New Method for Pattern Synthesizing of an Unequally Spaced Array with Dynamic Range Ratio Improvement

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Abstract

The amplitude-phase synthesis of an unequally spaced array is discussed in this study along with a new method that improves the amplitude dynamic range ratio with fewer elements. In this method, the Hankel matrix is established using the sampling points of a prescribed array factor. The array elements' locations are estimated using the eigenvalues of the Hankel matrix. The magnitude of the array currents is then calculated using the least-squares approach. By defining the reduction parameter, the amplitude dynamic range can be extremely reduced. Some theoretical and practical arrays are given to verify the performance of the proposed method. The obtained results are compared to those obtained by other methods, and simulation data.

Keywords: Antenna Array, Array Factor, Dynamic Range Ratio, Hankel Matrix, Phase Only

1. Introduction

Nowadays, antenna arrays have been widely employed for different targets, including electronic tracking, satellite communication systems, interference suppression, 5G cellular networks, directivity increase, and spatial multiplexing [1-4]. Typically, the assessment of the excitations that better satisfy the pattern synthesis requirements is an important challenge in array design problems [5-8]. To this end, several approaches have been introduced, considering the shape of the array, the character of the desired pattern, and the degrees of freedom that characterize the array excitations [9-11].

Synthesis of the radiation pattern of an array is done by changing both the amplitude and phase of the array elements. In array engineering, this problem is called amplitude-phase synthesis [12]. Nevertheless, in some applications like reflect-array antennas having either pencil or contoured beams, only the phase of the unit cells is the controllable parameter in the design procedure [13-14]. The magnitude of unit cells is predetermined by the other parameter in these applications. Additionally, changing the phase of the array elements is more practical and useful than changing their magnitudes. This procedure for pattern synthesis of an array is called phase-only synthesis.

Most of the introduced methods for radiation pattern synthesis cannot control the amplitude dynamic range. In [15], a null steering approach was introduced by changing only the excitation phases of a part of the array of elements. In this work, the required phase excitations are employed in the objective function. Then, by using the quasi-Newton method, the mean square error between the desired and synthesized pattern is minimized. In [16], it is tried to obtain the required array patterns with controlled nulls.

In [17], a new technique is introduced to determine the array weights with minimum amplitude dynamic range ratio. The array designed using this method can generate a pencil beam pattern with zero phases. Also, using it, the phases of the excitation coefficients can be continuously controlled. In [18], the autocorrelation matching technique is introduced to synthesize of the power pattern of an equally-spaced array. In [19], a teaching learning-based method is proposed for the synthesis of reconfigurable uniformly excited linear arrays. In this work, the modified quantum particle swarm optimization and symbiotic organisms search algorithms are employed in the synthesis procedure.

This article presents a new method for determining the amplitude and phase of the excitation coefficients of an unequally spaced array. In this work, it is tried to design a non-uniform array with a minimum amplitude dynamic range ratio. By establishing the Hankel matrix and calculating its eigenvalues, the location of the array elements is determined. The amplitude of excitation currents is calculated using the least square method. Then, by defining the reduction parameter, the amplitude dynamic range is reduced. Several practical arrays, including the sum and difference pattern, are investigated to confirm the effectiveness of the presented method.

2. Mathematical Formulation

Consider a linear, unequally-spaced array oriented along z-axis direction composed of N elements. Let I_n, d_n and k be the weight and location of the n^{th} element and the wave number defined by $2\pi/\lambda$ (λ is wavelength), respectively. By defining $u=\cos\theta$, in which θ is the elevation angle, the array factor $F(u)$ can be described as follows.

$$F = \sum_{n=1}^{N} I_n \exp\left(jkd_n u \right) \tag{1}$$

Assuming L is the array length, the prescribed array factor can be reconstructed using $2M+1$ samples with the uniform sampling step $\Delta \approx 1/(2L)$ over the interval $-1 \leq u \leq +1$ [20].

$$u_m = m\Delta = m/2L,\ m = -M,\ ...,\ 0,\ ...,\ M \tag{2}$$

The samples of the prescribed pattern can be expressed in vector form $\mathbf{V}=[F(m/2L)]_{1\times(2M+1)}$; $m=-M,\ldots,0,\ldots,M$. Two sample matrices **G**, **L** can be established using vector **V** as follows [20].

