On oscillatory rotating convection of a couple-stress fluid with chemical reaction

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Abstract

Newtonian fluids fail to accurately model fluid flow behavior in various physical scenarios due to their non-Newtonian nature. This article delves into an analytical investigation of how chemical reactions impact the onset of rotating convection in a couple-stress (CS) fluid. Utilizing linear stability theory, we derive equations for both stationary and oscillatory Rayleigh numbers. Graphical representations illustrate the influence of key parameters such as the CS fluid parameter, solutal Rayleigh number, Damkohler number, Lewis number, and Prandtl number on the onset of convection. The Lewis and Taylor numbers act to stabilize the system, with the Damkohler number exerting differing effects on oscillatory and stationary convection. Stationary instability is reached when the Taylor number is below 910.331, with oscillatory convection prevailing otherwise. If the Damkohler number is less than 1.76455, oscillatory instability occurs, while stationary convection dominates otherwise.

Keywords: Couple-stress fluid, Chemical Reaction, Linear stability analysis.

Nomenclature

P	Pressure;	Pr	Prandtl number;
R_T	thermal Rayleigh number;	R_S	solutal Rayleigh number;
β_T	thermal expansion coefficient;	β_S	solutal expansion coefficient;
ΔT	Temperature difference;	t	Time;
ΔS	concentration difference;	Dm	Damkohler number;
q	Wave number;	x, y, z	Coordinates;
Ta	Taylor number;	$\mathbf{V}$	Velocity;
D_v	solutal diffusivity;	Ω	Angular velocity;
T	Temperature;	d	Height of fluid;
g	acceleration due to gravity;	κ	thermal diffusivity;
μ	dynamic viscosity;	ρ	Density;
μ_1	CS viscosity;	S	concentration;
Le	Lewis number;	C_S	CS parameter;
ν	kinematic viscosity ;		

Superscripts

$'$ Perturbed quantity;

Subscripts

b	Basic state;	c	Critical value;
0	Reference value;		

Other symbols

$$\nabla^2 \qquad \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}; \qquad \nabla_h{}^2 \quad \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2};$$

1. Introduction

Thermal instability in Newtonian fluids garners significant attention, especially in addressing geological and industrial challenges. However, when dealing with non-Newtonian fluids, which are frequently encountered in various physical scenarios, Newtonian models fall short. Among non-Newtonian fluids, CS fluids stand out and find applications in fields like lubrication, pharmaceutical manufacturing, and medical science [1]-[14]. Stokes [15] pioneered the study of CS fluids, delving into their behavioral intricacies. This theoretical framework offers insights into the rheological characteristics of diverse complex fluids [16, 17, 18, 19].

Hsu and colleagues [20] delved into the influence of CS and surface roughness on non-Newtonian fluids. Sunil and colleagues [21] provided comprehen-

sive global non-linear stability findings for CS fluids. Shivakumara and colleagues explored the linear and weakly nonlinear stability of CS fluids in their work [22] [23]. Gaikwad and Kouser [24] delved into the thermal instability of CS fluids in a porous layer with an internal heat source. Additionally, Shivakumara and Naveen Kumar [25] examined triple diffusive convection in CS fluids, employing a Fourier series approach to investigate weakly nonlinear theory.

Srivastava and Bera [26] studied the onset of convection in a CS fluid within an anisotropic porous layer with a chemical reaction. Meanwhile, Malashetty and Biradar [27] explored the onset of double reaction-convection in an anisotropic porous layer. Ravi and colleagues [28] delved into the impact of cross diffusion on secondary convective instabilities in CS fluids. Wollkind and Frisch [29, 30] conducted the pioneering study on convection onset with reactive effects in a fluid layer. Their research considered convection in a horizontal layer of dissociating fluid. Subsequently, Bdzil and Frisch [31] delved into the thermal instability of variable density under the influence of gravitational forces in a fluid layer, examining chemical reactions in both equilibrium and non-equilibrium states within the quiescent phase. Steinberg and Brand [32] [33] were the first to explore thermoconvective instabilities in a binary mixture within a reactive porous medium. Meanwhile, Gatica, Viljoen, and Hlavacek [34] employed a variational approach to investigate oscillatory and monotonic instabilities within a porous layer, considering the impact of chemical reactions. Pritchard and Richardson [35] examined thermosolutal convection in a binary fluid confined within a porous layer. Their study assumed fixed temperatures and chemical equilibrium at the bounding surfaces, accounting for solubility variations with temperature. Using a linear stability analysis, these authors probed how solute precipitation or dissolution influenced the onset of convection. The Galerkin method was subsequently employed to advance this analysis and predict the initial bifurcation's structure.

On one hand, in their work cited as [36], Wang and Tan explored both stationary and oscillatory convection of thermosolutal instability in a porous medium with a reaction term. They focused on the Darcy-Brinkman model for a loosely packed, porous medium and investigated the relationship between the Lewis number, Darcy number, and the reaction term concerning the onset of double-diffusive convection. Additionally, in the presence of a first-order chemical reaction, Hill and Morad examined convective instabilities in an anisotropic porous medium, as documented in [37]. Their

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findings indicated that altering the balance between horizontal and vertical solutal diffusivities minimally affected the instability's behavior. In [38], Ward and colleagues conducted a comprehensive analysis of the onset of convection in an isotropic porous medium with a first-order chemical reaction. They employed stability analyses, time-dependent simulations, spectral and asymptotic methods, and stability analysis to explore this phenomenon. The authors also delved into the numerous bifurcations that emerge in steady-state solutions. Expanding upon the work of Pritchard and Richardson [35], Bushra Al-Sulaimi [39] extended the analysis by employing the energy method to investigate the nonlinear energy stability of the Darcy convection model with a reaction. A recent energy analysis of nonlinear convection in an anisotropic reactive porous medium was undertaken by Gautam and Narayana [40]. Their work demonstrated that altering the ratio of vertical to horizontal permeabilities had minimal impact on the instability's behavior when the solutal horizontal diffusivity exceeded the vertical diffusivity. Furthermore, in [41], Reddy and colleagues harnessed a chemical reaction to examine the onset of instability in a Maxwell fluid-saturated porous layer. They employed analytical and numerical techniques to assess the system's stability, resolving eigenvalue problems as part of their analysis. Most recently, in [42], Reddy and colleagues examined dissolution-driven convection in a porous medium influenced by the vertical axis of rotation and a magnetic field. They employed artificial neural network modeling to predict the critical Rayleigh number, subsequently comparing these predictions to simulation results.

Moreover, traditional studies of thermal convection in confined spaces typically involve the upward flow of a fluid heated from below, a phenomenon commonly known as Rayleigh-Benard convection. This natural process plays a crucial role in various contexts, including the Earth's magnetic field reversals [43]-[45], where rotating convection becomes a significant and influential factor. Rotation frequently exerts its influence on fluid dynamics, affecting geophysical flows, technological processes, and astrophysical phenomena [46], including the Earth's atmosphere [47] and oceans [48, 49, 50, 51].

The existing literature notably lacks an exploration of the impact of chemical reactions on rotating convection within a CS fluid. This paper addresses this research gap by investigating the linear stability theory of CS fluid in a horizontal layer featuring the Coriolis effect and chemical reactions. Our primary focus is on discerning the presence of instability and evaluating the effects of various physical parameters on linear instability.

