Fuzzy DEA Efficiency Analysis with the Acceptance Degree of the Violated Fuzzy Constraints

Monireh Jahani Sayyad Noveiri[1], Sohrab Kordrostami[1*]

[1]Department of Mathematics, Lahijan Branch, Islamic Azad University, Lahijan, Iran

[*]Corresponding author, Email: sohrabkordrostami@gmail.com, Mobile Number: +989111346054.

Abstract. In many real world applications, the performance of entities with undesirable outputs should be assessed while imprecise and vague information is presented. In this study, a fully fuzzy data envelopment analysis (FFDEA) approach is propounded to analyze the relative efficiency of decision making units (DMUs) in which the acceptance degree of decision maker that fuzzy constraints may be breached is incorporated. In order to achieve this purpose, the interval expectation of trapezoidal fuzzy numbers and the order relationship of trapezoidal fuzzy numbers are employed. Also, after converting the FFDEA model into an interval DEA model and then a bi-objective DEA model, parametric examination is applied to aggregate objectives. Therefore, input- and output-oriented FFDEA approaches with undesirable outputs are provided to estimate the relative efficiency of processes whilst the acceptance degree of the violated fuzzy constraints is considered. A dataset from the existing studies is used to clarify the introduced technique and to describe its applicability.

Keywords: Fuzzy data envelopment analysis, Acceptance degree, Trapezoidal fuzzy number, Efficiency, Undesirable outputs.

1. Introduction

Data envelopment analysis (DEA) is a popular technique used to assess the efficiency of decision making units (DMUs) that have multiple input-output measures. Standard DEA models treat all performance measures as precise and desirable, but in real-world studies, there are often variables that are undesirable and imprecise. Some DEA practitioners have investigated the uncertainty and undesirable outputs separately, but there have been few examinations that consider entities with both undesirable outputs and imprecise measures. Additionally, in certain cases, decision makers may not be able to meet specific fuzzy constraints, but they can accept them to a certain extent. Therefore, it is important to incorporate the approval degree of the decision maker in fuzzy problems.

Dong and Wan [1] proposed a trapezoidal fuzzy linear programming problem that integrated the acceptance degree of fuzzy constraints violated. They examined fuzzy objective constraints, fuzzy technological coefficients and fuzzy resources, whereas decision variables were assumed to be certain. Stanojevic et al. [2] extended the methodology of Dong and Wan [1] and dealt with a fully fuzzy linear program with approval degree of fuzzy constraints dissatisfied. Dong and Wan [3] also provided an approach to compute fuzzy multi-objective linear programming problems while considered the acceptance degree of fuzzy constraints violated.

Some studies such as [4-7] made the reviews of the DEA literature related to vagueness and fuzziness. Emrouznejad and Tavana [5] furnished the requisite contextual information pertaining to the subject matter at hand. The extant fuzzy DEA models were utilized in [5] to develop and articulate six divisions, viz. the tolerance approach, the α -level-based approach, the fuzzy ranking technique, the possibility approach, the fuzzy arithmetic, and the fuzzy random/Type-2 fuzzy set. Despite the advantages and disadvantages of different existing fuzzy DEA methods, there is a deficiency in the inclusion of the acceptance degree of decision maker that fuzzy constraints may be breached while according to Dong and Wan [1] it is necessary to consider. In the fuzzy DEA literature, it can also be found many extensions of fuzzy DEA approaches. Kordrostami et al. [8] analyzed the efficiency of the units in the presence of fuzzy and integer measures. The performance of multi-period systems with fuzzy measures was also addressed in [9]. Hatami-Marbini et al. [10] propounded fully fuzzy DEA models whilst all input-output measures were deemed as desirable ones. Wardana et al. [11] rendered an alternative approach, called the fuzzy DEA credibility constrained and RC index, to tackle uncertainty. Peykani et al. [12] presented an adjustable DEA methodology to cater to the varying needs and preferences of managers across different attitudes, including desirable measures. Puri and Yadav [13] presented a fuzzy DEA model based on α -cut approach to evaluate the efficiency of DMUs with fuzzy undesirable outputs. Moreover, Kordrostami et al. [14] provided a fuzzy expected value approach to estimate the relative efficiency of processes when both undesirable and fuzzy measures are presented. Puri and Yadav [15] developed the approaches of fully fuzzy DEA (FFDEA) and multi-component fully fuzzy DEA to assess the fuzzy technical efficiency with undesirable outputs. Ebrahimnejad and Amani [16] introduced a fuzzy DEA approach considering ideal points to estimate the fuzzy efficiency values of DMUs with undesirable outputs while performance measures and the efficiency variables are only deemed as triangular fuzzy numbers. Nasseri et al. [17] provided a fuzzy stochastic DEA approach with undesirable outputs to investigate the efficiency of entities with fuzzy and random input-output data. Also, Nasseri and Ahmadi Khatir [18] proposed a DEA technique to analyze the performance of two-stage systems with undesirable outputs in a fuzzy random environment. Peykani et al. [19] presented a new fuzzy network DEA model that utilizes an additive efficiency decomposition approach. This was achieved through the application of adjustable possibilistic programming and chance-constrained programming. Also, Peykani et al. [20] provided a comprehensive literature review of fuzzy chance-constrained DEA. Cinaroglu [21] employed a multi-stage fuzzy stochastic methodology to assess the efficiency of the Turkish healthcare system. Gholizadeh et al. [22] introduced a fuzzy data-driven scenario-based robust DEA technique to forecast and optimize the parameters of an electrical discharge machine. Izadikhah and Khoshroo [23] presented a modified enhanced Russell approach to measure the crisp efficiency of systems with undesirable outputs when there are fuzzy inputs and outputs. Chen et al. [24] proposed an integrated the best-worst method and DEA, including trapezoidal interval type-2 fuzzy for makeshift hospital selection. Nevertheless, the acceptance degree of decision maker that fuzzy constraints may be breached has not been included in the present fully fuzzy DEA approaches with undesirable outputs.

In practice, undesirable outputs can be generated in many real world problems. To confront this issue, there are some approaches in the existing literature that take undesirable outputs into account, see [25-29] for more information. Kordrostami et al. [30] provided models to incorporate the predefined variations of performance measures where undesirable outputs with strong and weak disposability are presented. In a prior study [31], the multi-period efficiency of processes with different perspectives of disposability, including strong and weak, for undesirable outputs has been investigated. Due to the popularity and acceptance of the weak disposable technology as mentioned in [32], the propounded FFDEA models are based upon considering weakly disposable undesirable outputs while incorporate the acceptance degree of the violated fuzzy constraints. As we know and Table 1 shows, the existing fully fuzzy DEA models with undesirable outputs have not incorporated the examination of the acceptance degrees of decision maker that fuzzy constraints may be dissatisfied. Selecting different acceptance degrees leads to disparate efficiency scores and increases the flexibility of decision making.

Table 1. A comparative review of fuzzy DEA

As can be found from Table 1, there is a scarcity of fully fuzzy DEA models with the acceptance degree of the violated fuzzy constraints and undesirable outputs. Considering fuzzy variables and data is preferable in modeling under uncertainty based on [1, 10, 45] while the majority of fuzzy DEA models are non-fully fuzzy approaches. Also, projection points have not been investigated in many fully fuzzy DEA approaches although they include beneficial information for making fuzzy decisions. Notice that none of existing DEA studies has incorporated the acceptance level of expert that the fuzzy constraints may be breached, except Chen et al. [24] that partially investigated it in the DEA multiplier form and did not addressed undesirable outputs and projection points. However, according to Dong and Wan [1, 3], incorporating the acceptance degree of decision maker is essential. Also, as shown in Table 1, there is a gap of investigating energy dependency of areas in the fully fuzzy environment while undesirable outputs and the acceptance degree of decision maker are considered.

Therefore, this paper focuses on elaborating input- and output-oriented fully fuzzy DEA models, containing the acceptance degree of the dissatisfied fuzzy constraints and weakly disposable undesirable outputs. All variables that are the efficiency variable, input-output measures and intensity variables are deemed as trapezoidal fuzzy numbers. To undertake this issue, the interval expectation of trapezoidal fuzzy numbers is used to gain the order relationship and they are employed to transform FFDEA models into the interval DEA models. Then interval DEA models are converted into bi-objective DEA models using the order relation between intervals and the existing approach to address interval objective programs. Afterwards, the bi-objective DEA models are substituted with parametric linear DEA models to unify objective functions. Overall, the contribution of this research is fourfold:

i. Proposing new envelopment fully fuzzy DEA models with the acceptance degree of the dissatisfied fuzzy constraints,
ii. Analyzing the fuzzy efficiency values using input- and output-oriented FFDEA approaches with undesirable outputs where the acceptance degree of the violated fuzzy constraints is considered,

iii. Estimating fuzzy input and output targets to improve the performance of DMUs under uncertainty,

iv. Applying a dataset from the literature to explain the introduced approach and to compare with some existing DEA approaches.

The rest of this paper is designed as follows. Some preliminaries and related concepts are provided in Section 2 that they are essential to obtain the profound insight of the proposed technique. Fully fuzzy DEA models considering the acceptance degrees of fuzzy constraints violated are introduced in Section 3. An application is given in Section 4 to clarify and show the suitability of the approach presented in this study. In Section 5, conclusions and some further remarks are drawn.

2. Preliminaries

In this section, firstly some basic notions and expressions about fuzzy numbers, interval order relation and interval objective problems are reviewed that have been derived from [1, 46]. Then modeling in the presence of weakly disposable undesirable outputs is discussed.

2.1. Concepts related to fuzzy numbers, interval order relation and interval objective problems

Fuzzy numbers are normal convex fuzzy subsets of the set of real numbers that their membership functions are piecewise continuous. The fuzzy subset $\tilde{A}$ of a universal set Y is characterized by the membership function $\mu_{\tilde{A}}: Y \to [0,1]$.

One of the most popular fuzzy numbers is the trapezoidal fuzzy number that is defined in the following way:

A trapezoidal fuzzy number $\tilde{A}$ is described with the value point (a_1, a_2, a_3, a_4) and the following the membership function:

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a_1)/(a_2-a_1), & a_1 \le x \le a_2, \\ 1, & a_2 \le x \le a_3, \\ (a_4-x)/(a_4-a_3), & a_3 \le x \le a_4, \\ 0, & otherwise. \end{cases} \tag{1}$$

The interval expectation of the trapezoidal fuzzy number $\tilde{A}$ denoted by $E(\tilde{A})$ is defined as $E(\tilde{A}) = [\frac{1}{2}(a_1+a_2), \frac{1}{2}(a_3+a_4)]$. Also, by considering $\tilde{A}$ and $\tilde{B}$ as trapezoidal fuzzy numbers, we have $E(\alpha_1\tilde{A}+\alpha_2\tilde{B}) = \alpha_1 E(\tilde{A}) + \alpha_2 E(\tilde{B})$, in which α_1 and α_2 are real values.

According to [1, 47], the fuzzy partial order relation for intervals can be described as follows:

Suppose $\tilde{b} = [\underline{b}, \overline{b}]$ and $\tilde{c} = [\underline{c}, \overline{c}]$ are intervals, that $-\infty < \underline{b} \le \overline{b} < +\infty$, $-\infty < \underline{c} \le \overline{c} < +\infty$. $\tilde{b} \le \tilde{c}$ is a fuzzy set with the following membership function:

$$\mu(\tilde{b} \le \tilde{c}) = \begin{cases} 1, & \overline{b} \le \underline{c}, \\ 1^-, & \underline{b} \le \underline{c} \le \overline{b} \le \overline{c}; r(\tilde{b}) > 0, \\ (\overline{c} - \overline{b}) / (2(r(\tilde{c}) - r(\tilde{b}))), & \underline{c} \le \underline{b} \le \overline{b} \le \overline{c}; r(\tilde{c}) > r(\tilde{b}), \\ 0.5, & r(\tilde{b}) = r(\tilde{c}); \underline{b} = \underline{c}, \end{cases} \tag{2}$$

in which 1^- is a fuzzy number less than one that means that the interval $\tilde{b}$ is weakly not more than the interval $\tilde{c}$. $r(\tilde{b})$ shows radius of the interval $\tilde{b}$ that is equal to $r(\tilde{b}) = (\overline{b} - \underline{b})/2$. Similarly, $r(\tilde{c})$ can be described. The membership function for $\tilde{b} \ge \tilde{c}$ can be analogously written that has been also represented in [1].