$$\mathbf{G} = \begin{bmatrix} V_2 & V_3 & \cdots & V_{M+1} \\ V_3 & V_4 & \cdots & V_{M+2} \\ \vdots & \vdots & \ddots & \vdots \\ V_{M+2} & V_{M+3} & \cdots & V_{2M+1} \end{bmatrix}_{(M+1)\times M} \tag{3}$$

$$\mathbf{L} = \begin{bmatrix} V_1 & V_2 & \cdots & V_{M} \\ V_2 & V_3 & \cdots & V_{M+1} \\ \vdots & \vdots & \ddots & \vdots \\ V_{M+1} & V_{M+2} & \cdots & V_{2M} \end{bmatrix}_{(M+1)\times M} \tag{4}$$

Combining these matrices gives the Hankel matrix **H** using the following equation [9].

$$\mathbf{H} = \left(\mathbf{G}^T\mathbf{L}\right)^{-1}\mathbf{L}^T\mathbf{G} \tag{5}$$

To save the number of elements, the singular value decomposition (SVD) of the Hankel matrix **H** can be carried out as follows.

$$\mathbf{H} = \mathbf{U}\mathbf{S}\mathbf{W}^T \tag{6}$$

in which **U** and **W** are the unitary matrices, and **S** is a diagonal matrix, including the singular values of **H**. The number of principal singular values of **H** equals the number of array elements. For many practical arrays, the number of nonzero singular values is less than the number of array elements. So, by regarding the Q number of the principal singular values, which corresponds to a new array with fewer elements, a low-rank approximation of **H** is achieved as follows [20].

$$\mathbf{H}_Q = \mathbf{U}\mathbf{S}_Q\mathbf{W}^T \tag{7}$$

where $\mathbf{H}_\mathrm{Q}$ is the low rank version of **H**. It is worthy to note that there is not a clear relationship between the primary numbers of array elements N and the reduced numbers of array elements Q. For each case study, after specifying the singular values of matrix **H**, only non-principle or non-zero singular values are remained and the other ones are withdrawn. It is shown in [21] that the fractional Fourier series of the prescribed pattern (F_r) can be expressed using the distinct eigenvalues ζ_n of the low-rank matrix $\mathbf{H}_\mathrm{Q}$, in which c_ns are constant coefficients.

$$F_r = \sum_{n=1}^{Q} c_n \exp\left(\zeta_n u\right) \tag{8}$$

By comparing equations (1) and equation (8), it is found that the locations of the elements of the new array (d_n) can be calculated using the phase of normalized eigenvalues ζ_n as follows.

$$\begin{cases} d_n = L\alpha_n/\pi \\ \alpha_n = \measuredangle\left(\zeta_n/\left|\zeta_n\right|\right) \end{cases}, \quad -\pi \le \alpha_n \le \pi \tag{9}$$

where L and α_n are the array length and the phase of normalized eigenvalues (in radian), respectively. After determining the array element's locations d_n, the weight of the elements I_n can be determined using the Least Square Method (LSM). To this end, the following system of equations is established.

$$\mathbf{AX} = \mathbf{B} \quad \rightarrow \quad \mathbf{X} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{B} \tag{10}$$

$$\mathbf{A} = \begin{bmatrix} e^{jkd_1u_1} & e^{jkd_2u_1} & \cdots & e^{jkd_Qu_1} \\ e^{jkd_1u_2} & e^{jkd_2u_2} & \cdots & e^{jkd_Qu_2} \\ \vdots & \vdots & \ddots & \vdots \\ e^{jkd_1u_P} & e^{jkd_2u_P} & \cdots & e^{jkd_Qu_P} \end{bmatrix} \tag{11}$$

$$\mathbf{X} = \begin{bmatrix} I_1 & I_2 & \dots & I_Q \end{bmatrix}^T \tag{12}$$

$$\mathbf{B} = \begin{bmatrix} F\left(u_1\right) & F\left(u_2\right) & \dots & F\left(u_P\right) \end{bmatrix}^T \tag{13}$$

in which $P \approx 5L/\lambda$ is the total number of the samples and is determined using the Nyquist theorem [5]. The second purpose of this paper is to reduce the amplitude dynamic range (ADR) defined by.

$$ADR = \frac{\max\left(|I_n|\right)}{\min\left(|I_n|\right)}, \quad n = 1, \ldots, Q \tag{14}$$

To this end, the absolute values of weights I_ns are modified to the new real weights w_ns so that the ADR of the new array is less than the primary array. The value of weights is determined using the following equation.

$$w_n = \eta\left[|I_n| - \min\left(|I_n|\right)\right], \quad n = 1, \ldots, Q \tag{15}$$

in which $0<\eta<1$ is the parameter that is used to reduce the ADR. In this case, the array factor is rewritten as follows.

