Ensuring sustainable strategies for achieving multi-commodity maximum flow on a fuzzy network under interdictions

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Abstract

This paper addresses the multi-commodity maximum flow network interdiction problem (MC-MFNIP) which involves two opposite sides with conflicting objectives in the presence of uncertain arc capacities. In this problem, one party, the follower tries to maximize the multi-commodity flow throughout the network, while the other party, the leader, attempts to minimize the total flow quantity that the follower achieves using the limited interdiction resources. To tackle this challenge, this study presents a fuzzy-based optimization model that, for the first time, considers the prioritization of multiple-source and multiple-sink nodes for commodities while addressing the uncertainties inherent in arc capacities for the MC-MFNIP. The formulated model addresses the MC-MFNIP, in which uncertain arc capacities are defined using triangular fuzzy numbers and considers the feasibility degrees that specify the level of risk the decision-maker is willing to accept. Following that, computational analyses are performed through a set of cases regarding the various sized networks and α-cut levels to test the model's performance and track changes in flow quantities. The model is efficient for devising fortification strategies against interdictions at an operational level since the model quickly provides reasonable information about the most critical (interdicted) arcs within seconds in all generated networks, ensuring tractability.

Keywords: *Maximum flow problem; Network interdiction problem; Fuzzy optimization model; Uncertainty, Triangular fuzzy numbers.*

1. Introduction

Today, when considering the increasing terror activities, the security of communication, transportation, or infrastructure networks is a crucial issue that needs to be emphasized. That being the case, the importance of the network interdiction problem (NIP) has increased for especially damaging and destructive activities on networks. NIPs focus on investigating the attitudes of two opposite sides named as follower and leader. The NIPs can be handled in terms of different concepts such as shortest path NIP, maximum flow NIP, sensor location problems, vulnerability analysis of a network, finding the most vital components on the network, etc. Overall, the attitudes of two opposite sides who have conflicting aims are formulated by following the game theory principles in these problems. The multi-commodity maximum flow network interdiction problems (MC-MFNIP) is one of them. In this problem, while the follower wants to maximize the multi-commodity flow throughout the network, the leader tries to minimize the total flow quantity that the follower has achieved considering the interdiction resource. Although most of the studies are handled with deterministic techniques (See: Table 1), there may be some uncertainties due to the nature of the problem. In this regard, stochastic, robust, and fuzzy models can be used to overcome uncertainty. Stochastic techniques handle the most common ones associated with the distribution of situations, while robust approaches focus on worst-case scenarios considering all possible situations. Besides, if there is no distribution information regarding uncertainties, fuzzy-based techniques can be used by defining the fuzzy sets to overcome the vagueness. In the existing literature regarding the MFNIP, the stochastic or robust models [1-7] are handled in almost all studies considering the vagueness, However, to the best of the authors' knowledge, there is no study in which any fuzzy approach is used to cope with the uncertainties.

Uncertain observations or possible confusion make it difficult to accurately express the capacities and flows in a network. Also, arc capacities can change over time. For this reason, representing these values with fuzzy numbers offers a rather more realistic approach and valid solution [8]. In this regard, this study aims to fill the gap in the literature by considering fuzzy arc capacities for the MFNIPs. In this study, the fuzzy-based optimization model is formulated to allow consideration of multiple-source & multiple-sink nodes for the commodities and prioritizing the commodities for the MC-MFNIP in the presence of fuzzy information about the arc capacities. That being the case, for different purposes, the computational studies are realized through the different-sized network instances generated hypothetically by taking into consideration different α-cut levels. To this end, after the presented model is applied on an illustrative example visually to explain the addressed problem, the analyses are performed to track and compare the changes in flow quantities considering the situations regarding prioritization of the commodities, or the multiple-source & multiple-sink nodes. Besides, the performance of the fuzzy-based optimization model is tested on the addressed networks considering the runtimes.

Being thus motivated, in this study, the following research questions are answered by solving the proposed fuzzy-based optimization model. By doing so, further knowledge is gained regarding the addressing of the MC-MFNIP under a fuzzy environment from different perspectives.

- What is the variability of the objective function values in terms of α-cuts by considering fuzzy arc capacities in the MC-MFNIP?
- What are the effects of prioritizations of commodities on the commodity-based flow quantities in the MC-MFNIP under a fuzzy environment?
- What are the effects of considering multiple-source & multiple-sink nodes on the objective function value in the MC-MFNIP under a fuzzy environment?

The key contributions of this study are highlighted as follows: (i) The formulation of an exact fuzzy-based optimization model, which, for the first time, considers prioritization and involves multiple-source and multiple-sink nodes for commodities, addressing the inherent uncertainties in arc capacities for the MC-MFNIP from a theoretical perspective. (ii) As for the managerial point of view, the model can provide the information dealing with interdicted arcs, called risky or most vital arcs, for any network problem regarding the MC-MFNIP under fuzzy environment to the authorities so that they can improve the reinforcement strategies. (iii) From practical relevance, the model gives reasonable solutions within seconds and produces information regarding the interdiction plan at the operational level. Therefore, this study intends to pave the way for future studies regarding any interdiction problem under a fuzzy environment.

The remaining part of the study is organized as follows. In Section 2, the existing academic literature regarding the MFNIP is reviewed and classified concerning the concept and methodology. In Section 3, the MC-MFNIP is defined, and the relevant optimization models are presented for the follower and leader separately. In Section 4, the computational studies are performed for different purposes. (i) The fuzzy-based optimization model is explained through an illustrative example. (ii) A sensitivity analysis is conducted by prioritizing the commodities to reveal the changes in the commodity-based flow quantities. Further, the total flow quantities are observed under varying numbers of source and sink nodes for the network. (iii) The performance of the model is tested. In Section 5; the Conclusion is provided by stressing the future directions of the study.

2. Literature review

The maximum flow problems (MFPs) deal with determining the maximum flow that can be provided from the source node to the sink node on a network that has capacitated arcs. Ford and Fulkerson [9] stated that with the theory they developed in their study, the capacity of the minimum cut-set equals the maximum flow on the network. The cut-set mentioned here is the set of links that, when eliminated, will cause disconnection in terms of the maximum flow between nodes on the network. Also, the capacity of this set equals the sum of the capacities of the disconnected arcs. These links are the most important components to provide maximum flow on a network. From this point of view, it is seen that the first studies of the NIPs were handled in this manner, namely focusing on the determination of the most vital components (node(s) and arc(s)) of a network. (For deep reviews, consider the studies conducted by Wollmer [10, 11]). Besides, there are also studies aiming to find activities that will cause maximum disconnection on the network [12]. NIP also consists of many scopes such as including minimizing the maximum flow or maximizing the shortest path on a given network, nuclear smuggling interdiction [13], national defense [14], interdiction facility locations [15], determining sensor locations [16, 17], infrastructure strengthening [18], project interdiction [19], interdiction on supply chain [20, 21], quickest flow over time network interdiction [22]. Wood [23] developed a "*min-max"* formulation of the MFNIP and reduced it to a single-level integer-programming model. Most of the studies before Wood's formulations have been case-based and cannot be generalized. Thanks to this solution approach, an increase has been observed in studies on the NIP. To demonstrate the position of this issue in the literature, the "network interdiction problem" and "maximum flow network interdiction problem" words are searched separately using the "title, abstract, keywords" section in the Scopus database. All studies in English between 1993 and 2023 years are selected and shown in Figure 1. In total, 379 and 56 documents are observed regarding the NIP and MFNIP, respectively. As evident in Figure 1, the volume of studies focusing on NIP has exhibited a notable increase, particularly in recent years. However, MFNIP investigations have shown a relatively stable trend, maintaining an annual average of merely 3-4 publications since 2007.

Insert Figure 1 Here

Fundamentally, the studies regarding the NIPs can be grouped into two main titles: (i) Shortest Path Network Interdiction Problems (ii) Maximum Flow Network Interdiction Problems (MFNIP). Some noteworthy studies on shortest path network interdiction problems are as follows: [24-28]. In the following, the existing academic literature regarding MFNIP is reviewed in a detailed manner due to the scope of this study, and they are presented comparatively within three main groups: (i) information about the structure of networks; (ii) information about interdiction processes; (iii) information about methodologies (See: Table 1). In these studies, the thesis named “Network Interdiction Models” prepared by Steinrauf [29], and the paper named “Deterministic Network Interdiction” prepared by Wood [23], can be shown as pioneering studies.

Table 1 reveals that many of the relevant studies present a single-objective linear optimization model for the MFNIP in which discrete arc interdictions are considered on directed networks that have a single sink and source node for a single commodity in a deterministic environment. Relatively few studies use stochastic and robust approaches to handle uncertainties in the MFNIP and most of these studies focus on a single commodity. Only Mirzae et al. [5] which is among the studies employing the stochastic approach, structure their study to deal with multi-commodity problems with multiple sources and sinks, and they also include both discrete and continuous interdiction decisions.

As well as in MFNIP, the use of stochastic techniques in handling uncertainty has become widespread in various fields, in recent years [30-33]. For researchers interested in these methodologies, it is advised to comprehensively examine the following studies, which include remarkable stochastic schemes: [34-37].

In the literature, it is observed that fuzzy-based optimization models are also proposed to overcome vagueness in network problems such as facility location [38], supply chain [39, 40]. In MFPs, mostly the situation where there is fuzzy information regarding the arc capacities has been considered by employing different fuzzy sets. (For in-depth research: [41].)

However, the literature review indicates that no study has addressed any MFNIP under a fuzzy environment. Therefore, the motivation for employing fuzzy approaches is to fill this gap by presenting a fuzzy-based optimization model to directly solve the MC-MFNIP.

Insert Table 1 Here

3. Problem statement and model formulations

MC-MFNIP focuses on the attitudes of two sides who have conflicting aims on a network. In the problem, while the follower tries to maximize the multi-commodity flow throughout the network, the leader attempts to minimize the total flow quantity that the follower has achieved considering the limited interdiction resource. To this end, the fuzzy-based optimization model considering the prioritization and multiple-source & multiple-sink nodes for the commodities for the MC-MFNIP in which there is fuzzy information about the arc capacities is introduced.

