Free Vibrations Analysis of Stepped Nanobeams Using Nonlocal Elasticity Theory

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Abstract. Free vibration of stepped nanobeams was investigated using Eringen's nonlocal elasticity theory. Beam analysis is based on Bernoulli-Euler theory and nanoscale analysis is based on Eringen's nonlocal elasticity theory. The system boundary conditions were determined as simple-simple. The equations of motion of the system were obtained using Hamilton's principle. For the solution of the obtained state equations, a multi-time scale, which is one of the perturbation methods, was used. The results part of the study, it is aimed to observe the nano-size effect and the effects of the step state. For this purpose, the natural frequency values of the first three modes of the system were obtained for different non-local parameter values, step rates, and step positions. When the results were examined, it was determined that the non-local parameter value, step ratio, and natural frequency were inversely proportional to each other. In addition, to strengthen the accuracy of the results, the results obtained were compared with the results of other studies in the literature conducted under the specified conditions, and a perfect agreement was observed. The current beam model, on the other hand, could help design and manufacture ICs such as nano-sensors and nano-actuators.

Keywords: Stepped Nanobeam, Vibration, Nonlocal Elasticity, Perturbation Method

1. Introductions

In today's technology, it is a fact that the expectations from the technology produced increase with the increase and expansion of the working disciplines. In an age when many new product updates are demanded, such as the sensitivity of the environmental conditions in which the product is operating and the superior material properties expected from the device, it is an inevitable expectation that the device sizes will decrease rapidly.
When we look at materials science, the mechanical properties of the material are encountered as the dimensions of the produced material become smaller. We can experience nano sensors, nano actuators, nanoresonators, and nanofluid carriers with new physical properties that we can describe as completely different and perfect. (see [1], [2], [3], [4], and [5]).

Modeling of the continuous mechanics of nanoscale structures must be replaced by theories that include measurements for non-classical phenomena [6]. Some of the theories are as follows. Modified the couple stress theory [7,8], micropolar elasticity theory [9], the strain gradient theory [10,11], surface elasticity theory [12], nonlocal stress, strain gradient, and surface energy together [13], doublet mechanics [14], and Eringen's nonlocal elasticity theory [15].

Undoubtedly, nanomechanics is one of the main areas of studies at nanoscale. The study area of nano mechanics includes force and displacement relationships, vibration and frequency analysis, and functional and strain characteristics of nanoscale structures. With the emergence of smart materials technology, the discovery of carbon nanotubes [16,17], in the scientific world, the addition of microelectromechanical systems (MEMS) [18–21], and nanoelectromechanical systems (NEMS) [22–26] to research topics has greatly increased the interest in nanomechanics. All studies within the scope of these studies deal with mathematical models such as bars, beams, and plates. Particularly within the scope of the study, the beam model is frequently used in vibration and frequency analysis. In the scientific world, it is possible to come across studies using numerous beam models in macro, micro, and nano dimensions. This is the same for the nanobeams discussed in this study, although less in number.

Eric W. Wong et al. In his general study on the mechanics of nanobeams, he conducted experimental studies on the strength, toughness, and flexibility of nanotubes and rods, and focused on determining the mechanical properties of various material samples under the atomic force microscope. The results of the study showed experimentally that nano-sized beams have very different mechanical properties compared to macro-sized beam [27]. Aydoğdu used Eringen's generalized nonlocal beam theory to study the bending, buckling, and free vibration of nanobeams. In the study, many different beam theories were used, including Euler-Bernoulli, Timoshenko, Reddy, and Aydoğdu. Non-local parameter effect, beam length effect is considered for models produced from all different theories. In the conclusion, he suggested that the data can be used for static and dynamic analysis of nanotubes [28]. Bagdatli et al. in their study, they analyzed the linear vibration of mid-supported nanobeam, which is frequently encountered in the structure of nanoelectromechanical systems. They used Eringen's nonlocal elasticity theory to include the nanoscale effect in the analysis. The distance of the center support from the starting point and the non-local parameter from Eringen's theory were the focal points in his studies. As a result of the study, they emphasized that with the increase of the non-local parameter value, more nano-sized structures are obtained and when the middle support is positioned at the midpoint of the beam, the maximum high stiffness and linear natural frequency value are obtained [29]. Khaniki, in his study, saw Eringen's two-stage local and non-local integral model as a reliable and well-designed theory form, stating that it would be correct to use it in modeling size-dependent effects. In light of this information, he investigated the vibration behavior of the double-layer nanobeam system in his work. The vibration behavior of the double-layer nanobeam is formulated for three different situations, in-phase vibration, out-of-phase vibration, and stabilization of the underlying layer [30]. And many more, nanostructure problems are addressed by using size-dependent theories [31–36].