$$F = \sum_{n=1}^{Q} w_n \exp\left(j\varphi_n\right) \exp\left(jkd_n u\right) \tag{16}$$

in which φ_n is the phase of the n^{th} excitation coefficient. By defining $kd_n u=\psi_n$, $\cos(\varphi_n-\psi_n)=C_n$ and $\sin(\varphi_n-\psi_n)=S_n$ the array factor is rewritten as.

$$F = \sum_{n=1}^{Q} \left(w_n C_n + j w_n S_n\right) \tag{17}$$

The radiated power of the array can be calculated as follows.

$$|F|^2 = \left|\sum_{n=1}^{Q} \left(w_n C_n + j w_n S_n\right)\right|^2 = \left(w_1 C_1 + \ldots + w_N C_N\right)^2 + \left(w_1 S_1 + \ldots + w_N S_N\right)^2 \tag{18}$$

After some algebraic manipulations, the array factor is reduced as.

$$|F|^2 = \sum_{n=1}^{Q} w_n^2 \left[C_n^2 + S_n^2\right] + 2\sum_{n=1}^{Q-1} w_n \sum_{m=n+1}^{Q} w_m \left[C_n C_m + S_n S_m\right] \tag{19}$$

Since $S_n^2+C_n^2=1$, the above equation is simplified to.

$$|F|^2 = \sum_{n=1}^{Q} w_n^2 + 2\sum_{n=1}^{Q-1} w_n \sum_{m=n+1}^{Q} w_m \left[C_n C_m + S_n S_m\right] \tag{20}$$

Substituting equation (21) into equation (20) leads to equation (22).

$$\begin{aligned} C_n C_m + S_n S_m &= \cos\left(\varphi_n - \psi_n\right)\cos\left(\varphi_m - \psi_m\right) + \sin\left(\varphi_n - \psi_n\right)\sin\left(\varphi_m - \psi_m\right) \\ &= \cos\left[\varphi_n - \varphi_m + \left(\psi_n - \psi_m\right)\right] \end{aligned} \tag{21}$$

$$|F|^2 = \sum_{n=1}^{Q} w_n^2 + 2\sum_{n=1}^{Q-1} \sum_{m=n+1}^{Q} w_n w_m \cos\left[\varphi_n - \varphi_m + \left(\psi_n - \psi_m\right)\right] \tag{22}$$

It should be noted that in the above equation, φ_ns are only unknown. The target is to minimize the following expression.

$$\min_{\varphi_n} \left\|\mathbf{F} - \mathbf{F}_d\right\|_2 \tag{23}$$

in which $\mathbf{F}_d$ is the vector that includes the samples of the desired pattern and ||.|| is the norm-2 operator. The problem's solution is not easily determined. To this end, the introduced technique in [19] is employed. In each iteration, equation (23) is used to compute the mean square error.

3. Results and Discussion

In this section, several arrays are studied and the results are compared to validate the performance of the proposed method.

3.1. Sum Pattern

In the first example, amplitude-phase synthesis of an equi-ripple pattern with SLL=-25 dB, and array length L=10λ is considered. Figure (1) shows the obtained results using the proposed and the conventional Dolph-Tschebyscheff method [22]. Although a small deviation of about 2.5 dB is seen in the first two side lobes, the accuracy of the obtained results in other regions is good. Also, the magnitude and phase of the obtained currents for both methods are plotted in Figure (2). The proposed method not only reduces the number of elements in the designed array when compared to the conventional method, but it also reduces the amplitude dynamic range of the designed array by about 21% when compared to the Dolph-Tschebyscheff technique. It is worthy of mentioning that the phases of I_ns in the Dolph-Tschebyscheff method are zero.

3.2. Difference Pattern

In the second example, amplitude-phase synthesis of a difference pattern with the first side lobe about -25 dB, and array length L=12λ is considered. Figures (3) and (4) show the obtained results and amplitude and phase of the array currents using the proposed and the conventional Bayliss method [22]. It is seen that the designed array using the proposed method has eight elements lower than the Bayliss array, and the desired and synthesized pattern are matched very well. The amplitude dynamic range of the proposed array is about 55% lower than the Bayliss array.

3.3. Flat-Top Pattern

In the third example, amplitude-phase synthesis of a flat-top pattern with non-zero values over the interval $|u|\leq 0.35$ and array length L=13λ is considered. The desired pattern and synthesized pattern using the proposed and Fourier methods [22] are plotted in Figure (5). It is seen that the accuracy of the proposed method with Q=19 is better than the Fourier method with N=26. The amplitude and phase of the obtained currents for both methods are depicted in Figure (6) and Figure (7), respectively. The results show that the amplitude dynamic range of the designed array is about 78% less than the Fourier method.