The models are formulated under the following key assumptions:

- Problems are defined on an undirected network, and flow is impossible between nodes that are not connected.
- Both sides are conscious and have equal information about the network.
- Each commodity has its specific source and sink node(s), and there is no rule between source-sink node pairs.
- There is a predetermined cost to interdict each arc, and interdiction costs are adjusted by the number of the required equipment to interdict corresponding arcs.
- The leader has a predetermined interdiction resource.

Sets, indices, parameters, and decision variables related to the problem are provided as follows.

Sets and indices:

N: set of nodes

A: set of arcs

K: set of commodities

$S_{(k)}$: set of source nodes for commodity k

$T_{(k)}$: set of sink nodes for commodity k

$s_{(k)}$: index for source node of commodity k $(s_{(k)} \in S_{(k)})$

$t_{(k)}$ index for sink node of commodity k $(t_{(k)} \in T_{(k)})$

i or j : index for any nodes $(i \neq j), (i \in N, j \in N, (i, j) \in A)$

k : index for commodity $(k \in K)$

Parameters:

w_k : weight or priority value of commodity k

$\tilde{u}_{ij}$: fuzzy arc capacity of arc (i, j)

r_{ij}: the interdiction cost for arc (i, j)

R : total interdiction resource

Decision variables:

x_{ijk} : the flow quantity from node i to node j for the k^{th} commodity

λ_{ik} : the dual variables related to the flow balance constraints

δ_{ij} : the dual variables related to the capacity constraints

γ_{ij} : {if arc (i, j) is interdicted, 1; otherwise, 0.}

3.1. The multi-commodity maximum flow network interdiction problem under fuzzy environment (MC-MFNIP)

The MC-MFNIP addresses the problem where while the follower tries to maximize the multi-commodity flow throughout an undirected network, the leader attempts to minimize the total flow achieved by the follower using limited interdiction resource in the presence of fuzzy information regarding the arc capacities. In this section, the fuzzy-based model of the sides is formulated to allow considering multiple-source & multiple-sink nodes for the commodities and prioritizing the commodities. In the addressed problem, the uncertain arc capacities are defined using triangular fuzzy numbers as depicted in Figure 2. In Figure 2, triangular fuzzy numbers are represented by three points: $\tilde{A}=(a,b,c)$ where "a" corresponds to the smallest likely value (lower bound), "b" the most likely value, and "c" the largest likely value (upper bound) for a fuzzy event [87]. Here, the fuzzy set $\tilde{A}$ is represented by $\tilde{A}=\{(x,\mu_A(x):x\in A)\}$ where $\mu_A(x):x\rightarrow[0,1]$ is a function that is called the membership function of $\tilde{A}$. In this study, the solution approach, developed by Jimenez et al. [88] and then modified by Parra et al. [89], is employed to solve the fuzzy-based optimization models.

Insert Figure 2 Here

3.1.1. The formulations of the follower's problem

In this subsection, the mathematical model of the follower's problem is formulated. The objective function (1) maximizes the total flow quantity on the network by considering each commodity. Here, w_k is the weight or priority value of commodity k. In cases where commodities do not have priority values, w_k is considered as 1 for each commodity. The constraint sets (2) and (4) state the flow-balancing regarding the source and sink nodes, respectively. In a similar way, the constraint set (3) deals with the flow-balancing regarding the remaining nodes. The constraint set (5) limits the sum of the total flow quantity realized for all k commodities on the arc (i,j) with the fuzzy arc capacity of the arc (i,j). The constraint set (5) implies the nonnegativity for the decision variables.

$$MaxZ=\sum_k w_k\left(x_{t_{(k)}s_{(k)}k}\right) \quad (1)$$

s.t. *Dual variables*

$$\sum_{s_{(k)}}\sum_j x_{s_{(k)}jk}-\sum_{s_{(k)}}\sum_j x_{js_{(k)}k}-x_{t_{(k)}s_{(k)}k}=0 \quad \forall k\in K \qquad :\lambda_{s_{(k)}k} \quad (2)$$

$$\sum_j x_{ijk}-\sum_j x_{jik}=0 \quad \forall i\in N-\cup_k\left(S_{(k)},T_{(k)}\right),\forall k\in K \qquad :\lambda_{ik} \quad (3)$$

$$\sum_{t_{(k)}} \sum_j x_{t_{(k)}jk} - \sum_{t_{(k)}} \sum_j x_{jt_{(k)}k} - x_{t_{(k)}s_{(k)}k} = 0 \quad \forall k \in K \qquad : \lambda_{t_{(k)}k}$$

(4)

$$\sum_k \left(x_{ijk} + x_{jik} \right) \le \tilde{u}_{ij} \qquad \forall (i,j) \in A \qquad : \delta_{ij}$$

(5)

$$x_{ijk} \ge 0 \qquad \forall (i,j) \in A, \forall k \in K \qquad (6)$$

To solve the optimization model, the constraint given in Eq. (5) is rearranged using the transformation approach presented by Jimenez et al. [88] and replaced with Eq. (5*). In this rearrange, the fuzzy capacity values ($\tilde{u}_{ij}$) on the right-hand side of the constraint in Eq. (5) is defuzzied depending on the parameter, and the constraint containing these defuzzied capacity values is represented by Eq. (5*) (where the symbol * only represents the revised version of Eq. (5)). By doing so, the following ordinary α-parametric dual linear program (7-12) is constructed to obtain α-acceptable solutions of the problem. Here, α represents the feasibility degrees that the decision-maker is willing to consider. Having a high feasibility degree demonstrates that the DM will not be willing to admit high risks in the violation of the constraints.

$$\sum_k \left(x_{ijk} + x_{jik} \right) \le (\alpha) \left(\frac{u^a{}_{ij} + u^b{}_{ij}}{2} \right) + (1-\alpha) \left(\frac{u^b{}_{ij} + u^c{}_{ij}}{2} \right) \quad \forall (i,j) \in A \qquad : \delta_{ij} \quad (5^*)$$

As previously mentioned, Wood [23] developed a "*min-max*" formulation for the MFNIP and simplified it into a single-level integer programming model. To this end, it is needed to derive the dual form of the follower's inner maximization formulation. This dual model of the follower problem is employed to reduce the leader's bi-level ("*min-max*") interdiction model to a single-level ("*min-min*") structure. The obtained dual model exhibits no functional differences from the primal follower model presented above. Since the objective function in the dual model, where the primal model’s objective function is maximized, becomes minimization, this approach is adopted.

Accordingly, with the help of this dual model, the leader’s bi-level model transforms into a single-level "*min-min*" structure.

Here, λ is the dual variable related to the constraint sets (2), (3), and (4), and δ is the dual variable dealing with the constraint set (5).

$$MinZ = \sum_{(i,j)\in A}\left((\alpha)\left(\frac{u^{a}{}_{ij}+u^{b}{}_{ij}}{2}\right)+(1-\alpha)\left(\frac{u^{b}{}_{ij}+u^{c}{}_{ij}}{2}\right)\right)\delta_{ij} \tag{7}$$

s.t.

$$\lambda_{ik}-\lambda_{jk}+\delta_{ij}\geq 0 \qquad \forall (i,j)\in A, \forall k\in K \tag{8}$$

$$\lambda_{jk}-\lambda_{ik}+\delta_{ij}\geq 0 \qquad \forall (i,j)\in A, \forall k\in K \tag{9}$$

$$\lambda_{t_{(k)}k}-\lambda_{s_{(k)}k}\geq w_k \qquad \forall k\in K, \forall s_{(k)}\in S_{(k)}, \forall t_{(k)}\in T_{(k)} \tag{10}$$

$$\lambda_{ik}: urs \qquad \forall i\in N, \forall k\in K \tag{11}$$

$$\delta_{ij}\geq 0 \qquad \forall (i,j)\in A \tag{12}$$

3.1.2. The formulation of the leader's problem

In this subsection, the leader's problem is formulated as a bi-level "*min-max*" fuzzy-based optimization model as follows:

$$MinMaxZ = \sum_k w_k\left(x_{t_{(k)}s_{(k)}k}\right) \tag{13}$$

s.t.

$$\sum_{s_{(k)}}\sum_j x_{s_{(k)}jk}-\sum_{s_{(k)}}\sum_j x_{js_{(k)}k}-x_{t_{(k)}s_{(k)}k}=0 \qquad \forall k\in K \tag{14}$$

$$\sum_j x_{ijk}-\sum_j x_{jik}=0 \qquad \forall i\in N-\cup_k\left(S_{(k)},T_{(k)}\right), \forall k\in K \tag{15}$$

$$\sum_{t_{(k)}}\sum_j x_{t_{(k)}jk}-\sum_{t_{(k)}}\sum_j x_{jt_{(k)}k}-x_{t_{(k)}s_{(k)}k}=0 \qquad \forall k\in K \tag{16}$$

$$\sum_k\left(x_{ijk}+x_{jik}\right)\leq\left((\alpha)\left(\frac{u^{a}{}_{ij}+u^{b}{}_{ij}}{2}\right)+(1-\alpha)\left(\frac{u^{b}{}_{ij}+u^{c}{}_{ij}}{2}\right)\right)\left(1-\gamma_{ij}\right) \quad \forall (i,j)\in A \tag{17}$$

$$\sum_{(i,j)\in A} r_{ij}\gamma_{ij}\leq R \tag{18}$$

$$x_{ijk}\geq 0 \qquad \forall (i,j)\in A, \forall k\in K \tag{19}$$

$$\gamma_{ij}\in\{0,1\} \qquad \forall (i,j)\in A \tag{20}$$

The inner maximization problem corresponds to the follower's problem consisting of the constraint sets (13-16). The constraint set (17) ensures that the capacity of the arc (i, j) for an interdicted arc (i, j) equals 0 when $\gamma_{ij}=1$ or the total flow quantity realized for all k commodities for the arc (i, j) is limited by the fuzzy capacity of arc (i, j) when $\gamma_{ij}=0$.