In the theory of continuum mechanics, carbon nanotubes or nanobeams are considered homogeneous and continuous macrostructures. The physical properties of the material nanostructure, such as lattice voids in the material structure or some linear and surface defects that disrupt the continuity of the system, such as steps or cracks, examples of which can be seen in Figure 1, are neglected. This situation is likely to cause problems at the point of transforming the design into practice. [37] in their study, he emphasized that advanced micro/nanosystems should be developed and their mechanical behavior should be predicted correctly, and it is not correct to say that nanobeams are discontinuous as in classical beams. Since the properties of the material differ depending on the size of the nano-size, not neglecting the physical properties in the examination of the mechanical behavior of nanomaterials will allow more accurate analysis results in real engineering applications. As a result of these defects existing in the nanostructure of the material, fatigue life may be shortened during their working life. They can reduce the structure's natural frequencies because it becomes more flexible. In this context, many studies on beams with stepped and cracked surfaces exist.

Figure 1.

Tekin et al. studied the free vibrations of three beam systems with n steps and different geometry sections at different points [38]. Z.R. Lu et al proposed a new approach to analyzing the free and forced vibrations of beams with multiple section stages using a composite element method. At the end of the study, they claimed that the proposed method for vibration analysis of the stepped beam showed the accuracy [39].

Xing-Jian et al. In their research, they investigated the vibration properties of stepped laminated composite Timoshenko beam. Plots of the natural frequencies and mode shapes of the T300/970 laminated stepped beam are given to show the effect of step position parameter applications on the dynamic behavior of the beam [40]. Adomian decomposition method is also frequently used for vibration analysis of stepped beams. Qibo et al. used the Adomian decomposition method (ADM) to investigate the free vibrations of a stepped Euler-Bernoulli beam consisting of two uniform sections [41]. In another study, Qibo used the Adomian decomposition method to investigate the free vibrations of Euler-Bernoulli beams with multiple cross-section steps [42]. In many studies, analysis of cracks formed on various structures is encountered [27,43,44].

When the international literature databases are examined, Jaan Lellep et al. It is observed that they are one of the rare researchers who have focused on stepped nanobeams with their studies in the last few years. In their first study on this subject, [13,45–47] investigated the free vibrations of beams and rods made of nanomaterials. In their results, they emphasized that the presence of step and crack in the nanobeam is very important and affects the frequency modes. Taima et al. investigated the free vibration of multi-step nanobeams using the dynamic stiffness matrix method. The results show that the dimensionless natural frequency parameter is inversely proportional to the non-local parameters, except for the first mode for unclamped boundary conditions[48]. Masih et al. The free lateral vibration of the Euler-Bernoulli nanobeam with multiple discontinuities was investigated in their study, claiming that the natural frequencies of nanobeams are affected by various discontinuities and boundary conditions. The management equations are developed using Eringen's nonlocal elasticity theory. Discussing the effects of crack severity, the stepwise ratio of cross-sectional area, crack, and step location, buckyball mass, and small-scale parameter on natural frequencies, they argued that their approach was composed of a series of examples that could be used as criteria for other studies [49].

The discontinuity caused by steps or cracks on the nanobeam hinders the application of the classical continuum theory in the studies, and it is seen that there are many different theories applied to nanobeam. Therefore, the theory of continuum mechanics is much preferred and can capture effects considered important for the nanoscale. [50].

At this point, Eringen's [15] nonlocal elasticity theory is one of the most successful studies. The small-scale effect in nano-scale systems offers an important solution in terms of reality in the examination of the system [51]. For this reason, in studies dealing with nanostructures, Eringen's nonlocal continuum mechanics, which also considers the size effect, has been applied. Eringen's theory has been utilized in the analysis of many nanoscale structures such as nanobeams[52], nanoplates [53,54], lattice structures of materials [55], carbon nanotubes [56], and nano-switches [57].

In this study, the cascade nanobeam is modeled based on nonlocal theory. In the authors' opinion, this work may help gain an insight into the cascading nanobeam behavior for use in nanodevices or systems. Unlike the other stepped nanobeams work, an infrastructure was created for the design of much more materials by working without dimensions. The perturbation method was preferred for the first time as an analytical solution. Thereby presenting the possibility of nonlinear analysis considered as the next study.

