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Entropy generation analysis for chemically reactive flow of Sutterby nanofluid considering radiation aspects

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KEYWORDS
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GDQM.

Abstract. Nanofluids show greater heat transfer rate and characteristics of mechanical friction diminution using nano-sized hard elements to fluid. Moreover, regarding the working of heat transfer fluid, nanofluid is widely used in areas of refrigeration, shipping, automobile, chemical industry, energy, electronics, air conditioning, computer, and many other areas to cope heat transference issues. The aforesaid utilizations motivated us to encounter entropy generation aspects for Sutterby nanofluid flow configured by permeable surface. Moreover, well-known Buongiorno's model capturing same attributes of Brownian and thermophoretic-diffusions is presented for modeling and investigation. Additionally, magnetohydrodynamics (MHD) as well as thermal radiation effects are the part of current work. Here, we have also considered the viscous dissipation aspects. Similarity variable are used to decrease set of nonlinear Partial Differential Equations (PDEs) into set of Ordinary Differential Equations (ODEs) then resolved numerically by using bvp4c algorithm, besides the pertinent parameters are addressed graphically. The physical aspect of fluid flow, temperature, concentration for variation of involved parameters is explained with the help of graphs.

1. Introduction

The nano fluids theory was firstly initiated by Choi [1]. In fact, nanofluids show greater heat transport and features of mechanical friction diminution via introducing nano solid particles to fluid. Nanofluid is more often used for shipping purpose, refrigeration, energy, chemical industry, electronics, air conditioning, computer, and many other regions to measure the heat transference phenomenon with lubrication requirements of great heat encumber of heat replacing structure which has simple association for economy enhancement, reduction and sustainability of heat replacing structure and has extensive applications prospects and potentially great competence economic value. By using the features of nanoparticles, Oztop and Abu Nada [2] studied convection on hot surfaces. For further expansion, Sheikholeslami et al. [3] conducted the same study on magnetic nanofluids. Khan et al. [4] deliberated the MHD flow of a Oldroyd-B nanofluid on a radially stretched convective sheet using heat

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generation/absorption. In recent years, the importance of nano-fluids in the field of engineering sciences is growing day by day. In present time, nano materials have inclusive range of uses in thermal phenomenon, such as crushing processes, heat exchangers, cooling engines, aerospace technology and machinery. Makinde et al. [5] explored the consequence of a radiative heat of variable viscous nanofluid have convective boundary conditions at the surface. Khan et al. [6] discussed the flow of nano-fluids through a surface of variable thickness. Adding nanoparticles to the base fluid is best for improving the efficiency of solar collectors. Mahanthesh et al. [7] explored the chemically reacting nanofluid flow characteristics used in colloidal analysis are taken into account using the dipole movement passing through the permeable vertical plate. Khan et al. [8] demonstrated effects of chemical procedures against magneto nanoparticle for the generalized Burgers fluid. Mahanthesh et al. [9] discussed the different aspects of the radiator source and the radiation process as the nanofluid flows with in the turntable. Furthermore, current investigations under nanofluid dynamics contain refs. [10–31].

Efficiency of any system can be affected through the production of entropy because it reduces the outcomes of the scheme. For well presentation of system, it is essential to minimize the entropy of system. Entropy-generation is always perceived in any irreversible system whereas remains fix in any reversible system. Moreover, 2nd law of thermodynamics show vital role for the optimization of entropy-generation. In (1996) the basic idea about entropy-generation minimization can be functional for designing lagging/storing methods, power generation, heat exchangers and preservation methods. Majority of the entropy generation treats the convection procedures which exhibit that the entropy generation is the outcome of liquid resistance as well as heat and mass transferences phenomenon. In pure conduction methods, few papers are dealing with entropy generation. The research of Khan et al. [32–34] noticeably verified that scholars and progresses in exhibiting and simulation of entropy-generation for dissipative cross fluid by quartic autocatalysis and transportation of radiative-heat in dissipative cross model with entropy-generation as well as activation energy. Recently, Shahzad et al. [35], Zhao et al. [36] and Qayyum et al. [37], Wang et al. [38] worked against entropy generation in fluid flow of viscous and non-Newtonian things focus on different geometries. The consequences are found using numerical as well as logical techniques for flow fields and show pictorially. Total entropy generation rate is achieved against stretchable Riga wall, chemical reaction and cylindrical surface. Khan et al. [39] reflected that heat and entropy-generation in flow featuring Robin condition. In recent times, Qayyum et al. [40], Shah et al. [41], Hussain et al. [42] and Waqas et al. [43] considered entropy-generation under non-linear thermal-radiation, first-order velocity slip and heat as well as mass transference in MHD stagnation-point flow of a tangent hyperbolic nanofluid respectively.

