Dynamic behavior and stability of a discretely supported plate with a heat-proof coating under the action of an arbitrarily directed moving load

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Abstract

The problem of dynamic deformation of a thin plate lying on an elastic base and discretely supported by a system of stiffening ribs under the action of an arbitrarily directed moving load is approximated. The load is considered as an infinite uniformly distributed normal force, the front of which moves with a constant velocity at an arbitrary angle to the longitudinal axis of the plate. The elasticity of the foundation is considered within the Winkler hypothesis, and the discreteness of the fin arrangement is specified using generalized functions. There are two variants of solving the problem: quasi-static and dynamic. In the first one, the curved surface of the plate depends only on its longitudinal coordinates, while in the second one, it also depends on time. When using the dynamic solution, in addition to the deformed state of the ribbed plate, the frequencies of its natural oscillations, which are the most important dynamic characteristics of the structure, are also determined as an incidental result. Examples are considered. The results of the work can be used to predict the stress-strain state of thin-walled structures, including those with functional coatings.

Keywords: arbitrary direction of movement, critical speeds of movement, discrete stiffeners, dynamic deformation, elastic base, moving load, natural frequencies of vibrations, thin plate, two solution methods.

Nomenclature

V - velocity vector
g - gravity acceleration
b - stiffness coefficient of elastic foundation
C - number of stiffeners
$\delta(y-y_i)$ - Dirac delta functions

EJ_i - bending stiffness
U_i - stringer deflection respectively
w_{mn} - unknown coefficients
$\phi_{mn}(x,y)$ - orthogonal forms of natural vibrations of a smooth plate expressed in trigonometric functions
K matrix - block diagonal
A and B - integration constants determined from the initial conditions
w_* - partial solution
ω - the frequency of oscillation

Introduction

The problem under consideration arises during the operation of aerospace systems flying at ultrahigh speeds. In this case, a pressure wave arises moving along their skin, causing a number of extremely unfavourable effects. They are associated with dynamic deformation of structural elements made in the form of thin plates or shells discretely reinforced with stiffeners (stringers), as well as with a possible loss of their stability. All this can lead to the destruction of the aircraft. As a result of the foregoing, studies on the subject under consideration have not only specific technical applications, but are also relevant in the design of new generation aircraft systems.

In the proposed work, the pressure wave is treated as a moving load, and the heat-shielding coating is considered as an elastic foundation. The moving load causes significant inertial forces, which can cause buckling of structural cladding elements. To avoid it, the plates are additionally reinforced by a system of discretely located stringers, which increase the "rigidity" of the structure. This is also partially facilitated by the presence of a heat-shielding coating. All these factors determine the scientific novelty of the work.

In the scientific literature, the problems of the action of moving loads on beams have been studied in sufficient detail. In [1], the action of a moving load on smooth plates without stringers was studied, and in [2] a similar problem was solved, but simpler. The monograph [3] solves in different formulations and variants of the problem of the action of a moving load on one-dimensional elements such as beams or rods. In the proposed article, for the first time, an attempt was made to consider the dynamic state of locally reinforced plates with a heat-shielding coating applied to them [3]. Such structural elements are widely used in space technology, and problems with their thermal insulation led to unpredictable consequences, an example of which is the disaster with the Space Shuttle Columbia, which killed six astronauts. The papers [4], [5] analyzed the possibility of dynamic stability of elastic systems. Articles related to the study of the stress-strain state under the action of a moving load are [6], [7], [8]. The novelty of the proposed work is in the fact that for the first time it considers the problem of dynamic stability of a ribbed plate under the action of a moving load, the front of which is arbitrarily directed relative to its axis (Figure 1). In the works of other authors [9], [10], [11], only smooth (without stringers) plates were considered in the case of load movement along their axes. A change in the direction of the front of movement of forces leads to a change in critical speeds, which is also studied in [12], [13]. Works [14], [15] summarize the previous studies.

