The effect of network structure on the opinion-aware influence maximization problem

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Abstract

The problem of influence maximization is finding the best nodes at the beginning of the diffusion process to maximize the affected nodes at the end. Although there has been a great deal of modeling in this area, no studies have examined how network structure, size, and seed nodes affect these models. The present study has investigated this issue by designing and conducting an experiment. Erdos-Renyi, small-world, and scale free networks with different sizes are examined in this work. Additionally, the variation between these structures and the number of seed nodes in the opinion-aware influence maximization (OAIM) problem's output has been statistically examined for 1440 networks. As a result, while confirming the effect of network structure and size on the success of promoting an opinion in the network, recommendations have been provided for the message sent by the beneficiary in the OAIM problem.

Keywords: Influence maximization, Networks Structure, Complex networks, OAIM, Genetic algorithm

1. Introduction

In the social network's literature, finding the most effective nodes among all nodes to maximize information dissemination is known as "***influence maximization***." With the expansion of research in this field, more applications of this issue became apparent. Applications of this issue in viral marketing, target advertising, sociology, and other fields have been considered by researchers[1-3].

Considering that the classic influence maximization was incapable of modeling viral marketing in the real world, Kermani et al. developed a model called ***opinion-aware influence maximization***[4]. To make the problem more realistic, they pursued two goals in their research:

maximize the spread of a particular opinion and optimize message content. They considered ideas for each node and believed these vectors were influential in spreading the message on the network. Considering the ideas of nodes and their degree of sociality is one of the advantages of Kermani et al.'s Research, but they have ignored the network structure and the number of network nodes in their research.

On the other hand, in the social network analysis literature, networks have been studied in terms of their node characteristics and structural specifications. Part of the Research in the past has worked on homophily [5, 6], node centralities [7, 8], and assorsativity[9], all of which can be categorized as node properties. However, other categories of studies have focused on the structural features of networks, and some of these features have led to new structures in networks. These structures are found in random, scale-free, and small-world networks, which we will discuss in detail. The main question we seek to answer in this study is whether the network's structure and the number of their nodes affect the optimality of the opinion-aware influence maximization (OAIM) or not. In this study, an experiment is designed in which networks with different structures and sizes are generated and the OAIM problem is implemented for them with different seed nodes; Then, the outputs are statistically analyzed. The novel aspect of the current work is the simultaneous measurement of the effects of the three aforementioned components in an influence maximization problem that is resolved using meta-heuristic algorithms, most notably a genetic algorithm. The detailed statistical analysis carried out on the data sets resulting from solving the problem for 1440 networks is another reason for the confirmation of the examined hypotheses, which can contribute significantly to the expansion of literature in this field.

This paper aims to investigate the effect of network structure, network size, and the number of seed nodes on the opinion aware influence maximization. The rest of the paper is organized as follows: The next section deals with the influence maximization problem and network structure literature review. The third section of the paper introduces the OAIM problem, methodology of this study, and implementation. The statistical analysis is presented in the fourth section, and in the last section results of this paper and recommendations are expressed.

2. Literature review

The present work is a study between complex networks and influence maximization fields, so a review of the literature is presented in two sections. In the first part, the literature is explained "*influence maximization*," and in the second part, the types of "*network structures"* are examined.

2.1 Influence maximization

The issue of influence maximization as one of the fields of social network analysis seeks to find a category of nodes with maximum influence spread to maximize the number of affected nodes[10, 11]. Domingos and Richardson[12] were the first to use the issue of influence maximization in the form of viral marketing in 2001, although Kempe et al. [13] were the first to formulate IM in 2003[14]. The influence maximization problem has applications in other areas such as rumor control[15, 16], network monitoring[17], and social recommendation[18]. One of the most important applications of IM can be found in viral marketing[12, 19].

In their study, Yuchen Li et al. addressed the challenges of the influence maximization problem as follows: 1. How to model the information dissemination process, 2. Intrinsic Complexity of IM Problem (NP-Hard) 3. Increasingly, online social networks have been contributing to the influence maximization of modeling in this area, and context-aware influence maximization problems have emerged as a result [10]. A comprehensive study on influence maximization problem, its framworks, performances, challenges and directions has been conducted in [20].

Context-aware influence maximization problems are actually the result of combining the classic influence maximization problem with other items such as location, time, or even belief. In fact, context-aware IM problems use the classic IM for the intended applications[10]:

1. ***Topic-aware influence maximization:*** TAIM expanse the classic IM by considering topics. In these models, social influence is measured by calculating the amount of information diffusion on a topic in the networks [21]. Some researchers have worked in this field: [22, 23], and [24-26].
2. ***Time-Aware Influence Maximization***: in classical influence maximization problems, it is assumed that the propagation process continues until the node is no longer affected, but this assumption is unreasonable because the diffusion process may not stop for a long time. This is the basis of time maximization models. These models are presented to impose time constraints on the diffusion process. (For further information, refer to:[27-30])
3. ***Location-Aware Influence Maximization:*** With the increasing popularity of location-based social networks (Twitter, Instagram, etc.) and word-of-mouth marketing based on location, this field has been considered. The basic concept of LAIM is to maximize the influence of the location-relevant users instead of any users in the classic IM settings.[31] and [32] Could be mentioned as some related studies research. Recently, a comprehensive survey on location-aware influence maximization problem has been published[33].
4. ***Dynamic Influence Maximization***: the influence maximization algorithms discussed so far are inherently static. Given the social graph $G = (V, E)$, they assume that the probability of propagation of P_e is constant for each $e \in E$. This issue is far from real-world social networks, so a new topic was formed in the literature called dynamic IM. For example, Agrawal et al. In their research presented an algorithm that maximizes the influence at time $t + h$ after finding the set of primary nodes at time t. [34] and [35] have investigated this issue.
5. ***Competitive Influence Maximization:*** Another topic in the literature is competitive IM. In matters of influence maximization in a competitive environment, some competitors seek to increase their impact on the Network by simultaneously disseminating their desired

phenomenon, the dissemination of each of which interferes with the dissemination of the other. In these matters, the situation is such that as the influence of a person is maximized, the influence of his rival is minimized[36-39].

2.2 Network structure

One of the networks science goals is to produce networks with the characteristics of real networks[40]. In their Research, Newman and Park examine the causality of why social networks are different from other networks[41]. In their research, they show that social networks show completely different patterns of correlation between adjacent vertices. While degrees in most social networks have a positive correlation, most non-social networks have a negative correlation. They also argue that social networks show a high level of clustering, while clustering in many non-social networks is not higher than expected due to the observed degree distribution. So, the positive value of correlation between the degree of nodes and a large amount of clustering coefficient are the characteristics of social networks. Taking into account the characteristics of social networks by researchers in this field and over time, researchers have proposed new structures. In the following, the structures intended for this research are examined.

2.2.1 Random networks

The simplest type of complex network is a random network [42]. Erdos and Renyi introduced the prototype of random networks in 1959. A random network they introduced consists of $\boldsymbol{N}$ nodes where each node pair is connected with probability $\boldsymbol{p}$[43]. It should be noted that random networks were also proposed independently by Gilbert[44]. Erdos and Renyi deliberated the minimum and maximum degree distributions, while the complete degree distribution in a random graph was later calculated by Bollobás[45, 46]. Nodes have a binomial distribution in random networks, but in large random networks, this distribution is a well Poisson approximation[40].

2.2.2 Small world networks

Although random graphs have some social network characteristics; however, they are deprived of others, such as high clustering[47]. Defining two components related to small-world networks seems necessary to understand these networks:

***Clustering coefficient*:** Suppose node i is connected to j in the given network. Also, node i is connected to node k. The clustering coefficient will be the probability of having edge jk[47].

Small-world effect: In networks, there is the "small-world effect," which portends that the average path length between two network nodes can be orders of magnitude smaller than the total number of nodes[42].