The objective of current study is to model the heat-mass transference aspects of magneto hydrodynamic. Sutterby nanofluid flow toward stretching sheet with simultaneous features of entropy generation and thermal radiation is under consideration. The non-linear ODEs of third-order are obtained via implementation of self-similar transformations. GDQM (Generalized Differential Quadrature Method) is used to tackle the equations of the problem. The comparison tables have been computed through both schemes. Besides, the graphs are exhibited to communicate the features of non-dimensional quantities.

2. Formulation

Here, we considered the incompressible Sutterby nanofluid flow over a porous sheet along with magnetic field. Due to less magnetic Reynolds number assumption electric-field influence is overlooked. Entropy generation effect is exploited to measure the temperature features. The energy equation is based on thermophoresis, thermal radiation and Brownian movement influences.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\left(1-\frac{\beta^2}{6}\frac{\partial u}{\partial y}\right)^n\frac{\partial^2 u}{\partial y^2}-\left(\frac{n\nu\beta^2}{6}\right)\left(1-\frac{\beta^2}{6}\frac{\partial u}{\partial y}\right)\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\beta_0^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\nu}{c_p}\left(1-\frac{\beta^2}{6}\frac{\partial u}{\partial y}\right)^n\left(\frac{\partial u}{\partial y}\right)^2+\alpha\frac{\partial^2 T}{\partial y^2}+\tau\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2+\tau D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}-\frac{1}{(\rho c)_f}\frac{\partial q_r}{\partial y}, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}+D_B\frac{\partial^2 C}{\partial y^2}-k_r^2\left(\frac{T}{T_\infty}\right)^m(C-C_\infty)\exp\left(-\frac{E_a}{KT}\right), \tag{4}$$

with,

$$u=U_w=cx,\ v=0,\ T=T_w,\ C=C_w \text{ at } y=0, \tag{5}$$

$$u=U_e=cx,\ T\rightarrow T_\infty,\ C\rightarrow C_\infty \text{ as } y\rightarrow\infty. \tag{6}$$

Transformations:

$$\eta = y\sqrt{\frac{c}{v}}, \quad u = cxf^{'}(\eta), \quad v = -\sqrt{cv}f(\eta),$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \tag{7}$$

where $f'(\eta)$, $\theta(\eta)$, and $\phi(\eta)$ represents the dimensionless velocity field, temperature field, and concentration field. Eq. (1) is satisfied identically for $u = cxf^{'}(\eta), v = -\sqrt{cv}f(\eta)$ using Relation (7) into the Eqs. (2)–(6) , we achieve the differential systems as follows:

$$\left(1 - \frac{\alpha}{6}f^{''}\right)^n f^{'''} - \frac{n\alpha}{6}\left[1 - \frac{\alpha}{6}f^{''}\right]^{n-1}$$

$$f^{''}f^{'''} - f'^2 + ff'' - Mf' = 0, \tag{8}$$

$$\left(1 + \frac{4}{3}R\right)\theta'' + Pr[f\theta' + N_b\theta'\phi' + N_t\theta'^2]$$

$$+ EcPr\left[1 - \frac{\alpha}{6}f^{''}\right]^n f''^2 = 0, \tag{9}$$

$$\phi^{''} + Sc\left[f\phi' + \frac{N_t}{N_b}\theta^{''} - \sigma(1 + \delta\theta)^m\phi\right.$$

$$\left. exp\left(- \frac{E}{1 + \delta\theta}\right)\right] = 0, \tag{10}$$

$$f(0) = 0, \quad f^{'}(0) = 1, \quad f^{'}(\infty) \to 0, \tag{11}$$

$$\theta(0) = 1, \quad \theta(\infty) \to 0, \tag{12}$$

$$\phi(0) = 1, \quad \phi(\infty) \to 0. \tag{13}$$

Here, $M=\frac{\sigma^* B_0^2}{\rho_f c}$, $Pr=\frac{\nu}{\alpha}$, $R=\frac{4\sigma^{**}T_\infty^3}{k_f m^*}$, $N_b=\frac{\tau D_B(C_w-C_\infty)}{v}$, $N_t=\frac{\tau D_T(T_w-T_\infty)}{\nu T_\infty}$, $Ec=\frac{c^2x^2}{c_p(T_w-T_\infty)}$, $Sc=\frac{\nu}{D_B}$, $\sigma=\frac{kr^2}{c}$, $E=\frac{E_a}{\kappa T_\infty}$, and $\delta=\frac{T_w-T_\infty}{T_\infty}$, magnetic parameter, Prandtl number, thermal radiation parameter, Brownian motion parameter, thermophoresis parameter, Eckert number, Schmidt number, dimensionless reaction rate, dimensionless activation energy, and temperature difference parameter respectively.

