Numerical Investigation of the Heat Transfer and Peristaltic Flow Through a Asymmetric Channel Having Variable Viscosity and Electric Conductivity

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Abstract:

The peristaltic flow of nanofluids is a topic of growing interest in fluid dynamics. This study investigates the effect of temperature-dependent viscosity and electric conductivity on the peristaltic flow of nanofluids. The mathematical model of the peristaltic flow is developed using the governing equations of continuity, momentum, and energy for a Newtonian fluid. Large wavelength and small Reynolds number assumptions are used to study peristaltic flow to simplify the equations of continuity, momentum, and energy. In this article, the nanofluids are assumed to be electrically conducting and temperature dependent, and the effects of Hartman number and Eckert number is studied. The resulting equations are solved using the Shooting Method. The results show that the temperature-dependent viscosity and electric conductivity significantly affect the peristaltic flow of nanofluids. The flow rate and pressure gradient decrease with increasing viscosity and conductivity while the temperature and heat transfer rate increase. Moreover, the nanofluid concentration and particle size significantly impact the flow characteristics. In conclusion, this study comprehensively analyses the peristaltic flow of nanofluids with temperature-dependent viscosity and electric conductivity. The results can be useful for understanding the behaviour of nanofluids in various applications, such as drug delivery systems, microfluidics, and thermal management.

Keywords:

Nanofluids; Heat transfer; Shooting Method; Peristaltic flow; Variable viscosity; Variable electric conductivity

1. Introduction:

The word "nanofluid" was first used by Choi [1]. Nanofluid is a colloidal suspension containing nanoparticles with sizes ranging from 1 to 100 nanometers dispersed in a base fluid, such as water, oil, or ethylene glycol [2-3]. The nanoparticles can be made of various materials, including metals, metal oxides, and carbon-based materials. Nanofluids exhibit unique properties due to the nanoparticles' high surface area to volume ratio, which can enhance the base fluid's thermal conductivity, electrical conductivity, and viscosity [4]. The study of nanofluids has gained considerable attention in recent years due to their potential applications in various fields such as energy, biomedical, and industrial sectors [5]. Nanofluids have been studied as a promising candidate for heat transfer applications in the energy sector, as they exhibit higher thermal conductivity than conventional fluids [6]. In the biomedical field, nanofluids have been studied as potential drug-delivery vehicles due to their ability to penetrate biological barriers [7].

The peristaltic flow of nanofluids is a field of fluid mechanics that combines the study of peristaltic motion, which is a type of fluid flow induced by periodic contraction and relaxation of a tube or channel, with the unique properties of nanofluids, which are colloidal suspensions containing nanoparticles dispersed in a base fluid [8]. The study of the peristaltic flow of nanofluids has gained considerable attention in recent years due to its potential applications in various fields, such as biomedical, industrial, and energy sectors [9]. The unique properties of nanofluids, such as high thermal conductivity and enhanced viscosity, can affect the behaviour of peristaltic flow. Additionally, the peristaltic motion can induce particle migration and concentration gradients in the fluid, leading to potential applications in drug delivery, microfluidics, and heat transfer [10].

Studying the peristaltic flow of nanofluids requires a multidisciplinary approach that combines fluid mechanics, nanoscience, and engineering. Various analytical and numerical techniques have been used to study the peristaltic flow of nanofluids, including perturbation methods, numerical simulations, and experimental measurements [11-12]. Understanding the peristaltic flow of nanofluids is of great importance for designing and optimising microfluidic devices and heat transfer systems. The potential applications of peristaltic flow of nanofluids in the biomedical field, such as drug delivery systems and targeted therapy, have also generated significant interest among researchers [13-14]. The study of the peristaltic flow of nanofluids is a rapidly growing field that presents numerous opportunities for research and development, with potential applications in various areas, including energy, industrial, and biomedical sectors [15-18]. The peristaltic flow of nanofluids with heat transfer is a field of fluid mechanics that combines the study of peristaltic motion with the unique properties of nanofluids, which are colloidal suspensions containing nanoparticles dispersed in a base fluid, and heat transfer [19]. The study of the peristaltic flow of nanofluids with heat transfer has gained considerable attention in recent years due to its potential applications in various fields, such as energy, biomedical, and industrial sectors [20]. Peristaltic motion induced by periodic contraction and relaxation of a tube or channel can enhance heat transfer due to the mixing of the fluid and the generation of vortices. Additionally, the unique properties of nanofluids, such as high thermal conductivity, can further enhance heat transfer in peristaltic flow. Studying the peristaltic flow of nanofluids with heat transfer requires a multidisciplinary approach that combines fluid mechanics, nanoscience, and heat transfer. Various analytical and numerical techniques have been used to study the peristaltic flow of nanofluids with heat transfer, including perturbation methods, numerical simulations, and experimental measurements. Understanding the peristaltic flow of nanofluids with heat transfer is of great importance for the design and optimization of microfluidic devices and heat transfer systems, as well as for the development of new materials for heat transfer applications [21-23].

The potential applications of the peristaltic flow of nanofluids with heat transfer in the energy sector, such as in designing cooling systems for electronic devices and developing thermal energy storage systems, have generated significant interest among researchers [24]. In the biomedical field, the peristaltic flow of nanofluids with heat transfer can potentially be used in hyperthermia treatment for cancer therapy [25]. The study of the peristaltic flow of nanofluids with heat transfer is a rapidly growing field that presents numerous opportunities for research and development, with potential applications in various areas, including energy, biomedical, and industrial sectors [26]. The peristaltic flow of nanofluids with heat transfer having temperature-dependent viscosity and electric conductivity is an advanced field of fluid mechanics that combines the study of peristaltic motion with the unique properties of nanofluids and their temperature-dependent viscosity and electric conductivity. The analysis of the peristaltic flow of nanofluids with heat transfer and variable properties has gained considerable attention in recent years due to its potential applications in various fields, such as energy, biomedical, and industrial sectors [27-29]. The properties of nanofluids, such as high thermal conductivity and enhanced viscosity, can affect the behaviour of peristaltic flow [30]. Additionally, the peristaltic motion can induce particle migration and concentration gradients in the fluid, leading to potential applications in drug delivery, microfluidics, and heat transfer [31]. However, the properties of nanofluids can vary significantly with temperature, which can affect the behaviour of peristaltic flow and heat transfer.

