Collective dynamics of a full heterogeneous network of neurons: experimental validation

Zeric Njitacke Tabekoueng,[1,2,*] Janarthanan Ramadoss,[3] Jules Tagne Fossi,[4] Hayder Natiq,[5] and Jan Awrejcewicz [2]

1) *Department of Electrical and Electronic Engineering, College of Technology (COT), University of Buea, P.O.Box 63, Buea (Cameroon).*

2) *Department of Automation, Biomechanics and Mechatronics, Lodz University of Technology, ul. Stefanowskiego 1/15, 90-537 Lodz, Poland.*

3) Centre for Artificial Intelligence, Chennai Institute of Technology, India

4) Laboratory of Energy-Electric and Electronic Systems, Department of Physics, Faculty of Science, University of Yaoundé I, P.O. Box 812, Yaoundé, Cameroon

5) Information Technology Collage, Imam Ja'afar Al-Sadiq University, 10001, Baghdad, Iraq

Corresponding author

Zeric Njitacke Tabekoueng; email: zerictabekoueng@yahoo.fr; phone : +237670310325

Other authors

Janarthanan Ramadoss ; email: janarthananr@citchennai.net

Jules Tagne Fossi; email: jules_fossi@yahoo.fr

Hayder Natiq; email: haydernatiq86@gmail.com

Jan Awrejcewicz; email: jan.awrejcewicz@p.lodz.pl

Abstract

This contribution introduces and investigates a small network of type-I, type-II, and type-III neurons. The considered network is made up of one Hindmarsh-Rose neuron (type-I), one FitzHugh-Nagumo neuron (type-II), and one Wilson neuron (type-III), all connected via a gap junction. The investigation of the stability in the presence of an external current revealed that the network is equilibrium-free; therefore, the network exhibits hidden collective behavior. The dynamical behavior of the model has been evaluated using the two-dimensional Largest Lyapunov Exponent (2D LLE), and it has been discovered that the network exhibits either regular or irregular firing patterns as the synaptic weights vary. It is also found that the network is able to exhibit the coexistence of firing activities involving coherent and incoherent spiking or coherent and incoherent bursting. Finally, the microcontroller integration of the set of considered neurons is presented, and the findings support those of the numerical simulations.

Keywords: Heterogeneous network; Hindmarsh–Rose neuron; FitzHugh–Nagumo neuron; Wilson neuron; Coexisting patterns; Microcontroller validation.

1. Introduction

Various phenomena such as wave formation, multiple resonance and propagation, synchronization, information patterns, and many others Njitacke et al. [1], are at the origin of many brain behaviors. Information patterns have recently drawn a lot of attention in neuro-engineering based on some of the well-known neuron models Ramadoss et al. [2]. Among these neural models, we can mention as an example the Hodgkin-Huxley neuron by Wu et al. [3], the Integrate and Fire (IF) neuron by Woo et al. [4], the FitzHugh-Nagumo (FHN) neuron by Izhikevich and FitzHugh [5], the Morris-Lecar (ML) neuron by Leigh et al. [6], the Wilson neuron by Wilson et al. [7], the Hindmarsh-Rose (HR) neuron [8-10], the Chay neuron by Lu et al. [11], and the Izhikevich neuron by Izhikevich et al. [12], which are used to study the collective behavior of the set of connected neurons. Also, several works have been devoted to the study of the energy consumption of the biological neuron based on the Helmholtz theorem in order to support the varieties of electrical activities generated by those models as can be seen in Yang et al. [13], Xie et al. [14] and Sun et al. [15]. Apart from the previously quoted classes of neurons, several works have been also devoted to the study Hopfield neurons type. For example in Lai et al. [16, 17] the authors designed some memristive Hopfield neural networks with the properties of generating multiscroll. Particularly in ref. [16] an application to images encryption was develop using the random sequences generated by their multiscroll system. As it can be seen in Lai et al. by [18, 19] not only HNN are suitable for data protection. From the quoted references it is obvious that even discrete map are highly used for such applications. Also a particular attention has been recently devoted to the study of discrete neurons model such as Chialvo neuron by Ma et al. [20] or of Ma et al. Rulkov [21] neuron. From the investigation of such model authors realized such type of neuron model has less computational time than the continuous neuron model.

In De et al. [22], the authors studied the dynamics of a model of the FHN neuron coupled with a model of the ML neuron via a gap junction. The authors discovered that for different intensities of coupling strength, their considered coupled neuron could exhibit behaviors such as quiescence, coherent spiking, coherent bursting, and burst synchronization. The coexistence of firing patterns involving periodic and chaotic bursting has been reported in the work of Li et al. [23]. They also discovered phase synchronization in two different sets of connected neurons using a memristive synapse. In addition, a circuit realization of the connected neurons was also provided to further support their study. A model of HR neuron linked with a model of FHN neuron based on a memristive synapse has been investigated in Njitacke et al. [24]. The results showed that the coupled model could exhibit oscillatory dynamics. Furthermore, the model could also exhibit extreme homogeneous multistability, which is materialized by the simultaneous existence of an uncountable number of identical attractors. Finally, the authors presented a control technique that enables them to control that homogeneous extreme multistability through the selection of the desired pattern. Another study has been addressed in Njitacke et al. [25], where the dynamical analysis followed by the control of extreme multistability were addressed in the model of HR neuron coupled with FHN neuron by a memristor synapse, having a multistable property. The collective behavior of the photosensitive FHN neuron coupled with the thermosensitive FHN neuron using a memristive synapse has been carried out in Fossi et al. [26]. The investigation by the authors showed that the set of considered neurons was capable to experience phase synchronization for some discrete values of the coupling strength. More importantly, the phenomenon of the coexistence of an infinite number of patterns as well as its control was also addressed. The collective dynamics of a small set of three non-identical neurons have been explored in Njitacke et al. [27]. In this quoted work, the network is built in the relay configuration, meaning two HR neurons are coupled through one FHN neuron based on the gap junction. The sufficient energy needed by the coupled neurons to provide electrical activity was established using the Helmholtz theorem. Also, the exploration of the dynamics of the model showed it was capable to support the simultaneous existence of up to three firing modes of oscillations for the same sets of the connection weight by exploiting three different initial states. Several studies have been conducted on independent neurons in Bao et al.[28] and Hou et al. [29] as well as identically coupled neurons in Wouapi et al. [30] and Zhou et al. [31].

Therefore, from these works, it is obvious that the study of heterogeneously connected neurons has retained attention in recent years. However, the study of a full heterogeneous network in the ring configuration is still to be presented. In this contribution, special emphasis will be given to the proposal of a full heterogeneous network comprised of one HR neuron, one FHN neuron, and a Wilson neuron, all coupled in a ring topology using gap junction. Afterward, the collective behavior will be investigated based on the 2D LLE diagrams. In Section II, a model of a network of three different neurons is introduced. In Section III, computational analysis tools are used to reveal the evolutional behaviors of that set of neurons. In Section IV, the microcontroller investigation of the set neuron is used for the validation of the results. The last section concludes this paper and gives an ideal of future work on such a model.