2.1. Physical quantities

Following relations are for the resistive force (C_{fx}) as well as local Nusselt number (Nu_x):

$$C_{fx} = \frac{\tau_w}{\rho U_w^2}, \tag{14}$$

$$Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \tag{15}$$

here the (τ_w, q_w) shows the (wall shear stress, wall heat flux) which are expressed as:

$$\tau_w = \mu\frac{\partial u}{\partial y}\left[1 - \frac{\beta^2}{6}\left(\frac{\partial u}{\partial y}\right)\right]^n, \tag{16}$$

$$q_w = -k\frac{\partial T}{\partial y} - \frac{16\sigma^{**}T_\infty^3}{3m^*}\frac{\partial T}{\partial y}. \tag{17}$$

Equating Eqs. (16) and (17) in the Eqs. (14) and (5), we have following non-dimensional forms of friction force and local Nusselt number:

$$C_{fx}Re_x^{1/2} = \left[1 - \frac{\alpha}{6}f^{''}(0)\right]^n f^{''}(0), \tag{18}$$

$$Nu_x Re_x^{-1/2} = -\left[1 + \frac{4}{3}R\right]\theta^{'}(0), \tag{19}$$

where $Re_x = \frac{xU_w}{\nu}$ indicates local Reynolds number.

3. Entropy generation rate

For Sutterby nanoliquid flow the entropy generation expression in dimensionless form is written as:

$$S_G = \frac{k_f}{T_\infty^2}\left[1 + \frac{16\sigma^*T_\infty^3}{3k_f m^*}\left(\frac{\partial T}{\partial y}\right)^2\right]$$

$$+ \frac{\mu}{T_\infty}\left(\frac{\partial u}{\partial y}\right)^2\left[1 - \frac{\beta^2}{6}\frac{\partial u}{\partial y}\right]^n$$

$$+ \frac{\sigma^* B_0^2 u^2}{T_\infty} + \frac{RD}{C_\infty}\left(\frac{\partial C}{\partial y}\right)^2 + \frac{RD}{T_\infty}\left(\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}\right). \tag{20}$$

Eq. (20) shows three factors (i) nanofluid resistance ir-reversibility, (ii) heat transference irreversibility and (iii) diffusive irreversibility. After using the transformation, Eq. (20) can be reduced into non-dimensional form written as:

$$N_G = \alpha_1\left[1 + \frac{4}{3}R\right]\theta^{'2} + Br\left[1 - \frac{\alpha}{6}f^{''}\right]^n$$

$$f''^2 + MBrf^{'2} + \frac{\alpha_2}{\alpha_1}L\phi^{'2} + L\theta^{'}\phi^{'}, \tag{21}$$

here $N_G=\frac{\nu T_\infty S_G}{\kappa c\,\Delta\, T}$, $Br=\frac{\mu U_w^2}{\kappa\,\Delta\, T}$, $\alpha_1=\frac{\Delta\, C}{C_\infty}$, $\alpha_2=\frac{\Delta\, T}{T_\infty}$, and $L=\frac{RD(C_w-C)}{k_f}$ represents the entropy-generation rate, Brinkman number, non-dimensional concentration, temperature ratio variable, and diffusive variable respectively.

The Bejan number (Be) obtained by Eq. (22) is shown in Box I.

4. Discussion

In this section, we have examined the influences of entropy generation for Sutterby fluid. Heat and mass transference features are explored by seeing features of Buongiorno's model. In order to resolve the governing equations, the numerical technique termed as bvp4c scheme is assimilated to integrate the governing ODEs.

$$Be = \frac{\alpha_1\left[1+\frac{4}{3}R\right]\theta'^2}{\alpha_1\left[1+\frac{4}{3}R\right]\theta'^2 + Br\left[1-\frac{\alpha}{6}f''\right]^n f''^2 + MBrf'^2 + \frac{\alpha_2}{\alpha_1}L\phi'^2 + L\theta'\phi'}. \tag{22}$$