Table 1 shows some of the main parameters of the obtained results, such as the total number of array elements Q, amplitude dynamic range ADR, reduction percent of dynamic range η_0, mean square error (MSE), and the required running time (t). The proposed method is implemented in MATLAB and it is run using a computer with CPU core i7 & 16G RAM. It can be seen that the proposed method can be used to design an unequally-spaced array with the minimum number of elements, minimum dynamic range, and lower mean square error.

4. Practical Array Example

To assess the performance of the proposed method in real applications, the previously designed array with equi-ripple array factor is simulated using the full-wave simulator, HFSS software. The simulated array is made of a half-wavelength dipole antenna, arranged along the x-axis.

Fig. 8 shows the simulation results of the mutual coupling between two adjacent elements. It is seen that the minimum and maximum coupling between the elements are about -20.9 dB and -19.4 dB, respectively.

The total radiation pattern of the implemented dipole array with Q=13 elements at standard planes of φ=0 and $\varphi=\pi/2$ are depicted in Figure (9) and Figure (10), respectively. The total radiation pattern includes both element and array factors. For an array-oriented along the x-axis, the value of the array factor in $\varphi=\pi/2$ is constant. So, the total radiation pattern in this plane is approximately the same as the element pattern.

In Figure (9), a deviation is seen in the broadside direction, and over a small range of elevation angles. This is because we only focused on optimizing the array factor in the theory section. Thus, the proposed array model has not considered mutual coupling between elements that cause a significant difference in the element radiation patterns. Hence, in real radiation problems, the mutual coupling reduces the accuracy of the proposed method. However, the proposed method can be used for the primary design of a practical array with acceptable accuracy. It is clear that for the real array, an optimization process should be applied using the full-wave simulator.

5. Conclusion

A new approach was proposed for amplitude and phase synthesis of the radiation pattern of an unequally-spaced antenna array. The main purpose of this work is to improve the amplitude dynamic range ratio with the reduced number of elements. In this paper, using the sample points of the desired pattern and establishing the Hankel matrix, first, the location of the array elements are evaluated, and then, by the least square method the excitation currents are determined. By defining the reduction parameter, the amplitude dynamic range is reduced and the phase of the array weights are calculated. Some practical arrays are studied to verify the performance of the proposed method.

References

[1] Alijani. M, G, H, Sheikh. S, Kishk. A, “Development of Closed-Form Formula for Quick Estimation of Antenna Factor”, 15th European Conference on Antennas and Propagation (EuCAP), (2021)

[2] Boozari, M., Khalaj-Amirhosseini, M. “Synthesis of unequally-spaced arrays using the fractional Fourier series”, Sci. Rep. **12**, (2022)

[3] Tamandani. A, Alijani. M, G, H, “Development of an analytical method for pattern synthesizing of linear and planar arrays with optimal parameters”, AEU-International Journal of Electronics and Communications **148**, (2022)

[4] Das, A., Mandal, D., Kar, R., “An optimal radiation pattern synthesis and correction of mutually coupled circular dipole antenna array”, Scientia Iranica, **29**, (2022)

[5] Tamandani. A, Alijani. M, G, H, “Development of a New Method for Pattern Synthesizing of the Concentric Ring Arrays with Minimum Number of Rings”, AEU-International Journal of Electronics and Communications **152**, (2022)

[6] Jamunaa, D., Mahanti, G.K., Hasson, F.N., “Multi-verse optimization algorithm for optimal synthesis of phase-only reconfigurable linear array of mutually coupled parallel half-wavelength dipole antennas placed at finite distances from the ground plane”, Scientia Iranica, **29**, (2022)

[7] Gong. Y, Xiao. S, Wang. B, Z, "Synthesis of Sparse Planar Arrays with Multiple Patterns by the Generalized Matrix Enhancement and Matrix Pencil", IEEE Transactions on Antennas and Propagation **69**, (2021)

[8] Alijani. M, G, H, Neshati. "Development of a New Method for Pattern Synthesizing of Linear and Planar Arrays Using Legendre Transform with Minimum Number of Elements", IEEE Transaction Antennas and Propagation **70**, (2022)

[9] Boozari, M., Khalaj-Amirhosseini, M. "Pattern synthesis of linear and ring arrays with minimum number of elements using FFT and Bessel transformation", Sci. Rep. **12**, (2022)

