Temperature dependent heat generation and variable viscosity features for viscoelastic fluid with homogeneous and heterogeneous (HH) chemical reactions

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Abstract: This investigation presents the heat and mass transfer phenomenon for the chemically reactive flow of second grade fluid subject to the homogeneous and heterogeneous (HH) chemical reactions. The viscosity of fluid is assumed to be temperature dependent instead of constant. The motivations for considering the viscosity as a function of temperature is justified with applications of metallurgical process, crude oil extraction, geothermal systems and machinery lubrication. Additionally, viscous dissipation and temperature dependent heat generation and absorption effects are also introduced to improve the thermal transportation phenomenon. The interaction of different new variables facilitates the problem into dimensionless form. The numerical achievements are predicted with implementing the Runge

Kutta (RK4) method. The physical onset behind the parameters have been reported. The tabular quantitative analysis is performed for different physical quantities.

Keywords: Heat transfer, viscoelastic fluid, temperature dependent viscosity, homogeneous/heterogeneous reactions, numerical solution.

Nomenclature

$\left(u,v\right)$ Velocity components	T Fluid temperature
$\bar{\bar{u}}_w\left(x\right)$ surface velocity	h_1 Heat transfer coefficient
Γ Medium permeability	k_1 Material parameter
β Second grade fluid parameter	α Thermal conductivity
β_0 Magnetic field strength	T_∞ Ambient fluid temperature
C_{f_x} Skin friction coefficient	T_f Fluid temperature
N_{u_x} Nusselt number	ρ density
c_p specific heat	λ viscosity parameter

1. Introduction

Widespread attention is devoted by investigators towards the rheology of non-Newtonian liquids in current research problems In many processing, engineering and industrial processes, the contribution of non-Newtonian materials is widely justified. To clarify the genuine performance of non-Newtonian fluid, diverse sorts of models are established in past. A second grade model is assumed to be updated class of linear materials attaining diverse rheology. The shear thinning along with shear thickening outcomes are predicted preferably by using this model. In recent contributions, the interest of scientists in grown exclusively in non-Newtonian materials. The reason behind their interest is the wide range of its applications in manufacturing processes. In many processes, these fluids are considered to flow over continuously moving surfaces. An individual fluid model is not enough to describe all the properties and qualities of each fluid. To overcome this deficiency and to describe its characteristics, various non-Newtonian fluid models are proposed which are delineated by rate, differential and integral type fluids. Their immense applications involve in industry such as in polymer processing industries, bio fluids dynamics and petroleum drilling and many others. The second grade fluid properties were elaborated in literature by scientistic with enrolment of many diverse consequences. Riaz et al. [1] expressed the optimized significance in heat transfer due to viscoelastic material. Ismail et al. [2] reported the inspiration of transient flow for second grade material numerically. Rehman et al. [3] observed the aspect of surface tension while exploring the different rheological mechanism of viscoelastic fluid. The joule heating determination regarding the optimized features of viscoelastic fluid has been visualized by Abbasi et al. [4]. Loganathan et al. [5] intended the mathematical exploration of Riga surface flow of second grade nanofluid numerically. Waqas et al. [6] investigated the bioconvection application for viscoelastic nanofluid with biofuel applications.

The heat transfer is the fundamental process in various engineering and industrial phenomenon. The source of heat transfer is associated to the exchange of energy in various systems or objects against the temperature difference. The heat transfer phenomenon present key important in the automotive engineering, cooling objects, HVAC systems, air conditionings, sterilization, industrial processes, aerospace engineering, petroleum industry etc. Different studies are reported in recent years to enhance and control the heat transfer phenomenon. For instance, Salawu et al. [7] predicted the heat transfer through contribution of ferromagnetic particles via Von Karman approach. Shamshuddin et al. [8] examined the contribution of variable viscosity while reporting the heat transfer aspects in nanofluid flow via rotating disk. Ferdows et al. [9] explained the assessment of internal heat transfer applications for power law fluid with attribution of exponential fluid viscosity. The heat transfer analysis in the MOS2 and Fe3O4 nanoparticles with ethylene glycol base liquid have been treated by Shamshuddin et al. [9]. Farajollahi et al. [10] suggested the heat transfer mechanism in conical turbulators by utilizing the tiny particles. Zheng et al. [11] performed the numerical computations for melting heat transfer problem due to shrinking space.

The fluid flow, the role of viscosity conveys an important role. It has been commonly noticed that the viscosity of fluid is subsequently fluctuated with phenomenon of heating, power pumping and pressure gradient. The dependence of viscosity of such factors is important and present many applications in aerodynamic processes, processing materials, extraction of petroleum etc. The process of thermo fluidic bearing with heat transfer is slightly dependent upon viscosity of material. Usually, the non-Newtonian materials possess variable viscosity. In more comprehensive way, the viscosity can be function of pressure, temperature and shear force.

However, the consideration of viscosity with temperature dependence is interesting and important to investigate. Numerous studies have been conducted to understand the effect of various models connected to varying viscosity. The inclined direction flow due to second grade material confining the Reynolds viscosity model was explored by Phuoc and Massoudi [12]. Aforesaid model was utilized for second grade material via vertical surface configuration have been evaluated by Massoudi et al. [13].

In a permeable medium, heat transfer impact in a viscoelastic fluid and temperature dependent viscosity effects on flow was considerate by Ramya et al. [14]. The viscosity taken by under the inverse relation of temperature distribution. Research in [15-18] refers different flow models with variable viscosity. However, the mutable viscosity effects to an uninterrupted uniform plate was described by Pop et al [19]. Two different variable viscosity models are discussed by Salahuddin and Arif [20]. Djebali et al. [21] observed the formulation of thermal boundary prediction for different similarity solutions.

The contemplation of electrically conducting material with impulsion of magnetic force acquires great significant from a technical perspective. Aforesaid problems have acquired abundant attention by several researchers. MHD flows has significant applications in metallurgical process, MHD pumps, plasma, chemical processes, engineering devices, generators etc. The term MHD was first introduced by Hannes Alfven in 1942. However, interest in the MHD flow begin in 1918, when Hartmann invented electromagnetic pump. MHD 2nd grade fluid was explored by Hsiao [22]. Pal and Mandal [23] investigated the effect of nonmaterial for updating the heating performances of electrically conducting material. Warke et al. [24] reported the behavior of magneto-micropolar suspension in heated porous layer.

Flow owing to a stretching sheet also arises in thermal and moisture behavior of materials, chiefly in processes such as in metallurgy, textile, paper production, glass and polymer sheets etc. Natural convection accompanied by variable viscosity and thermal conductivity due to wavy space was exhibited in research of Munir et al. [25]. Swain et al. [26] examined the magnetized flow of regarding the nanomaterials keeping variable consequences of thermal phenomenon.

