbb{K}$, Lemma 2 implies that Relation (29) can be rewritten as:

$$e(k)^T P e(k) > 1 \Rightarrow \exists i \in \mathbb{K} \text{ such that } (A_i e(k) + l_i)^T P$$

$$(A_i e(k) + l_i) - e(k)^T P e(k) \leq -\gamma. \quad (30)$$

Relation (30) means for each $e(k)$ satisfying $e(k)^T P e(k) > 1$ there exists an index i such that $(A_i e(k) + l_i)^T P(A_i e(k) + l_i) - e(k)^T P e(k) \leq -\gamma$. This is because outside the set $\mathcal{E}(P) = \{e(k) \in \mathbb{R}^n | e(k)^T P e(k) \leq 1\}$ the switching function is not constant. According to the switching rule (5), we have:

$$(A_{\sigma(e(k))} e(k) + l_{\sigma(e(k))})^T P(A_{\sigma(e(k))} e(k)$$

$$+ l_{\sigma(e(k))}) \leq (A_i e(k) + l_i)^T$$

$$P(A_i e(k) + l_i). \quad (31)$$

From Relations (30) and (31) one can reach:

$$e(k)^T P e(k) > 1 \Rightarrow (A_{\sigma(e(k))} e(k) + l_{\sigma(e(k))})^T P$$

$$(A_{\sigma(e(k))} e(k) + l_{\sigma(e(k))})$$

$$- e(k)^T P e(k) \leq (A_i e(k) + l_i)^T$$

$$P(A_i e(k) + l_i) - e(k)^T P e(k) \leq -\gamma, \quad (32)$$

or equivalently:

$$e(k)^T P e(k) > 1 \Rightarrow v(A_{\sigma(e(k))} e + l_{\sigma(e(k))}) - v(e(k))$$

$$= v(e(k+1)) - v(e(k)) = \Delta v(e(k))$$

$$\leq (A_i e(k) + l_i)^T P(A_i e(k) + l_i)$$

$$- e(k)^T P e(k) \leq -\gamma, \quad (33)$$

which implies condition (b) in Lemma 1. Since $P = P^T \succ 0$, $v(e(k)) = e(k)^T P e(k)$ is positive in the whole state space unless $e(k) = 0_{n \times 1}$ which satisfies condition (c) of Lemma 1. Therefore, all conditions of Lemma 1 are satisfied. Since during proof of Theorem 1 no restriction is imposed on the selection of $e(k)$, i.e., $e(k) \in \mathbb{R}^n$, as a result $D = \mathbb{R}^n$ and according to Remark 3 the switched affine system (2) is globally practically stable under switching rule (5) and the proof is concluded.

2. Corrections to [2]

In Lemma 1 of [2], the proposed conditions in items

(c) and (d) do not imply the finite-time convergence of the trajectories according to item (c) of Definition 1. In fact, in the proof of Lemma 1, when we reach $\lim_{k\to\infty} v(e(k)) = h \geq 0$, the proposed condition in item (c) of Lemma 1, implies that

$$\lim_{k\to\infty} [v(e(k+1)) - v(e(k))] =$$

$$h - h = 0 \leq \lim_{k\to\infty} -\phi(\|e(k)\|) < 0. \tag{1}$$

By this, one can conclude that $\lim_{k\to\infty} \phi(\|e(k)\|) = 0$. Therefore, one cannot conclude that there exists $\gamma > 0$ such that $\phi(\| e(k) \|) \geq \phi(\gamma)$, $\forall k \in \mathbb{Z}_{\geq 0}$. As a result, the existence of a positive constant $\phi(\gamma)$ is not proved and Relation (4) in [2] is not verified.

2.1. Correction to the statement and proof of Lemma 1

The correct statement and proof for Lemma 1 of [2] are given in below.