[10] Khalaj-Amirhosseini, M., "Reducing the number of elements of linear antenna arrays using Fourier's coefficients equating method", Scientia Iranica, **30**, (2023)

[11] Boozari, M. Khalaj-Amirhosseini, M., "Development of an Analytical Method for Arbitrary Shaped Pattern Synthesis of Planar Arrays", Radioengineering, **30**, (2021)

[12] Alijani. M, G, H, Neshati. M, H, "Development a new technique based on least square method to synthesize the pattern of equally space linear arrays", International Journal of Engineering, Transaction B **32**(11), (2019)

[13] Rafaei-Booket. M, Mousavi. S.M., "New phase realization approach for implementation of broadband reflectarrays", International Journal of Information and Communication Technology Research **13**(1), (2021)

[14] Rafaei-Booket. M, Mousavi. S.M., "Efficient Analysis Method and Design Approach for Broadband Reflect-arrays with Isotropic/-Artificial Anisotropic Substrates" IET Microwaves, Antennas & Propagation **14**(10), (2020)

[15] Hu. J.L., Ma. Y., Lin. S.M., "Deep null steering by controlling excitation phases of part elements", International Journal of Electronics **83**, (1997)

[16] Abdulkader. A.J., Mohammed. J.R., Thaher. R.H., "Phase Only Nulling with Limited Number of Controllable Side Elements", Progress in Electromagnetics Research C **99**, (2020)

[17] Mahanti. G.K., Das. S., Chakraborty. A., "Design of Phase-Differentiated Reconfigurable Array Antennas with Minimum Dynamic Range Ratio", IEEE Antennas and Wireless Propagation Letters **5**, (2006)

[18] Khalaj-Amirhosseini. M., "Phase-Only Power Pattern Synthesis of Linear Arrays Using Autocorrelation Matching Method", IEEE Antennas and Wireless Propagation Letter **18**, (2019)

[19] Boozari, M. Khalaj-Amirhosseini, M., "Development of an allocation method to synthesis of unequally spaced arrays with minimum number of elements and mutual coupling considerations", International Journal of RF and Microwave Computer-Aided Engineering **32**, (2022)

[20] Alijani. M, G, H, Neshati. M, H, "A new non-iterative method for pattern synthesis of unequally spaced linear arrays", International Journal of RF and Microwave Computer-Aided Engineering, **29**, (2019)

[21] Fermo. L., Mee. C., Seatzu. S., "Parameter estimation of monomial-exponential sums in one and two variables" Applied Mathematics and Computation, **258**, (2015)

[22] Mailloux. R.J., “Phased Array Antenna Handbook”, Artech House, (2018)

Author Contributions

The authors contributed equally to this work

Conflict of Interest

The authors declare no competing interests.

Figure Captions

Figure (1): The synthesis results of sum pattern with SLL=25dB and ADR=2.12.

Figure (2): Magnitude of the sum pattern with SLL=25dB and ADR=2.12.

Figure (3): The synthesis results of the difference pattern with ADR=3.13.

Figure (4): Magnitude and phase of the currents of difference pattern with ADR=3.13.

Figure (5): The synthesis results of the flat-top pattern with ADR=41.16.

Figure (6): Magnitude of the currents of flat-top pattern with ADR=41.16.

Figure (7): Phase of the currents of flat-top pattern with ADR=41.16.

Figure (8): The simulation results of the mutual coupling in the implemented dipole array in HFSS.

Figure (9): The total radiation pattern of the implemented dipole array in HFSS at $\varphi=0$.

Figure (10): The total radiation pattern of the implemented dipole array in HFSS at $\varphi=\pi/2$.