2. Nonlocal Elasticity Theory

According to the nonlocal elasticity theory, the stress at a reference point in the body of the body depends not only on the stresses at that point but also on the stresses at all other points of the body. This observation is in line with the atomic theory of lattice dynamics and experimental observations on phonon dispersion. The classical (local) theory of elasticity is obtained when the effects of strains at points other than the reference point are neglected in the limit [58].

Nanobeam's material defaults to be a non-linear elastic material that conforms to a nonlocal elasticity theory. According to [15]. concepts, the constitutive equations for materials conforming to nonlocal elasticity be expressed as [15]:

$$\sigma_{ij}^{n}(x^{*}) = \iiint_{(V)} a\left(\left|x^{*\prime} - x^{*}\right|, \tau\right)\sigma_{ij}^{c}(x^{*\prime})dV \quad (1)$$

Eq. (1), σ_{ij}^{n} denotes the tension tensor at non-local elasticity, σ_{ij}^{c} the classical (Hooke) tension tensor, and V the volume. Here, a is the kernel function, which is assumed to express the effect of the stress state in $x^{*\prime} \in V$ and the stress-strain state in $x^{*} \in V$, and τ is the physical constant [15]. Different forms of kernel function $a(x^{*})$ in eq. (1) describe different approximate models of nonlocal elasticity. Suppose $a(x^{*})$ is a linear differential operator L function. In this case[15],

$$La\left(\left[x^{*\prime} - x^{*}\right]\right) = \delta\left(\left[x^{*\prime} - x^{*}\right]\right) \quad (2)$$

Here δ is Dirac's δ - function, has shown that the function can be obtained by taking a simple two-dimensional kernel function [15].

$$L(a) = (1 - (e_0 a)^2 \nabla^2) a(x^{*}) \quad (3)$$

Here ∇ is the laplace operator. Eq. (3), e_0 is a physical constant. a is the repetitive interatomic distance parameter (lattice size) in the lattice structure of nanomaterials. Eringen named the $e_0 a$ expression as a small-scale parameter and suggested that its value should be taken in $e_0 a < 2$ nanometer scales [15]. According to Eqs. (1) - (3) the constitutive equation of nonlocal elasticity can be determined as follows [15],

$$\left(1 - e_0 a \nabla^2\right)\sigma_{ij}^{n} = \sigma_{ij}^{c} \quad (4)$$

For homogeneous isotropic Euler Bernoulli beam [15],

$$\sigma(x^{*}) - (e_0 a)^2 \frac{\partial^2 \sigma(x^{*})}{\partial x^2} = E\varepsilon(x^{*}) \quad (5)$$

3. Materials and Methods

Hamilton's principle was used to obtain the equations of motion of the stepped nanobeam. First, the Lagrangian of the system $£ = T - V$ was found. According to Hamilton's principle, the variation of the time integral of the difference between the kinetic T and potential V energies of a system should be zero. Here, the difference between kinetic and potential energies is defined as "Lagrangian (£)".

$$T = \frac{1}{2}\int_{0}^{x_s} \rho A_1 \left(\frac{\partial w_1^{*}}{\partial t^{*}}\right)^2 dx^{*} + \frac{1}{2}\int_{x_s}^{L} \rho A_2 \left(\frac{\partial w_2^{*}}{\partial t^{*}}\right)^2 dx^{*} \quad (6a)$$

$$V=\frac{1}{2}\int_0^{x_s}\left(\begin{array}{l}EI_1\frac{\partial^2 w_1^*}{\partial x^{*2}}+(e_0a)^2 N\frac{\partial^2 w_1^*}{\partial x^{*2}}\\-(e_0a)^2\rho A_1\frac{\partial^2 w_1^*}{\partial t^{*2}}\end{array}\right)\frac{\partial^2 w_1^*}{\partial x^{*2}}dx$$

$$-\frac{1}{2}\int_0^{x_s}N\left(\frac{\partial w_1^*}{\partial x^*}\right)^2$$

$$+\frac{1}{2}\int_{x_s}^{L}\left(\begin{array}{l}EI_2\frac{\partial^2 w_2^*}{\partial x^{*2}}+(e_0a)^2 N\frac{\partial^2 w_2^*}{\partial x^{*2}}\\-(e_0a)^2\rho A_2\frac{\partial^2 w_2^*}{\partial t^{*2}}\end{array}\right)\frac{\partial^2 w_2^*}{\partial x^{*2}}dx$$

$$-\frac{1}{2}\int_{x_s}^{L}N\left(\frac{\partial w_2^*}{\partial x^*}\right)^2$$

