An integrated bi-objective model under warranty, technology level, and pricing by considering pre-sale and post-sale costs

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Abstract

Due to the imperfection of processes, the quality of some products may be unsatisfactory. Moreover, equipment failure can stop production for a while. Therefore, integrating the triple concepts of quality, maintenance, and inventory control has attracted attention. Triple concepts are the constituents of the pre-sale costs. Selling price and warranty are generally considered to maintain market share and maximize the producer's profit. Quality and maintenance in production should be considered to reduce the post-sale costs of the warranty. Despite interactions, integrating warranty with triple concepts has been neglected. We integrate the quadruple concepts in a bi-objective model to maximize the profit and minimize the pre-sale and post-sale costs under the free minimal repair warranty policy. A non-central chi-square (NCS) control chart monitors the mean and variance, simultaneously. The technology level is also considered for increasing product quality and reducing failures during the warranty period. Due to its high complexity, the model is solved by the particle swarm optimization algorithm. The proposed model is applied through a numerical example and three comparative studies. The results indicate the better performance of the NCS chart, the superiority of bi-objective optimization versus single-objective optimization, and the importance of integrating presale and postsale costs.

Keywords: Simultaneous process monitoring, Maintenance, Technology level, Pricing, Warranty

1. Introduction

The final profit of a company is mainly affected by the selling price and warranty. Price, as a reflection of the quality, should be determined so that consumers are encouraged to buy. The rapid development of technologies fills the market with various new products. Providing better services to consumers can help the survival of companies. Manufacturers have also concentrated on optimizing warranty policies and guaranteeing post-sales services. The related costs of warranty and services can be reduced by considering 1) maintenance policies, which reduce the failure rate and increase the product's lifecycle, and 2) the level of manufacturing technology (LMT), which impact on the cost of production, failure times, and the reliability of the product.

In the presale state, the economic production quantity (EPQ) model was introduced to reduce costs. However, it considers two unrealistic assumptions: 1) the equipment is faultless during the cycle, and 2) all manufactured products satisfy the consumer's expectations [1]. In real conditions, the process may break down or go to an out-of-control state (OCS) due to the increasing failure rate and occurring any assignable cause (AC). This decreases the quality of products and increases costs. Increasing product quality needs further attention to 1) maintenance policy (MP) for restoring the initial condition, and 2) statistical process monitoring (SPM) for detecting an AC quickly. Farahani and Tohidi [2] indicated the importance of integrated optimization because of interactions among the triple concepts, the constituents of the pre-sale costs. Reducing the post-sale costs of warranty needs attention to quality and maintenance in production. Despite such interactions, integrating warranty with triple concepts has been neglected.

This is the first study to incorporate warranty into an integrated model and consider the presale and postsale costs of the quadruple concepts. We consider a free minimal repair warranty (FMRrW) policy. Maximizing the profit by selling products is the optimality criterion, which requires minimizing the presale and postsale costs. Moreover, the post-warranty costs of consumers are considered. To reduce these costs, we consider LMT, SPM, and MP. In SPM, occurring an AC can lead to a change in the mean and/or variance. We apply a non-central chi-square (NCS) chart for its better performance. We examine the importance of using the NCS chart, optimizing based on the manufacturer and consumer perspectives, and integrating presale and postsale costs. Accordingly, Section 2 reviews the literature. Section 3 defines the problem at hand. The integrated model is proposed in Section 4. Then, a solution procedure is described. Section 6 investigates numerical studies and analyses. Then, we discuss the benefits and managerial insights. Finally, conclusions are mentioned.

2. Literature review

We review studies in which: 1) pricing has been integrated with at least one of the mentioned concepts, 2) reducing the number of failures has been addressed by considering maintenance and/or LMT factors, 3) the manufacturer, consumer, or combined perspectives have been addressed, and 4) occurring shifts have been considered in the mean or variance or both simultaneously.

For scheduling a flow shop cell, Foumani et al. [3] noticed the pre-sale and post-sale costs to control inspection times regardless of inventory, maintenance, and warranty concepts. For an imperfect process, Giri and Dohi [4] scheduled inspections to perform preventive maintenance (PM) for products sold under a free repair warranty (FRrW) contract, regardless of the selling price. To increase profit, producers should determine the selling price, since it often has the greatest impact on a consumer's decision. Tsao et al. [5] integrated inventory and pricing for high-tech products under a free replacement warranty (FRtW) policy. By integrating the effect of the sales price and non-renewable free minimal-repair warranty (NRFMRrW) in marketing and post-sales service, Xie [6] proposed a model to maximize profit. Chen et al. [7] modeled an imperfect process in which the selling price was a function of the free repair/replacement warranty (FRrRtW) period. Liu et al. [8] optimally determined the pricing and production strategies for a monopolistic manufacturer under a non-renewing FRtW (NRFRtW) during a two-period planning horizon. Salmasnia and Kohan [9] jointly optimized pricing, inventory control, and keeping investment in technology for deteriorating products. Wang et al. [10] presented a dynamic model by considering the influences of product price fluctuations and repair learning.

The effect of performing MPs and the right LMT to reduce product failure is important. While neglecting LMT, some studies have only implemented MPs to reduce product failure. LMT has a significant impact on production cost, failure times, and product reliability. Also, introducing a new product needs to consider critical technical variables such as design and reliability. Accordingly, Darghouth et al. [11] jointly optimized the design, warranty, and price of products sold under maintenance service contracts. Salmasnia and Hatami [12] developed an integrated model and considered FMRrW, maintenance planning, and the LMT for reducing product failure (see also[13-16]).

Most studies determine the optimal warranty strategy by minimizing the service cost of the producer while ignoring the consumer's perspective. Accordingly, Wu et al. [17] developed a profit-maximizing model for manufacturers based on a predetermined lifetime to optimally determine the selling price, warranty period, and production rate. Shafiee et al. [18] developed statistical models to estimate dealer warranty costs in which second-hand products were sold under a two-dimensional FRrRtW. Only a few studies have focused on the consumer's perspective. From this perspective, Lim et al. [19] optimally determined the duration of the maintenance period at the sale of a second-hand product under a prorated warranty (PRW). Some studies have investigated the hybrid perspective. Accordingly, Kim et al. [20] planned PM for products sold under the warranty period. Salmasnia and Yazdekhasti [21] provided a periodic PM strategy under a non-renewing warranty (NRW) policy by optimizing the manufacturer’s costs and consumer satisfaction level. Salmasnia and Baratian [22] considered maximizing manufacturer and buyer satisfaction to optimally determine postsale services (see also [23-25]). However, no research has examined the hybrid perspective under our defined framework.

Typically, the X-bar and the R charts are respectively used to detect ACs in the mean and variance [26]. However, their combination leads to poor performance in detecting small to medium shifts and not recognizing the type of shift. Costa and Rahim [27] were motivated to present an NCS chart for the simultaneous monitoring of the mean and variance. Then, Costa and Rahim [28] investigated the NCS chart with two-stage samplings. Costa and De Magalhaes [29] developed an adaptive type of NCS chart. Tsai et al. [30] presented the economic design (ED) of a two-stage NCS chart for dependent variables. By integrating the triple concepts of inventory control, MP, and SPM in an economic-statistical design (ESD), Salmasnia et al. [31, 32] used an NCS chart. However, the NCS chart has not been studied under our assumptions.

According to Table 1, no research has integrated the triple concepts by incorporating warranty into it and considering pricing and LMT. We aim to integrate the quadruple concepts and present a bi-objective model for optimizing the manufacturer's profit and the total pre-sale and post-sale costs. Postsale costs can be reduced by increasing the quality of products and reducing the frequency of failures during the warranty period. Accordingly, SPM, LMT, and MP are included. We apply the NCS chart to monitor both the mean and variance according to its better performance. Our proposed model is of type ESD that keeps desirable statistical criteria.

{Please insert Table 1 about here.}

3. Problem definition

An imperfect process begins in the in-control state (ICS). After a while, it may face: 1) an OCS, which causes a shift due to occurring an AC, and 2) equipment failure, which suddenly stops the process. In the first case, we use an NCS chart to monitor the mean and variance simultaneously. If the issued signal is correct, predictive maintenance (PRDM) is implemented to return the process to ICS. If the signal is false, the process continues without taking any maintenance action. In the second case, corrective maintenance (CM) is implemented to restore the process to its initial condition. When no warning signal and no equipment failure are observed until the cycle ends, the process may: 1) remain in ICS, and 2) go to an OCS and remain undetected. In the end, PM is performed or switched to PRDM. Figure 1 depicts five different scenarios along with their stock levels.

The LMT can affect the production cost and number of failures. It determines product reliability, the inverse of the design variable. For the occurrence of product failure over time, we relate the Weibull distribution to the design variable. Moreover, we relate the cost of manufacturing to the LMT. The higher the LMT, the higher the production cost and the lower the number of failures. From the manufacturer's viewpoint, getting only more profit can lead to a lower LMT and product reliability. However, consumers tend to buy high-quality products that need to consider a higher LMT. Therefore, it is better to determine the design variable by considering both viewpoints.

In this study, the FMRrW policy is granted by the manufacturer to the consumer, and the repair costs in the post-warranty period are considered. Therefore, the costs of repairing the product during the warranty and post-warranty periods are respectively borne by the manufacturer and the consumer. We also assume that fixing the product failure requires a minor repair to return its condition to that before the failure. Since it is expensive to repair failures during the product's lifecycle, we use a periodic PM policy to reduce the failure rate. Figure 2 represents the costs during the warranty and post-warranty periods.

By optimizing the producer's profit and customer's costs, we can determine the sample size (n), the sampling interval (h), the coefficient of control limit of the NCS chart (L), the number of sampling per manufacturing cycle (k), the level of maintenance in the product lifecycle (m), the design variable (ϖ), the sale price (P) and, the optimal goal values correspond to the multi-choice goal programming (GP) model (r_1, r_2).

{Please insert Figure 1 about here.}

{Please insert Figure 2 about here.}

4. Model description

Before presenting the proposed model, the basic assumptions, scenarios' calculations, and structures of costs are given.

4.1. *Notations* and *assumptions*

Table 2 indicates the applied notations. The following assumptions are considered:

1. The elapsed time before occurring an AC follows a Weibull distribution with scale parameter η>0 and shape parameter γ>0 as $f(x)=(\gamma/\eta)(x/\eta)^{\gamma-1}e^{-(x/\eta)^{\gamma}}$,
2. Initially, the characteristics of the process are in ICS as μ_0 and σ_0. When an AC occurs, it shifts both the mean and variance to OCS as $\mu_1= \mu_0\pm\delta\sigma_0$ and $\sigma_1=\Psi\sigma_0$, respectively,
3. Nonconforming items are produced in both ICS and OCS. The rate of producing those items in OCS is much higher than those in ICS,
4. The time duration until occurring a failure in ICS (OCS) follows the Weibull distribution with scale parameter $\eta_0(\eta_1)$ and shape parameter $\beta_0(\beta_1)$,
5. Performing all types of MPs returns the initial condition,
6. The time durations to implement PM, CM, and PRDM are negligible,
7. The duration to repair a failure in the warranty period is ignored because it is minimal compared to the average time between two consecutive failures,
8. Manufactured products are repairable and sold under an FMRrW policy,
9. In the warranty period, the time duration until the failure of conforming (nonconforming) items follows the Weibull distribution with scale parameter η_c (η_{nc}) and shape parameter $\lambda_c(\lambda_{nc})$. The failure rate of nonconforming items is much higher than that of conforming items ($r_{02}(t)>r_{01}(t)$).