The involvement of different chemical reactions such as homogeneous and heterogeneous, has great significance in chemical reacting processes. Basha et al. [27] scrutinized chemical feautres in viscoelastic material via numerical approach. The smooth conduction of these processes requires a special agent called catalyst. A complex structure develops between homogeneous/heterogeneous (HH) species near surface of catalyst. The application of these reactions involves in combustions and biochemical systems and at special rate they engage in the manufacturing and utilizing on catalytic surfaces and within the fluid. Initially in fluid flow problems these reactions were analyzed by Chaudhary and Merkin [28] by considering the stagnation point flow. The homogeneous reaction is applied in bulk through isothermal cubic autocatalator kinetics whereas the other one on the surface by first order kinetics. Shaw et al. [29] investigated these reaction effects on micropolar fluid in which they considered the flow over the permeable stretched surface. Sheikh and Zaheer [30] investigated its behavior on Casson fluid when the fluid is experiencing the uniform suction and depicted that its concentration increases due to uniform suction. Flow with fluids elasticity for an incompressible homogeneous second grade fluid was investigated by Cortell [31]. Due to the elastic properties of fluid the strain energy which is produced due to viscous dissipation is stored in the fluid and reflects in energy equation. Hakeen et al. [32] discussed the effects of elastic deformation of Walters B fluid where the fluid is flowing through permeable sheet, and they show the inverse behavior of fluid temperature due to elastic deformation. Ramzan et al. [33] addressed the role of autocatalytic chemical reactive species for nonlinear thermally developed flow due to curved surface. Prabhugouda et al. [34] presented the role of homogeneous/heterogeneous (HH) chemical reaction while focusing the thermal aspect of hybrid nanofluid. The applications of HH chemical reaction in optimized flow of hybrid nanofluid was visualized by Siddiqui et al. [35]. Vaidya et al. [36] discussed the HH chemical significance for hemodynamic flow. Iqbal et al. [37] addressed the viscous dissipation assessment of viscoelastic fluid by presenting the exact solution.

This communication aims to address the applications of homogeneous/heterogeneous (HH) chemical reaction for viscoelastic fluid flow with collective variable thermal features. The analysis has been performed when the viscosity of viscoelastic fluid is fluctuated with temperature change. The contribution of heat absorption and generation features are treated as exponential function of temperature gradient. Moreover, the analysis has been further extended by incorporating the viscous dissipation and magnetic force impact. The induced flow is considered due to linearly porous saturated space subject to the convective boundary conditions. The numerical outcomes are presented for flow model by using the BVP 4c approach. The onset of set of parameters is physically observed.

2.Physical description of problem

A second grade material due to two-dimensional motion against the stretching surface is observed. In the county $(y>0),$ the flow is restrained in the direction of a surface corresponding to the plane $(y=0)$ and the fluid is allowed to pass through the porous medium. Both the axis viz. x-axis and y-axis are considered as horizontal and vertical plane, respectively. The viscosity of fluid is supposed to be variable which changes due to temperature. The electrically conductive second grade fluid is inspired by the influence of magnetic field which is applied perpendicularly to sheet. Fig. 1 illustrating the flow pattern of current research model.

The governing flow associated expressions [37]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu^{*}(T)\frac{\partial u}{\partial y}\right)+k_{1}\left[\frac{\partial u}{\partial x}\frac{\partial^{2}u}{\partial y^{2}}+u\frac{\partial^{3}u}{\partial x\partial y^{2}}-\frac{\partial^{2}v}{\partial y^{2}}\frac{\partial u}{\partial y}+v\frac{\partial^{3}u}{\partial y^{3}}\right]-\sigma B_{0}^{2}u-\frac{\upsilon}{\Gamma}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^{2}T}{\partial y^{2}}+\frac{\mu^{*}(T)}{\rho c_{p}}\left(\frac{\partial u}{\partial y}\right)^{2}+\frac{\xi'''}{\rho c_{p}}-\frac{\delta k_{1}}{\rho c_{p}}\left[\frac{\partial u}{\partial y}\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)\right], \tag{3}$$

with associated boundary conditions

$$\left.\begin{aligned}&u=u_{w}\left(x\right)=cx,-k\frac{\partial T}{\partial y}=h_{1}\left(T_{f}-T\right),\ v=0,\ \text{at}\,y=0,\\&u,v\rightarrow 0,\ T\rightarrow T_{\infty}\ \text{at}\,y\rightarrow\infty.\end{aligned}\right\} \tag{4}$$

The defined boundary constraints are associated to stretching surface phenomenon. The flow associated to the stretched surface convey important applications in the manufacturing systems, chemical processes, coating production, polymer extrusions, fiber spinning, molten polymers, forming of sheet metals, biodiesel applications etc. In above defined problem, c is positive constant, $\beta_0,\ k_1$ represents magnetic field and second grade parameter, c_p is specific heat. α for thermal diffusivity. Γ is the medium permeability. δ represents elastic deformation parameter， h_1 stands for heat transfer coefficient. Fluid temperature is represented by T_f whereas ambient fluid temperature is represented by T_∞. ℓ_1 and ℓ_s be strength of homogeneous and heterogeneous chemical species, respectively. The heat source/sink with temperature dependent convective relations ξ''' is [37]:

$$\xi''' = \left(\frac{k\bar{\tilde{u}}_w}{xv}\right)\left[H\left(T_f - T_\infty\right) + E\left(T - T_\infty\right)\right], \tag{5}$$

where H and E be temperature dependent heat source/sink relations. If H, $E > 0$ then it relates to heat generation otherwise then heat absorption. The expression, defines the chemical reaction (homogeneous and heterogeneous) are

$$H + 2E \rightarrow 3E,\ rate = k_c \alpha \beta^2, \tag{6}$$

The isothermal, Ist-order reaction confederated with a catalyst surface is characterized as

$$H \rightarrow E,\ rate = l_s a. \tag{7}$$

where a and b is the amount of concentration in H and E (chemical species) and the term l_s shows rate constants. The governing equations which represent the current flow problem are listed as [37]:

$$u\frac{\partial a}{\partial x} + v\frac{\partial a}{\partial y} = D_H \frac{\partial^2 a}{\partial y^2} - l_1 ab^2, \tag{8}$$

$$u\frac{\partial b}{\partial x} + v\frac{\partial b}{\partial y} = D_E \frac{\partial^2 b}{\partial y^2} + l_1 ab^2, \tag{9}$$

$$\left.\begin{array}{l} D_H \dfrac{\partial a}{\partial y} = \ell_s a,\ D_E \dfrac{\partial b}{\partial y} = -\ell_s a, \quad \text{at } y = 0 \\ \qquad a \rightarrow a_0,\ b \rightarrow 0, \quad \text{at } y \rightarrow \infty. \end{array}\right\} \tag{10}$$

where D_H and D_E shows diffusion species coefficients of H and E.