Studying the peristaltic flow of nanofluids with heat transfer having temperature-dependent viscosity and electric conductivity requires a multidisciplinary approach that combines fluid mechanics, nanoscience, heat transfer, and electromagnetics. Various analytical and numerical techniques have been used to study the peristaltic flow of nanofluids with variable properties, including perturbation methods, numerical simulations, and experimental measurements [32-33]. Understanding the peristaltic flow of nanofluids with heat transfer and variable properties is of great importance for the design and optimization of microfluidic devices and heat transfer systems [34]. The potential applications of peristaltic flow of nanofluids with variable properties in the energy sector, such as in designing cooling systems for electronic devices and developing thermal energy storage systems, have generated significant interest among researchers. In the biomedical field, the peristaltic flow of nanofluids with variable properties can potentially be used in hyperthermia treatment for cancer therapy and drug delivery systems [35].

Studying the peristaltic flow of nanofluids with heat transfer having temperature-dependent viscosity and electric conductivity is an emerging and challenging field that presents numerous opportunities for research and development, with potential applications in various areas, including energy, biomedical, and industrial sectors. The temperature-dependent viscosity and electric conductivity of nanofluids can significantly affect the behaviour of peristaltic flow and heat transfer. The following are some of the results that have been observed in the study of the peristaltic flow of nanofluids with variable properties:

1. Temperature-dependent viscosity: The viscosity of nanofluids can increase significantly with increasing temperature due to Brownian motion and particle aggregation. This can affect the peristaltic motion and the generation of vortices, leading to changes in heat transfer characteristics. Furthermore, an increase in viscosity can also result in higher pressure drop, which can affect the pumping efficiency of the system.
2. Temperature-dependent electric conductivity: The electric conductivity of nanofluids can also vary with temperature due to changes in the mobility of ions and electrons. The peristaltic motion can induce electrical charges and create electric fields, leading to potential applications in electrokinetics and electroosmotic flow. The electric

conductivity can also affect the Joule heating and the overall heat transfer characteristics of the system.

3. Particle migration and concentration gradients: The peristaltic motion can induce particle migration and concentration gradients in the fluid due to the combined effects of fluid flow and electric fields. The migration and concentration of nanoparticles can affect the thermal conductivity and heat transfer characteristics of the system.
4. Non-Newtonian behavior: Nanofluids can exhibit non-Newtonian behavior due to the presence of nanoparticles and their interaction with the base fluid. The non-Newtonian behavior can affect the peristaltic motion and the overall heat transfer characteristics of the system.

The temperature-dependent viscosity and electric conductivity of nanofluids can significantly affect the behaviour of peristaltic flow and heat transfer. The study of the peristaltic flow of nanofluids with variable properties is an emerging field that presents numerous opportunities for research and development, with potential applications in various areas, including energy, biomedical, and industrial sectors.

2. Problem Formulation:

Flow of an incompressible nanofluid in a $l_1 + l_2$ width asymmetric channel. The flow is caused by the propagation of sinusoidal waves over the channel's non-conducting and flexible walls. The following are the mathematical formulas for channel walls [36] as seen in Figure. 1:

$$Y = W_1(\mathrm{X}, \mathrm{t}) = \mathrm{l}_1 + \delta_1 \cos\left[\frac{2\pi}{\lambda}(\mathrm{X} - \mathrm{st})\right],$$

$$Y = W_2(\mathrm{X}, \mathrm{t}) = -\mathrm{l}_1 - \delta_2 \cos\left[\frac{2\pi}{\lambda}(\mathrm{X} - \mathrm{st}) + \alpha\right], \tag{1}$$

In the case of rectangular coordinates, the length of the channel and the normal to its walls are used, while time coordinate is represented by t. The amplitudes of the upper and lower walls of the channel are denoted by the letters L1 and L2, respectively. These amplitudes may be used to calculate the Reynolds number, which is a dimensionless parameter that describes the ratio of inertial forces to viscous forces in the fluid. The Reynolds number is an important parameter in fluid mechanics, as it can determine the nature of the fluid flow behavior, such as laminar or turbulent flow. Overall, the choice of coordinate system and the parameters used to describe fluid flow behavior depend on the specific problem being analyzed and the physical system being studied, while the phase difference is denoted by the letters $\alpha \in [0, \pi]$ and the wavelength is denoted by the letterλ. Furthermore, the amplitudes δ_1, δ_2, the width of the channel $l_1 + l_2$ and the phase difference meets the following constraint: amplitudes δ_1, δ_2, width of the channel $l_1 + l_2$ .In addition, the magnetic field is present.

$$\sqrt{\delta_1 + \delta_2 + 2\delta_1\delta_2 \cos(\alpha)} \leq (l_1 + l_2) \tag{2}$$

It is assumed that constant wall temperature of the upper wall denoted T_0 and the constant temperature of the lower wall satisfies the following the condition $\mathrm{T}_0 \leq T_1$, When a magnetic field is present, it can exert a force on the charged particles in the fluid, which in turn can affect the fluid flow behavior. To model the effects of a magnetic field in a fluid, the magnetic field strength is typically described using the magnetic field intensity, denoted by the symbol $\mathrm{M} = [0; \mathrm{M}; 0]$. Temperature dependent viscosity and electric conductivity are two crucial properties that are at the heart of many scientific research studies. Viscosity is a measure of a fluid's resistance to flow, while electric conductivity is a measure of a material's ability to conduct electrical current. Both of these properties are highly dependent on temperature, which can have a significant impact on the behavior of materials and fluids. In many industrial and scientific applications, understanding the temperature-dependent behavior of viscosity and electric conductivity is essential. For example, in the petroleum industry, knowledge of viscosity is crucial for predicting the flow behavior of crude oil through pipelines. Similarly, in the field of materials science, understanding the electric conductivity of materials at different temperatures is important for developing new technologies such as solar cells and batteries. Research into the temperature-dependent behavior of viscosity and electric conductivity is ongoing, and new insights into these properties are continually being discovered. As scientists continue to deepen our understanding of these properties, we are able to better predict and control the behavior of materials and fluids, leading to advances in a wide range of industries and fields.