{Please insert Table 2 about here.}

4.2. *Scenarios' descriptions and calculations*

The system schedules a production rate of p to satisfy a demand rate of d where $p>d$ (Figure 1). The inventory level increases with a rate of p-d until the manufacturing cycle terminates. At this point, the inventory achieves the maximum level of $(p-d)T_p$. Then, consuming the inventory with a rate of d leads to inventory depletion at time pT_p/d. Therefore, the next production cycle begins. According to Figure 1, five scenarios may happen:

Scenario 1 (S_1). The process starts in ICS. However, equipment failure stops the process at $y_1\in(0, T_p)$. Implementing CM restores its initial condition. Since no AC occurs before the failure, the expected out-of-control time (OCT) equals zero. The occurrence probability and expected values of in-control time (ICT) and OCT are obtained as follows:

$$p\left(S_1\right)=\int_0^{T_p}\int_{y_1}^{\infty} g_1\left(y_1\right)f\left(x\right)dxdy_1=\int_0^{T_p} g_1\left(y_1\right)\bar{F}\left(y_1\right)dy_1 \quad (1)$$

$$E\left(T_{in1}\right)=E\left(y_1 \mid S_1\right)=\frac{\int_0^{T_p} y_1 g_1\left(y_1\right)\bar{F}\left(y_1\right)dy_1}{p\left(S_1\right)} \quad (2)$$

$$E\left(T_{out1}\right)=0 \quad (3)$$

Scenario 2 (*S_2*). The process remains in ICS throughout the cycle because neither failure nor AC occurs. Implementing PM increases the reliability of the process. The equations of S_2 are given as:

$$p\left(S_2\right)=\int_{T_p}^{\infty}\int_{T_p}^{\infty} g_1\left(y_1\right)f\left(x\right)dy_1dx=\bar{G}_1\left(T_p\right)\bar{F}\left(T_p\right)$$

(4)

$$E\left(T_{in2}\right)=T_p \quad (5)$$

$$E\left(T_{out2}\right)=0 \quad (6)$$

Scenario 3 (*S_3*). The process starts under ICS. An occurred AC at time *x* shifts both mean and variance. However, the control chart cannot detect it. Before the cycle ends, an equipment failure at time y_2 stops the process. CM is instantly implemented. The equations of S_3 are as follows:

$$p\left(S_3\right)=\int_0^{T_p} f\left(x\right)\bar{G}_1\left(x\right)\frac{\int_x^{T_p} g_2\left(y_2\right)\beta^{\left\lfloor \frac{y_2-x}{h}\right\rfloor}dy_2}{\bar{G}_2\left(x\right)}dx \quad (7)$$

$$E\left(T_{in3}\right)=E\left(\mathrm{x} \mid S_3\right)=\int_0^{T_p} xf\left(x\right)\bar{G}_1\left(x\right)\frac{\int_x^{T_p} g_2\left(y_2\right)\beta^{\left\lfloor \frac{y_2-x}{h}\right\rfloor}dy_2}{\bar{G}_2\left(x\right)p\left(S_3\right)}dx \quad (8)$$

$$E\left(T_{out3}\right)=E\left(\mathrm{y}_2 \mid S_3\right)-E\left(T_{in3}\right)=\int_0^{T_p} y_2 f\left(x\right)\bar{G}_1\left(x\right)\frac{\int_x^{T_p} g_2\left(y_2\right)\beta^{\left\lfloor \frac{y_2-x}{h}\right\rfloor}dy_2}{\bar{G}_2\left(x\right)p\left(S_3\right)}dx-E\left(T_{in3}\right) \quad (9)$$

Scenario 4 (*S_4*). The difference from S_3 is that the control chart detects the shift before the failure occurrence and time T_p. Performing PRDM restores the process to the initial condition. Depending on when the failure occurs in OCS, we face two cases:

***S_{41}*.** Equipment failure occurs during (x, T_p). An occurred AC at time *x* is identified during (x, T_p) before occurring the failure. The equations of S_{41} are as follows:

$$p\left(S_{41}\right)=\int_{0}^{kh} f(x)\bar{G}_1(x)\frac{\int_{x}^{T_p} g_2(y_2)\left(1-\beta^{\left\lfloor \frac{y_2-x}{h}\right\rfloor}\right)dy_2}{\bar{G}_2(x)}dx \tag{10}$$

$$E\left(T_{in41}\right)=E\left(x \mid S_{41}\right)=\int_{0}^{kh} xf(x)\bar{G}_1(x)\frac{\int_{x}^{T_p} g_2(y_2)\left(1-\beta^{\left\lfloor \frac{y_2-x}{h}\right\rfloor}\right)dy_2}{\bar{G}_2(x)\,p\left(S_{41}\right)}dx \tag{11}$$

$$E\left(T_{out41}\right)=\sum_{z=1}^{k} P_{z41}\left(\sum_{r=1}^{k-z+1} \beta^{r-1}(1-\beta)\left(r\times h-\ell_{z41}\right)\right) \tag{12}$$

where:

$$P_{z41}=\int_{(z-1)h}^{zh} f(x)\bar{G}_1(zh)\frac{\int_{zh}^{T_p} g_2(y_2)dy_2}{\bar{G}_2(x)}dx$$

$$\ell_{z41}=\int_{(z-1)h}^{zh} \left(x-(z-1)h\right)f(x)\bar{G}_1(x)\frac{\int_{x}^{T_p} g_2(y_2)dy_2}{\bar{G}_2(x)P_{z41}}dx$$

S_{42}. Equipment failure occurs during (T_p, ∞). An occurred AC at time x is detected before the manufacturing cycle ends. The equations are similar to those in S_{41}, with the difference that the integral limits for y_2 change to (T_p, ∞):

$$p\left(S_{42}\right)=\int_{0}^{kh} f(x)\bar{G}_1(x)\frac{\int_{T_p}^{\infty} g_2(y_2)\left(1-\beta^{\left\lfloor \frac{T_p-x}{h}\right\rfloor}\right)dy_2}{\bar{G}_2(x)}dx \tag{13}$$

$$E\left(T_{in42}\right)=E\left(x \mid S_{42}\right)=\int_{0}^{kh} xf(x)\bar{G}_1(x)\frac{\int_{T_p}^{\infty} g_2(y_2)\left(1-\beta^{\left\lfloor \frac{T_p-x}{h}\right\rfloor}\right)dy_2}{\bar{G}_2(x)\,p\left(S_{42}\right)}dx \tag{14}$$

$$E\left(T_{out42}\right)=\sum_{z=1}^{k} P_{z42}\left(\sum_{r=1}^{k-z+1} \beta^{r-1}(1-\beta)\left(r\times h-\ell_{z42}\right)\right) \tag{15}$$

where:

$$P_{z42}=\int_{(z-1)h}^{zh} f(x)\bar{G}_1(zh)\frac{\int_{T_p}^{\infty} g_2(y_2)dy_2}{\bar{G}_2(x)}dx$$

$$\ell_{z42}=\int_{(z-1)h}^{zh} \left(x-(z-1)h\right)f(x)\bar{G}_1(x)\frac{\int_{T_p}^{\infty} g_2(y_2)dy_2}{\bar{G}_2(x)P_{z42}}dx$$

Eventually, the equations of S_4 are calculated as follows:

$$p(S_4) = p(S_{41}) + p(S_{42}) \tag{16}$$

$$E(T_{in4}) = \frac{p(S_{41})}{p(S_4)} E(T_{in41}) + \frac{p(S_{42})}{p(S_4)} E(T_{in42}) \tag{17}$$

$$E(T_{out4}) = \frac{p(S_{41})}{p(S_4)} E(T_{out41}) + \frac{p(S_{42})}{p(S_4)} E(T_{out42}) \tag{18}$$

Scenario 5 (S_5). The difference from S_3 is in the non-occurrence of equipment failure until the cycle ends. The shift is identified while implementing PM. Thus, PM is switched to PRDM. The equations of S_5 are given as:

$$p(S_5) = \int_0^{T_p} f(x)\bar{G}_1(x) \frac{\int_{T_p}^{\infty} g_2(y_2)\beta^{\left\lfloor \frac{T_p - x}{h} \right\rfloor} dy_2}{\bar{G}_2(x)} dx \tag{19}$$

$$E(T_{in5}) = \int_0^{T_p} x f(x)\bar{G}_1(x) \frac{\int_{T_p}^{\infty} g_2(y_2)\beta^{\left\lfloor \frac{T_p - x}{h} \right\rfloor} dy_2}{\bar{G}_2(x)\, p(S_5)} dx \tag{20}$$

$$E(T_{out5}) = T_p - E(T_{in5}) \tag{21}$$

4.3. Manufacturer costs

This subsection describes the structure of costs imposed on the manufacturer in presale and postsale periods.

4.3.1. Pre-sale costs

The constituents of the pre-sale cost function are defined in the following subsections.

4.3.1.1. Production cost

We relate the cost of manufacturing a product to its LMT. The higher the LMT, the higher the production cost. Since the product design variable is the inverse of the LMT, the manufacturing cost should relate indirectly to the design variable. Calculating the cost of producing each unit is as follows [11]:

$$C_p = \psi_0 + \psi_1 \left(e^{\frac{\varpi_{\max} - \varpi}{\varpi - \varpi_{\min}}} \right) \tag{22}$$

where ϖ_{min} and ϖ_{max} are respectively the lower and upper limit of the design variable, and Ψ_0 and Ψ_1 are respectively the fixed cost of manufacturing the product and the sensitivity coefficient of the design variable.

4.3.1.2. Quality loss cost (QLC)

Manufacturer endures *QLC* in both ICS and OCS. The average manufactured items in each state should be multiplied by the *QLC* per unit, indicated by C_{in} for ICS and C_{out} for OCS, to calculate the average cost (note that $C_{in}=k_1C_p$ and $C_{out}=k_2C_p$). Then, it should be divided by the total manufactured items in one production cycle. The calculations for each scenario are as follows:

$$E\left(C_Q \mid S_1\right)=k_1\left(\psi_0+\psi_1\left(e^{\frac{\varpi_{\max}-\varpi}{\varpi-\varpi_{\min}}}\right)\right) \tag{23}$$

$$E\left(C_Q \mid S_2\right)=k_1\left(\psi_0+\psi_1\left(e^{\frac{\varpi_{\max}-\varpi}{\varpi-\varpi_{\min}}}\right)\right) \tag{24}$$

$$E\left(C_Q \mid S_3\right)=\frac{k_1 E\left(T_{in3}\right)\left(\psi_0+\psi_1\left(e^{\frac{\varpi_{\max}-\varpi}{\varpi-\varpi_{\min}}}\right)\right)+k_2 E\left(T_{out3}\right)\left(\psi_0+\psi_1\left(e^{\frac{\varpi_{\max}-\varpi}{\varpi-\varpi_{\min}}}\right)\right)}{E\left(T_{in3}\right)+E\left(T_{out3}\right)} \tag{25}$$

$$E\left(C_Q \mid S_4\right)=\frac{k_1 E\left(T_{in4}\right)\left(\psi_0+\psi_1\left(e^{\frac{\varpi_{\max}-\varpi}{\varpi-\varpi_{\min}}}\right)\right)+k_2 E\left(T_{out4}\right)\left(\psi_0+\psi_1\left(e^{\frac{\varpi_{\max}-\varpi}{\varpi-\varpi_{\min}}}\right)\right)}{E\left(T_{in4}\right)+E\left(T_{out4}\right)} \tag{26}$$

$$E\left(C_Q \mid S_5\right)=\frac{k_1 E\left(T_{in5}\right)\left(\psi_0+\psi_1\left(e^{\frac{\varpi_{\max}-\varpi}{\varpi-\varpi_{\min}}}\right)\right)+k_2 E\left(T_{out5}\right)\left(\psi_0+\psi_1\left(e^{\frac{\varpi_{\max}-\varpi}{\varpi-\varpi_{\min}}}\right)\right)}{E\left(T_{in5}\right)+E\left(T_{out5}\right)} \tag{27}$$

For the expected *QLC* per unit, we have:

$$E\left(C_Q\right)=\sum_{i=1}^{5} E\left(C_Q \mid S_i\right)\times p\left(S_i\right) \tag{28}$$

4.3.1.3. Sampling cost

The expected sampling cost per unit is achieved by multiplying the expected number of samples by the total fixed and variable sampling costs and dividing it by the total manufactured items in one cycle. The expected number of samples equals k in S_2 and S_5. In scenarios that stop before time T_p, it equals the sum of the expected number of samples in ICS and OCS. The calculations for each scenario are as follows:

$$E\left(C_{sampling} \mid S_1\right)=\frac{R_{in1}\left(C_f+C_v n\right)}{p.E\left(T_{in1}\right)} \tag{29}$$

$$E\left(C_{sampling} \mid S_2\right)=\frac{k\left(C_f+C_v n\right)}{p.T_p} \tag{30}$$

$$E\left(C_{sampling} \mid S_3\right)=\frac{\left(R_{in3}+R_{out3}\right)\left(C_f+C_v n\right)}{p.\left(E\left(T_{in3}\right)+E\left(T_{out3}\right)\right)} \tag{31}$$

$$E\left(C_{sampling} \mid S_4\right)=\frac{\left(R_{in4}+R_{out4}\right)\left(C_f+C_v n\right)}{p.\left(E\left(T_{in4}\right)+E\left(T_{out4}\right)\right)} \tag{32}$$

$$E\left(C_{sampling} \mid S_5\right)=\frac{k\left(C_f+C_v n\right)}{p.\left(E\left(T_{in5}\right)+E\left(T_{out5}\right)\right)} \tag{33}$$

where:

$R_{in_i}=\left\lfloor E(T_{in_i})/h\right\rfloor, \quad i=1,3,4,5$

$R_{out_i}=k-R_{in_i}, \quad i=5$

$R_{out_i}=\left\lfloor\left(E(T_{in_i})+E(T_{out_i})\right)/h\right\rfloor-R_{in_i}, \quad i=3,4$

For the expected sampling cost per unit, we have:

$$E\left(C_{sampling}\right)=\sum_{i=1}^{5} E\left(C_{sampling} \mid S_i\right)\times p\left(S_i\right) \tag{34}$$