The viscosity relations variable exponential factor is [7, 37]:

$$\mu^*(T) = \mu_o^* \exp\left[-\hat{a}\left(T - T_\infty\right)\right]. \tag{11}$$

Using the relation of Reynolds model [10]:

$$\mu^*(T) = \mu_0^* e^{-\lambda\theta}, \tag{12}$$

where λ represents viscosity parameter.

Intervolving new variables [37]:

$$\eta = \sqrt{\left(\frac{c}{v}\right)} y, u = cxf'(\eta), v = -\sqrt{cv}\, f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, a = a_o \varphi(\eta), b = b_o h(\eta) \tag{13}$$

The dimensionless set of equations in view of above transformations is:

$$(1-\lambda\theta)\,f''' + \beta\left(2ff''' - f^{2} - ff''''\right) - \left(f'\right)^{2} + ff'' - Mf' - \lambda f''\theta - a^{*}f' = 0, \tag{14}$$

$$\theta'' + \Pr f\theta' + Hf' + E\theta + Ec\Pr f''^{2}(1-\lambda\theta) - Ec\Pr\delta\beta f''\left(ff'' - ff'''\right) = 0, \tag{15}$$

$$\frac{1}{S_c}\varphi'' + f\varphi' - L\varphi h^{2} = 0, \tag{16}$$

$$\frac{1}{S_c}h'' + fh' + L\varphi h^{2} = 0, \tag{17}$$

associated boundary conditions will be

$$\begin{gathered} f(0)=0,\ f'(0)=1,\ f'(\infty)\to 0, \theta'(0) = -\gamma\left[1+\theta(0)\right], \\ \theta(\infty)\to 0, \varphi(\infty)\to 1, \varphi'(0)\to L_s\varphi(0) \end{gathered} \tag{18}$$

In above mathematical expression $\lambda(\text{viscosity parameter})$, M (Hartmann number), $\Pr(\text{Prandtl number})$, $S_c(\text{Schmidt number})$, a^{*} is the representation for permeability. $L(\text{chemical reaction parameter})$, $Ec(\text{ Eckert number}), \delta$ $(\text{elastic deformation parameter})$, β $(\text{second grade fluid parameter})$ and $\gamma(\text{conjugate parameter})$ which are defined as:

$$\left.\begin{aligned} &\lambda=\hat{a}\left(T_f - T_\infty\right),\ M = \frac{\sigma B_0^2}{c\rho},\ a^{*} = \frac{\upsilon}{c\Gamma},\ \Pr = \frac{\nu}{k_1},\ S_c = \frac{\nu}{D}, \\ &L = \frac{l_1 a_0^2}{c},\ Ec = \frac{u_w^2}{c_p\left(T_f - T\infty\right)},\ \beta = \frac{k_1 c}{\mu_o^{?}}\ ,\gamma = \frac{h_s}{k}\sqrt{\frac{\nu x}{c}}. \end{aligned}\right\} \tag{19}$$

The results for viscous fluid are obtained when $\beta = 0.$ Since H and E (chemical species) are believed to be of identical size, D_H and D_E are equivalent,

$$\varphi(\eta) + h(\eta) = 1, \tag{20}$$

therefore equations (16) and (17) takes the form

$$\frac{1}{Sc}\varphi'' + f\varphi' - L\varphi(1-\varphi)^2 = 0, \tag{21}$$

$$\varphi'(0) = L_s\varphi(0), \varphi(\eta)\to 1 \text{ as } \eta\to\infty. \tag{22}$$

Defining the stress component (τ_w) and heat flux rate (q_w) as [37]:

$$\tau_w = \mu^{*}(T)\frac{\partial u}{\partial y} + \alpha_1\left\{2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + v\frac{\partial^2 u}{\partial y\partial x} + u\frac{\partial^2 u}{\partial y\partial x}\right\}\Bigg|_{y=0},\ q_w = -k\frac{\partial T}{\partial y}\bigg|_{y=0}. \tag{23}$$

Defining skin friction coefficient C_{f_x} and Nusselt number N_{u_x} :

$$C_{f_x} = \frac{\tau_w}{\rho u_w^2},\ N_{u_x} = \frac{x q_w}{k\left(T_f - T\infty\right)}, \tag{24}$$

with:

$$(\mathrm{Re}_x)^{1/2} C_{fx} = (1 - \lambda\theta(0) + 3\beta) f''(0),\ (\mathrm{Re}_x)^{-1/2} Nu_x = -\theta'(0) = \gamma\left(1 + \theta\left(0\right)\right) \tag{25}$$

where $\mathrm{Re}_x = U_w(x)x/\nu$ be Reynolds number.

3. Numerical computational procedure

The complicated nature of dimensionless set of equations (14)-(15) and (21) present the nonlinearity in nature. Instead of exact solution, the numerical algorithm with RK-5 scheme is implemented. The first order expressions against formulated equations is attained with assumptions of $(y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8) = (f, f', f'', f''', \theta, \theta', \phi, \phi'),$ this yields following.

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \\ y_5' \\ y_6' \\ y_7' \\ y_8' \end{pmatrix} = \begin{pmatrix} y_2 \\ y_3 \\ y_4 \\ \left(\frac{1-\lambda y_5 + 2\beta y_2}{\beta y_1}\right) y_4 + \frac{1}{\beta y_1} y_3\left(y_1 - \lambda y_5\right) - \frac{1}{\beta y_1} y_2^2 + \frac{1}{\beta y_1} y_2\left(M + \frac{\upsilon}{c\Gamma}\right) - \frac{y_3^2}{y_1} \\ y_6 \\ -\Pr y_1 y_6 - H y_2 - E y_5 - E_c \Pr\left[y_3^2\left(1 - \lambda y_5\right) - \delta\beta y_3\left(y_2 y_3 - y_1 y_4\right)\right] \\ y_8 \\ -Scy_1 y_8 + L y_7\left(1 - y_7\right)^2 \end{pmatrix} \tag{26}$$

which yields the following along with initial conditions

$$\begin{pmatrix} y_1(0) \\ y_2(0) \\ y_3(0) \\ y_4(0) \\ y_5(0) \\ y_6(0) \\ y_7(0) \\ y_8(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ f''(0) \\ \theta(0) \\ -\gamma\left(1 + y_4(0)\right) \\ \theta'(0) \\ \varphi\left(0\right) \\ Ls\left(y_7(0)\right) \end{pmatrix} \tag{27}$$

The small step size of h=0.001 and error accuracy 10^{-6} has been followed. MATHEMTICA software is used to perform the computations.

4. Validation of results

For the confirmation and validity obtained numerical data, the comparison of results is worked out with analysis of Coretell [31] in table 1. The concluded observations are predicted for limiting case. A fine accuracy is noted for both computations.