2. Design of the coupled model

Neurons provide a means for the exchange, storage, learning, and processing of information in the brain. Those neurons are generally connected through their synapses, which can be electrical, chemical, or memristive. Among the wide classes of neurons studied in the previous research works, we have the famous 2D HR neuron as expressed in Eq.(1), which is considered as our type type-I neuron model.

$$\begin{cases} \dfrac{dx_1}{d\tau} = y_1 - a_1 x_1^3 + b_1 x_1^2 + i_1 \\ \dfrac{dy_1}{d\tau} = c_1 - d_1 x_1^2 - y_1 \end{cases} \tag{1}$$

That model was introduced by Hindmarsh and Rose [32]. Besides this model, we have the 2D FHN neuron model as provided in Eq.(2), and it is used as our type-II neuron model. It was introduced by Izhikevich and FitzHugh [5]. Finally, the third model considered in this work is the 2D Wilson neuron [7] of Eq. (3) used as our type-III neuron model.

$$\begin{cases} \dfrac{dx_2}{d\tau} = x_2 - b_2 x_2^3 - y_2 + i_2 \\ \dfrac{dy_2}{d\tau} = {}^1\!/\!_\varepsilon \left(a_2 + x_2 - c_2 y_2 \right) \end{cases} \tag{2}$$

$$\cdot \begin{cases} c_m \dfrac{dx_3}{d\tau} = -m_\infty \left(x_3 \right)\left(x_3 - E_{Na} \right) - g_k y_3 \left(x_3 - E_k \right) + i_3 \\ \tau_r \dfrac{dy_3}{d\tau} = -y_3 + r_\infty \left(x_3 \right) \end{cases} \tag{3a}$$

$$m_\infty \left(x_3 \right) = 17.8 + 47.6 x_3 + 33.8 x_3^2 \ , \ r_\infty \left(x_3 \right) = 1.24 + 3.7 x_3 + 3.2 x_3^2 \tag{3b}$$

From Eqs. 1, 2, 3, $x_i \left(i = 1,2,3 \right)$ are fast variables, corresponding to the membrane potentials of each neuron, $y_i \left(i = 1,2,3 \right)$ designate the parameters for recovery of the transmembrane current due to ions Na^+ or ions K^+, $i_i \left(i = 1,2,3 \right)$ represent external forcing currents, $m_\infty \left(x_3 \right)$ and $r_\infty \left(x_3 \right)$ denote the activation function of the Na^+ ion and state equation of the transmembrane current recovery parameter, respectively. From Table 1 of Xu et al. [33] E_{Na} is the reverse potential of Na^+ ion channel, c_m represents the membrane capacitor, E_k the reverse potential of K^+ ion channel, τ_r channel activation constant of K^+ ion and g_k the maximal conductance of K^+ ion channel. From these three described models of neurons, the full hybrid small network of Fig. 1 has been proposed. The topological configuration that takes into account all the neurons is bidirectionally coupled through a gap junction, and all three-coupled neurons are different, so they can perform different tasks. Therefore, from that topological connection, the mathematical model from which their collective behavior will be investigated is given in Eq. (4). The motivation of such topology come from the fact that The organization of neurons in the cerebellum, which is responsible for motor coordination, is an example of a neuron-connected heterogeneous network [34]. The structure of the hippocampus, which is involved in memory formation and spatial navigation, is another example of a neuron-connected heterogeneous network [35]. The connections between neurons in the olfactory bulb, which is responsible for processing smells, are also an example of a neuron-connected heterogeneous network [36].

$$
\begin{cases}
\dfrac{dx_1}{d\tau} = y_1 - a_1x_1^3 + b_1x_1^2 + i_1 + m_1\left(x_2 - x_1\right) + m_5\left(x_3 - x_1\right) \\
\dfrac{dy_1}{d\tau} = c_1 - d_1x_1^2 - y_1 \\
\dfrac{dx_2}{d\tau} = x_2 - b_2x_2^3 - y_2 + i_2 + m_2\left(x_1 - x_2\right) + m_3\left(x_3 - x_2\right) \\
\dfrac{dy_2}{d\tau} = 1/\varepsilon\left(a_2 + x_2 - c_2y_2\right) \\
c_m\dfrac{dx_3}{d\tau} = -m_\infty\left(x_3\right)\left(x_3 - E_{Na}\right) - g_ky_3\left(x_3 - E_k\right) + i_3 + m_4\left(x_2 - x_3\right) + m_6\left(x_1 - x_3\right) \\
\tau_r\dfrac{dy_3}{d\tau} = -y_3 + r_\infty\left(x_3\right)
\end{cases}
\tag{4a}
$$

$$
m_\infty\left(x_3\right) = 17.8 + 47.6x_3 + 33.8x_3^2, \quad r_\infty\left(x_3\right) = 1.24 + 3.7x_3 + 3.2x_3^2 \tag{4a}
$$

In the coupled network of Eq.4, $m_i\left(i = 1\cdots6\right)$ are the coupling weight of the considered topological connection. Based on previous work on the HR neuron, FHN neuron, and Wilson neurons, we would like to stress that Eq. 4, which presents the dynamic model of the three coupled neurons, is purely mathematical, and the parameters of the dimensionless model are defined as follows:

$a_1 = 1$, $b_1 = 3.05$, $c_1 = 1$, $d_1 = 5$, $i_1 = 0.4$, $a_2 = 0.77$, $b_2 = 1/3$, $c_2 = 0.8$ $\varepsilon = 13$, $i_2 = 0$, $i_3 = 0.1$ $c_m = 1$, $E_{Na} = 0.5$, $E_k = -0.95$, $g_k = 26$, $\tau_r = 5$, $\tau_\varphi = 0.5$ and $m_i(i = 1, \dots, 6)$.

From the set of parameters used for the investigation, it was found that the small network exhibited hidden collective behavior since all the equilibria of the model were complex instead of real.

3. Numerical findings

Using nonlinear analysis techniques including bifurcation graphs, phase portraits, time series, and basins of attractions, the suggested network's overall behavior will be examined in this part. The Runge-Kutta algorithm is used to compute each of these tools, and the values of the parameters and variables are chosen using the extended precision mode.

3.1. Two-parameters Lyapunov charts

In this section, the behavior of the considered network is studied, considering as control parameters the synaptic weights of the electrical coupling. Each two-parameter Lyapunov exponent graph is captured by simultaneously sweeping two synaptic weights and saving the value of the LLE at each iteration, as shown in Fig. 2. For each set of synaptic weights, the Lyapunov exponent chart is obtained by increasing both synaptic weights starting from the same fixed initial condition as provided in the caption of Fig. 2. From those maps, two main patterns can be recorded. On one hand, irregular patterns characterized by a positive Lyapunov exponent are found, and on the other hand, regular patterns characterized by a negative Lyapunov. From the two parameter diagrams in the plane (m_6, m_3) , it is clear that, for a suitable choice of m_3, the considered network can generate a wide range of irregular patterns when varying the synaptic m_6.

3.2.Bifurcation Diagrams and coexisting patterns

As said previously, a suitable choice of m_3, the considered network can generate a wide range of irregular patterns when varying the synaptic weight m_6. For example, when $m_3 = -0.1$. the 1D evolutional diagram of Fig.3 (a) is constructed. From that diagram, only a few windows of synaptic weight m_6 where regular patterns can be captured can be recorded. This bifurcation diagram result corroborates well with the evolution of the LLE shown in Fig. 3(b). In the same vein, the graphs in Fig. 4 were studied using a series of parameters leading to the plan (m_1, m_3) as shown in Fig. 2 when $m_1 = 1$. On these bifurcation diagrams, two sets of data are accumulated. The data set in magenta is recorded by increasing the tuning parameter with fixed initial conditions, while those in blue are obtained with the continuation technique.

