Investments in energy efficiency with government environmental sensitiveness: An application of geometric programming and game theory

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Abstract

To maintain a competitive advantage, manufacturers of household appliances should promote energy efficiency, considering the impact on customer purchasing behavior. Since energy efficiency and pricing policies influence customers' purchasing decisions, manufacturers confront significant challenges in balancing costs and demand since they must consider their profit-maximizing objective and government regulations. The Stackelberg game framework represents the interactions between the government, the leader, and a manufacturer, the follower, incorporating the government's involvement in environmentally dependent social welfare under a tax structure. This paper proposes closed-form equilibrium by utilizing a game theory approach and geometric programming (GP) to solve the government's and manufacturer's non-linear decision models. The analytical results offer insight into the management's approach to energy efficiency. The findings demonstrate that when clients' concerns about energy-saving grow, the ratio of net payoff to total manufacturer revenue always decreases. The outcomes imply that the manufacturer must allocate a more significant portion of the revenue to tax expenditures in markets with more price-sensitive clients.

Keywords: OR in energy; Energy efficiency; Government intervention; Game theory; Geometric programming.

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1. Introduction

The recent rise in energy consumption has intensified the significance of energy efficiency concerns in manufacturing energy-intensive products within sustainable development. Accordingly, the energy efficiency of these products has increased considerably, owing primarily to the recent enhancement of environmental awareness about more efficient products [1]. The higher the level of energy efficiency, the higher the manufacturer's cost per unit of production [2]. Energy efficiency catalyzes the development of new business models, innovative technologies, and services, giving manufacturers a competitive advantage that enables them to enter the global marketplace [3]. In addition to the energy efficiency level, the manufacturer seeks to set the product's selling price. Deciding on the efficiency measures, the rational manufacturer should examine the trade-off between the expense of improving efficiency and the increased demand coming with being more efficient [4].

Governments consider incentive energy programs to promote energy conservation and optimize social welfare [5,6]. Governments implement these initiatives in various ways, including providing manufacturers with a tax deduction or subsidizing energy-efficient products for manufacturers and clients [7]. As evidence, in 2007, Shandong, Henan, and Sichuan provinces established a rural household appliance incentive program. These products must meet the national first or second-level energy efficiency criteria and the national safety requirement. In 2012, the Chinese State Council unveiled a $4.1 billion incentive scheme to promote energy saving. This effort targeted water heaters, flat-screen TVs, washing machines, refrigerators, and air conditioners [8].

In practice, most governments consider energy conservation a crucial element of welfare maximization programs due to growing social concerns about the environment [9]. The government often provides these incentives directly to approved manufacturers, who then pass them on to consumers when buying a product to encourage manufacturers' investments in energy efficiency enhancement [8]. Efforts to improve energy efficiency do not always result in increased energy savings. Energy-saving from an energy-efficient product is smaller than engineering estimations due to a behavioral phenomenon; the rebound effect, negating this gain [10,11].

This paper's main objective is to present a mathematical model using the Stackelberg game to evaluate the impacts of different strategies used by energy-efficient appliance manufacturers and the government on their utility functions. Specifically, this research utilizing geometric programming (GP) seeks to analyze the obtained solution for manufacturers and governments concerning energy saving. The current work addresses the following significant challenges:

- How much energy efficiency is advantageous for the manufacturer to provide?
- What is the manufacturer's pricing policy to balance the cost and sales?
- What is the acceptable tax rate for the environmentally-conscious government?

Addressing the above three research questions based on the issues above, the manufacturing of energy-efficient products is explored in this research, comprising a government and a manufacturer. Our research is motivated by a real-world example from the electrical household appliance industry that a manufacturer employs two key marketing strategies: energy efficiency and pricing. Since the electrical household appliance industry is subject to government taxation, the manufacturer must pay tax and the cost of manufacturing. Taxation imposes financial constraints on the manufacturer of electrical household appliances instead of investing in energy efficiency enhancements as a competitive advantage. Energy efficiency programs increase consumer interest in purchasing appliances and manufacturing costs.

Due to these financial constraints, the manufacturer's pricing for energy-efficient appliances is challenging because clients do not want to pay a high price for them. Therefore, the government, conscious of the environmental benefits of energy-efficient appliances, should include energy conservation in its social welfare programs to encourage manufacturers to produce energy-efficient products. The government should strike a balance between tax revenue and energy conservation achieved via purchasing energy-efficient appliances regarding the rebound effect offsetting the advantages of these products. The government sets the tax rate while the manufacturer struggles with energy efficiency improvements and price decisions.

The remainder of the research is in the following manner. Section 2 provides theoretical background and contributions. The proposed model and notations are in Section 3, and Section 4 presents results in analytical form. Section 5 conducts analytical results, and Section 6 summarizes the paper, provides managerial insights, and discusses future studies. The appendix contains all lemma, theorem, and corollary proofs.

2. Literature review

This section mainly spans three research streams: pricing, energy efficiency, and government's regulatory policy, to emphasize the research's primary contributions,

2.1. Pricing

Due to the significance of selling price in a client's purchase decision, extensive efforts on pricing have been conducted recently [12]. R. Li and Teng [13] examine an evaluation of the freshness of products establishing a reference pricing for grocery store items. Jiang et al. [14] present a non-linear mixed-integer programming model for maximizing online retailers' profit through bundle discounts and instantaneous coupons. J. Zhang and Chiang [15] analyze different dynamic pricing strategies for durable goods, considering reference prices and saturation effects. Z. Li et al. [16] provide a two-period marketing strategy in a dual supply chain based on the Stackelberg game and three coupon issuing mechanisms. Considering strategic customers, C. Li et al. [17] investigate a two-period game for determining an e-commerce platform's price promotion strategy. Z. Zhang et al. [18] investigate the impact of pricing and coupon patterns on clients' redemption decisions in a duopoly market.

Song et al. [19] employ a differential game of innovation and pricing with a seller and a customer to examine how different model parameters affect innovation and pricing. Martonosi et al.

[20] use game theory to the COVID-19 vaccine pricing issue, helping the US CDC negotiate with vaccine manufacturers. To study the influence of co-creation and pricing in an experience-good market, R. Yang et al. [21] use a duopoly game model. A mean-field Stackelberg game structure allows Chaab et al. [22] to study product diffusion in a two-segment market considering pricing and promotion. To summarize, pricing is a concern in various contexts, including durable products, vaccination, and grocery. The above review investigates pricing from various perspectives, including reference pricing, dynamic pricing, and price promotion. Overall, the reviewed research neglected the pricing of energy-consuming households and the relation between pricing and energy efficiency.

2.2. Energy efficiency

In addition to the pricing strategy, manufacturers offer energy efficiency as a competitive advantage, determining purchase intention. Therefore, extensive researchers have studied energy efficiency recently [23]. W. Zhou and Huang [24] examine two contract types for energy-saving products in a monopoly with the government's limited funds. Zhou et al. [25] use game theory to explore the effects of energy performance contracting on two firms in Cournot competition. Safarzadeh and Rasti-Barzoki [9] study competition between efficient and inefficient manufacturers under the control of a government with tax and subsidy policies. Safarzadeh and Rasti-Barzoki [26] propose a multi-stage sustainable supply chain including an energy supplier, a manufacturer, and the government based on the Bertrand model. In a market with two competing firms, Huang et al. [8] investigate the optimal government subsidies for energy-efficient products.