Interval inequality relationships can be transformed into the satisfactory crisp equivalent forms as presented in [1, 48]. To illustrate in more details, assume $\alpha \in [0,1]$ shows the acceptance degree of the interval inequality constraint that may be dissatisfied. Therefore, $\overline{b}x \le \overline{c}$ and $\mu(\tilde{b}x \ge \tilde{c}) \le \alpha$ is a satisfactory crisp equal structure for relation $\tilde{b}x \le \tilde{c}$. In the same way, $\underline{b}x \ge \underline{c}$ and $\mu(\tilde{b}x \le \tilde{c}) \le \alpha$ is an acceptable crisp equal structure for relation $\tilde{b}x \ge \tilde{c}$.

Considering $\tilde{A}$ and $\tilde{B}$ as trapezoidal fuzzy numbers, the order relation between them is stated in the following:

i. $\tilde{A} \ge \tilde{B}$ iff $E(\tilde{A}) \ge E(\tilde{B})$; ii. $\tilde{A} \le \tilde{B}$ iff $E(\tilde{A}) \le E(\tilde{B})$; iii. $\tilde{A} = \tilde{B}$ iff $E(\tilde{A}) = E(\tilde{B})$.

Also, as stated in [1, 49], the below maximization and minimization problems with the interval functions

$$Max\{\sum_{i=1}^{m} \tilde{c}_i x_i \mid (x_1,...,x_m)^T \in X\}, \tag{3}$$

and

$$Min\{\sum_{i=1}^{m} \tilde{c}_i x_i \mid (x_1,...,x_m)^T \in X\}, \tag{4}$$

in which $\tilde{c}_i = [\underline{c}_i, \overline{c}_i]$ $(i = 1,...,m)$ are intervals, can be substituted with the following bi-objective problems, respectively.

$$Max\{(\sum_{i=1}^{m} \underline{c}_i x_i, \frac{1}{2}\sum_{i=1}^{m} (\underline{c}_i + \overline{c}_i) x_i) \mid (x_1,...,x_m)^T \in X\}, \tag{5}$$

and

$$Min\{(\sum_{i=1}^{m} \overline{c}_i x_i, \frac{1}{2}\sum_{i=1}^{m} (\underline{c}_i + \overline{c}_i) x_i) \mid (x_1,...,x_m)^T \in X\}. \tag{6}$$

X is the feasible set of $(x_1,...,x_m)$.

2.2. Crisp DEA model with weakly disposable undesirable outputs

Suppose there are n DMUs, denoted by DMU_j $(j = 1,...,n)$, that their performance is expected to be evaluated. Also, each DMU uses m inputs $x_{ij}(i = 1,...,m)$, produces s desirable outputs $y_{rj}(r = 1,...,s)$ and emits K undesirable outputs $z_{kj}(k = 1,...,K)$. Following Färe et al. [29], we have the below definition and model to deal with undesirable outputs.

Definition 1. Weak disposability of outputs signifies that if the output vector $v = (y, z)$ can be generated, $(\theta y, \theta z); 0 \le \theta \le 1$ can also be generated while the input vector x is specified.

Färe et al. [29] also presented the following technology to address weakly disposable undesirable outputs:

$T(x) = \{(y, z) \mid x \ge \lambda X, z = \lambda Z, y \le \lambda Y, \lambda \ge 0\}.$

Accordingly, the below precise input-oriented DEA model under the constant returns to scale assumption can be formulated to tackle weakly disposable undesirable outputs:

$$
\begin{aligned}
& \theta^* = Min \ \ \theta \\
& s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io}, \quad i = 1,...,m, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, \quad r = 1,...,s, \\
& \sum_{j=1}^{n} \lambda_j z_{kj} = z_{ko}, \quad k = 1,...,K, \\
& \lambda_j \ge 0, \ j = 1, 2, ..., n,
\end{aligned}
\tag{7}
$$

that λ_j is the intensity variable and θ^* is the efficiency value.

Because of the presence of imprecise input-output data in many situations, FFDEA approaches considering the acceptance degree of decision maker to violate fuzzy constraints are rendered in the next section to address the performance of entities with weakly disposable undesirable outputs.

3. A FFDEA approach with the acceptance degree of the violated fuzzy constraints

Assuming the presence of n DMUs ($DMU_j; j = 1,...,n$), each consists of m inputs denoted by trapezoidal fuzzy values $\tilde{x}_{ij} = (x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}); (i = 1,...,m)$, s fuzzy desirable outputs $\tilde{y}_{rj} = (y_{rj1}, y_{rj2}, y_{rj3}, y_{rj4})$; $(r = 1,..., s)$ and K fuzzy undesirable outputs $\tilde{z}_{kj} = (z_{kj1}, z_{kj2}, z_{kj3}, z_{kj4})$; $(k = 1,..., K)$. Also, the maximum reduction of inputs is shown by the trapezoidal fuzzy number $\tilde{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4)$ and $\tilde{\lambda}_j = (\lambda_{j1}, \lambda_{j2}, \lambda_{j3}, \lambda_{j4})$ is the fuzzy intensity weights. Actually, all factors are deemed as trapezoidal fuzzy numbers. By regarding the constant returns to scale assumption and the weak disposability of undesirable outputs, the fully fuzzy DEA model (8) is provided to analyze the efficiency of DMUs.

As can be seen, the FFDEA model (8) is based on input orientation and the maximum input reduction.

$$\begin{aligned}
& Min\ \ \tilde{\theta} \\
& s.t. \sum_{j=1}^{n} \tilde{\lambda}_j \tilde{x}_{ij} \le \tilde{\theta} \tilde{x}_{io}, \qquad i = 1,\dots,m, \\
& \quad \sum_{j=1}^{n} \tilde{\lambda}_j \tilde{y}_{rj} \ge \tilde{y}_{ro}, \qquad r = 1,\dots,s, \\
& \quad \sum_{j=1}^{n} \tilde{\lambda}_j \tilde{z}_{kj} = \tilde{z}_{ko}, \qquad k = 1,\dots,K, \\
& \quad \tilde{\lambda}_j \ge 0,\ j = 1,2,\dots,n.
\end{aligned} \tag{8}$$

$(\tilde{x}_{io}, \tilde{y}_{ro}, \tilde{z}_{ko})$ in model (8) show components of the unit under evaluation, i.e. DMU_o. Due to statements revealed in the prior section, model (8) is equal to the below interval problem. To illustrate in more details, trapezoidal fuzzy expressions $\sum_{j=1}^{n} \tilde{\lambda}_j \tilde{x}_{ij}$, $\sum_{j=1}^{n} \tilde{\lambda}_j \tilde{y}_{rj}$, $\sum_{j=1}^{n} \tilde{\lambda}_j \tilde{z}_{kj}$, $\tilde{\theta}$, $\tilde{\theta}\tilde{x}_{io}$, $\tilde{y}_{ro}$ and $\tilde{z}_{ko}$ are substituted with their interval expectations.

$$\begin{aligned}
& Min\ \ E(\tilde{\theta}) \\
& s.t. \quad E(\sum_{j=1}^{n} \tilde{\lambda}_j \tilde{x}_{ij}) \le E(\tilde{\theta} \tilde{x}_{io}), \qquad i = 1,\dots,m, \\
& \qquad E(\sum_{j=1}^{n} \tilde{\lambda}_j \tilde{y}_{rj}) \ge E(\tilde{y}_{ro}), \qquad r = 1,\dots,s, \\
& \qquad E(\sum_{j=1}^{n} \tilde{\lambda}_j \tilde{z}_{kj}) = E(\tilde{z}_{ko}), \qquad k = 1,\dots,K, \\
& \qquad \tilde{\lambda}_j \ge 0,\ j = 1,2,\dots,n.
\end{aligned} \tag{9}$$

In other words, according to the interval expectation definition and arithmetic operations on fuzzy numbers, we have:

$$\begin{aligned}
& Min\ \ [\frac{1}{2}(\theta_1+\theta_2), \frac{1}{2}(\theta_3+\theta_4)] \\
& s.t. \quad \left[\frac{1}{2}\sum_{j=1}^{n}(\lambda_{j1}x_{ij1}+\lambda_{j2}x_{ij2}), \frac{1}{2}\sum_{j=1}^{n}(\lambda_{j3}x_{ij3}+\lambda_{j4}x_{ij4})\right] \le \left[\frac{1}{2}(\theta_1 x_{io1}+\theta_2 x_{io2}), \frac{1}{2}(\theta_3 x_{io3}+\theta_4 x_{io4})\right],\ i=1,\dots,m, \\
& \qquad \left[\frac{1}{2}\sum_{j=1}^{n}(\lambda_{j1}y_{rj1}+\lambda_{j2}y_{rj2}), \frac{1}{2}\sum_{j=1}^{n}(\lambda_{j3}y_{rj3}+\lambda_{j4}y_{rj4})\right] \ge \left[\frac{1}{2}(y_{ro1}+y_{ro2}), \frac{1}{2}(y_{ro3}+y_{ro4})\right],\ r=1,\dots,s, \\
& \qquad \left[\frac{1}{2}\sum_{j=1}^{n}(\lambda_{j1}z_{kj1}+\lambda_{j2}z_{kj2}), \frac{1}{2}\sum_{j=1}^{n}(\lambda_{j3}z_{kj3}+\lambda_{j4}z_{kj4})\right] = \left[\frac{1}{2}(z_{ko1}+z_{ko2}), \frac{1}{2}(z_{ko3}+z_{ko4})\right],\ k=1,\dots,K, \\
& \qquad 0 \le \lambda_{j1} \le \lambda_{j2} \le \lambda_{j3} \le \lambda_{j4},\ j=1,2,\dots,n, \varepsilon \le \theta_1 \le \theta_2 \le \theta_3 \le \theta_4,\ \theta_4 \le 1, \varepsilon > 0.
\end{aligned} \tag{10}$$

Model (10) can be replaced by the following bi-objective DEA model using the order relations and details provided in [1, 2] that were partially explained in the preceding section.