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The paper is organized as follows: Section 2 provides a comprehensive mathematical formulation. Section 3 delves into the intricacies of linear stability analysis. Section 4 showcases the key findings elucidated in this article, along with detailed discussions. Lastly, Section 5 offers the conclusion of our study.

2. Mathematical basis

We commence by establishing the mathematical framework for the problem under consideration. Our focus is on a horizontal layer of CS fluid, which is confined within the region $z \in (0, d)$. This layer maintains a constant temperature of T_0 at the upper boundary and $T_0 + \Delta T$ ($\Delta T > 0$) at the lower boundary. Additionally, the layer exhibits solutal concentration levels of S_0 at the upper boundary and $S_0 + \Delta S$ ($\Delta S > 0$) at the lower boundary, as illustrated in Figure 1.

The dimensional governing equations for this system can be precisely formulated, as documented in [23, 25, 28, 41, 49].
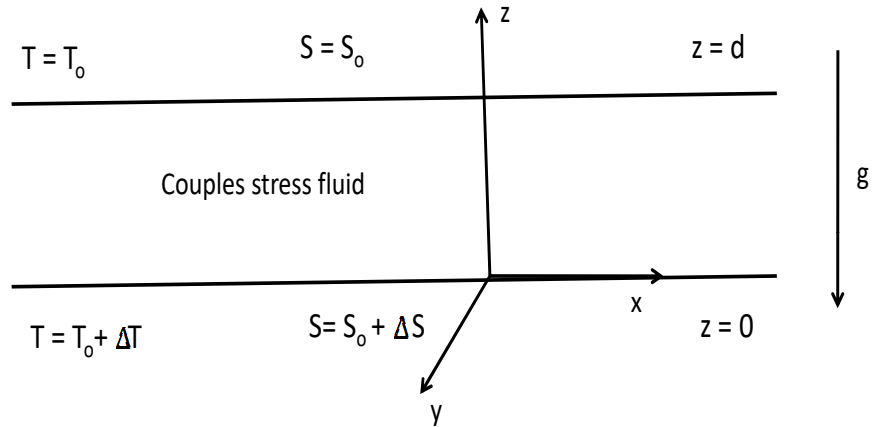

Figure 1: Physical configuration of the problem

$$\nabla \cdot \mathbf{V} = 0, \tag{1}$$

$$\rho_0 \frac{\partial \mathbf{V}}{\partial t} = -\nabla P + \mu \nabla^2 \mathbf{V} - \mu_1 \nabla^4 \mathbf{V} + \rho \mathbf{g} + 2\rho_0 \Omega (\mathbf{V} \times \widehat{e_z}), \tag{2}$$

$$\frac{\partial T}{\partial t} + (\mathbf{V} \cdot \nabla) T = \kappa \nabla^2 T, \tag{3}$$

$$\frac{\partial S}{\partial t} + (\mathbf{V} \cdot \nabla) S = D_v \nabla^2 S + \chi (S_{eq}(T) - S), \tag{4}$$

$$\rho = \rho_0 (1 - \beta_T (T - T_0) + \beta_S (S - S_0)). \tag{5}$$

Assuming the equilibrium solute concentration as a linear function of the temperature, i.e., $S_{eq}(T) = S_0 + \pi(T - T_0)$, and considering chemical equilibrium at the boundaries, we can obtain $\phi = \frac{\Delta S}{\delta T}$. The conduction state is marked by

$$T_b = \left(1 - \frac{z}{d}\right) \Delta T + T_0, \tag{6}$$

$$S_b = \left(1 - \frac{z}{d}\right) \Delta S + S_0. \tag{7}$$

In the fundamental state, we overlay minor perturbations in the following manner:

$$\begin{aligned}
\mathbf{V} &= \mathbf{V}_b + \mathbf{V}', \\
T &= T_b(z) + T', \\
S &= S_b(z) + S', \\
P &= P_b(z) + P', \\
\rho &= \rho_b(z) + \rho'(x, y, z, t).
\end{aligned} \tag{8}$$

To delve deeper, let us introduce the following (non-dimensional) parameters:

$$
\begin{cases}
(x', y', z') = d(x^*, y^*, z^*), \\
\mathbf{V'} = \frac{\kappa}{d}\mathbf{V}^*, \\
t' = \frac{d^2}{\kappa}t^*, \\
T' = (\Delta T)T^*, \\
S' = (\Delta S)S^*, \\
P' = \frac{\rho_0 \kappa^2}{d^2} P^*.
\end{cases}
$$

We obtain the subsequent non-dimensional equations (upon removing the asterisks):

$$\nabla \cdot \mathbf{V} = 0, \tag{9}$$

$$\frac{1}{Pr}\frac{\partial \mathbf{V}}{\partial t} = -\frac{\nabla P}{Pr} + \nabla^2 \mathbf{V} - C_S \nabla^4 \mathbf{V} + (R_T\theta - R_S S)\hat{e}_z + Ta(\mathbf{V} \times \hat{e}_z), \tag{10}$$

$$\frac{\partial \theta}{\partial t} + (\mathbf{V} \cdot \nabla)\theta = \omega + \nabla^2\theta, \tag{11}$$

$$\frac{\partial S}{\partial t} + (\mathbf{V} \cdot \nabla)S = \omega + \frac{1}{Le}\nabla^2 S + D_v(\theta - S), \tag{12}$$

where

$$R_T = \frac{\beta_T g \Delta T d^3}{\kappa\nu}, \quad R_S = \frac{\beta_S \Delta S d^3}{\kappa\nu}, \quad C_S = \frac{\mu_1}{\mu d^2},$$

$$Pr = \frac{\mu}{\rho_0\kappa}, \quad Le = \frac{\kappa}{D_v}, \quad Ta = \frac{2\rho_0 \Omega d^2}{\mu},$$

$$Dm = \frac{d^2\chi}{\kappa}.$$

It is important to note that all of the terms used in the aforementioned equations have definitions in the nomenclature.

3. Linear stability analysis

3.1. Equations

Let us re-consider Eqs. (9) - (12) as

$$\frac{1}{Pr}\frac{\partial \mathbf{V}}{\partial t} = -\frac{\nabla P}{Pr} + \nabla^2\mathbf{V} - C_S\nabla^4\mathbf{V} + (R_T\theta - R_SS)\widehat{e}_z + Ta^{1/2}(\mathbf{V}\times\widehat{e}_z),$$
(13)

$$\frac{\partial \theta}{\partial t} = w + \nabla^2\theta,$$
(14)

$$\frac{\partial S}{\partial t} = w + \frac{1}{Le}\nabla^2 S + Dm(\theta - S).$$
(15)

By extracting the third components of the curl from Eq. (13), we attain

$$\left(\frac{1}{Pr}\frac{\partial}{\partial t} - \nabla^2 + C_S\nabla^4\right)\omega_z - Ta^{1/2}\frac{\partial w}{\partial z} = 0,$$
(16)

$$\left(\frac{1}{Pr}\frac{\partial}{\partial t} - \nabla^2 + C_S\nabla^4\right)\nabla^2 w - \left(R_T\nabla_h^2\theta - R_S\nabla_h^2 S\right) + Ta^{1/2}\frac{\partial\omega_z}{\partial z} = 0. \quad (17)$$

where ω_z represents the z-component of vorticity $((\nabla\times\mathbf{V})\,\widehat{e}_z)$, while w corresponds to the z-component of velocity, and

$$\nabla_h^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

is the horizontal Laplacian operator. By removing ω_z from Eqs. (16) and (17), one obtains

$$\left(\frac{1}{Pr}\frac{\partial}{\partial t} - \nabla^2 + C_S\nabla^4\right)^2\nabla^2 w - \left(\frac{1}{Pr}\frac{\partial}{\partial t} - \nabla^2 + C_S\nabla^4\right)\left(R_T\nabla_h^2\theta - R_S\nabla_h^2 S\right)$$
$$+ Ta\frac{\partial^2 w}{\partial z^2} = 0.$$
(18)