Broek et al. [27] study energy-saving policies for a production system as an M / G /1 queue, considering the time and energy required. Safarzadeh and Rasti-Barzoki [23] formulate a sustainable supply chain consisting of an energy supplier and an energy-efficient manufacturer based on Cournot competition. Rasti-Barzoki and Moon [28] use the Stackelberg game in the problem that car manufacturers determine a car's price and its level of energy efficiency. Using a multi-stage game, Safarzadeh, Rasti-Barzoki, Hejazi, and Piran [7] model energy efficiency in a duopolistic supply chain with one energy supplier and two manufacturers. For the improved Boussinesq equation, J. Yan et al. [29] developed a novel linearized energy-conserving Crank-Nicolson finite volume element technique. W. Wang et al. [30] examine the output feedback control of the asymmetric hydraulic servo system for energy reserves. This paper proposes analytic modeling of a manufacturer to maximize its payoff. Our critical dissimilarity based on real-world practice investigates a non-linear model using GP to consider the price and energy efficiency elasticities. Specifically, previous studies have neglected the sensitivity of production cost to the pricing policy and energy efficiency elasticities.

2.3. *Government's regulatory policy*

Recently, green awareness has grown, prompting governments to increase regulations on environmental issues [4,31]. Xu et al. [32] investigate the effect of two principal regulations, including cap-and-trade and carbon tax rules, on carbon emissions, business profit, and social welfare. Shao et al. [33] examine a government granting buyers a subsidy or a price discount incentive to encourage the adoption of electric cars. Mahmoudi and Rasti-Barzoki [6] analyze three scenarios considering that the government sets a trade-off between profit and environmental objectives utilizing a two-population evolutionary game. Hafezi and Zolfagharinia [34] consider a firm deciding on product types, market prices, and quality dimensions based on government regulations. Q. Zhang et al. [2] use the differential game to investigate a supply chain comprising a retailer and a manufacturer jointly engaging in energy efficiency.

Examining the encouragement strategies of a social welfare maximizer government on total research and development investment is the concern of Nielsen et al. [5]. Kong et al. [35] study the effect of various governmental policies on electric vehicle distribution, including product subsidy, carbon emission trading, and license plate limitation due to system dynamics. Ramandi & Bafruei [36] explore a government regulating pollution by subsidizing and penalizing suppliers. Y. Zhang et al. [37] use the Stackelberg game to describe the regulator's and manufacturer's decisions considering a tax policy, a subsidy policy, and a tax-subsidy policy. Jung and Feng [38] examine a government that continuously improves social welfare while subsidizing green technologies. Bian et al. [39] utilize game theory to investigate the interactions between environmental subsidy policies, emissions reduction, and other government-manufacturer decisions.

Guo et al. [40] propose that the government provide two types of subsidies to reduce disruption: manufacturer and supplier subsidies. Zaman and Zaccour [41] determine the optimal subsidy levels to encourage consumers to upgrade their vehicles quickly in a two-period game between strategic consumers and the government. J. Lee and Choi [42] investigate a supply chain with carbon-emitting suppliers and buyers and a social planner assigning the carbon footprint to supply chain participants. Rasti-Barzoki and Moon [43] examine two strategies for achieving four key government sustainability goals: electric vehicle sales, CO2 emissions reduction, revenue generation, and customer surplus. Cai et al. [44] suggest three environmental taxes to maximize social welfare, reduce waste at source, and enhance environmental performance.

The cited studies examined the government's regulatory policy through various mechanisms, including subsidy, carbon tax, and cap-and-trade. They have not examined the government taxation on the product as the best response to energy-efficient manufacturer strategies. Additionally, energy-efficient products' demand and production costs have not been non-linear. This paper reflects energy conservation as a part of social welfare, lacking in related literature. Few papers, such as Rasti-Barzoki and Moon [43], considering energy conservation a part of social welfare, have neglected the non-linear nature of demand.

2.4. Research gaps and contributions

To summarize, this paper contributes the following to the preceding literature. The first innovative approach of this research is incorporating energy conservation as a component of a government's utility function while also accounting for the rebound effect. This research gap is lacking in the related literature but is prevalent in the real world where governments care about the environment. The Second novel aspect of this study is enabling the government to impose a tax rate on the manufacturer, which is customary in many countries and industries in the real world. The impact of the government's tax rate on energy-saving decisions made by manufacturers of energy-consuming appliances is lacking to the best of our knowledge. Third, GP is incorporated into the game theory to consider non-linear demand and production cost functions, increasing the modeling's realism.

3. Problem description and model formulation

This problem includes a monopolistic manufacturer of energy-consuming household appliances under the control of one government. The manufacturer-government relationship is studied using the Stackelberg game, where the manufacturer produces energy-consuming household appliances and the government is in charge of tax policy. Due to the government's tax, the profit-maximizing electrical appliances manufacturer (denoted by M) optimizes pricing (P) and energy efficiency (E), affecting production cost $C(E,D)$ and product demand $D(E,P)$. An environmentally conscious government seeking to promote social welfare (SW) finds it challenging to determine the tax rate T on the manufacturer who invests in environmental aspects by improving energy efficiency E. The manufacturer's specification of the product's price and energy efficiency influences customers' willingness to purchase energy-efficient household appliances D, as shown by demand elasticities. GP as a powerful mathematical tool for solving non-linear problems is used to characterize the model's exponential nature of demand elasticities.

3.1. Decision model of the manufacturer

As with other firms, the manufacturer pays a specific tax amount on each item sold and bases the decisions on the tax rate determined by the government (T); therefore, the manufacturer is considered the follower in the Stackelberg game. The firm manufacturing household appliances make pricing and energy efficiency decisions to maximize its payoff $\pi_M(E,P)$ by growing demand $D(E,P)$ and decreasing costs $C(E, D)$. While sales management is critical for revenue generation via pricing and demand enhancement, production management is necessary for cost control and meeting customer requirements like efficiency [45,46].

Higher energy efficiency increases demand by enabling customers to save money on their energy bills. Whereas energy efficiency grows due to an innovation process, as do manufacturing costs (including R&D and technology inputs). The manufacturer should balance production cost and demand promotion by adopting energy efficiency, considering that both demand and production costs are efficiency-sensitive. To this end, we define the impact of joint price and energy efficiency on market demand by modeling product demand as follows:

$$D(E.P) = kE^{\gamma}P^{-\beta} \tag{1}$$

where γ specifies energy efficiency elasticity of demand and indicates the relationship between the amount demanded of the energy-consuming appliance and its energy efficiency. An improvement in the energy efficiency of household appliances will result in increased demand for the product. β is a measure of the extent to which a change in the price of the demand affects the number of sales called the demand's price elasticity. Similar to [47], the current work considers a price-elastic demand that rises at a diminishing rate as price P lowers ($\beta > 1$). k is a given scale coefficient k, γ, and β are all deterministic positive parameters ($\beta, \gamma, k > 0$).

Elasticity is a known concept among researchers as a dimensionless parameter that simplifies data analysis. It is defined empirically as the estimated coefficient in a linear regression equation when the dependent and independent variables are in natural logarithms. These parameter values are assessable based on historical demand, price, and energy efficiency data using linear regression to the logarithm of the exponential demand function [48].

A significant elasticity implies a more price-sensitive demand and energy efficiency, and a higher energy efficiency elasticity γ indicating clients tend to conserve energy more. The higher the price elasticity β is, the more price-sensitive clients are, and the more influential the price is in purchasing behavior. A multitude of factors shapes elasticities; for example, price elasticity β associated with brand loyalty [49], and energy efficiency elasticity γ is linked with environmental consciousness [50].