(6b)

Here, ρ represents the density of the stepped nanobeam, A_1 and represents the cross-sectional areas of the stepped nanobeam. E is the modulus of elasticity of the stepped nanobeam, I_1 and is the moment of inertia. L is defined as the length scale parameter of the stepped nanobeam, x_s is step place, and N is the axial force. $(\,)^*$ represents dimensional parameters. The equations of motion and boundary conditions before and after the step of the stepped nano beam were found as follows, using Hamilton's:

$$EI_1\frac{\partial^4 w_1^*}{\partial x^{*4}}+\rho A_1\left(\frac{\partial^2 w_1^*}{\partial t^{*2}}-(e_0a)^2\frac{\partial^4 w_1^*}{\partial t^{*2}\partial x^{*2}}\right)=\frac{EA_1}{2\left[x_s+(L-x_s)/\left(\frac{r_2}{r_1}\right)^2\right]}\left[\int_0^{x_s}\left(\frac{\partial w_1^*}{\partial x^*}\right)^2dx^*+\int_{x_s}^{L}\left(\frac{\partial w_2^*}{\partial x^*}\right)^2dx^*\right]\left(\frac{\partial^2 w_1^*}{\partial x^{*2}}-(e_0a)^2\frac{\partial^4 w_1^*}{\partial x^{*4}}\right) \quad (7)$$

$$EI_2\frac{\partial^4 w_2^*}{\partial x^{*4}}+\rho A_2\left(\frac{\partial^2 w_2^*}{\partial t^{*2}}-(e_0a)^2\frac{\partial^2 w_2^*}{\partial t^{*2}\partial x^{*2}}\right)=\frac{EA_1}{2\left[x_s+(L-x_s)/\left(\frac{r_2}{r_1}\right)^2\right]}\left[\int_0^{x_s}\left(\frac{\partial w_1^*}{\partial x^*}\right)^2dx^*+\int_{x_s}^{L}\left(\frac{\partial w_2^*}{\partial x^*}\right)^2dx^*\right]\left(\frac{\partial^2 w_2^*}{\partial x^{*2}}-(e_0a)^2\frac{\partial^4 w_2^*}{\partial x^{*4}}\right) \quad (8)$$

For Simple-Simple Support,

$$\begin{array}{ll}\frac{\partial^2 w_1^*(0)}{\partial x^{*2}}=0, & \delta w_1^*(x_s)=\delta w_2^*(x_s),\\ \delta w_1^*(0)=0, & \frac{\partial(\delta w_1^*(x_s))}{\partial x^*}=\frac{\partial(\delta w_2^*(x_s))}{\partial x^*},\\ \frac{\partial^2 w_2^*(L)}{\partial x^{*2}}=0, & -EI_1\frac{\partial^2 w_1^*(x_s)}{\partial x^{*2}}+EI_2\frac{\partial^2 w_2^*(x_s)}{\partial x^{*2}}=0,\\ \delta w_2^*(L)=0 & EI_1\frac{\partial^3 w_1^*(x_s)}{\partial x^{*3}}-EI_2\frac{\partial^3 w_2^*(x_s)}{\partial x^{*3}}=0\end{array} \quad (9)$$

Dimensionless parameters are associated with dimensional values marked with an asterisk and equations are nondimensionalized

$$x=\frac{x^*}{L},\ w_{1,2}=\frac{w_{1,2}^*}{R_{1,2}},\ t=\beta t^*,\ \gamma=\frac{e_0a}{L},$$

$$\alpha=\frac{r_2}{r_1},\ \eta=\frac{x_s}{L},\ \beta=\frac{1}{L^2}\sqrt{\frac{EI_1}{\rho A_1}}$$

(10)

α is a dimensionless parameter that indicates the ratio of the radio of the steps at eq. (10). γ is a dimensionless non-local parameter. η is a dimensionless parameter expressing the step location. R is the parameter expressing the radius of inertia of the circular cross-section stepped beam.

