PARALLEL CONTROL STRUCTURE BASED SLIDING MODE CONTROLLER FOR SECOND ORDER UNSTABLE PROCESSES WITH DEAD-TIME

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Abstract: As one or more poles are located on the right side of the s-plane, the unstable processes are challenging to control. The presence of dead time in such systems makes control much more difficult. This work focuses on the control of unstable processes with dead-time using sliding mode control in a parallel control structure. Two controllers for set-point tracking and load-disturbance rejection are designed with a proportional-integral-derivative-acceleration based sliding surface. The parameters of the continuous and discontinuous control laws are obtained using the particle swarm optimization technique. An objective function is constituted in terms of a performance measure (integral absolute error). The proposed sliding mode controller design in a parallel structure gives enhanced set-point tracking and load disturbance rejection. Illustrative examples demonstrate the superiority of the proposed controller over earlier reported work in this realm, especially in terms of load rejection. Furthermore, the robustness of the proposed controller is also investigated by including perturbations in the parameters. The obtained results clearly show how well the suggested controller works.

Keywords: sliding mode control, unstable process with dead-time, parallel control structure, particle swarm optimization, SOPDT process

List of symbols

t_0	: Time delay
τ_1, τ_2	: Time-constants
λ	: Tuning parameter
$r(t)$	: Reference command
$e(t)$	: Error signal
$x(t)$	: Process output
$U(t)$	: Controlled input
K	: Plant/process gain
l	: Dead-time

$s(t)$: Sliding surface

$k_1, k_2, k_3, k_p, k_D, \delta$: Sliding mode controller tuning parameters

u_{cc} : Continuous control law

u_{dc} : Discontinuous control law

1. Introduction

1.1 Overview of controllers for unstable processes

It is well known that obtaining appropriate closed-loop step responses for processes with unstable dynamics is difficult [1]. Furthermore, the presence of dead time makes it more challenging to control such unstable processes. Various control solutions have been presented by researchers in the field of control engineering who have focused on how to handle unstable plants that have dead time. A relatively common and effective controller for unstable delayed plants is the proportional-integral-derivative (PID) controller [2]–[5]. Recently, some internal model control based multi-loop P/PD/PID controllers have been presented by [6]–[8] to deal with unstable integrating/first/second order plus dead-time processes. However, most of the aforementioned schemes are complex as they use multiple controllers. Furthermore, there are instances where the P/PD/PID tuning rule is insufficient to address process perturbations and load disturbances. The sliding mode-based control (SMC) is well recognized in this context for its ability to overcome modeling errors in the plant and unidentified disruptions. It has been extensively shown to be beneficial for nonlinear and time-varying systems with a nominal model as well as a perturbed model. These characteristics make SMC widely used in a variety of industries, including the chemical industry, process control, power system, and biomedical processing.

1.2 Literature survey of sliding mode control schemes

Researchers in the process control field have become fascinated with SMC for the regulation of industrial processes over the past two decades [9]–[11]. SMC has been utilized by Camacho and Smith [9] to control stable chemical processes, based on the first-order plus dead-time (FOPDT) model that was found using an open-loop step test. However, their approach has a significant overshoot and a lengthy settling period. They have developed a sliding surface having two unknown parameters. By combining a novel sliding surface with four unknown parameters with the SMC approach, Kaya [10] has demonstrated enhanced performance over [9]. In addition, Kaya has used a stable FOPDT-based plant model; nonetheless, the parameters of Kaya were determined using a closed-loop test with relay feedback. Camacho and Rojas [11] has extended the work of [9] using a PID type surface selection to regulate the integrating process and multiple input multiple output operations.

The aforementioned approaches are based on the short-delay FOPDT model, and when used with delay-dominant processes, their performance degrades. Researchers have employed sliding mode controller (SMCr) with modified control structures to regulate the delay-dominant processes [12]–[16]. A dynamic SMCr with modified SP for longer dead time processes with inverse response dynamics has been given by [12]. By transforming higher-order processes into FOPDT models, their methodology—which was predicated on the FOPDT process model—can be applied to them. SMC with a generalized predictive control (GPC) has been used by Parte et al. [13] to regulate the FOPDT process model. Smith-predictor (SP) architecture and SMC have been integrated by Camacho and Cruz [14] to regulate first order delay dominating processes with a delay-to-time-constant ratio (t_0/τ) of up to 2.5. Camacho et al. [15] employed the internal model control scheme-based sliding mode controller (SMCr) for processes with high t_0/τ relationships. Their approach was based on the FOPDT model, and the Nelder-Mead search algorithm was used to find the controller settings. In order to increase load-disturbance responsiveness for large dead-time processes, Mehta and Kaya [16] suggested an SP-based SMCr that uses the particle swarm optimization approach.

All of the aforementioned methods were limited to an SMC for stable processes. In order to control unstable open-loop FOPDT processes, a new SMC approach has been presented by Rojas et al. [17]. However, if the t_0/τ ratio is greater than 1, their approach is inappropriate. Sivaramakrishnan et al. [18] have succeeded in extending the t_0/τ ratio to 1.8. However, there was a little improvement in load disturbance rejection performance. For unstable elevated delay FOPDT systems having enhanced load-disturbance rejection, Mehta and Rojas [19] have provided an SP-based SMC. In their research, a cuckoo search algorithm was used to obtain the controller parameter.

According to the literature reviewed above, the majority of researchers have only provided SMC designs for the FOPDT process model. An approximated FOPDT model is insufficient to accurately depict the dynamics of higher-order unstable plants since these plants may also experience information suppression in addition to unstable dynamic delays [20]. In this direction, Siddiqui et al. [21] introduced an SMCr for unstable second-order plus dead-time (SOPDT) processes with a novel sliding surface, where the control parameters were determined using the root-locus approach and a metaheuristic grass-hoper algorithm. Recently, Reference [22] introduced an SMC strategy to regulate a second-order inverse process with variable dead-time.

1.3 Key research gaps and motivation

The application of SMC approaches in the process industry is given in Table I based on the literature review and Ref. [1], and the following inference can be made:

- SMC is used fairly infrequently in process control, and the majority of the effort is focused on stable processes. Since most industrial processes are inherently unstable, handling them requires special attention.
- Regardless of whether the system is stable or unstable, the model utilized for SMCr design is often of the FOPDT type. Their approach has been utilized to analyze higher-order processes by transforming them into the FOPDT model.
- The SMC has been implemented with improved control structures utilizing the FOPDT model to increase performance in complex processes.
- There hasn't been much attention given to controlling the unstable SOPDT process.

