Blasius and Sakiadis flow of titania-copper-water based hybrid nanofluid flow: An artificial neural network modeling

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Abstract: The addition of different nanoparticles in conventional fluid with proper quantity gives the hybrid fluids which have higher thermo-physical properties. The geometry of the hybrid nanoparticles has substantial impacts in numerous engineering and bio-medical applications. Blasius and Sakiadis flows are considered under under the impact of viscous dissipation phenomenon. Both flows are exemplified at the surface of laminar boundary layer conditions in water-based hybrid nanofluid comprising copper and Titania with Prandtl number 6.2. The heat transport is executed by the implication of intelligent computing paradigm through process of Artificial Levenberg Marquardt back propagated neural networks. The nonlinear PDE's which governs the fluid flow are organized into set of nonlinear ODE's by using the similarity functions. Runge-Kutta-Fehlberg's fourth-fifth order (RKF-45) based shooting scheme is utilized to solve the reduced ODE's. The larger buoyancy parameter values enhanced the velocity over stationary surface with constant free-stream velocity for Blasius flow and vertical movement of planer surface is moved in stagnant free-stream for Sakiadis flow. The thermal and concentration profiles are reduced against higher buoyancy values for both the cases.

Keywords: Blasius-Sakiadis flow; hybrid nanofluids; heat mass transportations; moving plate

Nomenclature

(u,v)	Velocity components	D	Diffusion co-efficient
(x,y)	Coordinate axes	m_1	Velocity ratio parameter
μ	Dynamic viscosity	Ec	Eckert number
k	Thermal conductivity	T	Fluid temperature
ρ	Density of base fluid	C	Fluid concentration
f	Dimensionless velocity profile	$\Pr$	Prandtl number
Re	Local Reynolds number	Sc	Schmidt number
θ	Non-dimensional temperature	**Subscript**	
(ρC_p)	Heat capacitance	f	Fluid
Gr	Grashof number	bf	Base fluid
g	Gravity	hnf	Hybrid nanofluid
ϕ_1, ϕ_2	Solid volume fractions of nanoparticles	s_1, s_2	Solid particle
ν	Kinematic viscosity	∞	Ambient
χ	Concentration profile	m	Surface/wall

λ^*	Porosity parameter		

1. Introduction

The boundary layer flow through surfaces with movement is a classical problem. The fluid flow via motionless or movable surfaces has fantastic manufacturing and technical applications like plate drag calculation in the absence of incidence, polymer and metal extrusion, metallic sheets cooling, thermal constituents traveling among a feed and spring-operated roller, etc. The boundary layer velocity is observed by the raise in the way of motion of the plate. The conventional problem of moving fluid through a motionless horizontal plate was discoursed by Blasius [1]. In divergence to this, Sakiadis [2] scrutinized the new class of boundary-layer flow on continuous limited surfaces. The Blasius and Sakiadis problems in fluid flow with nanoparticles through moving plates have more practical significance and this was scrutinized by Bachok et al. [3] on taking account of heat flow. Heat transfer coefficients and Skin friction have increased as a result of nanoparticle addition in base water fluid; these coefficients rise noticeably as the volume fraction of nanoparticles rises. Hafeez et al. [4] addressed the non-Fourier boundary-driven Maxwell liquid flowing over the moving sheet near a stagnant point. For a variety of fluidic scenarios, buoyancy effects as well as other physical phenomena are computed and the numerical results are provided. Stagnation point boundary-driven non-Newtonian liquid flow with distinct nanoparticles is reported by Devi et al. [5].

Due to the presence of frictional heating brought on by the fluid's increased contact with the solid phase and the wall, and the internal heating brought on by the mechanical force required to extrude the fluid, the effect of viscous dissipation is vital. A dynamic role for viscous dissipation is played in the fluid flow issue for various geometries. There have been several investigations done on the effects of viscous dissipation on nanofluid flows. Ali et al. [6] addressed the magnetized effect in nano-fluid by adopting homogeneous-heterogeneous type chemical reactions. They discussed the impacts of the magnetic field, non-Darcy, viscous dissipation, and convection parameters on the velocity, volume fraction of nanoparticles, temperature, and mass and heat transfer rates. The nanomaterial utilization to address the magnetic field influences on fluid flow is made by Sheikholeslami and Ganji [7]. Ramesh et al. [8] gave the modeling of nanofluid flow through microchannel. The 2D stagnation flows with the influence of radiation are illustrated by Shafiq et al. [9]. There calculated results suggest that whereas temperature distribution exhibits growing behavior for greater homogenous heat parameters, velocity distribution exhibits a decaying behavior for higher estimations of the magnetic component. The radiative heat transport in fluid flow with nano-particles is illustrated by Madhukesh et al. [10]. They made a finding that the improved nanoliquid showed enhanced heat transfer for an opposing flow scenario when measured against enhanced radiation parameter values.

An inclusion of nanoparticles into ordinary fluid is referred as “nanofluid” [11]. Copper, carbon, aluminium, and other substances that are chemically stable are employed to create the nanoparticles. The promise of nanofluids technology and concentrating on its specialized industrial applications has been growing in recent years. Materials with high thermal conductivity transfer heat more quickly. Examples of such materials include metals (copper), natural diamonds, graphene, silver, gold, aluminum (pure), aluminum (alloys), lead, and stainless steel. Titania nanoparticles, sometimes referred to as titanium dioxide nanoparticles, are particles of titanium dioxide (TiO_2) that are on the nanoscale. Due to their tiny size, vast surface area, and quantum effects, they have distinct physical and chemical characteristics. Titania nanoparticles are highly photocatalytic, or able to accelerate chemical processes in the presence of light. Because of this ability, they are beneficial in a variety of applications, including the management of air pollution and the purification of water. Copper nanoparticles are tiny particles of elemental copper (Cu). Because of their tiny size and enhanced surface-to-volume ratio, they have different characteristics when compared to bulk copper. Due to the great electrical conductivity that copper nanoparticles possess, they are well suited for use in conductive inks, printed electronics, and electrical interconnects. Animasaun et al. [12] studied the unsteady water-based hybridized nanofluids with wall stretching velocity and bioconvection on wedges. They observed that, due to the increased ternary hybrid nanofluids velocity being perpendicular to or parallel to the wedge, increasing stretching at the wall of the wedge. Sheikholeslami and Jafaryar [13] examined using complex helical turbulators and *CNT* nanoparticles, thermal analysis of a solar concentrator system. They noticed that the impact of adding nanoparticles is 1.72 times greater for the largest flow rate Q than for $Q = 8$. Sheikholeslami and Jafaryar [14] explored the efficiency of an energy storage device with vase-shaped fins made of paraffin has been improved with nanoparticles. Steel was used to make the fins, and paraffin characteristics were enhanced by adding SiO_2 nanoparticles. A single-phase approach must be used to forecast the characteristics of NEPCM. Sheikholeslami [15] investigated the solar system provided with a hybrid nanofluid and innovative turbulator. They showed that as the pitch ratio decreases, hybrid nanofluid mixing becomes better, the convective factor increases by about 6.43%, and as a result, pumping power increases by around 30.67%. Sheikholeslami [16] considered the storage of solar energy via double pipe employing nanoparticles for the melting expedition. They proposed the combine thermal storage with a solar concentrating unit. It is observed that the melting duration decreases by around 4.79% when *ZnO* nanoparticles are added, helping to improve the conduction mode.

Researchers have lately experimented with hybrid nanofluids, which are created by suspending distinct nanoparticles in a composite or mixture form, as a continuation of their work on nanofluids. Hybrid nanofluids aim to further enhance the features of drop in pressure and heat transmission by balancing the advantages and disadvantages of distinct suspensions. This is made feasible by the nanomaterials' combined effects, good aspect ratio, and enhanced thermal network. Sheikholeslami [17] analyzed energy storage systems with paraffin and *ZnO* nano-powders mixture with porous media. They noticed that applying a wire mesh technique can be a good way to speed up the process, and combining *ZnO* nanomaterial can be seen as an appropriate addition to this. On horizontal surfaces, Animasaun et al. [18] reported the impact of convective and unsteady acceleration on the Darcy flow of ternary-hybrid nanofluid. They concluded that the leading-edge accretion caused the higher convective acceleration. The analysis of hydrothermal for parabolic solar unit through nanomaterial and wavy absorber pipe is reported by Sheikholeslami et al. [19]. Xiu et al. [20] examined the ternary-hybridized nanofluids when the nanoparticle volume is large and small because of dual stretching on wedge surfaces. They observed that the dispersion of species that make up each water-based ternary hybrid nanofluid grows with time, regardless of how big or small the volume of the nanoparticles is. Sheikholeslami [21] analyzed the melting process of paraffin over heat storage employing nanoparticles with a configuration of honeycomb. They proposed a unique honeycomb shape for a heat storage system. In order to get a higher condition mode and quicker charging, alumina nanoparticles are added to paraffin.

