DEVELOPING A CONTINUOUS DYNAMIC MULTI CRITERIA DECISION MAKING MODEL FOR RANKING COMMODITY GROUPS IN IRAN RAILWAYS

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ABSTRACT

Freight transport is a key enabler for the growth of the national industry and the opportunity to leverage Iran’s geographical position for freight transit. The main challenge facing Iran railways is prioritizing commodity groups. Given the dynamic conditions of rail transport in Iran, the use of dynamic multi-criteria decision-making models is inevitable. This study aims to develop a model that prioritizes alternatives based on the desirability they create over a finite future horizon. In all previous studies on dynamic multi-criteria decision-making, the behavior of alternatives with respect to criteria has been extracted periodically. The challenging subject for the implementation of these models is the correct choice of period length in which the information is extracted. Otherwise, this may be associated with the loss of information between periods. Therefore, we decided to develop models, which consider the behavioral changes of alternatives with respect to criteria, continuously. The models contain nine commodity groups as alternatives and "tonnage", "ton-kilometer" and "average revenue per ton-kilometer" as criteria. The findings derived from implementing the model reveal that minerals are poised to attain the highest rank in the future. Furthermore, the subsequent ranks are anticipated to be occupied by Petroleum Products and industrial materials.

Keywords: Continuous Dynamic Multi-Criteria Decision-Making, Freight Transport, Iran Railways, Dynamic Multi-Criteria Decision-Making, Continuous Time, Ranking

1. INTRODUCTION

The Iranian railway network has more than 11,400 km of railway tracks that can be used by freight trains in the form of dedicated or mixed use with passenger services [1]. Iranian freight rail system is located at the crossroad of international freight corridors, connecting Asia to Europe, especially the North-South Industrial Corridor and New Southern Silk Road. The Islamic Republic of Iran Railways (IRIR) is a state-owned company under the supervision of the Ministry of Road and Urban Development, which was founded in 1916. IRIR owned infrastructure, traction and all the wagons; until the majority of wagons were privatized in 2005. Thenceforth, IRIR focused on developing and implementing strategies, marketing, determining tariffs and monitoring rail network.

Recently, rail transportation has attracted considerable attention in Iran; a 37 percent increase in freight tonnage and a 46 percent increase in ton-kilometer over the past ten years have led to the goal of a 30 percent modal share of rail freight transportation in the country's macro policies [1].

In order to achieve this goal, IRIR must determine the desirability of each commodity group by properly analyzing the past and correctly predicting the future. Investing, marketing and providing facilities and discounts can be aimed to attract more commodities that are most desirable. Therefore, the challenge facing IRIR can be considered as a multi-criteria decision-making (MCDM) problem. Since in classical MCDM models, only the information collected at the time of decision-making is considered and since cross-sectional information alone is not decisive to determine the desirability of a commodity group, so we have to look for models that have a glance at the past behavior of alternatives.

Multi-criteria decision-making has its roots in operations research, a field that aims to create mathematical and computational tools to assist decision-makers in their subjective evaluation of performance criteria [2]. Multi-criteria decision-making involves selecting the most desirable alternative from a finite number of available alternatives based on their characteristics with respect to the criteria [3]. In other words, classical MCDM which is also called static MCDM [4] is a method that evaluates the aggregated performance of alternatives considering a set of criteria. The alternatives are subsequently ranked, with those demonstrating the highest aggregate performance being selected [5]. Different MCDM problems called dynamic multi-criteria decision-making (DMCDM) problems, take time into account. This leads to considering the past and future performance of the alternatives [6,7,8,9,10].

As mentioned, the usage of DMCDM models for the Iran railway is inevitable. According to many previous studies, in DMCDM models, the information gathered at different time periods generally influences the final decision [3,10,11,12,13]. However, selecting the appropriate intervals between periods so that the least information is lost, guarantees the accuracy of these models. Furthermore, rail freight transport is performed continuously and is affected by factors such as seasonal changes, exchange rate fluctuations, etc., which do not necessarily coincide; therefore, collecting information at certain points in time may lead to losing too much information. Therefore, previous studies in the field of dynamic multi-criteria decision-making, which either predicted the information in different future periods or considered it as scenarios with different probabilities, do not meet the needs of IRIR.

Given this preamble, we were actuated to develop a dynamic multi-criteria decision-making model that considers both past and the future by examining the historical behavior of alternatives and predicting future in the form of continuous-time functions. The main elements of implementing such a model for the Iran railway include 9 commodity groups as alternatives and "tonnage", "ton-kilometer" and "average revenue per ton-kilometer" as criteria.

The process of extending the model involves a series of fundamental steps. Initially, experts' ideas are utilized to extract relevant criteria and alternatives. Subsequently, historical data on the alternatives is collected. The functional behavior of the alternatives is extracted, taking into account the identified criteria. Then, continuous dynamic simple additive weighting (SAW) and continuous dynamic technique for order of preference by similarity to ideal solution (TOPSIS) are implemented, which will be comprehensively described in the subsequent sections. Lastly, the alternatives are ranked based on the scores derived from the proposed methodologies.

The rest of the paper is structured as follows: background information and related works are introduced in section 2, and then in section 3 we present two continuous dynamic multi-criteria decision-making models, we move on with implementing the models for IRIR in section 4, and conclude with section 5.

2. RELATED WORKS

Dynamic multi-criteria decision-making was first introduced by Kornbluth [14]. He discussed time-dependent weights in MCDM and presented an analysis of empirical data associated with dynamic decision-making as a game.

Since then, there has been growing interest in considering the time element in MCDM. Some studies have examined DMCDM, conceptually. For example, Vo et al. [15] presented a conceptual incorporation of MCDM and system dynamics modeling for urban infrastructure. Pais and Ribeiro [16] described in detail the architecture of dealing with Dynamic MADM problems and made some suggestions to address the uncertainty and importance of the criteria in these problems. Yu and Chen [17] interpreted the mechanism of human brain behavior as DMCDM and stated that not only may the elements of decision-making change over time, but they may also interact.