Introducing normal modes involves expressing perturbations in the following form:

$$(w,\ \theta,\ S) = (W(z),\ \theta(z),\ S(z))\,e^{i(lx+my)+\sigma t},$$
(19)

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where l and m are the wave numbers along the x and y directions, respectively, with $q = \sqrt{l^2 + m^2}$ and σ is a complex parameter. Substituting the above expressions into Eqs. (18), (14) and (15), we obtain

$$A_1^2(D^2 - q^2)W - A_1(R_S S - R_T\theta)q^2 + TaD^2W = 0, \tag{20}$$

$$\sigma\theta = W + (D^2 - q^2)\theta, \tag{21}$$

$$\sigma S = W + \frac{1}{Le}\left(D^2 - q^2\right)S + Dm\left(\theta - S\right), \tag{22}$$

$$W = \theta = S = 0 \text{ at } z = 0,\ 1, \tag{23}$$

where $A_1 = \left(\frac{\sigma}{Pr} - (D^2 - q^2) + C_S(D^2 - q^2)^2\right)$, $D = \frac{d}{dz}$ and $q^2 = l^2 + m^2$. We assume that the solutions to W, θ and S are under the following form:

$$\begin{pmatrix} W(z) \\ \theta(z) \\ S(z) \end{pmatrix} = \begin{pmatrix} W_0 \sin(\pi z) \\ \theta_0 \sin(\pi z) \\ S_0 \sin(\pi z) \end{pmatrix}, \tag{24}$$

which satisfy the boundary conditions in Eq. (23). On substituting Eq. (24) into Eqs. (20) - (22), one obtains

$$\begin{pmatrix} A_2^2(-\delta^2) - Ta\pi^2 & A_2 R_T q^2 & -A_2 R_S q^2 \\ 1 & (-\delta^2 - \sigma) & 0 \\ 1 & Dm & \left(-\frac{1}{Le}\delta^2 - Dm - \sigma\right) \end{pmatrix} \begin{pmatrix} W \\ \theta \\ S \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \tag{25}$$

where $A_2 = \left(\frac{\sigma}{Pr} + \delta^2 + C_S\delta^4\right)$, $\delta^2 = \pi^2 + q^2$. For the nontrivial solution of the above matrix Eq. (25), the determinant of above matrix is zero, from which one obtains the following approximate instability threshold:

$$R_{T_c} = \frac{(i\omega + \delta^2)(\pi^2 Pr^2 Ta + \delta^2 A_3^2)}{Prq^2 A_3} + \frac{LeR_S(Dm + i\omega + \delta^2)}{DmLe + iLe\omega + \delta^2}, \tag{26}$$

being

$$A_3 = i\omega + Pr\delta^2 + C_S Pr\delta^4. \tag{27}$$

3.2. Stationary Convection

Substituting $\omega = 0$ allows us to ascertain the critical Rayleigh number for the initiation of stationary convection (denoted as $R_{T_c}^{sc}$). This results in

$$R_{T_c}^{sc} = \frac{\pi^2 Ta + \delta^6(1 + C_S\delta^2)^2}{q^2(1 + C_S\delta^2)} + \frac{LeR_S(Dm + \delta^2)}{DmLe + \delta^2}. \tag{28}$$

For Newtonian fluid, in the absence of the Coriolis effect and chemical reaction, the above Rayleigh number reduces to

$$R_{T_c}^{sc} = \frac{\delta^6}{q^2}, \tag{29}$$

which is well agree with the results obtained by Chandrasekhar [52].

3.3. Oscillatory Convection

Now, let us delve into the examination of the real and imaginary components of R, with a requirement for the imaginary part of R to be nullified. When we substitute ω^2 into the real part of R, we arrive at the thermal Rayleigh number, denoted as $R_{T_c}^{oc}$, signifying oscillatory convection.

4. Discussion

In this article, we examine the eigenvalue problem related to the stability of thermosolutal convection in a CS fluid and the impact of Coriolis and chemical reactions. We highlight some features of the sets of expansion functions that can be used for an analytical analysis of this problem using the spectral methods that we applied.

The behavior of the thermal critical Rayleigh number at the onset of stationary convection $(R_{T_c}^{sc})$ versus the Dm with different values of $Le = 2, 4, 6$ is displayed in Fig. 2, with the other parameters held constant at $R_S = 800$, $C_S = 0.04$ and $Ta = 50$. This figure makes it clear that the Damkohler number has a destabilizing effect on a fluid layer because the Dm increases as $R_{T_c}^{sc}$ decreases.