$$
\begin{aligned}
&Min \quad \frac{1}{2}(\theta_3+\theta_4)\\
&Min \quad \frac{1}{4}(\theta_1+\theta_2+\theta_3+\theta_4)\\
&s.t. \quad \sum_{j=1}^{n}(\lambda_{j3}x_{ij3}+\lambda_{j4}x_{ij4}) \le (\theta_3 x_{io3}+\theta_4 x_{io4}), \qquad i=1,...,m,\\
&(1-\alpha_i)\sum_{j=1}^{n}(\lambda_{j1}x_{ij1}+\lambda_{j2}x_{ij2})+\alpha_i\sum_{j=1}^{n}(\lambda_{j3}x_{ij3}+\lambda_{j4}x_{ij4}) \le (1-\alpha_i)(\theta_1 x_{io1}+\theta_2 x_{io2})+\alpha_i(\theta_3 x_{io3}+\theta_4 x_{io4}), i=1,...,m,\\
&\quad \sum_{j=1}^{n}(\lambda_{j1}y_{rj1}+\lambda_{j2}y_{rj2}) \ge (y_{ro1}+y_{ro2}), \qquad r=1,...,s,\\
&(1-\beta_r)\sum_{j=1}^{n}(\lambda_{j3}y_{rj3}+\lambda_{j4}y_{rj4})+\beta_r\sum_{j=1}^{n}(\lambda_{j1}y_{rj1}+\lambda_{j2}y_{rj2}) \ge (1-\beta_r)(y_{ro3}+y_{ro4})+\beta_r(y_{ro1}+y_{ro2}), \qquad r=1,...,s,\\
&\sum_{j=1}^{n}(\lambda_{j3}z_{kj3}+\lambda_{j4}z_{kj4}) \le (z_{ko3}+z_{ko4}), \qquad k=1,...,K,\\
&(1-\gamma_k)\sum_{j=1}^{n}(\lambda_{j1}z_{kj1}+\lambda_{j2}z_{kj2})+\gamma_k\sum_{j=1}^{n}(\lambda_{j3}z_{kj3}+\lambda_{j4}z_{kj4}) \le (1-\gamma_k)(z_{ko1}+z_{ko2})+\gamma_k(z_{ko3}+z_{ko4}), \quad k=1,...,K,\\
&\sum_{j=1}^{n}(\lambda_{j1}z_{kj1}+\lambda_{j2}z_{kj2}) \ge (z_{ko1}+z_{ko2}), \quad k=1,...,K,\\
&(1-\gamma_k)\sum_{j=1}^{n}(\lambda_{j3}z_{kj3}+\lambda_{j4}z_{kj4})+\gamma_k\sum_{j=1}^{n}(\lambda_{j1}z_{kj1}+\lambda_{j2}z_{kj2}) \ge (1-\gamma_k)(z_{ko3}+z_{ko4})+\gamma_k(z_{ko1}+z_{ko2}), \quad k=1,...,K,\\
&\quad 0 \le \lambda_{j1} \le \lambda_{j2} \le \lambda_{j3} \le \lambda_{j4}, j=1,2,...,n, \varepsilon \le \theta_1 \le \theta_2 \le \theta_3 \le \theta_4, \theta_4 \le 1, \varepsilon > 0,
\end{aligned}
\tag{11}
$$

in which $\alpha_i \in [0,1), \forall i$, $\beta_r \in [0,1], \forall r$ and $\gamma_k \in [0,1], \forall k$ and they show that acceptance degrees of decision maker for violating interval inequality constraints. Parameters within $[0, 0.5)$, $(0.5,1]$ and 0.5 show the risk aversion of decision maker, the risk liking of decision maker and risk neutrality of decision maker, respectively. We consider the different acceptance degrees for constraints in modeling. Also, the first objective of model (11) is the right side of the objective interval of model (10) while the middle point of the objective interval of model (10) is presented as the second objective of model (11).

To solve model (11), we use the weighted sum method because of its simplicity and popularity. To more illustrate, we employ an additional parameter $\mu \in [0,1]$ to compute bi-objective problem (11) and to integrate the objectives by following [2]. Actually, μ denotes a degree of importance to the objectives that is determined by managers and decision makers.

$$
\begin{aligned}
&E_I^* = Min \quad \mu\, \frac{1}{2}(\theta_3+\theta_4)+(1-\mu)\frac{1}{4}(\theta_1+\theta_2+\theta_3+\theta_4)\\
&s.t. \quad \sum_{j=1}^{n}(\lambda_{j3}x_{ij3}+\lambda_{j4}x_{ij4}) \le (\theta_3 x_{io3}+\theta_4 x_{io4}), \qquad i=1,\dots,m,\\
&(1-\alpha_i)\sum_{j=1}^{n}(\lambda_{j1}x_{ij1}+\lambda_{j2}x_{ij2})+\alpha_i\sum_{j=1}^{n}(\lambda_{j3}x_{ij3}+\lambda_{j4}x_{ij4}) \le (1-\alpha_i)(\theta_1 x_{io1}+\theta_2 x_{io2})+\alpha_i(\theta_3 x_{io3}+\theta_4 x_{io4}), i=1,\dots,m,\\
&\qquad \sum_{j=1}^{n}(\lambda_{j1}y_{rj1}+\lambda_{j2}y_{rj2}) \ge (y_{ro1}+y_{ro2}), \qquad r=1,\dots,s,\\
&(1-\beta_r)\sum_{j=1}^{n}(\lambda_{j3}y_{rj3}+\lambda_{j4}y_{rj4})+\beta_r\sum_{j=1}^{n}(\lambda_{j1}y_{rj1}+\lambda_{j2}y_{rj2}) \ge (1-\beta_r)(y_{ro3}+y_{ro4})+\beta_r(y_{ro1}+y_{ro2}), \qquad r=1,\dots,s,\\
&\sum_{j=1}^{n}(\lambda_{j3}z_{kj3}+\lambda_{j4}z_{kj4}) \le (z_{ko3}+z_{ko4}), \qquad k=1,\dots,K,\\
&(1-\gamma_k)\sum_{j=1}^{n}(\lambda_{j1}z_{kj1}+\lambda_{j2}z_{kj2})+\gamma_k\sum_{j=1}^{n}(\lambda_{j3}z_{kj3}+\lambda_{j4}z_{kj4}) \le (1-\gamma_k)(z_{ko1}+z_{ko2})+\gamma_k(z_{ko3}+z_{ko4}), \quad k=1,\dots,K,\\
&\sum_{j=1}^{n}(\lambda_{j1}z_{kj1}+\lambda_{j2}z_{kj2}) \ge (z_{ko1}+z_{ko2}), \quad k=1,\dots,K,\\
&(1-\gamma_k)\sum_{j=1}^{n}(\lambda_{j3}z_{kj3}+\lambda_{j4}z_{kj4})+\gamma_k\sum_{j=1}^{n}(\lambda_{j1}z_{kj1}+\lambda_{j2}z_{kj2}) \ge (1-\gamma_k)(z_{ko3}+z_{ko4})+\gamma_k(z_{ko1}+z_{ko2}), \quad k=1,\dots,K,\\
&\qquad 0 \le \lambda_{j1} \le \lambda_{j2} \le \lambda_{j3} \le \lambda_{j4},\ j=1,2,\dots,n, \varepsilon \le \theta_1 \le \theta_2 \le \theta_3 \le \theta_4, \theta_4 \le 1, \varepsilon > 0.
\end{aligned} \tag{12}
$$

Definition 2. The unit under consideration is called efficient in model (12) if and only if $E_I^* = 1$ for given $\alpha_i, \beta_r, \gamma_k$ and μ. Otherwise, it is inefficient.

Also, the fuzzy efficiency can be appended as the optimal value $(\theta_1^*, \theta_2^*, \theta_3^*, \theta_4^*)$ resulted from model (12)

Theorem 1. The optimal value $(\lambda_{j1}^*, \lambda_{j2}^*, \lambda_{j3}^*, \lambda_{j4}^*, \theta_1^*, \theta_2^*, \theta_3^*, \theta_4^*)$ obtained from solving model (12) is a weak Pareto solution of model (11).

Proof. For specific values $\alpha_i, \beta_r, \gamma_k$ and μ, assume $(\lambda_{j1}^*, \lambda_{j2}^*, \lambda_{j3}^*, \lambda_{j4}^*, \theta_1^*, \theta_2^*, \theta_3^*, \theta_4^*)$ is the optimal solution of model (12) but not the weak Pareto solution of model (11). Accordingly, there is $(\lambda'_{j1}, \lambda'_{j2}, \lambda'_{j3}, \lambda'_{j4}, \theta'_1, \theta'_2, \theta'_3, \theta'_4)$ in model (11) that $(\frac{1}{2}(\theta'_3+\theta'_4), \frac{1}{4}(\theta'_1+\theta'_2+\theta'_3+\theta'_4))$ is less than $(\frac{1}{2}(\theta_3^*+\theta_4^*), \frac{1}{4}(\theta_1^*+\theta_2^*+\theta_3^*+\theta_4^*))$. As can be found from models (11) and (12), their constraints are similar. Therefore, $(\lambda'_{j1}, \lambda'_{j2}, \lambda'_{j3}, \lambda'_{j4}, \theta'_1, \theta'_2, \theta'_3, \theta'_4)$ is a feasible solution for model (12) that entails a contradiction due to the following statement:

$$
\mu\frac{1}{2}(\theta'_3+\theta'_4), +(1-\mu)\frac{1}{4}(\theta'_1+\theta'_2+\theta'_3+\theta'_4)) < \mu\frac{1}{2}(\theta_3^*+\theta_4^*), +(1-\mu)\frac{1}{4}(\theta_1^*+\theta_2^*+\theta_3^*+\theta_4^*).
$$

Actually, it indicates $(\lambda_{j1}^*, \lambda_{j2}^*, \lambda_{j3}^*, \lambda_{j4}^*, \theta_1^*, \theta_2^*, \theta_3^*, \theta_4^*)$ is not the optimal solution of model (12). Consequently, the optimal value achieved from model (12) is a weak Pareto solution of model (11). □

In the next section, the aforementioned approach is rewritten to analyze the performance in output orientation.

3.1 Output-oriented FFDEA model with the acceptance degree of the violated fuzzy constraints

By taking into consideration notations and symbols explained before and contemplating the trapezoidal fuzzy variable $\tilde{\varphi}=(\varphi_1,\varphi_2,\varphi_3,\varphi_4)$ as the maximum expansion of desirable outputs, the next output-oriented FFDEA model with acceptance degrees of the violated fuzzy constraints is introduced:

$$
\begin{aligned}
& Max \ \ \tilde{\varphi} \\
& s.t. \sum_{j=1}^{n} \tilde{\lambda}_j \tilde{x}_{ij} \le \tilde{x}_{io}, \quad i=1,\dots,m, \\
& \sum_{j=1}^{n} \tilde{\lambda}_j \tilde{y}_{rj} \ge \tilde{\varphi}\tilde{y}_{ro}, \quad r=1,\dots,s, \\
& \sum_{j=1}^{n} \tilde{\lambda}_j \tilde{z}_{kj} = \tilde{z}_{ko}, \quad k=1,\dots,K, \\
& \tilde{\lambda}_j \ge 0, \quad j=1,2,\dots,n.
\end{aligned}
\tag{13}
$$

Model (13) equals to the interval DEA model (14) owing to before-mentioned concepts.

$$
\begin{aligned}
& Max \ \ E(\tilde{\varphi}) \\
& s.t. \quad E(\sum_{j=1}^{n} \tilde{\lambda}_j \tilde{x}_{ij}) \le E(\tilde{x}_{io}), \quad i=1,\dots,m, \\
& E(\sum_{j=1}^{n} \tilde{\lambda}_j \tilde{y}_{rj}) \ge E(\tilde{\varphi}\tilde{y}_{ro}), \quad r=1,\dots,s, \\
& E(\sum_{j=1}^{n} \tilde{\lambda}_j \tilde{z}_{kj}) = E(\tilde{z}_{ko}), \quad k=1,\dots,K, \\
& \tilde{\lambda}_j \ge 0, \quad j=1,2,\dots,n.
\end{aligned}
\tag{14}
$$

The equivalent form of model (14) is written as follows:

$$
\begin{aligned}
& Max \ [\frac{1}{2}(\varphi_1+\varphi_2),\frac{1}{2}(\varphi_3+\varphi_4)] \\
& s.t. \ \left[\frac{1}{2}\sum_{j=1}^{n}(\lambda_{j1}x_{ij1}+\lambda_{j2}x_{ij2}),\frac{1}{2}\sum_{j=1}^{n}(\lambda_{j3}x_{ij3}+\lambda_{j4}x_{ij4})\right] \le \left[\frac{1}{2}(x_{io1}+x_{io2}),\frac{1}{2}(x_{io3}+x_{io4})\right], \ i=1,\dots,m, \\
& \left[\frac{1}{2}\sum_{j=1}^{n}(\lambda_{j1}y_{rj1}+\lambda_{j2}y_{rj2}),\frac{1}{2}\sum_{j=1}^{n}(\lambda_{j3}y_{rj3}+\lambda_{j4}y_{rj4})\right] \ge \left[\frac{1}{2}(\varphi_1 y_{ro1}+\varphi_2 y_{ro2}),\frac{1}{2}(\varphi_3 y_{ro3}+\varphi_4 y_{ro4})\right], \ r=1,\dots,s, \\
& \left[\frac{1}{2}\sum_{j=1}^{n}(\lambda_{j1}z_{kj1}+\lambda_{j2}z_{kj2}),\frac{1}{2}\sum_{j=1}^{n}(\lambda_{j3}z_{kj3}+\lambda_{j4}z_{kj4})\right] = \left[\frac{1}{2}(z_{ko1}+z_{ko2}),\frac{1}{2}(z_{ko3}+z_{ko4})\right], k=1,\dots,K, \\
& 0 \le \lambda_{j1} \le \lambda_{j2} \le \lambda_{j3} \le \lambda_{j4}, \ j=1,2,\dots,n, \varphi_1 \le \varphi_2 \le \varphi_3 \le \varphi_4, \varphi_1 \ge 1.
\end{aligned}
\tag{15}
$$