Demand is considered positive and this function is derived from the classic exponential function for demand $D(t) = kP^{-\beta}$ that D decreases as the selling price increases [50]. Equation 1 is achievable by incorporating the positive effect of the energy efficiency level of household appliances into the classic exponential demand function. To avoid costly inventory and backlog, the manufacturer produces a quantity equal to demand, according to the finding of [45] for a production system. Similar to [47], the unit production cost is an exponential function of production size [51] and, as indicated in energy literature, a growing function of energy efficiency level [2,52]. While previous researchers, such as [52], considered a linear relationship between production cost and energy efficiency, this article formulates the relationship as a non-linear function. While the unit production cost decreases as production volume (demand) grows, it increases as energy efficiency improves, implying that every energy efficiency improvement results in a higher production cost. Thus, the unit production cost is defined as a function of energy efficiency and batch size (demand), i.e.;

$$C(E,D) = \theta E^{h} D^{-\alpha} \tag{2}$$

Where h measures the sensitivity of production cost to the energy efficiency level of the energy-consuming household appliances and measures how the production cost changes in response to a change in energy efficiency ($h > 0$). θ is a positive scaling coefficient, and the exponent α symbolizes the production size (demand) elasticity of production unit costs with a value of $0 < \alpha < 1$, which [47] recommends a small number for it.

Batch production is ordinary for producing household goods which need several components. Each batch of products requires the setting up of equipment and tools. Thus, manufacturing a batch of output incurs machine setup costs at the start of production and machine disassembly costs after the completion of the batch production. The cost per unit is calculable by dividing the total cost by the total number of units in the batch. The setup or preparation costs per batch stay fixed regardless of the batch size, and as the batch size grows, the portion of each unit coming from these shared costs goes down, and vice versa. The total number of units (demand) decreases production cost with discounting rate α. The unit production cost increases as energy efficiency improves, reflecting the R&D efforts and technological innovations required to improve product efficiency [28]. While each increase in energy efficiency raises production costs, achieving extra energy efficiency becomes more challenging and thus more expensive when the current level of energy efficiency is high (exponential cost function).

Q. Zhang et al. [2] describe this correlation between energy efficiency level and costs based on the diminishing investment return. h denotes the unit production cost's sensitivity to the energy efficiency of household appliances. A greater h represents an increase in energy efficiency significantly increases production costs. Incorporating governmental involvement makes the issue more applicable to the real world and provides regulators with further insights. In addition to the production cost $C(E,\ D)$, the manufacturer has to pay tax *TD*, and the tax rate is the government's decision variable. The manufacturer's objective is to maximize the net payoff by determining the selling price and energy efficiency level. The selling price and energy efficiency level affect demand and, hence, the production cost. Subtracting the manufacturer's costs from the revenue results in net payoff as

Manufacturer's net payoff= Total revenue_ Production cost _Tax payment

$$\text{Max } \pi_M(E,P) = PD - CD - TD. \tag{3}$$

Substituting Equation (2) for production cost function into Equation (3) manufacturer's net payoff $\pi_M(E,P)$ is rewritten as:

$$\text{Max } \pi_M(E,P) = PD - \theta E^h D^{-\alpha+1} - TD. \tag{4}$$

3.2. Decision model of the government

This paper addresses the issue of producing energy-efficient household appliances from the perspective of a rational, welfare-maximizing regulator, not just the manufacturer. To optimize social welfare, the government, as the leader in the Stackelberg game, decides to impose a tax scheme on the other player, the manufacturer. Tax revenues and energy conservation are two items of the government's utility function for social welfare as follows:

$$\text{Max } SW = TD + (1-r)euED, \quad (5)$$

where SW denotes social welfare and TD represents the government's tax revenue obtained by multiplying the tax on each product unit by demand. The second item $(1-r)euED$ is derived from the idea of social welfare's ecological element in [53] and [54]. Due to [7], energy conservation positively affects social welfare. Energy conservation-related issues (e.g., regional power outages) caused by electrical overload, environmental damage and pollutant emissions linked with power production, and resource restrictions are significant concerns in contemporary society. Energy-efficient appliances benefit society by using fewer resources at a cheaper cost and saving the environment by emitting less pollution.

The government maximizes social welfare, and the utility function of the government includes the collecting of taxes. The total tax revenue is calculable by multiplying the tax rate per unit by the product's entire demand [7]. In addition, energy conservation, reinforced via energy-efficient appliances, is an essential element of the government's utility function and social welfare in this model denoted by $(1-r)euED$. The term $(1-r)euE$ denotes the amount of energy saved by each energy-efficient household appliance. Multiplying the value $(1-r)euE$ by demand results in the total energy saved using energy-efficient appliances. E signifies the manufacturer's energy efficiency level, u stands for the government's environmental value associated with each energy unit, and e is the amount of energy that each efficient unit product has reserved for each energy efficiency unit. r indicates the rate of residential segment rebound effects.

The more efficient an energy-consuming product gets, the smaller the energy savings will be than the engineering estimations based on an economic phenomenon called the rebound effect offsetting this gain. The rebound effect originates from how consumers modify their behavior so that lower marginal energy costs are compensated. They use the energy savings from more energy-efficient appliances to purchase more energy-intensive items or services. In this context, governments in many countries investigate rebound effects to express the significance of restrictive energy regulations for energy efficiency programs [3]. This paper considers the rebound effect rate r while modeling the environmental component in the utility function of a social welfare maximizer government. The value for the rebound effect rate is $0 < r < 1$ [7].

For instance, a family saves money by purchasing an energy-efficient appliance (e.g., refrigerator). They may use the savings to purchase more energy-intensive items (e.g., space heaters). One example of the rebound effect is individuals purchasing more efficient washing machines or dishwashers and leaving them half-full regularly. Another example is purchasing a larger energy-efficient refrigerator; although the new appliance is technologically more efficient than the predecessors, it consumes more power owing to its size. In the case of the more energy-efficient refrigerator, the consumer may choose more lighting, resulting in no tangible savings in the user's energy bill.

Consider how much more pleasant it will be for the consumers of energy-efficient air conditioners if they can lower the thermostat a few degrees in summer, even without energy savings. Suppose a household buys an energy-efficient television and then leaves it switched on longer and uses it more often. While energy-efficient appliances are technologically more efficient than conventional ones, the financial advantages of energy savings would not result in overall environmental benefits. Due to the rebound effect r,, this study attributes just a portion $\left(1-r\right)$ of energy savings (from energy-efficient appliances) to the energy conservation element of social welfare. In the meantime, households squander the rest of their energy savings without benefiting the environment [55].

3.3. The Stackelberg game

The Stackelberg game structure is used to study government-manufacturer interaction, with the government leading and the manufacturer following [9,26,37]. The government leads and imposes its tax strategy on manufacturers, the followers. The leader starts the game, and the follower sequentially takes the best energy efficiency and pricing action given tax information. The government aims to maximize social welfare after accounting for the manufacturer's rational moves [56].

4. Solution and discussions

In the Stackelberg game, the government leads while the manufacturer follows, and the Stackelberg equilibrium is derivable by backward induction. Firstly, the government assesses the manufacturer's best response to discovering the tax rate. The government then sets the tax rate to maximize social welfare based on the manufacturer's expected response. The manufacturer recognizes it and determines the energy efficiency and price equilibrium. Given the equations above, both the government and manufacturer models are non-linear problems. Since the demand and production costs rise exponentially; thus, this study solves the GP model. GP is a powerful mathematical optimization approach for solving non-linear exponential problems; therefore, various practical problems have been equivalent to GPs [57]. Employing backward induction, firstly, the follower's model is solved as a unique maximizer $\pi_M\left(E\left(T\right),P\left(T\right)\right)$, based on the given T. The manufacturer model is a signomial GP problem, so essential changes are required to convert the model to a posynomial GP problem from TP, as follows,

$$(TP)\min z^{-1}\left(or\max z\right)$$

Subject to

$$PD-\theta E^{h}D^{-\alpha+1}-TD\geq z, \tag{6}$$

where z *is* a nonzero positive. Dividing equation 6 into the total revenue, PD, rearranging the items, and substituting demand equivalent, equation 7 is achieved in posynomial GP form as follows.