Fig. 3 illustrates the relationship between $R_{T_c}^{sc}$ and Le for various values of C_S, while keeping other parameters constant. This figure highlights the positive correlation between the values of $R_{T_c}^{sc}$ and C_S as Le increases. Consequently, it can be inferred that increasing the value of Le stabilizes the system.
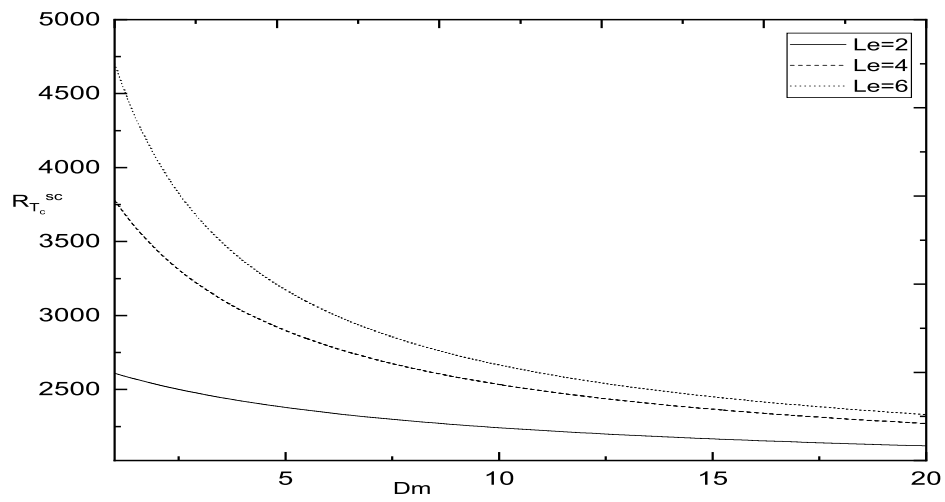
Figure 2: Change in the thermal critical Rayleigh number at the onset of stationary convection with the Damkohler number for distinct values of the Lewis number fixed at $R_S = 800$, $C_S = 0.04$ and $Ta = 50$.
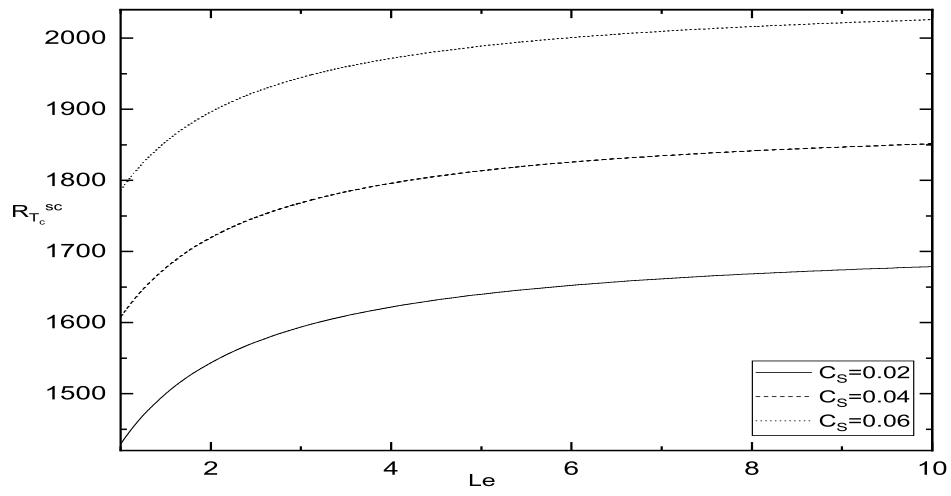
Figure 3: Change in the thermal critical Rayleigh number at the onset of stationary convection with the Lewis number for distinct values of the CS parameter fixed at $R_S = 500$, $Dm = 25$ and $Ta = 50$.
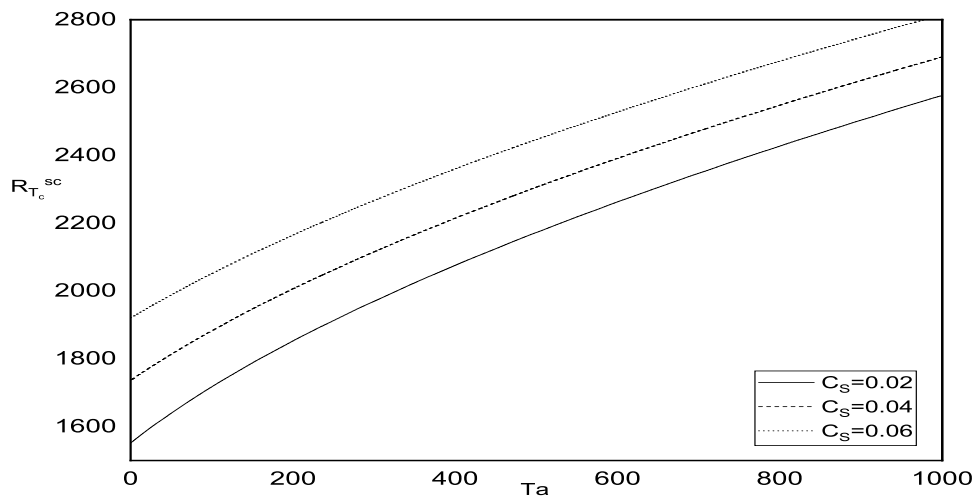
Figure 4: Change in thermal critical Rayleigh number at the onset of the stationary convection with the Taylor number for distinct values of the CS parameter fixed at $R_S = 500$, $Le = 5$ and $Dm = 25$.
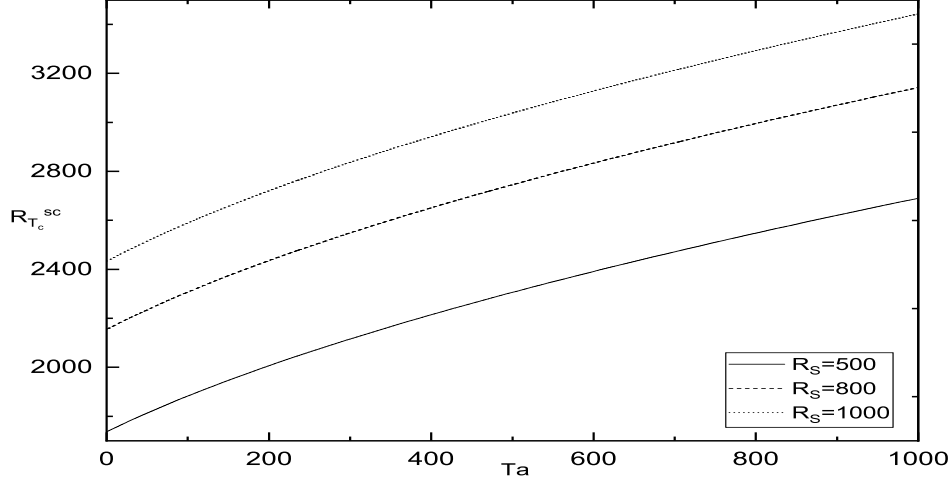
Figure 5: Change in the thermal critical Rayleigh number at the onset of stationary convection with the Taylor number for distinct values of the solute Rayleigh number fixed at $Dm = 25$, $C_S = 0.04$ and $Le = 5$.

Fig. 4 illustrates that $R_{T_c}^{sc}$ with Ta for different values of C_S and for fixed at $R_S = 500$, $Le = 5$ and $Dm = 25$. In addition, it shows that, as C_S increases, $R_{T_c}^{sc}$ increases. As a result, it should be highlighted that a rise in C_S's value stabilizes the system. An increase in the viscosity of a liquid, which slows down fluid motion, is represented by an increase in the values of C_S. As a result, the CS parameter stabilizes convection.

The graph in Figure 5 illustrates the relationship between $R_{T_c}^{sc}$ and Ta, while holding distinct values of R_S constant at Dm, C_S, Le. It is evident from this graph that as the value of Ta increases, both $R_{T_c}^{sc}$ and R_S also increase, suggesting a positive correlation. This implies that Ta exerts a stabilizing influence on the system. The introduction of vorticity into the fluid as it rotates results in an accelerated flow in horizontal planes. However, this rotational motion causes a reduction in fluid velocity perpendicular to these planes. Consequently, there is an observed increase in $R_{T_c}^{sc}$ with the rise in Ta.