Lemma 1. *System (2) is practically stable in the large in a given domain $D \subset \mathbb{R}^n$ containing the origin in the sense of Definition 3 if there exist a bounded set $\mathcal{V} \subset D$ and a scalar function $v(e(k)) : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ such that:*

(a) $0_{n\times 1} \in \mathcal{V}$;

(b) *if $e(k) \in \mathcal{V}$ then $e(k+1) = A_{\sigma(e(k))}e(k) + l_{\sigma(e(k))} \in \mathcal{V}$*

(c) *if $e(k) \in D - \mathcal{V}$ then $v(e(k + 1)) - v(e(k)) = \Delta v(e(k)) \leq -\gamma < 0$ where γ is a positive constant;*

(d) *$v(e(k)) > 0$ where $e(k) \in D - \mathcal{V}$.*

Proof. Conditions (a) and (b) are the same as conditions (a) and (b) of Definition 1 and therefore, fulfill the invariant property of the bounded set $\mathcal{V}$. To prove the attractiveness property of the set $\mathcal{V}$, according to condition (c) of Definition 1 it is necessary to show that starting any initial state $e(0) \in D - \mathcal{V}$, there exists a finite time $T = T(e(0)) > 0$ such that for $k \geq T$ the state $e(k)$ eventually enters within the set $\mathcal{V}$, i.e., $\exists T > 0$ such that $e(k) \in \mathcal{V}$ for $k \geq T$. We show this by contradiction. By using condition (c), one can write:

$$v(e(k)) = v(e(0)) + \sum_{n=0}^{k-1} (v(e(n+1) - v(e(n)))$$

$$\leq v(e(0)) - k\gamma. \tag{4}$$

The right side of Eq. (4) will be eventually negative when k takes large values. This leads to contradiction to the condition (d) where it is assumed $v(e(k))$ is positive definite on $D - \mathcal{V}$. Therefore, there exists a finite time $T = T(\epsilon) > 0$ such that for $k \geq T$ the state $e(k)$ eventually enters within the set $\mathcal{V}$. As a result, $\mathcal{V}$ is an invariant set of attraction according to Definition 1,

and therefore, according to Definition 3, system (2) is practically stable in the large on the domain D and under switching function $\sigma(e(k))$. Thus, the proof is completed. $\square$

Remark 2. *During foregoing modifications in Proof of Lemma 1, Relation (5) in [2] is omitted, and relation numbers in the range (6)-(27), must be reduced by 1. For instance, the relation number (27) must be changed to (26).*

In what follows, a part of Theorems 1 and 2 proofs established based on items (b) and (c) of Lemma 1 in [2] are modified as follows.

2.2. Correction to a part of the proof of Theorem 1

The correct proof of Theorem 1 for the implication of the attractiveness property of the set $\mathcal{V}$, in the paragraph after Inequality (27), page 1612 of [2], is given in below. In summary, a new Eq. (27) in this note is added, minor modifications are made in Eqs. (28)–(38) of [2] by changing "< 0" to "$\leq -\gamma$" in these relations, and arguments between (38) and (42) of [2] are omitted. In the following, a detailed explanation is given where all equation numbers are adjusted based on the Remark 2 of this note.

In the sequel, we plan to prove the attractiveness of the set $\mathcal{V}$ via matrix Inequality (12). From Inequality (12), one can conclude $\exists\gamma > 0$ such that:

$$\begin{bmatrix} \tau_{2h}P_h + \tilde{M}_{1h} & * \\ \tilde{M}_{2h} & \tilde{M}_{3h} - \tau_{2h} \end{bmatrix} \preceq -\gamma I_{n+1},$$

$$\forall h \in \mathbb{K}. \tag{27}$$

Pre-multiplying matrix Relation (27) by $[e(k)^T 1]$ and post-multiplying by $[e(k)^T 1]^T$, one can obtain:

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tau_{2h}P_h + \tilde{M}_{1h} & * \\ \tilde{M}_{2h} & \tilde{M}_{3h} - \tau_{2h} \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix}$$

$$\leq -\gamma \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq -\gamma, \qquad \forall h \in \mathbb{K}. \tag{28}$$

Relation (28) can be rewritten as Relation (29):

$$-\tau_{2h} \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} -P_h & * \\ 0_{1\times n} & 1 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} +$$