Recent years have seen a great deal of interest in thermal and mass transport. Mass and heat transfer happen instantaneously during procedures including evaporation, drying, energy transfer in cooling towers, and movement in desert coolers. Many natural processes and industrial uses, including polymer curing, materials chemical processing, pulp production, and insulated cable manufacturing, include natural convection processes employing combined mechanisms. Gowda et al. [22] described the chemical reactive mass transmission analysis of non-Newtonian nanomaterial flow with energy of activation. They achieved that the velocity gradient reduces and the efficiency of heat transmission improves as the porosity parameter values rise. Alsulami et al. [23] studied the wall jet flow of nanoparticles over a surface. They obtained that due to suction, the rate of heat transfer across a sheet surface rise. Kumar et al. [24] scrutinized the mass heat transportation on Casson fluid flowing over thin needle. The findings show that while an increase in the Dufour number reduces heat transmission, mass transfer exhibits the opposite tendency. Mass transport is improved by rising thermophoretic parameter values. The mass heat transfer of water-based nanomaterial flowing through permeable medium was addressed by Haq et al. [25]. The rising heat generation constraint values improved the temperature profile. Nagapavani et al. [26] inspected the thermal features of nanomaterial flow with the influence of a non-uniform heat generation/absorption. They noticed that heat transfer is subsequently increased with larger estimates of the heat generation/absorption parameter.

The fluid flow nature through moving plate is affected by a variety of variables, such as orientation, speed, and viscosity of the fluid. Many industrial processes, including hot rolling, continuous casting wire drawing, and hot extrusion, involve heat transfer and momentum from a hot moving surface to an ambient medium. Algehyne et al. [27] considered the Blasius-Sakiadis Casson hybridized nanofluid. The findings demonstrate that fluid transfer of heat is increased by rising the radiation constraint. Ferdows et al. [28] studied the nanofluid flow with heat generation and suction. Temperature is continuously increased by increasing Eckert numbers for both types of nanofluids, while it is significantly decreased by increasing radiation term values. Kumar et al. [29] explored the transient thermal transmission induced by the exponential movement of sheet. The magnetic hybridized nanofluid flow with Joule dissipation is demonstrated by Khashi'ie et al. [30]. They noticed that the operation of heat transfer is improved and the critical value is extended by an increase in the magnetic parameter and suction's strength. Patel et al. [31] studied the boundary-driven flow of Williamson fluid. It demonstrates that as the values of Williamson parameter are increased, the velocity field is reduced.

Based on the literature review mentioned above, the viscous dissipation influences on the hybridized nanomaterial flow over static/moving plate have not been studied. To overcome this research gap, the present study explores boundary-layer laminar flow of static or moving vertical flat plate in a water-based hybrid nanofluid with the impact of viscous dissipation. The hybrid nanofluid flow with viscous dissipation through the static or moving vertical flat plate using numerical methods is noteworthy because it provides intuitions into fluid transport and heat transfer characteristics of these fluids, which can have practical applications in various fields. The insights that may be obtained by examining this movement have the potential to influence the creation of innovative technologies. It has the potential to serve as a source of inspiration for the design of innovative cooling systems, heat exchangers, and other solutions for thermal management that make use of nanofluids to increase both performance and efficiency.

2. Mathematical formulation

Contemplate the steady two-dimensional, boundary-layer laminar flow of static or moving vertical flat plate in a water-based hybrid nanofluid with the impact of viscous dissipation (see Fig. 1). Assuming that the plate travels in the same or reverse direction to the free-stream with steady velocities. Assume that the plate temperature is at consistent value T_m.

The equations representing the fluid flow and energy equation for the thermal distribution inside the moving plate are first attained by using the boundary layer estimates. Under these circumstances, the governing equations are described as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{hnf}}{\rho_{hnf}}\left(\frac{\partial^2 u}{\partial y^2}\right)+\frac{g(\rho\beta)_{hnf}(T-T_\infty)}{\rho_{hnf}}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{hnf}}{(\rho C_p)_{hnf}}\left(\frac{\partial^2 T}{\partial y^2}\right)+\frac{\mu_{hnf}}{(\rho C_p)_{hnf}}\left(\frac{\partial u}{\partial y}\right)^2, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_{hnf}\left(\frac{\partial^2 C}{\partial y^2}\right). \tag{4}$$

Boundary conditions are given by

$$y=0: u=U_w, v=0, T=T_m, C=C_m,$$

$$y\to\infty: u=U_\infty, T=T_\infty, C=C_\infty. \tag{5}$$

Dimensionless stream function ψ and similarity coordinate η are as follow:

$$\eta=\sqrt{\frac{U}{\nu_f x}}y, \qquad \psi=\sqrt{\nu_f xU}\,f(\eta),$$

where the composite velocity is defined as $U=U_w+U_\infty$. Further, we defined $u=\frac{\partial\psi}{\partial y}$, $v=-\frac{\partial\psi}{\partial x}$, then we have $u=Uf'(\eta)$, $v=\frac{1}{2}\sqrt{\frac{\nu_f U}{x}}(\eta f'(\eta)-f(\eta))$, $\theta(\eta)=\frac{T-T_\infty}{T_m-T_\infty}$, $\chi(\eta)=\frac{C-C_\infty}{C_m-C_\infty}$ which satisfies equation (1).

The Thermal Characteristics of hybrid nanofluids are given by:

$$\frac{\rho_{hnf}}{\rho_f}=(1-\phi_2)\left[(1-\phi_1)+\phi_1\frac{\rho_{s_1}}{\rho_f}\right]+\phi_2\frac{\rho_{s_2}}{\rho_f},$$

$$\frac{(\rho C_p)_{hnf}}{(C_p\rho)_f}=(1-\phi_2)\left[(1-\phi_1)+\phi_1\left(\frac{(\rho C_p)_{s_1}}{(\rho C_p)_f}\right)\right]+\phi_2\frac{(\rho C_p)_{s_2}}{(\rho C_p)_f},$$

$$\frac{(\rho\beta)_{hnf}}{(\rho\beta)_f}=(1-\phi_2)\left[(1-\phi_1)+\phi_1\left(\frac{(\rho\beta)_{s_1}}{(\rho\beta)_f}\right)\right]+\phi_2\frac{(\rho\beta)_{s_2}}{(\rho\beta)_f},$$

$$\frac{k_{hnf}}{k_{bf}}=\frac{-2\phi_2(k_{bf}-k_{s_2})+k_{s_2}+2k_{bf}}{\phi_2(k_{bf}-k_{s_2})+k_{s_2}+2k_{bf}},\frac{k_{bf}}{k_f}=\frac{k_{s_1}+2k_f-2\phi_1(k_f-k_{s_1})}{k_{s_1}+2k_f+\phi_1(k_f-k_{s_1})},$$

$$\mu_{hnf}=\frac{\mu_f}{(1-\phi_2)^{2.5}(1-\phi_1)^{2.5}},D_{hnf}=(1-\phi_2)^{2.5}(1-\phi_1)^{2.5}D_f.$$

(6)