With varying values of Le, Fig. 6 gives a visual representation of the critical wave number at the onset of stationary convection (q_c^{sc}) versus the
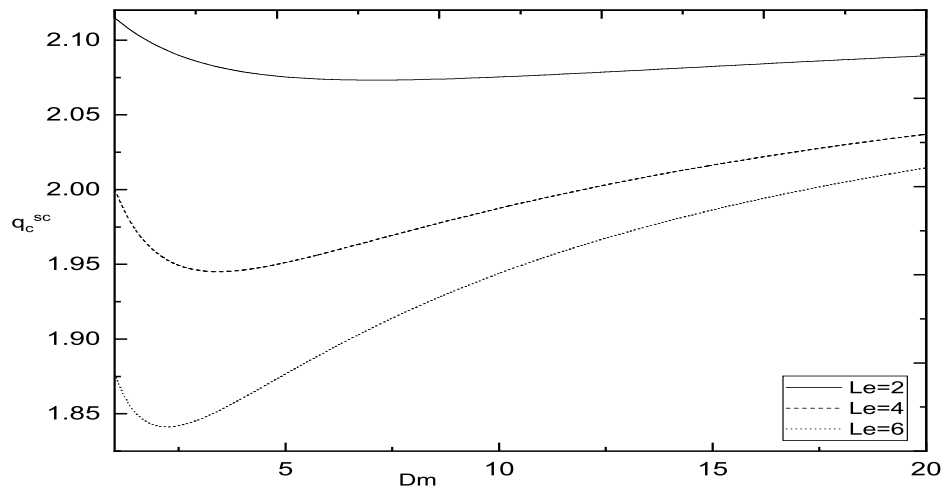
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Figure 6: Change in the thermal critical wave number at the onset of stationary convection with the Damkohler number for distinct values of the Lewis number fixed at $R_S = 800$, $C_S = 0.04$ and $Ta = 50$.
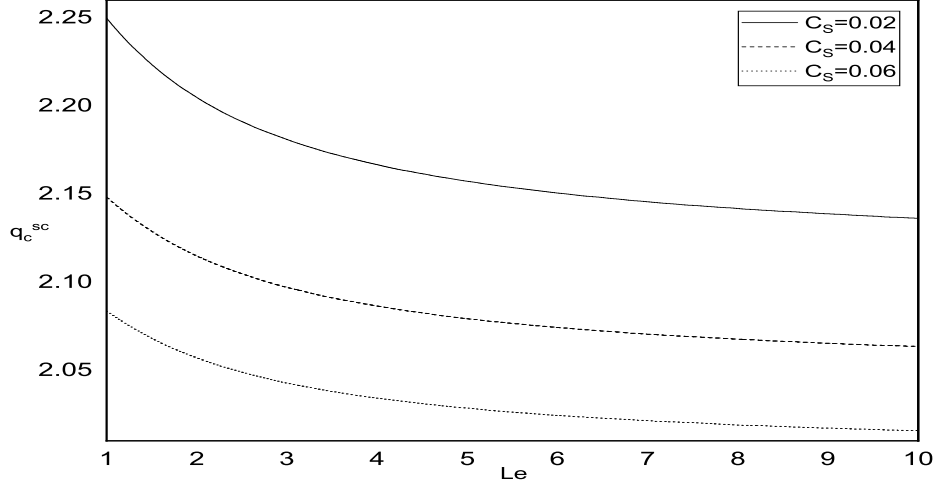
Figure 7: Change in the thermal critical wave number at the onset of stationary convection with the Lewis number for distinct values of the CS parameter fixed at $R_S = 500$, $Dm = 25$ and $Ta = 50$.

Damkohler number (Dm). This figure makes it clear that, up to a certain point, the critical wave number decreases with an increasing value of $Dm = Dm^*$, and increases with Dm thereafter. Consequently, q_c^{sc} is a non-monotonic function of Dm.

Fig. 7 illustrates the relationship between q_c^{sc} and Le for different values of C_S $(0.02, 0.04, 0.06)$, with the remaining parameters held constant at $R_S = 500$, $Dm = 25$, $Ta = 50$. Within this figure, q_c^{sc} exhibits a decreasing trend as both Le and C_S increase, establishing q_c^{sc} as a decreasing function of Le and C_S. Furthermore, Figs. 8 and 9 highlight the upward trajectory of q_c^{sc} concerning Ta, portraying its variation across different parameter values.

The behavior of the thermal critical Rayleigh number at the onset of oscillatory convection $R_{T_c}^{oc}$ versus Dm with different values of Le is depicted in Fig. 10, the remaining parameters are held fixed. This figure unequivocally demonstrates that Dm increases as $R_{T_c}^{oc}$ decreases. Therefore, Dm has a destabilizing effect on the system. Fig. 11 represents the variation of $R_{T_c}^{oc}$ versus Le for the different values of C_S fixed at $R_S = 500$, $Dm = 25$, $Ta = 50$ and $Pr = 0.0001$. This figure clearly shows the stabilizing effect of Le on the
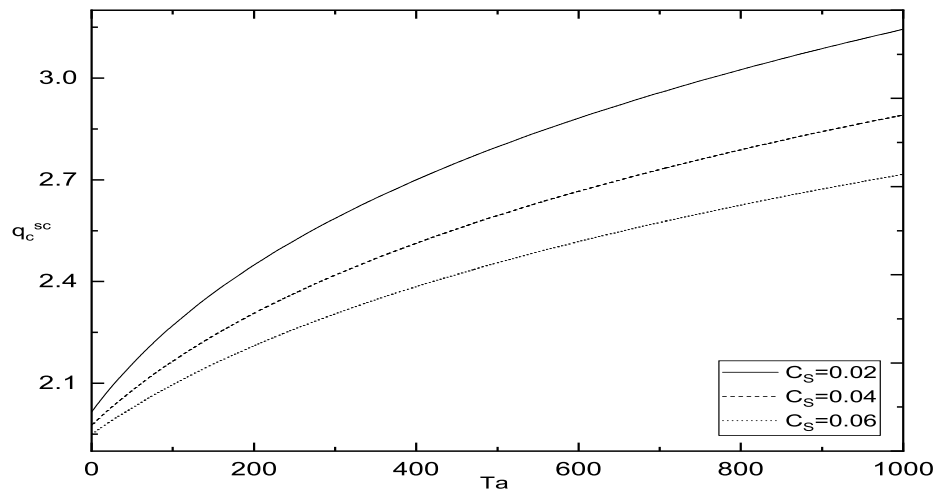
Figure 8: Change in the thermal critical wave number at the onset of stationary convection with the Taylor number for distinct values of the CS parameter fixed at $R_S = 500$, $Le = 5$ and $Dm = 25$.
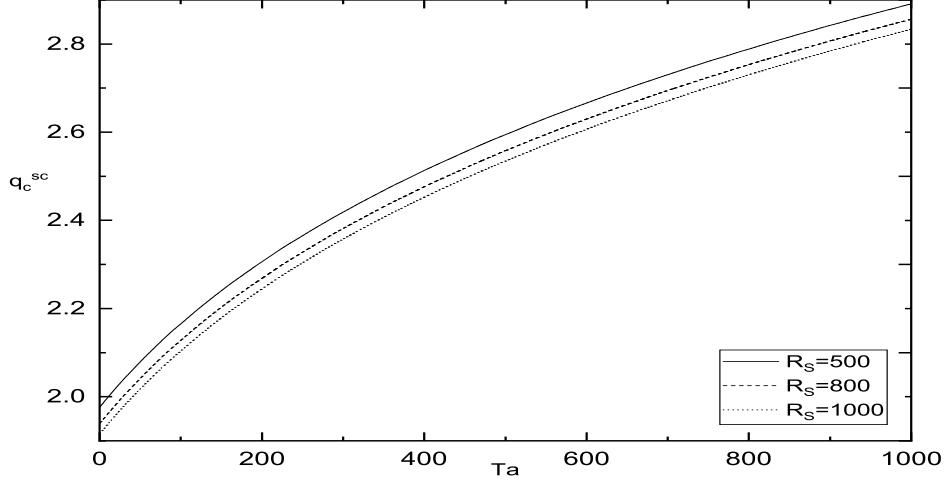
Figure 9: Change in the wave with the Taylor number for distinct values of the solute Rayleigh number fixed at $Dm = 25$, $C_S = 0.04$ and $Le = 5$.

system. As the Lewis number decreases, the distinction between solutal and thermal diffusivities diminishes, rendering it more challenging for inherently double diffusive processes, like the initiation of oscillatory convection, to transpire.

Fig. 12 shows that $R_{T_c}^{oc}$ with Ta for distinct values of $C_S = 0.02; 0.04; 0.06$ and fixed values of R_S, Dm, Le and Pr. This figure shows that as C_S increases, $R_{T_c}^{oc}$ increases. Hence, Ta and C_S have a stabilizing effect on the system.