4.3.1.4. Maintenance cost

This cost is obtained by dividing the sum of the false alarm cost and the cost of the implemented MP by the whole production. In S_1 and S_3, CM is implemented due to equipment failure. Due to occurring AC in S_4 and S_5, PRDM is performed to restore the initial condition. PM is implemented in S_2 due to the absence of failure and AC. The calculations for each scenario are as follows (CM and PRDM costs are linear functions of delay in shift detecting (ξ)):

$$E\left(C_M \mid S_1\right)=\frac{\rho_0+\dfrac{R_{in1}C_{falsealarm}}{ARL_0}}{p.\mathrm{E}\left(T_{in1}\right)} \tag{35}$$

$$E\left(C_M \mid S_2\right)=\frac{C_{PM}+\dfrac{k.C_{falsealarm}}{ARL_0}}{p.T_p} \tag{36}$$

$$E\left(C_M \mid S_3\right)=\frac{\rho_0+\rho_1.E\left(T_{out3}\right)+\dfrac{R_{in3}C_{falsealarm}}{ARL_0}}{p.\left(E\left(T_{in3}\right)+E\left(T_{out3}\right)\right)} \tag{37}$$

$$E\left(C_M \mid S_4\right)=\frac{\rho_0'+\rho_1'.E\left(T_{out4}\right)+\dfrac{R_{in4}C_{falsealarm}}{ARL_0}}{p.\left(E\left(T_{in4}\right)+E\left(T_{out4}\right)\right)} \tag{38}$$

$$E\left(C_M \mid S_5\right)=\frac{\rho_0'+\rho_1'.E\left(T_{out5}\right)+\dfrac{R_{in5}C_{falsealarm}}{ARL_0}}{p.\left(E\left(T_{in5}\right)+E\left(T_{out5}\right)\right)} \tag{39}$$

where the average run length (ARL) in ICS (ARL_0), indicates the number of samples obtained prior to a false out-of-control alarm (ARL_1 indicates the number of samples obtained to detect an occurred AC). It depends on n and L. Moreover, the expected number of false alarms is obtained by multiplying the average number of sampling in ICS

(R_{in}) by the probability of Type-I error (α=1/ARL_0)). For the expected maintenance cost per unit, we have:

$$E(C_M) = \sum_{i=1}^{5} E(C_M \mid S_i) \times p(S_i) \tag{40}$$

4.3.1.5. Inventory holding cost (IHC)

Figure 1 showed the inventory behavior in the imperfect production. For a production lot of size pT_p, the expected *IHC* per item is achieved by $C_h(p\text{-}d)T_p/2d$ where C_h represents the *IHC* per unit per time unit [4]. This is extended for each scenario as follows:

$$E(IHC \mid S_1) = \frac{C_h(p-d)E(T_{in1})}{2d} \tag{41}$$

$$E(IHC \mid S_2) = \frac{C_h(p-d)E(T_{in2})}{2d} \tag{42}$$

$$E(IHC \mid S_3) = \frac{C_h(p-d)\left(E(T_{in3}) + E(T_{out3})\right)}{2d} \tag{43}$$

$$E(IHC \mid S_4) = \frac{C_h(p-d)\left(E(T_{in4}) + E(T_{out4})\right)}{2d} \tag{44}$$

$$E(IHC \mid S_5) = \frac{C_h(p-d)\left(E(T_{in5}) + E(T_{out5})\right)}{2d} \tag{45}$$

For the expected *IHC* per unit, we have:

$$E(IHC) = \sum_{i=1}^{5} E(IHC \mid S_i) \times p(S_i) \tag{46}$$

4.3.1.6. Set-up cost (SC)

In the traditional EPQ model, *SC* in each production cycle is obtained by multiplying the number of set-ups (D/pT_p) by the cost of each set-up (A). The set-up is done only once at the beginning of the cycle. Calculating the expected *SC* for each scenario is as follows (the production run length (T_p) is computed by the sum of ICT and OCT):

$$E(SC \mid S_1) = \frac{A}{p.E(T_{in1})} \tag{47}$$

$$E(SC \mid S_2) = \frac{A}{p.T_p} \tag{48}$$

$$E(SC \mid S_3) = \frac{A}{p\left(E(T_{in3}) + E(T_{out3})\right)} \tag{49}$$

$$E(SC \mid S_4) = \frac{A}{p\left(E(T_{in4}) + E(T_{out4})\right)} \tag{50}$$

$$E(SC \mid S_5) = \frac{A}{p\left(E\left(T_{in5}\right) + E\left(T_{out5}\right)\right)} \quad (51)$$

For the total expected *SC* per item, we have:

$$E(SC) = \sum_{i=1}^{5} E(SC \mid S_i) \times p(S_i) \quad (52)$$

Eventually, the expected total cost (*ETC*) per item in the presale period is given by:

$$ETC_{pre-sale} = C_p + E(C_Q) + E(C_{sampling}) + E(C_M) + E(IHC) + E(SC) \quad (53)$$

4.3.2. Post-sale (warranty) costs

The product is sold under FMRrW policy with a warranty period of *w*. The manufacturer undertakes to repair defective items free of charge from purchase until the warranty period ends. Therefore, the costs of repairing product failure and performing PM actions are considered during the warranty period. Before calculating the warranty cost, we need to calculate the expected number of non-conforming items during a production cycle (θ_1 and θ_2 show the probability of non-conforming items produced in ICS and OCS, respectively):

$$E(N \mid S_1) = \theta_1 . p . E\left(T_{in1}\right) \quad (54)$$

$$E(N \mid S_2) = \theta_1 . p . T_p \quad (55)$$

$$E(N \mid S_3) = \theta_1 . p . E\left(T_{in3}\right) + \theta_2 . p . E\left(T_{out3}\right) \quad (56)$$

$$E(N \mid S_4) = \theta_1 . p . E\left(T_{in4}\right) + \theta_2 . p . E\left(T_{out4}\right) \quad (57)$$

$$E(N \mid S_5) = \theta_1 . p . E\left(T_{in5}\right) + \theta_2 . p . \left(T_p - E\left(T_{in5}\right)\right) \quad (58)$$

In each scenario, the proportion of produced nonconforming items during a cycle is calculated:

$$\alpha_1(T_p) = \theta_1 \quad (59)$$

$$\alpha_2(T_p) = \theta_1 \quad (60)$$

$$\alpha_3(T_p) = \frac{E(N \mid S_3)}{p\left(E\left(T_{in3}\right) + E\left(T_{out3}\right)\right)}$$

(61)

$$\alpha_4(T_p) = \frac{E(N \mid S_4)}{p\left(E\left(T_{in4}\right) + E\left(T_{out4}\right)\right)} \quad (62)$$

$$\alpha_5(T_p) = \frac{E(N \mid S_5)}{p\left(E\left(T_{in5}\right) + E\left(T_{out5}\right)\right)}$$

(63)

To control the costs by reducing the frequency of failure during the warranty period, periodic planned maintenance is used, as explained in the following subsections.

4.3.2.1. Modeling the maintenance strategy

Suppose at discrete times τ_1, τ_2, ..., τ_j, a series of PM actions with level m, $0 \leq m \leq M$ is performed on the product where m=0 indicates no PM implementation and $m=M$ indicates PM implementation at its highest level. By increasing the PM level, the deterioration effect decreases. This reduction depends on the level of maintenance and the age reduction factor $\delta'(m)$. For a given level after the j^{th} PM action, the virtual age of the product is obtained by:

$$v_j = v_{j-1} + \delta'(m)(\tau_j - \tau_{j-1}), \quad 0 \leq m \leq M, \; j \geq 2 \tag{64}$$

where v_0=0, τ_j is the actual age of the product at the time of PM implementation, and $\delta'(m)$ is a descending function of m as $\delta'(m)=(1+m)e^{-m}$. When m increases, the effect of wear decreases. If $m=M$, then $\delta'(M)=0$, and the product is restored to the initial condition. If m=0, then $\delta'(m)=1$ and $v_j=\tau_j$, $j \geq 1$, and PM activity is not effective in age reduction. The PM level is assumed the same during the warranty period. The virtual age of the product at a time t is obtained by:

$$v(t) = v_{j-1} + t - \tau_{j-1}, \; \tau_{j-1} \leq t \leq \tau_j, \; j = 1,2,... \tag{65}$$

Since failures are minimally repaired in a negligible time, the failure rate at time t is obtained by:

$$r\left[v(t), \varpi\right] = r\left(v_{j-1} + t - \tau_{j-1}, \varpi\right), \; \tau_{j-1} \leq t \leq \tau_j, \; j = 1,2,...,u_1 \tag{66}$$

where u_1 is the number of PM actions during the warranty period, and ϖ is the design variable that indicates the product's reliability [11].

4.3.2.2. Cost of PM by considering the learning effect in the warranty period

The cost of PM activity at level m in the warranty period is as follows [11]:

$$Cpm(m, j) = a + b(m) + c(j), \; j = 1,2,...,u_1; 0 \leq m \leq M \tag{67}$$

where a is the fixed cost and $b(m)$ is the cost of the improvement by the level m:

$$b(m) = \frac{\omega.m}{1 - e^{(-\phi[M-m])}}, \; j = 1,2,...,u_1; \; 0 \leq m \leq M, \; \omega > 0 \; and \; \phi > 0. \tag{68}$$

where ω is the contribution of the improvement level to the investment and ϕ indicates the degree of convergence from level m to M. If m=0, no improvement is achieved, and $b(m)$=0. If $m=M$, the product is restored to its initial condition, and $b(m) \rightarrow \infty$. Moreover, the cost of acquiring the necessary knowledge and practical skills for doing PM activities is expressed according to the learning curve:

$$c(j) = c_1 . j^{\frac{\log(0.8)}{\log(2.0)}}, \quad j = 1,2,... \tag{69}$$

where c_1 denotes the learning cost of the first PM activity (a and $c(j)$ are independent of m).

For the periodic PM cost in a specific warranty period, we have:

$$CPM_w(m, j) = \sum_{j=1}^{u_1} Cpm(m, j) \tag{70}$$

4.3.2.3. Modeling product failure based on LMT in the warranty period

LMT has a significant effect on the frequency of product failure and its reliability. The scale parameters of Weibull distribution for the time duration until the failure of conforming and non-conforming items, which are a function of the LMT, are respectively expressed as follows [11]:

$$\eta_c = \varpi\gamma_c \tag{71}$$

$$\eta_{nc} = \varpi\gamma_{nc} \tag{72}$$

where γ_c and γ_{nc} are positive constant values that can be estimated using historical warranty information. The failure rate during the warranty period with a PM activity at level m is calculated as follows:

$$r_m(t,\varpi) = \begin{cases} r_0(t,\varpi) & 0 \le t \le \tau_1 \\ r_0(\nu_j + t - \tau_j, \varpi) & \tau_j \le t \le \tau_{j+1},\ j = 1,2,\dots,(u_1 - 1) \\ r_0(\nu_{u_1} + t - \tau_{u_1}, \varpi) & \tau_{u_1} \le t \le W \end{cases} \tag{73}$$

where $u_1 = \lfloor W/\Delta \rfloor$ and $\tau_j = j\Delta$ for $j=1,\dots,u_1$.

Since the failure rates of conforming and non-conforming items are not the same, the expected failure frequency of those during the warranty period will respectively be equal to $\int r_{m1}(t,\varpi)dt$ and $\int r_{m2}(t,\varpi)dt$ by considering $0 \le t \le W$. The expected warranty cost per unit for the manufacturer in each scenario is obtained as follows:

$$\begin{aligned} E(C_W \mid S_i) = {} & C_r\left((1-\alpha_i(T_p))\int_0^{\tau_1} r_{01}(t,\varpi)dt + \alpha_i(T_p)\int_0^{\tau_1} r_{02}(t,\varpi)dt \right) + \\ & C_r\left((1-\alpha_i(T_p))\sum_{j=1}^{u_1-1}\int_{\tau_j}^{\tau_{j+1}} r_{01}(\nu_j + t - \tau_j,\varpi)dt + \alpha_i(T_p)\sum_{j=1}^{u_1-1}\int_{\tau_j}^{\tau_{j+1}} r_{02}(\nu_j + t - \tau_j,\varpi)dt \right) + \\ & C_r\left((1-\alpha_i(T_p))\int_{\tau_{u_1}}^{W} r_{01}(\nu_{u_1} + t - \tau_{u_1},\varpi)dt + \alpha_i(T_p)\int_{\tau_{u_1}}^{W} r_{02}(\nu_{u_1} + t - \tau_{u_1},\varpi)dt \right) + \\ & CPM_w(m,j),\ i = 1,2,3,4,5 \end{aligned} \tag{74}$$

For the ETC per item in the postsale period, we have:

$$ETC_{post-sale} = \sum_{i=1}^{5} E(C_W \mid S_i) \times p(S_i) \tag{75}$$

Eventually, the ETC per item for the manufacturer is computed by:

$$ETC_M = ETC_{pre-sale} + ETC_{post-sale} \tag{76}$$

4.4. Consumer costs

Consumer costs are obtained from the total purchase cost and post-warranty period costs. The latter includes the costs of periodic PM and repairing product failure. Those are calculated as follows (inspired by [11]):

$$Cpm_{pw}(m,j) = Cpm(m,j).(1+\alpha_p) \tag{77}$$

$$C_{rpw} = C_r(1+\alpha_r) \tag{78}$$

where α_p and α_r are profit margins associated with each PM and repair actions performed during the post-warranty period, respectively.