As a result, a SOPDT model would be preferable over a FOPDT model for describing unstable higher-order plants since it offers more accurate information on plant dynamics. Furthermore, SMC is not proposed in a two-degree-of-freedom parallel control structure (2DOF-PCS) to control unstable processes, as per the best knowledge of the authors.

1.4 Contributions

With these motivations, this paper presents an SMC for controlling the unstable SOPDT in the 2DOF-PCS scheme. In this proposed method, two separate SMCs are developed for closed-loop set-point tracking and load disturbance rejection. To achieve satisfactory closed-loop system performance, a sliding surface comprising three control parameters is used to control the SOPDT process. The control parameters are obtained from the set of tuning equations using the SOPDT model and the particle swarm optimization (PSO) algorithm. To demonstrate the benefits of the suggested technique over several current methods mentioned in the literature, some examples are simulated. Highlights of this work are summarized as follows:

- As per the best knowledge of the authors, SMCR is designed for the first time in a parallel control structure for unstable SOPDT.
- Two controllers dedicated to set-point tracking and load-disturbance rejection in the SMC scheme are proposed, which offer two degree of freedom.
- The proposed controller gives satisfactory results, especially in terms of load disturbance rejection, over recently reported methods.
- The effectiveness of the proposed controller is studied with the inclusion of perturbations.

The organization of the paper is as follows: Section 2 presents the SMC design in a parallel scheme. Section 3 provides the optimal tuning of the controller parameters. The simulation results of the suggested approach are shown in Section 4, along with a comparison to previously published methods. Finally, some conclusions are presented in Section 5.

2 SMC design in parallel control structure

Sliding mode control is a powerful control strategy that may be used to produce a robust closed-loop system even when there are disturbances and parametric uncertainties [1]. The main objective of SMC is to drag the system from its initial state to a selected surface so that it may slide to the desired value. Defining a sliding surface and creating a control rule that pulls the states of the system as fast as feasible to that surface are the two phases in a SMCr design. As illustrated in Figure 1, the SMC scheme in a parallel control structure contains two controllers (G_{c1}) and (G_{c2}), which are intended to perform the functions of load-disturbance (d) rejection and set-point $r(t)$ tracking, respectively [23]. G_{p1} represents the controllable plant model, while G_{p2} represents the process model. For perfect modeling, $G_{p1} = G_{p2}$ is selected. U_1 and U_2 are the control signals of both controllers G_{c1} and G_{c2}, respectively. Both controllers, G_{c1} and G_{c2}, are designed using the same SMC technique for 2DOF in PCS.

The primary goal of SMC is to transition a system from its original state to a chosen surface so that it may glide to the desired value. The design of an SMC involves two steps: defining a sliding surface and developing a control rule that draws the system's states as soon as possible towards that surface. In this paper, the following sliding surface $s(t)$, which is a proportional-integral-derivative-acceleration (PIDA) controller type, has been selected.

$$s(t) = k_1 e(t) + k_2 \int_0^t e(t)\,dt + k_3 \frac{de(t)}{dt} + \frac{d^2 e(t)}{dt^2}. \tag{1}$$

In Equation (1), $e(t)$ is tracking error and k_1, k_2 and k_3 are tuning parameters that control how well the system performs on a sliding surface. The goal of control is to always bring the controlled variable to its reference value; therefore, both $e(t)$ and its derivative must be zero. As soon as the reference value is attained, Equation (1) indicates that $s(t)$ has achieved a constant value, which means that $e(t)$ is zero at $t > 0$. To maintain $s(t)$ at a constant value, it is desired to make

$$\frac{ds(t)}{dt} = k_1 \frac{de(t)}{dt} + k_2 e(t) + k_3 \frac{d^2 e(t)}{dt^2} + \frac{d^3 e(t)}{dt^3} = 0. \tag{2}$$

It is necessary to construct a control rule $u(t)$ that ensures the controlled variable will always equal its reference value and satisfies Equation (2) once the sliding surface has been determined. Control law $u_t(t)$ has two additive parts: first one is continuous control law, $u_{cc}(t)$, and second is discontinuous control law, $u_{dc}(t)$. Hence,

$$u_t(t) = u_{cc}(t) + u_{dc}(t) \tag{3}$$

$u_{cc}(t)$ is given by [1]:

$$u_{cc}(t) = f\left(x(t), r(t)\right) \tag{4}$$

where, $x(t)$ and $r(t)$ are functions of the controlled variable and the reference value, respectively. If the initial trajectory is not towards the sliding surface, the $u_{dc}(t)$ is responsible for deriving the system there [20]. This control rule is a nonlinear switching element, identical to an ideal relay or saturation relay. Due to the existence of finite delays and the physical constraints of actuators, it is very challenging to achieve high switching control utilizing these relays in practice. It may produce chattering across the sliding surface which is extremely unwanted [16]. High-frequency dynamics that are disregarded in system modeling may be excited by chattering, a high-frequency oscillation over the sliding surface. One method to lessen chattering is to choose, $u_{dc}(t)$ as illustrated below [21].

$$u_{dc}(t) = k_D \frac{s(t)}{\left|s(t)\right| + \delta} \tag{5}$$

where, k_D is the tuning variable that causes the system to enter sliding mode. The reduction of the chattering phenomenon is achieved by tuning parameters δ. In this work, following unstable SOPDT process models ($G_a(s)$ and $G_b(s)$) have been considered.

$$G_a(s) = \frac{X(s)}{U_t(s)} = \frac{Ke^{-ls}}{(\tau_1 s + 1)(\tau_2 s - 1)} \tag{6}$$

$$G_b(s) = \frac{X(s)}{U_t(s)} = \frac{Ke^{-ls}}{(\tau_1 s - 1)(\tau_2 s - 1)} \tag{7}$$

where, $X(s)$ is the output of the system, $U(s)$ is the input to the plant, K is the gain of the plant, l is delay of the plant, and τ_1 and τ_2 are time constants. The above equations have an exponential term (e^{-ls}), which is required to be rationalized for ease of manipulation. The time delay component is approximated in this study by the first-order Taylor series formula $\left(e^{-ls} = 1/(1+ls)\right)$. Hence, these equations may be rationalized as

$$G(s)=\frac{X(s)}{U_t(s)}=\frac{K}{a_3s^3+a_2s^2+a_1s+a_0d}\ , \tag{8}$$

where, for Equation (6) : $a_3=\tau_1\tau_2 l, a_2=l(\tau_2-\tau_1)+\tau_1\tau_2, a_1=(\tau_2-\tau_1-l)$ and $a_0=-1$;

and for Equation (7) : $a_3=\tau_1\tau_2 l, a_2=\tau_1\tau_2-l(\tau_2+\tau_1)+\tau_1\tau_2, a_1=(l-\tau_2-\tau_1)$ and $a_0=1$.