$$k^{-1}P^{\beta-1}E^{-\gamma}z+k^{-\alpha}\theta P^{\alpha\beta-1}E^{h-\alpha\gamma}+TP^{-1}\leq 1. \tag{7}$$

The dual of the related problem in equation 7 is as follows, and its degree of difficulty is zero because the primal problem of the manufacturer has three items and two variables. The problem's degree of difficulty is the number of dual variables minus the number of dual equality constraints. Zero degree of difficulty representing the dual system of linear equality constraints has full rank; thus, the dual problem has a unique feasible solution to obtain a closed-form solution. A closed-form solution is calculable via four linear dual constraints and four dual variables.

$$(DP)\max f(\mathrm{w})=\left(\frac{1}{\mathrm{w}_0}\right)^{\mathrm{w}_0}\left(\frac{k^{-1}\,\lambda}{\mathrm{w}_1}\right)^{\mathrm{w}_1}\left(\frac{k^{-\alpha}\theta\lambda}{\mathrm{w}_2}\right)^{\mathrm{w}_2}\left(\frac{T\lambda}{\mathrm{w}_3}\right)^{\mathrm{w}_3}$$

Subject to

$$\begin{cases} \mathrm{w}_0=1 \\ -\mathrm{w}_0+\mathrm{w}_1=0 \\ (\beta-1)\mathrm{w}_1+(\alpha\beta-1)\mathrm{w}_2-\mathrm{w}_3=0 \\ -\gamma\mathrm{w}_1+(h-\alpha\gamma)\mathrm{w}_2=0 \end{cases} \tag{8}$$

where for the feasibility of the dual problem, $\mathrm{w}_i>0$ is necessary; thus $\beta h+\alpha\gamma>h+\gamma$ and $h>\alpha\gamma$ (the proof of Lemma 1). It is essential to define supplementary variables $\delta_i=\frac{\mathrm{w}_i}{\lambda}$ ($0<\delta_i<1$)) to obtain primal problem's optimal solution $\left(P^*(T),E^*(T)\right)$ based on the optimal solution of the dual problem (W^*,λ^*) considering the procedure of [58]. Duffin et al. [58] address just one decision-maker and neglect to combine GP and game theory to present interaction between players; this paper's contributions to GP literature.

These variables δ_1, and δ_3 reflect the proportions of the manufacturer's items to the total revenue. δ_1 represents the proportion of net payoff, δ_2 expresses the proportion of production cost, and δ_3 indicates the proportion of tax to total manufacturer's revenue, respectively. The variables δ_1, δ_2, and δ_3 are as follows

$$\delta_1=k^{-1}P^{\beta-1}E^{-\gamma}z,$$

$$\delta_2 = k^{-\alpha}\theta P^{\alpha\beta-1}E^{h-\alpha\gamma},$$

$$\delta_3 = Tv^{-1}. \tag{9}$$

$\delta_1+\delta_2+\delta_3 \le 1$ (equivalently $\mathrm{w}_1+\mathrm{w}_2+\mathrm{w}_3 \le \lambda$) and as stated W. Lee and Kim [59], in optimality $\delta_1^*+\delta_2^*+\delta_3^*=1$, well-matched with the concept of payoff proportion. Given that $\mathrm{w}_1+\mathrm{w}_2+\mathrm{w}_3 \le \lambda$ and concerning $\mathrm{w}_i>0$, the result is $\lambda>0$ Therefore, $0<\delta_i$ based on $\delta_i=\frac{\mathrm{w}_i}{\lambda}$, $\mathrm{w}_i>0$, and $\lambda>0$. Furthermore, $\delta_i<1$, based on $0<\delta_i$, and $\delta_1+\delta_2+\delta_3\le 1$. To sum it up, we have $0<\delta_i<1$. Lemma 1 represents the results of calculating δ_i^* based on the optimal solutions of the dual problem for the manufacturer (W^*,λ^*).

Lemma 1. *The manufacturer should spend a portion of the total revenue generated from the energy-efficient product sales on production expenses and taxes, leaving a residual as a net payoff. Net payoff as a percentage of total revenue, production costs as a percentage of total revenue, and tax as a percentage of total revenue are, respectively*

$$\delta=\begin{bmatrix}\delta_1^*\\ \delta_2^*\\ \delta_3^*\end{bmatrix}=\begin{bmatrix}\dfrac{h-\alpha\gamma}{\beta h}\\ \dfrac{\gamma}{\beta h}\\ \dfrac{h(\beta-1)+\gamma(\alpha-1)}{\beta h}\end{bmatrix}. \tag{10}$$

Lemma 2. *The best responses of an energy-efficient manufacturer (in terms of energy efficiency and price) to the tax rate determined by a government are as follows*

$$P^*(T)=\frac{T}{\delta_3},$$

$$E^*(T)=\left(k^{-\alpha}\delta_2^{-1}\theta\left(\frac{T}{\delta_3}\right)^{\alpha\beta-1}\right)^{1/(\alpha\gamma-h)}. \tag{11}$$

For a given government tax rate *T*, the manufacturer regarding its payoff function π_M decides on the best responses, including the optimal product's energy efficiency $E^*(T)$ and optimal selling price $P^*(T)$ functions of the tax rate *T*. Due to Lemma 2, the manufacturer increases the price as the government raises the tax rate to compensate for the increased payments, which is frequent in the real world. The manufacturer finds the optimal price $P^*(T)$ and product's energy efficiency $E^*(T)$ for a given tax rate T of the government using the manufacturer's model defined in Lemma 2, respectively. Then, the government optimizes social welfare by using the pair $\left(P^*(T), E^*(T)\right)$. Therefore,

$$\text{Max } SW = TD + (1-r)euED$$

Subject to

$$P^* = \frac{T}{\delta_3} \qquad ,$$

$$E^* = \left(k^{-\alpha}\delta_2^{-1}\theta\left(\frac{T}{\delta_3}\right)^{\alpha\beta-1}\right)^{1/(\alpha\gamma-h)} , \tag{12}$$

and thus, substituting equation 11 into equation 5 yields

$$\text{Max } SW = \left(\frac{k^{-h}\theta}{\delta_2\delta_3^{\beta h-\gamma}}\right)^B T^{B(\beta h-\gamma)+1} + (1-r)eu\left(\frac{\theta k^{-h-\frac{\alpha}{B}}}{\delta_2\delta_3^{A}}\right)^B T^{AB}. \tag{13}$$

where for notational convenience, the identities A and B are in the Appendix. The government's problem in equation 13 is a posynomial GP problem, and the degree of difficulty is zero. Theorem 1 represents a closed-form solution for the government solution.

Theorem 1. *The government, in the role of the Stackelberg game's leader, imposes the following optimal tax rate*

$$T^* = \left(\frac{\omega_2^{*}SW^*}{(1-r)eu}\right)^{1/AB}\left(\frac{\theta k^{-h-\frac{\alpha}{B}}}{\delta_2\delta_3^{A}}\right)^{-B/AB} = \delta_3\left(\frac{\omega_2^{*}SW^*}{(1-r)eu}\right)^{1/AB}\left(\frac{\theta k^{-h-\frac{\alpha}{B}}}{\delta_2}\right)^{-1/A} . \tag{14}$$

Substituting Equation 14 in Lemma 2, Corollary 1 is achieved.