Modeling product failure based on technology level in the post-warranty period is similar to subsection 4.3.2.3, but according to the interval (W, LC), the failure rate during the post-warranty period is calculated as follows:

$$r_m(t,\varpi)=\begin{cases} r_0(t,\varpi) & W \le t \le \tau_1 \\ r_0\left(v_j+t-\tau_j,\varpi\right) & \tau_j \le t \le \tau_{j+1}\ ,\ j=\left(u_1+1\right),\ldots,\left(u_{2-1}\right) \\ r_0\left(v_{u_2}+t-\tau_{u_2},\varpi\right) & \tau_{u_2} \le t \le LC \end{cases} \tag{79}$$

where $u_2=\lfloor(LC-W)/\Delta\rfloor$ denotes the number of PM actions in the post-warranty period. Moreover, $\tau_j=W+j\Delta$ for $j=1,\ldots,u_2$. Thus, the expected post-warranty in each scenario is obtained as follows:

$$\begin{aligned} E\left(C_{PW}\mid S_i\right)=&\,C_{rpw}\left(\left(1-\alpha_i\left(T_p\right)\right)\int_W^{\tau_{u_1+1}} r_{01}\left(v_{u_1}+t-\tau_{u_1},\varpi\right)dt+\alpha_i\left(T_p\right)\int_W^{\tau_{u_1+1}} r_{02}\left(v_{u_1}+t-\tau_{u_1},\varpi\right)dt\right) \\ &+C_{rpw}\left(\left(1-\alpha_i\left(T_p\right)\right)\sum_{j=u_1+1}^{u_2-1}\int_{\tau_j}^{\tau_{j+1}} r_{01}\left(v_j+t-\tau_j,\varpi\right)dt+\alpha_i\left(T_p\right)\sum_{j=u_1+1}^{u_2-1}\int_{\tau_j}^{\tau_{j+1}} r_{02}\left(v_j+t-\tau_j,\varpi\right)dt\right) \\ &+C_{rpw}\left(\left(1-\alpha_i\left(T_p\right)\right)\int_{\tau_{u_2}}^{LC} r_{01}\left(v_{u_2}+t-\tau_{u_2},\varpi\right)dt+\alpha_i\left(T_p\right)\int_{\tau_{u_2}}^{LC} r_{02}\left(v_{u_2}+t-\tau_{u_2},\varpi\right)dt\right) \\ &+\sum_{j=u_1+1}^{u_2} Cpm_{pw}\left(m,j\right);\ i=1,2,3,4,5 \end{aligned}$$

(80)

For the expected post-warranty cost per cycle, we have:

$$ETC_C=\sum_{i=1}^{5}E\left(C_{PW}\mid S_i\right)\times p(S_i) \tag{81}$$

Finally, ETC for the consumer, which is the sum of the purchase and post-warranty costs, is obtained as follows:

$$TC_C=P+ETC_C \tag{82}$$

4.5. Proposed Model

By considering the manufacturer's profit and consumer's costs per unit as objective functions, the model is proposed as follows:

$$Max\ TP_M=P-ETC_M$$

$$Min\ TC_C=P+ETC_C \tag{83}$$

$$s.t:$$

$$1\le n\le n' \tag{83.1}$$

$$ARL_0>ARL_l \tag{83.2}$$

$$ARL_1<ARL_u \tag{83.3}$$

$k \geq k'$ (83.4)

$0 \leq m \leq M$ (83.5)

$\varpi_{\min} < \varpi < \varpi_{\max}$ (83.6)

$P_{\min} < P < P_{\max}$ (83.7)

$n, k \in N^{+}, h, L, \varpi, P > 0, m \in \text{int}$ (83.8)

where the sample size should not exceed n' for economic reasons. To reduce the number of false alarms, ARL_0 should be greater than ARL_l. To instantly detect the shift, ARL_1 should be smaller than ARL_u. To guarantee the continuity of the process, the number of samplings in a production cycle must be greater than k'. For the level of PM implementation during the product life, the lower limit indicates that PM is not performed, and the upper limit indicates the PM implementation to restore the initial condition. For the design variable, while the lower bound implies higher product reliability, the upper bound indicates an acceptable level of reliability. To justify the price of competitors and the market, the product's price cannot exceed P_{max}.

5. Solution Procedure

The purpose is to find the optimal decision variables ($n,h,m,k,\varpi,P,L,r_1,r_2$) by optimizing the manufacturer's profit and consumer's costs. To convert the model into a single-objective model, we use a GP method. The complexities of the proposed model prevent it from being solved by exact methods. Using meta-heuristic algorithms in this situation helps to discover near-optimal solutions in a reasonable time. Among those, the genetic algorithm [23], and the particle swarm optimization (PSO) algorithm [33] have typically been used to solve similar models. We provide a solution procedure using PSO due to its unique search mechanism, simplicity of concepts, computational efficiency, and ease of implementation [9, 12, 34].

5.1. Revised multi-choice GP (RMCGP) method

GP, as an efficient technique for multi-objective decision-making, minimizes the deviation between the objectives and the optimal solution. In multi-choice GP (MCGP), some multiplicative binary variables are considered to facilitate the modeling on the right-hand side of constraints. RMCGP can be easily understood by practitioners because of removing multiplicative binary variables [35]. Moreover, the revised version is linear programming, while MCGP is integer programming (see [36] as an application of RMCGP). In RMCGP, a goal value (r_i) is determined by separate optimization of each objective function (it can also be obtained using historical data). Also, the upper limit ($g_{i,max}$) and lower limit ($g_{i,min}$) are set for r_i to create an interval instead of a fixed goal value. Then, the total of undesirable deviations (TUDs) from the goals is minimized. The proposed model is rewritten as follows:

$$
\begin{aligned}
&\min \ d_1^- + e_1^- + d_2^+ + e_2^+ \\
&s.t: \\
&\frac{TP_M}{g_{1,\max}} - d_1^+ + d_1^- = \frac{r_1}{g_{1,\max}} \\
&\frac{r_1}{g_{1,\max}} - e_1^+ + e_1^- = 1 \\
&g_{1,\min} \le r_1 \le g_{1,\max} \\
&\frac{TC_C}{g_{2,\min}} - d_2^+ + d_2^- = \frac{r_2}{g_{2,\min}} \\
&\frac{r_2}{g_{2,\min}} - e_2^+ + e_2^- = 1 \\
&g_{2,\min} \le r_2 \le g_{2,\max} \\
&d_i^+, d_i^-, e_i^+, e_i^- \ge 0 \qquad i = 1,2
\end{aligned}
\tag{84}
$$

where d_i^+ and d_i^- are respectively the positive and negative deviations from r_i for the i^{th} objective function. e_i^+ and e_i^- are respectively the positive and negative deviations of r_i from $g_{i,max}$, and $g_{i,min}$. In the first objective function, the undesirable deviation is that d_1^- takes a positive value. We considered limits $g_{1,min}$, and $g_{1,max}$ for the variability of this goal value. For the manufacturer, the more r_1 moves towards $g_{1,max}$ means more desirability, and the more it moves towards $g_{1,min}$ means less desirability. The undesirable deviation for r_1 is that e_1^- takes a positive value. In the second objective function, the undesirable deviation is that d_2^+ takes a positive value. The goal value r_2 can also vary between $g_{2,min}$, and $g_{2,max}$. Thus, the undesirable deviation for r_2 is that e_2^+ takes a positive value. Therefore, d_1^-, e_1^-, d_2^+, and e_2^+ must be present in the above objective function.

5.2. Solution algorithm

PSO considers a particle as a solution in feasible space. Each particle is represented by its position $X_i^t=[n,h,m,k,\varpi,P,L,r_1,r_2]$ and velocity V_i^t in the iteration t. Only n, m, and k are discrete variables. The primary value of continuous decision variables is randomly generated according to a uniform distribution. For three discrete variables, three random numbers of R_1, R_2, and R_3 are generated according to $U\sim(0,1)$. The primary values of those are created as follows:

$$n = \min((n_{\min} + \text{floor}((n_{\max} - n_{\min} + 1) \times R_1)), \text{n}_{\max}) \tag{85}$$

$$k = \min((k_{\min} + \text{floor}((k_{\max} - k_{\min} + 1) \times R_2)), k_{\max}) \tag{86}$$

$$m = \min((m_{\min} + \text{floor}((m_{\max} - m_{\min} + 1) \times R_3)), m_{\max}) \tag{87}$$

According to Figure 3, the PSO parameters are initially defined, including inertia weight, recognition, and social learning factors, population size (N_p), and iteration number. The positions and velocities of N_p particles are primarily set. At this step, the personal best (*pbest*) of each particle is set to its position (*pbest* is the best value experienced by that particle). The global best (*gbest*) is set to the best solution observed so far by considering all *pbest* values. Next, the searching process is continued for some iterations by updating the positions and velocities, and consequently, the *pbest* and *gbest* values. Finally, it checks to reach a preset number of iterations to stop. If so, the latest *gbest* is the best solution. Otherwise, the procedure continues from the second step.

{Please insert Figure 3 about here.}

6. Experimental results

First, a numerical example is presented. Then, three studies are conducted to evaluate the proposed model. Finally, the effect of some parameters on results is investigated through sensitivity analysis.

6.1. Numerical example

The information of an industrial example is presented (refer to [4, 11, 20 37]). The industrial equipment manufacturer intends to market his product with an FMRrW policy. The number of working days is 125. The decision-maker has set the acceptable lower limit for manufacturer profit at $80 and the acceptable upper limit for consumer costs at $480. Assume that the production process starts in ICS. The occurrence of AC leads to a shift. Table 3 shows the complete information about this example. To reduce the trend of product failure, the manufacturer implements a periodically planned PM, including five discrete levels at Δ=0.33 a year in the warranty period. After performing PM with any level of m=0,1,…,5, the corresponding reduction in product life is obtained by $\delta'(m)$. The proposed model is rewritten as follows:

$$Max\ TP_M = P - ETC_M$$
$$Min\ TC_C = P + ETC_C$$
$$s.t:$$
$$1 \le n \le 20$$
$$ARL_0 > 100$$
$$ARL_1 < 10$$
$$k \ge 3$$
$$0 \le m \le 5$$
$$0.01 < \varpi < 0.4$$
$$190 < P < 240$$
$$n, k \in N^+, \quad h, L, \varpi, P > 0, \quad m \in \text{int}$$

After converting the model into a single-objective one, we solved it by PSO, and obtained the following results (TUDs was 0.18):

$$\{n^*, h^*, m^*, k^*, \varpi^*, P^*, L^*, r_1^*, r_2^*\} = \{7, 1.2, 3, 25, 0.22, 225.57, 21.4, 93, 392\},$$
$$TP_M{}^* = 93.7, \quad TC_C{}^* = 392.56.$$

{Please insert Table 3 about here.}

6.2. Comparative studies

First, the performance of the NCS chart is compared with the X-bar and R charts. Then, simultaneous optimization based on the manufacturer and consumer perspectives is compared with single-objective optimization based on each separate perspective. Finally, the importance of integrating presale and postsale costs is examined by comparing the proposed model with a model in which those costs are minimized separately. Extensive comparisons are presented by generating an orthogonal-array Taguchi design. Table 4 shows the corresponding level planning, and Table 5 includes sixteen trials (L_{16}) of input parameters (C_v, k_1, k_2, W, C_h). Each trial is solved five times, and the best result is recorded.

{Please insert Table 4 about here.}
{Please insert Table 5 about here.}

6.2.1. Comparing the performance of control charts

Assume the proposed model using an NCS chart and X-bar and R charts as model (A) and model (B), respectively. We equate the process costs in ICS for both monitoring techniques and calculate the process costs in OCS. First, we solve the trials with the model (A). In model (B), we set h, k, m, ϖ, P, r_1, and r_2 equal to their optimal values in the model (A) and try to equalize the Type-I error in both models so that the same costs are obtained for them in ICS. Then, we determine the optimal values of n, $L_{X\text{-}bar}$, and L_R so that the total cost of the model (B) is minimized. In the design of X-bar and R charts, the probability of combined Type-I error, combined power, and combined Type-II error are equal to:

$$\alpha = \alpha_{\bar{x}} + \alpha_R - \alpha_{\bar{x}}.\alpha_R$$

$$p = p_{\bar{x}} + p_R - p_{\bar{x}}.p_R$$

$$\beta = \beta_{\bar{x}}.\beta_R$$

The NCS chart shows more efficiency in all trials (Table 6). While the expected ICS costs in both models have almost been equaled, the expected OCS costs in model (B) were higher. The NCS chart reduces the percentage of OCT by discovering the OCS faster. Therefore, fewer non-conforming items are produced. As a result, an average of 19.04% improvement in the QLC and 53.66% improvement in TUDs has been achieved using the model (A). Figures 4 to 6 depict these facts. Moreover, the performance of the two monitoring techniques is compared in terms of different shifts in the mean and standard deviation. Figures 7 and 8 indicate the better performance of the NCS chart in smaller shifts. As the shift increases, the performance difference between the two techniques decreases. In Figure 9, the improvement rate for each fixed δ decreases with the increase of Ψ. Also, an increase in δ relatively decreases the improvement rate.

{Please insert Table 6 about here.}
{Please insert Figure 4 about here.}
{Please insert Figure 5 about here.}

{Please insert Figure 6 about here.}
{Please insert Figure 7 about here.}
{Please insert Figure 8 about here.}
{Please insert Figure 9 about here.}

6.2.2. Comparison of bi-objective optimization with single-objective optimizations

In this regard, the following models are introduced:

- Model (A) is the same proposed model,
- Model (M) aims at maximizing the manufacturer's profit, and
- Model (C) aims at minimizing consumer costs.