5. Results and discussion

The findings of second grade fluid on a stretching sheet in the presence of a viscosity and chemical reaction are investigated in this section. The physical structures of the plotted figures and true mechanism of flow impressed by the dominant dimensionless parameters are also exposed in this section, by keeping $\lambda = 1, a^* = \gamma = \delta = 0.5$ fixed. The effects of Hartman number M on axial component of velocity are manifested in figure 2. The up surging strength of M is linked to a progressive resistive force known as the Lorentz force, which is activated by the interplay of electric and magnetic fields and resists fluid movement. Second grade fluid parameter β plays an important role in improving velocity of fluid. Figure 3 relates with the attitude of fluid parameter β on velocity. Wider $\beta,$ drops the viscosity of the liquid and accordingly velocity boosted. Such outcomes are physically associated to the rheology of viscoelastic fluid. Figures (4-5) display the behavior of velocity and temperature profile for viscosity parameter $\lambda.$ The velocity profile and heat transfer pattern gradually reduces when λ is higher. Such trend is accounted due to the variable fluid viscosity. Figures (6-7) exposed the impact of coefficient of internal heating generation and absorption H and E on temperature profile, the rise in temperature is due to internal heat generation. The presence of internal heat transfer enhanced the thermal pattern. Figure 8 displays the temperature contour for distinct values of Prandtl number $\Pr.$ It shows that the temperature profile falls as $\Pr$ enhanced. The slower rate of thermal diffusivity against larger $\Pr$ is depicted which report a declining in heating transportation. The reducing observations in temperature due to larger $\Pr$ is associated to the low thermal diffusivity. Figure 9 exhibits uniform rise in Ec causes less significant in heat dissipation means increase in fluid temperature. The physical consequence behind such trend is associated to the fluid kinetic energy. Figure 10 observed the declining assessment in temperature due to elastic deformation increases. The decrement in temperature profile is due to elastic deformation rate which retained larger magnitude. Figure 11 illustrates the influence of conjugate constant for Newtonian heating γ. It is observable that internal temperature of the flow rises for upper values of γ resulting the enhancement in temperature field. The temperature fluctuation due to second grade parameter β in the presences of variable viscosity is clearly shown in figure12. The slower heating transportation with increasing material parameter is observed. This trend is physically associated to the normal stresses which controls the increment in thermal phenomenon. Figure13 clarifies that for different values of magnetic force, temperature increases. The Lorentz force which possesses resistive nature appeared due to implementation of larger magnetic force. However, an increase in M, reduces the concentration level slightly in figure 14, referred to momentum diffusivity. The results exhibited for reaction parameter L against ϕ have been reported via figure 15 that displays for up surging values of L, the concentration field drops. The physical dynamic of Schmidt constant Sc on ϕ has been observed via Fig. 16. The decreasing outcomes for ϕ due to larger Sc is reflected. The physical behavior due to Sc is related to the fact of lower mass diffusivity.

The numerical framework presented in table 2 present the change in skin friction and wall temperature gradient. The wall shear force enhanced with λ, β and a^*. The Nusselt coefficient declined with $\Pr$, Ec, β and λ. The reducing numerical outcomes for skin fraction coefficient are noted for a^* and δ. Also, same performance of δ for Nusselt number. Growing values of γ, displays rise in Nusselt number.

6. Concluding remarks

The flow due second grade fluid with heat transfer analysis due to elongating surface with elastic deformation have been studied numerically. The homogeneous and heterogeneous reaction effects are also inspected. Runge Kutta numerical computations are predicted for formulated flow model. The verification for obtained solution is presented with excellent accuracy. The study reveals following major observations:

- By increasing magnetic parameter, the velocity and concentration of fluid reduces while temperature escalate.
- The temperature phenomenon drops for upsurging values of Prandtl number.
- An increment in Eckert number lead to enhancing the temperature gradient.
- Velocity profile upsurges with escalating values of viscoelastic parameter whereas reverse

results shown for temperature profile.

- For growing values of chemical reaction constant, the concentration field reduced.
- Both heat and mass transfer pattern reduce due to viscoelastic parameter.
- The wall shear force reduces due to Hartmann number and viscosity parameter.
- With enhancing viscoelastic parameter, the Nusselt number reduces.

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Table 1: Numerical verification of obtained solution with continuation of Cortell [31] when $\lambda = 0$.

M	β	Cortell [31]	Present results	Error
0.5	1.0	0.8660254	0.8660255	1.1×10^{-6}
1.0	1.0	1.0000000	1.0200000	2×10^{-2}
2.0	1.0	1.224745	1.224747	2×10^{-6}

Table 2: Numerical values of skin friction coefficient and Nusselt number for emerging parameters.

M	a^*	β	λ	δ	$-f''(0)$	Pr	Ec	β	λ	γ	δ	$-\theta'(0)$
2.0	0.2	0.4	2.0	0.1	1.36633	5.0	2.0	0.3	2.0	0.1	0.1	0.151835
3.0					1.35041	8.0						0.51071
4.0					0.395928	9.0						0.150519
	0.5				1.34382		1.0					0.151432
	1.0				1.35252		2.5					0.150427
	1.5				1.34766		3.0					0.150206
		0.6			1.96456			0.4				0.150482
		0.8			2.56906			0.6				0.149948
		1.0			3.17271			0.8				0.149345
			1.0		2.50479				0.5			0.285026
			1.5		2.55333				1.0			0.19959
			2.5		2.57414				1.5			0.167358
				0.1	2.46129					0.15		0.22704
				0.2	2.3383					0.2		0.304028
				0.3	2.21757					0.3		0.454654
											0.15	0.149963
											0.2	0.149216
											0.3	0.147723