Unlike real networks, clustering disappears with the increasing number of nodes in random networks. Therefore, small-world networks that are neither random nor regular networks have limited clustering while having a small-world effect. One of the small world networks developed by Watts and Strogatz was presented[48]. In this model, the network nodes are first arranged around the lattice; then, each node is connected to the k number of the nearest other nodes. Then, with a probability p, each edge is removed, and the edge is replaced so that the node degree distribution function does not change[42].

2.2.3 Scale-free networks

To describe some of the topological features of real networks, we need to construct networks that follow the power law degree distributions[46]. Scale-free networks are the most real networks compared to random and small-world networks. In these networks, the number of nodes increases over time, like the World Wide Web, which grows exponentially with adding new pages. In random networks and small worlds, it is assumed that the probability of connecting two nodes is uniform and random, but in scale-free networks, "preferential attachment" or "rich-get-richer" are considered[42]. The power law states that the number of nodes with a high degree is small, and vice versa.

3. Methodology

As mentioned in previous sections, kermani et al. considered some fundamental variables such as node's opinions, sociability, and decision variables[4]. Nevertheless, in their work, some important variables like network's structure and the network size are ignored that can be determinative. The focus of the present study is on non-considered variables and their effect on the output of the OAIM problem. In order to clarify the issue, opinion-aware influence maximization is explained in detail.

3.1 Opinion-aware influence maximization components

In their research, Kermani et al. developed a nonlinear bi-objective mathematical programming model intending to add people's opinions and their degree of sociality to the influence maximization problem[4]. The sets, indices, parameters, and variables used to formulate the OAIM problem are described in Table 1.Variables and Table 2.Notations.

Table1
Table 2

The model proposed by Kermani et al.[4] tries to maximize the spread of the desired opinion. This model also minimizes the number of initial infected, which can be referred to as a cost constraint. They convert their model to a linear single-objective model using ε -constraint method. After linearization, the model is as follows:

$$Max\sum_{i=0}^{n} p_{ir}(t), \qquad r \in \{1,\dots,q\} \qquad (1)$$

$t \rightarrow$ diffusion depth

Subject to:

$$\sum_{i=0}^{n} x_i(0) \le \varepsilon \tag{2}$$

$$l_{ij}(t+1) \le U.\tau_{ij}(t), \qquad \forall i, j \in N_i, t, \tag{3}$$

$$3-(x_i(t)+y_i(t+1)+z_j(t+1)) \le U.\tau_{ij}(t) \qquad \forall i, j \in N_i, t, \tag{4}$$

$$(l_{ij}(t+1)-1) \ge (x_i(t)+y_i(t+1)+z_j(t+1))-3, \qquad \forall i, j \in N_i, t, \tag{5}$$

$$l_{ij}(t+1) \ge x_i(t) \qquad \forall i, j \in N_i, t, \tag{6}$$

$$\sum_{i \in k_j} l_{ij}(t+1) \ge x_j(t+1)-x_j(t), \qquad \forall i, j \in K_j, j, t, \tag{7}$$

$$\sum_{s=1}^{q} (F_{is}^{+} + F_{is}^{-}) \le U.\tau_{ij}(t), \qquad \forall i, t \tag{8}$$

$$p_{is}(t+1) - \frac{m_s + p_{is}(t)}{2} = F_{is}^{+} + F_{is}^{-}, \qquad \forall i, t, s, \tag{9}$$

$$x_i(t)-x_i(t-1) \le U.(1-x_i(t)), \qquad \forall i, t, \tag{10}$$

$$x_i(t+1) \ge x_i(t), \qquad \forall i, t, \tag{11}$$

$$\sum_{j \in N_i} l_{ij}(t+2) \le U.(1-x_i(t)), \qquad \forall i, t, \tag{12}$$

$$(1-y_i(t+1))U.\varphi_i(t) \qquad \forall i, t, \tag{13}$$

$$\alpha_i(1-\frac{1}{2q}\sum_{s=1}^{q}(D_{is}^{+} + D_{is}^{-}))-\delta \le U.(1-\varphi_i(t)), \qquad \forall i, t, \tag{14}$$

$$y_i(t+1) \le U.\varphi_i(t) \qquad \forall i, t, \tag{15}$$

$$\delta - \alpha_i(1-\frac{1}{2q}\sum_{s=1}^{q}(D_{is}^{+} + D_{is}^{-})) \le U.(1-\varphi_i(t)) \qquad \forall i, t, \tag{16}$$

$$m_s - p_{is}(t+1) = (D_{is}^{+} + D_{is}^{-}) \qquad \forall i, t, s, \tag{17}$$

$$(1-z_j(t+1)) \le U.\theta_j(t), \qquad \forall i, t, \tag{18}$$

$$\alpha_i(1-\frac{1}{2q}\sum_{s=1}^{q}(E_{is}^{+} + E_{is}^{-}))-\delta' \le U.(1-\theta_j(t)) \qquad \forall j, t, \tag{19}$$

$$z_j(t+1) \le U.\theta_j(t), \qquad \forall j, t, \tag{20}$$

$$\delta' - \alpha_i(1-\frac{1}{2q}\sum_{s=1}^{q}(E_{is}^{+} + E_{is}^{-})) \le U.(1-\theta_j(t)) \qquad \forall j, t, \tag{21}$$

$$m_s - p_{js}(t) = E_{is}^{+} + E_{is}^{-}, \qquad \forall j, t, s, \tag{22}$$

$$\sum_{s=1}^{q} (H_{is}^{+} + H_{is}^{-}) \le x_i(t)-x_i(t-1), \qquad \forall i, t, \tag{23}$$

$$p_{is}(t+1) - p_{is}(t) = H_{is}^{+} + H_{is}^{-}, \qquad \forall i, t, s, \tag{24}$$

$$x_i(t), y_i(t), z_j(t), l_{ij}(t) \in \{0,1\}, \qquad \forall i, j, t \tag{25}$$

$$\tau_{ij}(t), \gamma_i(t), \omega_i(t), \theta_j(t), \varphi_i(t), \mu_j(t) \in \{0,1\}, \qquad \forall i, j, t \qquad (26)$$

$$F_{is}^{+}, F_{is}^{-}, D_{is}^{+}, D_{is}^{-}, E_{is}^{+}, E_{is}^{-}, H_{is}^{+}, H_{is}^{-} \geq 0, \qquad \forall i, s, \qquad (27)$$

$$-1 \leq p_{is}(t), m_s \leq 1, \qquad \forall i, s, t, \qquad (28)$$

Constraints 3 and 4 model the fact that if the sum of the variables used to "activate node i at time $\boldsymbol{t}$," "forward the message by node i at time $\boldsymbol{t + 1}$", and "select node j to forward message at time $\boldsymbol{t + 1}$ "are less than 3 the outgoing edges of node i is inactive. Furthermore, the fifth constraint is inverse propositions of numbers 3&4. Constraint 6 shows that if a node is active at the time $\boldsymbol{t}$, its outgoing links could be active or inactive. If a node is active at both $\boldsymbol{t}$ and $\boldsymbol{t + 1}$, then the incoming edges can be active or inactive at the time $\boldsymbol{t}$. this fact is modeled in constraints 7. Constraints 8, 9, and 10 show that if a node is active at the time $\boldsymbol{t}$ and inactive at its last time, then its updated opinion at the time $\boldsymbol{t + 1}$ would be an average of its opinion at time $\boldsymbol{t}$ and message's content propensity. Constraint 11 enforces the model to make as a progressive one. Constraint 12 models the fact that edges are coming out of a node are disabled after receiving a message at $\boldsymbol{t}$ and forwarding it at $\boldsymbol{t + 1}$.