The behavior of $R_{T_c}^{oc}$ versus Ta with different values of R_S is shown in Fig. 13. All other parameters are held constant. The enhancement of $R_{T_c}^{oc}$ with the increase in the value of Ta is shown in this figure. It is noted that an increment in the value of Ta makes the system stable.

In Fig. 14, $R_{T_c}^{oc}$ is plotted against the Prandtl number (Pr) for distinct values of $Dm = 8; 12; 16$ and for the fixed values of R_S, C_S, Le and Ta. This figure shows that $R_{T_c}^{oc}$ increases with an increasing value of $Dm = Dm^*$, and beyond $Dm = Dm^*$, $R_{T_c}^{oc}$ decreases with Dm. Hence, $R_{T_c}^{oc}$ is a non-monotonic function of Pr.

Fig. 15 depicts the critical wave number at the onset of oscillatory con-
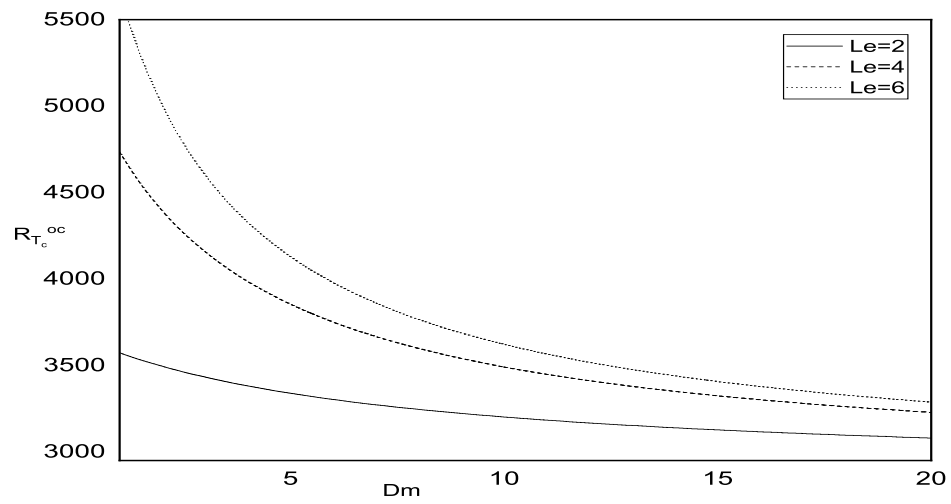
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Figure 10: Change in the thermal critical Rayleigh number at the onset of oscillatory convection with the Damkohler number for distinct values of the Lewis number fixed at $R_S = 800$, $C_S = 0.04$, $Ta = 50$ and $Pr = 0.0001$.
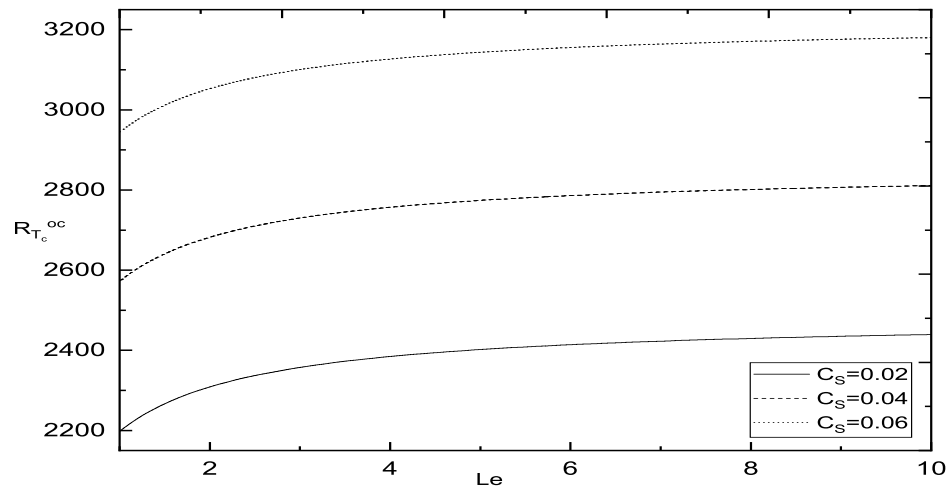
Figure 11: Change in the thermal critical Rayleigh number at the onset of oscillatory convection with the Lewis number for distinct values of the CS parameter fixed at $R_S = 500$, $Dm = 25$, $Ta = 50$ and $Pr = 0.0001$.
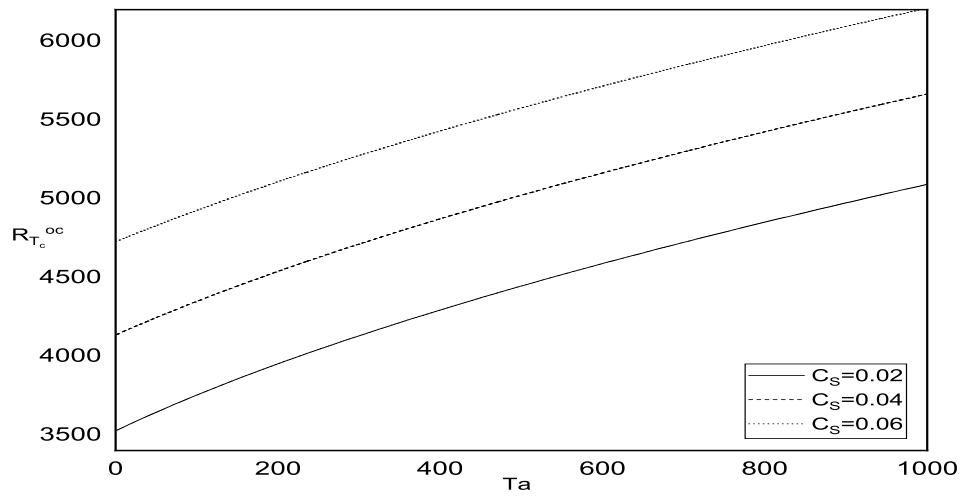
Figure 12: Change in the thermal critical Rayleigh number at the onset of oscillatory convection with the Taylor number for distinct values of the CS parameter fixed at $R_S = 500$, $Le = 5$, $Dm = 25$ and $Pr = 5$.
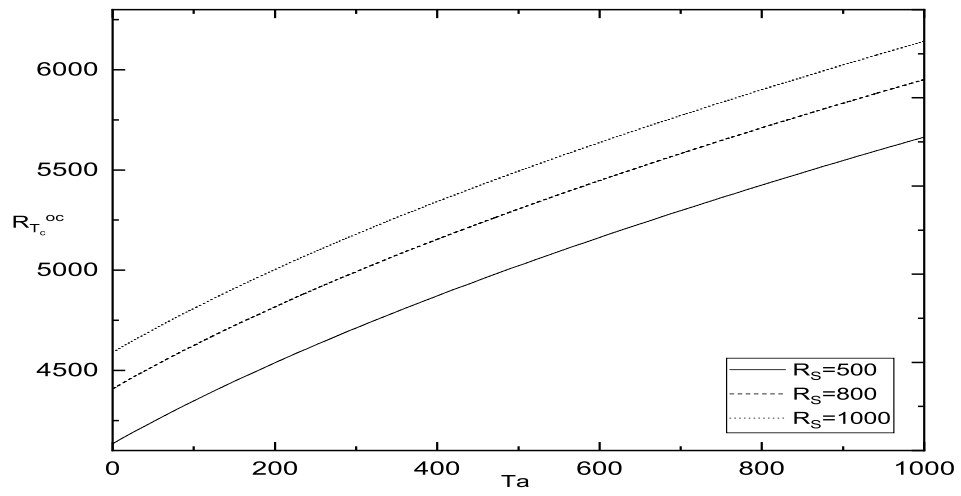
Figure 13: Change in the thermal critical Rayleigh number at the onset of oscillatory convection with the Taylor number for distinct values of the solute Rayleigh number fixed at $Dm = 25$, $C_S = 0.04$, $Le = 5$ and $Pr = 5$.