Equation (8) may be written in differential form as

$$Ku_t(t)=a_3\dddot{x}(t)+a_2\,\ddot{x}(t)+a_1\dot{x}(t)+a_0x(t). \tag{9}$$

Solving Equation (9) for the third-order differential term in conjunction with Equation (2), it may be written as

$$\frac{ds(t)}{dt}=\frac{1}{a_3}\left(-Ku_t(t)+a_2(t)+a_1\dot{x}(t)+a_0x(t)\right)+k_2r(t)-k_2x(t)-k_3\,\ddot{x}(t)-k_1\dot{x}(t)=0 \tag{10}$$

where, $e(t)=r(t)-x(t)$. Control law $u(t)$ can be obtained from Equation (10) as:

$$u_t(t)=\frac{\ddot{x}(t)(a_2-a_3k_3)+\dot{x}(t)(a_1-a_3k_1)+x(t)(a_0-a_3k_2)+a_3k_2r(t)}{K} \tag{11}$$

In the SMC technique, the total control law $u_t(t)$ relies on $u_{cc}(t)$ only when the system approaches the sliding surface. So, Equation (11) may further be simplified by putting $a_2-a_3k_3=0$, and

$$u_{cc}(t)=\frac{\dot{x}(t)(a_1-a_3k_1)+x(t)(a_0-a_3k_2)+a_3k_2r(t)}{K}. \tag{12}$$

From Equations (3), (5) and (12), control law $u_t(t)$ may be written as:

$$u_t(t)=\frac{\dot{x}(t)(a_1-a_3k_1)+x(t)(a_0-a_3k_2)+a_3k_2r(t)}{K}+k_D\frac{s(t)}{|s(t)|+\delta} \tag{13}$$

where,

$$s(t)=sgn(K)\left(k_1e(t)+\ k_2\int_0^t e(t)\,dt+\ k_3\frac{de(t)}{dt}+\frac{d^2e(t)}{dt^2}\right) \tag{14}$$

In Equation (14), a signum function, $sgn(K)$, is introduced for the controller to operate properly [16]. Since $sgn(K)$ depends on the plant gain, the controller action never switches [11]. The overall control law $u_t(t)$ will be obtained from Equation (13). This law is used to design both the controllers, G_{c1} and G_{c2}.

3 Optimal tuning of 2DOF-PCS SMC parameters

The improved performance of the controller is attributed to the appropriate choice of objective function, which is highly significant [20]. It is well acknowledged that when a controller is designed to minimize integral absolute error (IAE), it often produces good results [21]. The objective function J is defined in terms of IAE as

$$J = IAE = \int_0^{\infty} |e(t)| \, dt. \tag{15}$$

IAE evaluates the output control performance by minimizing its value as low as possible.

To solve the objective function without any major convergence problems, it is crucial to select the appropriate optimization approach. Various nature-aspired algorithms are available in the literature, like the genetic algorithm, particle swarm optimization, ant colony, cuckoo algorithm, etc. It has been demonstrated that the particle swarm optimization (PSO) technique produces the best possible combination of parameter values in a shorter amount of time [16]. Also, the motivation behind favoring this optimization is that it produced a good result in this study. PSO is a heuristic algorithm based on the social behavior of schools of fish and bird flocks. Members of fish schools or flocks of birds adhere to innate norms to move in unison so that they don't clash while searching for their food. As a result, it is expected that each member of the population, known as the swarm, would fly around the search area to locate the ideal solution in terms of its position. The principles governing how each member, known as the particle, in space changes its velocity and position were first inspired by behavioral models of flocking birds. Each time a better fitness value is found,

it stores and displays it as *pbest*. This iteration continues until it updates the two best values, one of which is *pbest* and the other is *gbest*, which represents the overall best value for the whole population. Each particle in PSO consists of the five variables $k_1, k_2, k_3, K_D, \delta$, and it updates the variables after each iteration to determine the *pbest* and *gbest*. The program then executes to reach an optimal outcome.

At time t, each i^{th} particle location is defined by $x_i^{'} = \left(x_{i1}, x_{i2,} x_{i3}, \ldots, x_{iD}\right)$ in search space (D), and the current velocity is defined as $v_i^{'} = \left(v_{i1}, v_{i2,} v_{i3}, \ldots, v_{iD}\right)$. If the fitness function, also called the objective function, needs to be minimized to get the optimal solution, then the position and velocity of each particle in the k^{th} iteration can be calculated as

$$v_{i,D}^{k+1} = wv_i^k + c_1 r_1 \left(pbest_{1,D} - x_{i,D}^k\right) + c_2 r_2 \left(gbest - x_{i,D}^k\right), \tag{16}$$

$$x_{i,D}^{k+1} = x_{i,D}^k + v_{i,D}^{k+1} \tag{17}$$

where, $i = 1, 2, 3, \ldots N$, w is the weight function, c_1 is called the cognitive parameter, which drags each particle to its $pbest$ location, c_2 is known as the social parameter, which gives the $gbest$ position for each particle, and r_1 and r_2 are the random sequences in the range $\left[0,1\right]$.

The goal of the controller design process is to choose controller parameter values from the search space that minimize the objective function under consideration. The fundamental block diagram for PSO algorithm-based controller tuning is shown in Figure 2. The PSO method initially assigns arbitrary values for k_1, k_2, k_3, K_D, δ and computes the J. This process continues until J reaches J_{min} or the specified number of iterations has been attained. To determine the optimal control parameter values, the PSO algorithm is developed with $N = 20$, $c_1 = c_2 = 2$, $w = 0.8$ in MATLAB 18a (2018) on Windows 10 with Intel i5 processor and 8GB RAM. To determine the ideal control settings for the required optimization problem, the simulation was conducted for 100 iterations.