The node (a person present in the network) forwards the message if the multiplication of the node's sociability and the degree of attractiveness of the message for the node are more than a specific limit ($\boldsymbol{\delta}$). This fact is modeled in constraints 13-17. Constraints 18-21 model the conditions for selecting a node to send a message by an active node (more explained). When a person in the network decides to send a message to others, then he chooses a person as the recipient whose level of sociality and the degree of attractiveness of the message for him is more than $\boldsymbol{\delta'}$. Constraints 23 and 24 state that people's opinions will not change in the Network if they do not receive a message. Equations 25-27 show the type of decision variables. To solve the OAIM problem, the authors suggest using genetic algorithms since the problem is NP-hard and using genetic algorithms to solve the problem on big data.

3.2 Hypothesis

As seen in the OAIM problem, the role of network structure is not considered. Given that the networks in which the opinion are propagated have a social structure, it is expected that the structure of the networks will play an essential role in the success of the propaganda of the opinions. According to this critical point, the first and main hypothesis of this research is presented as follows:

H1a: *The structure of social networks affects the success rate of propagating an opinion on those networks.*

Another thing that can affect the success of promoting an opinion on the network is the number of people in that network. On the one hand, the large number of people in the network can help spread the desired opinion. On the other hand, due to the differences in the people's opinions in the community, propagating a particular idea can become more difficult as the population grows. It seems that this point has not been considered in the research of Kermani et al. [4]. With this in mind, another research hypothesis can be presented as follows:

H1b: *Network size affects the success rate of promoting an idea on the Network.*

In the case of the OAIM problem, the authors sought to minimize the number of seed nodes to maximize the propensity to an opinion. The question is, will reduce the number of primary seed nodes succeed in propagating the idea? Given that more seed nodes can share the message in more areas of the network, it seems that another hypothesis can be put forward:

H1c: *The number of seed nodes in the OAIM problem affects the success of propagating an opinion in the Network.*

A designed experiment is introduced to test these hypotheses.

3.3 Data sets and implementation

The present work attempts to investigate the effect of different network structures, the number of network nodes and number of seed nodes on the spread of a desired opinion in the Opinion-aware influence maximization problem. For this purpose, networks with three structures are generated (random networks, small world networks, and scale-free networks) using the "*igraph*" package in *R*. These networks are made with three different numbers of nodes (n=100,150,200) to be able to check the effect of structure and number of nodes. Opinion-aware influence maximization problem for these networks is implemented and repeated experiment for different seed nodes (h=1, 2, 3, and 4). The OAIM problems output is categorized into two parts best fitness and best message. "best fitness" shows the amount of prevalence of the desired opinion in the network. Also, "best message" presented the best message content sent by the beneficiary to seed nodes for influence in network nodes' opinions. All the steps performed in the present study are shown schematically in Figure 1.Research Steps.

Figure1

In this experiment, 40 networks for each mode designed are generated. You can see the design of this experiment in Table 3.Experiments Design.

Table3

Generated networks sample can be seen in Figure 2.Erdos-Renyi generated networks sample, Figure 3.Scale-free generated networks sample and Figure 4.Small-world generated networks sample.

Figure 2

Figure 3

Figure4

Also, this experiment is performed for the different number of initial nodes (h = 1, 2, 3 and 4 initial nodes for start the diffusion process). The number initial nodes has been fixed like the other researches like[4, 49, 50]. The parameters used to implement the OAIM problem are given in Table 4.OAIM parameters below, the values used were selected following Kermani et al. [4]:

Table 4

As mentioned in the previous section, Kermani et al. developed a genetic algorithm to solve the OAIM problem. The proposed genetic algorithm generates 50 random chromosomes as the initial population. The chromosome they use to solve the OAIM problem consists of $(n + q)$ genes, the first part indicates the activation or inactivity of the network nodes, and the next part of genes indicates the tendency of the message content to different ideas. The mutation rate is assumed to be 0.4 and the crossover rate to be 0.6. The genetic algorithm process will continue until one of the stopping conditions occurs: (1) The number of iterations is more than 500 iterations. (2) The best-earned fitness in iteration *iter* is equal to the best-earned fitness in iteration iter-100 if iter > 100. In this experiment, the same settings were applied following Kermani et al. [4].

The output of this model is divided into two parts: (1) the best message (2) the best seed nodes that start the diffusion process. The optimized amount of opinion in the Network can be extracted as the best fitness from model solving. Therefore, after solving this problem, the outputs in two categories of "best fitness" and "best message" are collected for the constructed networks and used for statistical analysis. In this study, 1440 networks are generated and the OAIM problem is implemented for all of them to find the effect of network structure and the number of nodes on the optimality of OAIM also, the effect of the number of seed nodes on OAIM outputs is examined. The Figure 5.Sample results of OAIM problem implementation on Erdos - Renyi networks, Figure 6.Sample results of OAIM problem implementation on scale-free networks and Figure 7.Sample results of OAIM problem implementation on small-world networks show examples of solved networks furthermore the samples of outputs for these networks are in Table 5.Out puts of OAIM problem implementation on Erdos-Renyi networks, Table 6.Out puts of OAIM problem implementation on Scale-free networks and Table 7.Out puts of OAIM problem implementation on Small-world networks.

Figure5

Table5

Figure6

Table6

Figure7

Table7

After implementing the OAIM problem on 1440 networks, the obtained outputs were collected and analyzed. Table 8. Average OAIM problem outputs for 36 test modes summarizes the 36 experiment modes and their outputs. What is noteworthy is that the run time for scale-free networks was much shorter than for random and small-world networks. Also, the best fitness rate in scale-free networks has been much lower than in other networks. A more detailed analysis of the OAIM problem output data after implementation in 1440 networks is presented in the next section.

Table 8

4. *Statistical Analysis*

M * N experiments include experiments in which the number of levels of different agents is not the same. The present experimental study has 3 factors with different levels of first-factor network structure (3 levels), second-factor number of network nodes (3 levels), and third-factor number of seed nodes (4 levels) which are displayed as 3*3*4. The following is a variance analysis table and a table of the significance of the main and interaction effects of each factor for the variables "best fitness" and "best message1". Table 9.Factors informations shows information about statistical analysis factors, their different levels, and the number of observations. Table 10.Tests of Between-Subjects Effects also shows the results of multivariate analysis of variance (MONOVA).

Table9

Considering that the problem seeks to maximize the desired opinion (the first component of the opinion vector of the people presented in the Network), two problem-solving outputs are analyzed statistically, which include "Best fitness" and "Best message 1". According to Table 10.Tests of Between-Subjects Effects, the main effects of factors 1, 2, and 3 and the interaction of factor 1 in factor 3 on the variable's best fitness and best message 1 are significant because the significance value of these effects is less than 0.05. Also, the interaction of factor 1 in factor 2 on the best fitness variable is significant. As a result, we can confirm the hypotheses H1a, H1b, and H1c.

Table10

The interaction of factor 2 in factor 3 and factor 1 in factor 2 in factor 3 were insignificant. This can be explained by the fact that each of the main and interaction effects enters the model as an independent variable and the main effects have more weight in the impact. In a more detailed study, the Bonferroni post hoc test was performed for the main effects of factors 1, 2, and 3, which can be seen below. As seen in Table 11.Structures Multiple Comparisons, the mean difference between the best fitness variable at different levels of the structure factor is significant. Also, the significance of the mean difference of the variable best message 1 at different levels has been confirmed. The results of these comparisons are examined in the following.

Table 11

As can be seen in Table 12.Number of nodes multiple Comparisons, the mean difference between the best fitness variable at different levels of the number of nodes factor is significant. Also, the significance of the mean difference of the best message 1 variable at different levels of the number of 100 and 200 nodes has been confirmed.Table 13.Number of seed nodes multiple Comparisons also shows the post hoc test belonging to the number of seeds factor. Significant values of each level of this factor are visible.