2.1. Mathematical Modeling:

The peristaltic flow of nanofluids with heat transfer can be mathematically modeled using the Navier-Stokes equations for fluid flow and the energy equation for heat transfer. The governing equations are supplemented by the continuity equation and the Maxwell equations for the electric field.

The continuity equation is given as [37]:

$$\frac{\partial \rho}{\partial t} + \Delta(\rho u) = 0, \quad (3)$$

where ρ is the density, t is time, u is the velocity vector, and ∇ is the gradient operator.

The Navier-Stokes equations are [38]:

$$\frac{\partial u}{\partial t} + u.\Delta u = -(\frac{1}{\rho})\Delta p + \mu \Delta^2 u + f, \quad (4)$$

where p is the pressure, μ is the dynamic viscosity, and f is the body force per unit mass. For peristaltic flow, f is modeled as the wave velocity and amplitude.

The energy equation for heat transfer is [39]:

$$\rho c_p (\frac{\partial T}{\partial t} + \mathrm{u}.\Delta \mathrm{T}) = \Delta.(\mathrm{k}\,\Delta T) + \mathrm{Q}, \quad (5)$$

where C_p is the specific heat, T is the temperature, k is the thermal conductivity, and Q is the heat generation or absorption term.

For nanofluids, the viscosity and electric conductivity can be modeled as temperature dependent functions. The viscosity can be modeled using the Vogel-Fulcher-Tamman (VFT) Equation (6) [40]:

$$\mu(\mathrm{T}) = \mu_0 \exp[\frac{\Delta E}{k_b T - T_0}], \quad (6)$$

Where μ_0 is the viscosity at a reference temperature $T_0, \Delta E$ is the activation energy, $K_b$is the Boltzmann constant, and T is the temperature.

The electric conductivity can be modeled using the Arrhenius equation (7) [41]

$$\sigma(T) = \sigma_0 \exp[-\frac{Ea}{k_b}(\frac{1}{T} - \frac{1}{T_0})], \quad (7)$$

where σ_0 is the electric conductivity at a reference temperature T0, Ea is the activation energy, and T is the temperature.

The Maxwell equations for the electric field are [42]:

$$\Delta.\mathrm{E} = \rho_e / \varepsilon_0, \quad (8)$$

$$\Delta \times \mathrm{E} = -\frac{\partial B}{\partial t},$$

where E is the electric field, ρ_e is the charge density, ε_0 is the electric constant, and B is the magnetic field.

Governing Equations in Cartesian form is written as which is modeled on the basis of temperature depend

The governing equations for the continuity, momentum and energy equations are given below [43].

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{9}$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\mathrm{v}\frac{\partial u}{\partial x}\right)=\begin{cases}-\dfrac{\partial p}{\partial x}+2\left[\dfrac{\partial \mu(T)}{\partial T}\left(\dfrac{\partial T}{\partial x}\dfrac{\partial u}{\partial x}\right)+\mu(\mathrm{T})\dfrac{\partial^2 u}{\partial x^2}\right]+\\ \dfrac{\partial}{\partial Y}[\mu(\mathrm{T})(\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y})]-\sigma(\mathrm{T})\,\mathrm{M}_0{}^2\,u\end{cases} \tag{10}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+\mathrm{v}\frac{\partial v}{\partial x}\right)=\begin{cases}-\dfrac{\partial p}{\partial y}+2\left[\dfrac{\partial \mu(T)}{\partial T}\left(\dfrac{\partial T}{\partial y}\dfrac{\partial u}{\partial y}\right)+\mu(\mathrm{T})\dfrac{\partial^2 u}{\partial y^2}\right]+\\ \dfrac{\partial}{\partial x}[\mu(\mathrm{T})(\dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x})]\end{cases} \tag{11}$$

$$(\rho s)_{nf}\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+\mathrm{v}\frac{\partial T}{\partial y}\right)=\begin{cases}k_{nf}\left[\dfrac{\partial^2 T}{\partial x^2}+\dfrac{\partial^2 T}{\partial y^2}\right]+\sigma(T)M_0^2+\\ \mu(\mathrm{T})\left[(\dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x})^2+2(\dfrac{\partial u}{\partial x})^2+2(\dfrac{\partial u}{\partial x})^2\right]\end{cases} \tag{12}$$

The following equations are used to express the frames that relate unsteady and steady flow phenomena, which are referred to as the fixed frame and the moving frame, respectively:

$$\begin{cases}X=\bar{X}+st\\ Y=\bar{Y}+st\\ U(X,Y,t)=\bar{U}(\bar{\mathrm{X}},\bar{\mathrm{Y}})+\mathrm{s}\end{cases},\begin{cases}V(\mathrm{X},\mathrm{Y},\mathrm{t})=\mathrm{V}(\bar{\mathrm{X}},\bar{\mathrm{Y}})\\ \mathrm{P}(\mathrm{X},\mathrm{Y},\mathrm{t})=\bar{P}(\bar{\mathrm{X}},\bar{\mathrm{Y}})\ ,\\ \mathrm{T}(\mathrm{X},\mathrm{Y},\mathrm{t})=\bar{\mathrm{T}}(\bar{\mathrm{X}},\bar{\mathrm{Y}})\end{cases} \tag{13}$$