Corollary 1. *The manufacturer's equilibrium pricing and energy efficiency decisions are respectively*

$$P^* = \left(\frac{\omega_2^{*}SW^*}{(1-r)eu}\right)^{1/AB}\left(\frac{\theta k^{-h-\frac{\alpha}{B}}}{\delta_2}\right)^{-1/A} ,$$

$$E^* = \left(k^{-\alpha}\delta_2^{-1}\theta\left(\left(\frac{\omega_2^{*}SW^*}{(1-r)eu}\right)^{1/AB}\left(\frac{\theta k^{-h-\frac{\alpha}{B}}}{\delta_2}\right)^{-B/AB}\right)^{\alpha\beta-1}\right)^{1/(\alpha\gamma-h)} . \tag{15}$$

5. Analytical results

This section presents the sensitivity analyses of equilibrium solutions and examines the effect of three critical parameters on equilibrium solutions, including clients' willingness to conserve energy (γ), clients' price sensitivity (β), and production costs' sensitivity to the energy efficiency level (h). It demonstrates how decision variables change in response to variations in these key parameters. This paper establishes the sensitivity analysis approach of this GP problem to assess perturbations in variable exponents on Dembo's [60] technique. To do sensitivity analysis in this approach, we recast the objective function of the dual manufacturer problem in the following manner.

$$\max V(\delta)_M = \delta_1 \ln\left(\frac{k^{-1}}{\delta_1}\right) + \delta_2 \ln\left(\frac{k^{-\alpha}\theta}{\delta_2}\right) + \delta_3 \ln\left(\frac{T}{\delta_3}\right)$$

Subject to

$$\begin{cases} \delta_1 + \delta_2 + \delta_3 = 1 \\ (\beta - 1)\delta_1 + (\alpha\beta - 1)\delta_2 - \delta_3 = 0 \\ -\gamma\delta_1 + (h - \alpha\gamma)\delta_2 = 0 \end{cases} . \tag{16}$$

Let $\Psi = \left(\Psi_0^*, \left(\Psi_1^*\right), \left(\Psi_2^*\right)\right)$ be an optimized multiplier vector relating to the restrictions in Equation (16), and let δ^*, P^*, and E^* be optimal solutions of dual and primal programs for the manufacturer, respectively. Therefore, $\Psi = \begin{bmatrix} \Psi_0^* \\ \Psi_1^* \\ \Psi_2^* \end{bmatrix} = \begin{bmatrix} 1 - \ln f^* \\ \ln P^* \\ \ln E^* \end{bmatrix}$. Consider a particular exponent of the primal problem (γ, β or h) perturbed around a particular value. This section pertains to the process of discovering the optimal multipliers obtained from the primal or dual solution $\begin{bmatrix} \dot{\delta} \\ \dot{\Psi} \end{bmatrix}$ concerning the specified exponent (γ, β or h).

$\begin{bmatrix} \dot{\delta} \\ \dot{\Psi} \end{bmatrix}$ denotes the influence of perturbing important parameters (willingness to save energy, client price sensitivity, and production costs sensitiveness to the product's energy efficiency level) on altering the manufacturer's optimal solutions to the primal and dual problems. Optimal solutions to the primal problem reveal the price and product's energy efficiency for the manufacturer, and optimal solutions to its dual problem show the proportions of net payoff, production cost, and tax to total revenue. Based on [60], the manufacturer's primal unconstrained GP problem follows.

$$\begin{bmatrix} H_\delta^M & A_M{}^T \\ A_M & 0 \end{bmatrix} \begin{bmatrix} \dot{\delta} \\ \dot{\Psi} \end{bmatrix} = -\begin{bmatrix} \dot{g}_\delta^M + \dot{A}_M{}^T \Psi \\ \dot{A}_M \delta \end{bmatrix} \tag{17}$$

where $\delta = \begin{bmatrix} \delta_1^{\ *} \\ \delta_2^{\ *} \\ \delta_3^{\ *} \end{bmatrix}$, and A_M indicates the exponent matrix for the manufacturer's problem containing an extra row representing $\delta_1 + \delta_2 + \delta_3 = 1$. Therefore, $A_M \delta = e_0^M$ regarding e_0^M is a unit vector and ${A_M}^T$ is the matrix transpose of A_M, g_δ^M, and H_δ^M are respectively defined as $g_\delta^M = \frac{\partial\left(V\left(\delta\right)_M\right)}{\partial\delta}$ and $H_\delta^M = \frac{\partial(V\left(\delta\right)_M)^2}{\partial\delta^2}$. It is notable that $\dot{A}_M = \frac{\partial A_M}{\partial X}$, $\dot{A}_M{}^T = \frac{\partial {A_M}^T}{\partial X}$, and $\dot{g}_\delta^M = \frac{\partial g_\delta^M}{\partial X}$ considering *X* is a dummy variable encompassing the parameters (exponents) specified in Theorems 2 to 7.

Theorem 2. *Clients' propensity to save energy affects the percentage of net payoff, production cost proportion, and tax proportion of total manufacturer revenue, respectively* $\dot{\delta}_1 = \frac{-\alpha^2\gamma}{\beta h\left(h+\alpha\left(1-\gamma\right)\right)}$,

$\dot{\delta}_2 = \frac{\alpha\gamma}{\beta h\left(h+\alpha\left(1-\gamma\right)\right)}$, *and* $\dot{\delta}_3 = \frac{\alpha\gamma}{\beta h}\left(1+\frac{\alpha\left(h-\alpha\gamma+1\right)}{\beta h\left(h+\alpha\left(1-\gamma\right)\right)}\right)$.

Theorem 3. *Since* $\Psi_1^* = \ln P^*$ $\Psi_2^* = \ln E^*$ *the perturbation in clients' tendency to save energy affects the price and household appliance's energy efficiency (* $\dot{\Psi}_1 = \frac{\partial\Psi_1}{\partial\gamma}$ *and* $\dot{\Psi}_2 = \frac{\partial\Psi_2}{\partial\gamma}$ *) respectively*

$$\dot{\Psi}_1 = \frac{\left(h+\alpha\right)\left(h-\alpha\gamma\left(1-\alpha\right)\right)}{\gamma\beta\left(h-\alpha\gamma\right)\left(h+\alpha\left(1-\gamma\right)\right)^2} - \frac{\alpha\left(\left(h+\alpha\right)+\left(h-\alpha\gamma\right)\Psi_2^*\right)}{\beta\left(h+\alpha\left(1-\gamma\right)\right)\left(h-\alpha\gamma\right)} \tag{18}$$

$$\dot{\Psi}_2 = \left(\frac{\left(h+\alpha\right)\left(h-\alpha\gamma\left(1-\alpha\right)\right)}{\gamma\left(h-\alpha\gamma\right)\left(h+\alpha\left(1-\gamma\right)\right)^2}\right) + \frac{\alpha}{h+\alpha\left(1-\gamma\right)}\Psi_2^* \tag{19}$$

Theorem 4. *The impacts of household appliances' price elasticity on the net payoff percentage, production costs proportion, and the proportion of tax in entire energy-efficient manufacturer revenue are respectively* $\dot{\delta}_1 = \frac{\alpha\gamma-h}{\beta^2\left(h-\alpha\gamma+\alpha\right)}$, $\dot{\delta}_2 = \frac{-1}{\beta^2\left(h-\alpha\gamma+\alpha\right)}$, *and* $\dot{\delta}_3 = \frac{h-\alpha\gamma+1}{\beta^2\left(h-\alpha\gamma+\alpha\right)}$.