The constraint set (18) allows that the total cost of the interdicted arcs is smaller or equal to the interdiction resource. Constraint sets (19-20) are non-negativity and binary constraints for decision variables, respectively.

$$MinMinZ = \sum_{(i,j)\in A}\left((\alpha)\left(\frac{u^a{}_{ij}+u^b{}_{ij}}{2}\right)+(1-\alpha)\left(\frac{u^b{}_{ij}+u^c{}_{ij}}{2}\right)\right)(1-\gamma_{ij})\delta_{ij} \tag{21}$$

s.t.

$$\lambda_{ik}-\lambda_{jk}+\delta_{ij}\geq 0 \qquad \forall(i,j)\in A, \forall k\in K \tag{22}$$

$$\lambda_{jk}-\lambda_{ik}+\delta_{ij}\geq 0 \qquad \forall(i,j)\in A, \forall k\in K \tag{23}$$

$$\lambda_{t_{(k)}k}-\lambda_{s_{(k)}k}\geq w_k \qquad \forall k\in K, \forall s_{(k)}\in S_{(k)}, \forall t_{(k)}\in T_{(k)} \tag{24}$$

$$\lambda_{ik}: urs \qquad \forall i\in N, \forall k\in K \tag{25}$$

$$\delta_{ij}\geq 0 \qquad \forall(i,j)\in A \tag{26}$$

$$\gamma_{ij}\in\{0,1\} \qquad \forall(i,j)\in A \tag{27}$$

To obtain the final version of the leader's model, the leader's bi-level "*min-max*" model is reduced to a single level "*min-min*" fuzzy-based optimization model that is a minimization model. In this transformation, the inner maximization problem is converted to its dual problem by fixing γ_{ij} temporarily, and then releasing γ_{ij}.

After transformation, it is seen that the objective function (21) is nonlinear. For this reason, a new nonnegative decision variable β_{ij} is added to the model instead of the expression $(1-\gamma_{ij})\delta_{ij}$ for linearization of the model. Besides, the constraint set (32) is also added to the model, with M being a sufficiently big number, to ensure this linearization. It is possible to check that this constraint set (32) provides the linearization as follows:

- For, $\gamma_{ij}=0$, β_{ij} should be equal to δ_{ij}, by equality of $\left(\beta_{ij}=(1-\gamma_{ij})\delta_{ij}\right)$. When these expressions are substituted in the constraint set (32), $\left(\beta_{ij}+M*0-\delta_{ij}\geq 0\right)$, yields $\beta_{ij}\geq\delta_{ij}$ Since the objective function of the model is minimization, the program forces this constraint set to be $\beta_{ij}=\delta_{ij}$
- For, $\gamma_{ij}=1$, β_{ij} should be equal to zero, by equality of $\left(\beta_{ij}=(1-\gamma_{ij})\delta_{ij}\right)$. When these expressions are substituted in the constraint set (32), $\left(\beta_{ij}+M*0-\delta_{ij}\geq 0\right)$, yields

$\beta_{ij} \geq \delta_{ij} - M$. As M is a sufficiently big number, the right-hand side of the constraint set gets negative values. In this case, since the objective function of the model is also minimization, the program forces β_{ij} to be zero, which is the best value it can get for this constraint set.

Thus, it is seen that the conversion "$\left(\beta_{ij} = \left(1-\gamma_{ij}\right)\delta_{ij}\right)$" is met for both $\gamma_{ij} = 0$ and $\gamma_{ij} = 1$ in the constraint set (32). Hence, it can be said that the new constraint set (constraints set (32)) added through this conversion guarantees linearization.

Overall, the final version of the leader's model is presented as follows.

$$MinZ = \sum_{(i,j)\in A}\left(\left(\alpha\right)\left(\frac{u^a{}_{ij} + u^b{}_{ij}}{2}\right) + \left(1-\alpha\right)\left(\frac{u^b{}_{ij} + u^c{}_{ij}}{2}\right)\right)\beta_{ij} \tag{28}$$

s.t.

$$\lambda_{ik} - \lambda_{jk} + \delta_{ij} \geq 0 \qquad \forall (i,j)\in A, \forall k \in K \tag{29}$$

$$\lambda_{jk} - \lambda_{ik} + \delta_{ij} \geq 0 \qquad \forall (i,j)\in A, \forall k \in K \tag{30}$$

$$\lambda_{t_{(k)}k} - \lambda_{s_{(k)}k} \geq w_k \qquad \forall k \in K, \forall s_{(k)} \in S_{(k)}, \forall t_{(k)} \in T_{(k)} \tag{31}$$

$$\beta_{ij} + M\gamma_{ij} - \delta_{ij} \geq 0 \qquad \forall (i,j)\in A \tag{32}$$

$$\sum_{(i,j)\in A} r_{ij}\gamma_{ij} \leq R \tag{33}$$

$$\lambda_{ik} : urs \qquad \forall i \in N, \forall k \in K \tag{34}$$

$$\delta_{ij} \geq 0 \qquad \forall (i,j)\in A \tag{35}$$

$$\beta_{ij} \geq 0 \qquad \forall (i,j)\in A \tag{36}$$

$$\gamma_{ij} \in \{0,1\} \qquad \forall (i,j)\in A \tag{37}$$

The proposed fuzzy-based optimization model is formulated in a general form considering the prioritization and multiple-source & multiple-sink nodes for the commodities for the MC-MFNIP in which there is fuzzy information about the arc capacities. In the situation where the importance of the commodities with respect to each other is different, it is recommended that each w_k denoting the importance of the k^{th} commodity is defined as $0 \leq w_k \leq 1$. Here, the sum of weights of the importance of all commodities should equal 1. Similarly, each w_k should equal 1 for the situation where the commodities are not prioritized.

4. Computational studies

In this section, the computational studies are performed for three purposes: (i) The proposed fuzzy-based optimization model, given in subsection 3.1.2, is applied visually through an illustrative example to express the MC-MFNIP under a fuzzy environment. (ii) A sensitivity analysis is conducted by prioritizing the commodities to reveal changes in the flow quantity of commodities and total flow quantity. Besides, changes in the flow quantity are observed under varying numbers of source and sink nodes for the network. (iii) The model performance is tested on different-sized networks generated randomly by considering varying α-cut levels. In this regard, the networks $G(15,30)$, $G(20,43)$, $G(48,117)$, $G(80,205)$ and $G(120,317)$ are generated hypothetically and given in [90]. In the study, all networks are of rectangular grid type. $N = n_1 \times n_2$ denotes the total number of nodes in the networks in which n_1 represents the number of nodes at vertical, while n_2 represents the number of nodes at horizontal. These networks can be defined as 3×5, 5×4, 8×6, 8×10 and 10×12 respectively when considering grid structure.

All computational experiments are performed on a PC with a 2.7 GHz, i5 7200U processor, and 8 GB of RAM by using the optimization program IBM ILOG CPLEX 12.7 Optimization Studio. By benefiting from this program's high-performance mathematical programming solvers for linear programming, solutions are obtained in quite short times, ranging from 1-12 seconds depending on the model size for all experiments. In all experiments, the interdiction cost of each arc is assumed as 1-unit.

4.1. An illustrative example

The proposed fuzzy-based optimization model is visually applied to the network $G(15,30)$, as depicted in the graphical representation in, to clearly express the MC-MFNIP. In this example, prioritization for commodities is ignored and it is considered that each commodity has its single source and sink nodes. As depicted in Figure 3, different colors represent different commodities. For example, the source and sink nodes defined for commodity 2, indicated in purple, are 2 and 15, respectively. In this analysis, the optimization model is run until the complete interdiction of the flow is achieved for each α-cut level, and the results are summarized in Table 2. Figure 4 visualizes the alterations in the objective function values associated with each interdiction resource level with respect to α-cut levels. As illustrated in Figure 4, regardless of the interdiction resource level, it is clearly observed that the maximum flow rate decreases as the α-cut value increases. Furthermore, all results pertaining to commodity-based flow on the network are visualized for each interdiction resource level with respect to an α-cut level of 0.5 in Figure 5. Figure 5 visually represents the interdicted and unused arc information, flow routes for each commodity, as well as the total flow quantity and individual flow quantities for each commodity within the network $G(15,30)$. As depicted in Figure 5, where $R=0$ represents the commodity-based flow throughout the network when there is no interdiction on arcs, and $R=1$ represents the commodity-based flow throughout the network when the interdiction resource equals 1 unit. The analysis presented in Figure 5 is performed only for α-cut level: 0.5, while the results of the same computational analysis regarding the flow quantities of commodities for all α-cut levels are visualized in Figure 6 and numerically provided in Table 3.

Insert Figure 3 Here

Insert Table 2 Here

Insert Figure 4 Here

Insert Figure 5 Here

Triangular fuzzy number's upper and lower bounds are becoming narrow as α-cut level increases [91, 92]. Therefore, when considering a certain interdiction resource level, it is observed that the objective function values and commodity-based flow quantities are decreased as the α-cut value increases due to reducing the flexibility of the solution of model with increasing as α-cut level (See: Figure 5). Besides, it is revealed that all flows can be interdicted at $R = 5$. It is worth noting that as the interdiction resource level increases, the flow of commodity, which provides the most contribution to the objective function value or total flow quantity, is interdicted when looking at the commodity-based flow. Figure 6 demonstrates a decrease in the flow of commodities as the level of the interdiction resource increases. Similar to the degradation in the objective function value observed in Figure 4 as the α-cut level increases, Figure 6 also shows a worsening of the flow quantities of commodities as the α-cut level increases.

Insert Figure 6 Here

Insert Table 3 Here

4.2. Sensitivity analysis

In this section, a sensitivity analysis is performed by prioritizing the commodities to reveal the changes in the flow quantity of commodities and total flow quantity, unlike the previous illustrative example considering as equal the importance of the commodities. To this end, a set of scenarios regarding the prioritized commodities also including equal status priority is generated randomly and the weights of the commodities are given in Table 4. These scenarios are applied for the network $G(15,30)$ given in Figure 3.

According to each interdiction resource level, the obtained experimental results in terms of commodity-based flows are visualized for each scenario with respect to α-cut level=0.5 and given in Figure 7. For each interdiction resource level with respect to α-cut levels, all experimental results including information about the interdicted arcs as well as total flow quantity and commodity-based flow quantities are provided in [90].