In the proposed paper, we consider the same problem of moving load on a ribbed plate, but in a more general formulation, when the pressure wave front is arbitrarily directed towards its longitudinal axis (Figure 1), and the plate is supported on an elastic base imitating a heat-shielding covering. In two variants the critical load velocities are determined, and in the dynamic solution the spectrum of natural frequencies of oscillations is also found. Both options for solving the problem ultimately lead to the same results for critical velocities, but the quasi-static approach is preferable, because it is less labor intensive. On the other hand, in the dynamic version of the solution, as a by-product, the spectrum of natural oscillation frequencies of the structure is additionally determined, which makes it possible to avoid its falling into resonant operating modes.

Theoretical basis

The solution of the problem is based on two approaches: quasi-static and dynamic. In both of them, the problem is reduced to a partial differential plate bending equation solved by the Bubnov method. In the quasi-static version of the solution, the problem is ultimately reduced to a system of linear algebraic equations and the critical velocity is determined from the condition that its determinant is zero. In the dynamic variant of solution, the problem is reduced to a system of differential equations, but already in usual derivatives, solved by numerical methods. Besides, in order to determine the critical mode of force movements, here we additionally employ the dynamic criterion of stability whereby the critical loads are determined under the condition of equality to zero of the plate natural frequencies of oscillations. The elasticity of the base (heat shield) is considered within the framework of the Winkler hypothesis, and the discreteness of the supporting ribs is specified using generalized Dirac functions. The formulas for the lower critical velocities are obtained in closed form in the one-manifold approximation.

Methodology

We consider a thin elastic rectangular plate lying on an elastic base with stiffness coefficient b and discretely supported by a system of elastic stiffening ribs (stringers), which are conventionally shown as solid lines in the Figure 1. We assume that the neutral lines of these ribs lie in the median surface of the plate. Therefore, they can be considered as one-dimensional elastic inclusions. The plate is referred to the Cartesian coordinate system 0xyz. An infinite uniformly distributed inertial load of intensity q moving with constant velocity V at an arbitrary angle alpha to the x-axis of the plate as a linear force acting at the front of its motion. It is conventionally represented in the Figure 1.

The velocity vector V is decomposed into components $V_X = V\cos\alpha$ and $V_Y = V\sin\alpha$ in the direction of the *x*- and *y*-axes, respectively. Then the distance travelled by the load element in the direction of these axes in time *t* will be $x = V_X t$ and $y = V_Y t$. Neglecting the weight of the plate, the load on the plate consists of the gravitational and inertial components of the moving forces, in the following form [5]:

$$q - \frac{q}{g}\left(\frac{\partial^2 w}{\partial t^2} + 2V_X\frac{\partial^2 w}{\partial x\partial t} + 2V_Y\frac{\partial^2 w}{\partial y\partial t} + V_X^2\frac{\partial^2 w}{\partial x^2} + 2V_X V_Y\frac{\partial^2 w}{\partial x\partial y} + V_Y^2\frac{\partial^2 w}{\partial y^2}\right), \tag{1}$$

where *g* is the gravity acceleration.

The components in this formula containing mixed derivatives correspond to Coriolis forces and are neglected in practical applications. Solving the problem in the contact statement, we mentally separate the stiffening ribs (stringers) from the plate and replace their influence distributed along the interaction lines of bodies $y = y_i$, $(y_i = 1, 2, ...C)$ in the median surface of the plate by the normal interaction reactions $p_i(x)$ directed along the *z*-axis. Since the stiffness of the plate in the tangential *x* and *y* directions is much greater than in the normal to its surface direction, the tangential contact reactions are further neglected. The problem then reduces to a partial differential bending equation of the plate [4] with discontinuous coefficients at an unknown deflection *w*, which is taking the following form:

$$D\nabla^2\nabla^2 w + bw = q - \frac{q}{g}\left(\frac{\partial^2 w}{\partial t^2} + V_X{}^2\frac{\partial^2 w}{\partial x^2} + V_Y^2\frac{\partial^2 w}{\partial y^2}\right) - \sum_i^C p_i\delta(y - y_i), \tag{2}$$

where *b* is the stiffness coefficient of elastic foundation, *C* is number of stiffeners, and the Dirac delta functions $\delta(y - y_i)$ define the stringer location coordinates along the *y*-axis. Each of the stringers considered as an elastic bending beam is referred to a rectangular coordinate system *0xz* (Figure 1). Their balance equations in projection onto the *z*-axis are:

$$EJ_i\frac{d^4U_i}{dx^4} = p_i, (i = 1, 2, \; ....C), \tag{3}$$

where EJ_i and U_i are bending stiffness and stringer deflection respectively. The other designations are traditional for this paper. The mass of the stringer itself, as well as the inertia of its movement, by analogy with the plate, are neglected. At each of the contact lines the condition of equality of deflections of both bodies $w(x, y_i) = U_i, (i = 1, 2, \ldots C)$ is fulfilled, but we consider that the deformed states caused by the contact reactions of neighboring stringers do not interfere with each other. Then by substituting the contact reaction p_i from (3) into the bending equation of the plate (2), we obtain the solving equation of the problem:

$$D\nabla^2\nabla^2 w + bw + \frac{q}{g}\left(\frac{\partial^2 w}{\partial t^2} + V_X{}^2\frac{\partial^2 w}{\partial x^2} + V_Y^2\frac{\partial^2 w}{\partial y^2}\right) + \sum_i^C EJ\frac{\partial^4 w}{\partial x^4}\delta(y - y_i) = q\,. \tag{4}$$

The equation (4) is a partial derivative equation with discontinuous coefficients with unknown deflections w in the direction of the y-axis, which is due to the presence of Dirac delta functions in the last term of its left-hand side defining the coordinates of stringer arrangement. The problem will be solved in both quasi-static and dynamic formulations. Since equation (4) contains discontinuous coefficients, its exact solution cannot be obtained. Therefore, we will use Bubnov's method, which has its own peculiarities in both variants of the problem formulation.

Results and discussion

Quasistatic solution

We consider that the shape of the curved surface of the plate depends only on its longitudinal coordinates *x* and *y* and does not depend on the time *t*. As a consequence, equation (4) takes a simpler form:

$$D\nabla^2\nabla^2 w + bw + \frac{q}{g}V^2\left(\cos^2\alpha\frac{\partial^2 w}{\partial x^2} + \sin^2\alpha\frac{\partial^2 w}{\partial y^2}\right) + \sum_i^C EJ\frac{\partial^4 w}{\partial x^4}\delta(y - y_i) = q\,. \tag{5}$$

To solve the problem, Bubnov method is used (in some sources, Bubnov-Galerkin method), but in different versions of its use. This method is well known and widely used in solving problems of mechanics.

Its essence is reduced to minimizing the residual of the solution of differential equation (2) using the approximation of unknowns in the form of series (6). This approach can be interpreted either as a variational problem or as a method of reducing a differential equation to a system of linear algebraic equations, the order of which is determined by the number of terms of the series of unknown deflections preserved in the expansion.

To solve it by the Bubnov method, we present the deflection of the plate in the form of an expansion in the given coordinate functions $\phi_{mn}(x.y)$.

$$w = \sum_m^K\sum_n^L w_{mn}\phi_{mn}(x, y)\,, \tag{6}$$

where w_{mn} are the unknown coefficients, $\phi_{mn}(x, y)$ are the orthogonal forms of natural vibrations of a smooth plate expressed in trigonometric functions. Applying the Bubnov method procedure to equation (5), we reduce it to a coupled system of $K \times L$ linear algebraic equations with respect to the coefficients in decompositions (6). In matrix form, it has the following form:

$$\left[K - V^2 M\right]W = F, \tag{7}$$

$$K = \left[k_{mn}^{kl}\right],\quad M = \left[m_{mn}^{kl}\right],\quad W = \{w_{kl}\},\quad F = \{f_{kl}\}. \tag{8}$$