4. Perturbation Analysis

In this section, the approximate solution is obtained by the perturbation method. The multi-scale method, which is one of the perturbation methods, is applied to the solution. The following expansion can be suggested for the displacement functions [59].

$$y_1(x,t:\varepsilon)=\varepsilon^0 y_{10}(x,T_0,T_1)+\varepsilon y_{11}(x,T_0,T_1) \quad (11)$$

$$y_2(x,t:\varepsilon)=\varepsilon^0 y_{20}(x,T_0,T_1)+\varepsilon y_{21}(x,T_0,T_1) \quad (12)$$

ε is a small parameter used in calculations. $T_0=\varepsilon^0 t$ is a fast time scale, $T_1=\varepsilon t$ is a slow time scale. According to the time derivative expressions are written in terms of new time variables,

$$\partial/\partial t=D_0+\varepsilon D_1$$
$$\partial^2/\partial t^2=D_0{}^2+2\varepsilon D_0 D_1$$
$$\text{where, } D_n=\partial/\partial T \quad (13)$$

After expansion, the first and second terms of the expansion are separated as follows:

Order (ε^0)

$$y_{10}^{iv}+D_0^2 y_{10}-\gamma^2 D_0^2 y_{10}''=0 \quad (14)$$

$$y_{20}^{iv}+\frac{1}{\alpha^2}D_0^2 y_{20}-\frac{\gamma^2}{\alpha^2}D_0^2 y_{20}''=0 \quad (15)$$

Order (ε)

$$\begin{aligned}&y_{11}^{iv}+D_0{}^2 y_{11}+2D_0D_1 y_{10}-2\gamma^2 D_0D_1 y_{10}''-\gamma^2 D_0{}^2 y_{11}''\\&=\Gamma_1\left[\int_0^\eta (y_{10}'^2)dx+\alpha^2\int_\eta^1 (y_{20}'^2)dx\right]y_{10}''\\&-\Gamma_1\gamma^2\left[\int_0^\eta (y_{10}'^2)dx+\alpha^2\int_\eta^1 (y_{20}'^2)dx\right]y_{10}^{iv}\\&+F\cos\Omega t-2\mu D_0 y_{10}\end{aligned} \quad (16)$$

$$\begin{aligned}&y_{21}^{iv}+\frac{1}{\alpha^2}D_0{}^2 y_{21}+\frac{2}{\alpha^2}D_0D_1 y_{20}-2\frac{\gamma^2}{\alpha^2}D_0D_1 y_{20}''\\&-\frac{\gamma^2}{\alpha^2}D_0{}^2 y_{21}''\\&=\Gamma_2\left[\int_0^\eta (y_{10}'^2)dx+\alpha^2\int_\eta^1 (y_{20}'^2)dx\right]y_{20}''\\&-\Gamma_2\gamma^2\left[\int_0^\eta (y_{10}'^2)dx+\alpha^2\int_\eta^1 (y_{20}'^2)dx\right]y_{20}^{iv}\\&+F\cos\Omega t-2\mu D_0 y_{20}\end{aligned}$$
(17)

Where,

$$\Gamma_1=\frac{1}{2\left(\eta+\frac{(1-\eta)}{\alpha^2}\right)}, \text{ and } \quad \Gamma_2=\frac{1}{2\alpha^4\left(\eta+\frac{(1-\eta)}{\alpha^2}\right)}$$

The equations in the ε^0 Order give the linear equation of motion and the linear frequency equation of the system. The equations in ε Order show the effects coming from the nonlinear part. The boundary conditions can be represented as

$$\begin{aligned}&y_{10}(0)=0, \qquad y_{20}(1)=0\\&y_{11}(\eta)=\alpha y_{21}(\eta), \qquad y_{11}'(\eta)=\alpha y_{21}'(\eta)\\&y_{11}''(0)=0, \qquad y_{21}''(1)=0\\&y_{11}''(\eta)=\alpha^5 y_{21}''(\eta), \qquad y_{11}'''(\eta)=\alpha^5 y_{21}'''(\eta)\end{aligned} \quad (18)$$

5. Linear Problem

The first perturbation order ε^0 is given in Eqs. (14) and (15); The solution can be represented as

$$\begin{aligned}&y_{10}(x,T_0,T_1)=\\&A_1(T_1)e^{i\omega T_0}Y_1(x)+\bar{A}_1(T_1)e^{-i\omega T_0}\bar{Y}_1(x)\end{aligned} \quad (19)$$

$$\begin{aligned}&y_{20}(x,T_0,T_1)=\\&A_2(T_1)e^{i\omega T_0}Y_2(x)+\bar{A}_2(T_1)e^{-i\omega T_0}\bar{Y}_2(x)\end{aligned}$$
(20)

If eqs. (19) and (20) are applied to eqs. (14) and (15),

$$Y_1^{iv}(x)-\omega^2 Y_1(x)+\gamma^2\omega^2 Y_1''(x)=0 \quad (21)$$

$$Y_2^{iv}(x)-\frac{1}{\alpha^2}\omega^2 Y_2(x)+\frac{1}{\alpha^2}\gamma^2\omega^2 Y_2''(x)=0$$
(22)