5. *Conclusion*

As mentioned, this study aims to investigate the effect of network structure, the number of network nodes, and the initial nodes of the propagation process on the OAIM problem. According to the MANOVA analysis given in the previous section, the effect of the studied factors on the problem was confirmed. Also, in the post-hoc tests that we performed, the effects of different levels of factors were examined. As can be seen in Figure 8.Main effects plot for Best fitness, the amount of institutionalization of the opinion in scale-free networks is significantly less than Erdos-Renyi and Small-world networks. It can also be said that if the out-of-network beneficiary intends to institutionalize a particular belief in the Network, he or she should promote that belief in networks that have a structure similar to a random structure or a small world. Also, with the increase in the number of network nodes and seed nodes, the amount of institutionalization of an opinion in the network increases.

Figure 8

Another conclusion that can be drawn from this research and can be seen in Figure 9.Main effects plot for Best message is that if a beneficiary outside the network wants to spread an idea in the network and he/she faces a small-world or random network, his/her sent message content can be more radical than scale-free network. And it means that if a beneficiary intends to propagate an opinion on scale-free networks (that are more like real-world networks than other networks), the content of her/his sent message must be less radical.

Figure 9

This is true even though the hypothesis that the effects of the three factors mentioned above on the OAIM problem were confirmed. In order to advance the IM literature, it is still necessary to look into how network structure, network size, and the number of seed nodes affect other types of influence maximization problems.

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Tables:

Figures:

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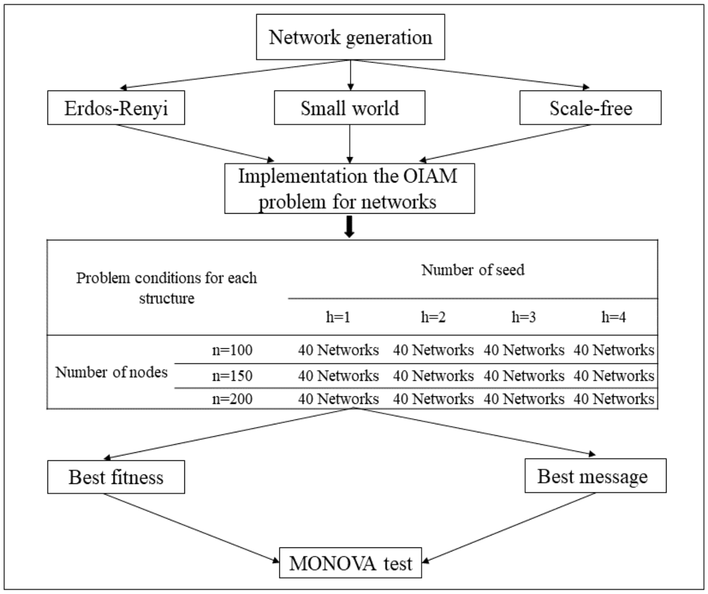