Insert Table 4 Here

The significance of commodities is clearly revealed in Figure 7. For instance, when comparing scenario 1 with scenario 2, it becomes evident that commodity 3 holds greater importance in scenario 1, while commodity 2 is more crucial in scenario 2. This significance is particularly pronounced for $R=0$, as shown in Figure 7. It can be deduced that the leader tends to interdict the flow of these commodities first due to their critical role in the objective function, resulting in relatively higher contributions. As the interdiction resource level increases, the complete interdiction of highly important commodities can occur at relatively lower resource levels. It is noteworthy that, even though commodity 1 holds greater importance in scenarios 5 and 6, its flow quantity for $R=0$ is lower compared to commodities 2 and 3. This discrepancy is because the maximum flow quantity for commodity 1 has already been reached. Furthermore, the significance of commodities 2 and 3 is prominently observed for $R=0$ in scenarios 5 and 6, aligning with the information provided in Table 4.

Insert Figure 7 Here

In another analysis, it is aimed to track the changes in the objective function value (total flow quantity) while varying numbers of source and sink nodes within the network $G(48,117)$. To this end, some possible scenarios are constructed by considering a different number of source and sink nodes for each commodity in the addressed network. Information about these sources and sink nodes is provided in Table 5 for each scenario. The experimental results are illustrated for each interdiction resource level with respect to α-cut levels in Figure 8. Additionally, rest of the results are provided in [90].

Insert Table 5 Here

As seen in Figure 8, it is worth noting that the objective function value decreases as the α-cut level increases in all R levels due to the narrowing of the fuzzy arc capacity limit values. Moreover, it is evident that the number of multiple-source and multiple-sink nodes directly influences to the objective function value as it affects the alternative relationships between source-sink nodes for the flow of commodities.

In Figure 8, as expected, until $R=8$, scenario 1 exhibits lower objective function values than other scenarios, where all flow is interdicted for $R=8$ in scenario 1. Similarly, scenario 4 demonstrates higher objective function values than other scenarios because of the greater number of alternative relationships between source-sink nodes. While scenarios 3 and 4 follow a similar trend in terms of objective function value until $R=5$, scenario 4 consistently maintains the highest objective function value compared to other scenarios. Furthermore, this analysis makes it somewhat challenging to draw clear conclusions about Scenarios 2 and 3. However, scenario 3 has higher objective function values than scenario 2 until $R=9$, and for $R=11$, all flow is interdicted in scenario 3, while in scenario 2, all flow is interdicted for $R=13$. In this context, it can be concluded that for this analysis, Scenario 2, which includes more sink nodes, is more resilient against interdictions compared to Scenario 3, which includes more source nodes. Therefore, Scenario 4, which incorporates both more source and sink nodes, proves to be the most resilient against interdictions.

Insert Figure 8 Here

4.3. Testing of the model performance

Besides the network $G(15,30)$ handled for the illustrative example, the performance of the fuzzy-based optimization model is tested on the networks $G(15,30)$, $G(20,43)$, $G(48,117)$, $G(80,205)$ and $G(120,317)$ in terms of average runtimes (in seconds) on both row and column basis.

The network $G(20,43)$ consists of 20 nodes and 43 arcs, and four commodities, which have their source/sink nodes, flowing on this network. Similarly, four, five, and five commodities flow on networks $G(48,117)$, $G(80,205)$ and $G(120,317)$ respectively. In all instances, the interdiction resource is increased until the total maximum flow quantity equals zero for α-cut levels: 0.00; 0.25; 0.50; 1.00. The experimental outcomes are summarized in terms of attained runtimes and objective function values for the different interdiction resource levels with respect to different α-cut levels for all generated instances (See: Tables 6-9). As expected, the objective function value decreases as the α-cut level increases in all R levels since the limit values of fuzzy arc capacities are narrowed. Besides, the objective function value decreases as the R level increases. Finally, the solutions are attained in seconds changing from 1 sec to 12 sec in all generated network instances.

Insert Table 6 Here

Insert Table 7 Here

Insert Table 8 Here

Insert Table 9 Here

5. Conclusion and future directions

This study addresses the MC-MFNIP for the situation involving fuzzy information regarding arc capacities. To this end, the fuzzy-based optimization model that considers the prioritization, and multiple-source & multiple-sink nodes for the commodities for the MC-MFNIP to cope with the uncertainties on the arc capacities. The proposed model is run for different purposes, and computational analyses are performed through a set of problems regarding the different-sized networks and α-cut levels to test the model performance and track the changes in the flow quantities under various interdiction resource level. Further, thanks to the different angles of the model, the situations regarding the prioritization of commodities and the inclusion of multiple-source & multiple-sink nodes are investigated through a set of scenarios. According to the experimental results, the proposed model can solve all generated networks and prove interdiction plans within seconds. Besides, it is deduced that the presence of the multiple-source and multiple-sink nodes provides resilience against interdictions. It is realized that the leader tends to interdict the flow of the commodity which has high importance first, as its contribution to the objective function is relatively high. Further, in the proposed fuzzy optimization model, it is observed that an increase in the α-cut value results in a deterioration (decrease) of the objective function value related to maximum flow. When analyzing the network $G(20,43)$, an average decrease of approximately 43% in the objective function is noted, while for the network $G(48,117)$, there is an average decrease of around 30%. In the case of the network $G(80,205)$, the average decrease is about 35%, and for the network $G(120,317)$, there is an average decrease of around 30%. However, it is important to emphasize that the rate of decrease in the objective function varies between 23% and 50% in all analyses.

The novelty of this research is presented from several perspectives as follows:

(i) From a theoretical standpoint, this study introduces a novel fuzzy-based optimization model that considers prioritization, multiple-source, and multiple-sink nodes for commodities, addressing the uncertainties inherent in arc capacities for the MC-MFNIP for the first time.

(ii) From a managerial perspective, the model provides valuable information about interdicted arcs, often referred to as risky or critical arcs. This information is beneficial for decision-makers, enabling them to develop effective fortification strategies for any network related to the MC-MFNIP in a fuzzy environment.

(iii) From a practical standpoint, the model offers efficient and tractable solutions, providing interdiction plans within seconds. Consequently, this research aims to serve as a pioneering contribution for researchers and academicians investigating interdiction problems in fuzzy environments, offering valuable insights for future studies.

This study can be extended in several interesting future directions: (i) The development of a data-driven optimization model is a promising avenue to manage uncertain arc capacity information. (ii) The MC-MFNIP can be developed as a multi-objective optimization problem, aiming to minimize both the total flow quantity achieved by the follower and the interdiction resource simultaneously. (iii) It would be intriguing to explore scenarios involving asymmetric information, where the leader and follower possess varying levels of knowledge about the network. (iv) Last but not least, the inclusion of multiple interdiction resources, with considerations for interdiction success rates or costs, could enrich the model. This extension would allow for the study of partial interdictions of arcs by incorporating diverse interdiction resources into the model.

Statements and Declarations:

On behalf of all authors, Gökhan ÖZÇELİK (Corresponding Author) states that there is no conflict of financial or non-financial interests directly or indirectly.

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Figure and Table Captions

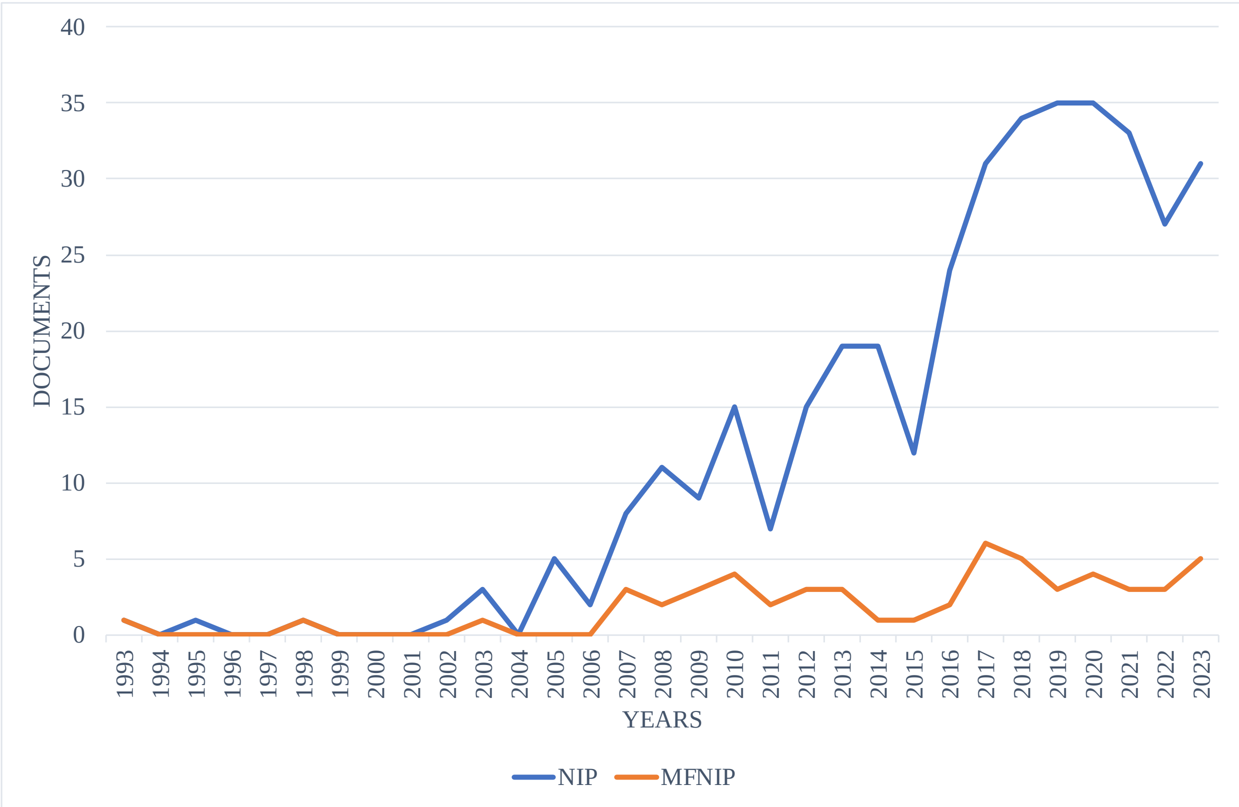
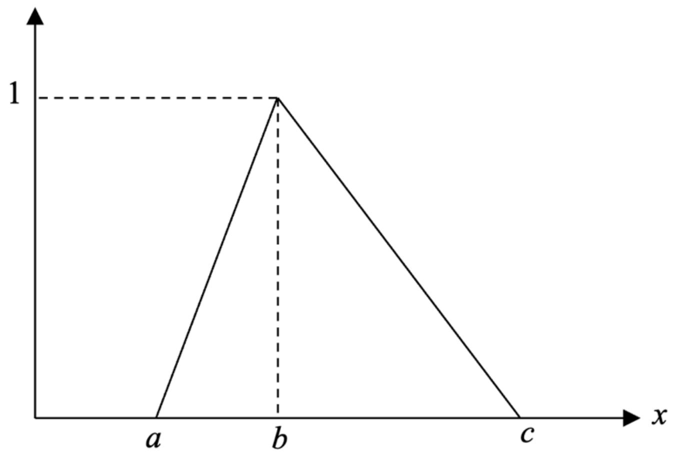


Figure 1. The trends related to NIP and MFNIP.



$$\mu_A(x) = \begin{cases} 0, & x < 0 \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{c-x}{c-b}, & b \le x \le c \\ 0, & x > c \end{cases}$$

Figure 2. Graphical representation of triangular fuzzy number.