The other group of studies considered dynamics through the use of fuzzy concepts in multi-criteria decision-making models [3,13,18-30]. These studies indicated the elements of the decision matrix as gray, triangular, trapezoidal or other types of fuzzy numbers and by introducing operators, they aggregated decision matrices of different periods.

In 2011, an article was published by Campanella and Ribeiro [7] that formed the basis of several subsequent articles. While the necessity of getting feedback from step to step was emphasized by defining a historical set in this study, an algorithm was designed that determines the ranking of alternatives based on the rank obtained from the classical MCDM in the current period and the ranking of the previous period. A few months later, this model was implemented for the problem of supplier selection [8]. Zulueta et al. [10] proposed a new discrete-time variable index to handle differences in the temporal behavior of alternatives, which are not discriminated against in Campanella and Ribeiro's dynamic approach.

Jassbi et al. [31] investigated an MCDM model for group decision-making using Campanella and Ribeiro's framework. Then, in the same year, they [32] extended Campanella and Ribeiro's framework to consider future knowledge. The extended model was implemented for data from a real car manufacturer company.

Hashemkhani Zolfani et al. [33] applied the impact of future knowledge on current decision-making by defining scenarios. In this way, by considering different scenarios of the future (which allows the set of criteria and alternatives to be different), they determined the most effective criteria and the most applicable alternatives in the present. As well, they introduced unpredictable scenarios as wild-cards and presented four conceptual steps to cope with them.

Whereas Hashemkhani Zolfani et al. [34] stated that the methodology proposed by Jassbi et al. is only applicable for the near future, they presented a new concept and approach in the MADM field called perspective multi-attribute decision-making. Taking into account the limiters that may occur in the future; they formed the decision matrix and implemented the decision model based on the probabilities of each limiter.

Watrobski et al. [35] used the framework introduced by Campanella and Ribeiro in the field of media marketing management. The framework has also been developed by Liu et al. [36] for bipolar linguistics term sets.

Benitez et al. [37] proposed a dynamic decision model based on AHP for maintenance planning, where instead of extracting weights from experts' opinions, they are determined stochastically. Thong et al. [38] suggested a dynamic TOPSIS model in dynamic interval-valued neutrosophic sets. This model handled historical data including the change of criteria, alternatives and decision-makers during periods based on Campanella and Ribeiro's framework.

The combination of fuzzy concepts and Campanella and Ribeiro's framework can be seen in Tao et al. [3] article. In their two-stage method, in addition to ranking the alternatives using the alternative queuing method, they also calculate the reliability of each decision-maker.

While introducing the concept of internal patterns in the dynamic multi-criteria decision-making model, Yao et al. [6] have included two indicators of fluctuation and tendency as two patterns influencing decision-making over time, and then a model to consolidate the dynamic multi-criteria decision-making framework of Campanella and Ribeiro and the indicators fluctuation and tendency have been presented. They believe that the new model works better than the framework that does not consider internal patterns.

To provide a concise and comprehensive summary of the literature review, we have compiled the findings from relevant studies into Table 1.

From previous studies, it is observable that the current research is the first attempt to investigate continuous dynamic multi-criteria decision-making. The present research represents the pioneering exploration of continuous dynamic multi-criteria decision-making, as prior studies have not delved into this area. In this study, two innovative models are developed, taking into account the time-dependent nature of the alternatives' behavior with respect to criteria. These models are constructed based on the SAW method, known for its straightforwardness in MCDM, and the TOPSIS, as one of the extensively utilized multi-criteria decision analysis approaches.

3. DYNAMIC MULTI-CRITERIA DECISION-MAKING BASED ON CONTINUOUS CHANGES OVER THE TIME

As reviewed in section 2, the previous studies on consideration of dynamics in multi-criteria decision-making problems work as shown in Figure 1. The ultimate goal is to select the alternative that is most desirable in the present, while feedback from the past and future periods has influenced the current ranking.

Where $t_{ps},...,t_{P1}$ indicate past time periods, t_c represents current time (when the decision is being made) and $t_{F1},...,t_{Fr}$ specify future time periods.

This approach enables decision-makers to identify the most desirable alternative in the present while taking into account information from specific points in the past or future. However, this approach overlooks the behavior of alternatives between these selected points, which can result in a loss of crucial information. To reduce this issue, it is essential to minimize the temporal gap between these points, effectively reducing the duration of each period. In other words, to avoid missing important information in dynamic scenarios, it becomes vital to consider the problem continuously, ensuring a comprehensive assessment of alternative behavior over time.

In contrast, the approach proposed in this paper builds upon Figure 2, allowing for the selection of alternatives that offer the highest desirability not only in the present but also over a specific time in the future. Notably, the impact of past information is also taken into account when past information is used to estimate the future behavior of alternatives.

To further illustrate the distinction between the two approaches, an example can be examined. For instance, consider a scenario where a company adjusts the prices of its products only when a new product is launched. In this case, the first approach adequately captures the behavior of alternatives in terms of price since the period is long enough to accommodate the changes.

However, if the prices of alternatives, such as digital currencies, experience daily fluctuations or even multiple changes within a single day, the second approach is necessary to effectively account for the continuous and frequent price changes.

In this paper, the second approach is expanded upon by introducing the development of two models. These models serve to enhance the application of the continuous dynamic multi-criteria decision-making framework. The basis of the two models discussed below is the same and the difference lies in the methodologies. The first model is based on the SAW method and the second model is developed using the TOPSIS concept.

The preliminary stage of these models is to predict the future behavior of alternatives with respect to criteria as time-dependent functions, based on past information.

As with classic MCDM models, $C=\{C_1,C_2,...,C_n\}$ indicates the criteria and the vector $W=\{w_1,w_2,...,w_n\}$ contains their weights. Therefore, the set of alternatives is defined as $A=\{A_1,A_2,...,A_m\}$. So, the dynamic decision matrix will be as follows:

$$D=\begin{bmatrix} f_{11}(t) & f_{12}(t) & \cdots & f_{1n}(t) \\ f_{21}(t) & f_{22}(t) & \cdots & f_{2n}(t) \\ \vdots & \vdots & \vdots & \vdots \\ f_{m1}(t) & f_{m2}(t) & \cdots & f_{mn}(t) \end{bmatrix}$$

Where $f_{ij}(t)$ indicates the dynamic behavior of alternative A_i with respect to criterion C_j.