Using similarity variables and boundary conditions equation (1-4) are reduced as follows:

$$\varepsilon_1 f'''+\frac{1}{2}ff''+\varepsilon_2 gr\theta=0, \quad (7)$$

$$\varepsilon_3\frac{k_{hnf}}{k_f}\left(\frac{1}{Pr}\right)\theta''+\frac{1}{2}f\theta'+\varepsilon_4 Ec(f'')^2=0, \quad (8)$$

$$(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}\left(\frac{1}{Sc}\right)\chi''+\frac{1}{2}f\chi'=0, \quad (9)$$

where

$$\varepsilon_1=\frac{1}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}\left[(1-\phi_2)\left[(1-\phi_1)+\phi_1\frac{\rho_{s1}}{\rho_f}\right]+\phi_2\frac{\rho_{s2}}{\rho_f}\right]},$$

$$\varepsilon_2=\frac{(1-\phi_2)\left[(1-\phi_1)+\phi_1\left(\frac{(\rho\beta)_{s1}}{(\rho\beta)_f}\right)\right]+\phi_2\frac{(\rho\beta)_{s2}}{(\rho\beta)_f}}{(1-\phi_2)\left[(1-\phi_1)+\phi_1\frac{\rho_{s1}}{\rho_f}\right]+\phi_2\frac{\rho_{s2}}{\rho_f}},$$

$$\varepsilon_3 = \frac{1}{\left(1-\phi_2\right)\left[\left(1-\phi_1\right)+\phi_1\left(\frac{\left(\rho Cp\right)_{s1}}{\left(\rho Cp\right)_f}\right)\right]+\phi_2\frac{\left(\rho Cp\right)_{s2}}{\left(\rho Cp\right)_f}},$$

$$\varepsilon_4 = \frac{1}{\left(1-\phi_1\right)^{2.5}\left(1-\phi_2\right)^{2.5}\left[\left(1-\phi_2\right)\left[\left(1-\phi_1\right)+\phi_1\left(\frac{\left(\rho Cp\right)_{s1}}{\left(\rho Cp\right)_f}\right)\right]+\phi_2\frac{\left(\rho Cp\right)_{s2}}{\left(\rho Cp\right)_f}\right]}.$$

Reduced boundary conditions are:

$$f(0)=0,\ f'(0)=m_1,\theta(0)=1,\chi(0)=1.$$

$$f'(\infty)=1-m_1,\theta(\infty)=0,\chi(\infty)=0. \tag{10}$$

Dimensionless parameters are:

$$\mathrm{Re}=\frac{Ux}{\nu_f},\ \ m_1=\frac{U_w}{U},\ \ Gr=\frac{g\beta_f(T_m-T_\infty)x^3}{\nu_f{}^2},\ \ gr=\frac{Gr}{\mathrm{Re}^2},$$

$$Ec=\frac{U^2}{C_p(T_m-T_\infty)},\ \ \mathrm{Pr}=\frac{\nu_f(\rho C_p)_f}{k_f},\ \ Sc=\frac{\nu_f}{D_f}.$$

The engineering quantities namely the skin-friction coefficient, the local Nusselt and Sherwood numbers by using the similar variables are reported as:

$$\mathrm{Re}^{1/2}C_f=\frac{1}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}}f''(0),\ \ \mathrm{Re}^{-1/2}Nu=-\frac{k_{hnf}}{k_f}\theta'(0),\ \ \mathrm{Re}^{-1/2}Sh=-\chi'(0).$$

3. Results and discussion

In this section, boundary-layer laminar flow of static or moving vertical flat plate in a water-based hybrid nanofluid is scrutinized by considering viscous dissipation into the account. The stationary surface has uniform free stream velocity $m_1=0$ (Blasius flow) and uniformly moving plane surface moving in a stagnant free stream $m_1=1$ (Sakiadis flow). The numerical analysis of velocity, thermal and concentration gradients subjected to selected values of pertinent parameters are displayed with the help of graphs. Tables 1 is constructed for comparative table of present results with existing work and obtained a good arrangement with each other. Thermo physical features of water, Cu and TiO_2 are displayed in Table 2.

Figs. 2-4 scrutinize the impact of buoyancy parameter over velocity, thermal and concentration profile. Keen observation is made by varying values of Grashof number. Grashof number is directly proportional to the buoyancy parameter, hence enhance in value of Grashof number shoot-ups the value of buoyancy parameter. It can be observed that rise in value of buoyancy parameter increases the velocity (Fig. 2) of the fluid motion in stationary surface with a uniform free stream velocity $m_1 = 0$ and vertically moving plane surface moving in a stagnant free stream $m_1 = 1$ and decreases the thermal (Fig. 3) and concentration (Fig. 4) rate in both the fluid flows.

Fig. 5 explains the influence of Ec subjected to temperature profile. The graph is depicted for numerous values of Ec over $\eta.$ It is noted that an enhancement in Ec boosted the temperature gradually. The reason behind this is that the liquid viscosity attains energy from the flow which transforms it into internal energy and leads to rise in thermal profile for both Blasius-Sakiadis flows. Fig. 6 illustrates the influence of Sc over concentration gradients for stationary surface with a uniform free stream velocity $m_1 = 0$ and vertically moving plane surface moving in a stagnant free stream $m_1 = 1.$ Result outcome reveals that enhance in value of Sc declines the concentration gradient. Schmidt number is defined as the ratio of viscosity and mass diffusivity. Higher values of Sc decline the mass diffusivity of the fluid, which leads to decline in concentration rate. Fig. 7 illustrated the energy transmission rate (Nusselt number) for various values of volume fraction against Eckert number. From the graph, it is noticed that the increase in value of Ec gradually declines the heat transfer rate in both the flows.

Figs. 8 and 9 illustrate the designed model of convergence of validation, testing, and training progressions against epochs indexes on $\mathrm{Re}^{-1/2} Nu$ for two various cases $m_1 = 0$ and $m_1 = 1$. The best performance can be seen in epochs 21 and 346, with MSE values of $1.6195e-11$ and $8.9417e-12$, respectively. Figs. 10 and 11 depict the convergence efficiency, correctness, and accuracy in solving the specified flow model on $\mathrm{Re}^{-1/2} Nu$ for two various cases $m_1 = 0$ and $m_1 = 1$. Mu is the gradient and step size in these instances, as well as during training gradient in verdict another vector. The graph depicts the corresponding gradient values for the expected flow. The graph also illustrates that as the epoch grows, the values of gradient and Mu decrease, the greater network testing and training can yield greater convergence of the findings for the smallest Mu and gradient value. The error dynamics examination yields error histograms (Figs. 12 and 13) on $\mathrm{Re}^{-1/2} Nu$ for two various cases $m_1 = 0$ and $m_1 = 1$, respectively. In addition to illuminating the error box of reference for the planned flow, a thorough examination of the error histogram will make it abundantly clear how many error values are far higher than the zero axis. Figs. 14 and 15 show the regression analysis for training, validation, testing, and total data. Throughout the operation, the output and target values are considered to be connected with a regression value of $R = 1$.

The overall performance of the ANN model for predicting the $\mathrm{Re}^{-1/2} Nu$ for two various cases $m_1 = 0$ and $m_1 = 1$ is $1.127561115472368e-11$ and $9.419129810612661e-12$, respectively, test performance for predicting the $\mathrm{Re}^{-1/2} Nu$ for two various cases $m_1 = 0$ and $m_1 = 1$ is $1.346232195441596e-11$ and $1.402290466594002e-11$ respectively, train performance for predicting the $\mathrm{Re}^{-1/2} Nu$ for two various cases $m_1 = 0$ and $m_1 = 1$ is

$9.757268756916470e-12$ and $m_1 = 8.537426668595055e-12$ respectively, validation performance for predicting the $\mathrm{Re}^{-1/2} Nu$ for two various cases $m_1 = 0$ and $m_1 = 1$ is $1.619474277676912e-11$ and $8.941725659927694e-12$ respectively.

4. Conclusions

The Blasius-Sakiadis copper-titania hybrid nanofluid flow induced due to movement of plate is illustrated. The numerical scheme is adopted for the illustration of the results in present case. The results are displayed for both Blasius $(m_1 = 0)$ and Sakiadis $(m_1 = 1)$ flow situations. To build heat transfer systems that are more effective, it is helpful to have a better understanding of the heat transfer properties of hybrid nanofluid flows of copper and titanium. Researchers are able to determine the factors that contribute to better convective heat transfer by focusing their attention on this particular flow. This allows them to develop more efficient heat exchangers and cooling systems. Results reveals that the rise in buoyancy parameter augmented the velocity profile in stationary surface having constant free-stream velocity $m_1 = 0$ and vertical moving planer surface in stagnant free-stream $m_1 = 1$. The increase in value of Ec gradually declines the heat transmission rate in both flow cases. The graph also illustrates that as the epoch grows, the gradient values and Mu decrease. Greater network training and testing can yield greater convergence of the findings for the smaller Mu and gradient values.

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Figure Captions

Fig. 1: Physical illustration of present problem.

Fig. 2: Influence of gr on f' when $m_1 = 0$ and $m_1 = 1$.

Fig. 3: Influence of gr on θ when $m_1 = 0$ and $m_1 = 1$.

Fig. 4: Influence of gr on χ when $m_1 = 0$ and $m_1 = 1$.

Fig. 5: Influence of Ec on θ when $m_1 = 0$ and $m_1 = 1$.