The continuation technique consists of increasing the control parameter while considering the initial condition of the next iteration as the final state of each iteration. These obtained diagrams enable us to support the simultaneous appearance of firing patterns observed in the considered model. For two discrete values of the control parameter m_3 chosen on those diagrams, the basins of attraction in Fig. 5 have been determined. From those diagrams, two domains of initial conditions associated with each coexisting pattern can be recorded. Among them, the blue region is associated with a regular pattern and the magenta region with an irregular pattern. Using a discrete value of the control parameter $m_3 = 1.41$ coexisting phase portraits of Figs. 6 (a1), (b1), and (c1) related to the HR neurons, FHN neurons, and Wilson neurons have been captured. From their corresponding coexisting time series in Figs. 6 (a2), (b2), and (c2), it is obvious that the coexisting patterns found for that set of parameters involve periodic (blue) and chaotic spiking (magenta). Using a discrete value of the control parameter $m_3 = -0.33$ coexisting phase portraits of Figs. 7 (a1), (b1), and (c1) related, respectively, to the HR neurons, FHN neurons, and Wilson neurons have been captured. From their corresponding coexisting time series in Figs. 7 (a2), (b2), and (c2), it can be concluded that the coexisting patterns found for that set of parameters involve periodic (blue) and chaotic bursting (magenta).

4. Experimental validation

The implementation of this model of coupled neurons was designed around an STM32F407ZGT6 microcontroller board (see Fig. 8). The used microcontroller is a 32-bit ARM® Cortex®-M4 RISC core and has the property of operating at a high frequency of around 168 MHZ. More importantly, it has some specialized functions such as timers, PWM, ADC, and DAC, just to name a few. With a suitable configuration environment, the Arduino IDE can be used to provide the code that will run and be uploaded to the STM board to enable the reproduction of the behavior of the coupled neurons. Since this implementation approach does not use usual electronic components, it can be considered a digital implementation of the considered set of coupled neurons.

We employ the fourth-order Runge-Kutta method with a time step of 0.005 to build a digital version of our model of coupled neurons that was previously presented with the STM32F407ZGT6 microcontroller. The microcontroller's internal DAC makes it possible to retrieve analog quantities that reflect the system state variables at the output of the pins. As seen in Figs. 9(a) and 9(b), a quick data acquisition module built using an Arduino Mega board is used to display the received analog values on a computer.

We can reproduce the complex dynamic behaviors generated in coupled neurons as described by the ODE equations using this experimental setup of the Fig. 9. Figs. 10 and 11 show experimental results demonstrating the simultaneous existence of firing patterns in the set of considered neurons, which are consistent with the numerical simulation results shown in Figs. 6 and 7, respectively. Based on these findings, this microcontroller approach is a fast and useful tool for implementing neural circuits.

5. Conclusion

The investigation of the collective behavior of a set of three linked non-identical neurons has been carried out in this contribution. From the stability analysis of the model, we found that it was an equilibrium-free type. The collective behaviors of the networks were characterized in terms of their regular and irregular dynamics using 2D LLE, bifurcation diagrams, phase diagrams, time series, and regions of attraction. For some particular value of the gap junction strength, we discover that the model could support coexisting patterns involving coherent and incoherent spiking or coherent and incoherent bursting. The results of this coexisting behavior have been further confirmed using a microcontroller implementation of the proposed network. Information coding in a set of homogeneous neurons has recently attracted attention in neuro-engineering. So the investigation of that phenomenon (information coding) in a heterogeneous network of neurons will be the topic of our next investigation.

6. Nomenclature

Designation	Function
$x_i\left(i=1,2,3\right)$	Membrane potential of each neuron
$y_i\left(i=1,2,3\right)$	Recovery variable for the transmembrane current due to ions Na^+ or ions K^+,
$i_i\left(i=1,2,3\right)$	External currents
$m_\infty\left(x_3\right)$	the activation function of the Na^+ ion
$r_\infty\left(x_3\right)$	state equation of the transmembrane current recovery parameter
E_{Na}	reverse potential of Na^+ ion channel
c_m	the membrane capacitor
E_k	reverse potential of K^+ ion channel
τ_r	channel activation constant of K^+ ion
g_k	maximal conductance of K^+ ion channel

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Compliance with ethical standards

Conflict of interest

The authors declare that they have no conflict of interest.

Data availability

The data used to support the findings of this study are available from the corresponding author upon request.

References.

1. Njitacke Z.T., Ramadoss J., Takembo C.N., et al. "An enhanced FitzHugh–Nagumo neuron circuit, microcontroller-based hardware implementation: Light illumination and magnetic field effects on information patterns" *Chaos, Solitons & Fractals*, 167, 113014 (2023).
2. Ramadoss J., Takembo C.N., Karthikeyan A., et al. "Effect of external excitation on the isolated and collective dynamics of a generic FitzHugh–Rinzel neuron" *The European Physical Journal Plu*s, 138, 962 (2023).
3. Wu F., Ma J., Zhang G. "Energy estimation and coupling synchronization between biophysical neurons" *Science China Technological Sciences*, 63, pp. 625-36 (2020).
4. Woo J., Kim S.H., Han K., et al. "Characterization of dynamics and information processing of integrate-and-fire neuron models" J*ournal of Physics A: Mathematical and Theoretical,* 54, 445601 (2021).]
5. Izhikevich E.M., FitzHugh R. "Fitzhugh-nagumo model" *Scholarpedia*, 1, 1349 (2006).
6. Leigh A.J., Heidarpur M., Mirhassani M. "A High-Accuracy Digital Implementation of the Morris-Lecar Neuron with Variable Physiological Parameters" *IEEE Transactions on Circuits and Systems II: Express Briefs*. (2022).
7. Wilson H.R. "Simplified dynamics of human and mammalian neocortical neurons" Journal of theoretical biology, 200 375-88 (1999).
8. Usha K., Subha P. "Hindmarsh-Rose neuron model with memristors" *Biosystems*, 178, pp. 1-9 (2019).
9. Lu L., Jia Y., Xu Y., et al. "Energy dependence on modes of electric activities of neuron driven by different external mixed signals under electromagnetic induction" *Science China Technological Sciences*, 62, pp. 427-40 (2019).
10. Qin H., Ma J., Jin W., et al. "Dynamics of electric activities in neuron and neurons of network induced by autapses" *Science China Technological Sciences,* 57, pp. 936-46 (2014).
11. Lu L., Yi M., Liu X. "Energy-efficient firing modes of chay neuron model in different bursting kinetics: *Science China Technological Sciences,* 65, pp. 1661-74 (2022).
12. Izhikevich E. "Simple Model of Spiking Neurons" IEEE Transactions on Neural Networksvol, (2003).
13. Yang F., Guo Q., Ma J. "A neuron model with nonlinear membranes" *Cognitive Neurodynamics*, pp. 1-12 (2023).
14. Xie Y., Yao Z., Ma J. "Formation of local heterogeneity under energy collection in neural networks" *Science China Technological Sciences*, 66, pp. 439-55 (2023).
15. Sun G., Yang F., Ren G., et al. "Energy encoding in a biophysical neuron and adaptive energy balance under field coupling" *Chaos, Solitons & Fractals,* 169, 113230 (2023).
16. Lai Q., Wan Z., Zhang H., et al. "Design and analysis of multiscroll memristive hopfield neural network with adjustable memductance and application to image encryption" *IEEE transactions on neural networks and learning systems* (2022).
17. Lai Q., Wan Z., Kuate P.D.K. "Generating grid multi-scroll attractors in memristive neural networks" *IEEE Transactions on Circuits and Systems I: Regular Papers,* 70, pp. 1324-36 (2022).