4 Simulations, Results and Discussion

In this section, the proposed SMC technique has been validated through various examples of dead-time unstable processes having one unstable pole and two unstable poles with dead-time. The simulation results taken from the suggested technique demonstrate the advantages, especially in terms of fast settling time and load disturbance (LD) rejection for nominal and perturbed systems, as compared to the results obtained from recently reported methods. The controller effort of the proposed technique is also compared with recent methods, and it is found to be better or comparable. A comparison of performance measures *IAE,* integral square error (*ISE*), integral time absolute error (*ITAE*) and *TV* for each example is shown in Table II.

Example 1 (An SOPDT with one unstable pole)

An unstable SOPDT process is based on an article by Siddiqui et al. [21]. They have shown improved results over the results obtained from previous studies done by Mehta and Rojas [19] and Atic and Kaya [24]. The example is described by the following transfer function:

$$G_1(s) = \frac{e^{-0.5s}}{(2s-1)(0.5s+1)}. \tag{18}$$

This example is studied with the proposed strategy, and the control parameters for (G_{c1}) and (G_{c2}) are found as ($k_1 = 7.2$, $k_2 = 4$, $k_3 = 3.5, K_D =$ 241.7 and $\delta = 107.4$) and ($k_1 = 5.1$, $k_2 = 4.1$, $k_3 = 3.5, K_D = -4.46$ and $\delta = -95.7$), respectively. Siddiqui et al. [21] have found the controller settings as $k_1 = 2$, $k_2 = 0.45$, $k_3 = 3.5, K_D =$12.38 and $\delta = 2.69$ by their method. By using a relay feedback test, Atic and Kaya [24] approximated this process into the FOPDT model ($e^{-0.918s} / 2.69s - 1$). The closed-loop response and controller output are displayed in Figure 3 and Figure 4, respectively. It is observed from Figure 3 that the suggested strategy gives a faster set-point tracking as compared to the techniques of Siddiqui et al. [21] and Atic and Kaya [24]. There is a significant improvement in the LD rejection response with the proposed technique. The controller's effort is vital in any closed-loop control system. Figure 4 shows that the control effort of the suggested technique is comparable. The performance indices *IAE, ITAE, ISE,* and *TV* are shown in Table II. A lower value of *IAE, ITAE, ISE,* and *TV* shows the superiority of the proposed method.

For robustness analysis, a perturbation change of +10% in process gain (K), time-constant (τ_2), and delay (l) of the nominal process is introduced simultaneously. The perturbed process considering these perturbed values may be written as $G_1(s) = \frac{1.1e^{-0.55s}}{(2.2s-1)(0.5s+1)}$. The process output and the controller output for the perturbed process by the suggested strategy and the methods considered for comparison are shown in Figure 5 and Figure 6, respectively. It is evident from Figure 5 that closed-loop performance is more robust with the proposed method. The method of Siddiqui et al. fails to track the set-point and LD rejection is also poor. The controller response in Figure 6 and the lesser value of *TV* from Table II indicate that controller effort is smaller with the proposed method. It is also observed from the table that the performance indices *IAE, ISE, ITAE* and *TV* for the proposed method are lower than those of Siddiqui et al. [21] and Atic and Kaya [24]. Hence, it may be concluded that the proposed method gives quite satisfactory results for both nominal and perturbed systems.

Noise may come from the industrial process itself, control valves, and control equipment. In order to justify the usefulness of the present technique under noisy conditions, the simulation of the example is done while taking into account white noise (noise power of 0.0015, seed equal to 1, and sampling time of 0.001 s). Figure 7 displays the system output in noisy situations. From Figure 7, it has been noted that the suggested technique performs well even in noisy environments.

Example 2 (An SOPDT with two unstable poles)

From the literature, it is observed that very few examples of unstable SOPDT were studied in process control with the application of a sliding mode controller. So, this example is taken from the literature of Raza and Anwar [4] and Cho et al. [25], in which they have designed a PID controller for the process defined by the transfer function

$$G_2(s) = \frac{e^{-0.2s}}{(3s-1)(s-1)}. \tag{19}$$

The control parameters of Raza and Anwar [4] for PID controller design are $K_p = 2.39, K_i = 0.87, K_d = 8.49$. The proposed method is applied for the $G_2(s)$, and control parameters for (G_{c1}) and (G_{c2}) are found to be ($k_1 = 13.5$, $k_2 = 12.3$, $k_3 = 3.5$, $K_D = 70.1$, $\delta = 33.9$) and ($k_1 = 2.96$, $k_2 = 80.4$, $k_3 = 3.5$, $K_D = 97.9$, $\delta = 97.5$).The process model is simulated with a set-point change of magnitude 1 at $t = 0$ at using these controller settings and a disturbance change of two unit step at $t = 25s$. The process output and controller output from the proposed method and the methods of Raza and Anwar [4] and Cho et al. [25] are shown in Figure 8 and Figure 9. The figures shows that the suggested technique performs the best among the methods provided by [4] and [25]. The overshoot is less for the set-point response with the suggested technique. Furthermore, the settling time of the set-point response of the suggested method is faster than that of Raza and Anwar [4] and Cho et al. [25]. Also, excellent LD rejection is observed with the suggested method. The performance indices *IAE, ITAE, ISE,* and *TV* are shown in Table II. Integral errors are lesser for the proposed method as compared to [4] and [25], but the *TV* value is higher for the proposed method. The reason behind higher *TV* is that Raza and Anwar [4] and Cho et al. [25] have used PID controllers, and the suggested method is based on SMC, which has a chattering phenomenon.

To show the robustness of the suggested SMC, perturbation changes of +20% in both the gain and the dead-time i.e. $G_2(s) = \frac{1.2e^{-0.24s}}{(3s-1)(s-1)}$ are considered. The responses for the process output and controller output under these perturbations are shown in Figure 10 and Figure 11. The figures show that the set-point response of the perturbed system by the proposed method is comparable, while the LD rejection of the proposed technique outperforms the methods given by Raza and Anwar [4] and Cho et al. [25]. The performance indices are shown in Table II for evaluation of the suitability of the proposed technique. The integral errors *IAE, ISE,* and *ITAE* of perturbed systems for the set-points are lower in the proposed method.