Theorem 5. *Considering the relation between multipliers and the manufacturer's decisions, including energy efficiency and pricing, the perturbation in price elasticity influences the manufacturer's decision as follows*

$$\dot{\Psi}_1 = -\frac{\Psi_1^*}{\beta} - \frac{1}{\beta^2} + \frac{\alpha-1-h}{\beta^2\left(h-\alpha\gamma+\alpha\right)} + \frac{h\left(\alpha-1\right)}{\beta^2\gamma\left(h-\alpha\gamma+\alpha\right)^2} - \frac{\left(h-\alpha\gamma+1\right)\left(h\left(h-\alpha\gamma+1\right)+\left(h-\gamma\left(\alpha-1\right)\right)\left(h-\alpha\gamma+\alpha\right)\right)}{\beta^2\left(h-\alpha\gamma+\alpha\right)^2\left(h\left(\beta-1\right)+\gamma\left(\alpha-1\right)\right)}, \tag{20}$$

$$\dot{\Psi}_2 = \frac{\alpha-1}{\beta\left(h-\alpha\gamma+\alpha\right)}\left(\frac{h\left(h\left(\beta-1\right)+\gamma\left(\alpha-1\right)\right)+h\gamma\left(h-\alpha\gamma+1\right)}{\gamma\left(h-\alpha\gamma+\alpha\right)\left(h\left(\beta-1\right)+\gamma\left(\alpha-1\right)\right)} + \frac{\alpha+h\beta-\gamma}{h\left(\beta-1\right)+\gamma\left(\alpha-1\right)}\right) \tag{21}$$

Theorem 6. *When the production cost sensitiveness to energy efficiency alters, the percentage of net payoff, production cost proportion, and tax proportion of overall manufacturer revenue, change as* $\dot{\delta}_1 = \frac{\gamma\alpha}{\beta h\left(h-\alpha\gamma+\alpha\right)}$, $\dot{\delta}_2 = \frac{-\gamma}{\beta h\left(h-\alpha\gamma+\alpha\right)}$ *and* $\dot{\delta}_3 = \frac{\gamma\left(1-\alpha\right)}{\beta h\left(h-\alpha\gamma+\alpha\right)}$.

Theorem 7. *Changes in the production cost sensitivity to energy efficiency lead to alterations in the price and energy efficiency of household appliances, as follows*

$$\dot{\Psi}_1 = -\frac{2h-\alpha\gamma+\alpha}{\beta h\left(h-\alpha\gamma+\alpha\right)^2}\left(1-\frac{2\gamma\left(\alpha-1\right)}{\left(h\left(\beta-1\right)+\gamma\left(\alpha-1\right)\right)} + \frac{\alpha\gamma\left(\alpha-1\right)}{\left(h-\alpha\gamma\right)} + \frac{\alpha\gamma\left(\alpha-1\right)}{\left(h\left(\beta-1\right)+\gamma\left(\alpha-1\right)\right)}\right) - \frac{\Psi_2^*}{\beta\left(h-\alpha\gamma+\alpha\right)} \tag{22}$$

$$\dot{\Psi}_2 = \frac{-\Psi_2^*}{\left(h-\alpha\gamma+\alpha\right)} - \frac{2h-\alpha\gamma+\alpha}{h\left(h-\alpha\gamma+\alpha\right)^2}\left(1-\frac{\gamma\left(\alpha-1\right)}{\left(h\left(\beta-1\right)+\gamma\left(\alpha-1\right)\right)} + \frac{\alpha^2\gamma}{\left(h-\alpha\gamma\right)} + \frac{\alpha\gamma\left(\alpha-1\right)}{\left(h\left(\beta-1\right)+\gamma\left(\alpha-1\right)\right)}\right) \tag{23}$$

5. Conclusion

This study presents a non-linear GP mathematical model to analyze how energy efficiency and pricing strategies affect customers' desire to acquire energy-consuming household equipment. To this purpose, the problem, comprising a government and a manufacturer, is studied. The government imposes a tax on the manufacturer, and the manufacturer decides on energy efficiency and the product's price. In addition to addressing government-manufacturer interaction, this research considers the consequences of decisions using the Stackelberg game. The government is environmentally sensitive and considers energy conservation as part of social welfare, assessed on the rebound effect. The manufacturer's energy efficiency, and pricing strategies, and the government's tax rate are optimized.

When clients are more concerned about the energy efficiency of household appliances, managers of manufacturing enterprises will benefit from reduced profit margins owing to increased investment in energy-efficient product production expenses. As clients become more concerned about energy efficiency, governments prefer to reduce tax ratios to encourage energy-efficient manufacturers to invest in the expenses of producing energy-efficient appliances. Whenever the clients are sensitive in terms of pricing, the government increases the tax ratio to compel household appliance manufacturers to produce more energy-efficient products. In this case, the manufacturer chooses lower investment in the production cost of energy-efficient products and even lower marginal profit to satisfy price-sensitive clients. When production costs are susceptible to energy efficiency, it is more advantageous for manufacturers to pick more significant profit margins. Since production costs are susceptible to energy efficiency, manufacturers should compensate for the increased proportion of production costs with a more significant profit margin. Since increasing energy efficiency increases manufacturing costs significantly, the government often levies a lower tax on energy-efficient manufacturers to incentivize innovation in energy efficiency development.

This research can be expanded in various ways in the future, as follows. Future research is also essential to explore how energy efficiency affects clients' purchasing behavior differently based on their environmental knowledge and loyalty; nonetheless, this effect is similar across all clients in this paper. One can examine the problem by considering the manufacturer a government enterprise. The supply chain can be extended to more than two competing manufacturers when one produces energy-efficient products and the other conventional ones to make the problem more practical. A potentially challenging issue is the competition in the worldwide market between two energy-efficient manufacturers operating under the auspices of two different governments. Another field of further research will be to study strategic customers and a farsighted energy-efficient manufacturer.

Further investigations are necessary to study the effects of the rebound effect on government decision-making when the rebound effect is not predictable and deterministic. Another is Expanding on this issue by addressing the government's minimum energy efficiency criteria for eco-labeling. Additional research is required to examine and compare various government programs relating to energy conservation. It will be intriguing to examine the effect of new concepts such as energy blockchain, internet of things (IoT), or energy-efficient smart homes on energy efficiency improvement of the appliances and how they can benefit governments and manufacturers of energy-intensive products. Future investigations can evaluate the manufacturer’s environmental responsibilities.

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Biography

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Appendix

Proof of Lemma 1. Considering Equation 8, we have

$$(DP)\max f(\mathrm{w})=\left(\frac{1}{\mathrm{w}_0}\right)^{\mathrm{w}_0}\left(\frac{k^{-1}\lambda}{\mathrm{w}_1}\right)^{\mathrm{w}_1}\left(\frac{k^{-\alpha}\theta\lambda}{\mathrm{w}_2}\right)^{\mathrm{w}_2}\left(\frac{T\lambda}{\mathrm{w}_3}\right)^{\mathrm{w}_3}$$

Subject to

$$\begin{cases} \mathrm{w}_0=1 \\ -\mathrm{w}_0+\mathrm{w}_1=0 \\ (\beta-1)\mathrm{w}_1+(\alpha\beta-1)\mathrm{w}_2-\mathrm{w}_3=0 \\ -\gamma\mathrm{w}_1+(h-\alpha\gamma)\mathrm{w}_2=0 \end{cases}, \tag{A.1}$$

Solving the dual system of linear equality constraints mentioned in Equation 8, the following optimal values for dual variables are achievable.

$$w_1^*=1 \ ,$$

$$w_2^*=\frac{\gamma}{h-\alpha\gamma},$$

$$w_3^*=\frac{\beta h+\alpha\gamma-h-\gamma}{h-\alpha\gamma}$$

$$\lambda^*=w_1^*+w_2^*+w_3^*,$$

$$\lambda^*=\frac{\beta h}{h-\alpha\gamma}, \tag{A.2}$$

where the dual variables are $\mathrm{w}_1,\ \mathrm{w}_2,\ \mathrm{w}_3>0$ to have a feasible dual solution [56], and thus $h>\alpha\gamma$ and $\beta h+\alpha\gamma>h+\gamma$ regarding the parameter range for energy efficiency elasticity defined in the paper ($\gamma>0$).