According to Table 7, model (A) has better performance. In model (M), the maximum selling price is determined, and in model (C), the minimum P is determined. In comparison, its value in model (A) is between the obtained values in single-objective models. To gain more profit, the manufacturer uses the lowest LMT. Therefore, the design variable ϖ, the inverse of the LMT, reaches its upper limit. In contrast, since the consumer tends to buy a higher quality product, ϖ reaches its lower limit. The model (A) is closer to reality since ϖ has a value between its upper and lower limits.

By replacing the optimal variables of the model (M) and model (C) in the model (A), 56.40% and 83.36% improvement in the TUDs are obtained, respectively. The highest and lowest improvement in the TUDs in model (A) compared to model (M) is observed in the 9th and 16th trials, respectively. By decreasing W and k_2 in the 16th trial compared to the 9th trial, manufacturer costs decrease, and profits increase. The manufacturer offers the product at a more reasonable price, and the consumer incurs a lower cost. Therefore, due to the increase in the manufacturer's profit and the decrease in the consumer's cost, the deviation of these functions has decreased compared to the 9th trial. Compared to the consumer's perspective, the greatest and least improvement in the TUDs is in the second and third trials, respectively. With the increase of k_1, k_2, and C_h in the third trial compared to the second trial, the manufacturer's costs and the selling price increase. With the increase of W, a lower cost is imposed on the consumer in the post-warranty period. Therefore, with the increase in the manufacturer's profit and the decrease in the consumer's costs, the deviation of these functions decreases. Figure 10 depicts the TUDs.

{Please insert Table 7 about here.}
{Please insert Figure 10 about here.}

6.2.3. Comparison of models with and without integrating pre-sale and post-sale costs

To investigate the effect of integrating the pre-sale and post-sale costs in model (A), comparisons are made with model (D), in which the post-sale costs are ignored. In Table 8, the TUDs in the model (A) have improved by an average of 21.28%. Model (A) is more sensitive about ET_{out}, and, the ratio of ET_{in} to production time increases. The reason is that model (A) considers the warranty cost and QLC. Since QLC is a decreasing function of the mentioned ratio, it has been accompanied by a decrease in all trials of the model

(A). $ETC_{pre\text{-}sale}$ values in the model (A) indicate a decreasing trend. QLC has had the greatest impact on it. Moreover, since model (A) considers $ETC_{post\text{-}sale}$, reductions are obtained in all trials. According to Equation (76), ETC_M (A) is decreased compared to ETC_M (D). The reduction in ETC_M and the increase in the sale price have increased the manufacturer's profit in the model (A). Despite the 10.70% increase in TC_c (A) due to the increase in post-warranty costs, the profit is increased by about 39.16% using the proposed model by integrating pre-sale and post-sale costs.

{Please insert Table 8 about here.}

6.3. Sensitivity analysis

We investigate the effect of C_v, k_1, k_2, W, and C_h on the objective functions through sensitivity analysis. Based on the generated sixteen trials according to the L_{16} design, the average profit and cost objective functions for each level were recorded in Tables 8 and 9, respectively. For example, 101.25 in Table 9 indicates the average profit in trials 1, 2, 3, and 4, where C_v is at level 1. The Delta line shows the difference between the highest and lowest profits for each parameter. By ranking it, the influence of the parameters on the profit can be seen in the Rank line. C_v and C_h have the greatest and least impact on the manufacturer's profit, respectively (see Table 9). Similarly, W has the most, and k_1 has the least effect on the consumer's cost (see Table 10).

In Table 9, the increase in k_1 has increased the profit because the manufacturer increases the price to compensate for the incurred cost. In Table 10, increasing the warranty period reduces the consumer's cost, and higher k_2 increases it. Since the post-warranty period decreases with the increase of W, the number of failures and the need to perform PM are reduced, and the customer bears a lower cost. The increase in k_2 increases the manufacturer's costs. The manufacturer compensates for it by increasing the price. Therefore, the consumer's cost increases. To investigate the effect of five parameters on the improvement in TUDs, the arithmetic average of the improvement in all four levels of those parameters has been calculated (see Table 11). Accordingly, k_2 is the most effective parameter on TUDs, while C_h has almost no effect.

{Please insert Table 9 about here.}
{Please insert Table 10 about here.}
{Please insert Table 11 about here.}

7. Discussion

Increasing competition in the market and paying attention to the warranty issue have motivated most manufacturers to supply high-quality products while reducing production costs. Consumers are also looking for products with a warranty to minimize maintenance and repair costs. Besides, production processes are always subject to breakdowns and failures. Such disturbances can lead to increased costs and reduced quantity and quality in production. Any production system that faces disruptions, is looking for high-quality and cost-effective production, and intends to supply the product with a warranty can find this research helpful.

Salmasnia et al. [31] modeled the triple concepts and used an adaptive NCS chart for monitoring. Accordingly, integrating the triple concepts significantly reduced the costs, and the adaptive NCS chart outperformed the fixed-parameter chart. Salmasnia et al. [32] applied another adaptive NCS chart in a similar model while considering random failures and multiple ACs. Accordingly, using the NCS chart decreased the cost function, and considering the system failure improved the financial terms. However, none considered warranty, pricing, and LMT. We took the importance of integrating the triple concepts and using the NCS chart for granted. By considering pricing and LMT, we modeled the quadruple concepts, including a warranty policy. We compared models with and without integrating pre-sale and post-sale costs. Despite the increase in post-warranty costs, the profit was significantly increased when integrating pre-sale and post-sale costs. Moreover, simultaneous optimization based on the manufacturer and consumer perspectives caused performance improvement and the least TUDs.

While maintaining both manufacturer and consumer perspectives, the proposed model gives the decision-maker the advantage of reaching a selling price at a reasonable LMT. Considering post-warranty costs can play a key role in profit maximization. Moreover, the NCS chart reflects significant benefits, which can help practitioners reduce the QLC and TUDs.

8. Conclusions

The goal of was to maximize the manufacturer's profit by minimizing the total pre-sale and post-sale costs, subject to statistical constraints. To reduce these costs and increase the quality, we considered LMT, MP, and SPM. To further conform the product to customer expectations, the mean and variance were monitored by an NCS chart. We used the FMRrW policy and considered the post-warranty and customer-related costs. To convert the bi-objective model into a single-objective one, we used the RMCGP method. We solved the model by the PSO due to its high complexity.

According to the results, the better performance of the NCS chart was noticeable in smaller shifts. QLC and TUDs were considerably improved. Considering both manufacturer and consumer perspectives caused performance improvement and the least TUDs. Integrating presale and postsale costs significantly increased the total profit. The variable cost of sampling, warranty period, and constant coefficients of QLC in OCS respectively had a significant effect on the first and second objective functions and the TUDs.

This was the first attempt to integrate the quadruple concepts in a unified model. We limited the monitoring techniques to the NCS chart, while using adaptive control charts may increase the performance and reduce the costs. Another restriction was the type of warranty policy, while there is a variety of warranty policies. Instead of the periodic type, a non-periodic MP can be implemented since the failure rate and associated costs increase over time. If the demand rate is higher than the production rate, the system should take an appropriate deficit policy to control inventory levels. Alternative distribution, instead of Weibull, may well express the failure process.

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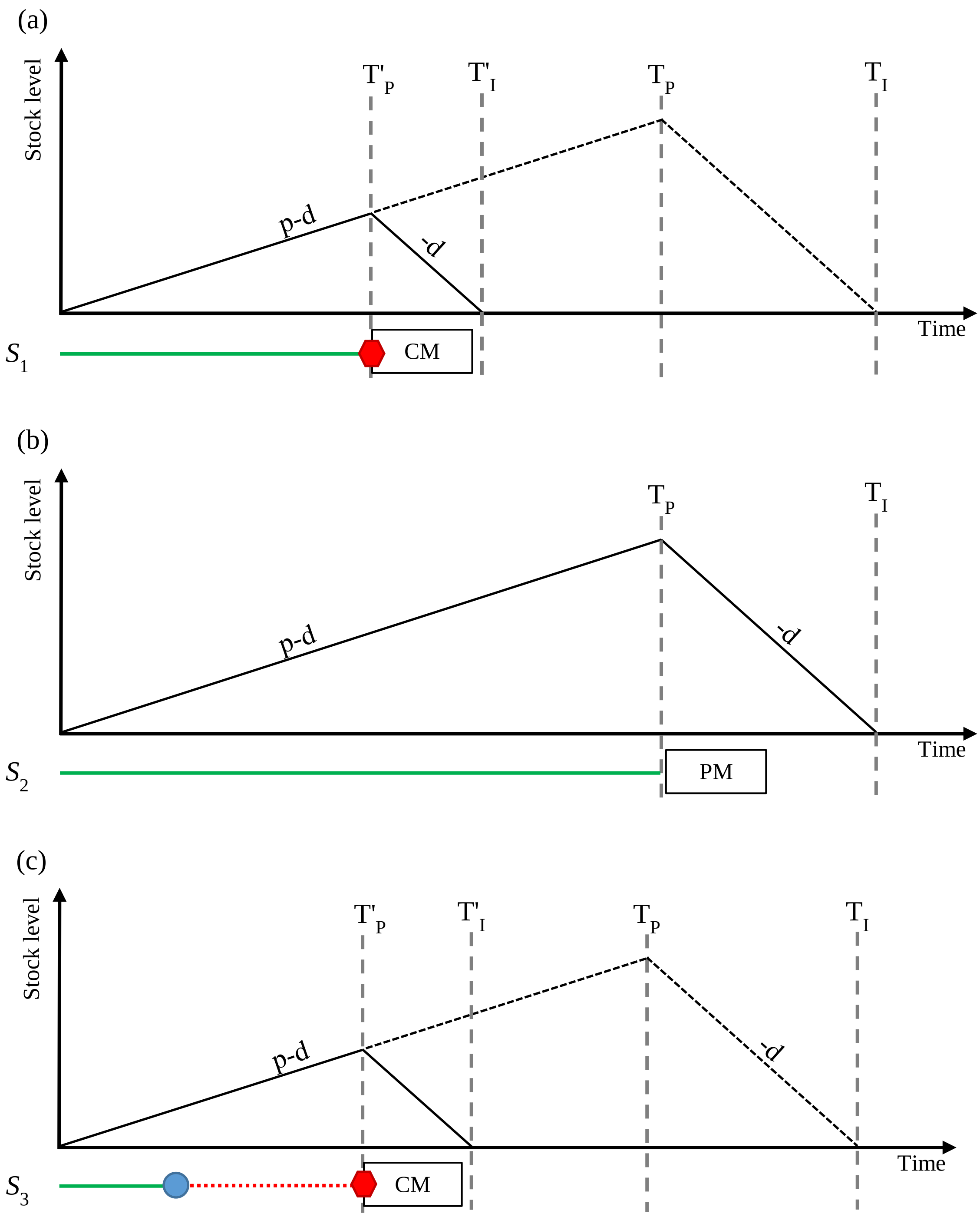
(a)
Stock level
T'P
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TP
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-d
Time
S1
CM
(b)
Stock level
TP
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Time
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(c)
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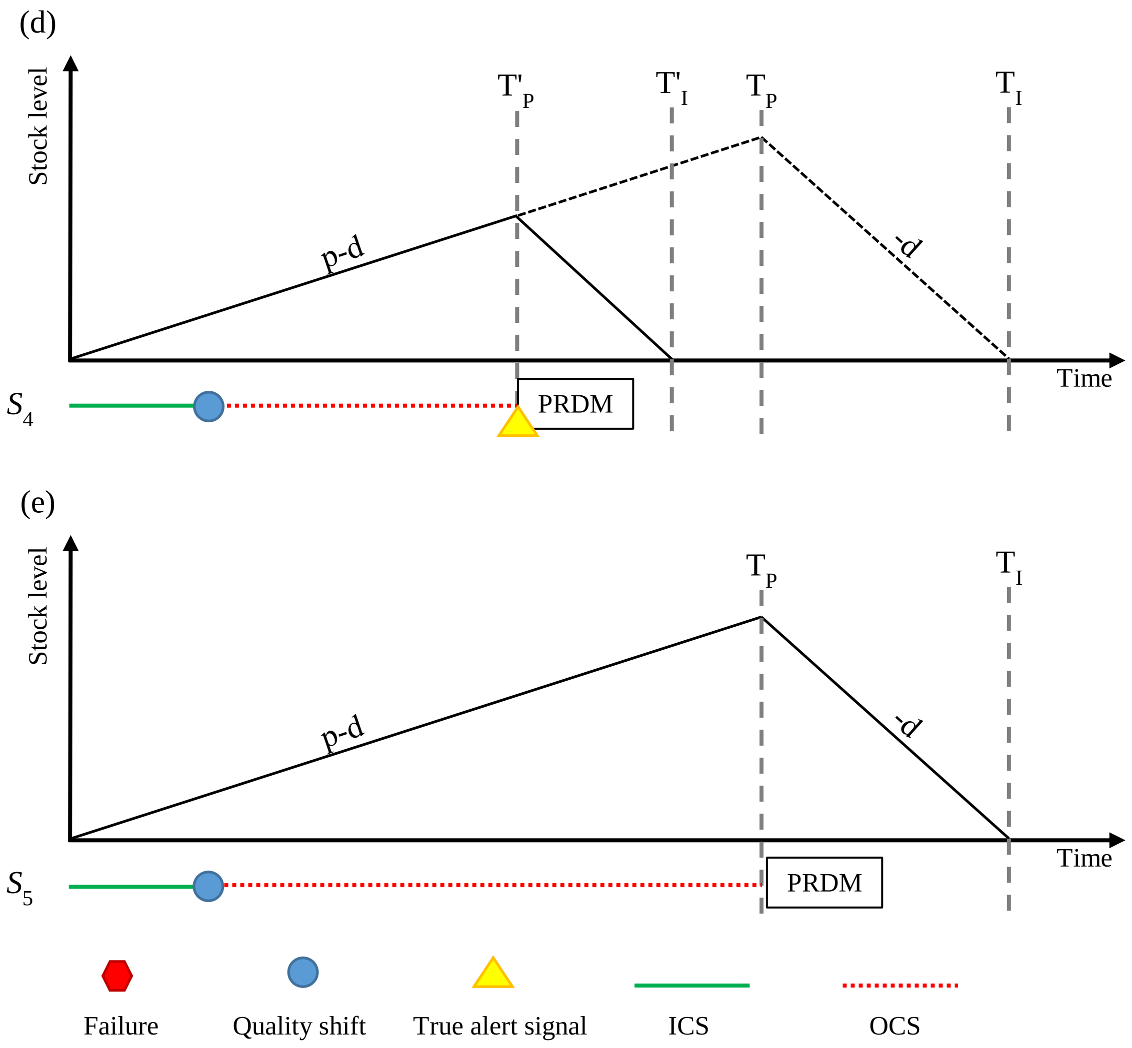