18. Lai Q., Yang L., Liu Y. "Design and realization of discrete memristive hyperchaotic map with application in image encryption" *Chaos, Solitons & Fractals*, 165, 112781 (2022).
19. Lai Q., Hu G., Erkan U., et al. "A novel pixel-split image encryption scheme based on 2D Salomon map" *Expert Systems with Applications*, 213, 118845 (2023).
20. Ma M., Xiong K., Li Z., et al. "Dynamic behavior analysis and synchronization of memristor-coupled heterogeneous discrete neural networks" *Mathematics*, 11, 375 (2023).
21. Ma M., Lu Y., Li Z., Sun Y., et al. Multistability and phase synchronization of Rulkov neurons coupled with a locally active discrete memristor *Fractal and Fractional*, 7, 82 (2023).
22. De S., Balakrishnan J. "Burst mechanisms and burst synchronization in a system of coupled type-I and type-II neurons" *Communications in Nonlinear Science and Numerical Simulation*, 90, 105391 (2020).
23. Li Z., Zhou H., Wang M., Ma M. "Coexisting firing patterns and phase synchronization in locally active memristor coupled neurons with HR and FN models" *Nonlinear Dynamics*, 104, pp. 1455-73 (2021).
24. Njitacke Z.T., Awrejcewicz J., Telem A.N.K., et al. "Complex dynamics of coupled neurons through a memristive synapse: extreme multistability and its control with selection of the desired state" *IEEE Transactions on Circuits and Systems II: Express Briefs*, (2022).
25. Njitacke Z.T., Shankar M.S,. Fozin F.T., et al. "Coexistence of infinitely many patterns and their control in heterogeneous coupled neurons through a multistable memristive synapse" *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 32, 053114 (2022).
26. Fossi J.T., Deli V., Njitacke Z.T., et al. "Phase synchronization, extreme multistability and its control with selection of a desired pattern in hybrid coupled neurons via a memristive synapse" *Nonlinear Dynamics*, *pp.*1-18 (2022).
27. Njitacke Z.T., Awrejcewicz J., Ramakrishnan B., et al. "Hamiltonian energy computation and complex behavior of a small heterogeneous network of three neurons: circuit implementation" *Nonlinear Dynamics*, 107 pp., 2867-86 (2022).
28. Bao H., Liu W., Ma J., et al. "Memristor initial-offset boosting in memristive HR neuron model with hidden firing patterns" *International Journal of Bifurcation and Chaos*, 30, 2030029 (2020).
29. Hou Z., Ma J., Zhan X., et al. "Estimate the electrical activity in a neuron under depolarization field" C*haos, Solitons & Fractals*, 142, 110522 (2021).
30. Wouapi M.K., Fotsin B.H., Ngouonkadi E.B.M., et al." Complex bifurcation analysis and synchronization optimal control for Hindmarsh–Rose neuron model under magnetic flow effect" *Cognitive Neurodynamics*, 15, pp.315-47 (2021).
31. Zhou P., Yao Z., Ma J., et al. "A piezoelectric sensing neuron and resonance synchronization between auditory neurons under stimulus" *Chaos, Solitons & Fractals*, 145, 110751 (2021).
32. Hindmarsh J., Rose R. "A model of the nerve impulse using two first-order differential equations" *Nature*, 296, pp. 162-4 (1982).
33. Xu Q., Ju Z., Ding S., et al. "Electromagnetic induction effects on electrical activity within a memristive Wilson neuron model" *Cognitive Neurodynamics*, pp. 1-11 (2022).
34. Herculano-Houzel S. "Not all brains are made the same: new views on brain scaling in evolution" *Brain, behavior and evolution*, 78, pp. 22-36 (2011).
35. David A., Pierre L. "Hippocampal neuroanatomy" The hippocampus book. (2009).
36. Shepherd G.M. "The synaptic organization of the brain" *Oxford university press* (2003).

Figure Captions

Fig.1. Topological configuration of the coupled neurons

Fig.2. Two-dimensional (2D) charts showing the various behaviors of the coupled neurons in the various for different values of the electrical coupling weights based on the value of the associated (LLE). The plane (m_6, m_3) is obtained for $m_1 = 1, m_2 = 0.6, m_4 = -0.2$ *and* m_5=-0.1. The plane (m_1, m_3) is obtained for $m_2 = 0.6, m_4 = -0.2$, m_5=-0.1 and $m_6 = 1$,. The plane (m_1, m_6) is obtained for $m_3 = 0.1, m_2 = 0.6, m_4 = -0.2$ *and* m_5=-0.1. The plane (m_1, m_2) is obtained for $m_6 = 5, m_3 = 0.1, m_4 = -0.2$ *and* m_5=-0.1. These diagrams are obtained with fixed initial conditions $(1, 0$,0, 0.1,0.1,0).

Fig.3. (a) dynamic behavior of the membrane potential of the first neuron x_1 when the control parameter m_6 is varied. (b) is the graph of the LLE associated to (a). These diagrams are obtained for the parameters used to compute Fig.2 in the plane(m_6, m_3).

Fig. 4. (a) and (b) are bifurcation plots showing the peak of the membrane potential of the HR neuron when the gap junction strength m_3 is varied. The diagrams in magenta are obtained using the upward direction of the control parameter with fixed initial conditions, while those in blue are obtained with the continuation technique. The remaining parameter values and initial states are found in the caption of Fig.2 with the plane (m_1, m_3) for $m_1 = 1$.

Fig. 5. Cross-section of the basin of attractions in the plane $(y_2(0), x_3(0))$. (a) is obtained for a discrete value $m_3 = 1.41$ while (b) is obtained for a discrete value $m_3 = -0.33$. These basins of attraction are obtained when other initial conditions are set to zero.

Fig.6. Phase portraits, showing the simultaneous existence of the firing patterns in the three sets of connected neurons. The incoherent spikings in magenta are obtained with initial conditions $(1, 0, 0, 0.1, 0.1, 0)$. While the coherent spikings in blue are captured with the initial conditions $(0, 0, 0, 0.1, 0.1, 0)$ when $m_3 = 1.41$.

Fig.7. Phase portraits, showing the simultaneous appearance of the firing patterns in the coupled neurons. The incoherent burstings in magenta are obtained with initial conditions $(1, 0, 0, 0.1, 0.1, 0)$. While the coherent burstings in blue are captured with the initial conditions $(0, 0, 0, 0, 0.1, 0)$ when $m_3 = -0.33$.

Fig.8. The STM32F407ZGT6 microcontroller

Fig.9. Block diagram (a) and experimental setup (b) of the microcontroller-based realization of the coupled neurons model.

Fig.10. Experimental phase portraits showing the simultaneous appearance of firing patterns in the sets of considered neurons. The incoherent spikings in blue are captured with initial conditions (1.0, 0, 0.1, 0.1, 0). While the coherent spikings in yellow are captured with initial conditions (0.0, 0, 0.1, 0.1, 0) when $m_3 = 1.41$.

Fig.11. Experimental phase portraits showing the simultaneous appearance of firing patterns in coupled neurons, the chaotic burstings in blue are captured with initial conditions (1, 0, 0, 0.1, 0.1, 0). While the periodic burstings in yellow are captured with initial conditions (0, 0, 0, 0, 0.1, 0) when $m_3 = -0.33$.