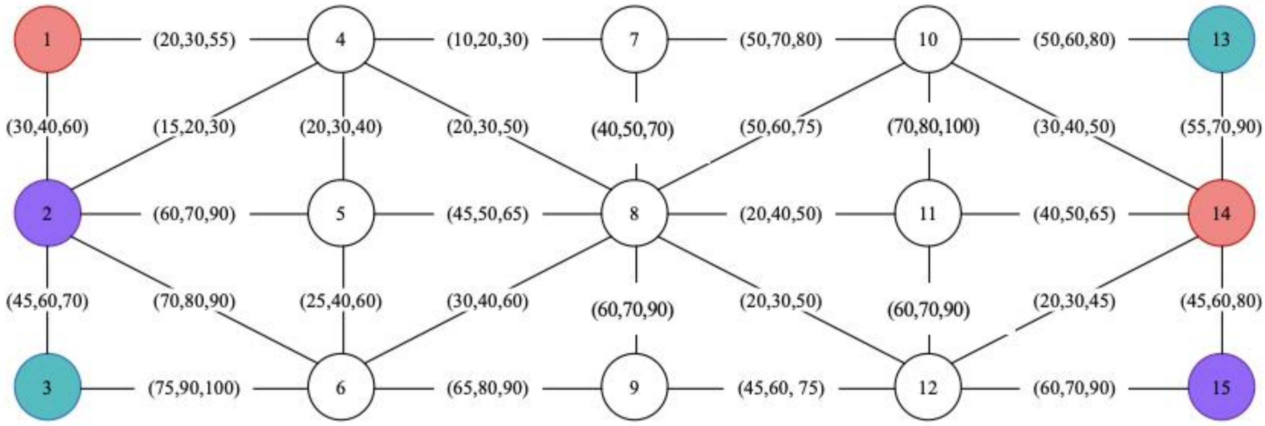


Figure 3. The visualization of the addressed network $G(15,30)$

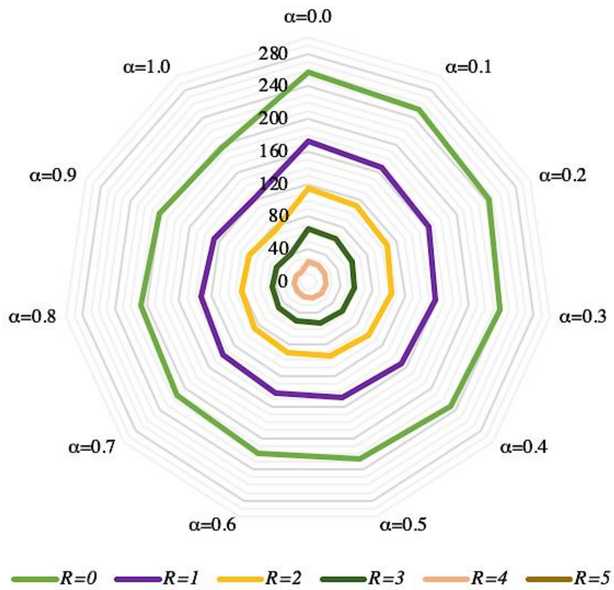


Figure 4. Visualizing alterations in objective function values (total flow quantity) for each interdiction resource level with respect to different α-cut levels.

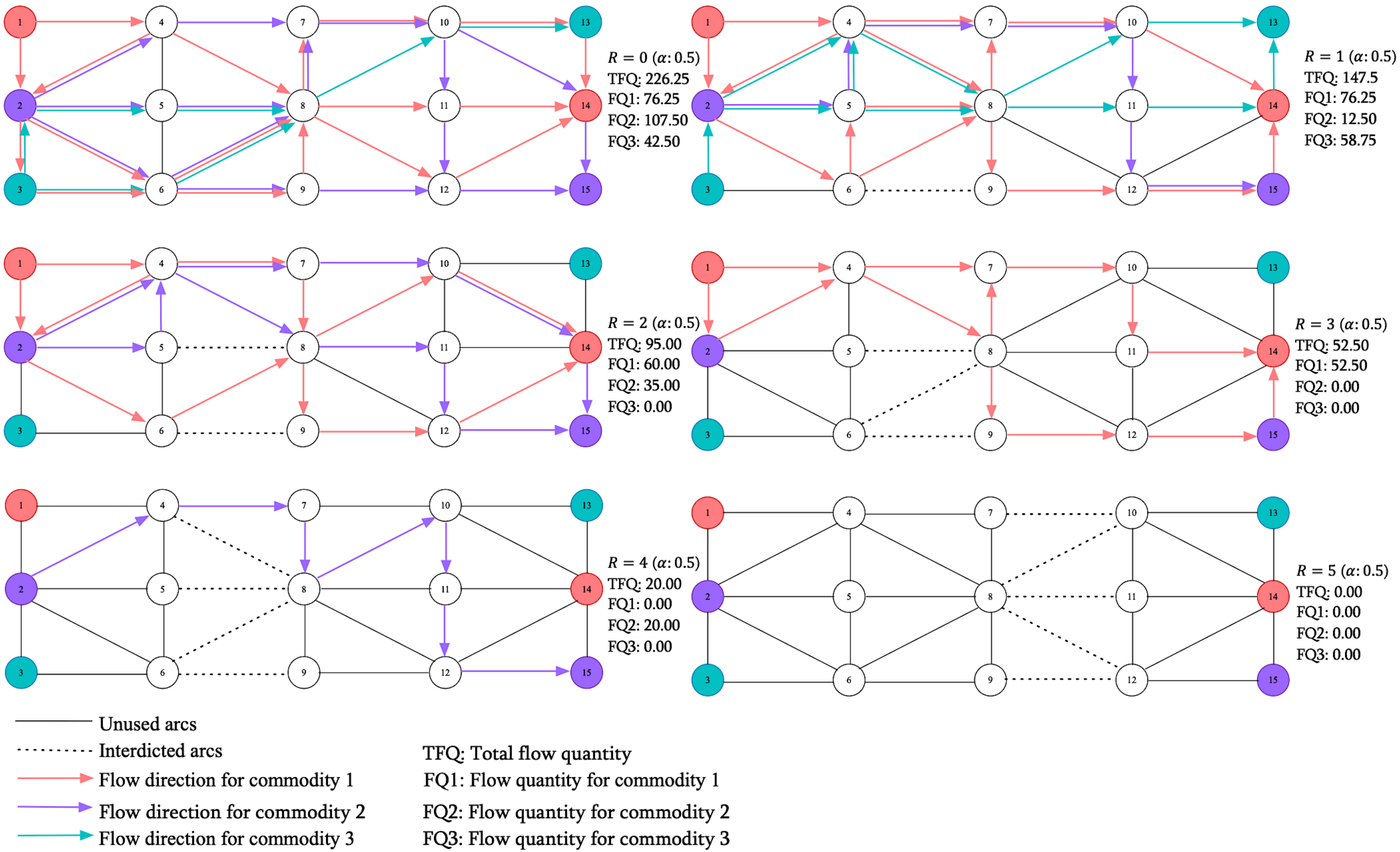


Figure 5. The commodity-based flows on the network $G(15,30)$ for α-cut level=0.5.