Figure 1. Graphical representation of five scenarios along with their inventory charts (T'_P and T'_I are the real manufacturing cycle time and the actual inventory cycle time, respectively)

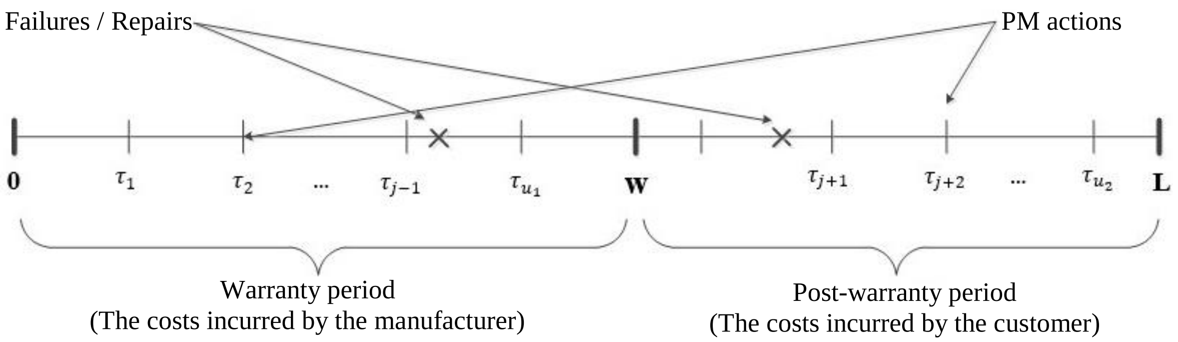

Figure 2. Distribution of costs during the warranty and post-warranty periods

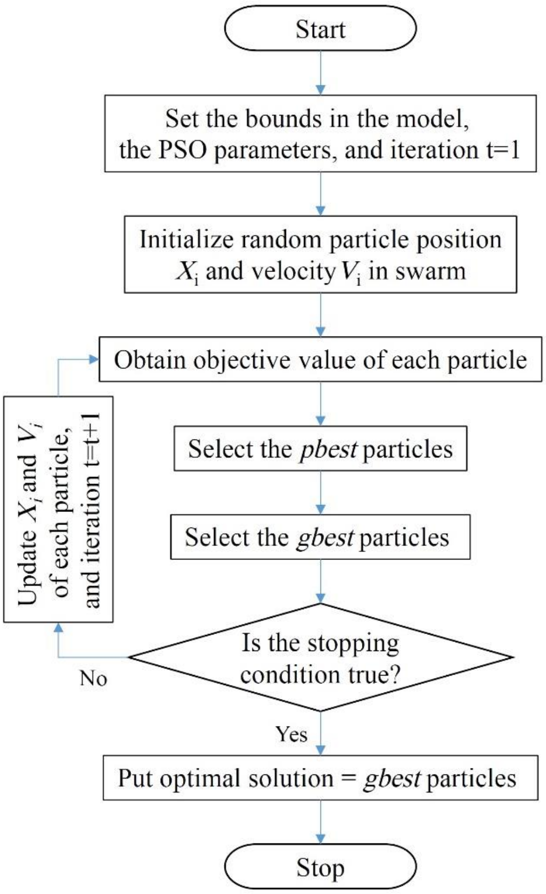

Figure 3. Graphical representation of PSO algorithm

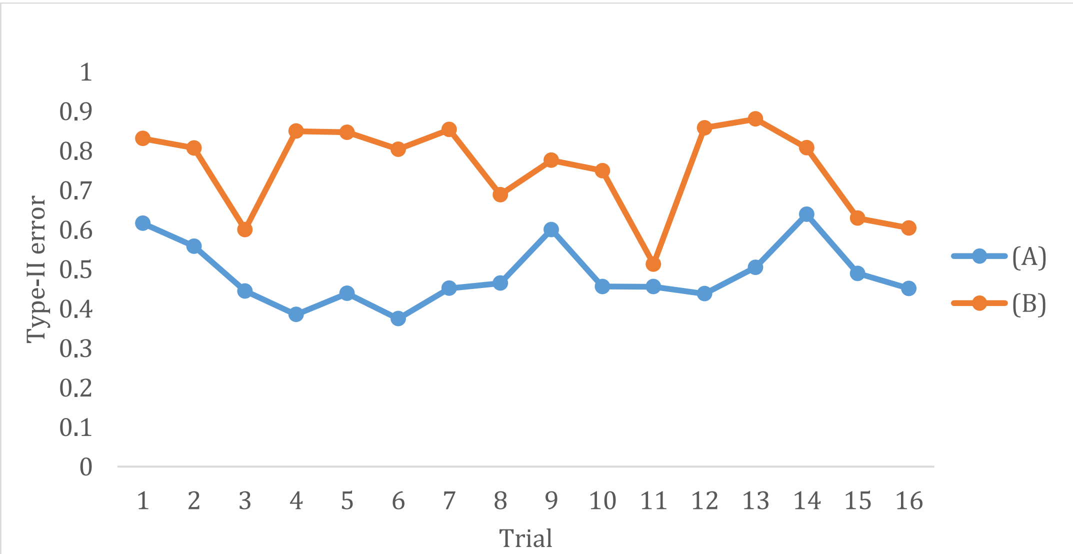

Figure 4. Comparison of Type-II error between model A and model B

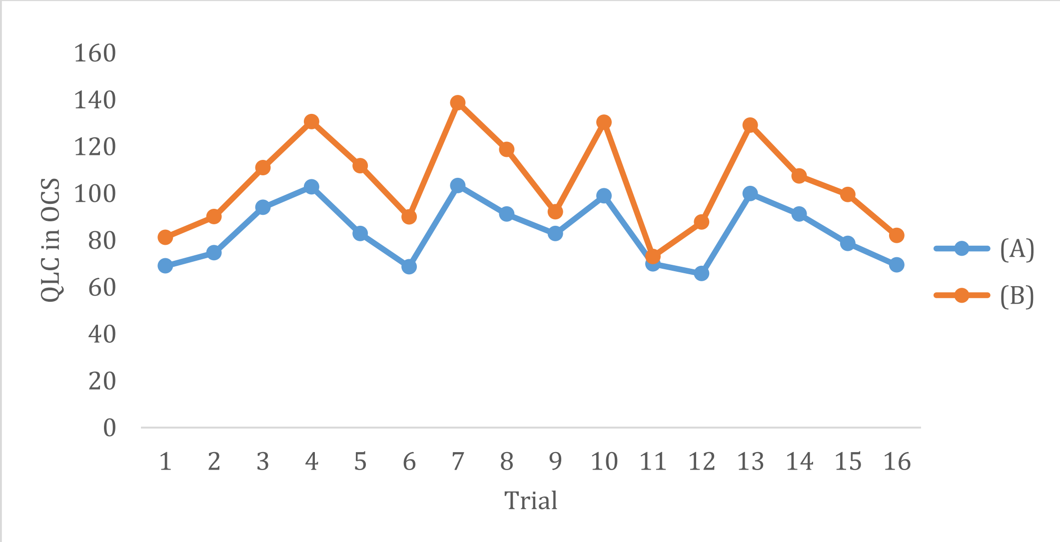

Figure 5. Comparing the QLC in OCS between model A and model B

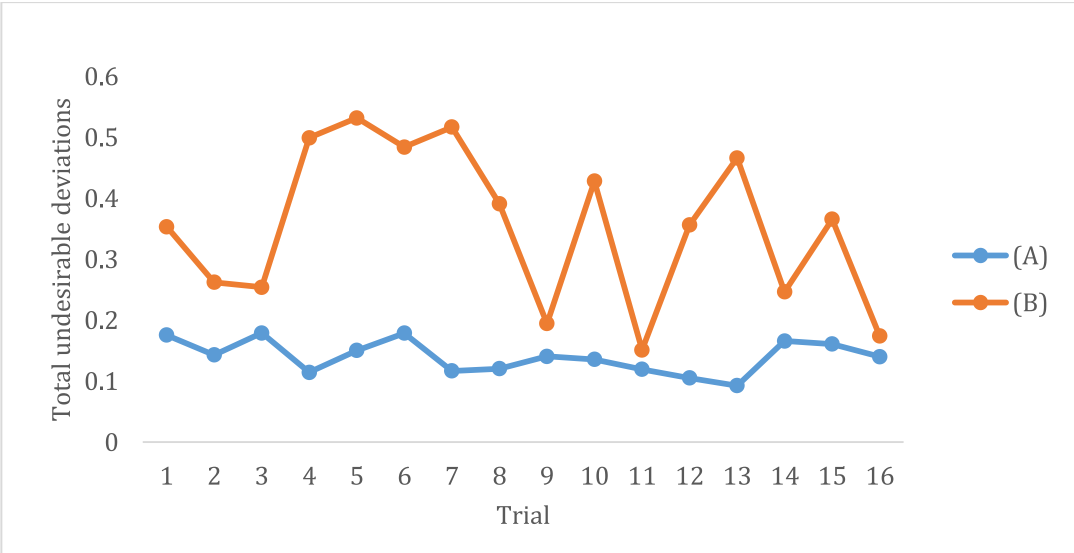

Figure 6. Comparison of total undesirable deviations between model A and model B

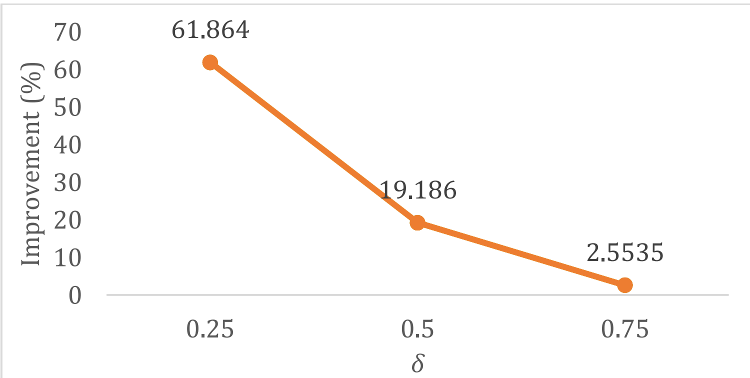

Figure 7. Average improvement rate (in total cost using NCS chart) versus the shift in the mean

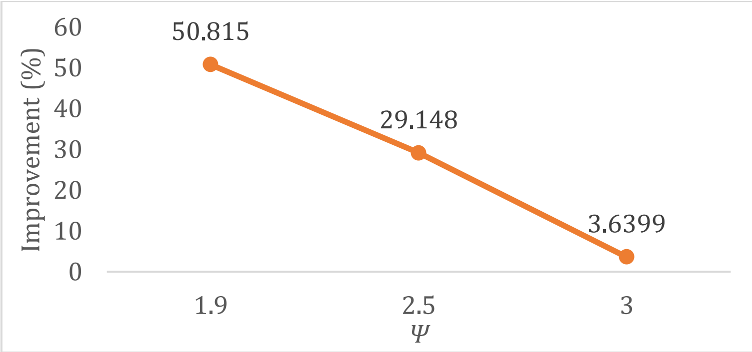

Figure 8. Average improvement rate (in total cost using NCS chart) versus the shift in the standard deviation