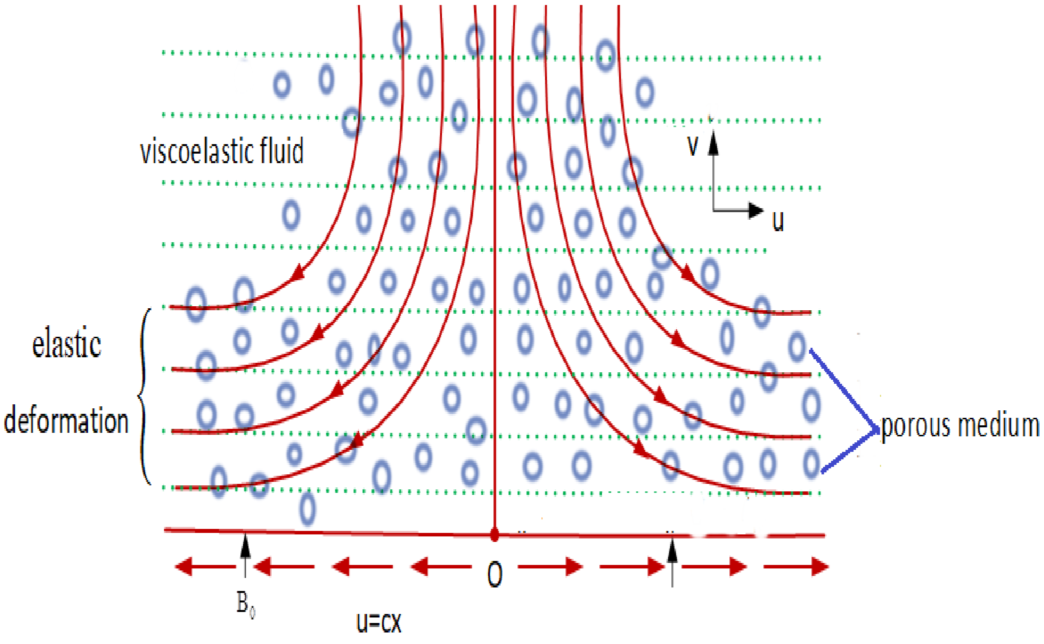

Fig. 1: Flow illustration of problem

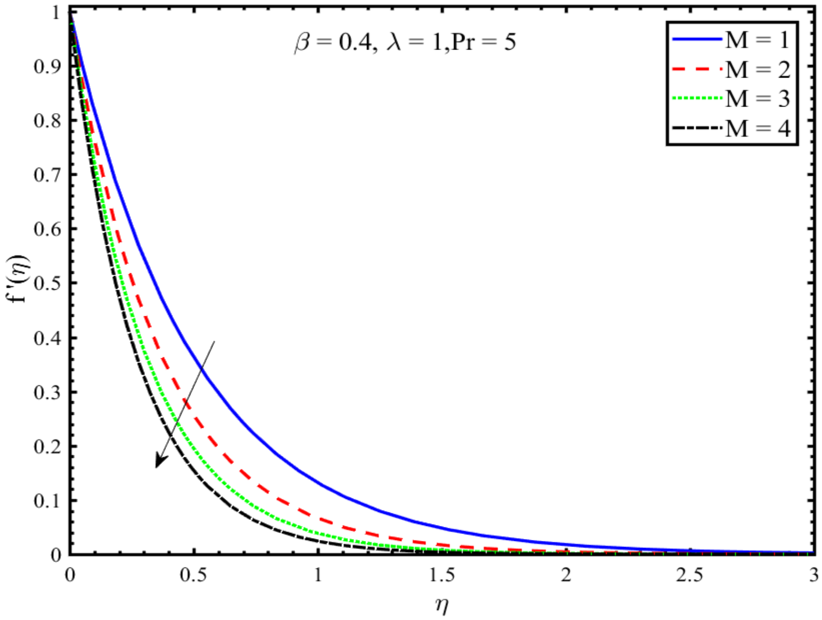

Fig. 2: Change in f' with M.

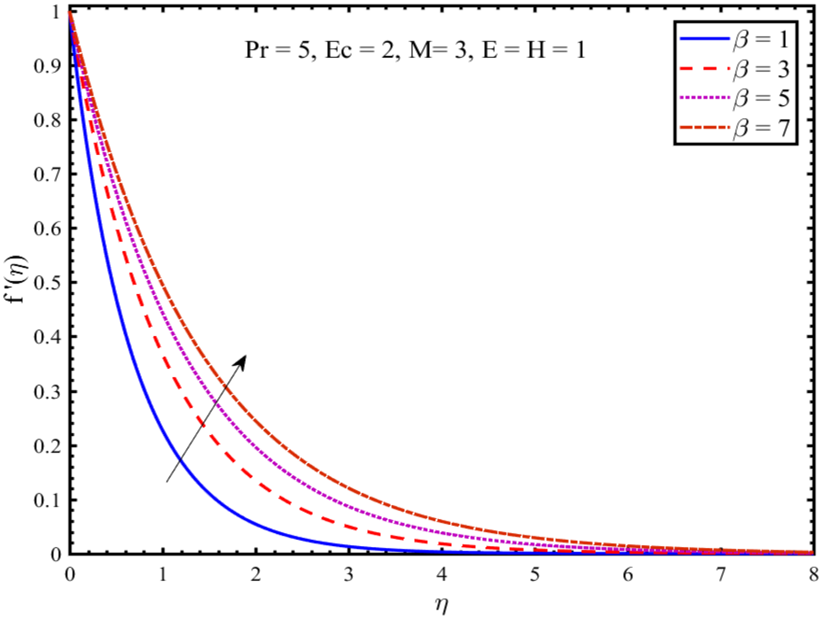

Fig. 3: Change in f' with β.

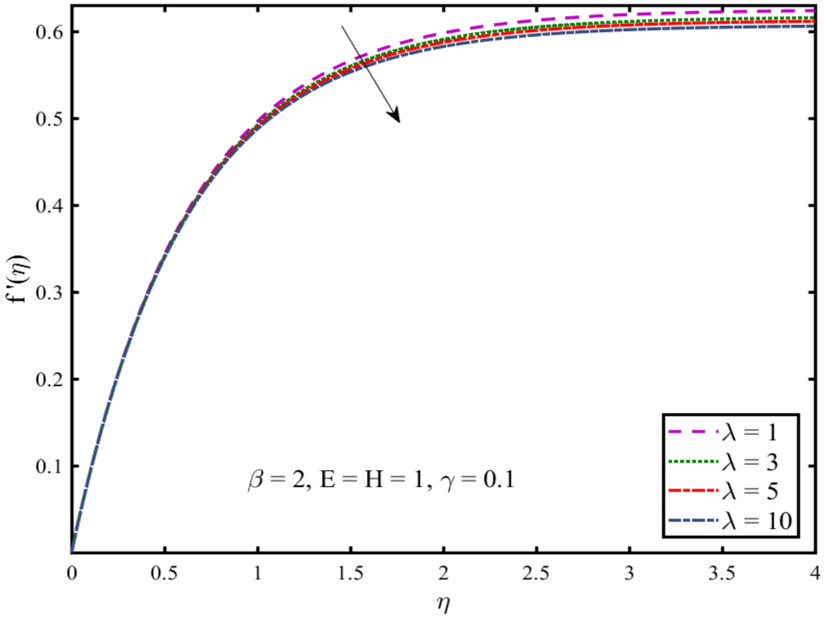

Fig. 4: Change in f' with λ.