Now, by using the order relationships and definitions briefly explained in this study that have also been presented in [1, 2], the interval DEA model (15) is substituted with the bi-objective DEA model (16).

$$
\begin{aligned}
&Max \quad \frac{1}{2}(\varphi_1+\varphi_2)\\
&Max \quad \frac{1}{4}(\varphi_1+\varphi_2+\varphi_3+\varphi_4)\\
&s.t. \quad \sum_{j=1}^{n}(\lambda_{j3}x_{ij3}+\lambda_{j4}x_{ij4}) \le (x_{io3}+x_{io4}), \quad i=1,...,m,\\
&(1-\alpha_i)\sum_{j=1}^{n}(\lambda_{j1}x_{ij1}+\lambda_{j2}x_{ij2})+\alpha_i\sum_{j=1}^{n}(\lambda_{j3}x_{ij3}+\lambda_{j4}x_{ij4}) \le (1-\alpha_i)(x_{io1}+x_{io2})+\alpha_i(x_{io3}+x_{io4}), i=1,...,m,\\
&\qquad \sum_{j=1}^{n}(\lambda_{j1}y_{rj1}+\lambda_{j2}y_{rj2}) \ge (\varphi_1 y_{ro1}+\varphi_2 y_{ro2}), \quad r=1,...,s,\\
&(1-\beta_r)\sum_{j=1}^{n}(\lambda_{j3}y_{rj3}+\lambda_{j4}y_{rj4})+\beta_r\sum_{j=1}^{n}(\lambda_{j1}y_{rj1}+\lambda_{j2}y_{rj2}) \ge (1-\beta_r)(\varphi_3 y_{ro3}+\varphi_4 y_{ro4})+\beta_r(\varphi_1 y_{ro1}+\varphi_2 y_{ro2}), \;\; r=1,...,s,\\
&\sum_{j=1}^{n}(\lambda_{j3}z_{kj3}+\lambda_{j4}z_{kj4}) \le (z_{ko3}+z_{ko4}), \quad k=1,...,K,\\
&(1-\gamma_k)\sum_{j=1}^{n}(\lambda_{j1}z_{kj1}+\lambda_{j2}z_{kj2})+\gamma_k\sum_{j=1}^{n}(\lambda_{j3}z_{kj3}+\lambda_{j4}z_{kj4}) \le (1-\gamma_k)(z_{ko1}+z_{ko2})+\gamma_k(z_{ko3}+z_{ko4}), \quad k=1,...,K,\\
&\sum_{j=1}^{n}(\lambda_{j1}z_{kj1}+\lambda_{j2}z_{kj2}) \ge (z_{ko1}+z_{ko2}), \quad k=1,...,K,\\
&(1-\gamma_k)\sum_{j=1}^{n}(\lambda_{j3}z_{kj3}+\lambda_{j4}z_{kj4})+\gamma_k\sum_{j=1}^{n}(\lambda_{j1}z_{kj1}+\lambda_{j2}z_{kj2}) \ge (1-\gamma_k)(z_{ko3}+z_{ko4})+\gamma_k(z_{ko1}+z_{ko2}), \quad k=1,...,K,\\
&\qquad 0 \le \lambda_{j1} \le \lambda_{j2} \le \lambda_{j3} \le \lambda_{j4}, \; j=1,2,...,n, \varphi_1 \le \varphi_2 \le \varphi_3 \le \varphi_4, \varphi_1 \ge 1.
\end{aligned} \tag{16}
$$

Similar to input-oriented FFDEA model, $\alpha_i \in [0,1], \forall i$, $\beta_r \in [0,1), \forall r$ and $\gamma_k \in [0,1], \forall k$ denote the acceptance degrees of constraints violated by the decision maker.

Model (16) can also be transformed into the following parametric linear DEA model by aggregation two objectives and using the parameter $\mu \in [0,1]$ in which μ shows the weighting factor of objectives.

$$
\begin{aligned}
&\varphi^* = Max \quad \mu\,\frac{1}{2}(\varphi_1+\varphi_2)+(1-\mu)\frac{1}{4}(\varphi_1+\varphi_2+\varphi_3+\varphi_4)\\
&s.t. \quad \sum_{j=1}^{n}(\lambda_{j3}x_{ij3}+\lambda_{j4}x_{ij4}) \le (x_{io3}+x_{io4}), \qquad i=1,...,m,\\
&(1-\alpha_i)\sum_{j=1}^{n}(\lambda_{j1}x_{ij1}+\lambda_{j2}x_{ij2})+\alpha_i\sum_{j=1}^{n}(\lambda_{j3}x_{ij3}+\lambda_{j4}x_{ij4}) \le (1-\alpha_i)(x_{io1}+x_{io2})+\alpha_i(x_{io3}+x_{io4}), i=1,...,m,\\
&\qquad \sum_{j=1}^{n}(\lambda_{j1}y_{rj1}+\lambda_{j2}y_{rj2}) \ge (\varphi_1 y_{ro1}+\varphi_2 y_{ro2}), \qquad r=1,...,s,\\
&(1-\beta_r)\sum_{j=1}^{n}(\lambda_{j3}y_{rj3}+\lambda_{j4}y_{rj4})+\beta_r\sum_{j=1}^{n}(\lambda_{j1}y_{rj1}+\lambda_{j2}y_{rj2}) \ge (1-\beta_r)(\varphi_3 y_{ro3}+\varphi_4 y_{ro4})+\beta_r(\varphi_1 y_{ro1}+\varphi_2 y_{ro2}), \;\; r=1,...,s,\\
&\sum_{j=1}^{n}(\lambda_{j3}z_{kj3}+\lambda_{j4}z_{kj4}) \le (z_{ko3}+z_{ko4}), \quad k=1,...,K,\\
&(1-\gamma_k)\sum_{j=1}^{n}(\lambda_{j1}z_{kj1}+\lambda_{j2}z_{kj2})+\gamma_k\sum_{j=1}^{n}(\lambda_{j3}z_{kj3}+\lambda_{j4}z_{kj4}) \le (1-\gamma_k)(z_{ko1}+z_{ko2})+\gamma_k(z_{ko3}+z_{ko4}), \quad k=1,...,K,\\
&\sum_{j=1}^{n}(\lambda_{j1}z_{kj1}+\lambda_{j2}z_{kj2}) \ge (z_{ko1}+z_{ko2}), \quad k=1,...,K,\\
&(1-\gamma_k)\sum_{j=1}^{n}(\lambda_{j3}z_{kj3}+\lambda_{j4}z_{kj4})+\gamma_k\sum_{j=1}^{n}(\lambda_{j1}z_{kj1}+\lambda_{j2}z_{kj2}) \ge (1-\gamma_k)(z_{ko3}+z_{ko4})+\gamma_k(z_{ko1}+z_{ko2}), \quad k=1,...,K,\\
&\qquad 0 \le \lambda_{j1} \le \lambda_{j2} \le \lambda_{j3} \le \lambda_{j4}, \; j=1,2,...,n, \varphi_1 \le \varphi_2 \le \varphi_3 \le \varphi_4, \varphi_1 \ge 1.
\end{aligned} \tag{17}
$$

In this case, the crisp efficiency is defined as $E_o^* = \dfrac{1}{\varphi^*}$.

Definition 3. The entity under investigation in model (17) is called efficient if and only if $E_o^* = 1$ for given $\alpha_i, \beta_r, \gamma_k$ and μ. Otherwise, it is inefficient.

Moreover, the fuzzy efficiency can be defined as the optimal value $(\dfrac{1}{\varphi_4^*}, \dfrac{1}{\varphi_3^*}, \dfrac{1}{\varphi_2^*}, \dfrac{1}{\varphi_1^*})$ achieved from model (17).

Theorem 2. The optimal value $(\lambda_{j1}^*, \lambda_{j2}^*, \lambda_{j3}^*, \lambda_{j4}^*, \varphi_1^*, \varphi_2^*, \varphi_3^*, \varphi_4^*)$ obtained from solving model (17) is a weak Pareto solution of model (16).

Proof. The same vein of Theorem 1 can be proved. □

It is clear that by changing $\alpha_i, \beta_r, \gamma_k$ and μ, on the basis of results, managers can be better apprised about the essence of the performance.

Furthermore, the projection points can be obtained in models (12) and (17) as follows:

$$\begin{aligned}
&(\bar{x}_{i1}, \bar{x}_{i2}, \bar{x}_{i3}, \bar{x}_{i4}) = (\sum_{j=1}^{n} \lambda_{j1}^* x_{ij1}, \sum_{j=1}^{n} \lambda_{j2}^* x_{ij2}, \sum_{j=1}^{n} \lambda_{j3}^* x_{ij3}, \sum_{j=1}^{n} \lambda_{j4}^* x_{ij4}),\\
&(\bar{y}_{r1}, \bar{y}_{r2}, \bar{y}_{r3}, \bar{y}_{r4}) = (\sum_{j=1}^{n} \lambda_{j1}^* y_{rj1}, \sum_{j=1}^{n} \lambda_{j2}^* y_{rj2}, \sum_{j=1}^{n} \lambda_{j3}^* y_{rj3}, \sum_{j=1}^{n} \lambda_{j4}^* y_{rj4}),\\
&(\bar{z}_{k1}, \bar{z}_{k2}, \bar{z}_{k3}, \bar{z}_{k4}) = (\sum_{j=1}^{n} \lambda_{j1}^* z_{kj1}, \sum_{j=1}^{n} \lambda_{j2}^* z_{kj2}, \sum_{j=1}^{n} \lambda_{j3}^* z_{kj3}, \sum_{j=1}^{n} \lambda_{j4}^* z_{kj4}),
\end{aligned} \tag{18}$$

in which $(\lambda_{j1}^*, \lambda_{j2}^*, \lambda_{j3}^*, \lambda_{j4}^*)$ show the optimal values obtained from models (12) and (17).

Notice that fuzzy constraints are not always satisfied because of their uncertainty nature. But decision makers can admit them among some degree of acceptance. Therefore, parameters α_i, β_r and γ_k are provided to address this issue. Accordingly, incorporating the acceptance degrees of breached fuzzy constraints in fuzzy decision-making problems such as fuzzy DEA models is rational and essential. A graphical design of the procedure is shown in Figure 1.

Fig.1. A graphical design of the approach

Note that the presented models can also be extended under the variable returns to scale (VRS) technology as illustrated in Appendix.

In the following, an example is provided to clarify the methods proposed in this research.

4. An application

In this section, an application dealt with in [50] is exposed to demonstrate the efficacy of the provided approach. Actually, the performance of 23 countries from the energy dependency case is addressed while all data, except substituted fuel that considered as real values, are deemed as triangular fuzzy numbers. As shown in Table 2, it has been considered one input, quantity of energy $(\tilde{x}_1)$, three desirable outputs, gross electricity $(\tilde{y}_1)$, average annual emissions $(\tilde{y}_2)$ and substituted fuel (y_3) and one undesirable output, CO_2 equivalent $(\tilde{z}_1)$.