The decision horizon is also assumed as $[t_C,t_F]$. In other words, we are deciding at time t_C and the goal is to rank the alternatives based on the desirability they create by t_F.

3.1. Continuous Dynamic SAW

The first model is to apply the concept of SAW in dynamic mode. The model includes the following steps:

Step 1- Calculating the area below the graph of each function $f_{ij}(t)$.

$$S_{ij} = \int_{t_C}^{t_F} f_{ij}(t)\,dt \qquad \forall\, i = 1,\dots,m \quad j = 1,\dots,n$$

(1)

S_{ij} represents the summation of the values that the alternative A_i satisfies the criterion C_j during the decision horizon.

Step 2 - Normalizing the areas

The best value and the worst value of the criterion C_j during the decision horizon are determined as follows:

$$f_j^{best} = \max_i \{ f_{ij}(t)\ ,\ t_C \le t \le t_F \} \qquad \textit{for positive criteria} \qquad (2)$$

$$f_k^{best} = \min_i \{ f_{ik}(t)\ ,\ t_C \le t \le t_F \} \qquad \textit{for negative criteria} \qquad (3)$$

So, the normalization step is performed according to equations (4) and (5):

$$S_{N_{ij}}^{+} = \frac{S_{ij}}{f_j^{best}(t_F - t_C)} \qquad \textit{for positive criteria} \qquad (4)$$

$$S_{N_{ik}}^{-} = \frac{f_k^{best}(t_F - t_C)}{S_{ik}} \qquad \textit{for negative criteria} \qquad (5)$$

Step 3 – The overall desirability of the alternatives is computed by equation (6).

$$\overline{S}_i = \sum_j w_j \times S_{N_{ij}}^{+} + \sum_k w_k \times S_{N_{ik}}^{-} \qquad (6)$$

The final ranking of alternatives is obtained by sorting the results of the previous step. The most desirable alternative is the one which yields the largest $\overline{S_i}$.

$$A^* = \{ A_i \mid \max_i(\overline{S}_i) \} \qquad (7)$$

3.2. Continuous Dynamic TOPSIS

The second model is based on the concept of the TOPSIS method. Similar to the best value of criteria, the worst values are also specified.

$$f_j^{worst} = \min_i \{ f_{ij}(t) \ , \ t_C \le t \le t_F \} \qquad \textit{for positive criteria} \tag{8}$$

$$f_k^{worst} = \max_i \{ f_{ik}(t) \ , \ t_C \le t \le t_F \} \qquad \textit{for negative criteria}$$
(9)

Continuous dynamic TOPSIS is performed as follows.

Step 1- Calculating the area between the behavioral functions and the best and the worst values for each criterion. This step, which is in line with calculating the distance of the alternatives from the best and worst solutions, in the TOPSIS, is obtained according to equations (10) to (13).

$$D_{ij}^{+} = \int_{t_C}^{t_F} [\, f_j^{best} - f_{ij}(t)] \, dt \qquad \textit{for positive criteria}$$
(10)

$$D_{ik}^{+} = \int_{t_C}^{t_F} [\, f_{ik}(t) - f_k^{best} \,] \, dt \qquad \textit{for negative criteria}$$
(11)

$$D_{ij}^{-} = \int_{t_C}^{t_F} [\, f_{ij}(t) - f_j^{worst} \,] \, dt \qquad \textit{for positive criteria}$$
(12)

$$D_{ik}^{-} = \int_{t_C}^{t_F} [\, f_k^{worst} - f_{ik}(t)] \, dt \qquad \textit{for negative criteria}$$
(13)

Step 2- This step includes normalizing the differences using equations (14) and (15).

$$D_{N_{ij}}^{+} = \frac{D_{ij}^{+}}{\sqrt{\sum_i (D_{ij}^{+})^2}} \tag{14}$$

$$D_{N_{ij}}^{-} = \frac{D_{ij}^{-}}{\sqrt{\sum_i (D_{ij}^{-})^2}} \tag{15}$$

Step 3- In this step, the weights of the criteria are involved and used as a factor to aggregate the differences.

$$\overline{D}_i^{+} = \sum_j w_j \times D_{N_{ij}}^{+} \tag{16}$$

$$\overline{D}_i^{-} = \sum_j w_j \times D_{N_{ij}}^{-} \tag{17}$$

Step 4- Just as in the TOPSIS method, an index is used to determine the proximity of an alternative to the best and avoidance of the worst solution, in this step the alternatives are ranked based on $\overline{D}_i$. The alternative with the largest value is the most desirable one.

$$\overline{D}_i = \frac{\overline{D}_i^-}{\overline{D}_i^+ + \overline{D}_i^-}$$
(18)

$$A^* = \{A_i \,|\, \max_i(\overline{D}_i)\}$$
(19)

3.3. Linear mode

To better illustrate the introduced models, this section considers the simplest case of behavioral functions. Assuming behavioral linear functions as $a_{ij}(t)+b_{ij}$, the decision matrix will be as follows.