5 Conclusion

In this work, a 2DOF-SMC design in PCS for unstable SOPDT has been presented. Two separate SMCs—set-point tracking and load disturbance rejection—are developed for PCS. The controller parameters are obtained using the PSO technique by minimizing an objective function consisting of *IAE*. The suggested approach is used to simulate several cases, and the results are effective under different load disturbances and parameter variations. The suggested technique is found to have lower integral errors (*IAE, ISE,* and *ITAE)* and *TV* than the previously reported SMC method. This approach may be extended to unstable processes with zeros and unstable higher-order processes. Future developments of this work could include the analysis of delay-dominant systems, the development of fractional-order IMC and Smith predictor controllers, and the use of complex optimization approaches.

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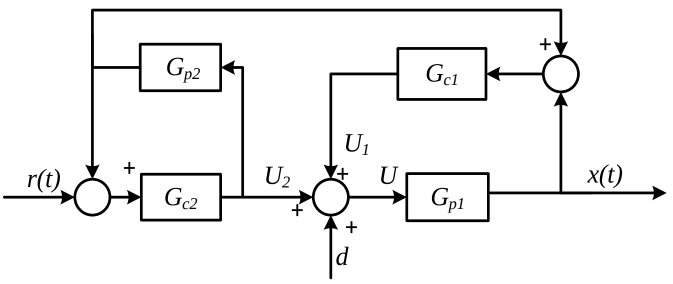


Figure 1. SMC scheme in parallel control structure

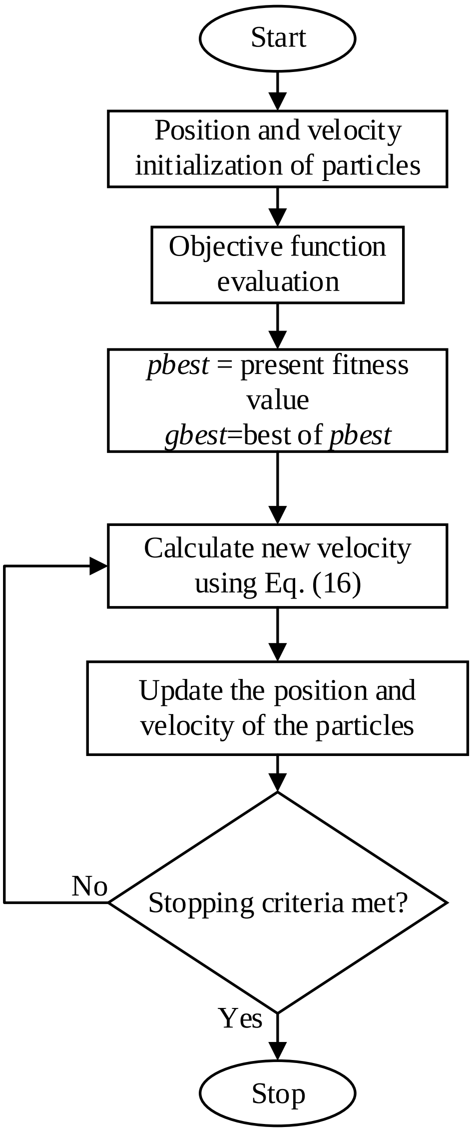


Figure 2. Flowchart of PSO algorithm

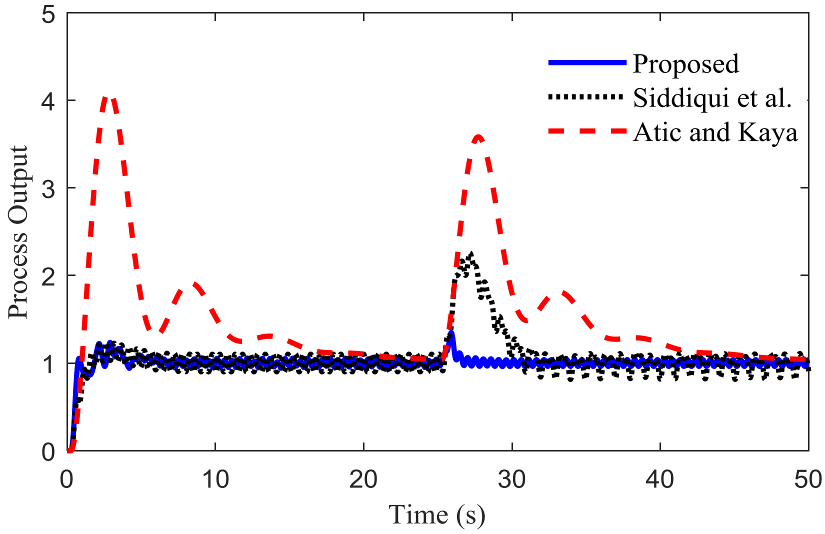
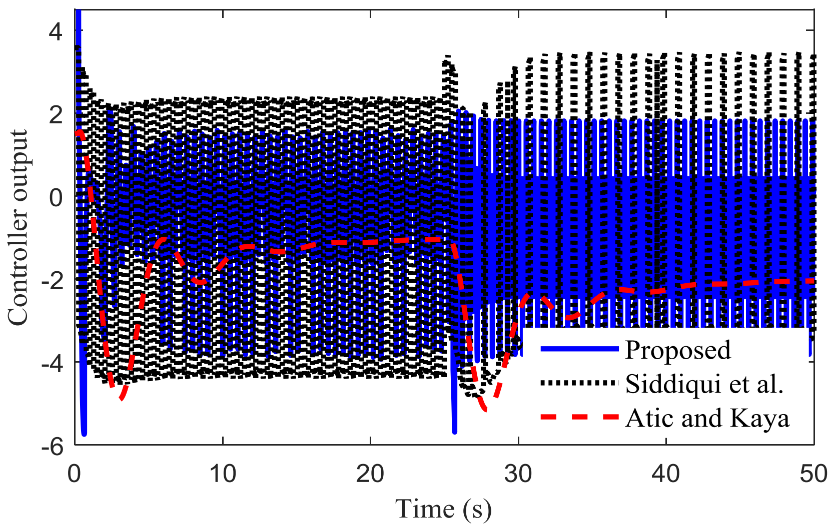