Box I

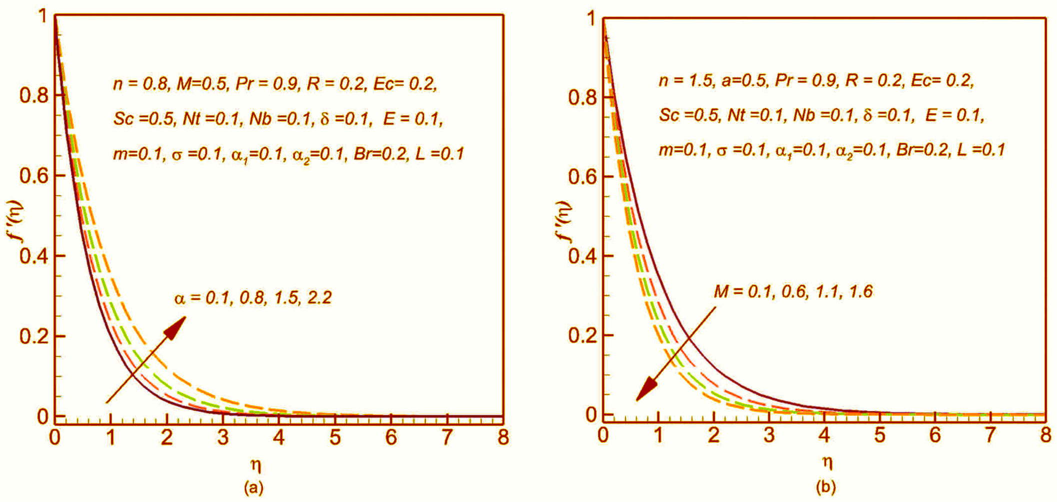

Figure 1. Impact of α and M on $f'(\eta)$.

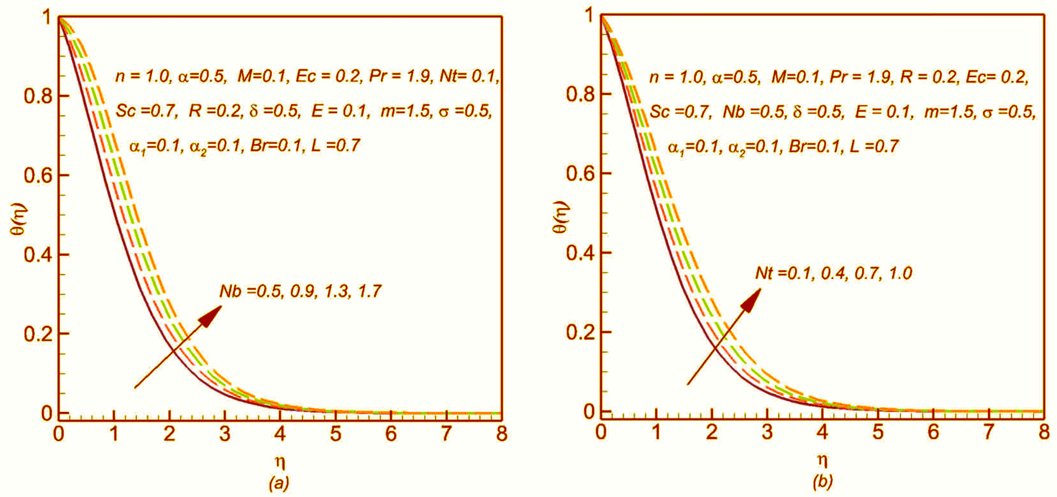

Figure 2. Effects of N_b and N_t upon $\theta(\eta)$.

4.1. Velocity distribution $f'(\eta)$

Figure 1(a) and (b) display the features of (α) and (M) upon $f'(\eta)$. In Figure 1(a) the behavior of Sutterby nanofluid parameter (α) against $f'(\eta)$ is exposed. Here, it is perceived that the fluid viscosity increases for larger value of α, therefore enlarged values of α produces greater resistive forces. So, decline performance is perceived in velocity profile $f'(\eta)$. Figure 1(b) is sketched to show the effects of Hartman number upon $f'(\eta)$. Same performance is detected for larger (M). Physically, Lorentz forces are associated with Hartman number, for larger values of Hartman number (M) yields more resistive forces in the transport phenomenon which declines the velocity field $f'(\eta)$.