In order to achieve primal problem's optimal solution $\left(P^*(T),\ E^*(T)\right)$ based on the optimal solution of the dual problem $(W^*,\ \lambda^*)$, defining supplementary variables as $\delta_i=\frac{\mathrm{w}_i}{\lambda}$ is required based on the solution approach provided by Duffin and Peterson [58] for GP problems. Due to Equation 9, **s**ubstituting $(W^*,\ \lambda^*)$, in $\delta_i=\frac{\mathrm{w}_i}{\lambda}$, yields

$$\delta_1^*=\frac{h-\alpha\gamma}{\beta h},$$

$$\delta_2^*=\frac{\gamma}{\beta h},$$

$$\delta_3^* = \frac{h(\beta-1)+\gamma(\alpha-1)}{\beta h}. \qquad \text{(Q.E.D.)}$$

Proof of Lemma 2. Substituting the $\delta_i = \frac{w_i}{\lambda}$ in Equation 9, we have

$$k^{-1}P^{\beta-1}E^{-\gamma}z = \frac{w_1^*}{\lambda^*},$$

$$k^{-\alpha}\theta P^{\alpha\beta-1}E^{h-\alpha\gamma} = \frac{w_2^*}{\lambda^*},$$

$$TP^{-1} = \frac{w_3^*}{\lambda^*} \qquad \text{(A.3)}$$

where right-hand sides in Equation A.3 have been achieved before as the solutions of the dual problem in Equation A.2. This system in Equation A.3 is linear in the logarithms of *E* and *P*, and it is simple to solve it, thus for P^* and E^* we have

$$P^*(T) = \frac{T}{\delta_3},$$

$$E^*(T) = \left(k^{-\alpha}\delta_2^{-1}\theta\left(\frac{T}{\delta_3}\right)^{\alpha\beta-1}\right)^{1/(\alpha\gamma-h)} \qquad \text{(Q.E.D.)}$$

Proof of Theorem 1. Equation 12 is solvable by substituting Equation 11 into Equation 5, which results in Equation 13 as follows

$$\text{Max } SW = \left(\frac{k^{-h}\theta}{\delta_2\delta_3^{\beta h-\gamma}}\right)^B T^{B(\beta h-\gamma)+1} + (1-r)eu\left(\frac{\theta k^{-h-\frac{\alpha}{B}}}{\delta_2\delta_3^{A}}\right)^B T^{AB}, \qquad \text{(A.4)}$$

where for notational convenience, the identifiers *A* and *B* are as follows

$$A = \alpha\beta\alpha\gamma - \alpha\beta h - \gamma - \alpha\gamma + h + \beta h,$$

$$B = \frac{1}{\alpha\gamma - h} \qquad \text{(A.5)}$$

Owing to the posynomial GP problem for the government in Equation A.4, with a zero degree of difficulty (Equation A.4 includes two terms and one variable), the closed-form solution is achievable. In order to solve the government's posynomial GP problem mentioned in Equation A.4, dual variables for each term are ω_1 and ω_2 thus the dual system of linear equality constraints is applied as follows

$$\left(B\left(\beta h-\gamma\right)+1\right)\omega_1 + AB\omega_2 = 0\,,$$

$$\omega_1 + \omega_2 = 1 \tag{A.6}$$

The following optimal values for dual variables ω_1 and ω_2 are obtained by solving the dual system of linear equality constraints mentioned in Equation A.6 and substituting A and B values

$$\omega_1^{\,*} = \frac{-\alpha\beta\alpha\gamma + \alpha\beta h + \gamma + \alpha\gamma - h - \beta h}{-\alpha\beta\alpha\gamma + \alpha\beta h + 2\alpha\gamma - 2h - \gamma}\,,$$

$$\omega_2^{\,*} = 1 - \omega_1^{\,*} = \frac{\alpha\gamma - h - 2\gamma + \beta h}{-\alpha\beta\alpha\gamma + \alpha\beta h + 2\alpha\gamma - 2h - \gamma} \tag{A.7}$$

Notably, in order to have a feasible dual solution, the values of ω_1 and ω_2 must stay positive, necessitating the use of some extra assumptions simultaneously. Consequently,

$$\alpha\gamma + \beta h > h + 2\gamma\,,$$

$$\alpha\beta h + 2\alpha\gamma > 2h + \gamma + \alpha\beta\alpha\gamma\,,$$

$$\text{and } \alpha\beta h + \gamma + \alpha\gamma h > h + \beta + \alpha\beta\alpha\gamma\,. \tag{A.8}$$

Achieving the optimal solution for the primal problem based on the optimal solution for the dual problem, we consider

$$U_1 = \left(\frac{k^{-h}\theta}{\delta_2\delta_3^{\,\beta h-\gamma}}\right)^{B} T^{B(\beta h-\gamma)+1}$$

$$U_2 = \left(1-r\right)eu\left(\frac{\theta k^{-h-\frac{\alpha}{B}}}{\delta_2\delta_3^{\,A}}\right)^{B} T^{AB}\,. \tag{A.9}$$

Therefore,

$$SW^{*} = \left(\frac{1}{\omega_1^{\,*}}\left(\frac{k^{-h}\theta}{\delta_2\delta_3^{\,\beta h-\gamma}}\right)^{B}\right)^{\omega_1^{*}} \left(\frac{1}{\omega_2^{\,*}}\left(1-r\right)eu\left(\frac{\theta k^{-h-\frac{\alpha}{B}}}{\delta_2\delta_3^{\,A}}\right)^{B}\right)^{\omega_2^{*}}\,, \tag{A.10}$$

and we have

$$U_1^{\,*} = \omega_1^{\,*}SW^{*} = \left(\frac{k^{-h}\theta}{\delta_2\delta_3^{\,\beta h-\gamma}}\right)^{B} T^{*B(\beta h-\gamma)+1}\,,$$

$$U_2^{\,*} = \omega_2^{\,*} SW^* = (1-r)eu\left(\frac{\theta k^{-h-\frac{\alpha}{B}}}{\delta_2 \delta_3^{\,A}}\right)^B T^{*AB}, \tag{A.11}$$

resulting in

$$T^* = \left(\frac{\omega_2^{\,*} SW^*}{(1-r)eu}\right)^{1/AB}\left(\frac{\theta k^{-h-\frac{\alpha}{B}}}{\delta_2 \delta_3^{\,A}}\right)^{-B/AB} = \delta_3\left(\frac{\omega_2^{\,*} SW^*}{(1-r)eu}\right)^{1/AB}\left(\frac{\theta k^{-h-\frac{\alpha}{B}}}{\delta_2}\right)^{-1/A} \quad \text{(QED.)}$$

Proof of Corollary 1. By substituting T^* from Theorem 1 into the $P^*(T)$ and $E^*(T)$ variables mentioned in Lemma 2, the following optimal values for manufacturer decisions, respectively pricing P^* and energy efficiency E^* are obtained as follows

$$P^* = \left(\frac{\omega_2^{\,*} SW^*}{(1-r)eu}\right)^{1/AB}\left(\frac{\theta k^{-h-\frac{\alpha}{B}}}{\delta_2}\right)^{-B/AB},$$

$$E^* = \left(k^{-\alpha}\delta_2^{\,-1}\theta\left(\left(\frac{\omega_2^{\,*} SW^*}{(1-r)eu}\right)^{1/AB}\left(\frac{\theta k^{-h-\frac{\alpha}{B}}}{\delta_2}\right)^{-B/AB}\right)^{\alpha\beta-1}\right)^{1/(\alpha\gamma-h)}. \quad \text{(QED.)}$$

Proof of Theorems 2. and 3. According to Dembo's [60] sensitivity analysis technique, the manufacturer's issue has the following exponent matrix

$$A_M = \begin{bmatrix} 1 & 1 & 1 \\ \beta-1 & \alpha\beta-1 & -1 \\ -1 & h-\alpha\gamma & 0 \end{bmatrix}, \tag{A.12}$$

and thus the matrix transpose of exponent matrix is

$$A_M^{\,T} = \begin{bmatrix} 1 & \beta-1 & -1 \\ 1 & \alpha\beta-1 & h-\alpha\gamma \\ 1 & -1 & 0 \end{bmatrix}. \tag{A.13}$$