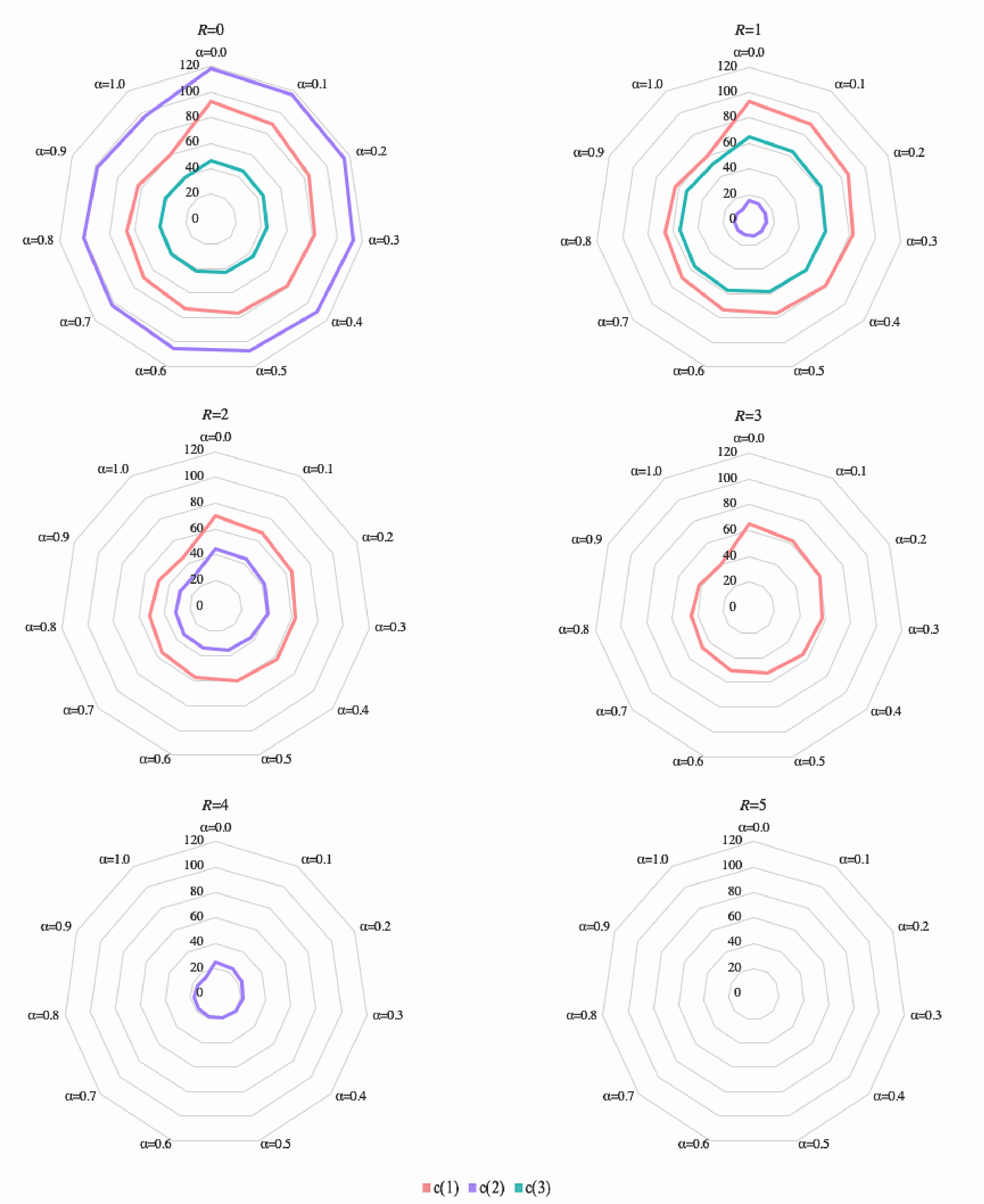


Figure 6. Visualizing changes in the commodity-based flow quantities for each interdiction resource level with respect to all α-cut levels.

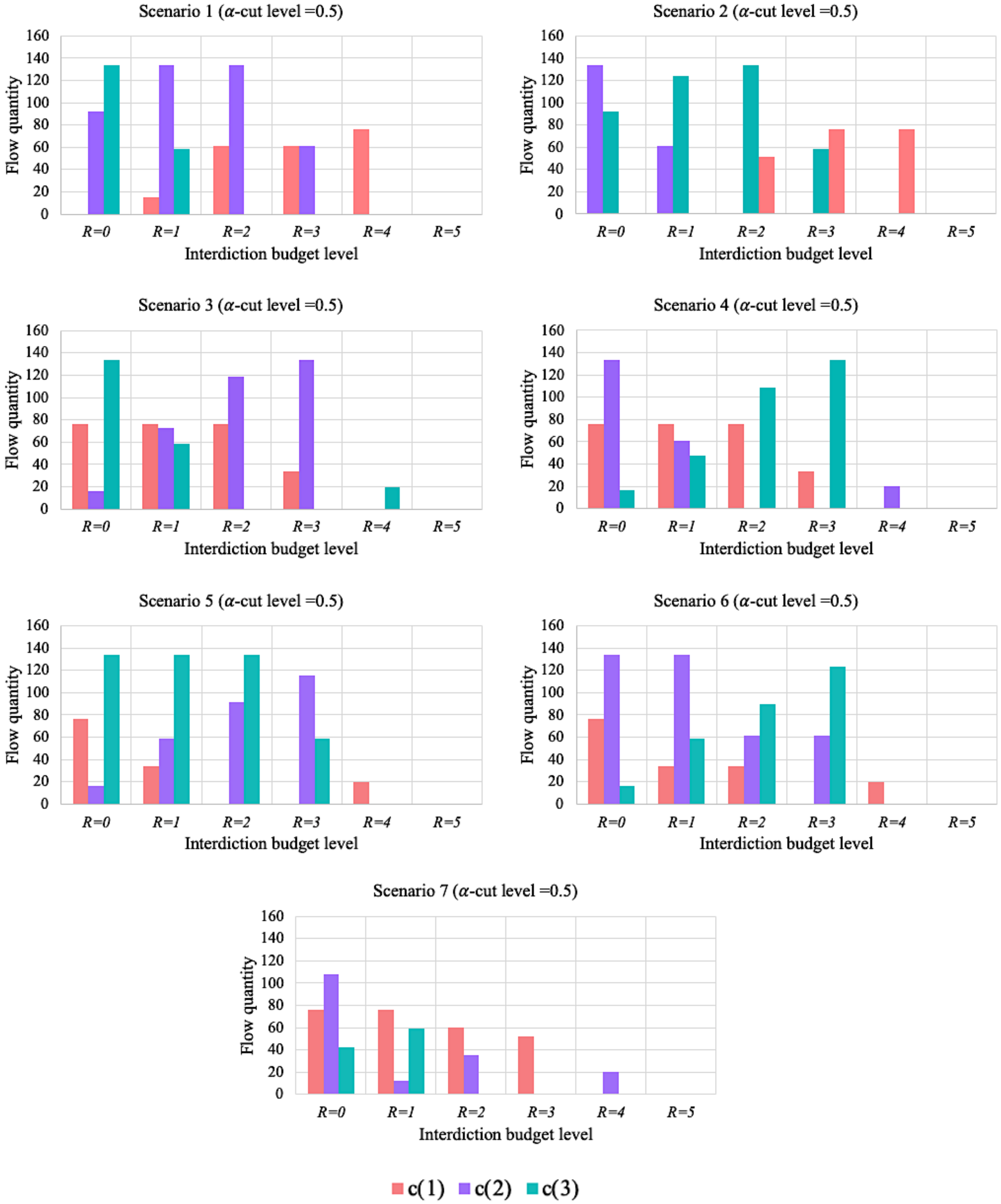


Figure 7. The graphs of commodity-based flow quantities regarding each scenario with respect to α-cut level=0.5.

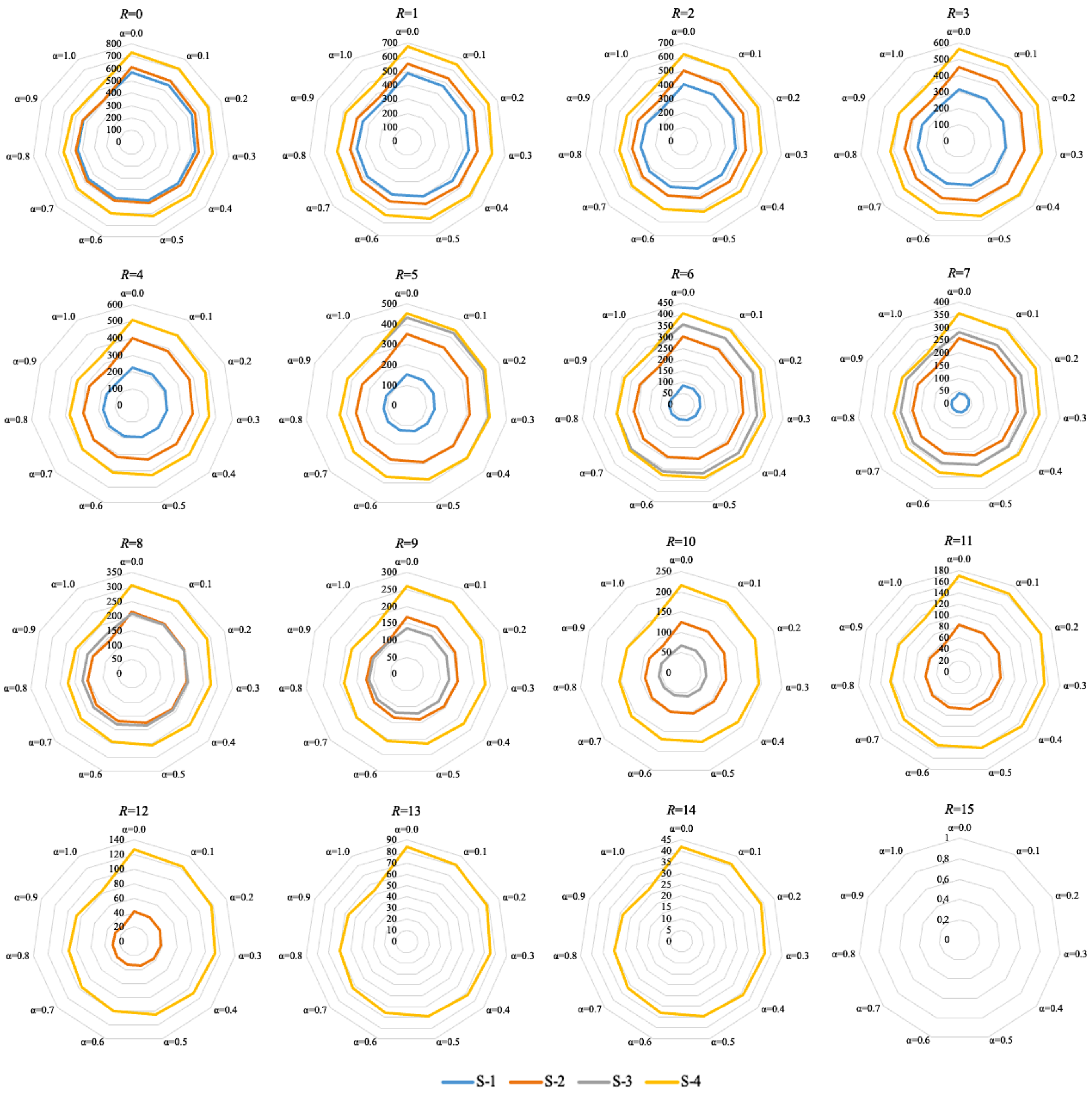


Figure 8. The experimental results for scenarios involving multiple-source and multiple-sink nodes related to each interdiction resource level with respect to α-cut levels.