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} a_{11}t+b_{11} & a_{12}t+b_{12} & \cdots & a_{1n}t+b_{1n} \\ a_{21}t+b_{21} & a_{22}t+b_{22} & \cdots & a_{2n}t+b_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1}t+b_{m1} & a_{m2}t+b_{m2} & \cdots & a_{mn}t+b_{mn} \end{bmatrix} \end{array}$$

To simplify further the problem, all the criteria are assumed as positive. Therefore, the steps of the Continuous Dynamic SAW model are executed as follows for linear mode.

$$S_{ij} = \int_{t_C}^{t_F} (a_{ij}t+b_{ij})\,dt = \frac{1}{2}a_{ij}(t_F^2 - t_C^2) + b_{ij}(t_F - t_C)$$
(20)

$$S_{N_{ij}}^+ = \frac{1}{f_j^{best}(t_F - t_C)}\left[\frac{1}{2}a_{ij}(t_F^2 - t_C^2) + b_{ij}(t_F - t_C)\right] = \frac{1}{f_j^{best}}\left[\frac{1}{2}a_{ij}(t_F + t_C) + b_{ij}\right]$$
(21)

$$\overline{S}_i = \sum_j \frac{w_j}{f_j^{best}}\left[\frac{1}{2}a_{ij}(t_F + t_C) + b_{ij}\right]$$
(22)

$$A^* = \{A_i \mid \max_i(\overline{S}_i)\}$$
(23)

4. IMPLEMENTING THE DMCDM MODELS FOR IRAN RAILWAYS

In this section, the previously proposed models are implemented based on the actual data of Iran Railways from 2011 to 2020.

In order to form the decision model, the main commodity groups are considered as alternatives: petroleum products, minerals, chemicals, cereals and agricultural products, industrial materials, oilseeds and edible oils, metals, sulfur and vehicles.

Moreover, the decision criteria are defined and weighed based on the insights of a group of industry experts in the rail sector. These experts, consist of a 15-member group of middle managers from the Iranian Railways in the financial and commercial departments. “tonnage”, “ton-kilometers” and “average revenue per ton-kilometer", which are all three of positive type, are included in the model as criteria with weights of 0.2, 0.3, and 0.5, respectively.

Table 2 shows the actual data of Iran Railways for the last ten years (2011 to 2020). Using this data, the behavioral functions of the alternatives with respect to criteria are identified, and then each of the two models is implemented to rank the alternatives over the next 5-year horizon (2021 to 2025).

4.1. Estimating behavioral functions

Predicting the future behavior of alternatives with respect to the criteria requires fitting functions to be specified. Discussing the details of determining appropriate fitting functions is not one of the objectives of this study and many related studies in this field can be referred to, however, for this purpose, we use the curve fitting toolbox to identify the appropriate functions based on data shown in Table 2.

First or second-degree polynomial functions are acceptable if the correlation coefficient values are more than +0.8 or less than -0.8. For example, the behavior of minerals with respect to the ton-kilometer criterion could be fitted by a linear function as Figure 3. Also, the behavior of “metals” with respect to the “tonnage”, could be interpreted by a quadratic polynomial function which is illustrated in Figure 4.

The fitting function of the above two diagrams was obtained as equations 24 and 25, respectively.

$$f(t) = 695t + 9660$$
(24)

$$f(t) = 54.2t^2 - 349.5t + 1855$$
(25)

Investigating the behavior of "Industrial Materials" with respect to "ton-kilometer" over ten years indicates the inefficiency of the linear polynomial function to interpret this seasonal relationship. Therefore, to fit the appropriate function, the Fourier series is used. The fitting curve and function are obtained as Figure 5 and equation (26).

$$f(t) = 853.8 - 140.7\times\cos(2.17t) + 89.22\times\sin(2.17t) - 39.34\times\cos(4.34t) + 225.4\times\sin(4.34t) \quad (26)$$

Table 3 summarizes the behavioral functions of alternatives that are needed to advance the models.

4.2. Implementing the models

After determining the behavioral functions, now we are looking to implement the models proposed in section 3. Considering the functions of Table 3, we have a decision matrix as follows.

$$\begin{array}{r c} & \begin{array}{ccc} \text{Tonnage} & \text{Ton-Kilometer} & \text{Revenue} \end{array} \\ \begin{array}{r} \textit{Petroleum Products} \\ \textit{Minerals} \\ \textit{Chemicals} \\ \textit{Cereal}\ldots \\ \textit{Industrial Materials} \\ \textit{Oil Seeds}\ldots \\ \textit{Metals} \\ \textit{Sulfur} \\ \textit{Vehicles} \end{array} & \begin{bmatrix} f_{11}(t) & f_{12}(t) & f_{13}(t) \\ f_{21}(t) & f_{22}(t) & f_{23}(t) \\ f_{31}(t) & f_{32}(t) & f_{33}(t) \\ f_{41}(t) & f_{42}(t) & f_{43}(t) \\ f_{51}(t) & f_{52}(t) & f_{53}(t) \\ f_{61}(t) & f_{62}(t) & f_{63}(t) \\ f_{71}(t) & f_{72}(t) & f_{73}(t) \\ f_{81}(t) & f_{82}(t) & f_{83}(t) \\ f_{91}(t) & f_{92}(t) & f_{93}(t) \end{bmatrix} \end{array}$$

The following elaborates on the steps to implement both models.

4.2.1. Continuous Dynamic SAW

Step 1- Calculating the area below the graph of the functions which represents the value that each alternative provides in relation to each criterion within [2021,2025].

$$S = \begin{array}{r c} \begin{array}{r} \textit{Petroleum Products} \\ \textit{Minerals} \\ \textit{Chemicals} \\ \textit{Cereal}\ldots \\ \textit{Industrial Materials} \\ \textit{Oil Seeds}\ldots \\ \textit{Metals} \\ \textit{Sulfur} \\ \textit{Vehicles} \end{array} & \begin{bmatrix} 13447.94 & 11054.37 & 2962.84 \\ 171658.02 & 91792.95 & 1865.23 \\ 358.91 & 665.56 & 1733.27 \\ 8985.17 & 9237.58 & 834.02 \\ 10025.75 & 4203.64 & 2065.51 \\ 2632.65 & 2860.44 & 1155.06 \\ 30340.87 & 12333.61 & 1616.72 \\ 2121.59 & 3267.44 & 1057.79 \\ 285.94 & 531.06 & 1830.24 \end{bmatrix} \end{array}$$

Step 2 – To normalize the areas, the best values of the criteria that will be provided by the alternatives during [2021,2025] are calculated.