$\Psi = \left(\Psi_0^*, \left(\Psi_1^*\right), \left(\Psi_2^*\right)\right)$ as an optimized multiplier vector relating with the restrictions regarding

$\Psi = \begin{bmatrix} \Psi_0^* \\ \Psi_1^* \\ \Psi_2^* \end{bmatrix} = \begin{bmatrix} 1-\ln f^* \\ \ln P^* \\ \ln E^* \end{bmatrix}$ onsidering that

$$\delta = \begin{bmatrix} \delta_1^* \\ \delta_2^* \\ \delta_3^* \end{bmatrix} = \begin{bmatrix} \dfrac{h-\alpha\gamma}{\beta h} \\ \dfrac{\gamma}{\beta h} \\ \dfrac{h(\beta-1)+\gamma(\alpha-1)}{\beta h} \end{bmatrix}, \tag{A.14}$$

$\begin{bmatrix} \dot{\delta} \\ \dot{\Psi} \end{bmatrix}$ represents the impact of perturbing important parameters on changing optimal solutions of primal and dual problems for the manufacturer as

$$\begin{bmatrix} \dot{\delta} \\ \dot{\Psi} \end{bmatrix} = \begin{bmatrix} \dot{\delta}_1 \\ \dot{\delta}_2 \\ \dot{\delta}_3 \\ \dot{\Psi}_0 \\ \dot{\Psi}_1 \\ \dot{\Psi}_2 \end{bmatrix} \tag{A.15}$$

Furthermore, for $g_\delta^M = \dfrac{\partial\left(V(\delta)_M\right)}{\partial\delta}$ and $H_\delta^M = \dfrac{\partial(V(\delta)_M)^2}{\partial\delta^2}$ respectively we have

$$g_\delta^M = \begin{bmatrix} \ln\left(\dfrac{k^{-1}}{\delta_1^*}\right)-1 \\ \ln\left(\dfrac{k^{-\alpha}\theta}{\delta_2^*}\right)-1 \\ \ln\left(\dfrac{T}{\delta_3^*}\right)-1 \end{bmatrix}, \tag{A.16}$$

$$H_\delta^M = \begin{bmatrix} -\delta_1^{-1} & 0 & 0 \\ 0 & -\delta_2^{-1} & 0 \\ 0 & 0 & -\delta_3^{-1} \end{bmatrix}. \tag{A.17}$$

According to Equations A.12, A.13, and A.16, respectively $\dot{A_M} = \frac{\partial A_M}{\partial \gamma}$, $\dot{A_M}^T = \frac{\partial A_M^T}{\partial \gamma}$, and $\dot{g_\delta^M} = \frac{\partial g_\delta^M}{\partial \gamma}$ are as follows

$$\dot{A_M} = \frac{\partial A_M}{\partial \gamma} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\alpha & 0 \end{bmatrix} \tag{A.18}$$

$$\dot{A_M}^T = \frac{\partial A_M^T}{\partial \gamma} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\alpha \\ 0 & 0 & 0 \end{bmatrix} \tag{A.19}$$

$$\dot{g_\delta^M} = \frac{\partial g_\delta^M}{\partial \gamma} = \begin{bmatrix} \frac{\alpha}{h-\alpha\gamma} \\ \frac{-1}{\gamma} \\ \frac{-(\alpha-1)}{h(\beta-1)+\gamma(\alpha-1)} \end{bmatrix} \tag{A.20}$$

Owing to the relationship proposed by Dembo [60] for the manufacturer's issue,

$$\begin{bmatrix} H_\delta^M & A_M^T \\ A_M & 0 \end{bmatrix} \begin{bmatrix} \dot{\delta} \\ \dot{\Psi} \end{bmatrix} = -\begin{bmatrix} \dot{g_\delta^M} + \dot{A_M}^T \Psi \\ \dot{A_M} \delta \end{bmatrix} \tag{A.21}$$

and calculating

$$-\begin{bmatrix} \dot{g_\delta^M} + \dot{A_M}^T \Psi \\ \dot{A_M} \delta \end{bmatrix} = \begin{bmatrix} \frac{-\alpha}{h-\alpha\gamma} \\ \frac{1}{\gamma} + \alpha\Psi_2^* \\ \frac{(1-\alpha)}{h(\beta-1)+\gamma(\alpha-1)} \\ 0 \\ 0 \\ \frac{\alpha\gamma}{\beta h} \end{bmatrix} \tag{A.22}$$

after Substituting H_{δ}^{M} , A_M , and ${A_M}^T$ considering Equations A.12, A.13, and A.17 in $\begin{bmatrix} H_{\delta}^{M} & {A_M}^T \\ A_M & 0 \end{bmatrix}$

and solving the system of Equations, elements of $\begin{bmatrix} \dot{\delta} \\ \dot{\Psi} \end{bmatrix}$ concerning γ are obtained as follow

$$\dot{\delta}_1 = \frac{-\alpha^2\gamma}{\beta h\left(h+\alpha\left(1-\gamma\right)\right)}$$

$$\dot{\delta}_2 = \frac{\alpha\gamma}{\beta h\left(h+\alpha\left(1-\gamma\right)\right)}$$

$$\dot{\delta}_3 = \frac{\alpha\gamma}{\beta h}\left(1+\frac{\alpha\left(h-\alpha\gamma+1\right)}{\beta h\left(h+\alpha\left(1-\gamma\right)\right)}\right)$$

$$\dot{\Psi}_1 = \frac{\left(h+\alpha\right)\left(h-\alpha\gamma\left(1-\alpha\right)\right)}{\gamma\beta\left(h-\alpha\gamma\right)\left(h+\alpha\left(1-\gamma\right)\right)^2} - \frac{\alpha\left(\left(h+\alpha\right)+\left(h-\alpha\gamma\right)\Psi_2^*\right)}{\beta\left(h+\alpha\left(1-\gamma\right)\right)\left(h-\alpha\gamma\right)}$$

$$\dot{\Psi}_2 = \left(\frac{\left(h+\alpha\right)\left(h-\alpha\gamma\left(1-\alpha\right)\right)}{\gamma\left(h-\alpha\gamma\right)\left(h+\alpha\left(1-\gamma\right)\right)^2}\right)+\frac{\alpha}{h+\alpha\left(1-\gamma\right)}\Psi_2^* \qquad \text{(QED.)}$$

Proof of Theorems 4. and 5. Similar to Dembo's [60] sensitivity analysis approach, mentioned for the demonstration of Theorems 2 and 3, the stages are repeatable to explore the impact of perturbations in price elasticity (β) on the manufacturer's decisions based on $\dot{A}_M = \frac{\partial A_M}{\partial \beta}$, $\dot{A}_M{}^T = \frac{\partial {A_M}^T}{\partial \beta}$, and $\dot{g}_{\delta}^{M} = \frac{\partial g_{\delta}^{M}}{\partial \beta}$ in order to find $\dot{\delta}_1 = \frac{\partial \delta_1}{\partial \beta}$, $\dot{\delta}_2 = \frac{\partial \delta_2}{\partial \beta}$, $\dot{\delta}_3 = \frac{\partial \delta_3}{\partial \beta}$, $\dot{\Psi}_1 = \frac{\partial \Psi_1}{\partial \beta}$ and $\dot{\Psi}_2 = \frac{\partial \Psi_2}{\partial \beta}$.

Proof of Theorems 6. and 7. As previously described for the proofs of Theorems 2 to 5, by deriving Equations A.12, A.13, and A.16, we have the same procedure for analyzing the influence of production cost sensitivity on energy efficiency (*h*).