Table 2. Dataset on energy dependency

To estimate the relative efficiency of these countries, model (12) is computed for different situations. Columns 2-8 of Table 3 show the findings for $\alpha_i = \beta_r = \gamma_k = \frac{1}{2}$ (decision maker is the risk neutral) and various rates of μ denoted as follows:

-Case 1: $\alpha_i = \frac{1}{2}, \beta_r = \frac{1}{2}, \gamma_k = \frac{1}{2}, \mu = 0$

-Case 2: $\alpha_i = \frac{1}{2}, \beta_r = \frac{1}{2}, \gamma_k = \frac{1}{2}, \mu = \frac{1}{10}$

-Case 3: $\alpha_i = \frac{1}{2}, \beta_r = \frac{1}{2}, \gamma_k = \frac{1}{2}, \mu = \frac{3}{10}$

-Case 4: $\alpha_i = \frac{1}{2}, \beta_r = \frac{1}{2}, \gamma_k = \frac{1}{2}, \mu = \frac{1}{2}$

-Case 5: $\alpha_i = \frac{1}{2}, \beta_r = \frac{1}{2}, \gamma_k = \frac{1}{2}, \mu = \frac{7}{10}$

-Case 6: $\alpha_i = \frac{1}{2}, \beta_r = \frac{1}{2}, \gamma_k = \frac{1}{2}, \mu = \frac{9}{10}$

-Case 7: $\alpha_i = \frac{1}{2}, \beta_r = \frac{1}{2}, \gamma_k = \frac{1}{2}, \mu = 1.$

Table 3. Results for different values μ

As can be seen, by increasing the value μ, the efficiency scores raise or are without change. For instance, these changes are depicted in Figure 2 specifically for Lithuania. As evidenced in this country, more values μ result more efficiency scores.

At this stage, we assess the efficiency scores using the provided approach and taking $\beta_r = \gamma_k = \mu = \frac{1}{2}$ and different values for α_i as follows:

-Case 8: $\alpha_i = 0, \beta_r = \frac{1}{2}, \gamma_k = \frac{1}{2}, \mu = \frac{1}{2}$

-Case 9: $\alpha_i = \frac{1}{10}, \beta_r = \frac{1}{2}, \gamma_k = \frac{1}{2}, \mu = \frac{1}{2}$

-Case 10: $\alpha_i = \frac{2}{10}, \beta_r = \frac{1}{2}, \gamma_k = \frac{1}{2}, \mu = \frac{1}{2}$

-Case 11: $\alpha_i = \frac{3}{10}, \beta_r = \frac{1}{2}, \gamma_k = \frac{1}{2}, \mu = \frac{1}{2}$

-Case 12: $\alpha_i = \frac{1}{2}, \beta_r = \frac{1}{2}, \gamma_k = \frac{1}{2}, \mu = \frac{1}{2}$

-Case 13: $\alpha_i = \frac{7}{10}, \beta_r = \frac{1}{2}, \gamma_k = \frac{1}{2}, \mu = \frac{1}{2}$

-Case 14: $\alpha_i = \frac{8}{10}, \beta_r = \frac{1}{2}, \gamma_k = \frac{1}{2}, \mu = \frac{1}{2}$

-Case 15: $\alpha_i = \frac{9}{10}, \beta_r = \frac{1}{2}, \gamma_k = \frac{1}{2}, \mu = \frac{1}{2}$.

Outcomes are displayed in Table 4. As can be found from Table 4, the performance reduces or is without variation by expanding α_i. For more clarity, the efficiency variations for five countries as samples are depicted in Figure 3. Actually, by increasing the risk level α_i, the efficiency scores reduce or will be without change. Therefore, the efficiency findings may change with the level variation of parameters. As presented in Tables 3 and 4, for different levels μ and α_i, ten countries, Austria, Estonia, Germany, Greece, Italy, Latvia, Poland, Slovakia, Sweden and United Kingdom were determined as efficient by the score one. Also, Belgium gained the least efficiency score in all cases investigated. It is evident that by using the designed approach, experts and managers can attain the substantial and flexible results in order to decision-making to enhance the performance.

Fig. 2. Efficiency scores for Lithuania

Expressions (18) are also applied to estimate the projection points of fuzzy input and output data. Results are revealed in Table 5 for $\alpha_i = \frac{1}{2}, \beta_r = \frac{1}{2}, \gamma_k = \frac{1}{2}$ and $\mu = \frac{1}{2}$. As can be seen the projection points of efficient countries are equal to themselves, but variations are visible in input-output values for inefficient countries. For instance, consider Spain in which its input changes from (3.230, 3.260, 3.356) into (3.102, 3.137, 3.188). It should be mentioned that there are also some changes for other inefficient countries that can be seen by comparing two Tables 2 and 5.

Table 4. Results for different values α

Fig. 3. Efficiency scores for five sample countries at different levels α_i

In this stage, model (7) is computed for midpoints of factors presented in Table 2 in order to compare the consequences gained from model (12) with the existing models. Achievements appear in the last column of Table 3. As shown, five countries, Estonia, Germany, Latvia, Poland and Sweden, are identified as efficient in model (7) whilst 10 countries were identified as efficient in the proposed method. Moreover, there are differences between the efficiency scores found for inefficient countries. Certainly, by using the proposed approach, managers will be more informed about the performance of countries and its changes when there are uncertainty in data and fuzzy constraint violations accepted with some degrees.

For investigation the performance of countries from the aspect of maximum expansion of the desirable outputs, model (17) is computed. Columns 9 and 10 of Table 3 show findings for two cases, $\alpha_i = \frac{1}{2}, \beta_r = \frac{1}{2}, \gamma_k = \frac{1}{2}, \mu = 0$ and $\alpha_i = \frac{1}{2}, \beta_r = \frac{1}{2}, \gamma_k = \frac{1}{2}, \mu = \frac{9}{10}$. Similar to results of model (12), 10 countries are efficient in this case and Belgium is the most inefficient country. However, there are dissimilarities in efficiency values specified by cases and models.

Table 5. Projection points of input-output measures

For further investigation, the consequences obtained from the approaches presented in [50, 51] are given in Table 6. Wang and Chen [51] provided the fuzzy expected value approach to analyze the efficiency of entities from different aspects when fuzzy inputs-outputs measures are presented. Ghasemi et al. [50] applied the expected value approach in generalized DEA model to integrate three approaches, including CCR (Charnes, Cooper and Rhodes), BCC (Banker, Charnes and Cooper) and FDH (Free Disposal Hull) under uncertainty.

Table 6. The efficiency values obtained from fuzzy approaches provided in [50, 51]

As can be seen in Table 6, three countries, including Germany, Latvia and Sweden were determined as efficient using the approach proposed by Wang and Chen [51] from optimistic point of view and also Ghasemi et al.'s approach [50]. These countries are also ascertained as efficient applying the introduced approaches of this study in all cases considered. Nevertheless, disparities can be seen among the efficiency scores obtained from approaches provided in [50, 51] with the introduced model under the investigated cases. It should be noted in models presented in [50, 51], undesirable outputs were not included and CO_2 equivalent was treated as an input. Also, projection points were not obtained. However, weakly disposable undesirable outputs have been incorporated in the presented models and fuzzy targets are achieved using the proposed models. Furthermore, in this research, the acceptance degree of the violated fuzzy constraints has been considered that leads to more flexibility.

Note that in this example, the findings of both models (12) and (17) have been given to more clarify and demonstrate the approaches. Nevertheless, decision makers can choice input- or output-oriented FFDEA models due to their ability to control the input and output measures and their purposes.

Also, it is apparent that in comparison with the performance analysis of DMUs taking precise data into account and also utilizing some existing fuzzy DEA approaches, the results of our proposed approach are more robust and rational when imprecise data and the approval degree of the contrary fuzzy constraints are detected.

Actually, the evaluation of organizational performance has long been a pivotal area of focus for managerial professionals. The rationale behind this pertains to identifying DMUs that exhibit desirable performance as a standard for inefficient DMUs. Furthermore, estimating targets for individual entities bears significant importance. DEA is an indisputably potent and efficacious methodology that can be applied for performance evaluation, prioritization, and comparison in a diverse range of scenarios. It is important to acknowledge that a number of real-world issues involve uncertain data and undesirable measures. Therefore, addressing the fuzzy performance and fuzzy targets of entities with undesirable outputs in a fuzzy environment is essential to obtain rational consequences. Also, in accordance with Dong and Wan [1], the incorporation of the decision maker's acceptance level regarding the potential violation of fuzzy constraints is a crucial aspect to contemplate. Accordingly, the present research suggests fully fuzzy DEA models in the envelopment form to analyze the performance of entities with fuzzy data and undesirable outputs while the decision maker's acceptance level has been included. Fuzzy targets related to inputs, desirable outputs and undesirable outputs are, moreover, estimated. To illustrate more, all data and variables were considered as fuzzy measures in the proposed technique and the weak disposability assumption was deemed for undesirable outputs. The present investigation has yielded a novel fully fuzzy DEA approach which exhibits the ability to be employed for all categories of decision makers and managers. Based on the application of the suggested framework in the preceding section's illustrative analysis, noteworthy outcomes can be discerned, which could offer useful insights for organizational decision-makers. As shown in the previous section, the proposed approach can be used to assess the energy efficiency of countries. By increasing the risk level, the efficiency values decreased or were without change. The variability in levels of parameters significantly impacted the efficiency outcomes. Furthermore, this methodology has the potential to be implemented in various applications where data is characterized by ambiguity and vagueness, and weakly disposable undesirable outputs are presented. Furthermore, managers will have the capacity to acquire pertinent information of fuzzy target points for making better decisions.

5. Conclusions

Due to the existence of imprecise information in many real world situations, a fully fuzzy DEA technique incorporating the acceptance degree of the violated fuzzy constraints was proposed in this paper to estimate the efficiency of entities. Weakly disposable undesirable outputs were taken into account owing to the presence of them in many real applications. Actually, input- and output-oriented methods were provided to analyze the performance when the acceptance degree of the violated fuzzy constraints is included. The planned FFDEA approach was transformed into the parametric linear DEA problem using the interval expectation of trapezoidal fuzzy numbers and the order relationship of trapezoidal fuzzy numbers. An example was also applied to clarify and demonstrate the utility of the disclosed plan in this study.

The present investigation is subject to several limitations which restrict its scope and generalizability. The outcomes of the models are contingent upon the selection of performance measures. Consequently, an alternative collection of outcomes and evaluations may be produced by utilizing a distinct array of indicators. The small number of DMUs is another limitation of this research. As alterations occur in the number of DMUs, there is a consequential modification in the corresponding levels of performance and targets. Therefore, it follows that an augmentation in the number of DMUs may potentially yield divergent outcomes and analyses. Actually, the energy dependency consideration of more countries with more indicators is essential to address.

This research renders a series of ideas to appraise the fuzzy energy efficiency of countries, encompassing the optimization of models, the definition of fuzzy targets, and the all-embracing evaluation. These concepts have the potential for broad applicability areas. The results show that the efficiency values might change when parameters, which are the weights of the objectives and acceptance degrees of violated fuzzy constraints, alter. This study is among primary works to analyze the relative efficiency of firms in a fully fuzzy environment that includes the acceptance degree of decision makers to violate the fuzzy constraints. It is clear that the acceptance degree of the decision maker can evince the preference of the manager and the risk standpoint and by incorporating it; the findings of the performance assessment more correspond with reality. The models proposed in this research were radial; therefore, they can be extended to non-radial and non-oriented fuzzy DEA models. An alternative interesting topic for future study is performance analysis of processes with fuzzy data in multiple periods of time and considering the acceptance degree of the violated fuzzy constraints. Also, the extension of the suggested approach to assess the efficiency of multi-stage systems can be undertaken as further research.

Appendix A.