$$f_j^{best} = \max_i \{ f_{ij}(t) \ \ , \ \ 2021 \le t \le 2025 \} = 41978$$
(27)

Therefore, we have:

$$F^{best} = \{41978, 20085, 690.10)$$

Accordingly, given that all three criteria are positive, the S_N matrix will be as follows.

$$S_N = \begin{array}{r} \textit{Petroleum Products} \\ \textit{Minerals} \\ \textit{Chemicals} \\ \textit{Cereal}\ldots \\ \textit{Industrial Materials} \\ \textit{Oil Seeds}\ldots \\ \textit{Metals} \\ \textit{Sulfur} \\ \textit{Vehicles} \end{array} \begin{bmatrix} 0.0641 & 0.1101 & 0.8587 \\ 0.8178 & 0.9140 & 0.5406 \\ 0.0017 & 0.0066 & 0.5023 \\ 0.0428 & 0.0920 & 0.2417 \\ 0.0478 & 0.0419 & 0.5986 \\ 0.0125 & 0.0285 & 0.3347 \\ 0.1446 & 0.1228 & 0.4685 \\ 0.0101 & 0.0325 & 0.3066 \\ 0.0014 & 0.0053 & 0.5304 \end{bmatrix}$$

Step 3 – Calculating the overall desirability by involving the weights of criteria leads to the $\overline{S}$ matrix.

$$\overline{S} = \begin{array}{r} \textit{Petroleum Products} \\ \textit{Minerals} \\ \textit{Chemicals} \\ \textit{Cereal}\ldots \\ \textit{Industrial Materials} \\ \textit{Oil Seeds}\ldots \\ \textit{Metals} \\ \textit{Sulfur} \\ \textit{Vehicles} \end{array} \begin{bmatrix} 0.4752 \\ 0.7081 \\ 0.2535 \\ 0.1570 \\ 0.3214 \\ 0.1784 \\ 0.3000 \\ 0.1651 \\ 0.2671 \end{bmatrix}$$

Based on $\overline{S}$, the most desirability during 2021 to 2025 will be created by the following commodity groups, respectively. Minerals, petroleum products, industrial materials, metals, vehicles, chemicals, oil seeds and edible oils, sulfur, cereal and agricultural products.

4.2.2. Continuous Dynamic TOPSIS

To implement the second model, it is necessary to determine the worst values of the criteria. Like the best values, the worst values are obtained according to the F^{worst} vector.

$$F^{worst} = \{21.49, 48.91, 126.36\}$$

Step 1- The area between the behavioral functions and the best and the worst values can be seen in the D^{+} matrix and D^{-} matrix, respectively.

$$D^{+} = \begin{array}{r} Petroleum\ Products \\ Minerals \\ Chemicals \\ Cereal\ldots \\ Industrial\ Materials \\ Oil\ Seeds\ldots \\ Metals \\ Sulfur \\ Vehicles \end{array} \begin{bmatrix} 196442.06 & 89370.63 & 487.69 \\ 38231.98 & 8632.05 & 1585.29 \\ 209531.09 & 99759.44 & 1717.25 \\ 207768.41 & 97157.56 & 2392.74 \\ 199864.25 & 96221.36 & 1385.02 \\ 207257.35 & 97564.56 & 2295.47 \\ 179549.13 & 88091.39 & 1833.80 \\ 200904.83 & 91187.42 & 2616.50 \\ 209604.06 & 99893.94 & 1620.28 \end{bmatrix}$$

$$D^{-} = \begin{array}{r} Petroleum\ Products \\ Minerals \\ Chemicals \\ Cereal\ldots \\ Industrial\ Materials \\ Oil\ Seeds\ldots \\ Metals \\ Sulfur \\ Vehicles \end{array} \begin{bmatrix} 13340.49 & 10809.80 & 2331.02 \\ 171550.57 & 91548.38 & 1233.41 \\ 251.45 & 420.99 & 1101.45 \\ 8877.72 & 8993.01 & 202.20 \\ 9918.29 & 3959.07 & 1433.69 \\ 2525.19 & 2615.88 & 523.24 \\ 30233.42 & 12089.04 & 984.90 \\ 2014.14 & 3022.88 & 425.97 \\ 178.48 & 286.49 & 1198.42 \end{bmatrix}$$

Step 2- This step involves normalizing the areas and the results are given in the D_N^{+} and D_N^{-} matrixes.

$$D_N^{+} = \begin{array}{r} Petroleum\ Products \\ Minerals \\ Chemicals \\ Cereal\ldots \\ Industrial\ Materials \\ Oil\ Seeds\ldots \\ Metals \\ Sulfur \\ Vehicles \end{array} \begin{bmatrix} 0.3438 & 0.3324 & 0.0870 \\ 0.0669 & 0.0321 & 0.2827 \\ 0.3667 & 0.3710 & 0.3062 \\ 0.3636 & 0.3614 & 0.4267 \\ 0.3497 & 0.3579 & 0.2470 \\ 0.3627 & 0.3629 & 0.4093 \\ 0.3142 & 0.3277 & 0.3270 \\ 0.3516 & 0.3392 & 0.4666 \\ 0.3668 & 0.3715 & 0.2889 \end{bmatrix}$$

$$D_N^- = \begin{array}{r} \textit{Petroleum Products} \\ \textit{Minerals} \\ \textit{Chemicals} \\ \textit{Cereal} \ldots \\ \textit{Industrial Materials} \\ \textit{Oil Seeds} \ldots \\ \textit{Metals} \\ \textit{Sulfur} \\ \textit{Vehicles} \end{array} \begin{bmatrix} 0.0761 & 0.1155 & 0.6434 \\ 0.9789 & 0.9783 & 0.3404 \\ 0.0014 & 0.0045 & 0.3040 \\ 0.0115 & 0.0323 & 0.1176 \\ 0.0566 & 0.0423 & 0.3957 \\ 0.0144 & 0.0280 & 0.1444 \\ 0.1725 & 0.1292 & 0.2718 \\ 0.0507 & 0.0961 & 0.0558 \\ 0.0010 & 0.0031 & 0.3308 \end{bmatrix}$$