The non-dimensional parameters are provided as follows:

$$\begin{cases} \xi = \dfrac{\bar{X}}{\lambda} \\ \upsilon = \dfrac{V}{\tau s} \\ w_2 = \dfrac{w_2}{L_1} \\ L = \dfrac{L_2}{L_1} \\ \gamma^* = \gamma(\bar{\mathrm{T}}_1 - \bar{\mathrm{T}}_0) \\ \Pr = \dfrac{\mu_0 (\mathrm{S}_p)_b}{k_b} \\ \mu_{nf}(\theta) = \dfrac{\mu_{nf}(\bar{\mathrm{T}})}{\mu_0} \end{cases} \qquad \begin{cases} \eta = \dfrac{Y}{L_1} \\ \tau = \dfrac{L}{\lambda} \\ \varepsilon_1 = \dfrac{\delta_1}{L_1} \\ p = \dfrac{p L_1^2}{\lambda s \mu_0} \\ \theta = \dfrac{T - T_0}{T_1 - T_0} \\ Ec = \dfrac{s^2}{(\mathrm{S}_p)_b (\bar{\mathrm{T}}_1 - \bar{\mathrm{T}}_0)} \end{cases} \qquad \begin{cases} u = \dfrac{\bar{U}}{s} \\ w_1 = \dfrac{W_1}{L_1} \\ \varepsilon_2 = \dfrac{\delta_2}{L_1} \\ Ha^2 = \dfrac{\sigma_0 M_0^2 L_1^2}{\mu_0} \\ \mathrm{Re} = \dfrac{\rho s L_1}{\mu_0} \end{cases} \tag{14}$$

The following dimension less equations are obtained by apply the assumptions of the long wavelength and low Reynolds number then also apply the stream function on the obtained governing equations such as,

$$u = \frac{\partial \psi}{\partial \eta}, \mathrm{v} = -\frac{\partial \psi}{\partial \zeta}, \tag{15}$$

Also substituting the values of the $\mu_{nf}(\theta), \sigma_{nf}(\theta)$ and κ_{nf} in the obtained governing equations such as

$$\frac{\partial p}{\partial \zeta} = A_1 \frac{\partial}{\partial \eta}((1-\beta^* \theta)\frac{\partial^2 \psi}{\partial \eta^2}) - (1+\gamma^* \theta)\,\mathrm{A}_2\, Ha^2 (\frac{\partial \psi}{\partial \eta} + 1), \tag{16}$$

$$\frac{\partial p}{\partial \eta} = 0, \tag{17}$$

$$A_3 (\frac{\partial^2 \theta}{\partial \eta^2}) + \mathrm{A}_1 \Pr Ec(1-\beta^* \theta)(\frac{\partial^2 \psi}{\partial \eta^2})^2 + \Pr EcHa^2 A_2 (1+\theta\gamma^*) \tag{18}$$

The values of the A_1, A_2 *and* A_3 written in the given,

$$A_1 = \frac{1}{(1-\chi)^{2.5}},$$
$$A_2 = \frac{\sigma_s + 2\sigma_0(1+\gamma^*\theta) + 2\chi(\sigma_s - \sigma_0(1+\gamma^*\theta))}{\sigma_s + 2\sigma_0(1+\gamma^*\theta) - \chi(\sigma_s - \sigma_0(1+\gamma^*\theta))}$$
$$A_3 = \frac{k_s + 2k_b + 2\chi(\mathrm{k}_s - \mathrm{k}_b)}{k_s + 2k_f - \chi(\mathrm{k}_s - \mathrm{k}_b)},$$

Equations (16-17) is simplified by removing the pressure gradient.

$$\left\{\begin{array}{l} A_1[1-\beta^*\theta\frac{\partial^3\psi}{\partial\eta^3} - \beta^*\frac{\partial^2\psi}{\partial\eta^2}\frac{\partial^2\theta}{\partial\eta^2} - 2\beta^*\frac{\partial^3\psi}{\partial\eta^3}] \\ -A_2 Ha^2\gamma^*\frac{\partial\theta}{\partial\eta}(\frac{\partial\psi}{\partial\eta}+1) - \\ [(1+\gamma^*\theta)A_2 Ha^2 \\ [(1+\gamma^*\theta)A_2 Ha^2(\frac{\partial\psi}{\partial\eta}+1)] \end{array}\right. = 0, \tag{19}$$

Dimensionless boundary conditions,

$$\psi = -\frac{F}{2}, \frac{\partial\psi}{\partial\eta} = 0, \theta = 1 at \eta = \omega_2,$$
$$\psi = -\frac{F}{2}, \frac{\partial\psi}{\partial\eta} = 0, \theta = 0 at \eta = \omega_1, \tag{20}$$

$F = K^* - L,$, where F and K^* respectively are the dimensionless mean flow rates in fixed and wave frames.

3. Solution Methodology:

The numerical approach implemented in Maple 2022 in which shooting technique is used to assess the final simplified Equations in (16), (17), (18) and (19), as well as the appropriate boundary conditions supplied in Equation. (20).