Table 1. The classification of relevant studies.

Authors	Information about structure of networks						Information about interdiction processes								Information about methodologies						
	Type of network		**Number of commodities**		**Number of source and sink nodes**		**Type of interdiction**				**Interdiction environment**				**Number of objectives**		**Solution methodology**				
	Directed	**Undirected**	**Single**	**Multi**	**Single**	**Multi**	**Edge (Arc)**	**Node**	**Discrete**	**Continuous**	**Deterministic**	**Stochastic**	**Robust**	**Fuzzy**	**Single**	**Multi**	**Linear Optimization**	**Nonlinear**	**Heusristic**	**Approximation/Pseudo-Approximation**	**Decomposition Approachs**
Steinrauf [29]		✓	✓		✓	✓	✓		✓		✓				✓		✓				
Wood [23]	✓	✓	✓	✓	✓	✓	✓		✓		✓				✓		✓				
Washburn and Wood [42]	✓	✓	✓		✓	✓	✓	✓		✓	✓				✓		✓				
Chern and Lin [43]	✓		✓			✓	✓			✓	✓				✓		✓				
Cormican et al. [44]	✓		✓		✓		✓		✓			✓			✓		✓			✓	✓
Whiteman [45]	✓		✓			✓		✓	✓		✓				✓	✓	✓				
Bingol [46]	✓		✓		✓		✓		✓		✓				✓		✓		✓	✓	
Burch et al. [47]		✓	✓		✓		✓		✓		✓				✓		✓				✓
Lim and Smith [48]	✓			✓	✓		✓		✓	✓	✓				✓		✓		✓		
Smith et al. [49]	✓			✓	✓		✓		✓	✓	✓				✓		✓				✓
Royset and Wood [50]	✓		✓			✓	✓		✓		✓					✓	✓				
Janjarassuk and Linderoth [51]	✓		✓		✓		✓		✓			✓			✓		✓		✓	✓	✓
Rocco and Ramirez-Marquez [52]	✓	✓	✓		✓	✓	✓		✓		✓				✓				✓		
Ramirez-Marquez and Rocco [53]	✓		✓		✓		✓		✓			✓			✓				✓		
Rocco et al. [54]	✓	✓	✓		✓	✓	✓		✓		✓					✓		✓	✓		
Altıner et al. [55]		✓	✓		✓		✓		✓		✓				✓		✓				
Rocco et al. [56]	✓	✓	✓		✓	✓	✓		✓		✓					✓		✓	✓		
Akgün et al. [57]		✓		✓	✓		✓		✓		✓				✓		✓			✓	
Lunday and Sherali [58]	✓		✓		✓		✓			✓	✓				✓		✓	✓	✓	✓	
Lunday and Sherali [59]	✓		✓		✓		✓		✓	✓	✓				✓		✓	✓			
Malaviya et al. [60]	✓		✓		✓		✓		✓		✓				✓		✓				
Zheng and Castanon [61]	✓		✓		✓		✓		✓		✓				✓		✓				✓
Zheng and Castanon [62]	✓		✓		✓		✓		✓			✓			✓		✓				✓
Granata et al. [63]		✓	✓		✓			✓	✓		✓				✓		✓				
Rad and Kakhki [64]	✓		✓		✓		✓		✓		✓				✓		✓				✓
Sullivan and Smith [65]		✓	✓			✓	✓		✓		✓				✓		✓				
Keshavarzi and Fathabadi [66]		✓	✓			✓	✓		✓	✓	✓				✓		✓				
Janjarassuk and Nakrachata-Amon [67]	✓		✓		✓		✓		✓			✓			✓				✓		
Baffier et al. [68]	✓		✓		✓		✓		✓		✓				✓		✓				
Guo et al. [69]	✓	✓	✓		✓		✓	✓	✓		✓				✓			✓	✓	✓	
Zhang and Fan [70]	✓			✓	✓		✓		✓	✓	✓				✓		✓				
Chestnut and Zenklusen [71]	✓		✓		✓		✓		✓				✓		✓		✓			✓	

Chestnut and Zenklusen [72]		✓	✓		✓		✓		✓		✓					✓				✓	
Naoum-Sawaya and Ghaddar [73]	✓			✓	✓		✓		✓		✓				✓		✓				
Soleimani-Alyar and Ghaffari-Hadigheh [74]	✓		✓		✓		✓		✓		✓				✓			✓			✓
Rad and Kakhki [75]	✓		✓		✓		✓		✓		✓				✓		✓				
Soleimani-Alyar and Ghaffari-Hadigheh [76]	✓		✓		✓		✓		✓			✓			✓		✓				
Baycik et al. [77]	✓			✓		✓	✓		✓		✓				✓		✓				
Lei at al. [1]	✓		✓		✓		✓		✓			✓			✓		✓				
Ashraf [78]	✓			✓		✓		✓	✓		✓				✓		✓		✓		
Özçelik and Gencer [20]	✓		✓		✓		✓		✓		✓					✓	✓				
Enayaty-Ahangar et al. [79]	✓		✓		✓		✓		✓		✓				✓		✓				✓
Zhang et al. [3]	✓		✓		✓		✓		✓				✓		✓		✓			✓	
Sadeghi and Seifi [2]	✓		✓		✓		✓		✓			✓			✓		✓				✓
Jabarzare et al. [21]	✓			✓		✓	✓		✓		✓				✓		✓		✓		✓
Wu et al. [80]		✓	✓			✓	✓		✓		✓				✓		✓				
Disser and Matuschke [4]	✓		✓		✓		✓		✓				✓		✓		✓				
Alsharirad [81]	✓		✓		✓		✓		✓		✓				✓		✓		✓	✓	
Shen et al. [82]	✓			✓	✓		✓		✓		✓				✓		✓				
Mirzaei et al. [5]	✓			✓		✓	✓		✓	✓		✓			✓		✓				✓
Boeckmann and Thielen [83]	✓		✓		✓		✓		✓		✓				✓					✓	
Kosmas et al. [84]	✓		✓		✓			✓	✓		✓				✓		✓				
Tayyebi et al. [85]	✓		✓		✓		✓			✓	✓				✓		✓				
Kosmas et al. [86]	✓		✓		✓			✓	✓		✓				✓		✓				
Tezcan and Maass [6]	✓		✓		✓		✓		✓			✓			✓				✓		
Najafi et al. [7]	✓		✓			✓	✓		✓			✓			✓		✓				✓
This study		✓		✓	✓	✓	✓		✓					✓	✓		✓				

Table 2. The objective values for each interdiction resource level with respect to α-cut levels.

Interdicted budget levels	Interdicted arcs	Objective Function Value											Average CPU times (in sec)
		α=0.0	α=0.1	α=0.2	α=0.3	α=0.4	α=0.5	α=0.6	α=0.7	α=0.8	α=0.9	α=1.0	
R=0	-	85.83	83.75	81.67	79.58	77.50	75.42	73.33	71.25	69.17	67.08	65.00	1.84
R=1	(6-9)	57.50	55.83	54.17	52.50	50.83	49.17	47.50	45.83	44.17	42.50	40.83	1.81
R=2	(5-8), (6-9)	38.33	37.00	35.67	34.33	33.00	31.67	30.33	29.00	27.67	26.33	25.00	1.76
R=3	(5-8), (6-8), (6-9)	21.67	62.50	60.00	57.50	55.00	52.50	50.00	47.50	45.00	42.50	40.00	1.80
R=4	(1-4), (2-5), (2-6), (3-6)	8.33	-	-	-	-	-	-	-	-	-	-	1,78
	(4-8), (5-8), (6-8), (6-9)	8.33	8.00	7.67	7.33	7.00	6.67	6.33	6.00	5.67	5.33	5.00	
R=5	(1-4), (2-4), (2-5), (2-6), (3-6)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1,82
	(7-10), (8-10), (8-11), (8-12), (9-12)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	

Table 3. The commodity-based flow quantities for each interdiction resource level with respect to all *α*-cut levels.

Interdicted budget levels	Interdicted arcs	The commodity-based flow quantities (wrt different α-cut levels)																	
		α=0.0			*α*=0.1			*α*=0.2			*α*=0.3			*α*=0.4			*α*=0.5		
		c(1)	c(2)	c(3)	c(1)	c(2)	c(3)	c(1)	c(2)	c(3)	c(1)	c(2)	c(3)	c(1)	c(2)	c(3)	c(1)	c(2)	c(3)
R=0	-	92.50	118.75	46.25	89.25	116.50	45.50	84.50	115.00	45.50	81.75	112.50	44.50	79.00	110.00	43.50	76.25	107.50	42.50
R=1	(6-9)	92.50	15.00	65.00	89.25	14.50	63.75	86.00	14.00	62.50	82.75	13.50	61.25	79.50	13.00	60.00	76.25	12.50	58.75
R=2	(5-8), (6-9)	70.00	45.00	0.00	67.50	43.50	0.00	65.00	42.00	0.00	62.50	40.50	0.00	62.75	36.25	0.00	60.00	35.00	0.00
R=3	(5-8), (6-8), (6-9)	65.00	0.00	0.00	62.50	0.00	0.00	60.00	0.00	0.00	57.50	0.00	0.00	55.00	0.00	0.00	52.50	0.00	0.00
R=4	(1-4), (2-5), (2-6), (3-6)	0.00	0.00	25.00	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	(4-8), (5-8), (6-8), (6-9)	0.00	25.00	0.00	0.00	24.00	0.00	0.00	23.00	0.00	0.00	22.00	0.00	0.00	21.00	0.00	0.00	20.00	0.00
R=5	(1-4), (2-4), (2-5), (2-6), (3-6)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(7-10), (8-10), (8-11), (8-12), (9-12)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Interdicted budget levels	Interdicted arcs	The commodity-based flow quantities (wrt different *α*-cut levels)														
		α=0.6			*α*=0.7			*α*=0.8			*α*=0.9			*α*=0.10		
		c(1)	c(2)	c(3)	c(1)	c(2)	c(3)	c(1)	c(2)	c(3)	c(1)	c(2)	c(3)	c(1)	c(2)	c(3)
R=0	-	73.00	105.25	41.75	69.75	103.00	41.00	66.50	100.75	40.25	63.25	98.50	39.50	60.00	96.25	38.75
R=1	(6-9)	73.00	12.00	57.50	69.75	11.50	56.25	66.50	11.00	55.00	63.25	10.50	53.75	60.00	10.00	52.50
R=2	(5-8), (6-9)	57.25	33.75	0.00	54.50	32.50	0.00	51.75	31.25	0.00	49.00	30.00	0.00	46.25	28.75	0.00
R=3	(5-8), (6-8), (6-9)	50.00	0.00	0.00	47.50	0.00	0.00	45.00	0.00	0.00	42.50	0.00	0.00	40.00	0.00	0.00
R=4	(1-4), (2-5), (2-6), (3-6)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	(4-8), (5-8), (6-8), (6-9)	0.00	19.00	0.00	0.00	18.00	0.00	0.00	17.00	0.00	0.00	16.00	0.00	0.00	15.00	0.00
R=5	(1-4), (2-4), (2-5), (2-6), (3-6)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(7-10), (8-10), (8-11), (8-12), (9-12)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

c(1), c(2) and c(3) represent the commodities

Table 4. The scenarios regarding the different weights of commodities.