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tilde{M}_{1h} & * \\ \tilde{M}_{2h} & \tilde{M}_{3h} \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq -\gamma,$$

$$\forall h \in \mathbb{K}. \tag{29}$$

Using S-procedure, from Relation (29) one can conclude Relation (30), $\forall e(k) \in \mathbb{R}^n$:

$$\left[\begin{array}{c} e(k) \\ 1 \end{array}\right]^T \left[\begin{array}{cc} -P_h & * \\ 0_{1 \times n} & 1 \end{array}\right] \left[\begin{array}{c} e(k) \\ 1 \end{array}\right] < 0$$

$$\Rightarrow \left[\begin{array}{c} e(k) \\ 1 \end{array}\right]^T \left[\begin{array}{cc} \tilde{M}_{1h} & * \\ \tilde{M}_{2h} & \tilde{M}_{3h} \end{array}\right] \left[\begin{array}{c} e(k) \\ 1 \end{array}\right] \leq -\gamma,$$

$$\forall h \in \mathbb{K}. \tag{30}$$

By substituting $\tilde{M}_{1h}$, $\tilde{M}_{2h}$, and $\tilde{M}_{3h}$ from Eqs. (16)–(18) into Relation (30) and after some algebra, one can reach:

$$e(k)^T P_h e(k) > 1 \Rightarrow \sum_{i \in \mathbb{K}} \beta_{hi} (\sum_{j \in \mathbb{K}} \lambda_{hj} [(A_j e(k) + l_j)^T$$

$$P_i (A_j e(k) + l_j) - e(k)^T P_j e(k)]) \leq -\gamma, \quad \forall h \in \mathbb{K}. \tag{31}$$

Since $\beta_{hi} \geq 0$, $i, h \in \mathbb{K}$, and $\sum_{i \in \mathbb{K}} \beta_{hi} > 0$ according to Lemma 2 and Relation (31) one can conclude:

$$e(k)^T P_h e(k) > 1 \Rightarrow \exists i \in \mathbb{K} \text{ such that } \sum_{j \in \mathbb{K}} \lambda_{hj}$$

$$[(A_j e(k) + l_j)^T P_i (A_j e(k) + l_j)$$

$$- e(k)^T P_j e(k)] \leq -\gamma, \forall h \in \mathbb{K}. \tag{32}$$

Again according to Lemma 2, since $\lambda_{hj} \geq 0, j, h \in \mathbb{K}$ and $\sum_{j \in \mathbb{K}} \lambda_{hj} > 0$, Relation (32) implies:

$$e(k)^T P_h e(k) > 1 \Rightarrow \exists i, j \in \mathbb{K} \text{ such that}$$

$$(A_j e(k) + l_j)^T P_i (A_j e(k) + l_j)$$

$$- e(k)^T P_j e(k) \leq -\gamma, \qquad \forall h \in \mathbb{K}. \tag{33}$$

Now, according to the definition of the set $\mathcal{V}$ in Eq. (3), we have:

$$e(k) \notin \mathcal{V} \Rightarrow \exists h \in \mathbb{K} \text{ such that } e(k)^T P_h e(k) > 1. \tag{34}$$

From Relations (33) and (34), one can infer:

$$e(k) \notin \mathcal{V} \Rightarrow \exists i, j \in \mathbb{K} \text{ such that } (A_j e(k) + l_j)^T$$

$$P_i (A_j e(k) + l_j) - e(k)^T P_j e(k) \leq -\gamma. \tag{35}$$

Since $e(k) \notin \mathcal{V}$, according to Relation (5) and item (2) in the switching Algorithm 1, one can conclude there exist $\sigma(e(k)), i, j \in \mathbb{K}$ satisfying the following expression:

$$(A_{\sigma(e(k))} e(k) + l_{\sigma(e(k))})^T P_i (A_{\sigma(e(k))} e + l_{\sigma(e(k))})$$

$$- e(k)^T P_{\sigma(e(k))} e(k) \leq$$

$$(A_j e(k) + l_j)^T P_i (A_j e(k) + l_j)$$

$$- e(k)^T P_j e(k). \tag{36}$$

From Relations (35) and (36) one can conclude that there exist $\sigma(e(k)), i \in \mathbb{K}$ such that:

$$e(k) \notin \mathcal{V} \Rightarrow \exists i, j, \sigma(e(k)) \in \mathbb{K} \text{ such that}$$