Figure 1.Research Steps

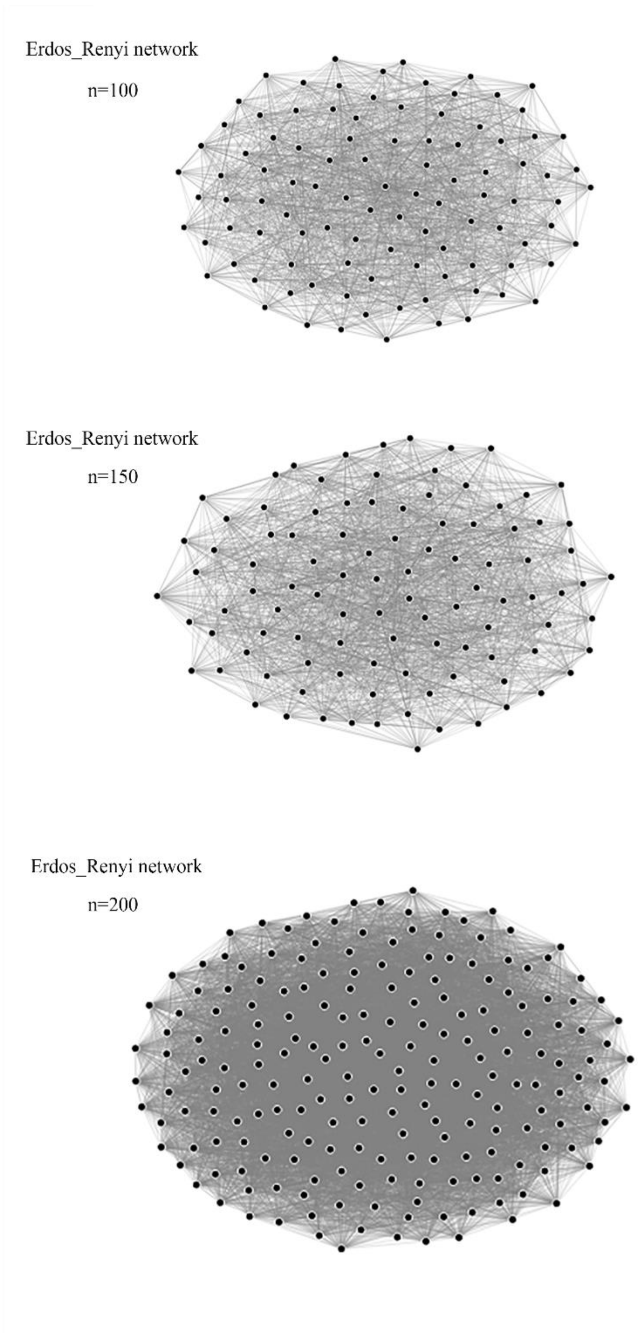

Figure 2.Erdos-Renyi generated networks sample

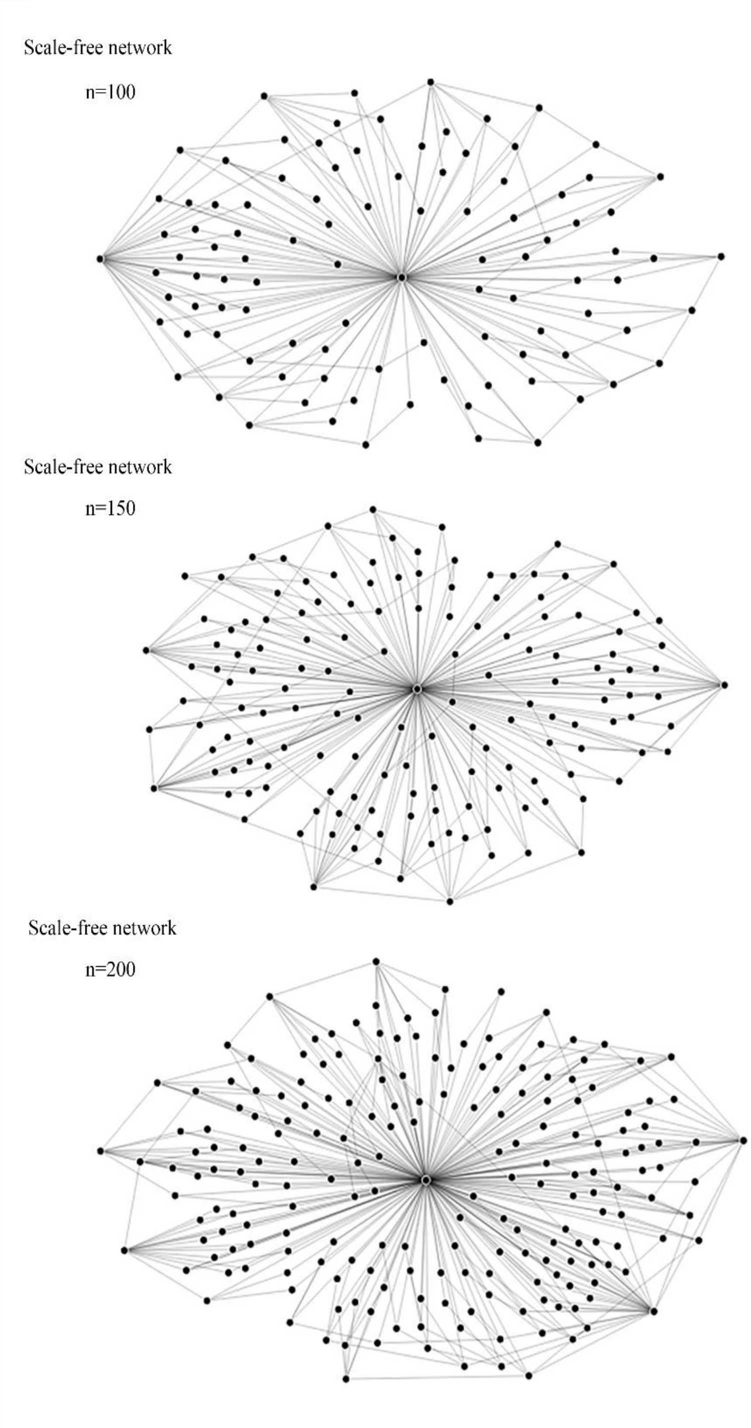

Figure 3.Scale-free generated networks sample

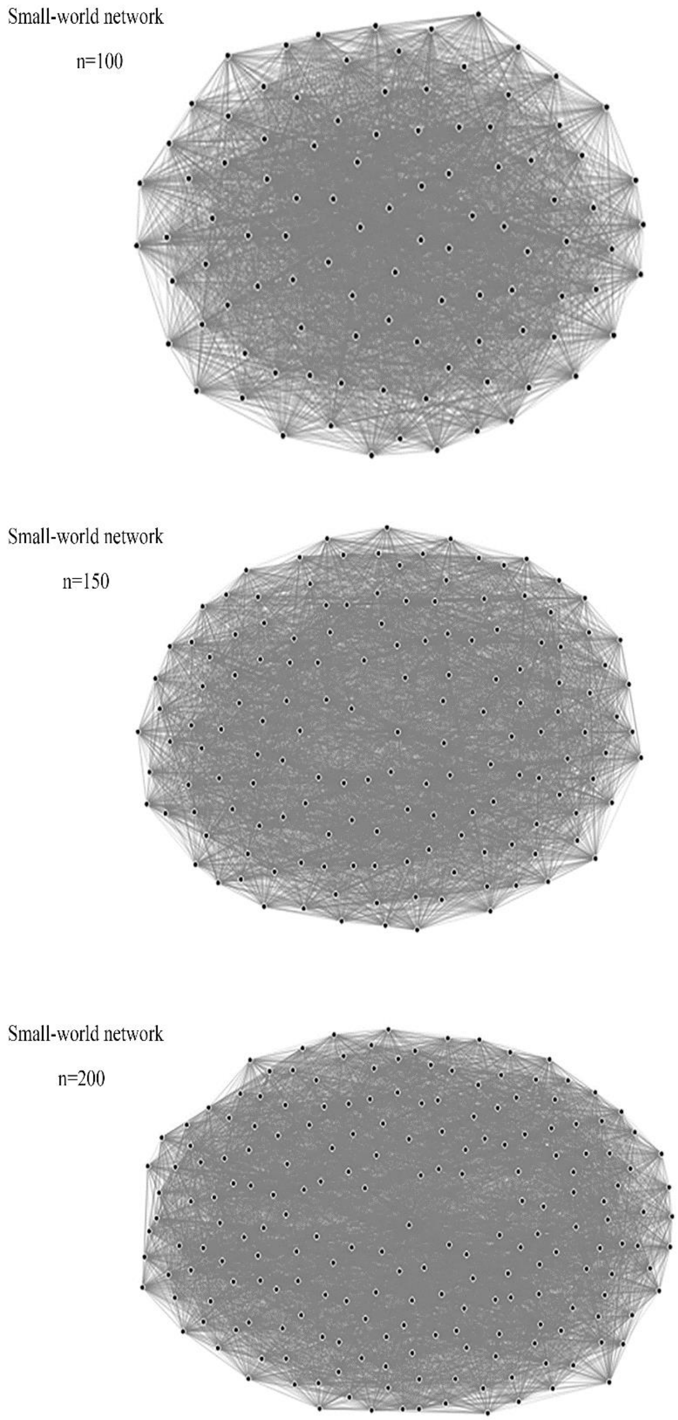

Figure 4.Small-world generated networks sample

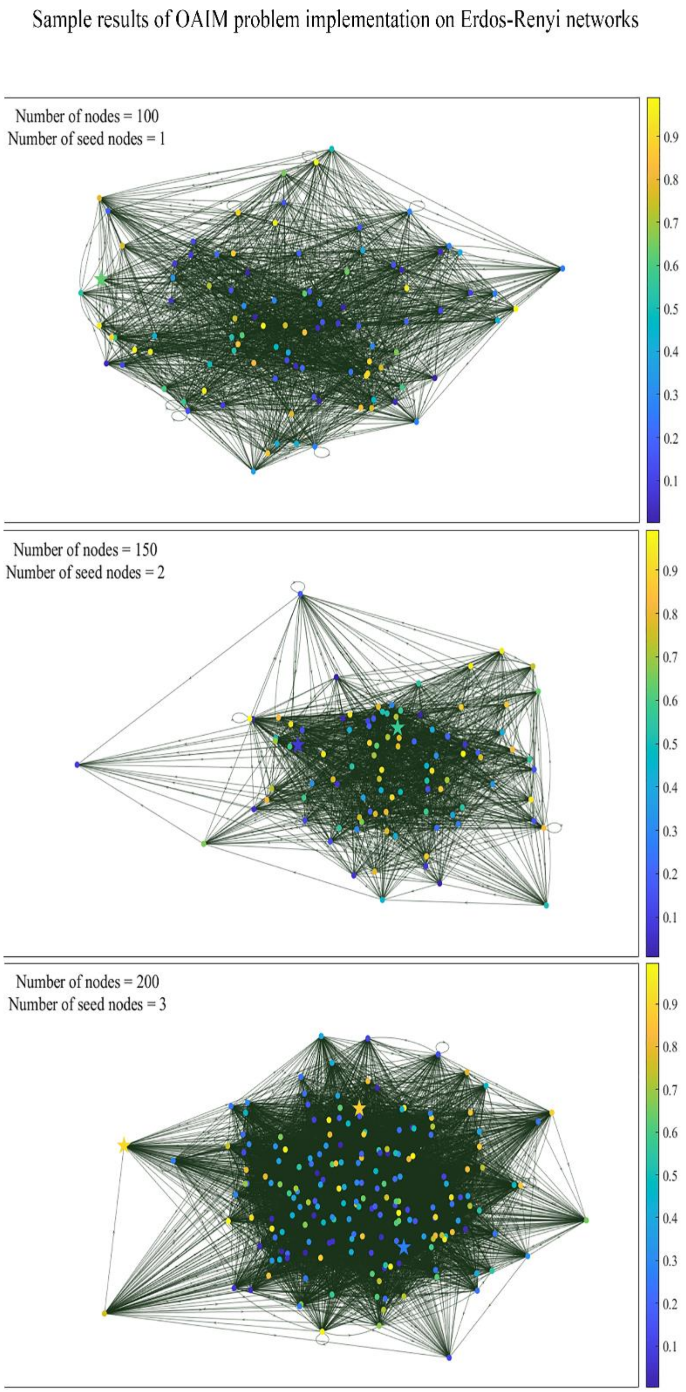

Figure 5.Sample results of OAIM problem implementation on Erdos - Renyi networks

Sample results of OAIM problem implementation on Scale-free networks

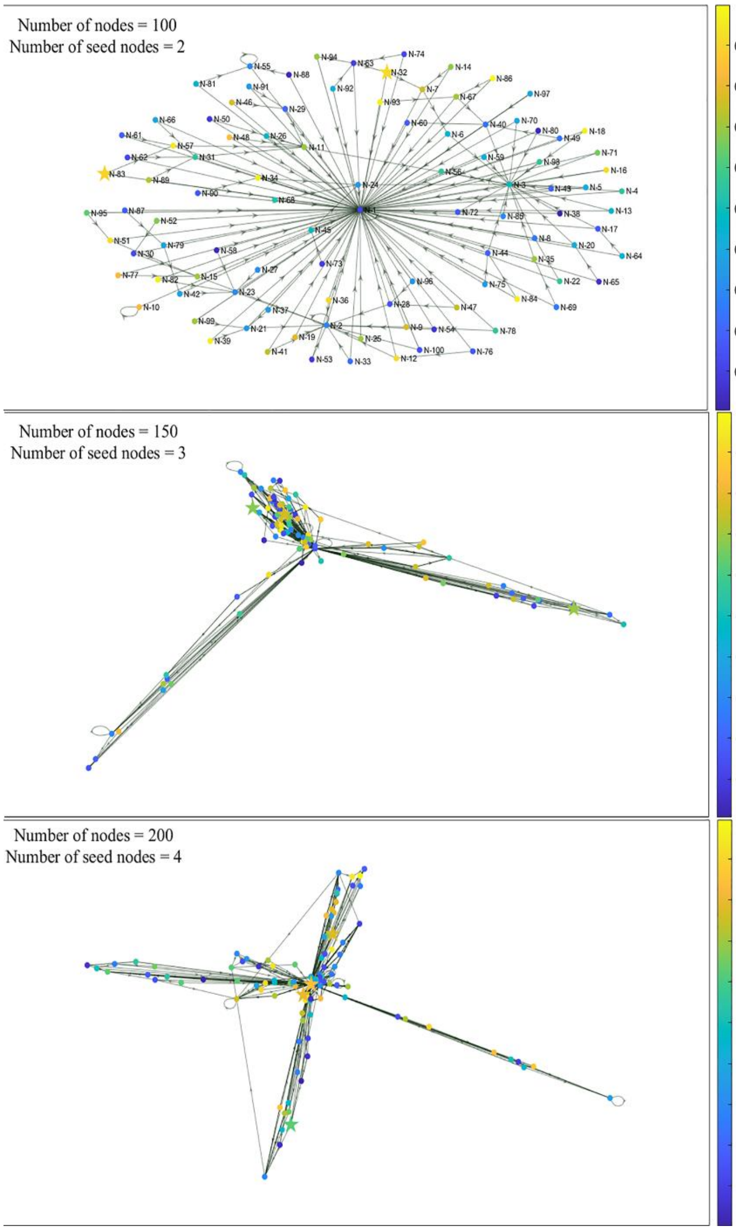

Figure 6.Sample results of OAIM problem implementation on scale-free networks

Sample results of OAIM problem implementation on Small-world networks

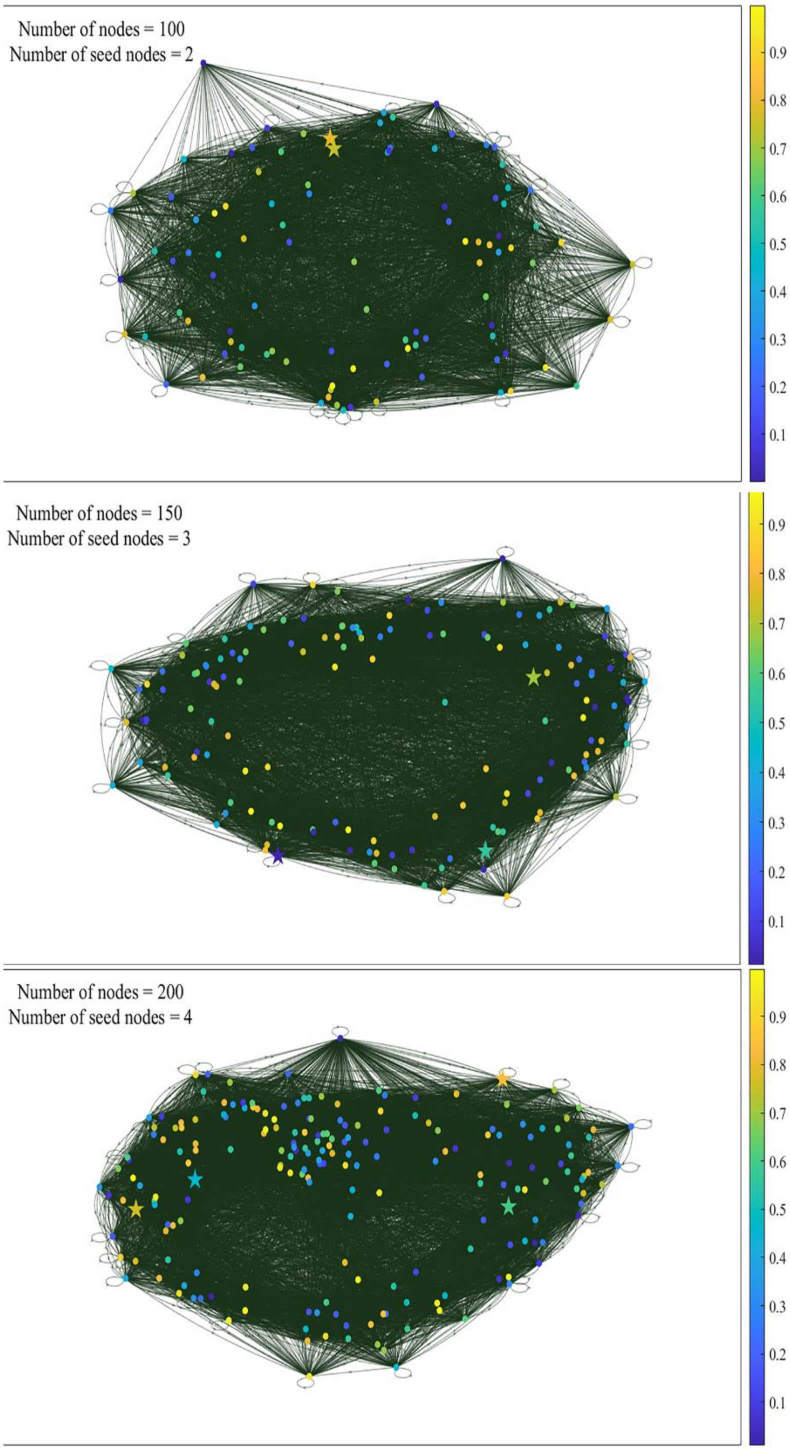

Figure 7.Sample results of OAIM problem implementation on small-world networks

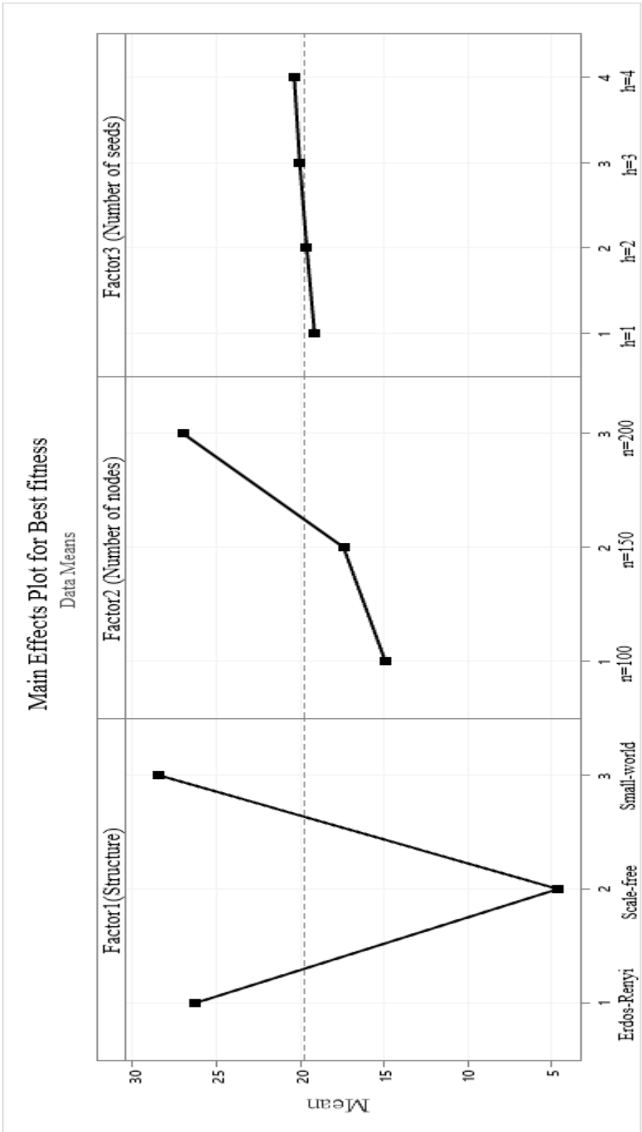

Figure 8.Main effects plot for Best fitness

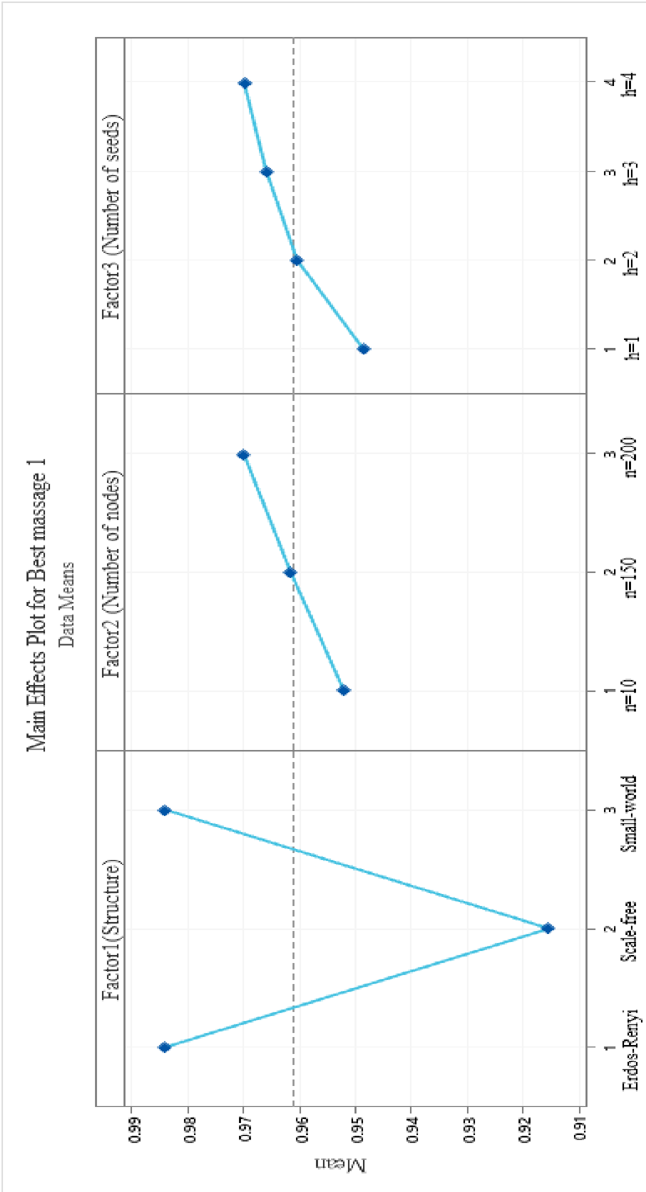

Figure 9.Main effects plot for Best message

Tables:

Table 1.Variables

Variables		
$X_i\ (t)$	1	if the node i receives a message at time period t
	0	Otherwise
$l_{ij}\ (t)$	1	if node i forwards the message to the node j at time period t
	0	Otherwise
$y_i\ (t)$	1	if the node i decides to forward the message at time t
	0	Otherwise