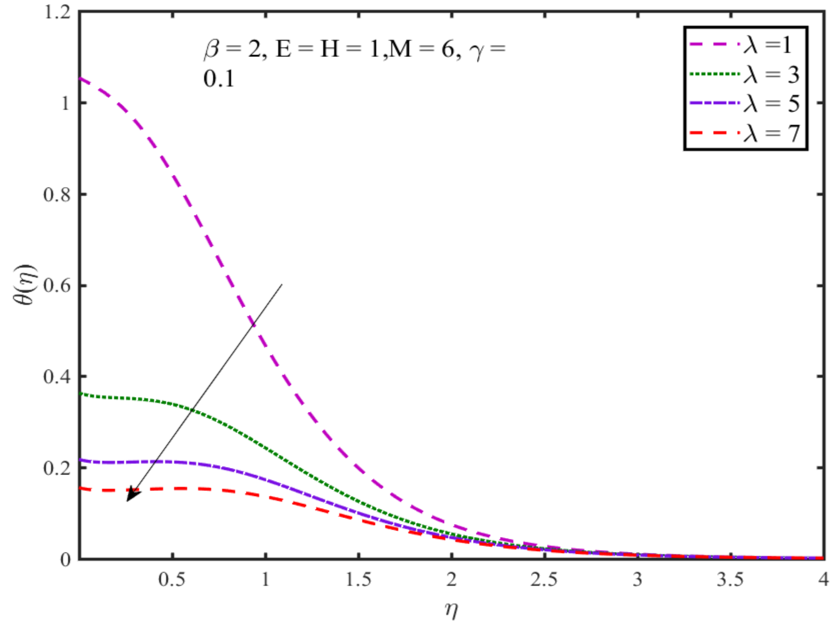

Fig. 5: Change in θ with λ.

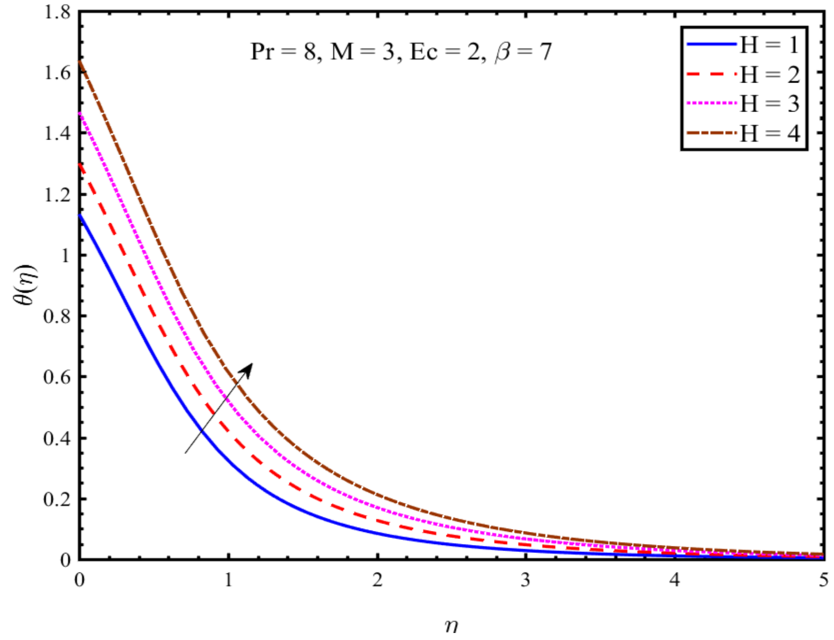

Fig. 6: Change in θ with H.

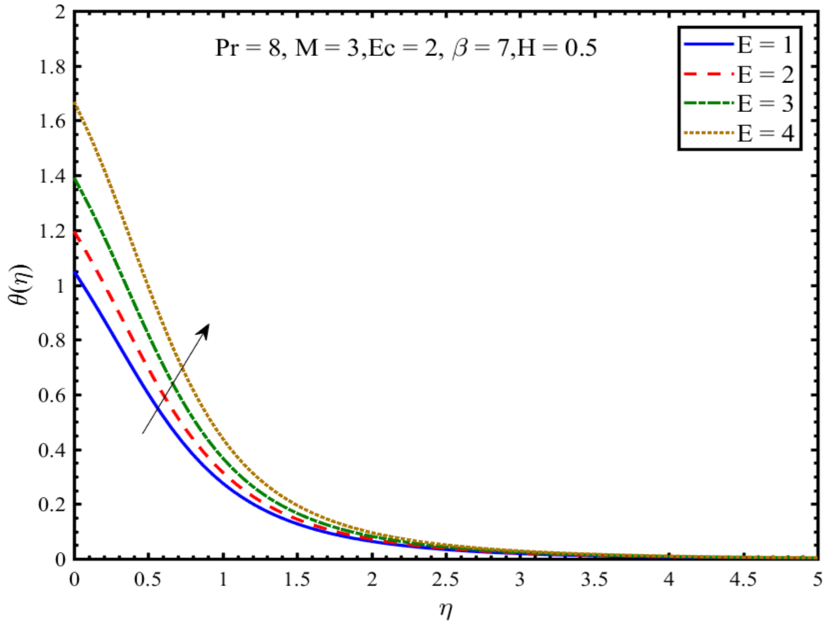

Fig. 7: Change in θ with Ec.

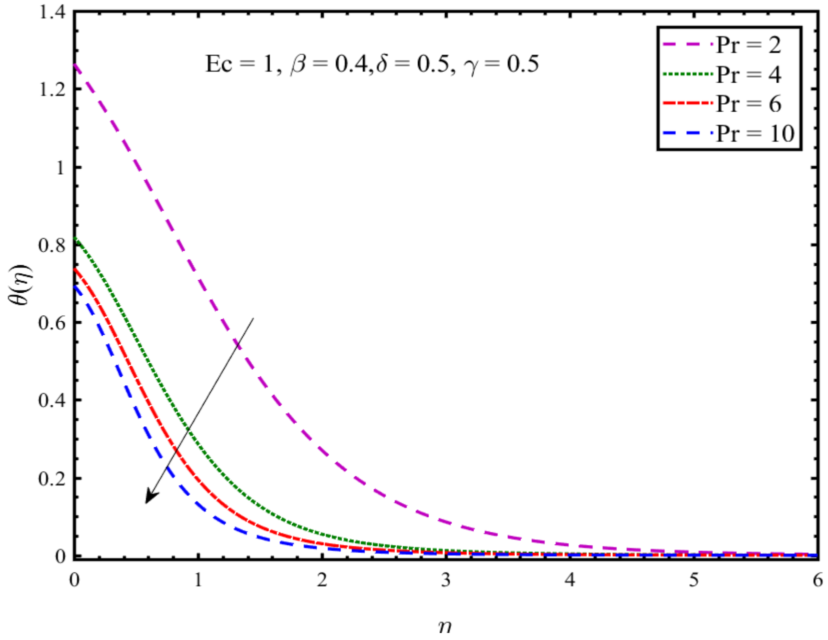

Fig. 8: Change in θ with Pr.