$$(A_{\sigma(e(k))} e(k) + l_{\sigma(e(k))})^T P_i (A_{\sigma(e(k))} e(k)$$

$$+ l_{\sigma(e(k))}) - e(k)^T P_{\sigma(e(k))} e(k) \leq (A_j e(k) + l_j)^T$$

$$P_i (A_j e(k) + l_j) - e(k)^T P_j e(k) \leq -\gamma, \tag{37}$$

or equivalently:

$$e(k) \notin \mathcal{V} \Rightarrow \exists i, j \in \mathbb{K} \text{ such that } v(e(k+1)) - v(e(k))$$

$$= \Delta v(e(k)) \leq (A_j e(k) + l_j)^T$$

$$P_i (A_j e(k) + l_j) - e(k)^T P_j e(k) \leq -\gamma, \tag{38}$$

which implies condition (c) in Lemma 1. Since $P_i = P_i^T \succ 0$, $v(e(k)) = e(k)^T P_{\sigma(e(k))} e(k)$ is positive in the whole state space unless $e(k) = 0_{n \times 1}$ which satisfies condition (d) of Lemma 1.

Therefore, according to Relations (38) and (26), all conditions of Lemma 1 are fulfilled. Since during the proof of Theorem 1 no restriction is imposed on the selection of $e(k)$, i.e., $e(k) \in \mathbb{R}^n$, as a result $D = \mathbb{R}^n$ and according to Remark 2 the switched affine system (2) is globally practically stable under switching Algorithm 1 and the proof is concluded.

Remark 3. *During the preceding modifications in a part of Theorem 1 proof, Relations (39)–(42) of [2] are omitted. Thus, the relation numbers in the range (43)–(50) of [2] must be reduced by 4. A new Eq. (47) is added through this note. Finally, all equations numbers in the range (51)–(68) of [2] are reduced by 3.*

2.3. Correction to a part of the proof of Theorem 2

The correct proof of Theorem 2 for the implication of the attractive property of the set $\mathcal{V}$, in the paragraph after Relation (50) of [2], is given in below. In summary, a new Relation (47) is added by this note, and minor modifications are made to the relations (51)–(60) of [2] by changing "< 0" to "$\leq -\gamma$" in these relations. In the following, a detailed explanation is given where all equation numbers are adjusted based on the Remark 3 of this note.

In the sequel, the attractive property of the set $\mathcal{V}$ is proved via conditions in Eq. (40). From Eq. (40), one can conclude $\exists \gamma > 0$ such that:

$$\left[\begin{array}{cc} \tau_{2hi} P_h + \tilde{M}_{1hi} & * \\ \tilde{M}_{2hi} & \tilde{M}_{3hi} - \tau_{2hi} \end{array}\right] \preceq -\gamma I_{n+1},$$

$$\forall i, h \in \mathbb{K}. \tag{47}$$

Pre-multiplying matrix Inequality (47) by $[e(k)^T 1]$ and post-multiplying by $[e(k)^T 1]^T$, one can obtain:

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tau_{2hi}P_h + \tilde{M}_{1hi} & * \\ \tilde{M}_{2hi} & \tilde{M}_{3hi} - \tau_{2hi} \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix}$$

$$\leq -\gamma \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq -\gamma,$$

$$\forall i, h \in \mathbb{K}. \tag{48}$$

Relation (48) can be rewritten as:

$$-\tau_{2hi} \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} -P_h & * \\ 0_{1 \times n} & 1 \end{bmatrix}$$

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix} + \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tilde{M}_{1hi} & * \\ \tilde{M}_{2hi} & \tilde{M}_{3hi} \end{bmatrix}$$

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix} \leq -\gamma, \qquad \forall i, h \in \mathbb{K}. \tag{49}$$

Using S-procedure, from Relation (49), one can reach to Relation (50), $\forall e(k) \in \mathbb{R}^n$:

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} -P_h & * \\ 0_{1 \times n} & 1 \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} < 0 \Rightarrow \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T$$

$$\begin{bmatrix} \tilde{M}_{1hi} & * \\ \tilde{M}_{2hi} & \tilde{M}_{3hi} \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \leq -\gamma,$$

$$\forall i, h \in \mathbb{K}. \tag{50}$$

By substituting $\tilde{M}_{1hi}$, $\tilde{M}_{2hi}$, and $\tilde{M}_{3hi}$ from Eqs. (44)–(46) into Relation (50) and after some algebra, one can reach:

$$e(k)^T P_h e(k) > 1 \Rightarrow \sum_{j \in \mathbb{K}} \lambda_{hj} [(A_j e(k) + l_j)^T P_i$$

$$(A_j e(k) + l_j) - e(k)^T P_j e(k)] \leq -\gamma,$$

$$\forall i, h \in \mathbb{K}. \tag{51}$$

Since $\lambda_{hj} \geq 0, j, h \in \mathbb{K}$, and $\sum_{j \in \mathbb{K}} \lambda_{hj} > 0$, according to Lemma 2, Relation (51) implies Relation (52):

$$e(k)^T P_h e(k) > 1 \Rightarrow \exists j \in \mathbb{K} \text{ such that } (A_j e(k)$$

$$+ l_j)^T P_i (A_j e(k) + l_j) - e(k)^T$$

$$P_j e(k) \leq -\gamma, \qquad \forall i, h \in \mathbb{K}. \tag{52}$$

According to the definition of the set $\mathcal{V}$ in Relation (3), we have

$$e(k) \notin \mathcal{V} \Rightarrow \exists h \in \mathbb{K} \text{ such that } e(k)^T P_h e(k) > 1. \tag{53}$$

Now from Relations (52) and (53), one can reach:

$$e(k) \notin \mathcal{V} \Rightarrow \exists j \in \mathbb{K} \text{ such that } (A_j e(k) + l_j)^T$$

$$P_i (A_j e(k) + l_j) - e(k)^T P_j e(k) \leq -\gamma,$$

$$\forall i \in \mathbb{K}. \tag{54}$$

Since $e(k) \notin \mathcal{V}$, according to Relation (54) and item (2) in the switching Algorithm 1, one can conclude there exists a $\sigma(e(k))$ satisfying the following expression:

$$(A_{\sigma(e(k))} e(k) + l_{\sigma(e(k))})^T P_i (A_{\sigma(e(k))} e + l_{\sigma(e(k))})$$

$$- e(k)^T P_{\sigma(e(k))} e(k) \leq (A_j e(k) + l_j)^T$$

$$P_i (A_j e(k) + l_j) - e(k)^T P_j e(k), \qquad \forall i \in \mathbb{K}. \tag{55}$$

From Relations (54) and (55), one can reach:

$$e(k) \notin \mathcal{V} \Rightarrow \exists j \in \mathbb{K} \text{ such that } (A_{\sigma(e(k))} e(k) + l_{\sigma(e(k))})^T$$

$$P_i (A_{\sigma(e(k))} e(k) + l_{\sigma(e(k))}) - e(k)^T P_{\sigma(e(k))} e(k) \leq$$

$$(A_j e(k) + l_j)^T P_i (A_j e(k) + l_j) - e(k)^T P_j e(k) \leq -\gamma,$$

$$\forall i \in \mathbb{K}, \tag{56}$$

or equivalently:

$$e(k) \notin \mathcal{V} \Rightarrow \exists j \in \mathbb{K} \text{ such that } v(e(k+1)) - v(e(k)) =$$

$$\Delta v(e(k)) \leq (A_j e(k) + l_j)^T$$

$$P_i (A_j e(k) + l_j) - e(k)^T P_j e(k) \leq -\gamma,$$

$$\forall i \in \mathbb{K}. \tag{57}$$

Similar to our argument in Theorem 1, the attractive property of the set $\mathcal{V}$ is inferred according to items (a), (c), and (d) of Lemma 1. Therefore, all conditions of Lemma 1 are fulfilled. Similar to Theorem 1, since no restriction is imposed on the selection of $e(k)$, namely, $e(k) \in \mathbb{R}^n$, therefore $D = \mathbb{R}^n$ and according to Remark 2 the switched affine system (2) is globally practically stable under switching Algorithm 1 and the proof is completed.

References

1. Hejri, M. "On the global practical stabilization of discrete-time switched affine systems: Application to switching power converters", *Scientia Iranica*, Transaction D, Computer Science and Electrical Engineering, **28**(3), pp. 1621–1642 (2021).
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Biography

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