$Z_j\ (t)$	1	if node j prone to receive the message at time t
	0	Otherwise
$P_{is}\ (t)$		The vector of ith node's opinion at the time t
M_s		The vector of the message's content propensity

Table 2.Notations

Notation		
Sets and Indices	i, j	Indices of source and destination nodes (social network members)
	t	Index of discrete time periods; t = 0; 1; 2; . . .: n
	s	Index of different opinions; s = 1; . . .; q.
	N_i	Out-degree of person i (i.e., the persons whose phone numbers i has in his/her contact list)
	K_i	In-degree of person i (i.e., the persons who have i's phone number in their contact lists
Parameters	α_i	Sociability score of node i
	δ	A threshold for forwarding messages by source nodes
	δ'	A threshold for selecting destination nodes to forward message
	P_{is}	The vector of the initial ith node's opinion
	U	An adequate large number

Table 3.Experiments Design

h=1,2,3,4		***Number of nodes***		
		100	150	200
Structures	Erdos-Renyi	40 networks	40 networks	40 networks
	Small-world	40 networks	40 networks	40 networks
	Scale-free	40 networks	40 networks	40 networks

Table 4.OAIM parameters

Parameters	Value
Threshold of forwarding message	$\delta = 0.2$
Threshold of selecting a destination	$\delta' = 0.2$
Desired opinion	$r = 1$
Number of initially infected nodes	$h = 1,2,3,4$