4. Results and Discussions:

The impacts of different physical factors on heat transfer coefficient, dimensionless velocity $u(\xi,\eta)$, temperature $\theta(\xi,\eta)$, and axial pressure gradient $\frac{dp}{d\xi}$ were investigated and described in this section. Figures (2-13) show the influence of the Hartmann number on the velocity, temperature and axial pressure gradient respectively, in order to better understand the phenomenon. Each of these graphs represents a nanofluid of water $(-Fe_3O_4)$. The Hartmann number (Ha) is a dimensionless parameter used to characterize the behavior of a conducting fluid in the presence of a magnetic field. It is defined as the ratio of the magnetic field strength (B) to the velocity of the fluid $u(\xi,\eta)$ times the square root of the fluid's electrical conductivity (σ) and density (ρ):

$Ha = \frac{B}{u(\xi,\eta)} * \sqrt{\rho * \sigma}$ The relationship between velocity and Hartmann number in Figure 2 can be analyzed by considering the effects of a magnetic field on the fluid flow. The magnetic field can cause the fluid to become more viscous and resistive, which in turn can affect the velocity of the fluid. In general, for a fixed magnetic field strength and electrical conductivity, as the velocity of the fluid increases, the Hartmann number decreases. This is because the denominator of the Hartmann number equation, $u(\xi,\eta) * \sqrt{\rho * \sigma}$ increases as the velocity increases, leading to a smaller Ha value. Conversely, for a fixed velocity and electrical conductivity, as the magnetic field strength increases, the Hartmann number increases. This is because the numerator of the Hartmann number equation, B, increases as the magnetic field strength increases, leading to a larger Ha value. The relationship between velocity and Hartmann number can have significant implications for the behavior of conducting fluids in magnetic fields. For example, at high Hartmann numbers, the magnetic field can dominate the fluid flow and suppress turbulence, while at low Hartmann numbers, the fluid flow can become turbulent and chaotic. Overall, the relationship between velocity and Hartmann number is complex and depends on the specific conditions of the fluid flow and magnetic field. However, understanding this relationship can help in predicting and controlling the behavior of conducting fluids in the presence of magnetic fields. The pressure gradient, in Figure 3 on the other hand, is a measure of how the pressure of a fluid changes with respect to distance. In general, a positive pressure gradient means that the pressure increases in the direction of flow, while a negative pressure gradient means that the pressure decreases in the direction of flow. The relationship between the pressure gradient $\frac{dp}{d\xi}$ and Hartmann number can be analyzed by considering the effects of a magnetic field on the fluid flow. The magnetic field can cause the fluid to become more viscous and resistive, which can in turn affect the pressure gradient of the fluid. In general, for a fixed magnetic field strength and electrical conductivity, as the pressure gradient increases, the Hartmann number decreases. This is because the denominator of the Hartmann number equation, $u(\xi,\eta) * \sqrt{\rho * \sigma}$ increases as the pressure gradient increases, leading to a smaller Ha value. Conversely, for a fixed pressure gradient and electrical conductivity, as the magnetic field strength increases, the Hartmann number increases. This is because the numerator of the Hartmann number equation, B, increases as the magnetic field strength increases, leading to a larger Ha value. The relationship between pressure gradient and Hartmann number can have significant implications for the behavior of conducting fluids in magnetic fields. For example, at high Hartmann numbers, the magnetic field can dominate the fluid flow and suppress turbulence, while at low Hartmann numbers, the fluid flow can become turbulent and chaotic. Similarly, changes in the pressure gradient can affect the behavior of the fluid, including its stability and the formation of vortices and eddies. The trapping phenomenon is a well-known phenomenon that occurs in magnetohydrodynamics (MHD), where a magnetic field is applied to a conducting fluid. The Hartmann number, which is a dimensionless parameter, is used to characterize the strength of the magnetic field. In this discussion, we will examine the trapping phenomenon for various values of the Hartmann number.The trapping phenomenon occurs when the magnetic field is strong enough to slow down the fluid flow in the direction perpendicular to the magnetic field lines, but not strong enough to completely stop it. This results in a build-up of fluid in Figure 4 in the direction perpendicular to the magnetic field, leading to the trapping phenomenon. The strength of the magnetic field is characterized by the Hartmann number, which is given by:

$$Ha = B_0 d\sqrt{(\sigma / \rho\mu)}$$

where B_0 is the strength of the magnetic field, d is the characteristic length scale of the system, σ is the electrical conductivity of the fluid, ρ is the density of the fluid, and μ is the magnetic permeability of the fluid. When the Hartmann number Ha is small, the magnetic field is not strong enough to cause trapping, and the fluid flows freely. As the Hartmann number increases, the magnetic field becomes stronger, and the fluid flow is slowed down. Eventually, a critical Hartmann number Ha is reached, at which point the trapping phenomenon occurs. Beyond this critical Hartmann number Ha, the fluid is trapped and does not flow freely anymore. The critical Hartmann number Ha depends on the specific system being studied, as well as the parameters of the system such as the fluid conductivity and density. However, in general, the critical Hartmann number increases with increasing fluid conductivity and decreasing density. In conclusion, the trapping phenomenon is a well-known phenomenon that occurs in magnetohydrodynamics when the magnetic field is strong enough to slow down the fluid flow in the direction perpendicular to the magnetic field lines, but not strong enough to completely stop it. The strength of the magnetic field is characterized by the Hartmann number Ha, and the critical Hartmann number Ha at which the trapping phenomenon occurs depends on the specific system being studied, as well as the parameters of the system such as the fluid conductivity and density. In general, the Hartman number is used to analyze the effect of magnetic fields on the behavior of conducting fluids. The Hartman number Ha varies with temperature because the electrical conductivity of the fluid depends on temperature as shown in the Figure 5. As the temperature $\theta(\xi,\eta)$ increases, the electrical conductivity generally increases as well. If we consider a system with a constant magnetic field strength and length scale, we can study how the Hartman number Ha changes with temperature $\theta(\xi,\eta)$ by looking at the other parameters in the equation. As the electrical conductivity σ increases with temperature, the Hartman number will increase as well, since it appears in the denominator of the equation. On the other hand, the fluid density ρ and kinematic viscosity ν generally decrease with increasing temperature $\theta(\xi,\eta)$, which would tend to decrease the Hartman number. Therefore, the net effect of temperature $\theta(\xi,\eta)$ on the Hartman number Ha would depend on the relative magnitudes of these factors. Without more specific information about the system in question, it is difficult to say more about how the Hartman number Ha might vary with temperature $\theta(\xi,\eta)$.