Step 3- Using the weights of the criteria for aggregating normalized distances obtains the $\overline{D}^+$ and $\overline{D}^-$ vectors.

$$\overline{D}^+ = \begin{array}{r} \textit{Petroleum Products} \\ \textit{Minerals} \\ \textit{Chemicals} \\ \textit{Cereal} \ldots \\ \textit{Industrial Materials} \\ \textit{Oil Seeds} \ldots \\ \textit{Metals} \\ \textit{Sulfur} \\ \textit{Vehicles} \end{array} \begin{bmatrix} 0.2120 \\ 0.1644 \\ 0.3378 \\ 0.3945 \\ 0.3008 \\ 0.3861 \\ 0.3246 \\ 0.4045 \\ 0.3293 \end{bmatrix} \qquad \overline{D}^- = \begin{array}{r} \textit{Petroleum Products} \\ \textit{Minerals} \\ \textit{Chemicals} \\ \textit{Cereal} \ldots \\ \textit{Industrial Materials} \\ \textit{Oil Seeds} \ldots \\ \textit{Metals} \\ \textit{Sulfur} \\ \textit{Vehicles} \end{array} \begin{bmatrix} 0.3716 \\ 0.6595 \\ 0.1536 \\ 0.0708 \\ 0.2219 \\ 0.0835 \\ 0.2092 \\ 0.0669 \\ 0.1665 \end{bmatrix}$$

Step 4- The final ranking of the commodity groups is attained by calculating the vector $\overline{D}$ as follows.

$$\overline{D} = \begin{array}{r} \textit{Petroleum Products} \\ \textit{Minerals} \\ \textit{Chemicals} \\ \textit{Cereal} \ldots \\ \textit{Industrial Materials} \\ \textit{Oil Seeds} \ldots \\ \textit{Metals} \\ \textit{Sulfur} \\ \textit{Vehicles} \end{array} \begin{bmatrix} 0.6368 \\ 0.8005 \\ 0.3127 \\ 0.1521 \\ 0.4245 \\ 0.1778 \\ 0.3919 \\ 0.1416 \\ 0.3358 \end{bmatrix}$$

Therefore, considering the past 10 years' data, minerals create the most desirability over 2021 to 2025. The next ranks belong to petroleum products, industrial materials, metals, vehicles, chemicals, oil seeds and edible oils, cereal and agricultural products and sulfur.

5. CONCLUSION

Multi-criteria decision-making is one of the widely used tools to select and rank alternatives. This tool can be dynamically upgraded in changeable situations. What has already been seen in previous studies is the consideration of dynamics, periodically.

However, this paper presents a novel approach by introducing two models based on the concepts of SAW and TOPSIS to address dynamic multi-criteria decision-making problems in a continuous manner.

Unlike previous research that primarily considers dynamics periodically, our models incorporate a continuous perspective by estimating the future behavior of alternatives based on their past performance. The contribution of this article lies in providing a continuous approach to dynamic multi-criteria decision-making, offering insights into the ranking of commodity groups within the context of Iranian Railways.

The models are implemented for the Iran railways, using the actual historical data for the past 10 years. The goal is to rank the commodity groups transported by Iranian Railways so that according to the extracted criteria, they will be most desirable during the years 2021 to 2025.

The results obtained from the models are largely consistent. Accordingly, the minerals, due to the high tonnage and ton-kilometer statistics in previous years, will have the first rank in the coming years. One of the noteworthy points is the position of the “vehicle” commodity group, which is obtained due to its behavior toward the income criterion.

Future research could focus on refining the models by incorporating additional criteria or considering other dynamic factors that might affect the desirability of alternatives. Furthermore, exploring the application of these models in different industries or sectors could provide valuable insights and extend the generalizability of the findings. Additionally, investigating the potential integration of other decision-making techniques or advanced optimization algorithms may enhance the accuracy and efficiency of the models in real-world scenarios. Also the

Overall, this study serves as a foundation for continuous dynamic multi-criteria decision-making and offers practical implications for decision-makers in the transportation domain.

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List of Captions

Figures and Tables

Table 1) Summary of Literature Review Studies

Category	Approach/Methodology	Practical Cases	Representative Articles
Conceptual	Time-dependent weights in MCDM as a game		[14]
	Incorporation of MCDM and system dynamics modeling	urban infrastructure	[15]
	Describing architecture of dealing with DMADM		[16]
	Interpreting the mechanism of human brain behavior as DMCDM		[17]
Fuzzy Approach	Combining the grey number and Minkowski distance function to apply TOPSIS	subcontractor selection	[3]
	Using intuitionistic fuzzy variable and two aggregation operators: DIFWA and UDIFWA	prioritizing the agroecological regions in China	[13]
	Using interval-valued picture fuzzy number in decision matrix Applying IVPFGWHM operator	selecting logistics company	[19]
	Introducing the grey incidence analysis and power weight Heronian aggregation operator	manufacturing industry	[20]
	Combining Delphi method and fuzzy set theory	development of Hendourabi Island (Iran)	[21]
	Using generalized trapezoidal fuzzy numbers in group decision making	supplier segmentation	[22]
	Using fuzzy entropy and fuzzy programming based on fuzzy programming	ranking the alternative locations for future nuclear power plants in Iran	[23]
	Provide preferences by using multi-granular fuzzy linguistic modelling	buying smart phone	[24]
	Using fuzzy AHP	rank the risks of a tunneling project	[25]
	Using fuzzy AHP to determine the weights of criteria and fuzzy TOPSIS to rank alternatives	ranking diets based on individual characteristics	[26]
	Using orthopair fuzzy sets in decision matrix	software selection problems	[27]
	Using triangular fuzzy numbers in decision matrix and TIFN-WAA operator	evaluating the options for investment	[28]
	Using intuitive fuzzy numbers and implementing the EDAS multi-criteria decision-making model	choosing an action plan during floods in China	[29]
	Using the Pythagorean fuzzy decision-making model	Determining the priority of people in COVID19 vaccine distribution	[30]