To reformulate model (7) under the VRS, $\sum_{j=1}^{n}\lambda_j = 1$ is added to it. Also, the FFDEA model (8) is redefined as follows:

$$
\begin{aligned}
& Min\ \ \tilde{\theta} \\
& s.t. \sum_{j=1}^{n} \tilde{\lambda}_j \tilde{x}_{ij} \le \tilde{\theta}\tilde{x}_{io}, \qquad i = 1,\dots,m, \\
& \quad \sum_{j=1}^{n} \tilde{\lambda}_j \tilde{y}_{rj} \ge \tilde{y}_{ro}, \qquad r = 1,\dots,s, \\
& \quad \sum_{j=1}^{n} \tilde{\lambda}_j \tilde{z}_{kj} = \tilde{z}_{ko}, \qquad k = 1,\dots,K, \\
& \quad \sum_{j=1}^{n} \tilde{\lambda}_j = \tilde{1}, \\
& \quad \tilde{\lambda}_j \ge 0,\ j = 1,2,\dots,n.
\end{aligned} \tag{A.1}
$$

In which $\tilde{1} = (1,1,1,1)$. Furthermore, model (A.1) is equal to the subsequent interval problem:

$$
\begin{aligned}
& Min\ \ E(\tilde{\theta}) \\
& s.t. \quad E(\sum_{j=1}^{n} \tilde{\lambda}_j \tilde{x}_{ij}) \le E(\tilde{\theta}\tilde{x}_{io}), \qquad i = 1,\dots,m, \\
& \qquad E(\sum_{j=1}^{n} \tilde{\lambda}_j \tilde{y}_{rj}) \ge E(\tilde{y}_{ro}), \qquad r = 1,\dots,s, \\
& \qquad E(\sum_{j=1}^{n} \tilde{\lambda}_j) = E(\tilde{1}), \\
& \qquad \tilde{\lambda}_j \ge 0,\ j = 1,2,\dots,n.
\end{aligned} \tag{A.2}
$$

Also, model (10) can be substituted with the following model under the VRS according to the interval expectation definition and arithmetic operations on fuzzy numbers:

$$
\begin{aligned}
&Min\ \ [\tfrac{1}{2}(\theta_1+\theta_2),\tfrac{1}{2}(\theta_3+\theta_4)]\\
&s.t.\ \ \left[\frac{1}{2}\sum_{j=1}^{n}(\lambda_{j1}x_{ij1}+\lambda_{j2}x_{ij2}),\frac{1}{2}\sum_{j=1}^{n}(\lambda_{j3}x_{ij3}+\lambda_{j4}x_{ij4})\right]\le\left[\frac{1}{2}(\theta_1x_{io1}+\theta_2x_{io2}),\frac{1}{2}(\theta_3x_{io3}+\theta_4x_{io4})\right],\ i=1,\dots,m,\\
&\quad\left[\frac{1}{2}\sum_{j=1}^{n}(\lambda_{j1}y_{rj1}+\lambda_{j2}y_{rj2}),\frac{1}{2}\sum_{j=1}^{n}(\lambda_{j3}y_{rj3}+\lambda_{j4}y_{rj4})\right]\ge\left[\frac{1}{2}(y_{ro1}+y_{ro2}),\frac{1}{2}(y_{ro3}+y_{ro4})\right],\ r=1,\dots,s,\\
&\quad\left[\frac{1}{2}\sum_{j=1}^{n}(\lambda_{j1}z_{kj1}+\lambda_{j2}z_{kj2}),\frac{1}{2}\sum_{j=1}^{n}(\lambda_{j3}z_{kj3}+\lambda_{j4}z_{kj4})\right]=\left[\frac{1}{2}(z_{ko1}+z_{ko2}),\frac{1}{2}(z_{ko3}+z_{ko4})\right],\ k=1,\dots,K,\\
&\quad\left[\frac{1}{2}\sum_{j=1}^{n}(\lambda_{j1}+\lambda_{j2}),\frac{1}{2}\sum_{j=1}^{n}(\lambda_{j3}+\lambda_{j4})\right]=\left[\frac{1}{2}(2),\frac{1}{2}(2)\right]\\
&\quad 0\le\lambda_{j1}\le\lambda_{j2}\le\lambda_{j3}\le\lambda_{j4},\ j=1,2,\dots,n,\varepsilon\le\theta_1\le\theta_2\le\theta_3\le\theta_4,\ \theta_4\le 1,\varepsilon>0.
\end{aligned}
\tag{A.3}
$$

Similarly, model (A.3) can be replaced by the following bi-objective DEA model:

$$
\begin{aligned}
&Min\ \ \frac{1}{2}(\theta_3+\theta_4)\\
&Min\ \ \frac{1}{4}(\theta_1+\theta_2+\theta_3+\theta_4)\\
&s.t.\ \ \sum_{j=1}^{n}(\lambda_{j3}x_{ij3}+\lambda_{j4}x_{ij4})\le(\theta_3x_{io3}+\theta_4x_{io4}),\qquad i=1,\dots,m,\\
&(1-\alpha_i)\sum_{j=1}^{n}(\lambda_{j1}x_{ij1}+\lambda_{j2}x_{ij2})+\alpha_i\sum_{j=1}^{n}(\lambda_{j3}x_{ij3}+\lambda_{j4}x_{ij4})\le(1-\alpha_i)(\theta_1x_{io1}+\theta_2x_{io2})+\alpha_i(\theta_3x_{io3}+\theta_4x_{io4}),i=1,\dots,m,\\
&\quad\sum_{j=1}^{n}(\lambda_{j1}y_{rj1}+\lambda_{j2}y_{rj2})\ge(y_{ro1}+y_{ro2}),\qquad r=1,\dots,s,\\
&(1-\beta_r)\sum_{j=1}^{n}(\lambda_{j3}y_{rj3}+\lambda_{j4}y_{rj4})+\beta_r\sum_{j=1}^{n}(\lambda_{j1}y_{rj1}+\lambda_{j2}y_{rj2})\ge(1-\beta_r)(y_{ro3}+y_{ro4})+\beta_r(y_{ro1}+y_{ro2}),\qquad r=1,\dots,s,\\
&\sum_{j=1}^{n}(\lambda_{j3}z_{kj3}+\lambda_{j4}z_{kj4})\le(z_{ko3}+z_{ko4}),\qquad k=1,\dots,K,\\
&(1-\gamma_k)\sum_{j=1}^{n}(\lambda_{j1}z_{kj1}+\lambda_{j2}z_{kj2})+\gamma_k\sum_{j=1}^{n}(\lambda_{j3}z_{kj3}+\lambda_{j4}z_{kj4})\le(1-\gamma_k)(z_{ko1}+z_{ko2})+\gamma_k(z_{ko3}+z_{ko4}),\quad k=1,\dots,K,\\
&\sum_{j=1}^{n}(\lambda_{j1}z_{kj1}+\lambda_{j2}z_{kj2})\ge(z_{ko1}+z_{ko2}),\quad k=1,\dots,K,\\
&(1-\gamma_k)\sum_{j=1}^{n}(\lambda_{j3}z_{kj3}+\lambda_{j4}z_{kj4})+\gamma_k\sum_{j=1}^{n}(\lambda_{j1}z_{kj1}+\lambda_{j2}z_{kj2})\ge(1-\gamma_k)(z_{ko3}+z_{ko4})+\gamma_k(z_{ko1}+z_{ko2}),\quad k=1,\dots,K,\\
&\sum_{j=1}^{n}(\lambda_{j3}+\lambda_{j4})\le 2,\\
&(1-\kappa)\sum_{j=1}^{n}(\lambda_{j1}+\lambda_{j2})+\kappa\sum_{j=1}^{n}(\lambda_{j3}+\lambda_{j4})\le 2,\\
&\sum_{j=1}^{n}(\lambda_{j1}+\lambda_{j2})\ge 2,\\
&(1-\kappa)\sum_{j=1}^{n}(\lambda_{j3}+\lambda_{j4})+\kappa\sum_{j=1}^{n}(\lambda_{j1}+\lambda_{j2})\ge 2,\\
&\quad 0\le\lambda_{j1}\le\lambda_{j2}\le\lambda_{j3}\le\lambda_{j4},\ j=1,2,\dots,n,\varepsilon\le\theta_1\le\theta_2\le\theta_3\le\theta_4,\theta_4\le 1,\varepsilon>0.
\end{aligned}
\tag{A.4}
$$

That $\kappa\in[0,1]$ indicates the acceptance degrees of decision maker for violating interval inequality constraints. Its interpretation is similar to the aforementioned parameters, i.e. within $[0,0.5)$, $(0.5,1]$ and 0.5 denote the risk aversion of expert, the risk liking of expert and risk neutrality of expert, individually.

In the same way, an additional parameter $\mu\in[0,1]$ is used to compute bi-objective problem (A.4) and to integrate the objectives as follows:

$$
\begin{aligned}
&E_{II}^{*} = Min \quad \mu\,\frac{1}{2}(\theta_3+\theta_4)+(1-\mu)\frac{1}{4}(\theta_1+\theta_2+\theta_3+\theta_4)\\
&s.t. \quad \sum_{j=1}^{n}(\lambda_{j3}x_{ij3}+\lambda_{j4}x_{ij4}) \le (\theta_3 x_{io3}+\theta_4 x_{io4}), \qquad i=1,\dots,m,\\
&(1-\alpha_i)\sum_{j=1}^{n}(\lambda_{j1}x_{ij1}+\lambda_{j2}x_{ij2})+\alpha_i\sum_{j=1}^{n}(\lambda_{j3}x_{ij3}+\lambda_{j4}x_{ij4}) \le (1-\alpha_i)(\theta_1 x_{io1}+\theta_2 x_{io2})+\alpha_i(\theta_3 x_{io3}+\theta_4 x_{io4}), i=1,\dots,m,\\
&\qquad \sum_{j=1}^{n}(\lambda_{j1}y_{rj1}+\lambda_{j2}y_{rj2}) \ge (y_{ro1}+y_{ro2}), \qquad r=1,\dots,s,\\
&(1-\beta_r)\sum_{j=1}^{n}(\lambda_{j3}y_{rj3}+\lambda_{j4}y_{rj4})+\beta_r\sum_{j=1}^{n}(\lambda_{j1}y_{rj1}+\lambda_{j2}y_{rj2}) \ge (1-\beta_r)(y_{ro3}+y_{ro4})+\beta_r(y_{ro1}+y_{ro2}), \qquad r=1,\dots,s,\\
&\sum_{j=1}^{n}(\lambda_{j3}z_{kj3}+\lambda_{j4}z_{kj4}) \le (z_{ko3}+z_{ko4}), \qquad k=1,\dots,K,\\
&(1-\gamma_k)\sum_{j=1}^{n}(\lambda_{j1}z_{kj1}+\lambda_{j2}z_{kj2})+\gamma_k\sum_{j=1}^{n}(\lambda_{j3}z_{kj3}+\lambda_{j4}z_{kj4}) \le (1-\gamma_k)(z_{ko1}+z_{ko2})+\gamma_k(z_{ko3}+z_{ko4}), \quad k=1,\dots,K,\\
&\sum_{j=1}^{n}(\lambda_{j1}z_{kj1}+\lambda_{j2}z_{kj2}) \ge (z_{ko1}+z_{ko2}), \quad k=1,\dots,K,\\
&(1-\gamma_k)\sum_{j=1}^{n}(\lambda_{j3}z_{kj3}+\lambda_{j4}z_{kj4})+\gamma_k\sum_{j=1}^{n}(\lambda_{j1}z_{kj1}+\lambda_{j2}z_{kj2}) \ge (1-\gamma_k)(z_{ko3}+z_{ko4})+\gamma_k(z_{ko1}+z_{ko2}), \quad k=1,\dots,K,\\
&\sum_{j=1}^{n}(\lambda_{j3}+\lambda_{j4}) \le 2,\\
&(1-\kappa)\sum_{j=1}^{n}(\lambda_{j1}+\lambda_{j2})+\kappa\sum_{j=1}^{n}(\lambda_{j3}+\lambda_{j4}) \le 2,\\
&\sum_{j=1}^{n}(\lambda_{j1}+\lambda_{j2}) \ge 2,\\
&(1-\kappa)\sum_{j=1}^{n}(\lambda_{j3}+\lambda_{j4})+\kappa\sum_{j=1}^{n}(\lambda_{j1}+\lambda_{j2}) \ge 2,\\
&\qquad 0 \le \lambda_{j1} \le \lambda_{j2} \le \lambda_{j3} \le \lambda_{j4},\ j=1,2,\dots,n, \varepsilon \le \theta_1 \le \theta_2 \le \theta_3 \le \theta_4, \theta_4 \le 1, \varepsilon > 0.
\end{aligned} \tag{A.5}
$$

Definition 4. The unit under consideration is called efficient in model (A.5) if and only if $E_{II}^{*}=1$ for given $\alpha_i,\beta_r,\gamma_k$, κ and μ. Otherwise, it is inefficient.