Eqs. (23) and (24) can be used to solve Eqs. (21) and (22)

$$Y_1(x)=c_{11}e^{ir_{11}x}+c_{12}e^{ir_{12}x}+c_{13}e^{ir_{13}x}+c_{14}e^{ir_{14}x}$$
$$=c_{11}\left(e^{ir_{11}x}+\frac{c_{12}}{c_{11}}e^{ir_{12}x}+\frac{c_{13}}{c_{11}}e^{ir_{13}x}+\frac{c_{14}}{c_{11}}e^{ir_{14}x}\right) \quad (23)$$

$$Y_2(x)=c_{21}e^{ikr_{21}x}+c_{22}e^{ikr_{22}x}+c_{23}e^{ikr_{23}x}+c_{24}e^{ikr_{24}x}$$
$$=c_{21}\left(e^{ikr_{21}x}+\frac{c_{22}}{c_{21}}e^{ikr_{22}x}+\frac{c_{23}}{c_{21}}e^{ikr_{23}x}+\frac{c_{24}}{c_{21}}e^{ikr_{24}x}\right) \quad (24)$$

Where, $k=\frac{1}{\sqrt{\alpha}}$

the scattering equations are obtained as follows.

$$r_{1n}^4-\gamma^2\omega^2 r_{1n}^2-\omega^2=0 \qquad n=1\ ,\ 2\ ,$$
(25)

$$r_{2n}^4 k^4-\frac{\gamma^2}{\alpha^2}k^2\omega^2 r_{2n}^2-\frac{1}{\alpha^2}k^2\omega^2=0 \qquad (26)$$

r_n roots can be obtained numerically after all the constant data are entered numerically. At this stage, to see the effects of the boundary conditions in the linear problem, a coefficient matrix is created by substituting the boundary conditions in equations (25) and (26). The values that make the determinant of this matrix zero are the natural frequencies of the system.

6. Results And Discussions

First, it was aimed to strengthen the accuracy of the results obtained from the study. Accordingly, the results were compared with similar studies in the literature (Table 1). When the table is examined, when the non-local parameter value is $\gamma=0$, the system is considered as a classical beam. In other cases, $\gamma=0.1-0.2-0.3-0.4-0.5$, the nanoscale effect is observed. In both cases, the results showed excellent agreement with the literature studies.

Figure 2.

The first three mode values of the stepped nanobeam with various step ratios versus nonlocal parameters are shown in Fig. 2. When the figure is examined, it is seen that the natural frequency values decrease as the non-local parameter value increases. The graphic is drawn by selecting the step location $\eta=0.4$. For this reason, it is seen that as the step ratio α increases, the frequency values of the beam increase, that is, its stiffness increases.

Table 1.

To prove the correctness of the solutions to the study's linear problem, the values are compared with the values of [48], and [60]. A stepped beam with a simple-simple boundary condition, $\alpha=1$ was chosen and the first three fundamental frequency values of the beam are compared with other nanobeam values for different small-scale parameters in Table 1 and show good agreement with the other two studies. From these comparisons, it is also probable to observe that the frequency values increase as the small-scale parameter increases. It is seen that this opinion is in parallel with the views of other many studies working at the nanoscale [28], [50], and [61].

Table 2.

In Table 2, the first three mode frequency values corresponding to different values for the stepped nanobeam are given. When the natural frequency values are examined, it is seen that the increase in the non-local parameter value causes the natural frequency values to decrease. Based on this situation, it can be concluded that there is a decrease in the stiffness of the material as the non-local parameter value increases. In addition, it is possible to examine the effect of the step location on the natural frequency in the table 2. It is seen that the values increase as the step location moves away from the starting point. This is because the step ratio $\alpha < 1$ is selected. The stiffness of the stepped beam increases as the step location moves away from the starting note. This causes the natural frequency to increase. This shows that the presence of stages in the beams is an important factor for the system.

Figure3.

In Figure 3, the changes in the fundamental frequency values of the stepped beam according to the step location are plotted for different non-local parameter values $\gamma = 0.1 - 0.3 - 0.5$. While obtaining the values, the step ratio was chosen $\alpha = 0.8$. The following results can be obtained from the graph. It is seen that the natural frequency values increase as the step location moves away from the starting point. This is true for any non-local parameter value. This is due to the step ratio being less than $\alpha < 1$. The stiffness of the stepped beam increases as the step location moves away from the starting point. In addition, it is seen that the natural frequency decreases with the increase of the non-local parameter value. This situation leads to the conclusion that the non-local parameter (small-scale parameter) weakens the rigidity of the object.