Fig. 6: Change in concentration profile against Sc when $m_1 = 0$ and $m_1 = 1$.

Fig. 7: Change in behaviour of Nusselt number over Ec for numerous values of $\phi_1 = \phi_2 = 0.05, 0.1, 0.15, 0.2, 0.25$ when $m_1 = 0$ and $m_1 = 1$.

Fig. 8: An examination of the MSE's outcomes about when $\mathrm{Re}^{-1/2}\, Nu$ for $m_1 = 0$.

Fig. 9: An examination of the MSE's outcomes about $\mathrm{Re}^{-1/2}\, Nu$ for $m_1 = 1$.

Fig. 10: An investigation into the transitional states about $\mathrm{Re}^{-1/2}\, Nu$ for $m_1 = 0$.

Fig. 11: An investigation into the transitional states about $\mathrm{Re}^{-1/2}\, Nu$ for $m_1 = 1$.

Fig. 12: Analysis of Error Histograms on $\mathrm{Re}^{-1/2}\, Nu$ for $m_1 = 0$.

Fig. 13: Analysis of Error Histograms on $\mathrm{Re}^{-1/2}\, Nu$ for $m_1 = 1$.

Fig. 14: Regression analysis on $\mathrm{Re}^{-1/2} Nu$ for $m_1 = 0$.

Fig. 15: Regression analysis on $\mathrm{Re}^{-1/2} Nu$ for $m_1 = 0$.

Table Captions

Table 1: Comparison of $f''(0)$ values for some reduced cases.

Table 2: Thermophysical characteristics of water, Cu and TiO_2 [36-40].

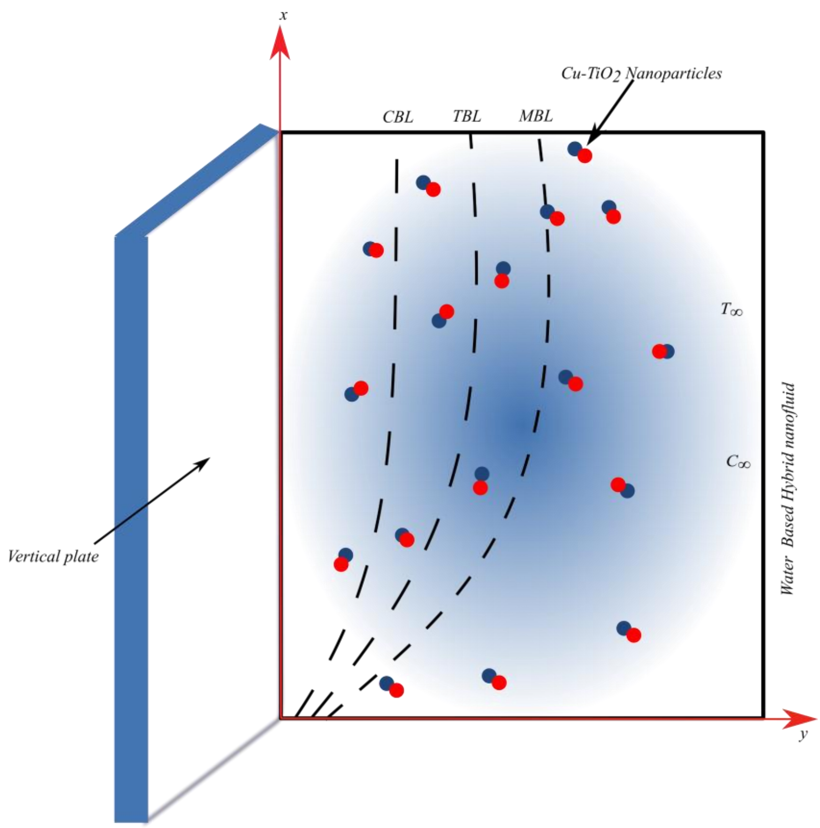

Fig. 1: Physical illustration of present problem.

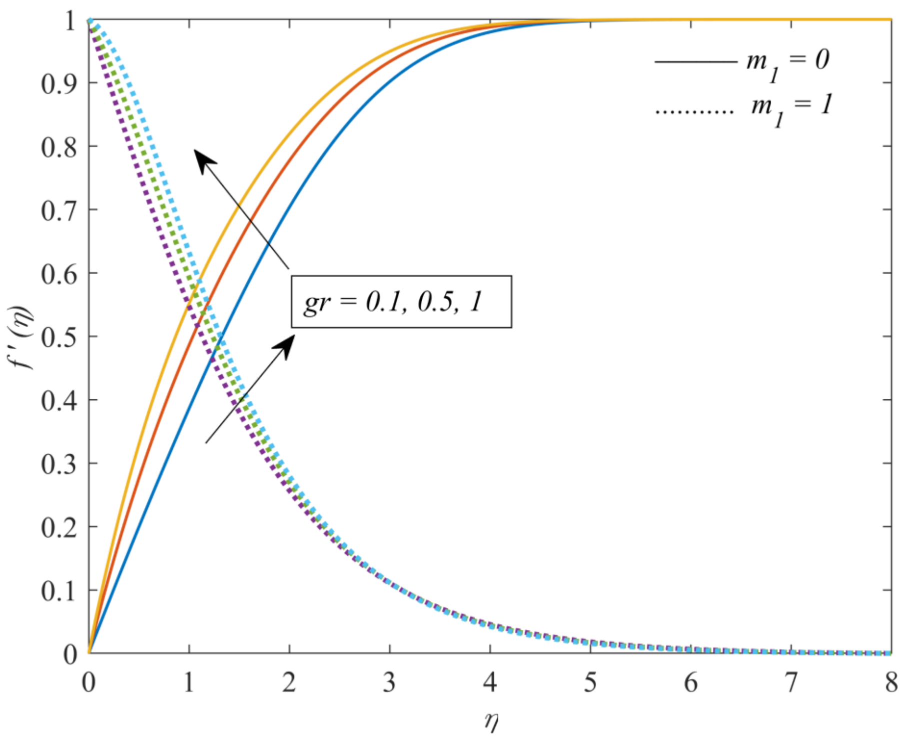

Fig. 2: Influence of gr on f' when $m_1 = 0$ and $m_1 = 1$.

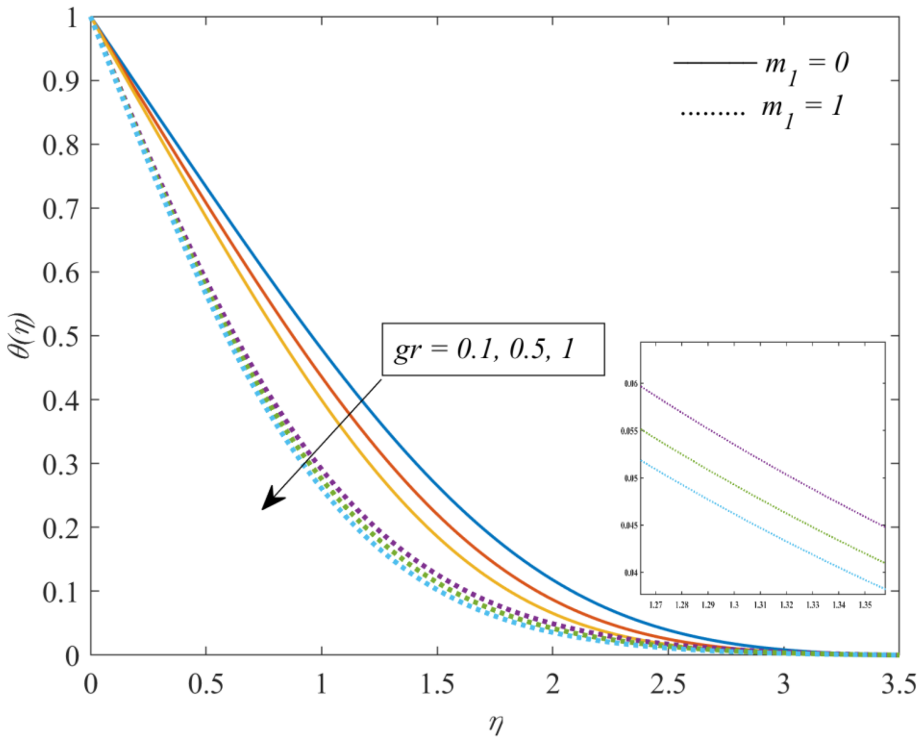

Fig. 3: Influence of gr on θ when $m_1 = 0$ and $m_1 = 1$.