The dimensionality of the stiffness matrices *K*, masses *M* and vectors *W*, *F* is determined by the number of row terms stored in the expansion (6). The elements of matrices *K*, *M* and vector *F* are:

$$k_{mn}^{kl} = \nabla^2\nabla^2\phi_{mn}\cdot\phi_{kl}dS + D\int_S b\int_S \phi_{mn}\phi_{kl}dS + \sum_{i=1}^C EJ_i\int_S \delta(y - y_i)\frac{\partial^4\phi_{mn}}{\partial x^4}\phi_{kl}dS,\,. \tag{9}$$

K matrix is block diagonal, due to the existence of a discontinuity in its coefficients only along the *y*-axis. The formula for the mass ratio m_{mn}^{kl} contains a modulus notation because the minus sign, which is obtained by differentiating trigonometric functions $\phi_{mn}(x,y)$, is already considered in equation (7). The system of equations (7) is solved by linear algebra methods for a given speed *V*.

To clarify the qualitative features of the behavior of the plate under the action of the moving load, we consider a model example in single term approximation for a plate with a single stringer located along the *x*-axis at *y=0* (*C=1*). Then the amplitude deflection value based on the solution of equations (7) will be:

$$w_1 = \frac{f_1}{k_{11} - V^2 m_{11}} . \tag{10}$$

The coefficients included in this formula are calculated from equation (9) at $\phi_{mn} = \phi_{kl} = \phi$. The second term in the denominator of this formula, which takes into account the massiveness and velocity of the load, enters with a minus sign. Consequently, with increasing mass or velocity, plate deflections will increase rapidly. For some values of these quantities, the denominator (10) is zero. This indicates that even at low shear load intensities an unrestricted increase in plate deflection due to centrifugal forces is possible and this phenomenon can be interpreted as a loss of plate stability. The speed at which the denominator turns to zero can be called critical.

$$V_{KP}^2 = \frac{k_{11}}{m_{11}} = \frac{D\int\limits_S \nabla^2\nabla^2\phi\cdot\phi dS + b\int\limits_S \phi^2 dS + EJ\int\limits_S \frac{\partial^4\phi}{\partial x^4}\delta(y-0)\phi dS}{\frac{q}{g}\left|\sin^2\alpha\int\limits_S \frac{\partial^2\phi}{\partial x^2}\phi dS + \cos^2\alpha\int\limits_S \frac{\partial^2\phi}{\partial y^2}\phi dS\right|} . \tag{11}$$

If *b=EJ=α=0* in the resulting expression, formula (11) is the same as the expression for the critical load velocity on a smooth plate [5].

When solving the problem in high approximations from the system of equations (7) by equating its determinant (12) to zero, the whole spectrum of critical velocities can be determined. However, only the minimum speed is of practical importance.

$$\det\left[K - V^2 M\right] = 0 . \tag{12}$$

Dynamic solution

In this case, the deflection of the plate depends not only on the spatial coordinates *x* and *y*, but also on the time *t*. According to the Bubnov method, we consider the deflection of the plate in the form of a decomposition of the given functions:

$$w = \sum_m^K \sum_n^L w_{mn}(t)\phi_{mn}(x,y) , \tag{13}$$

where $w_{mn}(t)$ are the unknown functions of time, $\phi_{mn}(x,y)$ are the orthogonal forms of natural vibrations of a smooth plate expressed in trigonometric functions. Substituting expansion (13) into equation (2) and applying the Bubnov method procedure on spatial coordinates to it, we arrive at a system of second order differential equations of motion of the plate with respect to functions $w_{mn}(t)$. In matrix form, it has the following form:

$$M\ddot{W} + KW = F . \tag{14}$$

$$\begin{gathered} K = \left[k_{mn}^{kl}\right], \quad M = \left[m_{mn}^{kl}\right], \\ W = \left\{w_{kl}(t)\right\}, F = \left\{f_{kl}\right\} \end{gathered} . \tag{15}$$

f_{kl} coefficients are calculated according to (9). In equations (14) and below, the points above the desired functions denote their time derivatives. The system (14) is integrated numerically under given initial conditions. By analogy with the quasi-static approach, we also consider the solution of this problem in the monomial approximation. Then the solving equation of the problem is:

$$\ddot{w}_1 + \omega^2 w_1 = \frac{f_{11}}{m_{11}} . \tag{17}$$

Solution of (17) is:

$$w_1 = A\sin\omega t + B\cos\omega t + w_*, \tag{18}$$

A and B are the integration constants determined from the initial conditions, w_* is the partial solution depending on the specific form of the right-hand side (17), ω is the frequency of oscillation. According to the dynamic stability criterion, the critical state of the system is realized when the frequency of the natural oscillations $\omega = \sqrt{k_{11}/m_{11}}$ is zero. From this ratio, which reduces to the condition $k_{11} = 0$, the critical speed can be determined:

$$V_{KP}^2 = \frac{D\int\limits_S \nabla^2\nabla^2\phi\cdot\phi dS + b\int\limits_S \phi^2 dS + EJ\int\limits_S \frac{\partial^4\phi}{\partial x^4}\delta(y-0)dS}{\frac{q}{g}\left|\sin^2\alpha\int\limits_S \frac{\partial^2\phi}{\partial x^2}\phi dS + \cos^2\alpha\int\limits_S \frac{\partial^2\phi}{\partial y^2}\phi dS\right|}. \tag{19}$$

This formula is exactly the same as the expression for the critical velocity obtained earlier in the quasi-static version of the problem.

We consider a plate with length *a* and width *2d*, supported by a single stringer positioned at its central point along the *x*-axis (Figure 1). All edges of the plate are freely supported. Since the lowest (main) critical load is mainly realized under the simplest form of stability loss, we solve the problem in the monomial approximation. With the boundary conditions considered, we select the approximating function in expansions (6) or (13) as $\phi = \sin(\pi x/a)\cos(\pi y/2d)$. The design is characterized by the following dimensionless parameters:

$$\begin{gathered} a/d = 4, h/a = 0,01, \\ J^* = J/a^4 = 5, b^* = bh/E\,. \\ q^* = q/E = 10 \end{gathered} \tag{20}$$

The Figure 2 shows the relations between the dimensionless square of the critical velocity $V_{KP} = V_{KP}^{2*} = V_{KP}^2/gh$ and the angle of inclination of the moving load front to the *x*-axis of the plate. Angle $\alpha = 0$ corresponds to the movement of the load along the *x*-axis and angle $\alpha = \pi/2$ corresponds to the movement of the load along the *y*-axis. The lower curve corresponds to a smooth plate and the upper curve to a ribbed plate. The presence of a stringer "stiffens" the plate and, as a consequence, the critical speed of the load is increased.

For a smooth plate with aspect ratio $a/2d = 2$, the critical loads in the different directions are not very different from each other, which agrees with the results given in the monograph [3]. When the forces move along the *x*-axis the critical velocities are much higher than in any other direction, due to the presence of a sufficiently strong stringer on this axis. So, for example, for a smooth plate without ribs with an inclination angle of the shock wave front of 0 degrees (movement along the x-axis), the dimensionless critical velocities in the proposed solution and in the solution from [2] give values of 15 and 120, respectively. For an inclination angle of $\frac{\pi}{2}$ (movement along the y-axis), the same speeds are 50 and 43. The given figures indicate good agreement between the results for a smooth plate. There are simply no solutions to problems for ribbed structures, so there is nothing to compare with.

There are a significant number of papers on this topic.