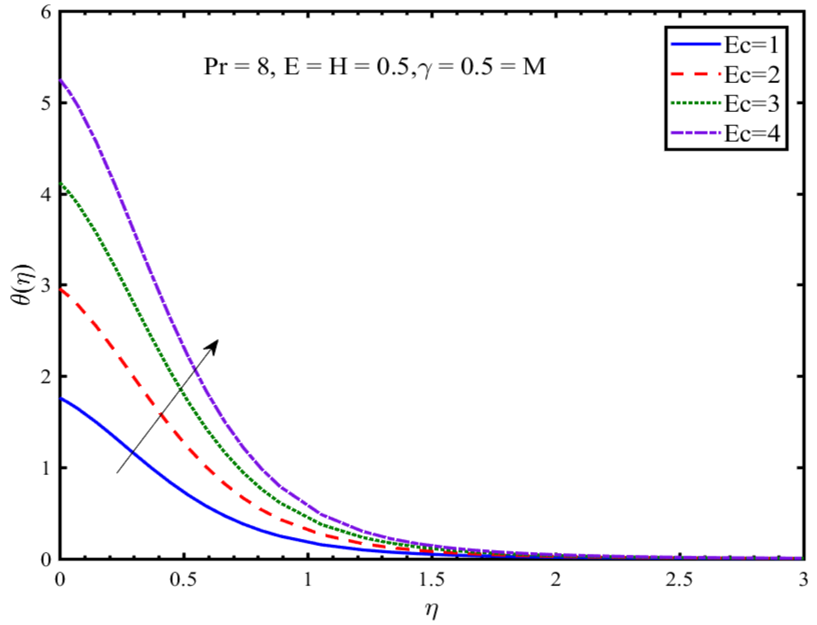

Fig. 9: Change in θ with Ec.

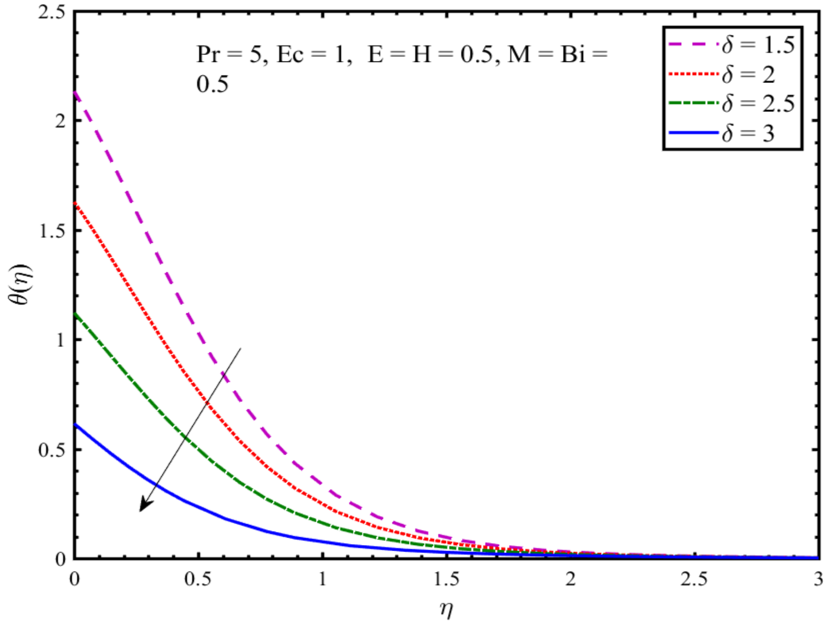

Fig. 10: Change in θ with δ.

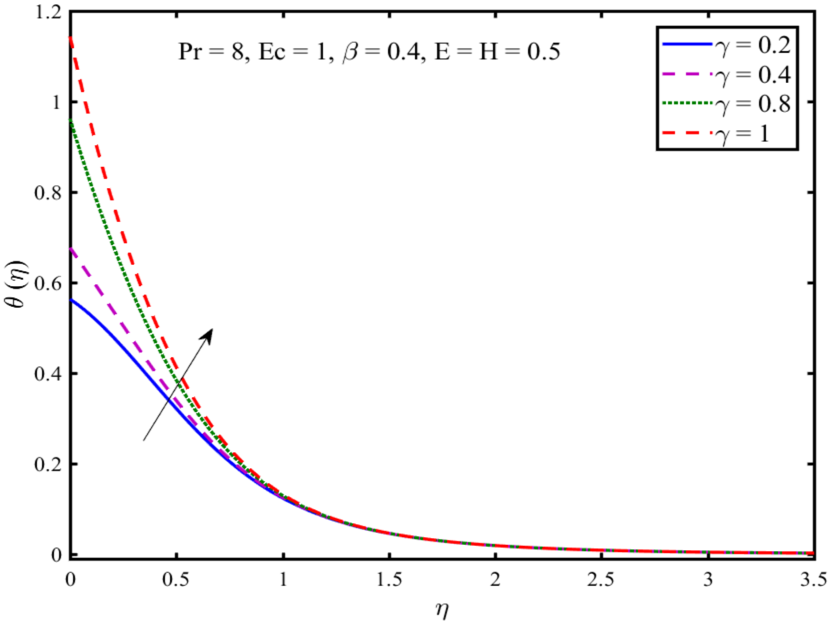

Fig. 11: Change in θ with γ.

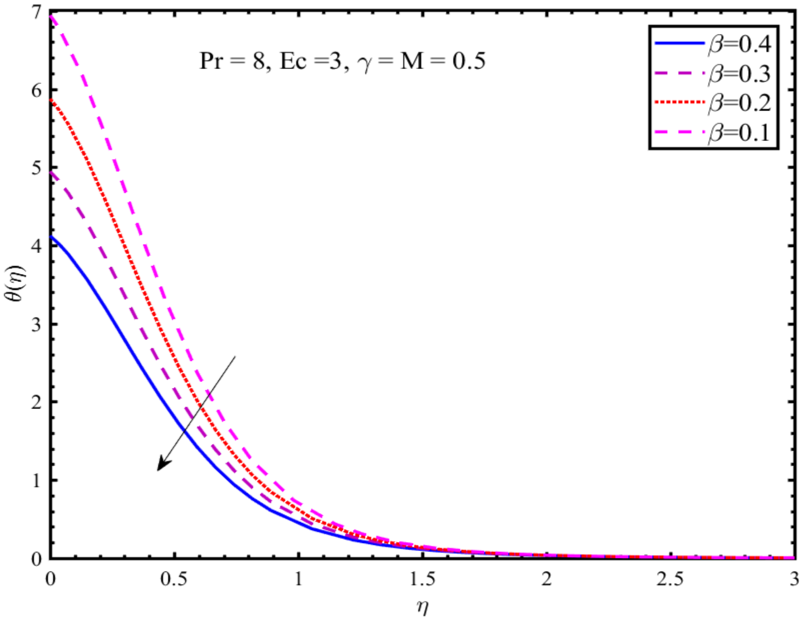

Fig. 12: Change in θ with β.