Tables Captions

TABLE 1: Comparison of the obtained results

Figures

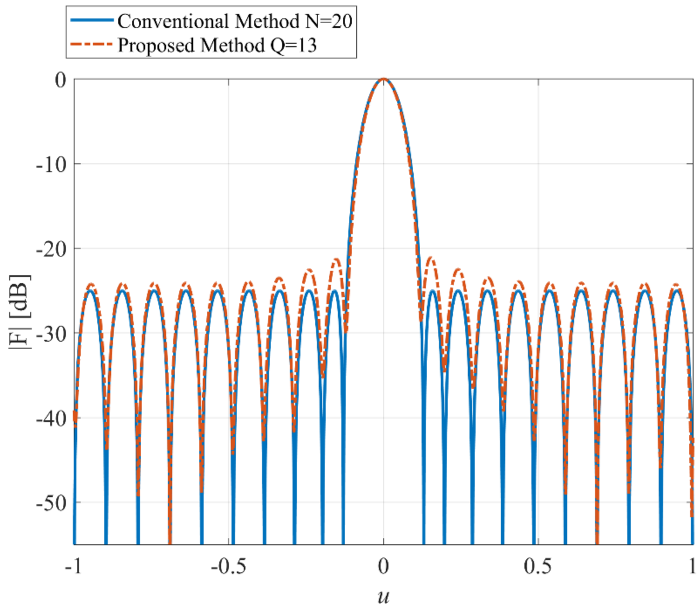
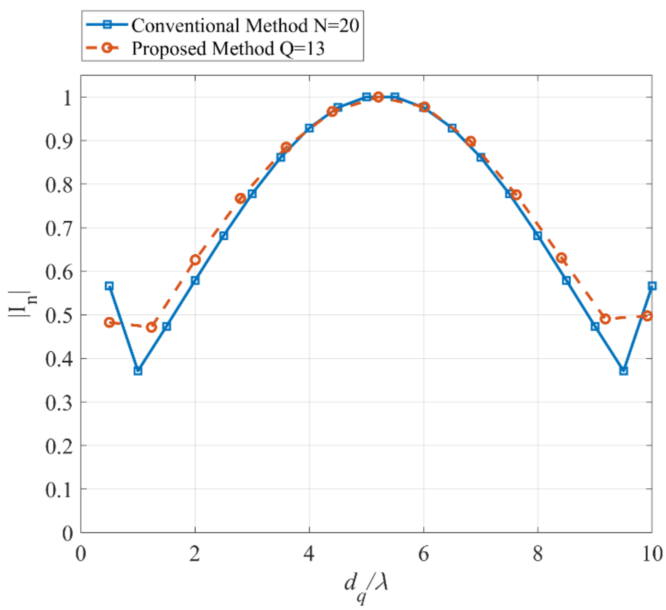


Figure (1)



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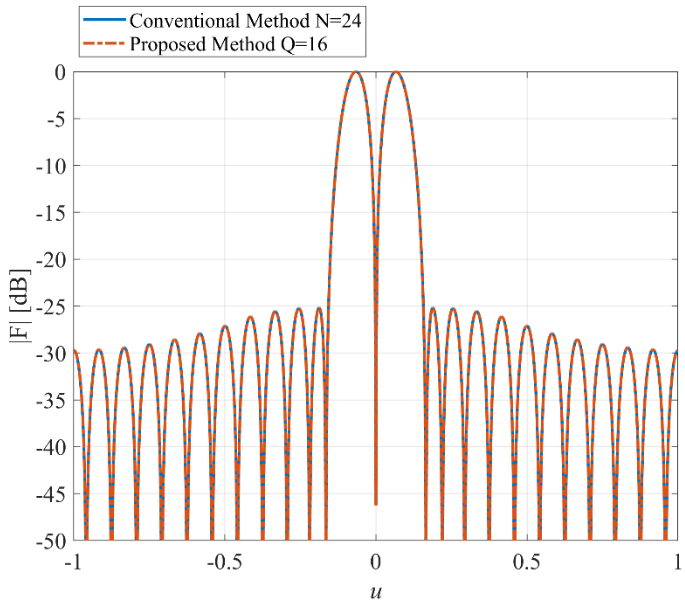

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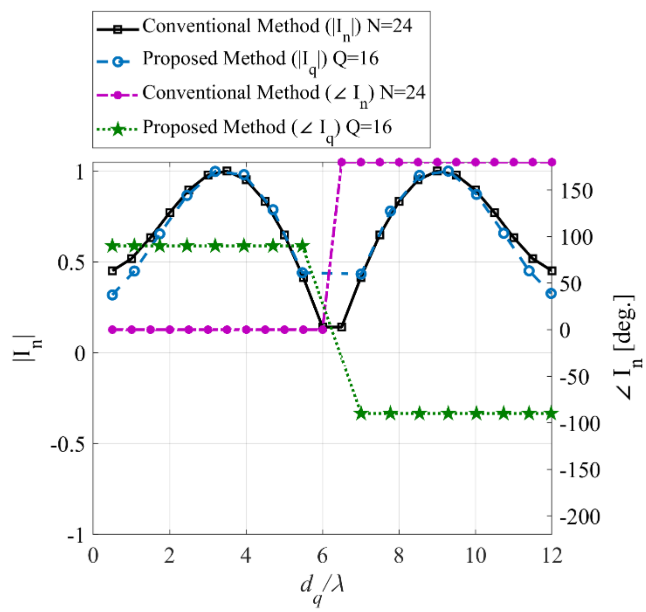