Category	Approach/Methodology	Practical Cases	Representative Articles
Dynamic Periodic MCDM	Dealing with decision making in time periods through introducing the history function Ranking alternatives by combining classic MCDM in each period and history function	picking a site for landing an helicopter	[7]
	A case study based on the proposed method in [6]	supplier selection	[8]
	Ranking alternatives with same score based on [6]	supplier selection	[10]
	Extending the framework proposed in [6] for group decision making problems	choosing a hotel	[31]
	Adding the impact of future predictions on framework proposed in [6]	supplier selection	[32]
	Considering multiple scenarios of the future for implementing the model proposed in [31]	evaluating the best choices to expand the future study plan of a university	[33]
	Introducing perspective multi criteria decision making		[34]
	A case study based on the proposed method in [6]	media marketing management	[35]
	Extending the framework proposed in [6] for bipolar linguistics term sets	supplier selection	[36]
	Proposing a dynamic AHP with stochastic weights	maintenance planning	[37]
	Proposing a dynamic TOPSIS model in dynamic interval-valued neutrosophic sets	ranking students of a university in Vietnam.	[38]

Table 3) Behavioral Functions of Alternatives with Respect to Criteria

Alternatives	Behavioral function with respect to "Tonnage"
Petroleum Products	$f_{11}(t) = 2838 + 139.4\cos(0.7065t) - 272.7\sin(0.7065t) + 271.9\cos(1.413t) + 111.5\sin(1.413t)$
Minerals	$f_{21}(t) = 143.1t^2 - 751.9t + 21050$
Chemicals	$f_{31}(t) = 56.96\exp(-((t-7.453)/3.012)^2) + 136.3\exp(-((t-1.02)/1.154)^2) + 158.1\exp(-((t-3.375)/0.8905)^2)$
Cereal and Agricultural Products	$f_{41}(t) = 19.36t^2 - 131.1t + 370.3$
Industrial Materials	$f_{51}(t) = 31.32t^2 - 507.5t + 4219$
Oil Seeds and Edible Oils	$f_{61}(t) = 533.3 - 4.62\cos(0.462t) - 40.68\sin(0.462t) - 125.8\cos(0.924t) - 8.386\sin(0.924t)$
Metal	$f_{71}(t) = 54.2t^2 - 349.5t + 1855$
Sulfur	$f_{81}(t) = 154.1\exp(-((t-6.862)/1.134)^2 + (2.42e+10)\exp(-((t-1079)/252.4)^2)$
Vehicles	$f_{91}(t) = 209.9 - 10.02\cos(0.306t) - 196.3\sin(0.306t) - 121\cos(0.612t) - 18.81\sin(0.612t)$
Alternatives	**Behavioral function with respect to "Ton-Kilometer"**
Petroleum Products	$f_{12}(t) = 2310 + 170.7\cos(0.7435t) - 327.3\sin(0.7435t) + 26.26\cos(1.478t) + 376.1\sin(1.478t)$
Minerals	$f_{22}(t) = 695t + 9660$
Chemicals	$f_{32}(t) = 95.34 + 4.78\cos(0.5864t) - 60.03\sin(0.5864t) - 30.84\cos(1.1728t) + 1.824\sin(1.1728t)$
Cereal and Agricultural Products	$f_{42}(t) = 20.4t^2 - 144.6t + 423.9$
Industrial Materials	$f_{52}(t) = 302\exp(-((t-2.045)/0.8778)^2) + 9446\exp(-((t-1226)/780.5)^2)$
Oil Seeds and Edible Oils	$f_{62}(t) = 58.16 + 15.49\cos(0.485t) - 32.56\sin(0.485t) - 133.1\cos(0.97t) - 50.61\sin(0.97t)$
Metal	$f_{72}(t) = 12.08t^2 - 7.972t + 655.5$
Sulfur	$f_{82}(t) = 245.3\exp(-((t-6.871)/1.126)^2) + 784.1\exp(-((t-28.37)/37.22)^2)$
Vehicles	$f_{92}(t) = 73.73\exp(-((t-5.345)/2.453)^2) + (2.03\times10^{16})\exp(-((t-311.7)/52.13)^2)$
Alternatives	**Behavioral function with respect to "Average Revenue"**
Petroleum Products	$f_{13}(t) = 39.02t + 104.8$
Minerals	$f_{23}(t) = 18.16t + 146.3$
Chemicals	$f_{33}(t) = 20.36t + 92.37$
Cereal and Agricultural Products	$f_{43}(t) = 186.6 + 5.508\cos(0.5884t) - 34.43\sin(0.5885t) + 25.81\cos(1.1768t) - 3.79\sin(1.1768t)$
Industrial Materials	$f_{53}(t) = 22.75t + 128.8$
Oil Seeds and Edible Oils	$f_{63}(t) = 227.9 - 30.19\cos(0.5458t) - 57.19\sin 0.5458t)$
Metal	$f_{73}(t) = 15.59t + 128.6$

Sulfur	$f_{83}(t) = 235.2 - 31.86\cos(0.5246t) - 6.99\sin(0.5246t) + 19.8\cos(1.0492t) - 29.35\sin(1.0492t)$
Vehicles	$f_{93}(t) = -0.2721t^2 - 20.12t + 157.7$

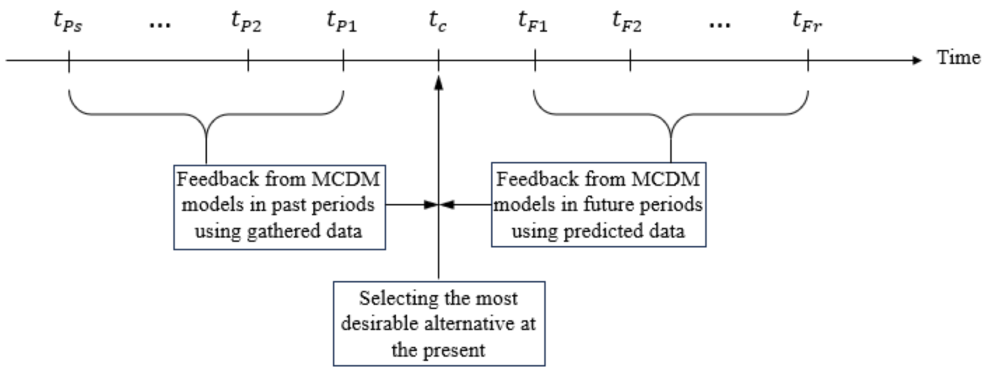

Figure 1) Dynamic multi-criteria decision-making models based on periodically extracted information