4.2. Temperature field $\theta(\eta)$

Figure 2(a) and (b) points the behavior of (N_b) and (N_t) versus $\theta(\eta)$. Figure 2(a) shows the performance of Brownian motion parameter versus thermal field $\theta(\eta)$. A growing behavior is detected for larger value of (N_b). Physically, when Brownian motion parameter increases, particles of Sutterby nanofluid collide rapidly due to which temperature field boosted. Figure 2(b). designates the impact of (N_t) upon thermal profile $\theta(\eta)$. Here, greater thermophoresis parameter (N_t) yields

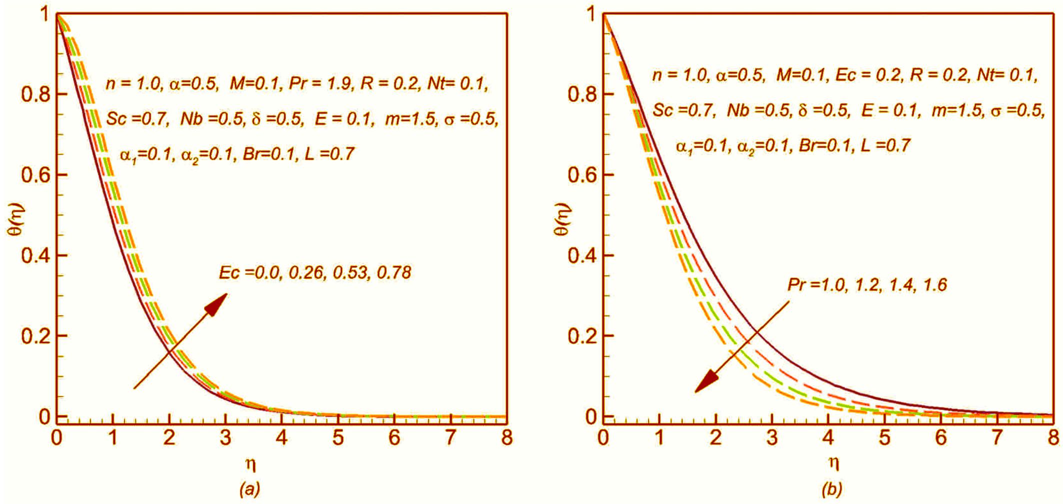

Figure 3. Influence of Ec and Pr against $\theta(\eta)$.

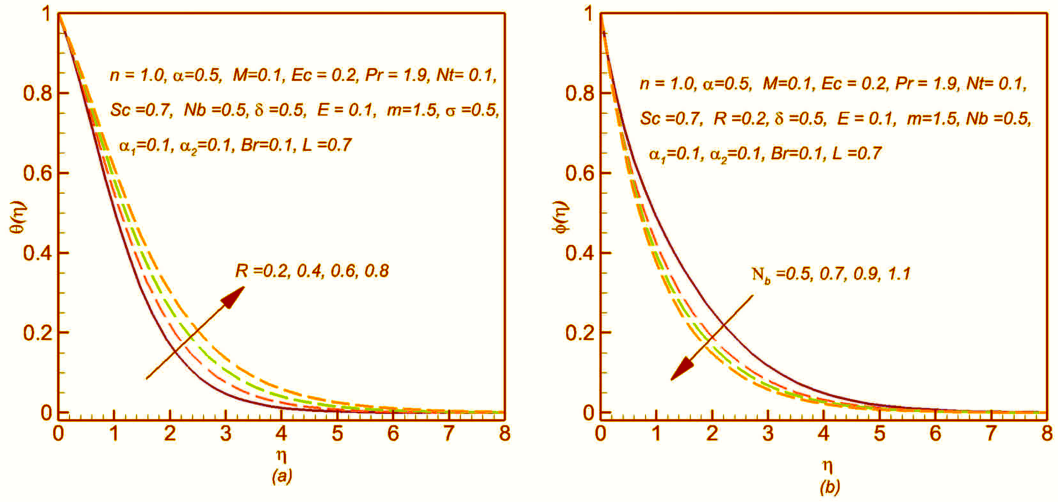

Figure 4. Effects of R and N_B upon $\theta(\eta)$ and $\phi(\eta)$.

larger $\theta(\eta)$. From the physical point of view, in thermophoresis impression insufficient fluid particles are dragged away from hot segment toward the cold segment. Thus, huge nano-materials are move away from the concentrated area which increase the nanofluid temperature. Here, Figure 3(a) exhibits the features of (Ec) upon temperature field. Basically, Eckert number is the relationship between enthalpy difference and K.E (Kinetic Energy). Actually, when the value of (Ec) is augmented temperature of nanofluid is boosted. It expands change of K.E into I.E (Internal Energy) via work-done compared to the viscid nanofluid stresses. Enlargement in Eckert number as a result loss within heat as of plate to the nanofluid. Thus, greater energy dissipation produces larger temperature field. Figure 3(b) exhibited the significance of Prandtl number (Pr) on thermal field $\theta(\eta)$ of fluid. Because of inverse proportion among Prandtl number (Pr) plus thermal diffusivity temperature profile $\theta(\eta)$ displays decaying behavior for greater Pr.