Furthermore, the fuzzy efficiency can be defined as the optimal value $(\theta_1^*,\theta_2^*,\theta_3^*,\theta_4^*)$ obtained from model (A.5).

Theorem 3. The optimal value $(\lambda_{j1}^*,\lambda_{j2}^*,\lambda_{j3}^*,\lambda_{j4}^*,\theta_1^*,\theta_2^*,\theta_3^*,\theta_4^*)$ obtained from solving model (A.5) is a weak Pareto solution of model (A.4).

Proof. Similar to Theorem 1, it can be conveniently proved. □

Notice that model (A.5) is input-oriented FFDEA under VRS with the acceptance degree of the violated fuzzy constraints. Analogously, under VRS, the output-oriented FFDEA model with the acceptance degree of the violated fuzzy constraints can be provided by extending models (13)-(17).

Conflict of interest

The authors declare that they have no conflict of interest.

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Biographies

Monireh Jahani Sayyad Noveiri received her PhD in Applied Mathematics from Lahijan Branch, Islamic Azad University. Her research interests include operations research, data envelopment analysis (DEA), Inverse DEA, supply chains and fuzzy theory. Jahani Sayyad Noveiri’s papers have appeared in a wide series of journals such as Applied Energy, Journal of Industrial and Management Optimization, Soft Computing, IMA Journal of Management Mathematics, Environment, Development and Sustainability, etc.

Sohrab Kordrostami is a full professor in applied mathematics (operations research field) department in Islamic Azad University, Lahijan branch. He completed his Ph.D. degree in Islamic Azad University of Tehran, Iran. His research interests include performance management with special emphasis on the quantitative methods of performance measurement, and especially those based on the broad set of methods known as Data Envelopment Analysis, (DEA). Kordrostami’s papers have appeared in a wide series of journals such as Applied mathematics and computation, Journal of the operations research society of Japan, Journal of Applied mathematics, International journal of advanced manufacturing technology, International journal of production economics, Optimization, International Journal of Mathematics in Operational research, Journal global optimization, etc.

Table Captions

Table 1. A comparative review of fuzzy DEA
Table 2. Dataset on energy dependency
Table 3. Results for different values μ
Table 4. Results for different values α
Table 5. Projection points of input-output measures
Table 6. The efficiency values obtained from fuzzy approaches provided in [50, 51]

Figure Captions
Fig. 1. A graphical design of the approach
Fig. 2. Efficiency scores for Lithuania
Fig. 3. Efficiency scores for five sample countries at different levels α_i

Table 1. A comparative review of fuzzy DEA

Study	Undesirable outputs	Determining projection points	Fully fuzzy DEA	ADDM	Envelopment form	Application Area
Puri and Yadav [13]	Y	N	N	N	N	Banking sector
Puri and Yadav [33]	Y	N	Y	N	N	-
Wu et al. [34]	Y	N	N	N	Y	Thermal power firms
Kordrostami et al. [14]	Y	N	N	N	N	Banking sector
Khalili-Damghani et al. [35]	Y	N	N	N	Y	Emerging markets for international banking
Ignatius et al. [36]	Y	N	N	N	N	Energy dependency case of European union member countries
Arya and Yadav [37]	N	N	Y	N	N	Health sector
Izadikhah and Khoshroo [23]	Y	Y	N	N	Y	Barley production farms
Hatami-Marbini et al. [10]	N	Y	Y	N	Y	Suppliers of raw materials
Zhou et al. [38]	Y	N	N	N	Y	Sustainable supply chains
Heydari et al. [39]	N	N	Y	N	Y	Airline industry
Arana-Jiménez et al. [40]	N	Y	Y	N	Y	Suppliers of raw materials
Ren et al. [41]	Y	Y	N	N	Y	Pallet rental industry
Mozaffari et al. [42]	Y	N	Y	N	N	Petrochemical sector
Jahani Sayyad Noveiri and Kordrostami [43]	Y	N	Y	N	Y	Health sector
Ebrahimnejad and Amani [16]	Y	N	N	N	N	-
Chen et al. [24]	N	N	Y	Y	N	Health sector
Arana-Jiménez et al. [44]	N	Y	N	N	Y	-
This research	Y	Y	Y	Y	Y	Energy dependency

	case of European union member countries

ADDM: Acceptance degrees of decision maker, Y: Yes and N: No

Table 2. Dataset on energy dependency

Country	Input	Desirable outputs			Undesirable output
	$\tilde{x}_1$	$\tilde{y}_1$	$\tilde{y}_2$	y_3	$\tilde{z}_1$
Austria	(4.105,4.130,4.143)	(59.038,61.363,64.980)	(0.3043,0.3088,0.3426)	29.7	(3.853,3.859,4.088)
Belgium	(5.501,5.567,5.719)	(3.960,4.359,5.391)	(0.5091,0.5231,0.5885)	4.6	(5.482,5.570,5.931)
Cyprus	(2.503,2.544,2.615)	(0.080,0.105,0.241)	(0.0530,0,0540,0.0555)	4.6	(5.129,5.168,5.931)
Czech Republic	(4.384,4.445,4.545)	(5.278,5.400,6.535)	(0.7843,0.8141,0.8609)	8.5	(9.118,9.143,9.958)
Denmark	(3.575,3.742,3.828)	(24.109,26.276,26.757)	(0.2551,0.2890,0.2967)	9.9	(4.127,5.369,5.892)
Estonia	(4.195,4.263,4.361)	(2.744,2.770,5.642)	(0.1054,0.1282,0.1510)	22.8	(11.869,12.645,17.731)
Finland	(6.531,6.934,7.151)	(25.214,26.613,30.189)	(0.3793,0.3940,0.4073)	30.3	(8.043,8.074,9.179)
France	(4.396,4.450,4.468)	(12.655,13.210,13.641)	(1.2194,1.2219,1.3136)	12.3	(2.339,2.355,2.595)
Germany	(4.148,4.173,4.201)	(12.144,14.079,15.187)	(4.6013,4.6655,4.9795)	9.8	(5.663,5.681,6.609)
Greece	(2.807,2.812,2.821)	(6.221,9.788,12.606)	(0.6710,0.6905,0.7343)	8.2	(6.153,6.167,6.650)
Hungary	(2.698,2.709,2.716)	(4.447,5.026,6.174)	(0.2480,0.2552,0.2895)	7.7	(2.867,2.872,3.226)
Ireland	(3.664,3.671,3.719)	(10.202,10.817,12.493)	(0.1981,0.2014,0.2238)	5.0	(4.510,4.562,4.636)
Italy	(3.082,3.114,3.162)	(15.417,16.020,19.090)	(2.1412,2.1485,2.4016)	8.9	(3.377,3.528,3.618)
Latvia	(1.967,2.018,2.063)	(40.230,41.122,46.793)	(0.0269,0.0276,0.0293)	34.3	(1.646,1.649,1.985)
Lithuania	(2.594,2.622,2.635)	(3.890,4.590,5.196)	(0.0574,0.0610,0.0633)	17.0	(2.495,3.088,3.667)
Netherlands	(4.949,5.065,5.164)	(7.060,7.440,8.455)	(0.7804,0.8028,0.8203)	4.1	(5.102,5.122,5.552)
Poland	(2.449,2.534,2.564)	(3.632,4.112,5.195)	(2.0363,2.0363,2.1277)	8.9	(5.981,5.982,6.661)
Portugal	(2.349,2.469,2.544)	(28.999,29.543,34.383)	(0.2931,0.3063,0.3181)	24.5	(3.326,3.334,3.766)
Slovakia	(3.399,3.424,3.485)	(16.570,16.609,18.308)	(0.2352,0.2425,0.2689)	10.3	(5.691,5.694,5.904)
Slovenia	(3.693,3.697, 3.836)	(26.492,28.110,33.534)	(0.0856,0.0870,0.0927)	16.9	(4.101,4.266,4.285)
Spain	(3.230,3.260,3.356)	(19.523,20.841,24.492)	(1.6183,1.6667,1.8543)	13.3	(3.548,3.683,3.805)
Sweden	(5.417,5.544,5.597)	(49.794,52.610,53.601)	(0.1852,0.1912,0.2095)	47.3	(2.418,2.421,2.594)
United Kingdom	(3.671,3.724,3.764)	(5.063,5.356,6.250)	(2.4735,2.5119,2.7460)	2.9	(3.453,3.467,3.501)

Table 3. Results for different values μ

Countries	Input-oriented efficiency							Output-oriented efficiency		Model (7)
	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 1	Case 6	
Austria	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8536
Belgium	0.4422	0.4509	0.4509	0.4509	0.4509	0.4509	0.4509	0.1647	0.1647	0.3752
Cyprus	0.8300	0.8332	0.8333	0.8333	0.8334	0.8334	0.8334	0.2580	0.2580	0.6850
Czech Republic	0.9097	0.9111	0.9111	0.9111	0.9111	0.9111	0.9111	0.4470	0.4470	0.7551
Denmark	0.6743	0.6872	0.6872	0.6872	0.6872	0.6872	0.6872	0.4497	0.4559	0.7372
Estonia	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Finland	0.5768	0.5987	0.5987	0.5987	0.5987	0.5987	0.5987	0.3450	0.3450	0.5304
France	0.4589	0.4645	0.4648	0.4651	0.4655	0.4658	0.4660	0.8562	0.8582	0.3853
Germany	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Greece	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8939
Hungary	0.4957	0.4981	0.4990	0.5000	0.5010	0.5020	0.5024	0.2863	0.2863	0.4332
Ireland	0.9838	0.9841	0.9846	0.9851	0.9856	0.9861	0.9863	0.9469	0.9469	0.5237
Italy	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8049
Latvia	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Lithuania	0.5796	0.5839	0.5847	0.5855	0.5863	0.5871	0.5875	0.4329	0.4329	0.6035
Netherlands	0.4510	0.4632	0.4632	0.4632	0.4632	0.4632	0.4632	0.2470	0.2492	0.4169
Poland	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Portugal	0.9422	0.9448	0.9448	0.9448	0.9448	0.9448	0.9448	0.8593	0.8624	0.8965
Slovakia	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.7327
Slovenia	0.9082	0.9091	0.9091	0.9091	0.9091	0.9091	0.9091	0.8898	0.8898	0.6548
Spain	0.9580	0.9586	0.9586	0.9586	0.9586	0.9586	0.9586	0.9375	0.9375	0.7645
Sweden	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
United Kingdom	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.6256