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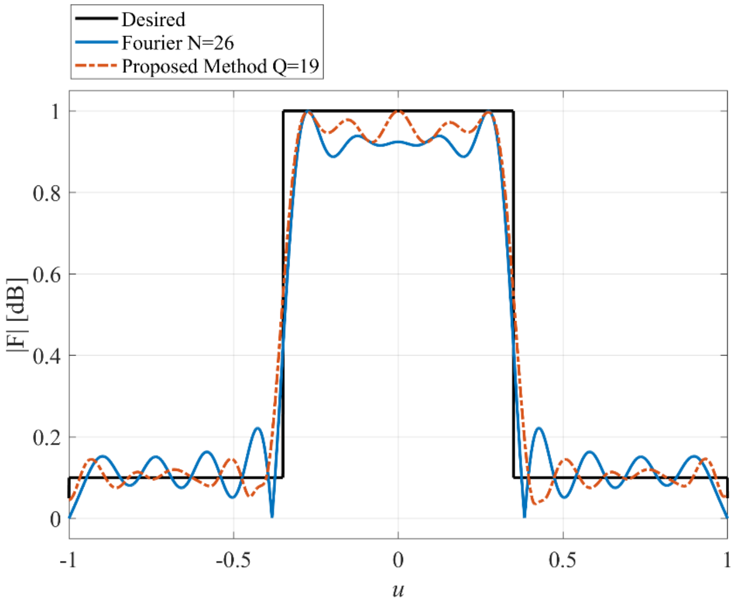


Figure (5)

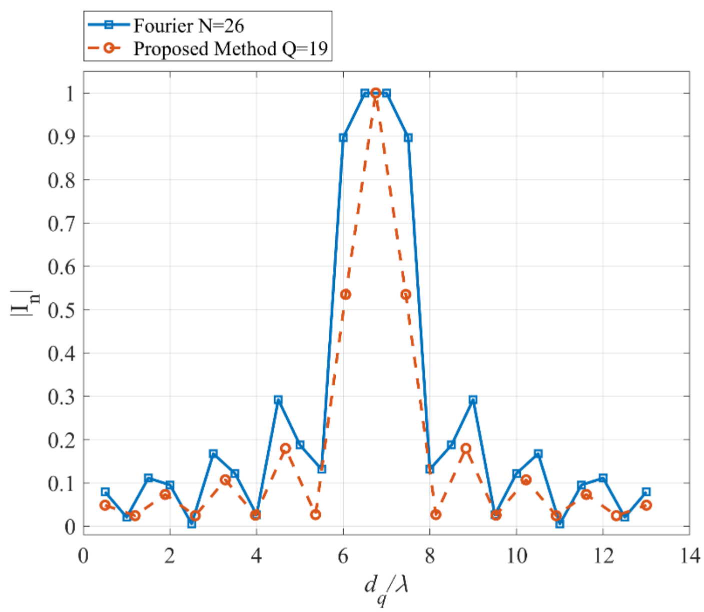
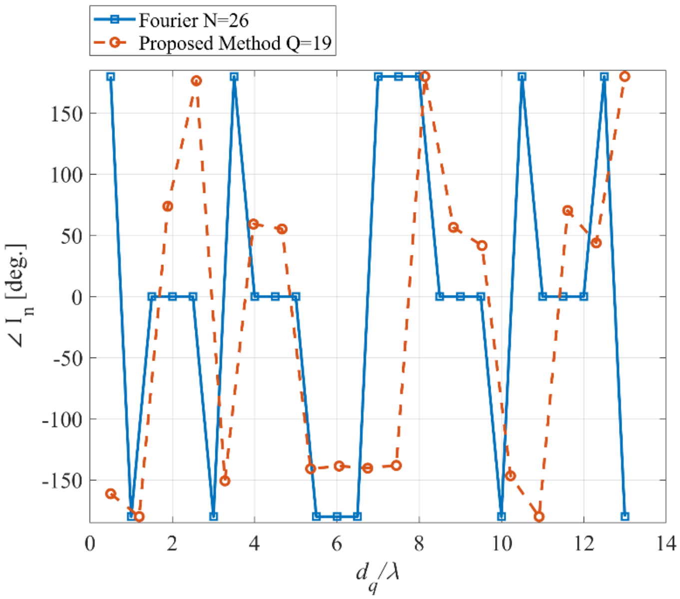


Figure (6)