Figure 4.

In Figure 4, the changes in the fundamental frequency values of the stepped beam according to the step ratio are drawn for different step locations. While obtaining the values, the non-local parameter value was chosen $\gamma = 0.2$. The following results can be obtained from the graph. First, if the step ratio is $\alpha = 1$, all-natural frequency values are equal since the step location becomes unimportant. Then, as the step ratio moves away from one, different natural frequency values are seen according to the step location. This situation becomes more evident as the step ratio moves away from the value of $\alpha = 1$. For example, the case where the step location $\eta = 0.2$ can be examined. As the step ratio approaches $\alpha = 0.5 \to 3$, there are serious increases in the natural frequency value. This is due to the increase in stiffness because of the growth of the thick part of the stepped beam. In the case where the step location is taken $\eta = 0.8$, it is observed that there is an inverse situation to $\eta = 0.2$. In other words, as the step ratio approaches $\alpha = 0.5 \to 3$, the natural frequency value decreases. This is the result of the decrease in the stiffness of the beam because of the decrease in the thick part of the stepped beam.

7. Conclusions

In the present study, the linear vibration behavior of the stepped nanobeam is investigated. The results are presented in graphs and tables. As expected, the reduction of the natural frequency of mode shapes is observed with the increase of the nonlocal parameter. It is seen that the step position and ratios contribute significantly to the natural frequency. It has been shown that determining the desired frequency range or distancing at specific frequencies can be carried out easily by changing the positions of the step. There are very few cascade nanobeam studies in the literature, and they have all been written in the last few years. Therefore, it is evaluated that this study will be a new light for this area.

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Figures and Tables Captions

Figure 1. Samples of Stepped Nano Beam Scanned by Scanning Electron Microscopy [62]

Figure 2. First three dimensionless frequencies of stepped nanobeam with various step ratios versus nonlocal parameter

Table 1 Comparison of the first three fundamental frequencies according to different non-local parameters

Table 2 The first three frequencies are for different step locations and nonlocal parameters

Figure 3. First dimensionless frequencies of stepped nanobeam with various the nonlocal parameter for versus step locations

Figure 4. First dimensionless frequencies of stepped nanobeam with various step locations versus step ratios

Figures and Tables

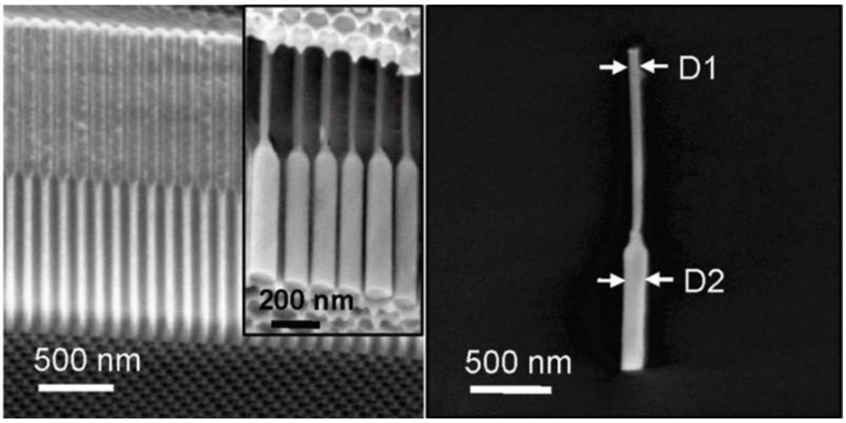

Figure 1.

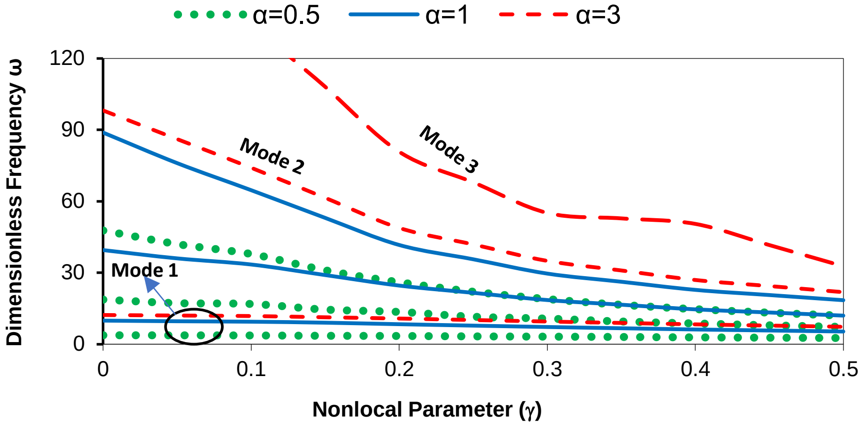

Figure 2.