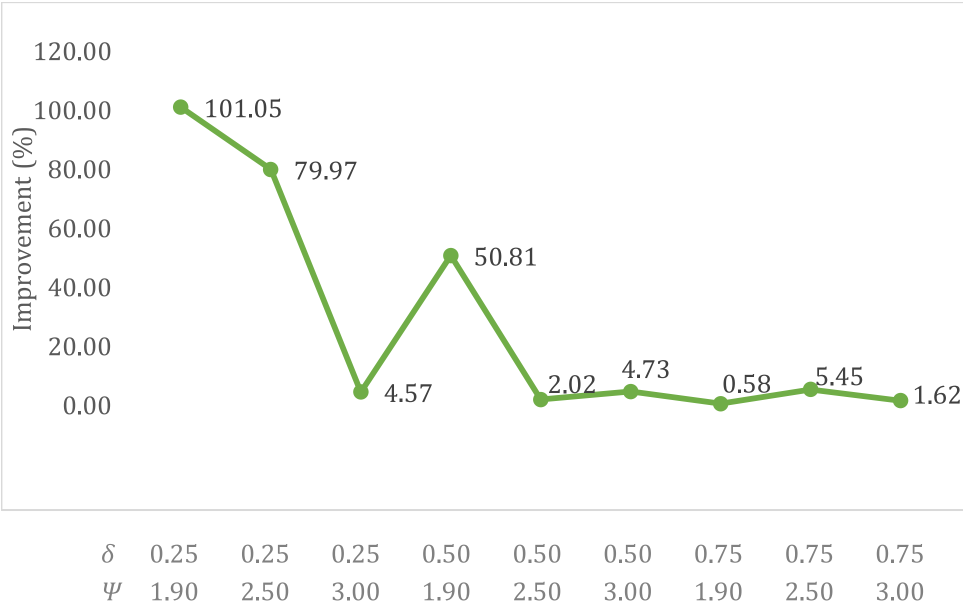

Figure 9. Improvement rate (in total cost using NCS chart) versus shifts in mean and standard deviation

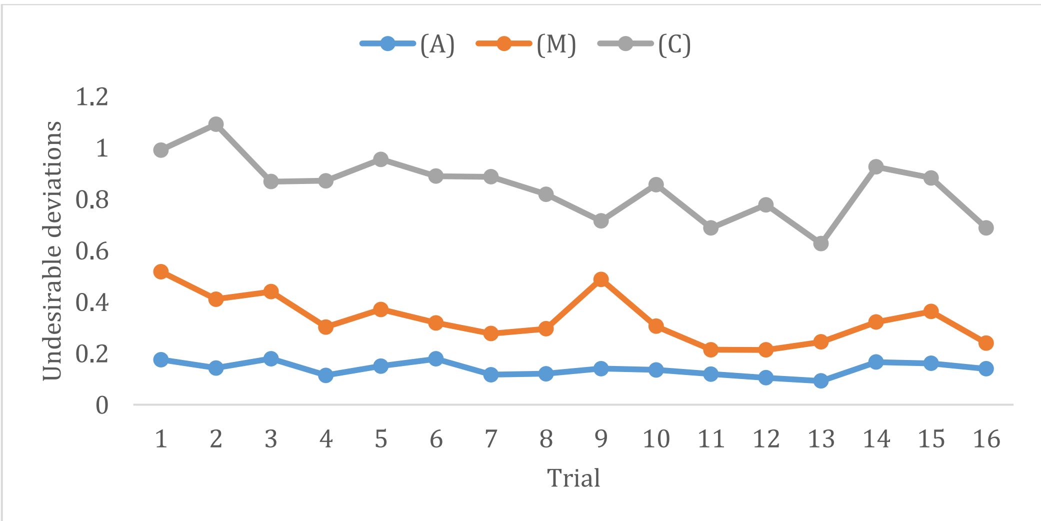

Figure 10. Comparison of total undesirable deviations among models A, M and C

Table 1. Summarized literature review

Paper	Integration				Pricing	Technology/Reliability	Failure			Chart		Shift		Design	
	Inventory	Maintenance	SPM	Warranty			Process	Product	Distribution	X-bar & R	NCS	Mean	Variance	ED	ESD
[4]	✓	✓	-	FRrW	-	-	✓	✓	Wbl[2]	-	-	-	-	-	-
[5]	✓	-	-	FRtW	✓	-	-	-	-	-	-	-	-	-	-
[6]	✓	-		NRFMRrW	✓	✓	-	✓	Wbl	-	-	-	-	-	-
[7]	✓	✓	-	FRrRtW	✓	-	-	✓	Exp[3]	-	-	-	-	-	-
[8]	-	-	-	NRFRtW	✓	-	-	✓	Exp	-	-	-	-	-	-
[9]	✓	-	-	-	✓	-	-	✓	Wbl	-	-	-	-	-	-
[10]	-	✓	-	TDEW[1]	✓	✓	-	✓	Wbl	-	-	-	-	-	-
[11]	-	✓	-	FRtW	✓	✓	-	✓	Wbl	-	-	-	-	-	-
[13]	-	-	-	FRrRtW	✓	✓	-	✓	Exp	-	-	-	-	-	-
[14]	-	✓	-	-	-	✓	-	✓	Exp	-	-	-	-	-	-
[15]	-	✓	-	NRFRtW	-	-	-	✓	Wbl	-	-	-	-	-	-
[12]	-	✓	-	FMRrW	✓	✓	-	✓	Wbl	-	-	-	-	-	-
[16]	-	✓	-	TDEW	✓	✓	-	✓	Wbl	-	-	-	-	-	-
[17]	✓	-	-	FRtW	✓	-	-	✓	Wbl	-	-	-	-	-	-
[18]	-	-	-	FRrRtW	✓	-	-	✓	Exp	-	-	-	-	-	-
[19]	-	✓	-	PRW	-	-	-	✓	Wbl	-	-	-	-	-	-
[20]	-	✓	-	NRFRtW	-	-	-	✓	Wbl	-	-	-	-	-	-
[21]	-	✓	-	NRW	-	-	-	✓	Wbl	-	-	-	-	-	-
[23]	-	-	-	FRtW/PRW	✓	-	-	✓	Wbl	-	-	-	-	-	-
[22]	-	✓	-	FRtW	-	-	-	✓	Wbl	-	-	-	-	-	-
[24]	-	✓	-	RrW	-	-	-	✓	Wbl	-	-	-	-	-	-
[25]	-	✓	-	FRrW	-	-	-	✓	NHPP[4]	-	-	-	-	-	-
[26]	-	-	✓	-	-	-	✓	-	Wbl	✓	-	✓	✓	✓	-
[27]	-	-	✓	-	-	-	✓	-	-	-	✓	✓	✓	-	-
[28]	-	-	✓	-	-	-	✓	-	-	-	✓	✓	✓	-	-
[29]	-	-	✓	-	-	-	✓	-	-	-	✓	✓	✓	-	-
[30]	-	-	✓	-	-	-	✓	-	Exp	-	✓	✓	✓	✓	-
[31]	✓	✓	✓	-	-	-	✓	-	Wbl	-	✓	✓	✓	-	✓
[32]	✓	✓	✓	-	-	-	✓	-	Wbl	-	✓	✓	✓	-	✓
This paper	✓	✓	✓	FMRrW	✓	✓	✓	✓	Wbl	-	✓	✓	✓	-	✓

1. Two-dimensional extended warranty
2. Weibull
3.Exponential
4. Non-homogeneous Poisson Process

Table 2. Notations

Notation	Description
Abbreviations	
AC	Assignable cause
ARL	Average run length
CM	Corrective maintenance
ED	Economic design
EPQ	Economic production quantity
ESD	Economic-statistical design
ETC	Expected total cost
FMRrW	Free minimal repair warranty
GP	Goal programming
ICS	In-control state
ICT	In-control time
IHC	Inventory holding cost
LMT	Level of manufacturing technology
MP	Maintenance policy
NCS	Non-central chi-square
OCS	Out-of-control state
OCT	Out-of-control time
PM	Preventive maintenance
PRDM	Predictive maintenance
PSO	Particle swarm optimization
QLC	Quality loss cost

RMCGP	Revised multi-choice GP
SC	Set-up cost
SPM	Statistical process monitoring
TUDs	Total of undesirable deviations
Decision variables	
n	The sample size
h	The sampling interval
L	The coefficient of control limit for NCS chart
k	The number of sampling per manufacturing cycle
m	PM level ($0 \leq m \leq M$)
ϖ	Design variable that indicates the reliability of the product
P	Sale price of the product
r_1, r_2	The goal of manufacturer and consumer functions, respectively
Indicator	
i	Index of scenarios (S_i indicates the i^{th} scenario)
j	Index of *PM* level implemented on the product in the warranty period
z	Index of sampling interval
Time parameters	
T_p, T_I	The production run length and the inventory cycle length
T_{in}, T_{out}	The time that the process is in ICS and OCS in the manufacturing cycle, respectively
Cost parameters	
C_r, C_{rpw}	The cost of each minimal repair actions over the interval $[0,W)$ and $[W, LC)$, respectively
Cpm, Cpm_{pw}	The cost of PM activity on the product over the interval $[0,W)$ and $[W, LC)$, respectively
$C_{CM}=\rho_0+\rho_1\xi$	The CM cost in both ICS and OCS where $\rho_0>0$, $\rho_1\geq 0$ and ξ denotes the shift detection delay time (consider $\xi=0$ in ICS)
$C_{PRDM}=\rho'_0+\rho'_1\xi$	The PRDM cost where $\rho'_0>0$, $\rho'_1\geq 0$ and ξ denotes the shift detection delay time
C_{PM}	The cost of PM activity on the process
$C_{falsealarm}$	The cost of false alarm
C_f	The fixed cost of sampling
C_v	The variable cost of sampling
C_{in}, C_{out}	The QLC values per unit item in ICS and OCS, respectively
k_1, k_2	The constant coefficients of QLC in ICS and OCS, respectively
C_p	The production cost
C_h	The IHC per item per in time unit
A	The SC
$E(IHC\|S_i)$	The expected IHC per unit item when scenario i occurs
$E(SC\|S_i)$	The expected SC per unit item when scenario i occurs
$E(C_Q\|S_i)$	The expected QLC per unit item when scenario i occurs
$E(C_{sampling}\|S_i)$	The expected sampling cost per unit item when scenario i occurs
$E(C_M\|S_i)$	The expected maintenance cost per unit item when scenario i occurs
$E(C_W\|S_i)$	The expected warranty cost per unit item when scenario i occurs
$E(C_{PW}\|S_i)$	The expected post-warranty cost per unit item when scenario i occurs
ETC_M	The ETC of manufacturer
ETC_C	The expected post-warranty cost per cycle for the consumer
$ETC_{pre\text{-}sale}$	The expected total pre-sale cost per unit item
$ETC_{post\text{-}sale}$	The expected total post-sale (warranty) cost per unit item
TP_M	The expected total profit of manufacturer
TC_C	The ETC of consumer
Process parameters	
D	The annual demand
d	The daily demand rate
p	The daily production rate ($p>d$)
δ	The size of the shift in the process mean
Ψ	The size of the shift in the process variance
R_{in}, R_{out}	The expected number of taken samples in ICS and OCS, respectively
ARL_0, ARL_1	The average run length during in ICS and OCS, respectively

ARL_l, ARL_u	The lower bound of ARL_0, the upper bound of ARL_1
$E(N\|S_i)$	The expected number of produced non-conforming items when scenario i occurs
$\alpha_i(T_p)$	The fraction of the produced non-conforming items in i^{th} scenario
x	The random variable of time-to-shift
$f(x)$	Time-to- shift probability density function (PDF)
$F(x)$	Time-to- shift cumulative distribution function (CDF)
y_1, y_2	The random variables of time-to-failure in ICS and OCS, respectively
$g_1(y_1)$, $G_1(y_1)$	Time-to-failure PDF and CDF when the process is in ICS
$g_2(y_2)$, $G_2(y_2)$	Time-to-failure PDF and CDF when the process is in OCS
θ_1, θ_2	The fraction of produced nonconforming items in ICS and OCS, respectively ($\theta_2>\theta_1$)
$r_{01}(t,\varpi)$, $r_{02}(t,\varpi)$	The failure rate of conforming and non-conforming items under no PM strategy $r_{02}(t)>>r_{01}(t)$
$r_{m1}(t,\varpi)$, $r_{m2}(t,\varpi)$	The failure rate of conforming and non-conforming items under PM level of m
u_1, u_2	The number of PM actions during the interval $[0,W)$ and $[W, LC)$, respectively
α_p, α_r	profit margin associated with each PM and repair actions performed during the post-warranty period, respectively
τ_j	The time of implementing the PM activity
$v_j(t)$	Virtual age of the item at time t
Δ	Time interval between two consecutive PM activities during the warranty period
$\delta'(m)$	Age reduction factor at PM level of m
p_z	The occurrence probability of the shift in the z^{th} sampling interval
l_z	The expected ICT given that shift occurs within z^{th} sampling interval
$P(S_i)$	The probability of happening i^{th} scenario
LC	Product's lifecycle
W	The warranty period

Table 3. Nominal values of the input parameters

Parameter	k_1	k_2	C_{PM}	C_f	C_v	ω	c_1
Value	0.8	10	240	30	5	0.005	5
Parameter	$C_{false\ alarm}$	C_r	u_1	W	M	ρ_0	ρ_1
Value	200	5	3	1	5	6000	800
Parameter	ρ'_0	ρ'_1	D	d	p	Δ	Ψ
Value	3000	400	10000	80	100	0.33	1.9
Parameter	δ	A	C_h	θ_1	θ_2	γ	η
Value	0.25	100	10	0.05	0.95	1.7	0.25
Parameter	β_0	η_0	β_1	η_1	λ_c	γ_c	λ_{nc}
Value	1.4	0.012	1.4	0.5	2	2	2
Parameter	γ_{nc}	u_2	LC	Ψ_0	Ψ_1	α_P	α_r
Value	6	21	8	15	2	0.3	0.28
Parameter	a	ϕ	ξ				
Value	1	0.1	-0.35				