The importance of the commodities			
	w(1)*	w(2)	w(3)
Scenario 1	0.100	0.300	0.600
Scenario 2	0.100	0.600	0.300
Scenario 3	0.300	0.100	0.600
Scenario 4	0.300	0.600	0.100
Scenario 5	0.600	0.100	0.300
Scenario 6	0.600	0.300	0.100
Scenario 7	0.333	0.333	0.333

*the importance of the commodity 1

Table 5. The scenarios regarding the different number of source and sink nodes.

Scenarios	Commodities	Source nodes	Sink nodes
1	1	{1}	{45}
	2	{4}	{48}
	3	{6}	{41}
	4	{8}	{42}
2	1	{1}	{37, 44, 45, 46}
	2	{4}	{40, 46, 47, 48}
	3	{6}	{33, 41, 42, 43}
	4	{8}	{34, 42, 43, 44}
3	1	{1, 2, 3, 4}	{45}
	2	{4, 5, 6, 7}	{48}
	3	{1, 2, 6, 8}	{41}
	4	{3, 6, 7, 8}	{42}
4	1	{1, 2, 3, 4}	{37, 44, 45, 46}
	2	{4, 5, 6, 7}	{40, 46, 47, 48}
	3	{1, 2, 6, 8}	{33, 41, 42, 43}
	4	{3, 6, 7, 8}	{34, 42, 43, 44}

Table 6. The results for the network $G(20,43)$

Interdiction resource levels	Interdicted arcs	Objective function values (wrt different α-cut levels)					Average CPU times (in sec)
		α=0.00	α=0.25	α=0.50	α=0.75	α=1.00	
R=0	-	269.00	242.75	216.50	190.25	164.00	1.97
R=1	(12-18)	225.00	202.50	180.00	157.50	135.00	1.91
R=2	(12-13), (12-18)	183.00	164.25	145.50	126.75	108.00	1.89
R=3	(12-13), (12-18), (17-18)	143.00	128.00	113.00	98.00	83.00	1.85
R=4	(2-8), (12-13), (12-18), (17-18)	103.00	91.75	80.50	69.25	58.00	1.87
R=5	(2-8), (8-12), (12-13), (12-18), (17-18)	65.00	57.50	50.00	42.50	35.00	1.90
R=6	(2-8), (7-8), (8-12), (12-13), (12-18), (17-18)	30.00	26.25	22.50	18.75	15.00	1.98
R=7	(3-4), (4-8), (8-9), (8-14), (13-14), (14-18), (18-19)	0.00	0.00	0.00	0.00	0.00	1.97
Average CPU times (in sec)		1.95	1.89	2.13	1.90	1.90	

Table 7. The results for the network $G(48,117)$

Interdiction resource levels	Interdicted arcs	Objective function values (wrt different α-cut levels)					Average CPU times (in sec)
		α=0.00	α=0.25	α=0.50	α=0.75	α=1.00	
R=0	-	572.00	530.75	489.50	448.25	407.00	3.43
R=1	(40-48)	483.00	445.50	408.00	370.50	333.00	3.44
R=2	(34-41), (34-42)	404.00	381.50	-	-	-	3.51
	(40-48), (47-48)	-	-	348.50	314.75	281.00	
R=3	(33-41), (34-41), (34-42)	313.00	294.25	275.50	256.75	238.00	3.54
R=4	(33-41), (34-41), (34-42), (40-48)	224.00	209.00	194.00	179.00	164.00	3.46
R=5	(33-41), (34-41), (34-42), (42-43), (40-48)	153.00	141.75	130.50	119.25	108.00	3.49
R=6	(33-41), (34-41), (34-42), (42-43), (40-48), (47-48)	86.00	78.50	71.00	63.50	56.00	3.53
R=7	(1-9), (33-41), (34-41), (34-42), (42-43), (40-48), (47-48)	42.00	38.25	34.50	30.75	27.00	3.50
R=8	(1-2), (1-9), (33-41), (34-41), (34-42), (42-43), (40-48), (47-48)	0.00	0.00	0.00	0.00	0.00	3.40
Average CPU times (in sec)		3.50	3.493	3.475	3.479	3.47	

Table 8. The results for the network $G(80, 205)$

Interdiction resource levels	Interdicted arcs	Objective function values (wrt different α-cut levels)					Average CPU times (in sec)
		α=0.00	α=0.25	α=0.50	α=0.75	α=1.00	
R=0	-	498.00	456.75	415.50	374.25	333.00	6.12
R=1	(8-16)	449.00	411.50	374.00	336.50	299.00	6.19
R=2	(6-7), (8-16)	401.00	367.25	333.50	299.75	266.00	6.23
R=3	(8-16), (77-78), (78-79) (2-3), (6-7), (8-16)	354.00 -	- 324.00	- 294.00	- 264.00	234.00 -	6.30
R=4	(6-7), (8-16), (77-78), (78-79)	306.00	279.75	253.50	227.25	201.00	6.19
R=5	(6-7), (8-16), (77-78), (78-79), (79-80)	259.00	236.50	214.00	191.50	169.00	6.22
R=6	(6-7), (8-16), (72-80), (77-78), (78-79), (79-80)	212.00	193.25	174.50	155.75	137.00	6.21
R=7	(6-7), (7-15), (8-16), (72-80), (77-78), (78-79), (79-80)	168.00	153.00	138.00	123.00	108.00	6.24
R=8	(1-9), (6-7), (7-15), (8-16), (72-80), (77-78), (78-79), (79-80)	124.00	112.75	101.50	90.25	79.00	6.25
R=9	(1-9), (6-7), (7-15), (8-15), (8-16), (72-80), (77-78), (78-79), (79-80)	82.00	74.50	67.00	59.50	52.00	6.18
R=10	(1-2), (1-9), (6-7), (7-15), (8-15), (8-16), (72-80), (77-78), (78-79), (79-80)	40.00	36.25	32.50	28.75	25.00	6.21
R=11	(1-2), (1-9), (6-7), (7-15), (8-15), (8-16), (70-78), (72-80), (77-78), (78-79), (79-80)	0.00	0.00	0.00	0.00	0.00	6.26
	Average CPU times (in sec)	6.27	6.22	6.21	6.22	6.24	

Table 9. The results for the network $G(120, 317)$

Interdiction resource levels	Interdicted arcs	Objective function values (wrt different α-cut levels)					Average CPU times (in sec)
		α=0.00	α=0.25	α=0.50	α=0.75	α=1.00	
R=0	-	658.00	613.00	568.00	523.00	478.00	10.96
R=1	(101-111)	581.00	539.75	498.50	457.25	416.00	11.11
R=2	(101-111), (102-112)	508.00	470.50	433.00	395.50	358.00	11.04
R=3	(101-111), (102-112), (110-120)	439.00	-	-	-	-	11.09
	(101-111), (102-111), (102-112)	-	405.25	371.50	337.75	304.00	
R=4	(101-111), (102-111), (102-112), (110-120)	370.00	343.75	317.50	-	-	11.30
	(101-111), (102-111), (102-112), (112-113)	-	-	-	290.00	260.00	
R=5	(101-111), (102-111), (102-112), (110-120), (119-120)	304.00	281.50	259.00	236.50	214.00	11.10
R=6	(101-111), (102-111), (102-112), (110-120), (112-113), (119-120)	245.00	226.25	207.50	188.75	170.00	11.04
R=7	(10-20), (101-111), (102-111), (102-112), (110-120), (112-113), (119-120)	188.00	173.00	158.00	143.00	128.00	11.08
R=8	(9-10), (10-20), (101-111), (102-111), (102-112), (110-120), (112-113), (119-120)	137.00	125.75	114.50	103.25	92.00	11.15
R=9	(101-111), (102-111), (102-112), (104-114), (110-120), (112-113), (113-114), (114-115), (119-120)	86.00	78.50	71.00	63.50	56.00	11.06
R=10	(1-11), (101-111), (102-111), (102-112), (104-114), (110-120), (112-113), (113-114), (114-115), (119-120)	42.00	38.25	34.50	30.75	27.00	11.08
R=11	(1-2), (1-11), (101-111), (102-111), (102-112), (104-114), (110-120), (112-113), (113-114), (114-115), (119-120)	0.00	0.00	0.00	0.00	0.00	11.00
Average CPU times (in sec)		11.08	11.07	11.08	11.09	11.11	

Biographies

Gökhan Özçelik received PhD degree in Industrial Engineering from Gazi University in 2016. He is currently an Associate Professor at Karadeniz Technical University, Trabzon Turkey. His research interests include optimization, multi-criteria decision making and fuzzy set and systems. He has published many papers in highly cited journals such as European Journal of Operational Research, Transportation Research Part E, Expert Systems with Applications, International Journal of Production Research, Journal of Cleaner Production etc.

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