Figure 14: Change in the thermal critical Rayleigh number at the onset of oscillatory convection with the Prandtl number for distinct values of the Damkohler number when $R_S = 1000$, $C_S = 0.04$, $Le = 5$ and $Ta = 50$.

Figure 15: Change in the thermal critical wave number at the onset of oscillatory convection with the Damkohler number for distinct values of the Lewis number fixed at $R_S = 800$, $C_S = 0.04$, $Ta = 50$ and $Pr = 0.0001$.
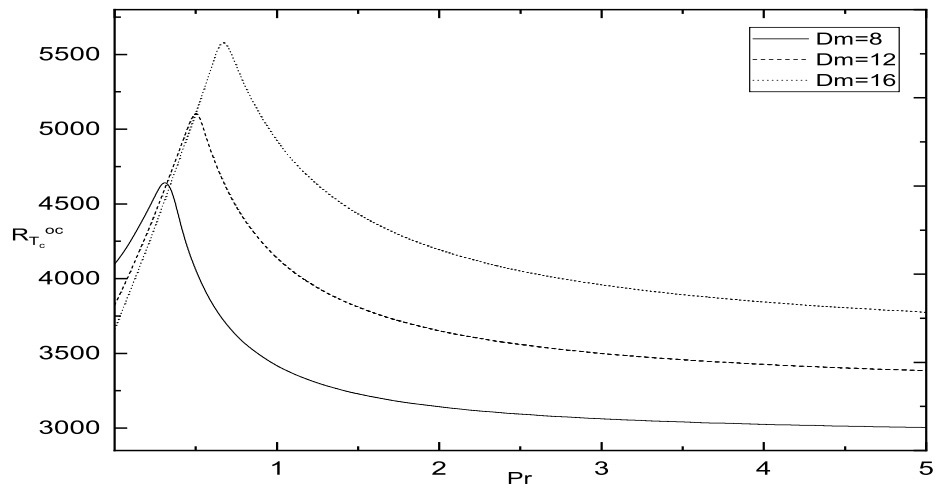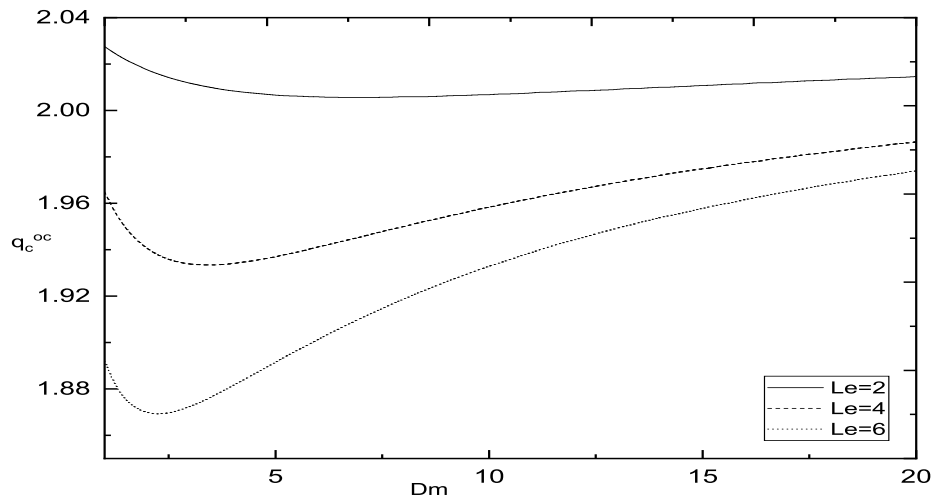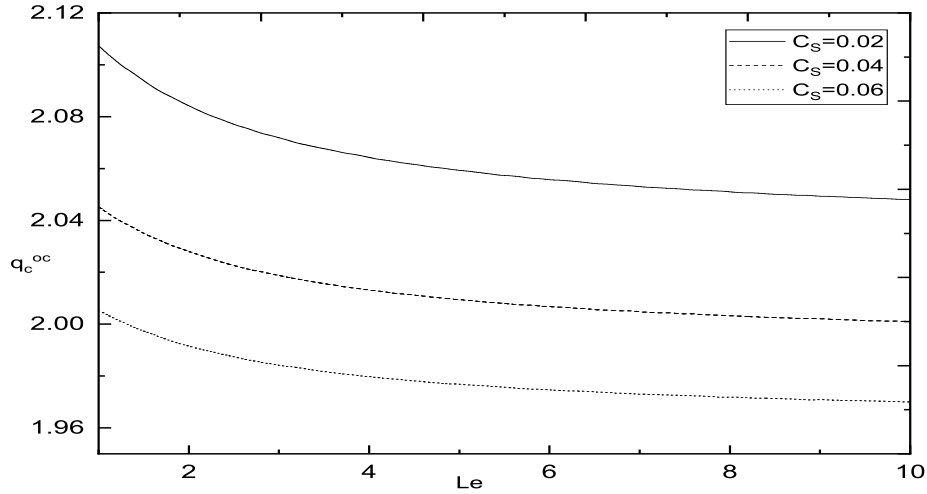
Figure 16: Change in the thermal critical wave number at the onset of oscillatory convection with the Lewis number for distinct values of the CS parameter fixed at $R_S = 500$, $Dm = 25$, $Ta = 50$ and $Pr = 0.0001$.

vection (q_c^{oc}) versus Dm for various values of Le. This figure shows that Dm increases but q_c^{sc} decreases and then increases, indicating q_c^{sc} is a non-monotonic function of Dm.

Fig. 16 depicts the linear instability thresholds with the q_c^{oc} versus Le for different values of C_S, with the remaining parameters held constant at $R_S = 500$, $Dm = 25$, $Ta = 50$, and $Pr = 0.0001$. In this figure, q_c^{oc} decreases as Le increases, but C_S decreases as q_c^{sc} increases.

Fig. 17 represents q_c^{oc} versus Ta by taking into account distinct values of C_S at $R_S = 500$, $Le = 5$, $Dm = 25$ and $Pr = 5$. The enhancement of q_c^{oc} with the enhancement in the value of Ta is observed. A similar trend of q_c^{oc} versus Ta can be observed in Fig. 18.