Figure 6 depicts the effect of the parameter velocity$u(\xi,\eta)$. The behaviour on the wall surfaces is diametrically opposed. The velocity profile rises as the distance between the higher wall and the ground increases. With an increase in the variable viscosity parameter Figure 7, the amount of the pressure gradient reduces in magnitude. Fluid is trapped in the variable viscosity case as compared to the constant velocity case. Impact of variable viscosity parameter on the dimensionless temperature $\theta(\xi,\eta)$ is captured in Figure 8. Temperature of the fluid $\theta(\xi,\eta)$ decreases by increasing the variable viscosity parameter.

Figure 9 is presented to see the variation in the velocity $u(\xi,\eta)$ with variable electric conductivity. Increase in electric conductivity results in velocity decrease in the vicinity center of channel. Because electrical conductivity enters in the momentum equation as a function of the Lorentz force, it may have the same effect as the Hartmann number. Figure 10 show that effect of variable electric conductivity parameter on magnitude of the pressure gradient $\frac{dp}{d\xi}$ is also similar to Hartmann number, that is, it decreases by increasing electric conductivity. Size of trapped bolus also decreases by increasing γ^* . Figure 11 is plotted to see the impact of electric conductivity on temperature. The graph exhibits decrease in temperature with the increase in γ^* .

Figure 12-13 are displayed to demonstrate the effect of Eckert number Ec on the velocity $u(\xi,\eta)$, axial pressure gradient $\frac{dp}{d\xi}$ and the temperature $\theta(\xi,\eta)$, respectively. Eckert number Ec appears in the energy equation when we incorporate the viscous dissipation effects. Due to viscous dissipation, kinetic energy is converted to heat energy in the flow process and increasing the Eckert number Ec increases the heat energy. Therefore, both velocity $u(\xi,\eta)$ and temperature $\theta(\xi,\eta)$ of the fluid enhances by increasing the Eckert number Ec. Magnitude of the axial pressure gradient $\frac{dp}{d\xi}$ decreases by increasing Eckert number Ec.

5. Conclusions

This paper investigates the peristaltic transport of nanofluid via an asymmetric channel under the influence of a magnetic field in which the viscosity and electric conductivity of the fluid vary. The effects of the many physical factors under consideration are analysed, and their behaviour is expanded via the use of graphs and tables. The following are the most significant findings:

- When nanoparticles are introduced to a base fluid, the rate of heat transmission rises significantly.
- The addition of 2 percent nanoparticles to the base fluid results in the greatest increase in heat transfer rate in the case of blood-Au nanofluid, which is 5.82 percent higher than the base fluid.
- The fluid's velocity falls as the Hartmann number and the variable electric conductivity parameter increase.
- Increases in the Hartmann number and the variable electric conductivity parameter both enhance the rate of heat transmission but increases in the variable viscosity parameter reduce the rate of heat transfer.
- When the variable viscosity parameter is increased, the magnitude of the pressure gradient increases, but the reverse tendency is seen when the variable electric conductivity parameter is increased.

- The size of the trapped bolus diminishes when the intensity of the magnetic field is increased.

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7. Appendices sections:

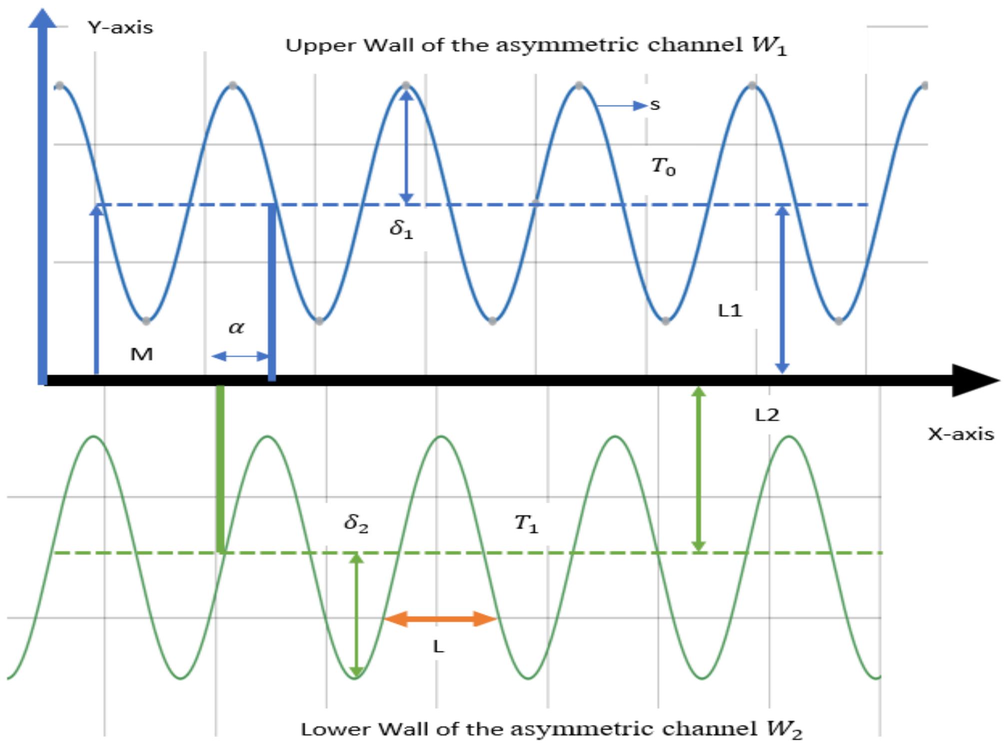


Figure 1.The problem's geometrical characteristics

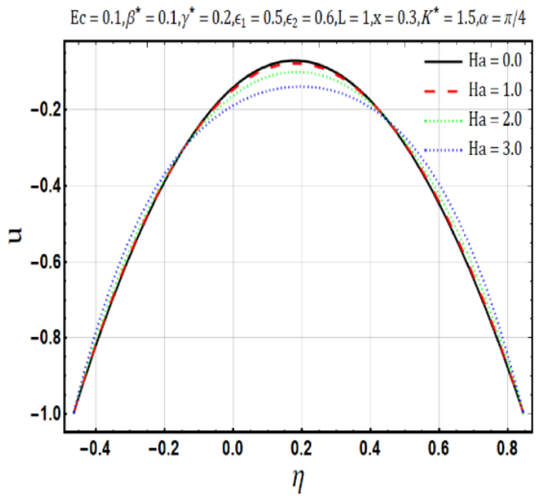


Figure.1 Variation in velocity $u(\xi,\eta)$ with Hartman number

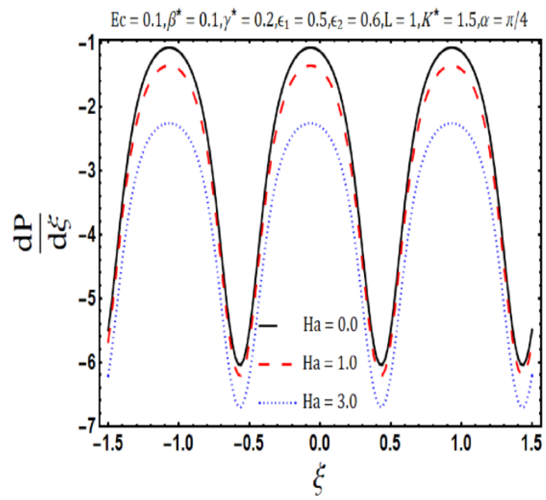


Figure.2 Variation in pressure gradient $\frac{dp}{d\xi}$ with Hartman number

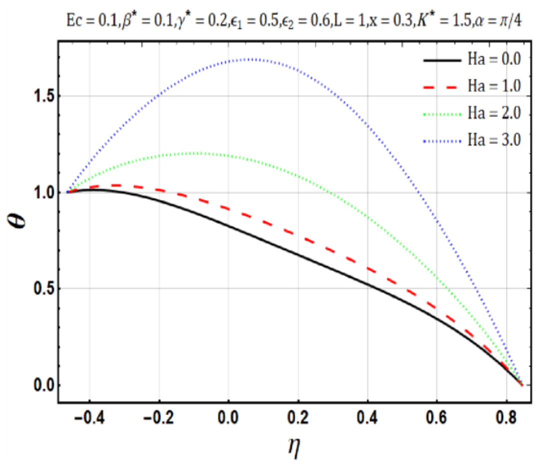
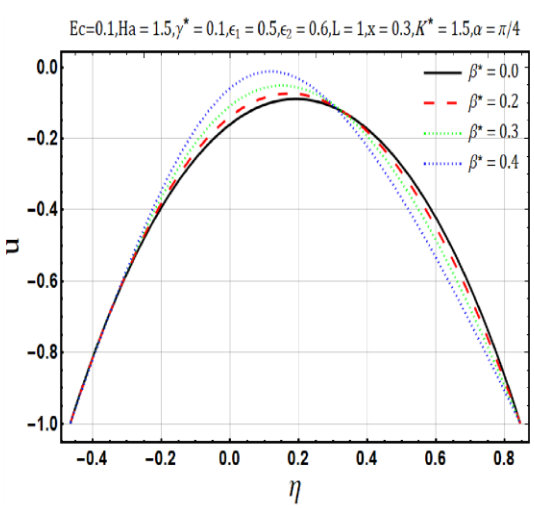