Figure 3. Process output for $G_1(s) = \dfrac{e^{-0.5s}}{(2s-1)(0.5s+1)}$



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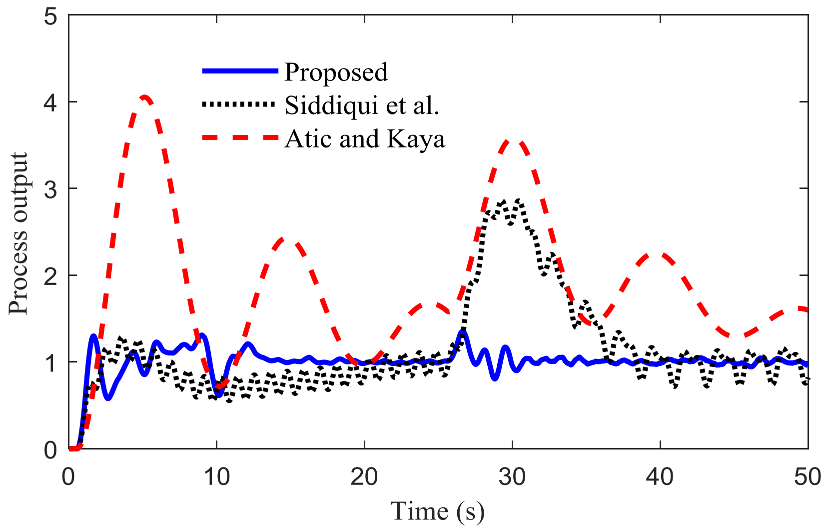


Figure 5. Process output perturbed process $G_1(s)=\dfrac{1.1e^{-0.55s}}{(2.2s-1)(0.5s+1)}$

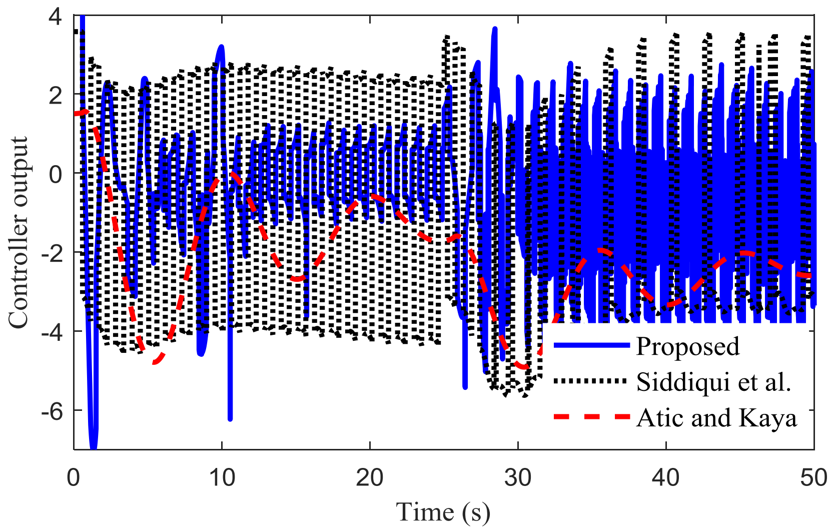
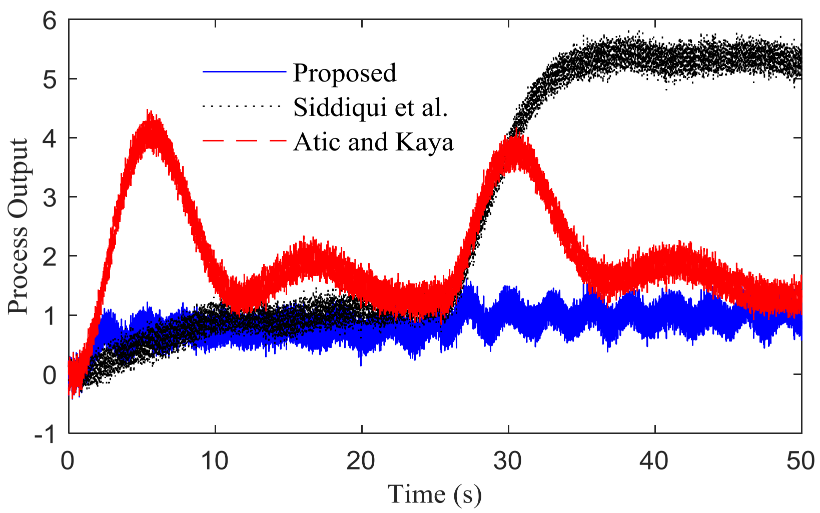


Figure 6. Controller output for perturbed process $G_1(s)=\dfrac{1.1e^{-0.55s}}{(2.2s-1)(0.5s+1)}$



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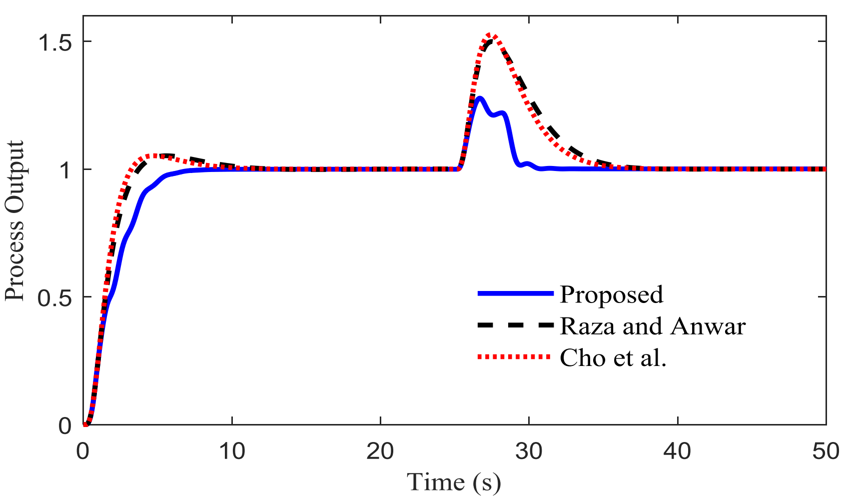