We give some examples of steady or oscillatory instability developing for constant values of physical parameters in Tables 1 and 2. According to Table 1, the stationary instability threshold occurs when $Ta \leq 910.331$, and convection is oscillatory when $Ta > 910.331$. From Table 2, we can see that the oscillatory instability threshold occurs when $Dm \leq 1.76455$, whereas the stationary convection dominates when $Dm > 1.76455$. To put it another
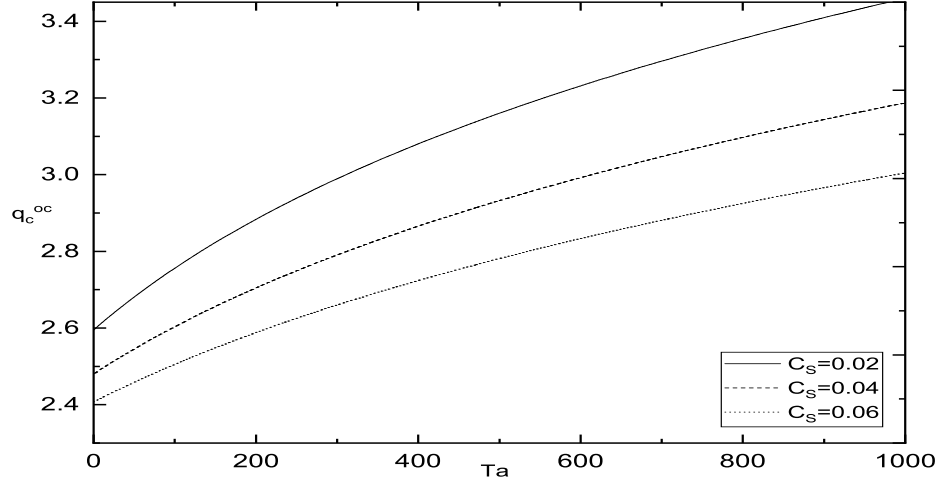
25

Figure 17: Change in the thermal critical wave number at the onset of oscillatory convection with the Taylor number for distinct values of the CS parameter fixed at $R_S = 500$, $Le = 5$, $Dm = 25$ and $Pr = 5$.

Ta	$R_{T_c}^{sc}$	q_c^{osc}	$R_{T_c}^{oc}$	q_c^{oc}	Instability
850	3953.117	2.672	4006.643	1.978	St.
860	3961.338	2.672	4005.879	1.978	St.
870	3969.509	2.682	4005.120	1.978	St.
880	3977.662	2.682	4004.364	1.978	St.
890	3985.747	2.691	4003.612	1.979	St.
900	3993.816	2.701	4002.865	1.979	St.
910	4001.834	2.702	4002.121	1.979	St.
910.331	4002.097	2.703	4002.097	1.979	St.
920	4009.822	2.710	4001.382	1.979	Os.
930	4017.775	2.714	4000.646	1.979	Os.
940	4025.683	2.720	3999.914	1.980	Os.
950	4033.571	2.726	3999.186	1.979	Os.

Table 1: Critical values of R_T for the case $Pr = 0.1$, $R_S = 800$, $Le = 5$, $Dm = 5$ and $C_S = 0.04$ (St. is for Stationary and Os. is for Oscillatory)
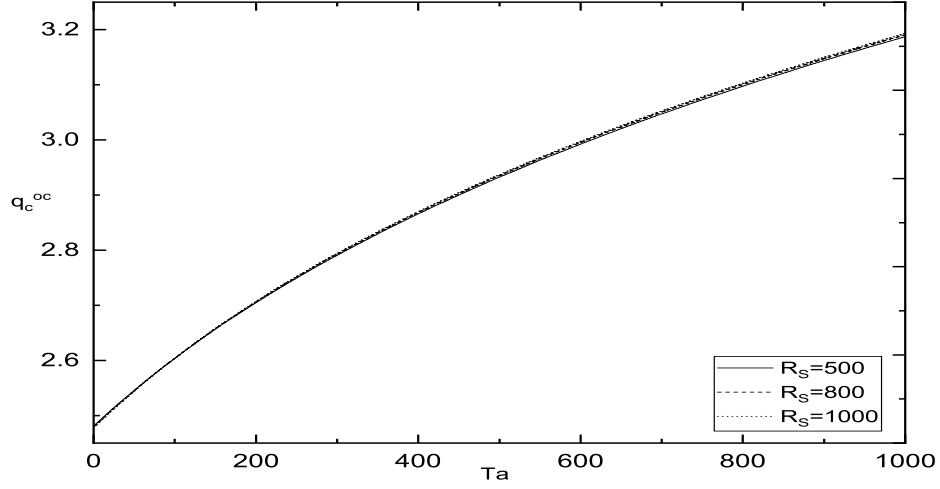
26

Figure 18: Change in the thermal critical wave number at the onset of oscillatory convection with the Taylor number for distinct values of the solute Rayleigh number fixed at $Dm = 25$, $C_S = 0.04$, $Le = 5$ and $Pr = 5$.

Dm	$R_{T_c}^{sc}$	q_c^{osc}	$R_{T_c}^{oc}$	q_c^{oc}	Instability
1	4489.224	2.177	3707.138	2.181	Os.
1.1	4429.914	2.169	3767.729	2.182	Os.
1.2	4373.429	2.162	3826.564	2.180	Os.
1.3	4319.584	2.156	3883.190	2.177	Os.
1.4	4268.209	2.150	3937.057	2.170	Os.
1.5	4219.144	2.146	3987.525	2.161	Os.
1.6	4172.243	2.142	4033.889	2.149	Os.
1.7	4127.372	2.138	4075.444	2.133	St.
1.76455	4099.427	2.137	4099.427	2.122	St.
1.8	4084.404	2.136	4111.601	2.115	St.
1.9	4043.223	2.133	4142.011	2.096	St.
2	4003.724	2.131	4166.667	2.077	St.

Table 2: Critical values of R_T for the case $Pr = 0.1$, $R_S = 800$, $Le = 5$, $Ta = 200$ and $C_S = 0.04$ (St. is for Stationary and Os. is for Oscillatory) .

way, oscillatory convection begins, but as soon as the value of Dm reaches a critical value (1.76455), it stops being oscillatory and the first bifurcation occurs as stationary convection.

5. Conclusion

In this article, we investigated the chemical reaction on the stability of thermo-solutal convection in a rotating CS fluid. Various parameters such as R_T, q, R_S, Dm, Ta, Le , C_S and Pr have been studied. The following is a summary of the findings:

Stationary convection:

- The Damkohler number has a destabilizing influence on a fluid layer.

- The Lewis, Taylor number and solute Rayleigh numbers have a stabilizing effect on the system.

- The critical wave number is a non-monotonic function of the Damkohler number.

- The critical wave number is a decreasing function of the Lewis number, solute Rayleigh number and CS parameters.

- The critical wave number is a increasing function of the Taylor number.

Oscillatory convection:

- As we saw in the case of stationary convection, the Damkohler number has a destabilizing effect on the fluid layer.

- The Lewis, Taylor and olute Rayleigh numbers have a stabilizing effect on the system.

- The critical Rayleigh number is a non-monotonic function of the Prandtl number.

- The critical wave number is a non-monotonic function of the Damkohler number.

- The critical wave number is a decreasing function of the Lewis number, solute Rayleigh number and CS parameter.

- The critical wave number is an increasing function of the Taylor number.

Furthermore:

- The stationary instability threshold occurs when $Ta \leq 910.331$ and convection is by oscillation when $Ta > 910.331$.

- The oscillatory instability threshold occurs when $Dm \leq 1.76455$, whereas the stationary convection dominates when $Dm > 1.76455$.

In future, we plan to explore the heat and mass transport by deriving the amplitude equation using weakly non-linear analysis. Also, we want to study the non-linear instability analysis using the energy method.

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