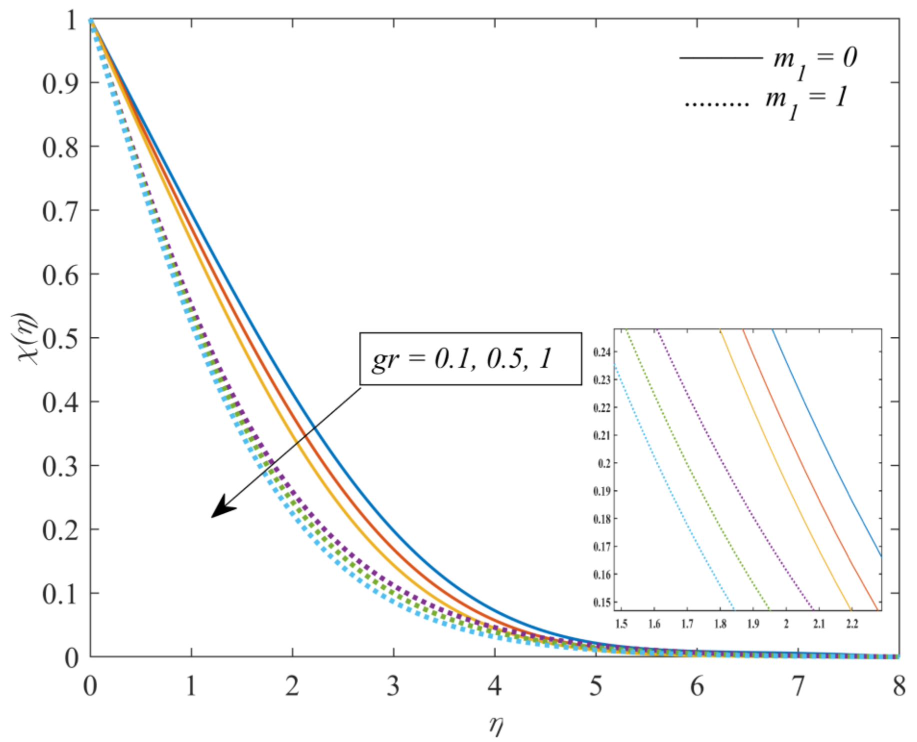

Fig. 4: Influence of gr on χ when $m_1 = 0$ and $m_1 = 1$.

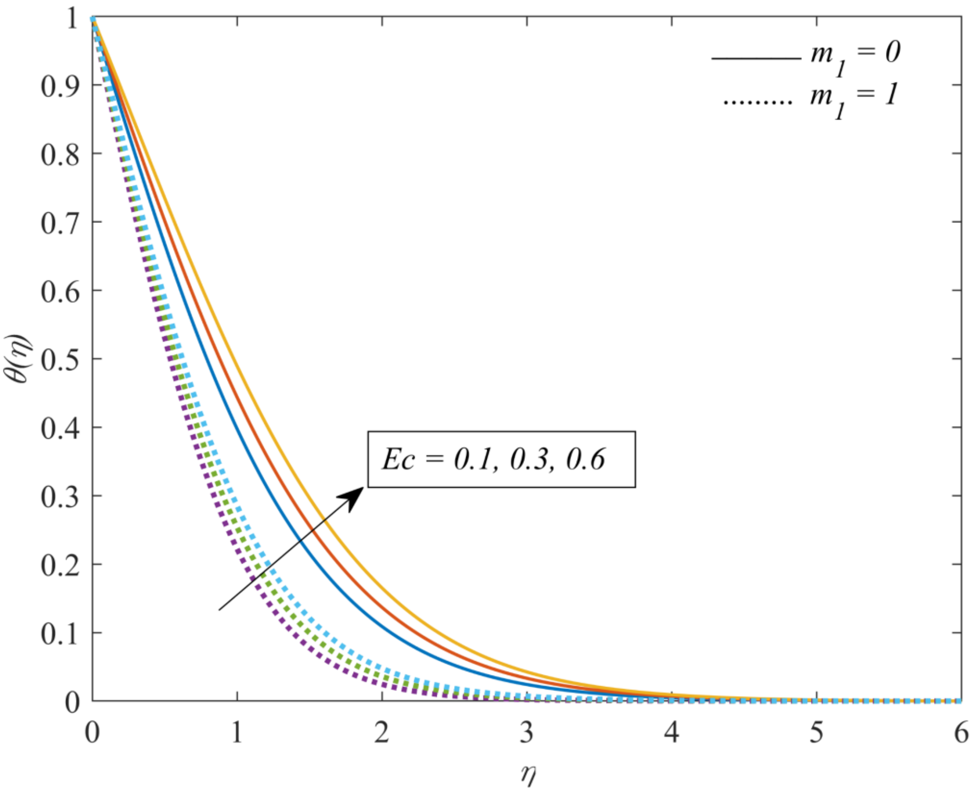

Fig. 5: Influence of Ec on θ when $m_1 = 0$ and $m_1 = 1$.

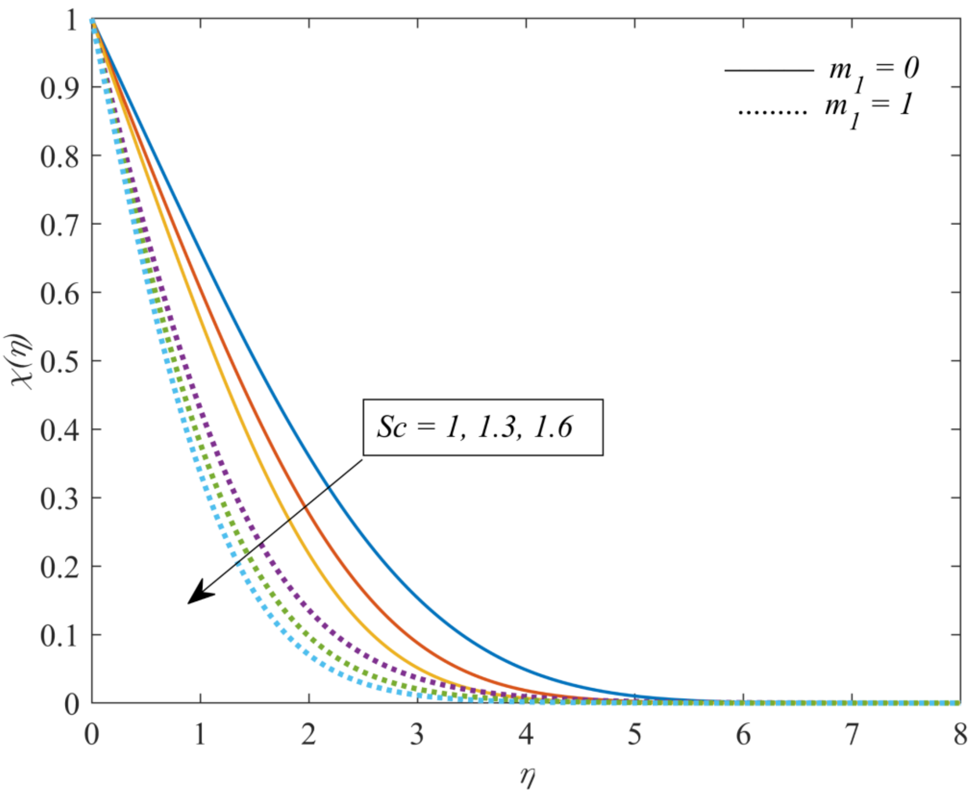

Fig. 6: Change in concentration profile against Sc when $m_1 = 0$ and $m_1 = 1$.