Figure.4 Variation in temperature $\theta(\xi,\eta)$ with Hartman



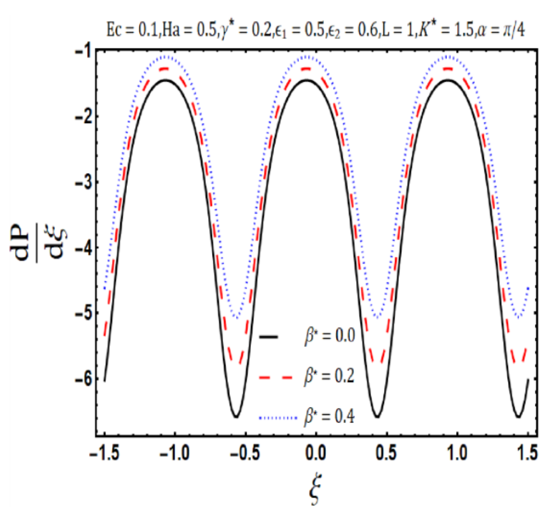


Figure.5 Variation in velocity $u(\xi,\eta)$with variable viscosity

Figure.6 Variation in pressure gradient $\frac{dp}{d\xi}$with variable viscosity

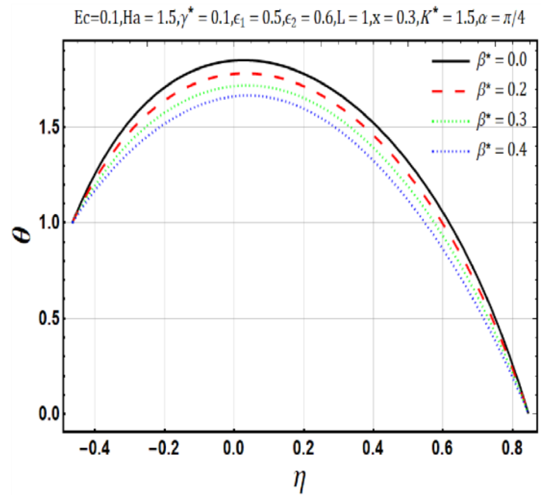


Figure.7 Variation in temperature $\theta(\xi,\eta)$with variable viscosity

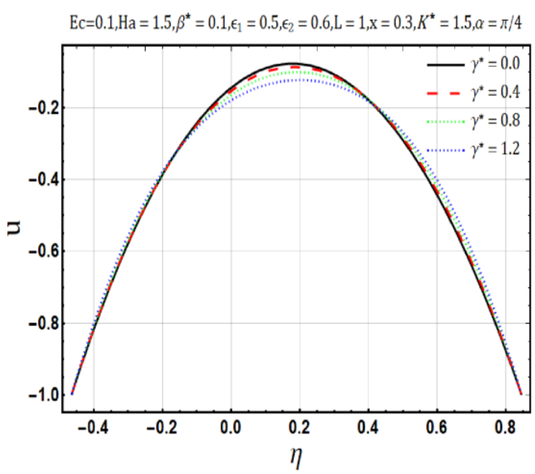


Figure.8 Variation in velocity $u(\xi,\eta)$with variable electric conductivity

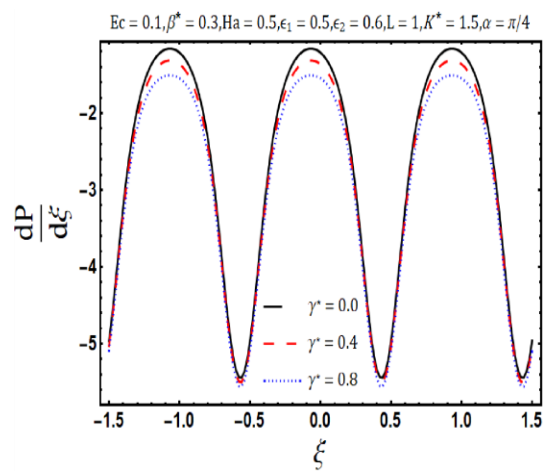


Figure.9 Variation in pressure gradient $\frac{dp}{d\xi}$with variable electric conductivity

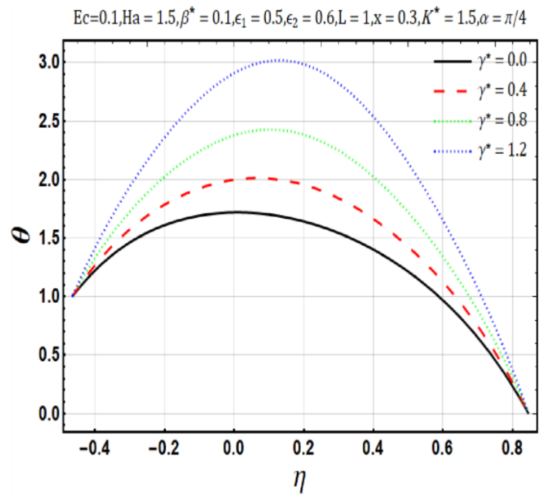


Figure.10 Variation in temperature $\theta(\xi, \eta)$for electric conductivity

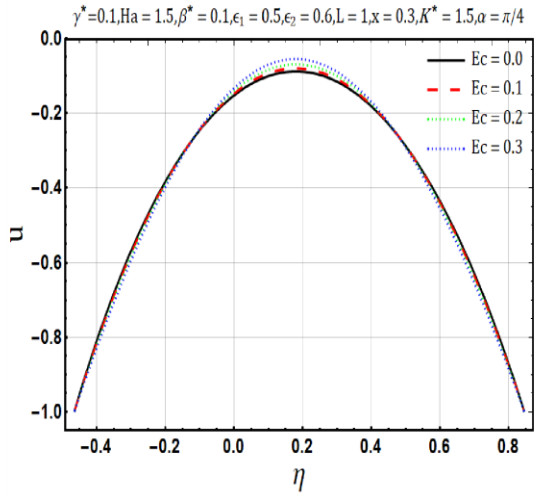


Figure.11 Variation in velocity $u(\xi, \eta)$for Eckert number

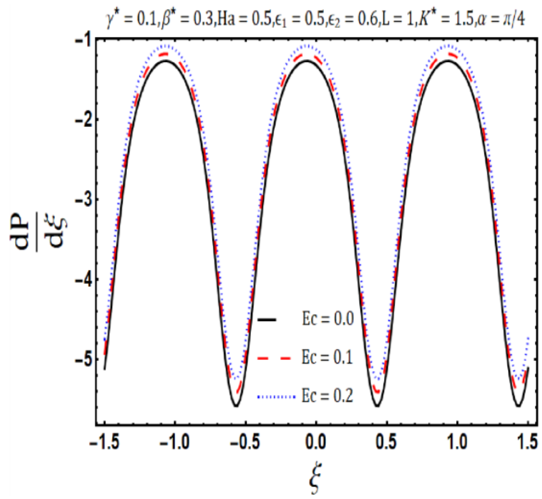


Figure.12 Variation in pressure gradient $\frac{dp}{d\xi}$for Eckert number

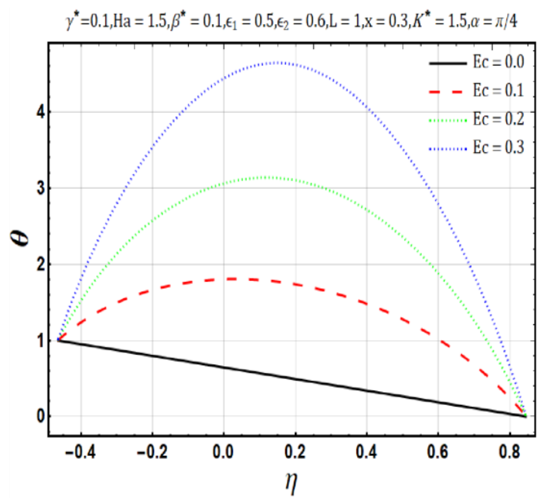


Figure.13 Variation in temperature $\theta(\xi, \eta)$for Eckert number