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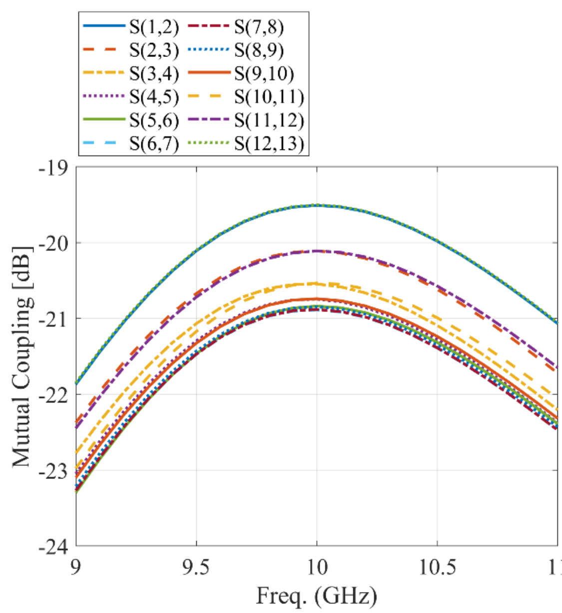

Figure (8)

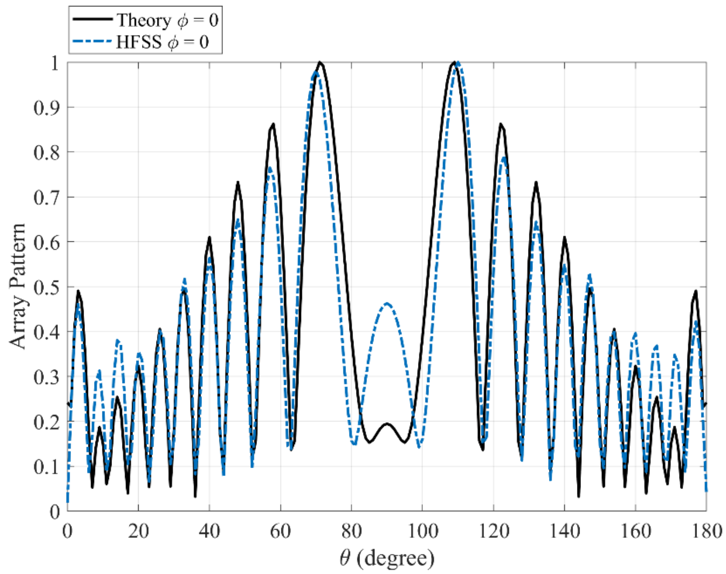

Figure (9)

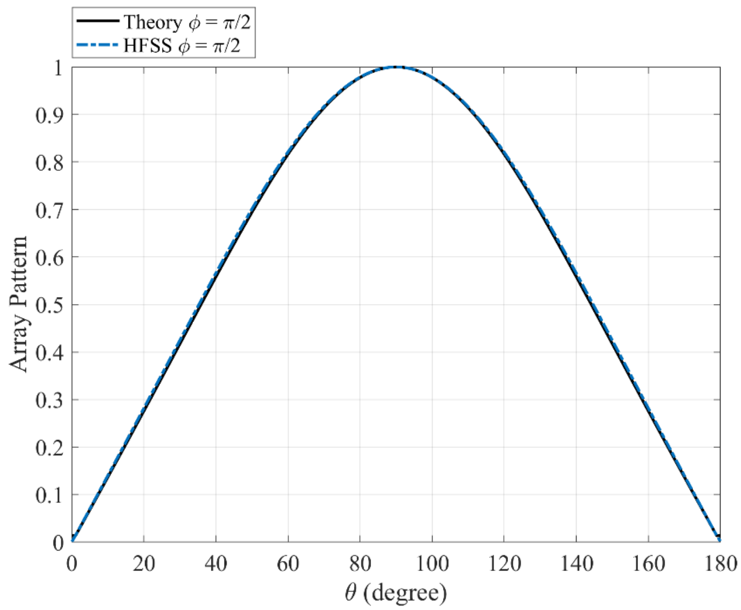

Figure (10)

			ADR	η_0 (%)	*MSE*	*t* (s)
Example I	Proposed Method	*Q*=13	2.12	21	1.8e-4	10.27
	Dolph-Tschebyscheff	*N*=20	2.69			
Example II	Proposed Method	*Q*=16	3.13	55	≈ 0	10.24
	Bayliss Method	*N*=24	7.04			
Example III	Proposed Method	*Q*=19	41.16	78	6.9e-3	10.62
	Fourier Method	*N*=26	187.6			

Biographies

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