The vector of the initial node's opinion	$P_{is}(0) \in [-1,1]$
Sociability score of node i	$\alpha_i \in [0,1]$

Table 5.Out puts of OAIM problem implementation on Erdos-Renyi networks

Network Type: Erdos-Renyi

Number of nodes	100	150	200
Number of seeds	2	3	4
Best fitness	21.0690	21.2313	29.3943
Best message	(0.993, -0.328,0.335)	(0.997, -0.265,0.324)	(0.996, -0.307,0.357)
Best seeds nodes numbers	No.9, No.28	No.4, No.15, No.32	No.10, No.31, No.49, No.153
Number of iteration (GA)	356	416	223
Run time	10 min	27 min	104 min

Table 6.Out puts of OAIM problem implementation on Scale-free networks

Network Type: Scale-free

Number of nodes	100	150	200
Number of seeds	2	3	4
Best Fitness	4/071	5/974	6/323
Best massage	(0.998,0.646,0.527)	(0.878,0.493,0.050)	(0.818,0.976,0.548)
Best Seeds	No.45, No.65	No.45, No.95, No.103	No.23, No.26, No.49, No.57
Number of Iteration (GA)	258	230	400
Run time	28 Sec	35 Sec	47 Sec

Table 7.Out puts of OAIM problem implementation on Small-world networks

Network Type: Small-world

Number of nodes	100	150	200
Number of seeds	2	3	4
Best Fitness	18/839	29/419	37/12660083
Best massage	(0.976,0.214, -0.642)	(0.978, -0.070, -0.113)	(0.997, -0.187, -0.349)
Best Seeds	No.1, No.2	No.25, No.26, No.46	No.27, No.52, No.91, No.92