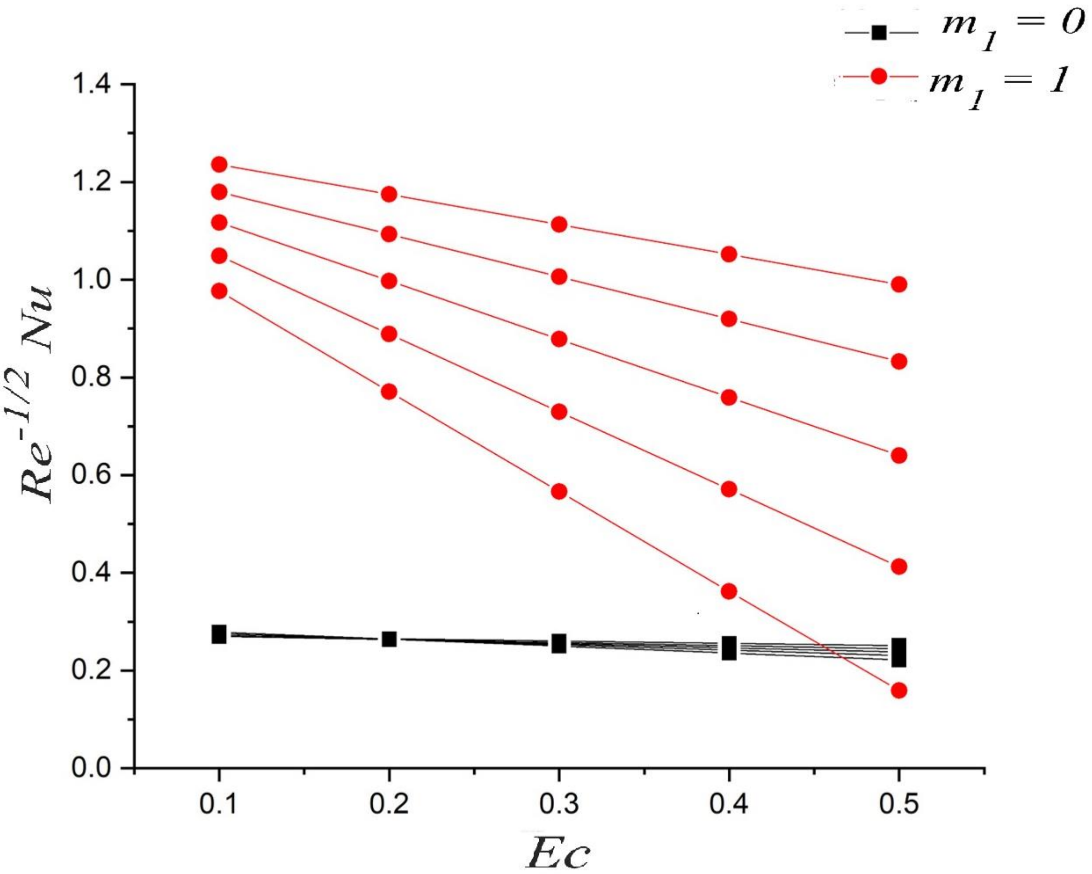

Fig. 7: Change in behaviour of Nusselt number over Ec for numerous values of

$\phi_1 = \phi_2 = 0.05, 0.1, 0.15, 0.2, 0.25$ when $m_1 = 0$ and $m_1 = 1$.

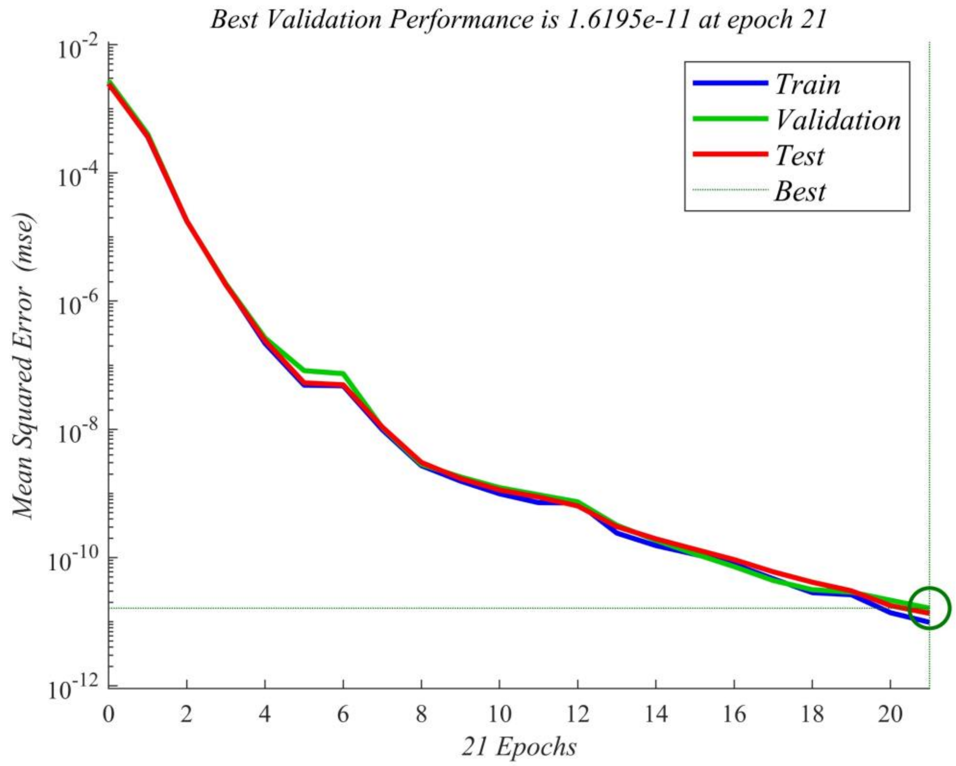

Fig. 8: An examination of the MSE's outcomes about when $\mathrm{Re}^{-1/2}\,Nu$ for $m_1 = 0$.

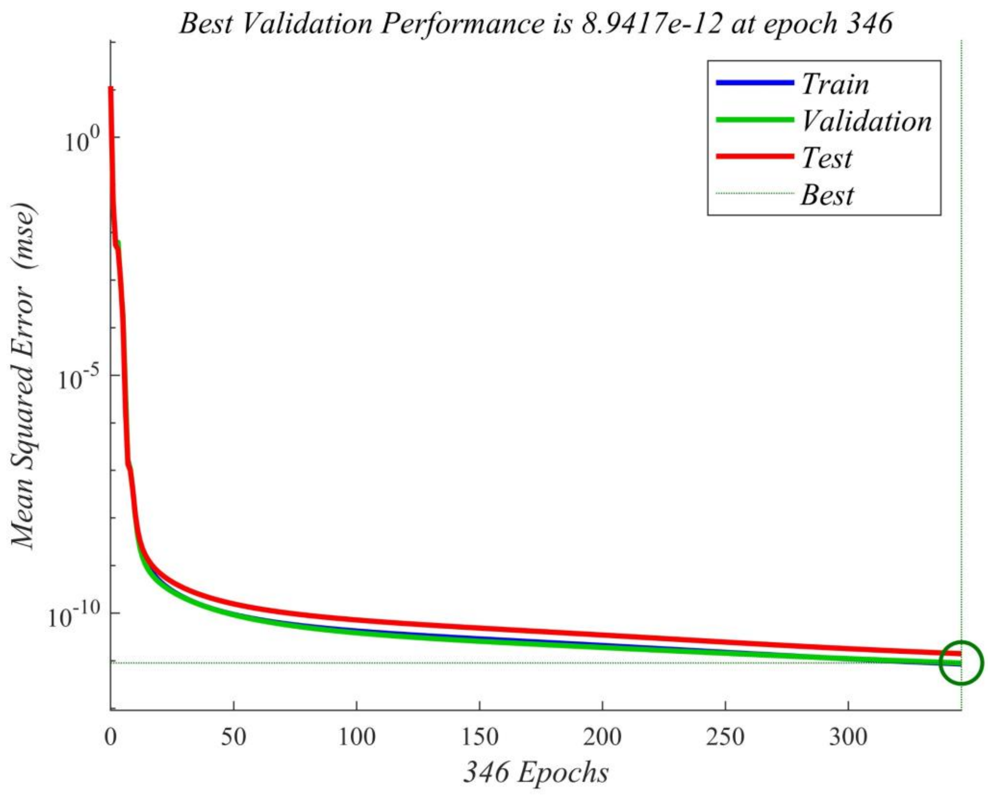

Fig. 9: An examination of the MSE's outcomes about $\mathrm{Re}^{-1/2}\, Nu$ for $m_1 = 1$.