4.3. Concentration profile $\phi(\eta)$

The effects of thermal radiation parameter (R) upon thermal field is exposed in Figure 4(a). Actually, growth in thermal radiation (R) boosts $\theta(\eta)$. Physically, radiation procedure yield extra heat within the working Sutterby nanofluid thus $\theta(\eta)$ as well as associated thermal layer thickness augments. Moreover, $\phi(\eta)$ and related concentration layer diminish when Brownian motion (N_b) is intensifies (see Figure 4(b)). Figure 5(a) is considered to analyze the impact of chemical reaction parameter (σ) upon concentration profile $\phi(\eta)$ it is observed that concentration profile boosts for larger calculation of chemical reaction parameter (σ). Furthermore, use of reactive species drops speedily for larger value of (σ). Moreover, in Figure 5(b) when value of diffusive variable (L) intensifies a diminishing performance is noticed for entropy generation N_G.

4.4. Entropy generation N_G and Bejan number (Be)

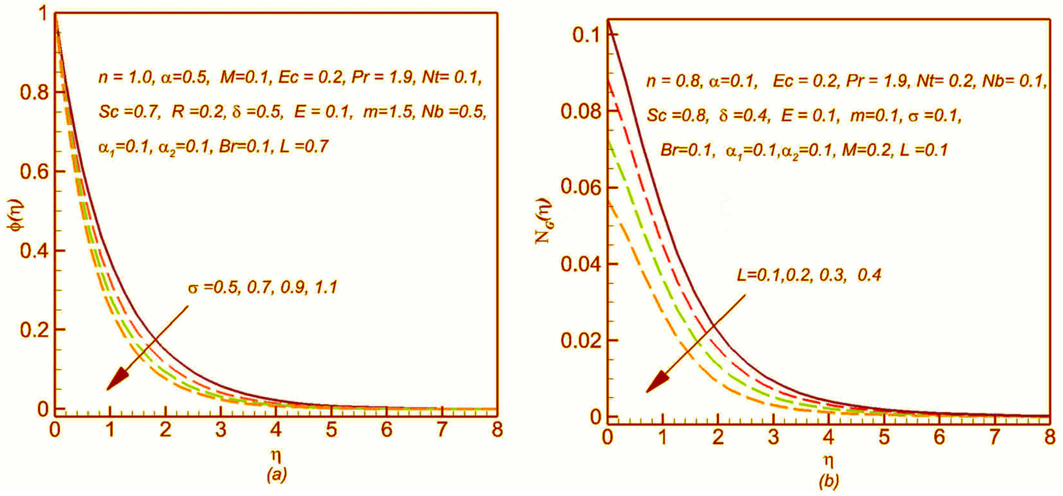

Figure 5. Impact of σ and L upon $\phi(\eta)$ and N_G.

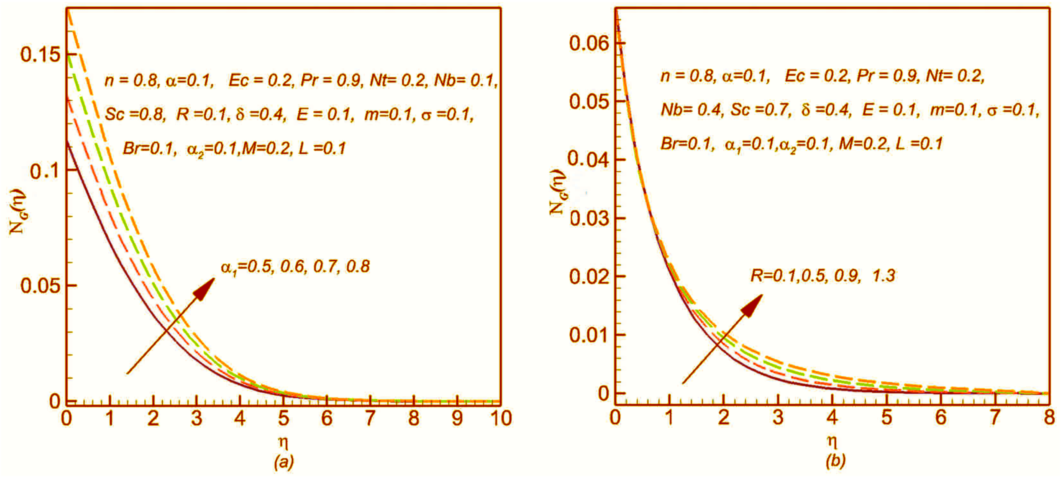

Figure 6. Impacts of α_1 and R upon N_G.