Conclusions

In this paper the problem of dynamic deformation and stability of a thin discrete stringer system of a plate lying on an imitating a heat-shielding coating elastic base is approximately solved in two variants. The main focus is on determining the main (lowest) critical load speed. For it, formulas are obtained in closed form in a one-term approximation. And in both versions of the problem these formulas coincide. This indicates that the solution is correct. Using the dynamic solution, expressions for the fundamental (lowest) natural frequency of the plate are also obtained in closed form. Numerical simulation results show that the presence of a stringer "stiffens" the plate and as a consequence leads to an increase in critical performance. The effect of the load direction on the critical velocity has been investigated. When forces move in the direction of the long side of the plate, the critical velocity is higher than along the short side of the plate. As follows from the above numerical experiments, the use of ribbed plates leads to higher results of critical loads compared to smooth ones (without stringers). The latter makes it possible in practice to avoid the loss of stability of the aircraft skin at operational flight speeds. From the point of view of practical applications, we should strive to ensure that the critical speeds of the moving load significantly exceed the operational speeds of the aircraft. Therefore, the variant of pressure wave motion along the long side of the plate is preferable to all other directions of its motion. The advantage of the proposed approach in application to practical engineering calculations is that the most dangerous lower (basic) critical loads (velocities) can be easily determined in a single-term approximation. The same applies to natural oscillation frequencies, which, as a side effect, are found when solving a problem in a dynamic formulation.

It is difficult to compare the results obtained in this work with the data of other authors, since there are definitely no theoretical studies of the problem in the case of an arbitrary front of the direction of the shock wave for a plate discretely supported by stringers and also bearing a heat-shielding coating. The same situation is about the comparison with the results of the experiment, which, if they exist, are most likely of a closed nature.

The results obtained can be used to predict the behavior of thin-walled structures, including those with functional coatings, under conditions of interaction with high-speed flows, when factors such as the direction of the front of aerodynamic loads, the formation of flow discontinuity surfaces, unsteady heating caused by both convective heat transfer, and the heat of chemical reactions (oxidation, catalysis). The advantages of the work, in addition to determining the main critical speeds of motion, are also the determination of natural oscillation frequencies, the knowledge of which makes it possible to avoid getting the aircraft into near-resonance flight modes that threaten unpredictable consequences. Thus, the work and its results are practical.

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Figure 1: Discretely supported plate with elastic ribs

Figure 2: Dimensionless critical speed as a function of the angle of inclination in the moving load front

Figure 1

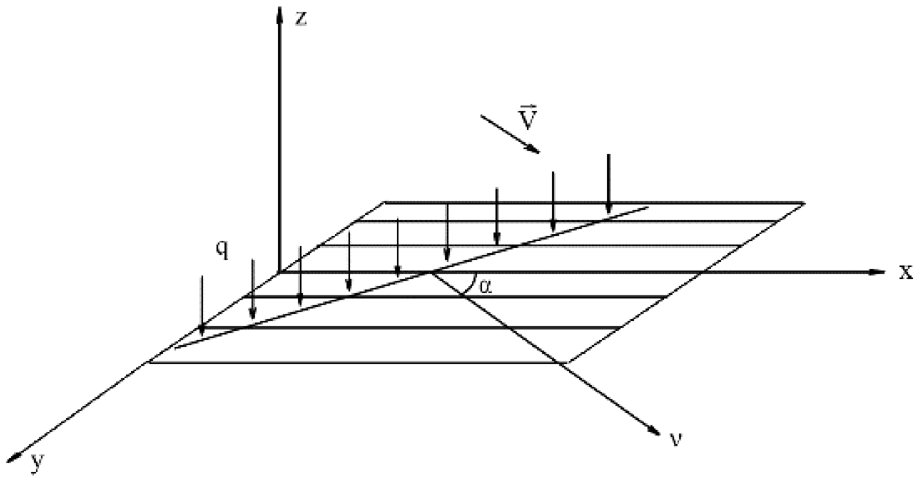

Figure 2