Table 1

				$\alpha=1$		$\eta=1$			
γ		ω_1			ω_2			ω_3	
	Present	[60]	[48]	**Present**	[60]	[48]	**Present**	[60]	(Taima et al, 2021)
0	9.8696	9.8696	9.8696	39.4784	39.4784	39.4787	88.8264	88.8264	88.8280
0.1	9.4158	9.4159	9.4159	33.4277	33.4277	33.4279	64.6414	64.6414	64.6420
0.2	8.3569	8.3569	8.3569	24.5823	24.5823	24.5824	41.6284	41.6284	41.6287
0.3	7.1824	7.1824	-	18.5015	18.5016	-	29.6180	29.6180	-
0.4	6.1455	6.1456	-	14.5951	14.5951	-	22.7743	22.7743	-
0.5	5.3003	5.3003	-	11.9744	11.9744	-	18.4389	18.4389	-

Table 2

α	η	γ	ω_1	ω_2	ω_3
0.5	**0.2**	**0**	3.499	14.401	34.034
		0.1	3.418	13.198	28.280
		0.2	3.205	10.850	20.373
		0.3	2.924	8.743	15.183
		0.4	2.632	7.160	11.922
		0.5	2.359	6.002	9.757
0.5	**0.4**	**0**	3.723	18.683	47.711
		0.1	3.642	16.860	37.832
		0.2	3.425	13.520	25.982
		0.3	3.138	10.709	18.922
		0.4	2.835	8.682	14.681
		0.5	2.550	7.234	11.929
0.5	**0.6**	**0**	4.778	27.241	50.684
		0.1	4.581	23.924	40.291
		0.2	4.288	18.349	27.770
		0.3	3.907	14.053	20.238
		0.4	3.514	11.134	15.690
		0.5	3.150	9.128	12.738
0.5	**0.8**	**0**	7.591	27.192	70.363
		0.1	7.330	24.065	53.581
		0.2	6.686	18.712	35.346

0.3	5.917	14.485	25.180
0.4	5.187	11.558	19.296
0.5	4.556	9.523	15.568

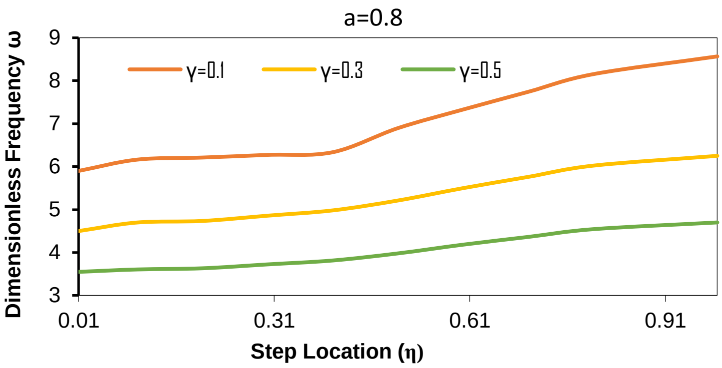

Figure 3.

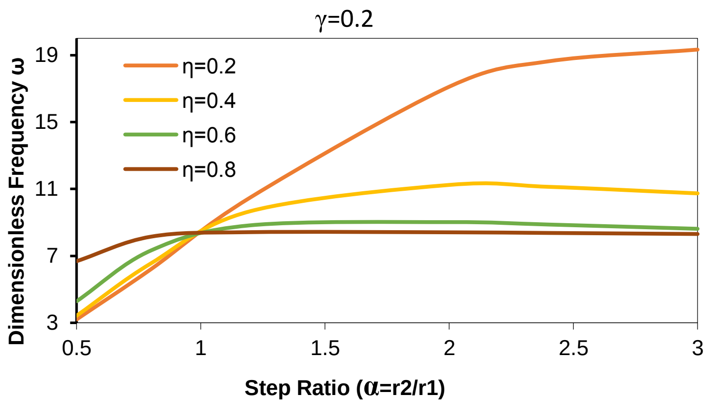

Figure 4.