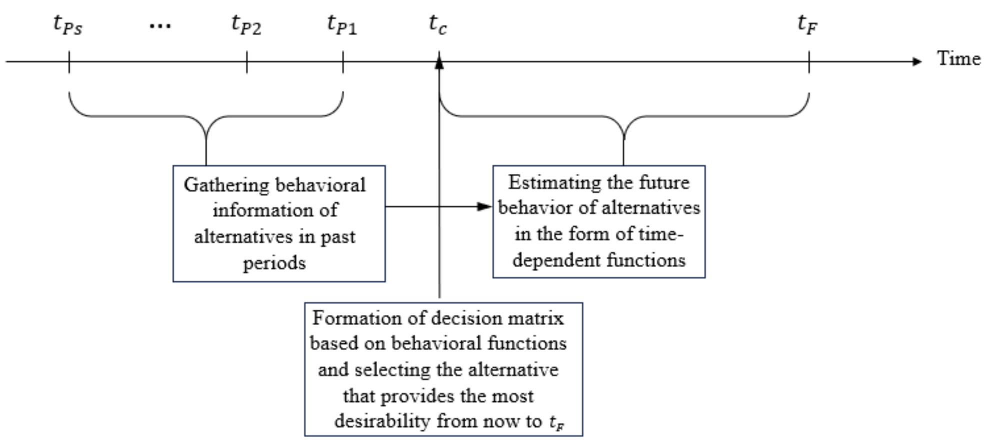

Figure 2) Dynamic multi-criteria decision-making models based on continuously predicted information

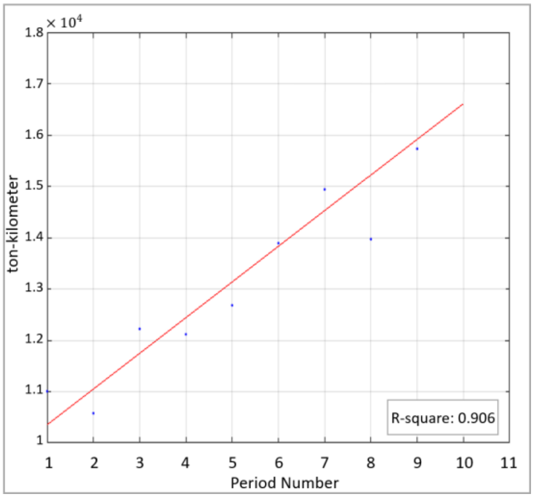

Figure 3) Minerals fitted curve with respect to ton-kilometers

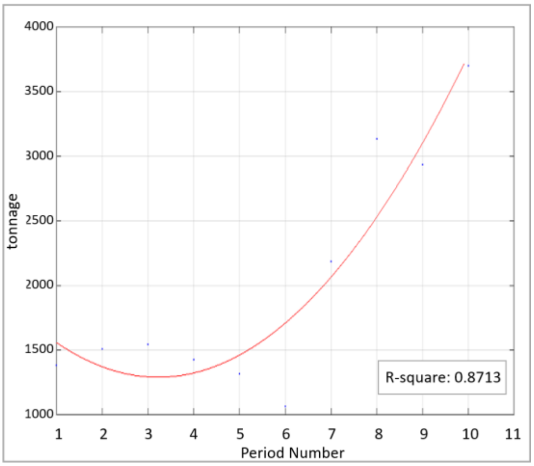

Figure 4) Metals fitted curve with respect to tonnage

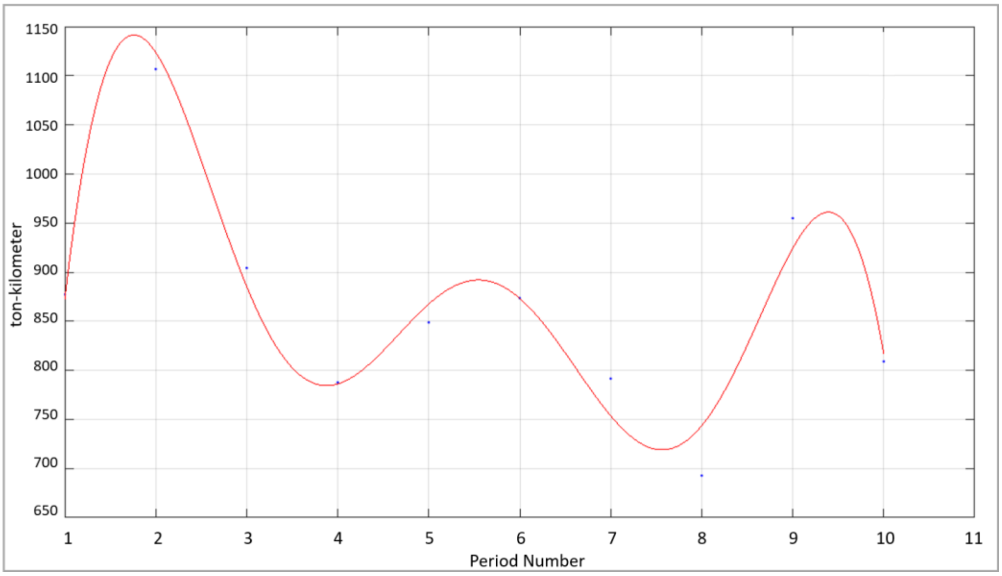

Figure 5) Industrial Materials fitted curve with respect to ton-kilometers