Figure 8. Process output for process $G_2(s)=\dfrac{e^{-0.2s}}{(3s-1)(s-1)}$

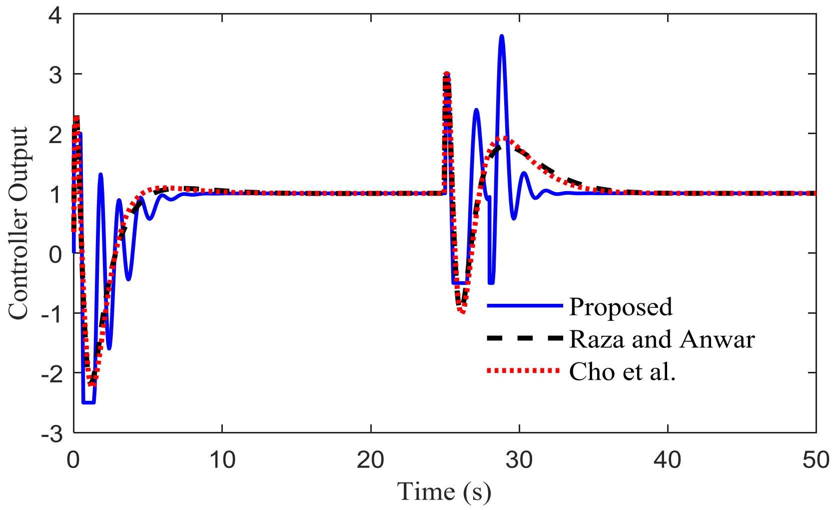

Figure 9. Controller output for process $G_2(s)=\dfrac{e^{-0.2s}}{(3s-1)(s-1)}$

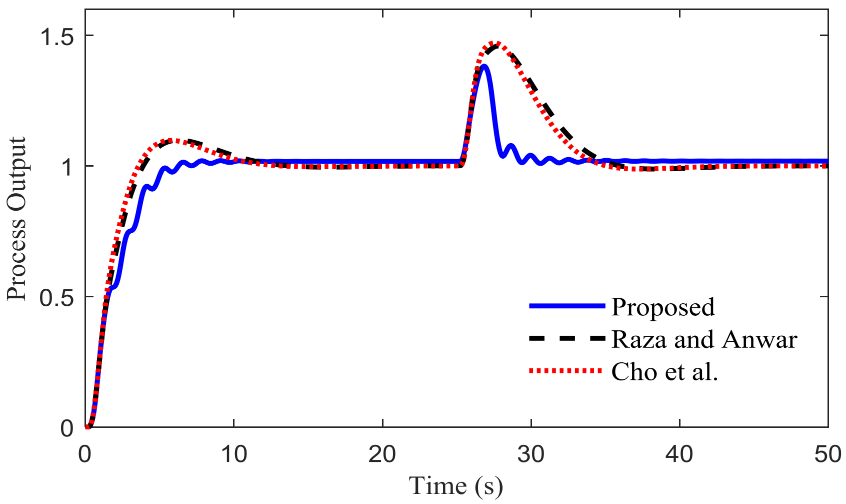

Figure 10. Process output for perturbed process i.e. $G_2(s)=\dfrac{1.2e^{-0.24s}}{(3s-1)(s-1)}$

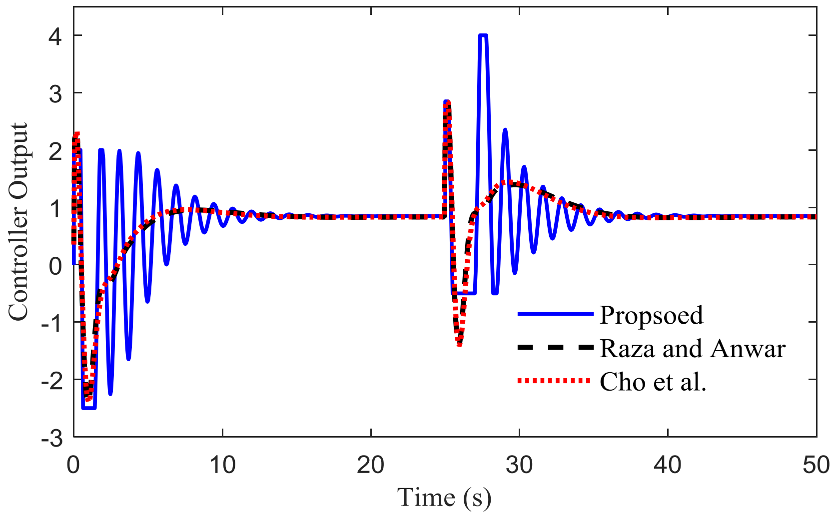

Figure 11. Controller output for perturbed process $G_2(s)=\dfrac{1.2e^{-0.24s}}{(3s-1)(s-1)}$