List of Figures

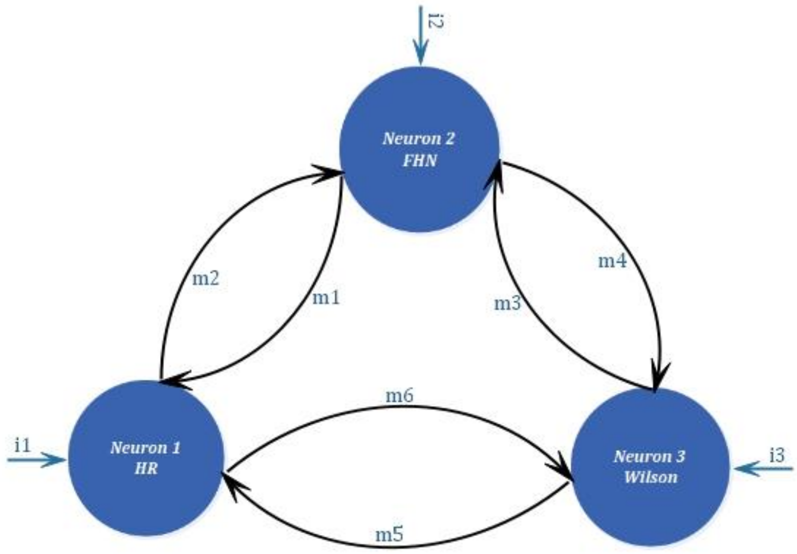

i2
Neuron 2
FHN
m2
m1
m4
m3
m6
i1
Neuron 1
HR
Neuron 3
Wilson
i3
m5

Fig.1.

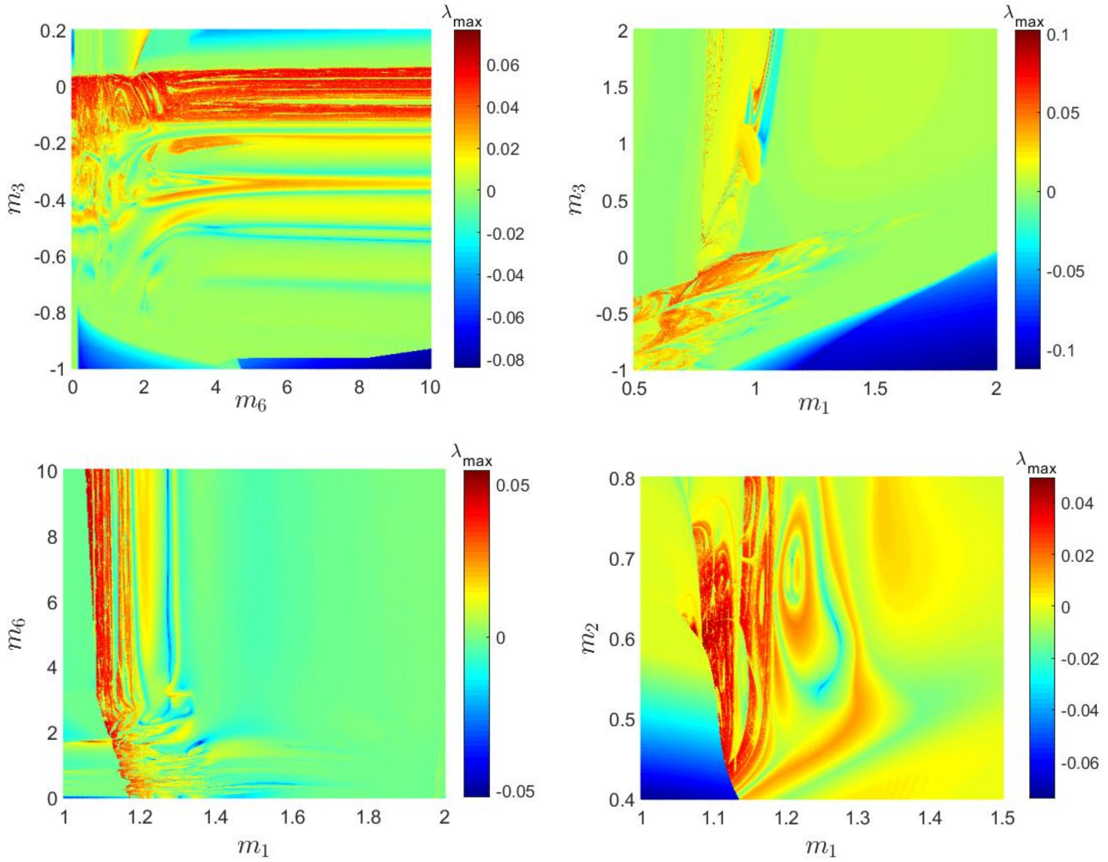

λ_{max}
m_3
m_6
m_1
m_2

Fig.2.

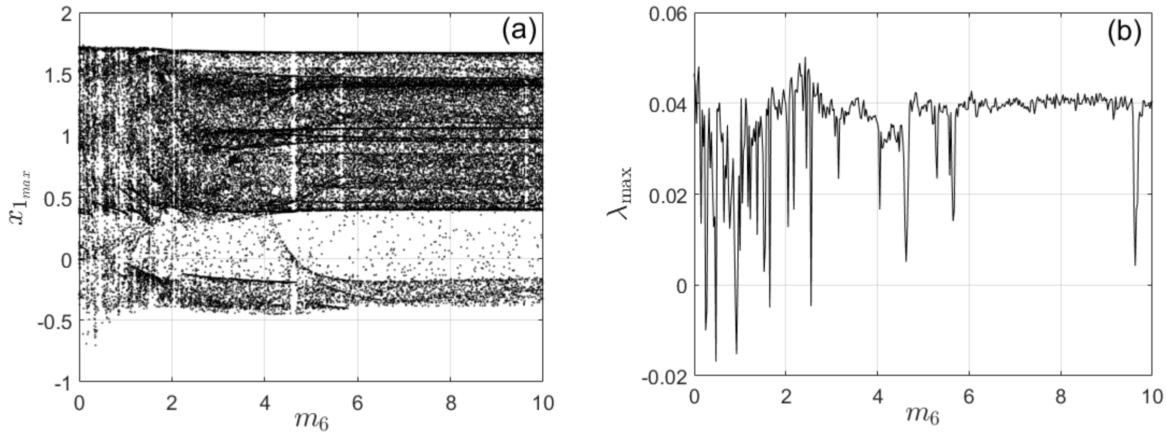


Fig.3.