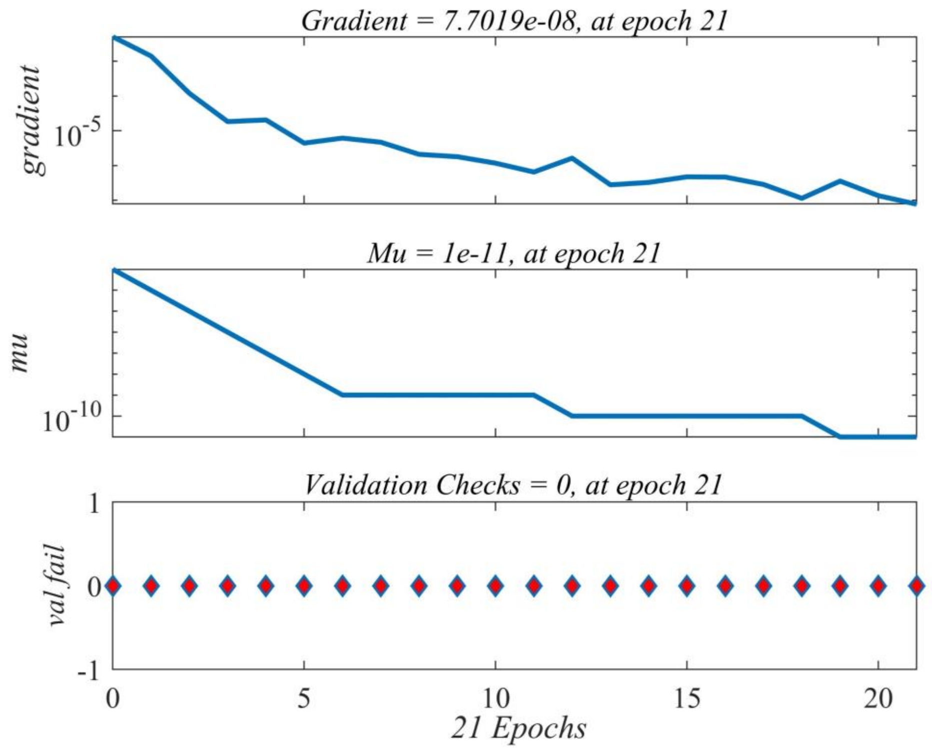

Fig. 10: An investigation into the transitional states about $\mathrm{Re}^{-1/2}\, Nu$ for $m_1 = 0$.

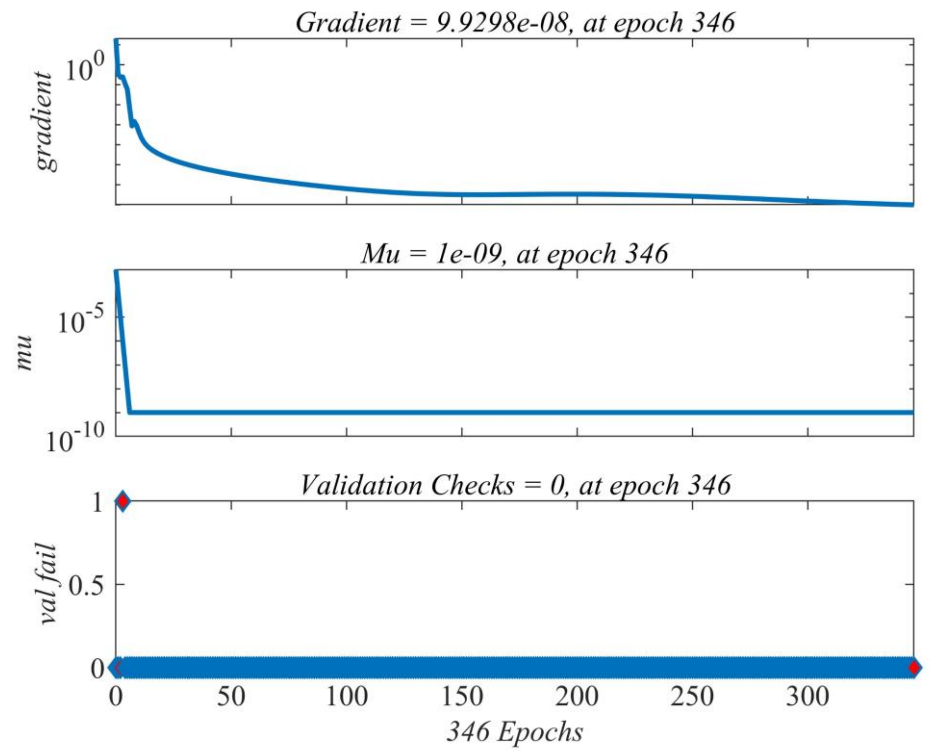

Fig. 11: An investigation into the transitional states about $\mathrm{Re}^{-1/2}\,Nu$ for $m_1 = 1$.

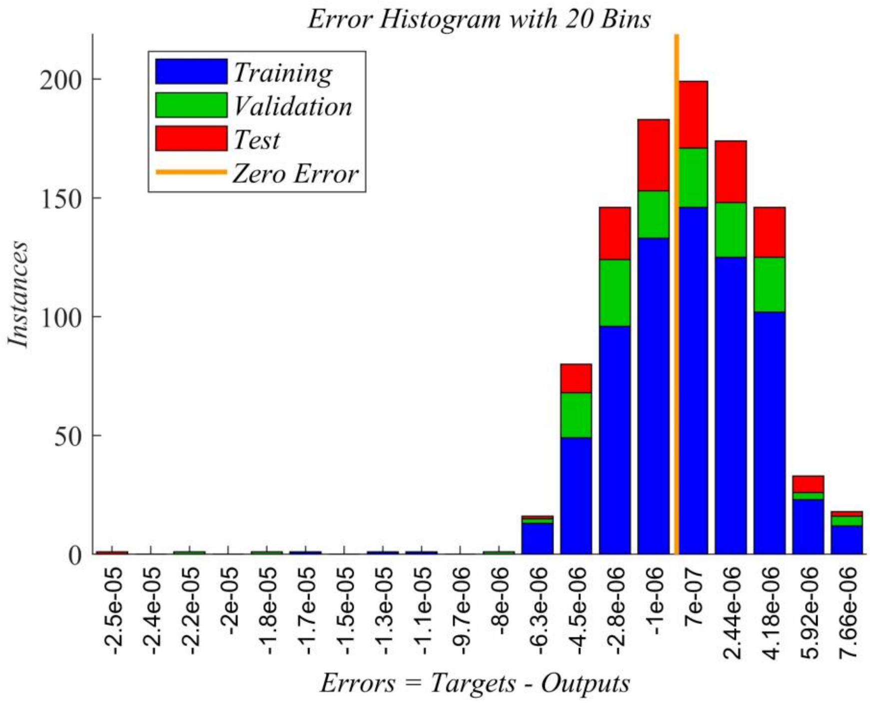

Fig. 12: Analysis of Error Histograms on $\mathrm{Re}^{-1/2}\,Nu$ for $m_1 = 0$.

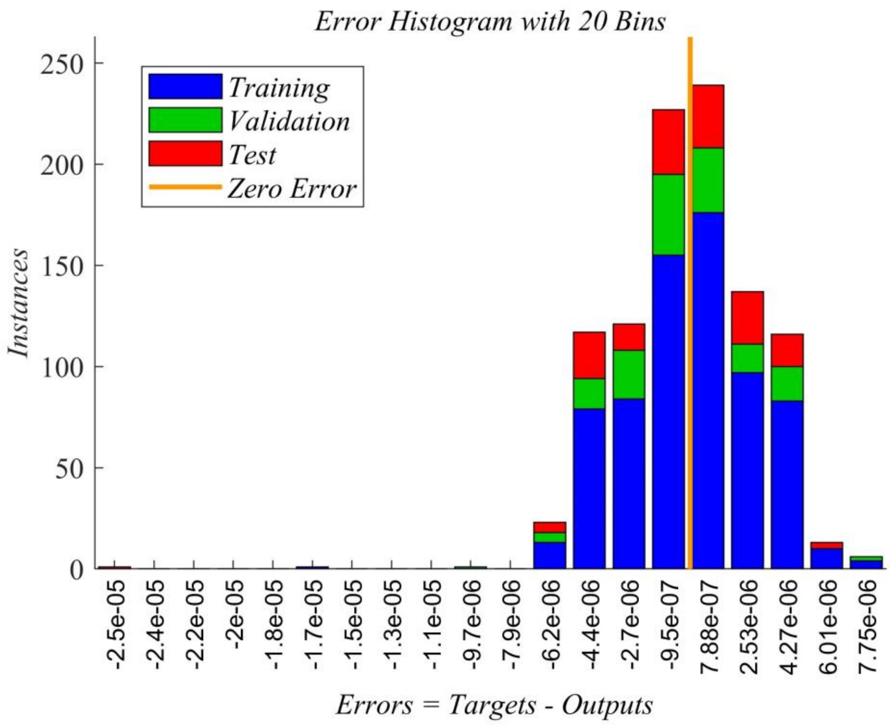

Fig. 13: Analysis of Error Histograms on $\mathrm{Re}^{-1/2}\,Nu$ for $m_1 = 1$.