Table I: An overview of SMC usage in process industry

Literature	**Process model Used**	**Type of sliding surface taken**	**Continuous control law ($u_c(t)$)**	**Technique for obtaining $u_c(t)$**	**Discontinuous control law ($u_d(t)$)**	**Technique for obtaining $u_d(t)$**	**Structure used**
Herrera et al. [12]	Stable FOPDT	PI	$\left(\frac{1}{2k_m(\lambda-\eta)}\right)* \left[\begin{matrix}(-\lambda_m\tau_m t_0+2\tau_m+t_0)\mathrm{Y}^* \\ +(\lambda_m\tau_m t_0 e^*(t)+2\mathrm{Y}^*-2k_m U_{DSMC}(t)\end{matrix}\right]\frac{1}{2}$	From tuning equations obtained for cirtical or overdamped system	$K_D sgn(\sigma)$	From tuning equations obtained for a cirtical or overdamped system	Complex due to use of compensator
Camacho [26]	Stable FOPDT	P	$\left[\frac{x_m^-(t)}{K}\right]$	From tuning equations having dead time, gain and time constant	$K_D \frac{s(t)}{\lvert s(t)\rvert+\delta}$	From tuning equations having dead time, gain and time constant	Complex due to use of Smith predictor (SP)
Camacho and Smith [9]	Stable FOPDT	PID	$\left(\frac{t_0\tau}{K}\right)\left[\frac{X(t)}{t_0\tau}+\lambda_0 e(t)\right]$	Using Nelder-Mead search algorithm	$K_D \frac{s(t)}{\lvert s(t)\rvert+\delta}$	Using Nelder-Mead search algorithm	Complex due to use of Iinoya and Altpeter compensator
Camacho et al. [27]	Stable FOPDT	PI	$\left(\frac{t_0\tau}{K}\right)\left[\frac{X(t)}{t_0\tau}+\lambda_0 e(t)\right]$	Using Nelder-Mead search algorithm	$K_D \frac{s(t)}{\lvert s(t)\rvert+\delta}$	Using Nelder-Mead search algorithm	Complex due to use of IMC
Kaya [10]	Stable FOPDT	PI-PD	$\left(\frac{TL}{Kk_4}\right)\left[\left(\frac{k_4}{TL}-k_2\right)c(t)+\mathrm{k}_2 r(t)\right]$	From the condition of the closed loop system to have an overdamped response	$k \tan h\left(\frac{s(t)}{\Omega}\right)$	From hyperbolic tangent function	Simple
Parte et al. [13]	Stable FOPDT	PID	$\frac{1}{K}X(k)+\frac{(t_0+\tau)^2}{4Kt_0\tau}e(k)$	From tuning equation and fitting technique	$K_D \frac{s(t)}{\lvert s(t)\rvert+\eta}$	From tuning equation and fitting technique	Complex due to use of GPC
Mehta and Kaya [16]	Stable FOPDT	PI	$\left(\frac{1}{k_m}\right)\left[\tau_m\lambda e(t)+y_m^-(t)\right]$	Using PSO optimization technique	$\alpha\lvert S(t)\rvert^{\beta}*$ $sign(S(t))$	Using PSO optimization technique	Complex due to use of SP
Camacho and Rojas [11]	Stable FOPDT and IFOPDT	PID	$\left(\frac{t_0\tau}{K}\right)\left[\frac{X(t)}{t_0\tau}+\lambda_0 e(t)\right]$	Using Nelder-Mead search algorithm	$K_D \frac{s(t)}{\lvert s(t)\rvert+\delta}$	Using Nelder-Mead search algorithm	Simple and complex both considered
Mehta and Rojas [19]	Unstable FOPDT	PI	$\left(\frac{1}{k_m}\right)\left[\tau_m\lambda e(t)+y_m^-(t)\right]$	Using cuckoo search optimization technique	$\alpha\lvert S(t)\rvert^{\beta}*$ $sign(S(t))$	Using cuckoo search optimization technique	Complex due to use SP based SMC
Camacho and Cruz [14]	IFOPDT	PID	$\left(\frac{1}{K}\right)\left[(1-\tau_m\lambda_1)\frac{dX_1(t)}{t_0\tau}+\tau_m\lambda_0 e(t)\right]$	From controllability relationship	$K_D \frac{s(t)}{\lvert s(t)\rvert+\delta}$	Nelder-Mead search algorithm	Complex due to use of SP based SMC
Rojas et al. [17]	Unstable FOPDT	PID	$\left(\frac{t_0\tau}{K}\right)\left[\begin{matrix}\left(\frac{\tau-t_0}{t_0\tau}-\lambda_1\right)\frac{dX(t)}{dt}- \\ \frac{X(t)}{t_0\tau}+\lambda_0 e(t)\end{matrix}\right]$	From tuning equations obtained for a cirtical or overdamped system	$K_D \frac{\sigma(t)}{\lvert\sigma(t)\rvert+\delta}$	Using Nelder-Mead search algorithm	Simple and fixed

Sivarama-krishnan et al. [18]	Unstable FOPDT	PID	$\left(\frac{t_0\tau}{K}\right)\left[-\frac{X(t)}{t_0\tau}+\lambda_0 e(t)\right]$	From tuning equations obtained for an overdamped system	$K_D\frac{s(t)}{\lvert s(t)\rvert+\delta}$	Estimated by minimising ISE using matlab least squares method	Simple
Siddiqui et al. [21]	Unstable SOPDT	PIDD	$\frac{1}{K}\left(-X(t)[c_0+k_2c_3]\right)+\frac{1}{K}k_2c_3R(t)$	Using Root Locus technique	$K_D\frac{s_s(t)}{\lvert s_s(t)\rvert+\delta}$	Using grass hoper optimization	Simple
Cárdenas et al. [22]	Unstable FOPDT	PID	$\left(\frac{\theta_a\tau_a}{K_a}\right)\left[\frac{X(t)}{\theta_a\tau_a}+\lambda_0 e(t)\right]$	Using Nelder-Mead search algorithm	$K_D\frac{s(t)}{\lvert s(t)\rvert+\delta}$	Using Nelder-Mead search algorithm	Simple
Proposed	Unstable SOPDT	PIDD	$\frac{1}{K}\left[\dot{x}(t)(c-ak_1)+x(t)(d-ak_2)+ak_2r(t)\right]$	Using PSO algorithm	$K_D\frac{s(t)}{\lvert s(t)\rvert+\delta}$	Using PSO algorithm	Parallel

Table II: Performance Indices

Method	**Controller Type**	**Nominal System**								**Perturbed System**							
		Set-Point Response				**Load Response**				**Set-Point Response**				**Load Response**			
		IAE	***ISE***	***ITAE***	***TV***	***IAE***	***ISE***	***ITAE***	***TV***	***IAE***	***ISE***	***ITAE***	***TV***	***IAE***	***ISE***	***ITAE***	***TV***
Example 1																	
Proposed	SMC	**2.9**	**1.1**	**41**	**458.4**	**4.0**	**0.4**	**96.6**	**543**	**7.2**	**2.1**	**143.7**	**542**	**7.5**	**1.7**	**187.2**	**627**
Siddiqui et al. [21]	SMC	4.9	1.4	85.8	683.8	9	5	125	372.6	7.6	2.3	134.6	627.8	10.8	6.8	163.9	350
Atic and Kaya [24]	PID	29	45.6	347.6	14.24	26.2	34.7	327.4	8.6	27	40.5	348.6	20.9	25.7	32	356	12.9
Example 2																	
Proposed * $\boldsymbol{F}_\mathbf{a}$	SMC	**0.67**	**0.14**	**1.6**	19.38	**0.07**	**0**	**0.9**	5.9	**1.9**	**0.3**	**28.6**	87.6	**0.08**	**0**	**0.9**	7.7
Raza and Anwar [4] ***** $\boldsymbol{F}_\mathbf{b}$	PID	2.4	0.4	25.2	10.2	1.1	0.2	4.5	4.8	2.2	0.29	23.8	10.3	2.4	0.8	10.3	9.7
Cho et al. [25] ***** $\boldsymbol{F}_\mathbf{c}$	PID	2.5	0.4	23.8	10.3	1.1	0.2	3.8	4.9	2.1	0.26	21.4	10.5	2.3	0.8	8.9	9.7

*Set-point filter used: $F_{\mathrm{a}} = \frac{1}{s+1}, F_{\mathrm{b}} = \frac{1.49s^2 + 2.44s + 1}{9.65s^2 + 2.7s + 1}, F_{\mathrm{c}} == \frac{1.49s^2 + 2.44s + 1}{9.65s^2 + 2.7s + 1}$.