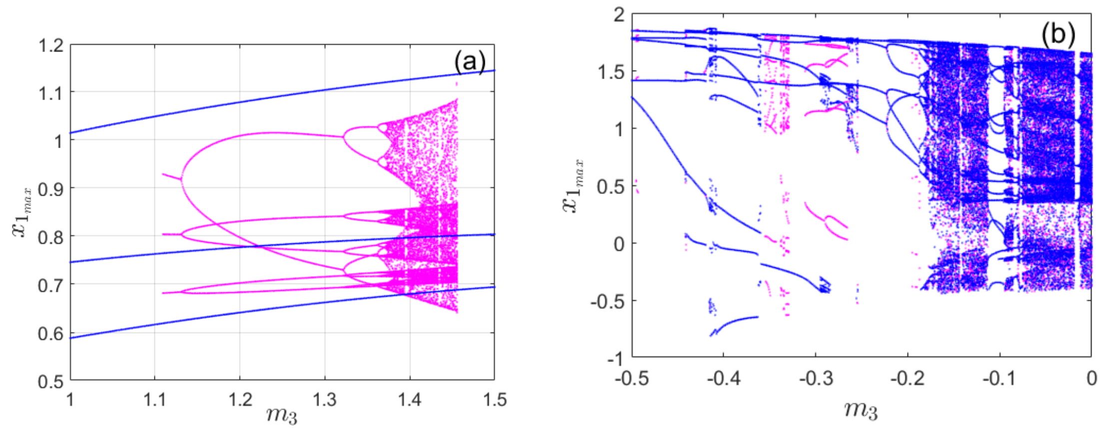
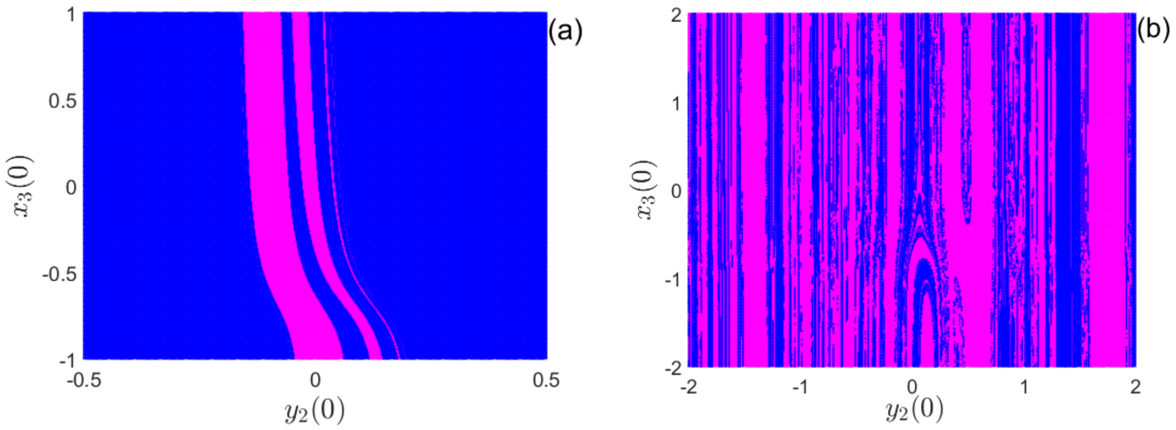


Fig. 4.



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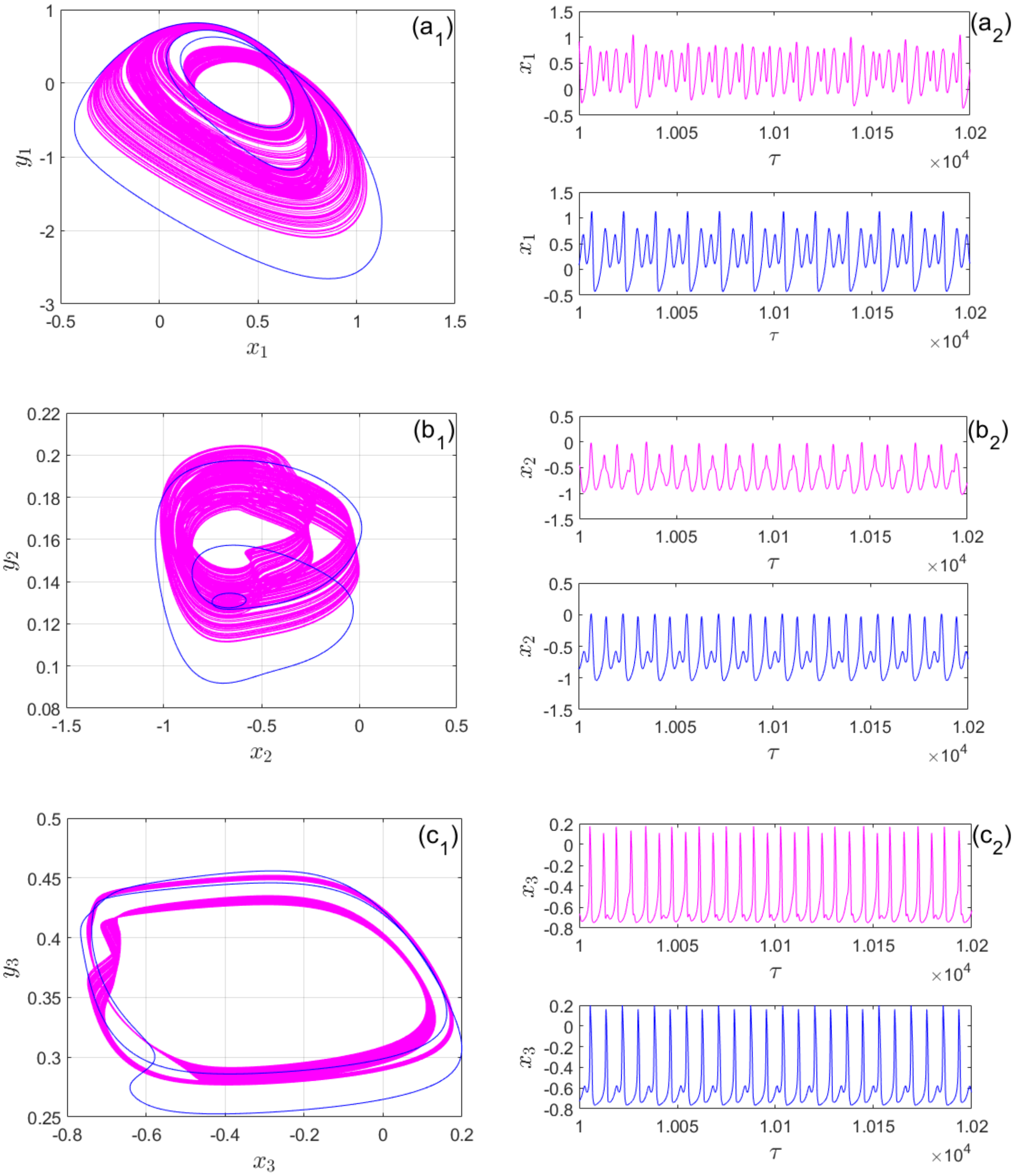
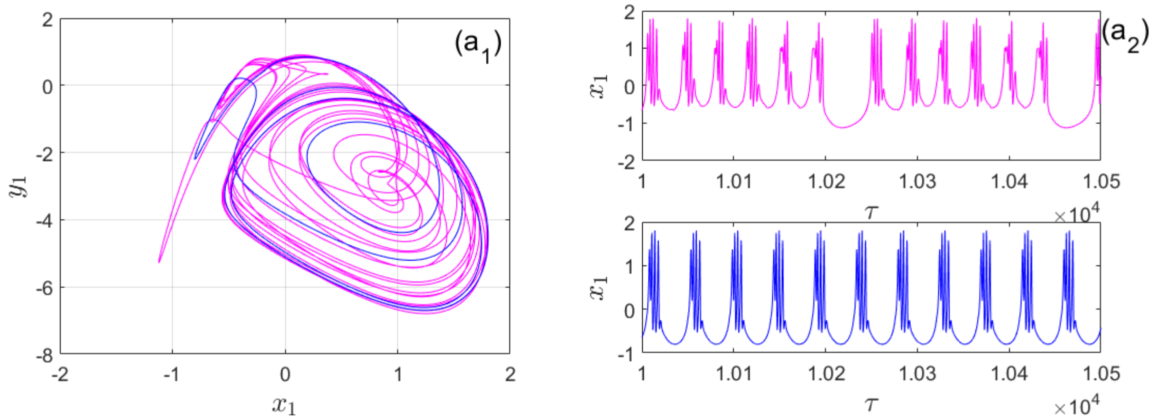


Fig.6.