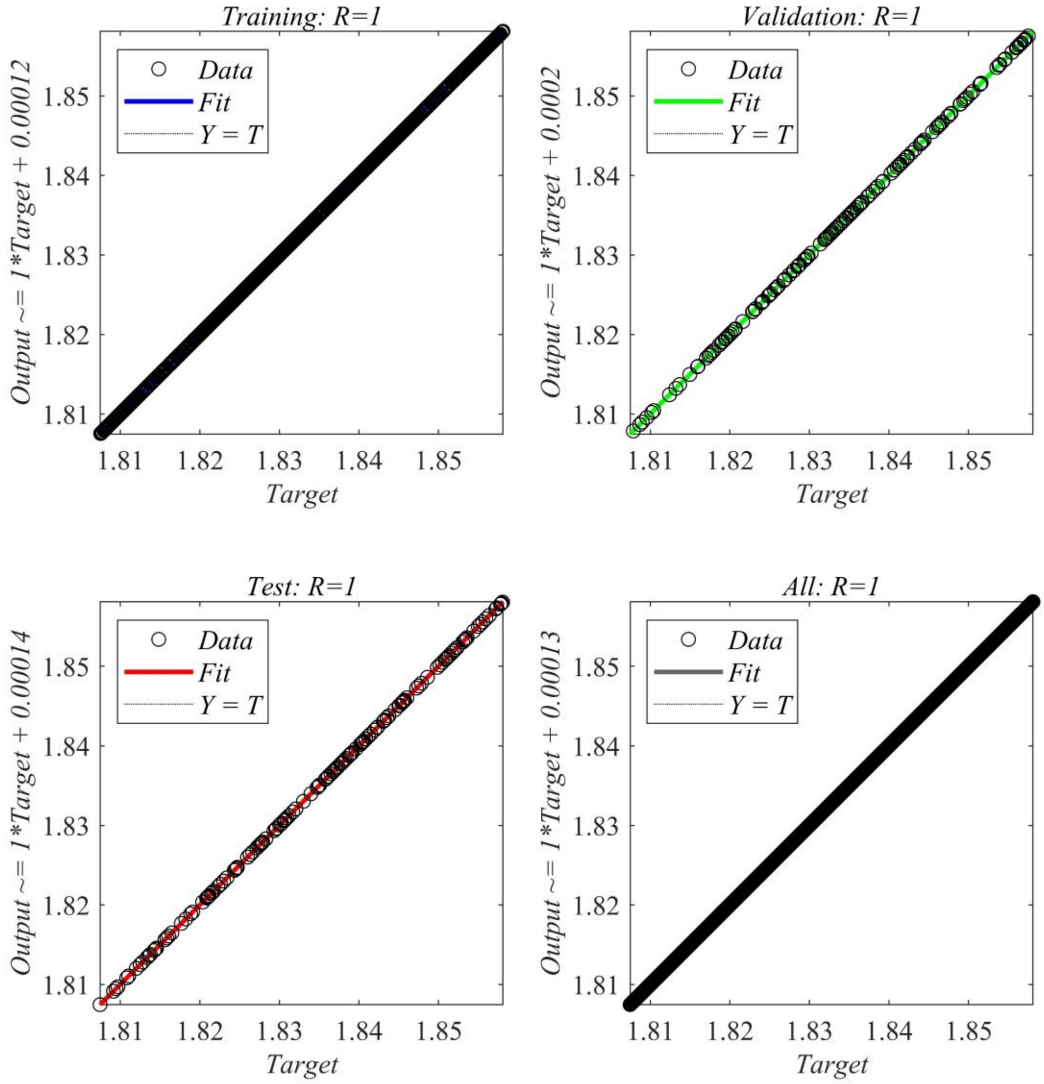

Fig. 14: Regression analysis on $\mathrm{Re}^{-1/2}\,Nu$ for $m_1 = 0$.

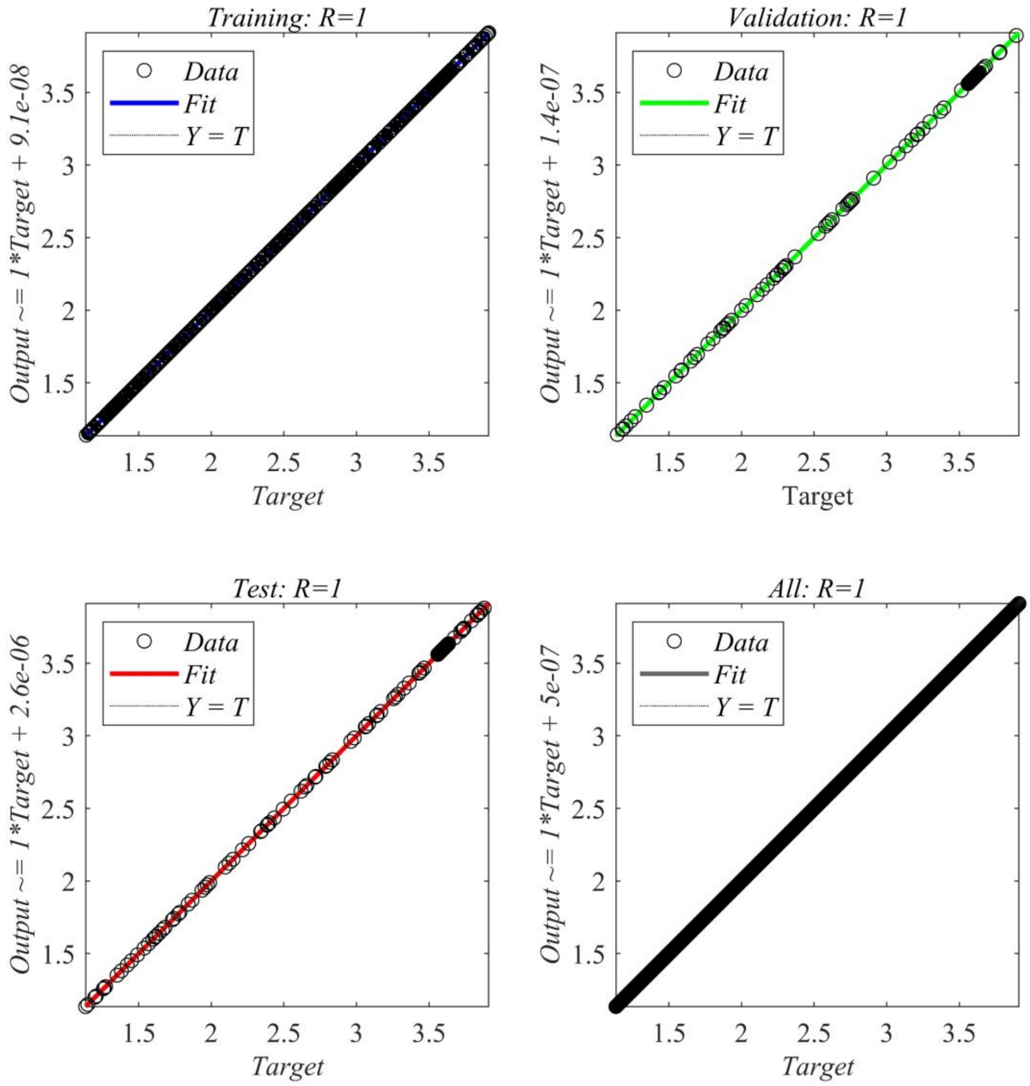

Fig. 15: Regression analysis on $\mathrm{Re}^{-1/2}\, Nu$ for $m_1 = 0$.

Table 1: Comparison of $f''(0)$ values for some reduced cases.

	$m_1 = 1$	$m_1 = 0$
Ref. [1]	-	0.332
Ref. [2]	-0.44375	-
Ref. [32]	-0.4438	0.3321
Ref. [33]	-0.4438	0.3321
Ref. [34]	-0.4438	0.3321
Ref. [35]	-0.443751	0.332057
Ref. [27]	-0.443832	0.332082
Present results	-0.443830	0.332078

Table 2: Thermophysical characteristics of water, Cu and TiO_2 [36-40].

Physical properties	Water	Cu	Cu
C_p	4,179	385	686.2
P	997.1	8,933	4,250
k	0.613	400	8.9538
β	21×10^{-5}	1.67×10^{-5}	0.9×10^{-5}

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