Table 4. Level planning of factors for generating the trials

Factor	A	B	C	D	E
Notation	C_v	k_1	k_2	W	C_h
Level 1	5	0.8	10	1.0	10
Level 2	7	0.9	12	1.5	15
Level 3	9	1.0	14	2.0	20
Level 4	11	1.1	16	2.5	25

Table 5. The generated trials through the Taguchi L_{16} design

Trial	A	B	C	D	E
1	1	1	1	1	1
2	1	1	1	1	2
3	1	1	1	1	3
4	1	2	2	2	1
5	1	2	2	2	2
6	1	2	2	2	3
7	1	3	3	3	1
8	1	3	3	3	2
9	1	3	3	3	3
10	2	1	2	3	1
11	2	1	2	3	2
12	2	1	2	3	3
13	2	2	3	1	1
14	2	2	3	1	2
15	2	2	3	1	3
16	2	3	1	2	1

Table 6. Comparing the results of model (A) and model (B)

Trial	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	All trials
Undesirable dev. (A)	0.18	0.14	0.18	0.11	0.15	0.18	0.12	0.12	0.14	0.14	0.12	0.11	0.09	0.17	0.16	0.14	
Undesirable dev. (B)	0.35	0.26	0.25	0.50	0.53	0.48	0.52	0.39	0.20	0.43	0.15	0.36	0.47	0.25	0.37	0.17	
Improvement (%)	50.14	45.42	29.53	77.15	71.62	63.02	77.37	69.05	27.69	68.22	20.53	70.51	80.04	32.79	56.01	19.54	53.66
$\%ET_{in}$ (A)	65.61	64.98	65.74	65.57	65.73	65.56	65.74	65.56	64.25	65.75	65.75	65.73	65.19	65.55	65.22	65.74	
$\%ET_{in}$ (B)	55.67	54.12	58.23	48.48	48.34	48.78	48.70	53.93	58.93	51.69	63.66	48.45	50.17	57.00	56.62	58.25	
Improvement (%)	17.86	20.07	12.90	35.25	35.97	34.40	34.99	21.56	9.03	27.20	3.28	35.67	29.94	15.00	15.19	12.86	22.57
$\%ET_{out}$ (A)	34.39	35.02	34.26	34.43	34.27	34.44	34.26	34.44	35.75	34.25	34.25	34.27	34.81	34.45	34.78	34.26	
$\%ET_{out}$ (B)	44.33	45.88	41.77	51.52	51.66	51.22	51.30	46.07	41.07	48.31	36.34	51.55	49.83	43.00	43.38	41.75	
Improvement (%)	22.42	23.67	17.98	33.17	33.66	32.76	33.22	25.24	12.95	29.10	5.75	33.52	30.14	19.88	19.82	17.94	24.45
Type-II error (A)	0.62	0.56	0.45	0.39	0.44	0.38	0.45	0.47	0.60	0.46	0.46	0.44	0.51	0.64	0.49	0.45	
Type-II error (B)	0.83	0.81	0.60	0.85	0.85	0.80	0.85	0.69	0.78	0.75	0.51	0.86	0.88	0.81	0.63	0.60	
Improvement (%)	25.87	30.86	25.83	54.71	48.17	53.36	47.07	32.51	22.55	39.20	11.11	48.95	42.61	20.82	22.26	25.17	34.44
ARL_1 (A)	2.61	2.26	1.80	1.63	1.78	1.60	1.82	1.87	2.50	1.84	1.84	1.78	2.02	2.77	1.96	1.82	
ARL_1 (B)	5.93	5.19	2.50	6.66	6.53	5.10	6.84	3.22	4.47	4.00	2.05	7.05	8.36	5.20	2.70	2.53	
Improvement (%)	56.04	56.34	27.98	75.56	72.70	68.63	73.35	41.90	43.96	53.99	10.52	74.74	75.85	46.64	27.40	27.89	52.09
$E(C_Q)$ (A)	68.99	74.62	94.03	102.80	82.80	68.69	103.40	91.11	82.78	98.94	69.90	65.78	99.90	91.17	78.64	69.43	
$E(C_Q)$ (B)	81.27	90.06	110.90	130.60	111.70	89.96	138.70	118.80	92.09	130.40	73.04	87.74	129.20	107.40	99.49	82.01	
Improvement (%)	15.11	17.14	15.21	21.29	25.87	23.64	25.45	23.31	10.11	24.13	4.30	25.03	22.68	15.11	20.96	15.34	19.04

Table 7. Comparing the results of model (A) with model (M) and model (C)

Trial	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	All trials
Undesirable dev. (A)	0.18	0.14	0.18	0.11	0.15	0.18	0.12	0.12	0.14	0.14	0.12	0.11	0.09	0.17	0.16	0.14	
Undesirable dev. (M)	0.52	0.41	0.44	0.30	0.37	0.32	0.28	0.30	0.49	0.31	0.21	0.21	0.25	0.32	0.36	0.24	
Improvement (%)	66.02	65.21	59.32	62.38	59.30	43.89	57.76	59.12	71.11	55.56	43.93	50.70	62.04	48.45	55.65	41.91	56.40
Undesirable dev. (C)	0.99	1.09	0.87	0.87	0.96	0.89	0.89	0.82	0.72	0.86	0.69	0.78	0.63	0.93	0.88	0.69	
Improvement (%)	82.24	86.89	79.40	86.93	84.19	79.89	86.81	85.24	80.31	84.11	82.56	86.50	85.17	82.07	81.77	79.68	83.36

Table 8. Comparing the results of model (A) and model (D)

Trial	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	All trials
Undesirable dev. (A)	0.18	0.14	0.18	0.11	0.15	0.18	0.12	0.12	0.14	0.14	0.12	0.11	0.09	0.17	0.16	0.14	
Undesirable dev. (D)	0.19	0.24	0.20	0.15	0.20	0.19	0.17	0.18	0.16	0.15	0.15	0.33	0.15	0.18	0.17	0.15	
Improvement (%)	5.26	41.67	10.00	26.67	25.00	5.26	29.41	33.33	12.50	6.67	20.00	66.67	40.00	5.56	5.88	6.67	21.28
$\%ET_{in}$ (A)	65.61	64.98	65.74	65.57	65.73	65.56	65.74	65.56	64.25	65.75	65.75	65.73	65.19	65.55	65.22	65.74	
$\%ET_{in}$ (D)	63.80	60.41	59.00	51.55	52.98	53.22	58.11	62.09	63.69	58.62	58.60	52.55	57.16	58.18	57.54	56.90	
Improvement (%)	2.84	7.56	11.42	27.20	24.07	23.19	13.13	5.59	0.88	12.16	12.20	25.08	14.05	12.67	13.35	15.54	13.81
$\%ET_{out}$ (A)	34.39	35.02	34.26	34.43	34.27	34.44	34.26	34.44	35.75	34.25	34.25	34.27	34.81	34.45	34.78	34.26	
$\%ET_{out}$ (D)	36.20	39.59	41.00	48.45	47.02	46.78	41.89	37.91	36.31	41.38	41.40	47.45	42.84	41.82	42.46	43.10	
Improvement (%)	5.00	11.54	16.44	28.94	27.12	26.38	18.21	9.15	1.54	17.23	17.27	27.78	18.74	17.62	18.09	20.51	17.60
$E(C_Q)$ (A)	79.96	85.55	107.60	116.90	93.96	80.97	116.50	105.60	91.72	110.20	84.08	77.97	109.90	102.80	91.77	84.91	
$E(C_Q)$ (D)	83.38	89.68	110.70	139.50	102.60	83.04	121.20	106.80	110.70	117.80	86.22	99.17	120.50	110.20	102.10	92.56	
Improvement (%)	4.10	4.61	2.80	16.20	8.42	2.49	3.88	1.12	17.15	6.45	2.48	21.38	8.80	6.72	10.12	8.26	7.81
$ETC_{pre\text{-}sale}$ (A)	120.70	125.85	150.10	159.70	138.90	123.20	156.40	143.90	131.30	147.80	129.70	116.80	151.70	148.50	130.70	126.50	
$ETC_{pre\text{-}sale}$ (D)	123.30	130.03	153.90	189.30	147.30	126.70	162.20	146.10	151.90	156.70	132.30	147.80	164.40	155.50	141.30	136.30	
Improvement (%)	2.11	3.21	2.47	15.64	5.70	2.76	3.58	1.51	13.56	5.68	1.97	20.97	7.73	4.50	7.50	7.19	6.63
$ETC_{post\text{-}sale}$ (A)	24.29	34.13	39.88	48.88	40.30	46.68	23.57	30.74	50.89	42.23	30.00	23.42	31.70	24.52	48.34	40.22	
$ETC_{post\text{-}sale}$ (D)	27.06	34.58	45.11	51.32	45.11	51.32	27.06	34.58	56.01	48.83	32.71	27.06	34.58	25.37	56.01	48.83	
Improvement (%)	10.24	1.30	11.59	4.75	10.66	9.04	12.90	11.10	9.14	13.52	8.28	13.45	8.33	3.35	13.69	17.63	9.94
ETC_M (A)	145.0	159.98	190	208.6	179.2	169.9	180.0	174.6	182.1	190.0	159.7	140.2	183.4	173.0	179.0	166.7	
ETC_M (D)	150.4	164.61	199	240.6	192.5	178.1	189.2	180.7	207.9	205.6	165.0	174.9	199.0	180.9	197.3	185.1	
Improvement (%)	3.59	2.81	4.52	13.30	6.91	4.60	4.86	3.38	12.41	7.59	3.21	19.84	7.84	4.37	9.28	9.94	7.40
P (A)	240.00	264.98	265.00	313.60	279.20	259.90	300.00	289.60	287.10	305.00	279.00	260.20	296.40	280.00	279.00	276.70	
P (D)	230.00	238.25	279.10	307.10	252.10	228.70	288.70	263.60	292.90	283.50	257.50	273.80	288.90	268.50	256.50	264.50	
Improvement (%)	4.35	11.22	-5.05	2.12	10.75	13.64	3.91	9.86	-1.98	7.58	8.35	-4.97	2.60	4.28	8.77	4.61	5.00
TP_M (A)	95.00	105.00	100.00	105.00	100.00	90.00	120.00	115.00	105.00	115.00	120.00	120.00	113.00	107.00	100.00	110.00	
TP_M (D)	79.59	73.64	80.07	66.49	59.69	50.67	99.51	82.92	85.00	77.99	92.41	98.91	89.89	87.61	59.22	79.41	
Improvement (%)	19.36	42.59	24.89	57.92	67.53	77.62	20.59	38.69	23.53	47.45	29.86	21.32	25.71	22.13	68.86	38.52	39.16
ETC_c (A)	154.00	152.11	96.72	87.29	94.33	77.94	153.40	125.80	101.70	110.80	114.30	171.30	134.10	166.30	82.85	93.35	
ETC_c (D)	137.60	107.05	78.69	57.38	78.69	57.38	137.60	107.00	64.18	87.74	100.30	137.60	107.00	125.50	64.18	87.74	
Improvement (%)	-11.92	-42.09	-22.91	-52.13	-19.88	-35.83	-11.48	-17.57	-58.46	-26.28	-13.96	-24.49	-25.33	-32.51	-29.09	-6.39	-26.90
TC_c (A)	394.00	417.09	386.70	400.90	373.50	337.90	453.40	415.40	388.80	415.80	394.00	431.50	430.60	446.30	361.90	370.10	
TC_c (D)	367.60	345.30	357.80	364.50	330.80	286.10	426.30	370.70	357.10	371.30	357.80	411.40	395.90	394.00	320.70	352.30	
Improvement (%)	-7.18	-20.79	-8.08	-9.99	-12.91	-18.11	-6.36	-12.06	-8.88	-11.98	-10.12	-4.89	-8.76	-13.27	-12.85	-5.05	-10.70

Table 9. Effects of the specified parameters on the manufacturer's profit

Factor	A	B	C	D	E
Level 1	101.25	103.25	103.75	110.5	106.25
Level 2	106.25	104.25	107.5	113.25	110
Level 3	115	110	106.75	106.25	105.75
Level 4	107.5	113.25	113.25	100	108
Delta	13.75	10	9.5	13.25	4.25
Rank	1	3	4	2	5

Table 10. Effects of the specified parameters on the consumer's cost

Factor	A	B	C	D	E
Level 1	399.5	397	379.75	431	396.75
Level 2	395	404.25	402.75	420	407.25
Level 3	413.25	404.5	409	386.5	396.5
Level 4	402.25	404.25	425.25	372.5	409.5
Delta	18.25	7.5	45.5	58.5	13
Rank	3	5	2	1	4

Table 11. Effects of the specified parameters on the TUDs

Factor	A	B	C	D	E
Level 1	0.153058	0.139911	0.153582	0.141042	0.148327
Level 2	0.141723	0.15594	0.13097	0.118934	0.135218
Level 3	0.125397	0.14413	0.151617	0.151403	0.138974
Level 4	0.139971	0.120167	0.114964	0.148769	0.13763
Delta	0.013086	0.023963	0.038618	0.03247	0.002411
Rank	4	3	1	2	5