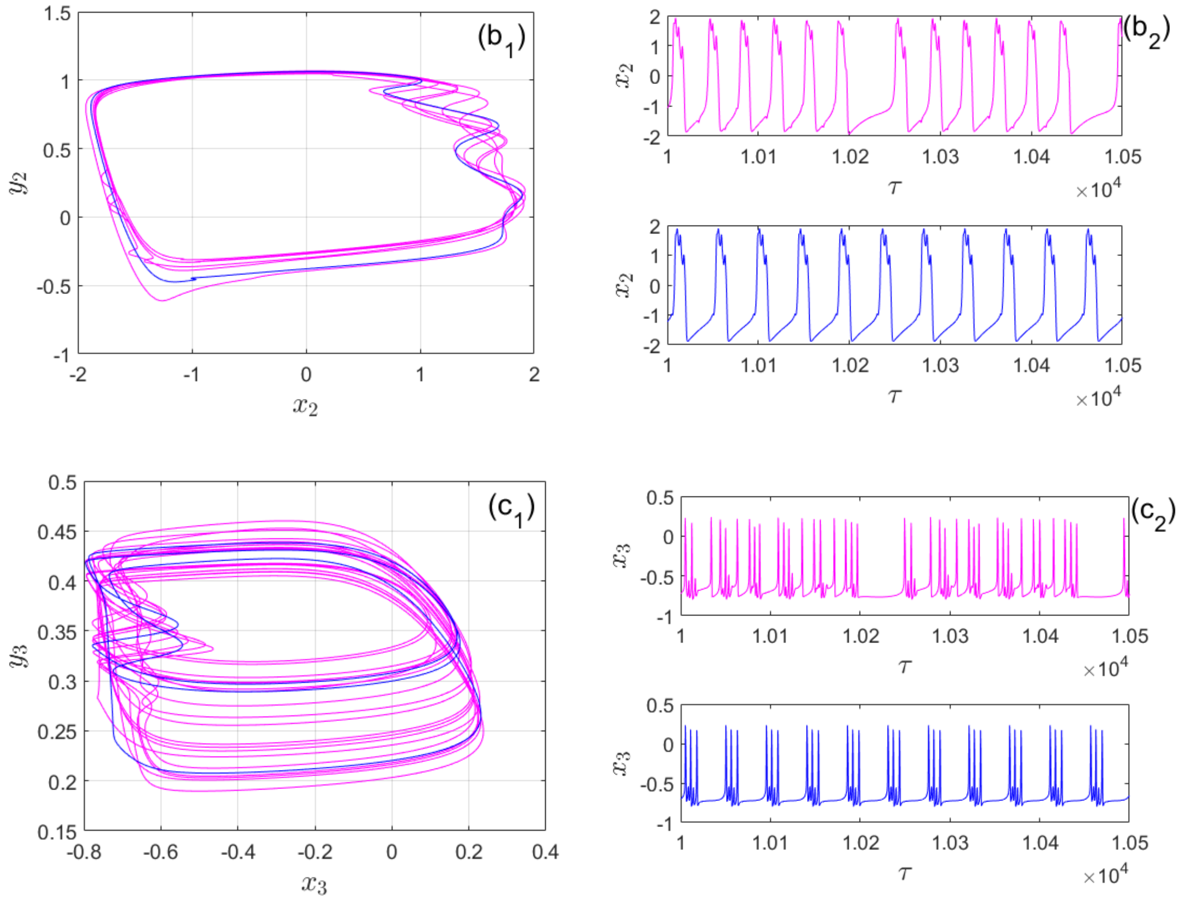
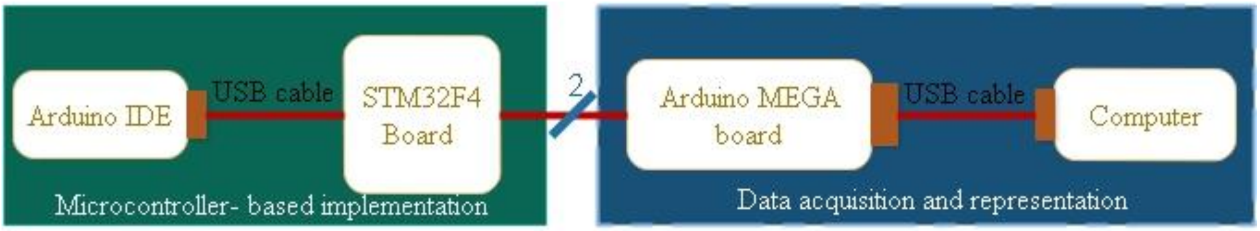

(b$_1$)
(b$_2$)
(c$_1$)
(c$_2$)
y_2
x_2
y_3
x_3
τ
$\times 10^4$

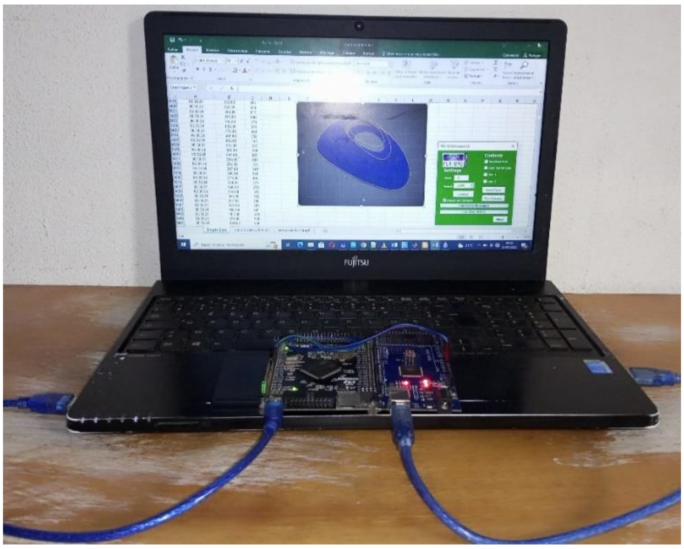
Fig.7.


STM32F407VET6
STM32F4VE
FLASH

Fig.8.

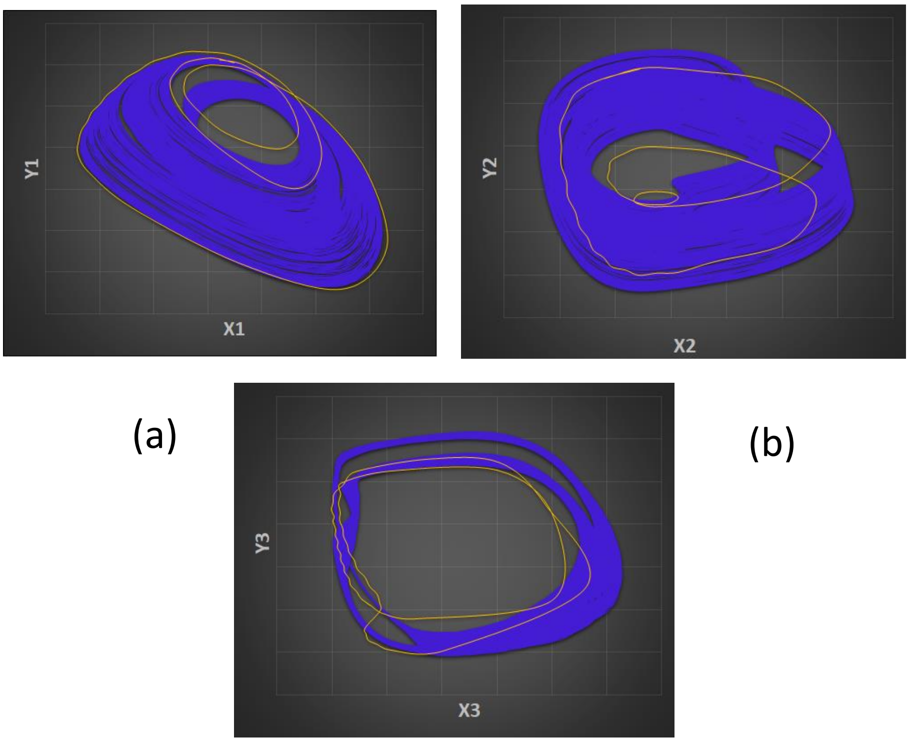


(a)



(b)

Fig.9.