Attributes of (α_1) and (R) on N_G shown in Figure 6(a) and (b). Figure 6(a) exhibits the effects of dimensionless temperature ratio variable (α_1) on N_G. Here, entropy generation N_G enhances for larger (α_1). Clearly, N_G increases when temperature of nanofluid enhances for larger dimensionless temperature ratio variable. Furthermore, Figure 6(b) demonstrated the influence of (R) versus entropy generation N_G. It is evaluated that the entropy generation N_G rises when thermal radiation parameter R is intensifies. Physically, when internal energy of nanofluid is increases then the entropy generation N_G augments. Figure 7(a) and (b) depicts the impacts of (Br) versus entropy generation rate N_G as well as on Bejan number (Be). Physically, Brinkman number has ability of transferring heat in flowing liquid toward the heat transmission within molecular conduction i.e in polymer processing. Heat transport in molecular conduction is more than the heat conduction in viscid effects. So, motion of nanofluid particles yields extra heat in the close layers which augments the entropy N_G and system disorderness (see Figure 7(a)). Figure 7(b) display that the Be drops for larger value of Brinkman number. The fact behind this trend is that greater Brinkman number corresponds to rise entropy rate which decays the Bejan number (Be). Figure 8(a) and (b) displays the effects of magnetic parameter on entropy generation rate N_G and Bejan number (Be). Therefore, it is perceived that N_G increases for growing value of magnetic parameter (M). Actually, increase in (M) yield further Lorentz force which rises the resistance in nanofluid flow results entropy generation rate N_G enhances (see Figure 8(a)). However, greater Hartmann number (M) displays decay in (Be). Here, nanofluid resistance isreversibility has strong impacts upon the heat and mass transmission irreversibility thus Bejan number (Be) decays (see Figure 8(b)).

4.5. Consequence of heat and mass transference rate

Table 1 represents the impacts of few physical parameters for heat as well as mass transference. It is seen

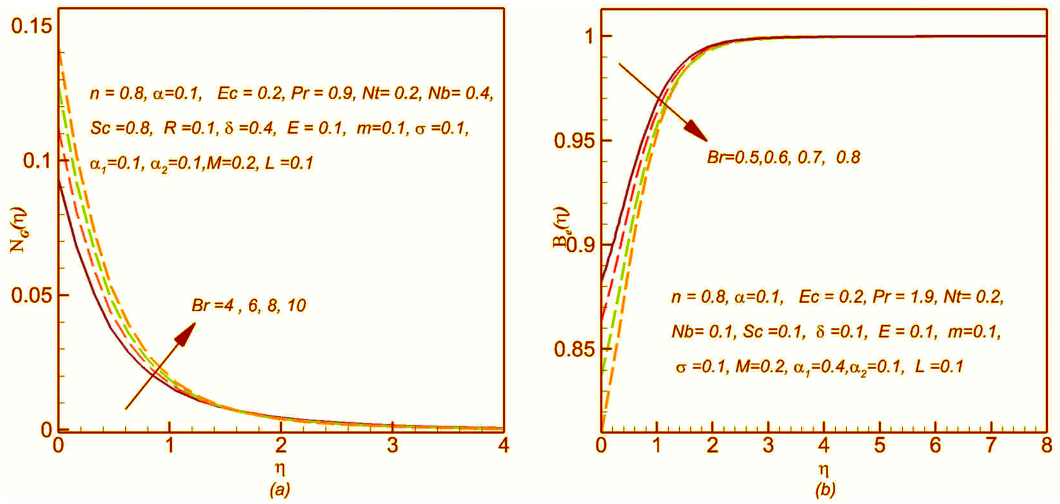

Figure 7. Effects of Br versus N_G and Be.