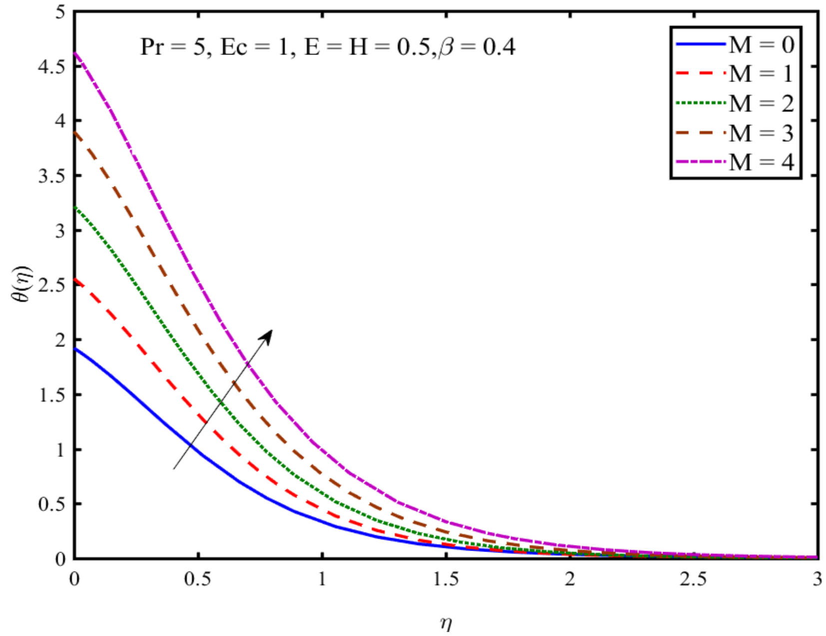

Fig. 13: Change in θ with M.

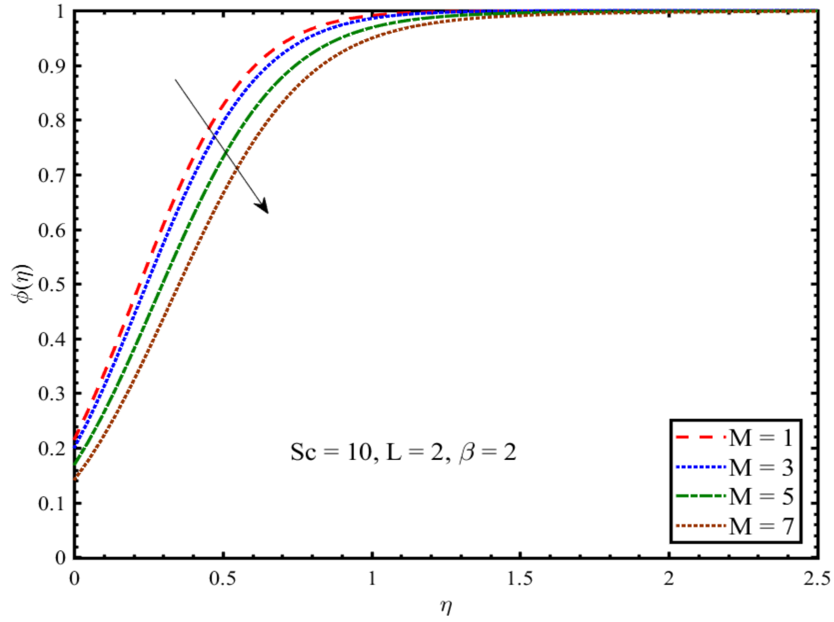

Fig. 14: Change in ϕ with M.

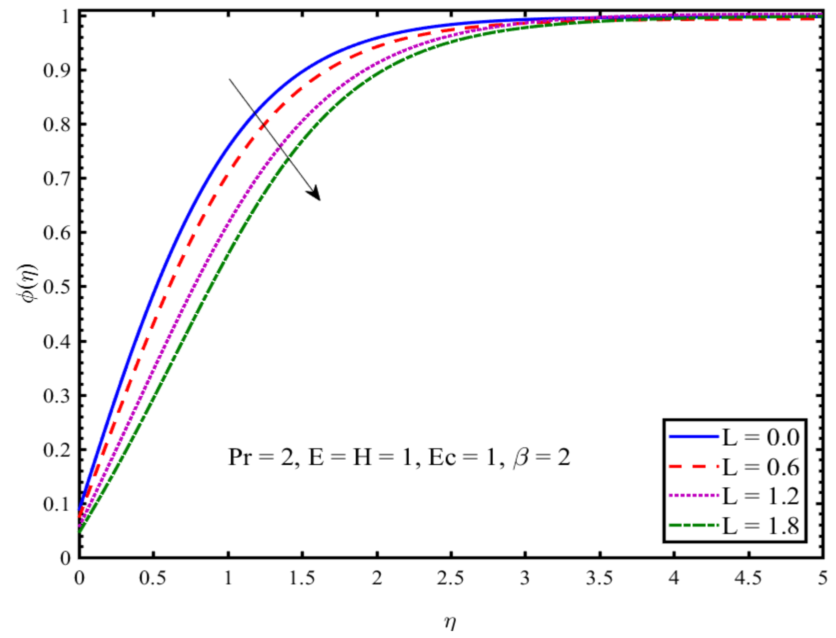

Fig. 15: Change in ϕ with L.

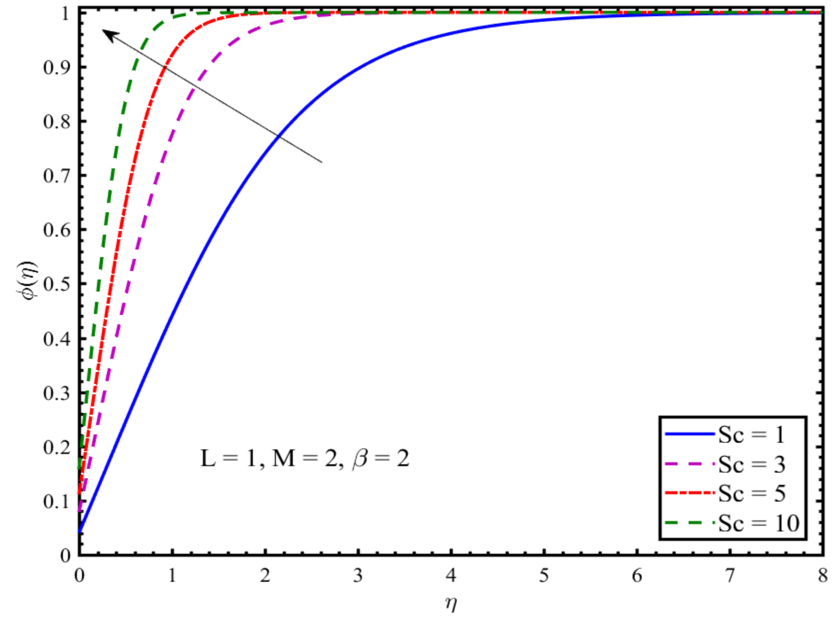

Fig. 16: Change in ϕ with Sc.

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