Table 4. Results for different values α

Countries	Input-oriented efficiency							
	Case 8	Case 9	Case 10	Case 11	Case 12	Case 13	Case 14	Case 15
Austria	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Belgium	0.4547	0.4540	0.4532	0.4525	0.4509	0.4494	0.4145	0.3676
Cyprus	0.8333	0.8333	0.8333	0.8333	0.8333	0.8333	0.7877	0.6922
Czech Republic	0.9153	0.9144	0.9136	0.9128	0.9111	0.9095	0.8834	0.7659
Denmark	0.6912	0.6904	0.6896	0.6888	0.6872	0.6855	0.6363	0.5659
Estonia	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Finland	0.6054	0.6040	0.6027	0.6013	0.5987	0.5960	0.5496	0.4884
France	0.4651	0.4651	0.4651	0.4651	0.4651	0.4651	0.4349	0.3870
Germany	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Greece	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Hungary	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.4686	0.4174
Ireland	0.9851	0.9851	0.9851	0.9851	0.9851	0.9851	0.9816	0.9644
Italy	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Latvia	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Lithuania	0.5855	0.5855	0.5855	0.5855	0.5855	0.5855	0.5480	0.4879
Netherlands	0.4666	0.4659	0.4652	0.4645	0.4632	0.4618	0.4274	0.3793
Poland	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Portugal	0.9559	0.9536	0.9514	0.9492	0.9448	0.9405	0.9182	0.8324
Slovakia	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Slovenia	0.9131	0.9123	0.9115	0.9107	0.9091	0.9076	0.8816	0.7609
Spain	0.9613	0.9607	0.9602	0.9596	0.9586	0.9575	0.9452	0.8888
Sweden	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
United Kingdom	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 5. Projection points of input-output measures

	Input	Desirable outputs			Undesirable output
Country	$\tilde{x}_1$	$\tilde{y}_1$	$\tilde{y}_2$	y_3	$\tilde{z}_1$
Austria	(4.105, 4.130, 4.143)	(59.038,61.363, 64.980)	(0.3043, 0.3088, 0.3426)	29.7	(3.853, 3.859, 4.088)
Belgium	(2.514, 2.519, 2.528)	(5.542, 8.694, 11.196)	(0.6241, 0.6412, 0.6813)	7.4	(5.520, 5.532, 5.969)
Cyprus	(2.068, 2.136, 2.164)	(2.898, 3.265, 4.229)	(1.5555, 1.5566, 1.6273)	7.9	(5.129, 5.168, 5.931)
Czech Republic	(4.016, 4.065, 4.090)	(7.960, 11.685, 14.997)	(1.7194, 1.7384, 1.8289)	12.6	(9.123, 9.138, 9.963)
Denmark	(2.500, 2.557, 2.612)	(24.901, 25.484, 29.686)	(0.2735, 0.2797, 0.2968)	27.1	(4.649, 4.847, 6.407)
Estonia	(4.195, 4.263, 4.361)	(2.744, 2.770, 5.642)	(0.1054, 0.1282, 0.1510)	22.8	(11.869, 12.645, 17.731)
Finland	(4.012, 4.139, 4.202)	(27.143, 28.190, 32.734)	(2.3173, 2.3184, 2.4233)	30.3	(8.046, 8.071, 9.182)
France	(2.032, 2.061, 2.094)	(16.194, 16.765, 19.404)	(1.2173, 1.2240, 1.3485)	12.3	(2.315, 2.379, 2.571)
Germany	(4.148, 4.173,4.201)	(12.144,14.079, 15.187)	(4.6013, 4.6655, 4.9795)	9.8	(5.663, 5.681, 6.609)
Greece	(2.807, 2.812, 2.821)	(6.221, 9.788, 12.606)	(0.6710, 0.6905, 0.7343)	8.2	(6.153, 6.167, 6.650)
Hungary	(1.310, 1.354, 1.372)	(5.931, 6.238,7.336)	(0.9046, 0.9048, 0.9455)	7.7	(2.868, 2.871, 3.227)
Ireland	(3.579, 3.620, 3.670)	(10.624, 10.806, 12.092)	(1.4454, 1.4695, 1.6083)	6.5	(4.532, 4.540, 4.658)
Italy	(3.082, 3.114, 3.162)	(15.417, 16.020, 19.090)	(2.1412, 2.1485, 2.4016)	8.9	(3.377, 3.528, 3.618)

Latvia	(1.967, 2.018, 2.063)	(40.239, 41.122, 46.793)	(0.0269, 0.0276, 0.0293)	34.3	(1.646, 1.649, 1.985)
Lithuania	(1.495, 1.527, 1.561)	(15.751,16.097, 18.749)	(0.0571, 0.0613, 0.0671)	17.0	(2.733, 2.867, 3.871)
Netherlands	(2.327, 2.346, 2.358)	(6.021, 8.479, 10.668)	(0.7852, 0.7980, 0.8424)	7.9	(5.107, 5.117, 5.557)
Poland	(2.449, 2.534, 2.564)	(3.632, 4.112, 5.195)	(2.0363, 2.0363, 2.1277)	8.9	(5.981, 5.982, 6.661)
Portugal	(2.278, 2.328, 2.364)	(28.590, 29.952, 34.457)	(0.5021, 0.5063, 0.5319)	25.9	(3.328, 3.332, 3.768)
Slovakia	(3.399, 3.424, 3.485)	(16.570,16.609, 18.308)	(0.2352, 0.2425, 0.2689)	10.3	(5.691, 5.694, 5.904)
Slovenia	(3.358, 3.390, 3.429)	(27.934,28.802, 30.707)	(0.3080, 0.3148, 0.3474)	16.9	(4.182, 4.185, 4.366)
Spain	(3.102, 3.137, 3.188)	(20.130, 20.738, 24.092)	(1.6478,1.6550, 1.8482)	13.3	(3.560, 3.671, 3.817)
Sweden	(5.417, 5.544, 5.597)	(49.794, 52.610, 53.601)	(0.1852, 0.1912, 0.2095)	47.3	(2.418, 2.421, 2.594)
United Kingdom	(3.671, 3.724, 3.764)	(5.063, 5.356,6.250)	(2.4735, 2.5119, 2.7460)	2.9	(3.453, 3.467, 3.501)

Table 6. The efficiency values obtained from fuzzy approaches provided in [50, 51]

DMU	Efficiency values achieved from [51]			The efficiency derived from [50]
	Optimistic	Pessimistic	Geometric Average	The CCR form $(\alpha = 10)$
Austria	0.761	3.557	1.646	-2.599
Belgium	0.147	1	0.383	-36.432
Cyprus	0.121	1	0.349	-35.254
Czech Republic	0.251	1.096	0.525	-26.644
Denmark	0.385	3.050	1.083	-14.651
Estonia	0.333	1	0.577	-15.871
Finland	0.300	2.474	0.861	-17.910
France	0.833	3.118	1.613	-2.770
Germany	1	2.073	1.440	0
Greece	0.359	1.576	0.752	-18.530
Hungary	0.238	1.971	0.685	-24.482
Ireland	0.183	1.005	0.429	-32.364
Italy	0.894	3.228	1.698	-1.971
Latvia	1	1	1	0
Lithuania	0.395	1.118	0.665	-13,042
Netherlands	0.234	1	0.484	-31.274
Poland	0.832	1.725	1.198	-3.022
Portugal	0.681	5.574	1.948	-5.026
Slovakia	0.287	1.893	0.737	-22.030
Slovenia	0.385	1.160	0.668	-14.964
Spain	0.752	4.455	1.830	-4.690
Sweden	1	1.756	1.325	0
United Kingdom	0.916	1	0.957	-1.096

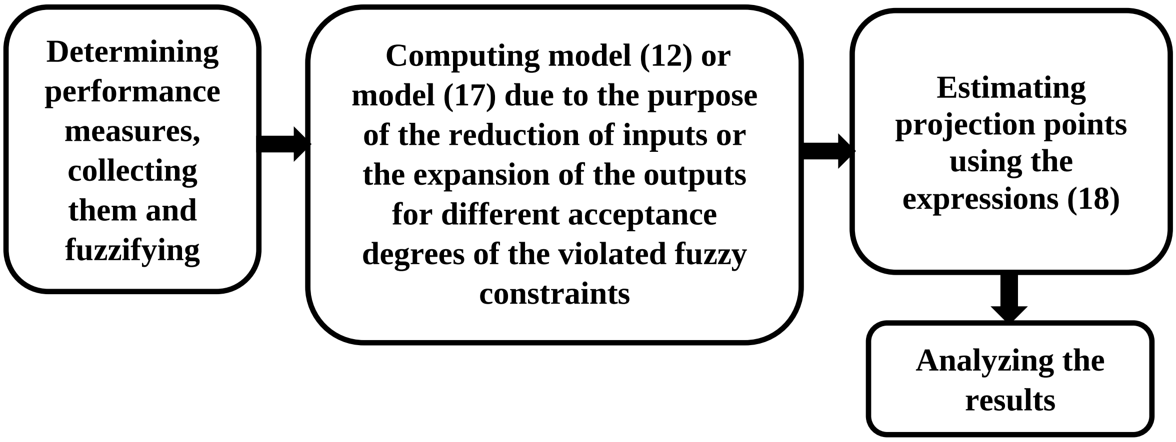

Fig.1. A graphical design of the approach

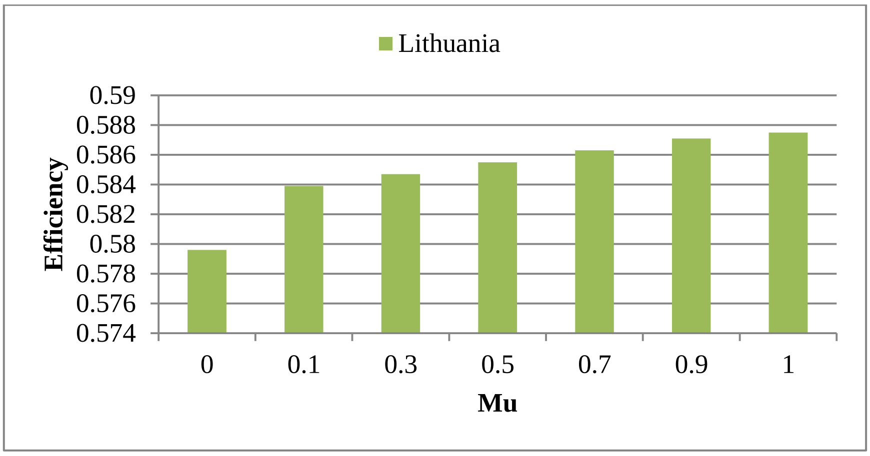

Fig. 2. Efficiency scores for Lithuania

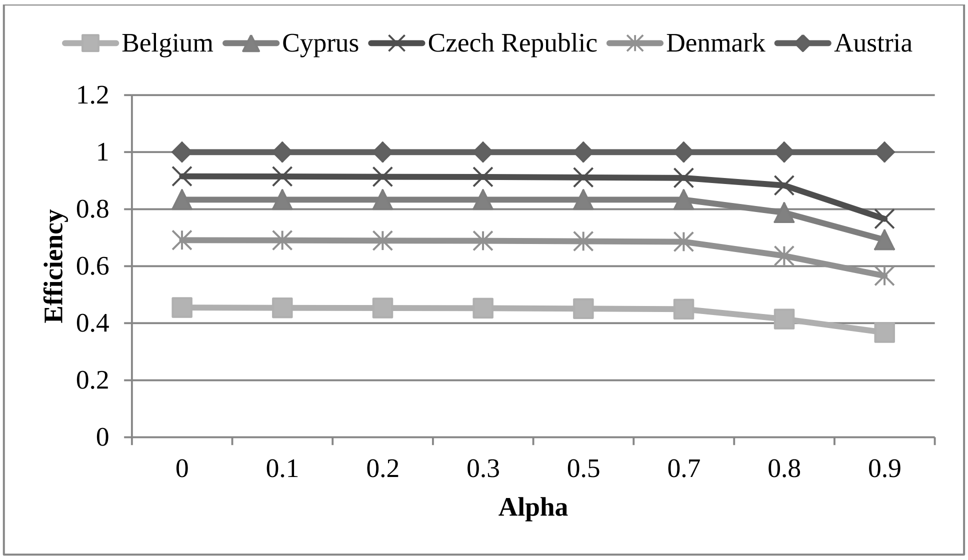

Fig. 3. Efficiency scores for five sample countries at different levels α_i