Number of Iteration (GA)	330	223	334
Run time	30 Minutes	97 Minutes	360Minutes

Table 8. Average OAIM problem outputs for 36 test modes

Number of seed nodes	networks size	Networks Structures	Average of Best fitness in 40 networks	Best message1	Best message2	Best message3	Run time average (sec)
1	100	Erdos-Renyi	20.2034279	0.981897	-0.07137	-0.06874	950
		Scale-free	2.5795	0.79698	0.140052	-0.15165	4
		Small world	20.214847	0.983779	-0.00872	-0.07243	893
	150	Erdos-Renyi	20.065324	0.984842	-0.04835	0.039983	485
		Scale-free	2.8995128	0.90692	-0.03132	-0.05492	3
		Small world	27.43057	0.984006	0.021713	0.009088	734.5
	200	Erdos-Renyi	38.450523	0.988183	0.027056	-0.02027	1440
		Scale-free	2.74854	0.92414	-0.0112	-0.06726	7
		Small world	37.747131	0.985658	-0.00402	0.037433	1200
2	100	Erdos-Renyi	20.2366293	0.985694	-0.06507	-0.03807	960
		Scale-free	3.8353173	0.8985621	-0.042382	-0.096155	5.9
		Small world	20.205077	0.981158	-0.01152	-0.04615	847
	150	Erdos-Renyi	20.10251616	0.984496	-0.0189	0.03695	530
		Scale-free	4.2508216	0.8963445	-0.096286	0.04399141	5.9
		Small world	27.413112	0.977731	0.030618	0.020345	613
	200	Erdos-Renyi	38.4538904	0.987165	0.049047	0.010499	3500
		Scale-free	4.32434654	0.9451425	0.0215783	-0.1486649	7.5
		Small world	37.676864	0.986754	0.01562	0.056974	1689
3	100	Erdos-Renyi	20.195292	0.985841	-0.09409	-0.03221	1498
		Scale-free	5.13738897	0.93929	1.0375E-05	0.0183569	5.5
		Small world	20.23102	0.9826247	0.0020829	-0.060882	1730
	150	Erdos-Renyi	20.07736	0.979058	-0.02607	0.038793	980
		Scale-free	5.44533	0.9372766	-0.0242504	0.04395221	9.5
		Small world	27.438905	0.98364286	-0.0108974	0.0019786	2130
	200	Erdos-Renyi	38.4649212	0.988138	0.047387	-0.05224	4928
		Scale-free	5.621856	0.91314106	-0.0718914	-0.20009	15.39
		Small world	37.652291	0.9838913	0.0037372	0.0724835	3734.7
4	100	Erdos-Renyi	20.18210599	0.979591	-0.06913	-0.0327	2180
		Scale-free	6.0284258	0.927808	0.013301	0.02736	11.1
		Small world	20.20626188	0.9808643	-0.028047	-0.073471	1495
	150	Erdos-Renyi	20.123222	0.975954	0.017629	0.004957	1440
		Scale-free	6.48946498	0.942601	0.006677	-0.00879	10.06
		Small world	27.43438	0.985385	0.042486	0.009093	2349
	200	Erdos-Renyi	38.416868	0.988017	0.010437	-0.001955	15870
		Scale-free	6.6408259	0.956588	0.031889	-0.15559	16.5
		Small world	37.701797	0.992088	-0.00777	0.041626	13783

Table 9.Factors informations

		Value Label	N
Factor1	1	Erdos-Renyi	480
	2	Scale-free	480
	3	Small-world	480
Factor2	1	n=100	480
	2	n=150	480
	3	n=200	480
Factor3	1	h=1	360
	2	h=2	360
	3	h=3	360
	4	h=4	360

Table 10.Tests of Between-Subjects Effects

Source	Dependent Variable	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	Best fitness	227090.623	35	6488.304	649.547	.000
	Best message 1	2.293	35	.066	6.686	.000
Intercept	Best fitness	563786.498	1	563786.498	56440.891	.000
	Best message 1	1330.273	1	1330.273	135734.902	.000
Factor1	Best fitness	165763.229	2	82881.615	8297.311	.000
	Best message 1	1.507	2	.753	76.874	.000
Factor2	Best fitness	38866.433	2	19433.216	1945.467	.000
	Best message 1	.077	2	.039	3.929	.020
Factor3	Best fitness	296.361	3	98.787	9.890	.000
	Best message 1	.093	3	.031	3.177	.023
Factor1 * Factor2	Best fitness	21560.391	4	5390.098	539.605	.000
	Best message 1	.092	4	.023	2.340	.053
Factor1 * Factor3	Best fitness	601.356	6	100.226	10.034	.000
	Best message 1	.208	6	.035	3.540	.002
Factor2 * Factor3	Best fitness	.632	6	.105	.011	1.000
	Best message 1	.112	6	.019	1.913	.076
Factor1 * Factor2 * Factor3	Best fitness	2.221	12	.185	.019	1.000
	Best message 1	.204	12	.017	1.733	.055

Error	Best fitness	14024.517	1404	9.989		
	Best message 1	13.760	1404	.010		
Total	Best fitness	804901.638	1440			
	Best message 1	1346.327	1440			
Corrected Total	Best fitness	241115.140	1439			
	Best message 1	16.053	1439			

Table 11.Structures Multiple Comparisons

Bonferroni

Dependent Variable	(I) Structure	(J) Structure	Mean Difference (I-J)	Std. Error	Sig.
Best fitness	Erdos-Renyi	Scale-free	21.58089540212*	.204011563175	.000
		Small-world	-2.19834771571*	.204011563175	.000
	Scale-free	Erdos-Renyi	-21.58089540212*	.204011563175	.000
		Small-world	-23.77924311783*	.204011563175	.000
	Small-world	Erdos-Renyi	2.19834771571*	.204011563175	.000
		Scale-free	23.77924311783*	.204011563175	.000
Best message 1	Erdos-Renyi	Scale-free	.06867443128*	.006390267396	.000
		Small-world	.00010763091	.006390267396	1.000
	Scale-free	Erdos-Renyi	-.06867443128*	.006390267396	.000
		Small-world	-.06856680036*	.006390267396	.000
	Small-world	Erdos-Renyi	-.00010763091	.006390267396	1.000
		Scale-free	.06856680036*	.006390267396	.000

Table 12.Number of nodes multiple Comparisons

Bonferroni

Dependent Variable	(I) Number of nodes	(J) Number of nodes	Mean Difference (I-J)	Std. Error	Sig.
Best fitness	100	150	-2.49293432094*	.204011563175	.000
		200	-12.05371294675*	.204011563175	.000
	150	100	2.49293432094*	.204011563175	.000
		200	-9.56077862581*	.204011563175	.000
	200	100	12.05371294675*	.204011563175	.000
		150	9.56077862581*	.204011563175	.000

Best message 1	100	150	-.00951409374	.006390267396	.410
		200	-.01790146131*	.006390267396	.015
	150	100	.00951409374	.006390267396	.410
		200	-.00838736757	.006390267396	.569
	200	100	.01790146131*	.006390267396	.015
		150	.00838736757	.006390267396	.569

Table 13.Number of seed nodes multiple Comparisons

Bonferroni

Dependent Variable	(I) Number of seeds	(J) Number of seeds	Mean Difference (I-J)	Std. Error	Sig.
Best fitness	1	2	-.46213329231	.235572261834	.300
		3	-.88055435190*	.235572261834	.001
		4	-1.20933055459*	.235572261834	.000
	2	1	.46213329231	.235572261834	.300
		3	-.41842105958	.235572261834	.456
		4	-.74719726227*	.235572261834	.009
	3	1	.88055435190*	.235572261834	.001
		2	.41842105958	.235572261834	.456
		4	-.32877620269	.235572261834	.978
	4	1	1.20933055459*	.235572261834	.000
		2	.74719726227*	.235572261834	.009
		3	.32877620269	.235572261834	.978
Best message 1	1	2	-.01185000756	.007378845202	.651
		3	-.01738904579	.007378845202	.111
		4	-.02138876106*	.007378845202	.023
	2	1	.01185000756	.007378845202	.651
		3	-.00553903823	.007378845202	1.000
		4	-.00953875351	.007378845202	1.000
	3	1	.01738904579	.007378845202	.111
		2	.00553903823	.007378845202	1.000
		4	-.00399971528	.007378845202	1.000
	4	1	.02138876106*	.007378845202	.023
		2	.00953875351	.007378845202	1.000
		3	.00399971528	.007378845202	1.000