(a)

(b)

(c)

Fig.10.

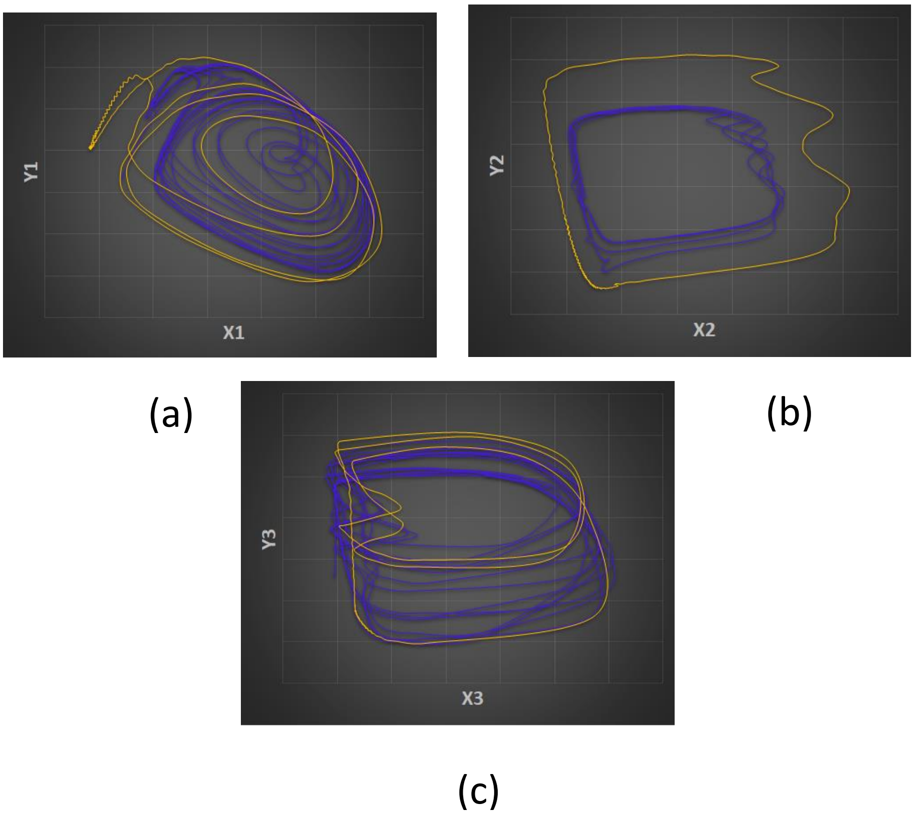


Fig.11.

Dr. Zeric Tabekoueng Njitacke was born in Bafoussam, Cameroon. In 2013, and 2015, he received his B.Sc. and M.Sc. degrees in applied physics with an option in electronics from the University of Dschang. In 2015, he graduated from the Higher Technical Teacher's Training College (HTTTC), University of Bamenda, with a DIPET 2 in Electrical and Electronics Engineering. He received his Ph.D. degree in Physics/Electronics from the University of Dschang in 2019. He was recruited as an assistant lecturer at the College of Technology, University of Buea, Cameroon, in 2019, where he is now an assistant professor. He worked as a postdoctoral fellow in the Department of Automation, Biomechanics, and Mechatronics at Lodz University of Technology, Poland, between 2021 and 2022. He is the author or co-author of more than 70 articles in peer-reviewed journals. He also serves as a reviewer for a number of prestigious international journals. His research interests include nonlinear systems and circuits, multistability and its control in electronic circuits, biological neurons and neural networks, and medical image encryption.

Dr. R. Janarthanan has currently working as a Professor of Computer Science and Engineering Department in Chennai Institute of Technology, Chennai, Tamil Nadu, India. He has completed B.E. (CSE) degree in 1995 from M.S. University, Tamil Nadu, India, MBA degree in 1997 from Madurai Kamarajar University, M.Tech. (CSE) degree in 2006 from Dr. MGR Educational and Research Institute, Chennai and received the Ph.D. degree from

Jadavpur University, West Bengal, India in 2018 under the guidance of Prof. Amit Konar. He has published 35 International & 15 National Journals. He is the author of more than 75 publications in top International Conference papers. He has filed 1 international and 14 Indian Patents. He is the author of 4 text books. He has more than 25 years of teaching experience. His area of interests is Artificial Intelligence & Soft Computing, Image Processing, Type 1 and Type 2 Fuzzy Logic / Sets, Computer Networks, Machine Learning, Reasoning, Data Analytics using Python and R and Software Engineering. He is life member of ISTE and member of IEEE & CSI.

Dr. Jules Tagne Fossi obtained a Master's degree in Physics Option Electronics, Electrotechnics, and Automation from the University of Dschang in Cameroon in 2017 and a PhD in Energy, Electrical, and Electronic Systems from the University of Yaoundé 1 in Cameroon in 2023. His research focuses on the modeling, numerical, and experimental analysis of neurological circuits and synchronization between neurons coupled via synapses.

Dr. Hayder Natiq received his B.S. and M.S. degrees from the Department of Applied Sciences, University of Technology, Baghdad, Iraq, in 2009 and 2014, respectively. He earned his Ph.D. degree from the University Putra Malaysia (UPM) in 2019. Currently, he serves as a senior lecturer in the College of Information Technology at Imam Ja'afar Al-Sadiq University in Baghdad, Iraq. His research interests include chaotic systems, chaos-based applications, and multimedia security.

Prof. Jan Awrejcewicz is the Head of the Department of Automation, Biomechanics and Mechatronics at Lodz University of Technology, Head of Ph.D. School on ’Mechanics’ (since 1996) and of graduate/postgraduate programs on Mechatronics (since 2006). He is also recipient of Doctor Honoris Causa (Honorary Professor) of Academy of Arts and Technology (Poland, Bielsko-Biala, 2014) and of Czestochowa University of Technology (Poland, Czestochowa, 2014), Kielce University of Technology (2019), National Technical University ”Kharkiv Polytechnic Institute” (2019), and Gdansk University of Technology (2019). ´ His papers and research cover various disciplines of mechanics, material science, biomechanics, applied mathematics, automation, physics and computer oriented sciences, with main focus on nonlinear processes. He is a corresponding member of Polish Academy of Sciences (since 2016) and recipient of numerous honors and awards, including the prestigious Alexander von Humboldt Award (twice). He authored/co-authored over 830 journal papers and refereed international conference papers and 53 monographs. JA is Editor-in-Chief of 3 international journals and member of the Editorial Boards of 90 international journals (23 with IF) as well as editor of 33 books and 37 journal special issues. For more information please visit www.abm.p.lodz.pl.