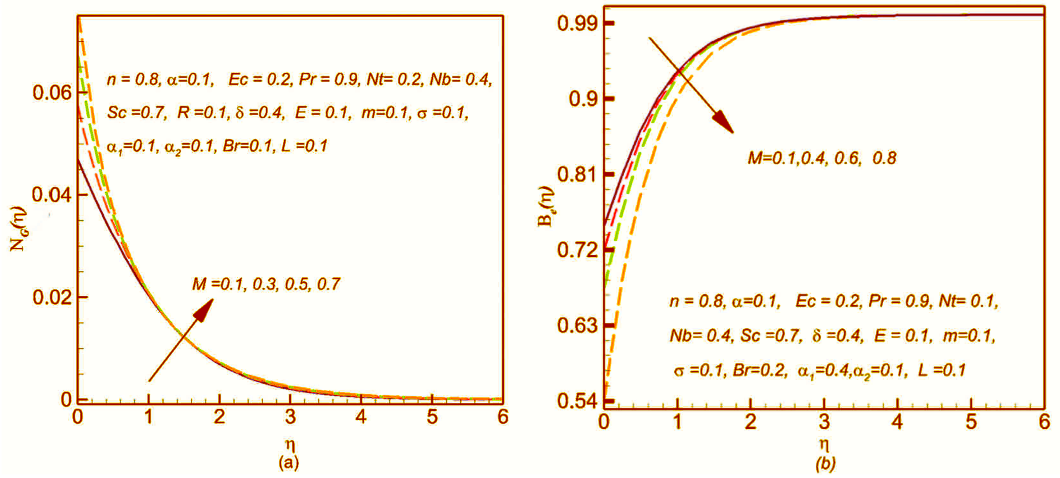

Figure 8. Consequence of M versus N_G and Be.

that heat transmission rate boosts for higher value of Pr, M, R, and Sc whereas it decreases for higher value of Ec.

5. Final remarks

Entropy generation rate within MHD mixed convective flow of Sutterby nanoliquid is inspected mathematically in the existence of viscous-dissipation plus thermal radiation. Following are the key points of current work:

- Sutterby nanofluid velocity profile is declining function of magnetic parameter;
- Intensifications within the values of Ec, R, Nb, and Nt increases the nanofluid temperature;
- Sutterby nanofluid concentration decreased for greater Nb;
- Greater Lorentz force give augmentation to the entropy generation while declines for Bejan number;
- As compare to Bejan number, higher Br give rise to entropy generation rate.

Nomenclature

u, v	Velocity components
x, y	Space coordinates
ρ	Density of fluid
ν	Kinematic viscosity
μ	Dynamic viscosity
B_0	Uniform magnetic field strength
$(\rho c)_f$	Heat capacity of fluid
τ	Ratio of heat capacity
$(\rho c)_p$	Effective heat capacity
σ^{**}	Stefan-Boltzmann constant
α	Thermal diffusivity
k_f	Thermal conductivity

Table 1. Values of Local Nusselt number and Sherwood number for different values of the parameters Pr, M, N_b, Sc and when $M = \lambda = N_t = \alpha = 0.1$.

$\boldsymbol{Pr}$	$\boldsymbol{M}$	$\boldsymbol{N_b}$	$\boldsymbol{Sc}$	$\boldsymbol{Ec}$	$\boldsymbol{-\theta'(0)}$	$\boldsymbol{-\phi'(0)}$
0.5	0.1	0.1	0.1	0.1	−0.27252	−0.340371
0.6					−0.30199	−0.339589
0.7					−0.35901	−0.325297
0.5	0.2				−0.28458	−0.330763
	0.3				−0.27805	−0.330543
	0.4				−0.272	−0.33035
		0.2			−0.28518	−0.337229
		0.3			−0.2788	−0.339316
		0.4			−0.27252	−0.340371
			0.2		−0.29141	−0.351474
			0.3		−0.29118	−0.372936
			0.4		−0.29096	−0.395071
				0.2	−0.27107	−0.332987
				0.3	−0.25047	−0.334961
				0.4	−0.22987	−0.336936

c_p	Specific heat capacity
D_B, D_T	Brownian, thermophoresis
T, C	Temperature, concentration
T_∞	Ambient temperature
C_∞	Ambient concentration
T_w	Surface temperature
C_w	Surface concentration
k_{r^2}	Reaction rate
E_a	Activation energy
κ	Boltzmann constant
k_c	Rate of chemical reaction
m	Fitted rate constant
c	Dimensional constant
η	Dimensionless variable
U_w	Stretching velocity
Pr	Prandtl number
M	Magnetic parameter
Nr	Buoyancy ratio parameter
$GDQM$	Generalized Differential Quadrature Method
R	Thermal radiation parameter
N_b	Brownian motion parameter
N_t	Thermophoresis parameter
Ec	Eckert Number
Sc	Schmidth Number
σ	Reaction rate
E	Activation energy
δ	Temperature difference parameter
τ_w	Wall shear-stress
q_w	Wall heat-flux
f	Dimensionless velocitie
θ	Dimensionless temperature
ϕ	Dimensionless concentration
N_G	Entropy generation rate
α_1	Temperature ratio parameter
α_2	Concentration ratio parameter
L	Diffusive variable
Br	Brinkman number
C_{fx}	Skin friction
Nu_x	Local Nusselt number
Re_x	Local Reynolds number

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