Effect of variable heat source and gravity variance on the convection in porous layer with temperature-dependent viscosity

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Abstract

The joint influences of variable heat source patterns and temperature-reliant viscosity on the onset of convective motion in porous beds in the presence of gravity variance have been investigated. The linear analysis is performed using normal mode analysis and the Galerkin technique is applied to analyze the impact of variable heating and changeable gravity field on the behavior of system stability. The exponential temperature-dependent viscosity is considered. We examined three different types of heat source and gravity variance function combinations: Convection is accelerated by increases in viscosity and the gravity variance parameter, but decelerated by increases in the heat source strength. It has been shown that the configuration is more stable when the gravity variance and heat source functions are combined in instance (ii), but less stable when they are combined in case (iii).

Keywords: variable gravity; changeable heat source; temperature-reliant viscosity; Galerkin technique; critical Rayleigh number.

1. Introduction

There are many natural and industrial contexts where the study of the convective motion in a fluid-saturated porous bed is applicable, including hydrology, building insulation, dispersion of pollutants in the environment, geothermal energy extraction and reservoirs, diffuse nuclear storage, space, food processing, and heat exchangers[1,2].

By using the linear stability technique, Straughan and Rionero [3] investigated the thermal convective problem in a porous bed with a changing internal heat source strength. Mahajan & Nandal [4] assert that different interior heating patterns have an impact on thermal convection. The Galerkin method and the linear stability theory were used by Mahabaleshwar [5] and Ananda et al. [6] to examine the effects of varying heat source patterns and gravity vectors. Gangadharaiah et al. [7] investigated penetrative connective motion in a porous bed. Suma et al. [8] investigated connective problems due to internal heating effects in a two-layered configuration with throughflow. Shivakumara et al. [9] examined the impacts of heat source strength in an anisotropic material with Marangoni effects. Different types of internal heating in a system of two layers configuration were studied by Gangadharaiah [10]. Manjunatha et al. [11] and Manjunatha and Sumithra [12-13] discussed constant heat sources in the presence of with and without magnetic field in a two-layer system. Nouri-Borujerdi et al. [14] looked at the impact of internal heating in a porous bed using a thermal model. Many scholars have looked at the effect of temperature-dependent viscosity issues on thermal convection in recent years (Gangadharaiah and Ananda [15], Barletta and Nield [16]]). In addition, the effect of temperature-dependent viscosity is vital for the regulation of convective mechanisms in science, geophysics, manufacturing, etc. However, the study of variable heat sources, gravity with temperature-dependent viscosity, is very restricted. Suma et al. [17] and Gangadharaiah et al.[18] examined the joint impact of the internal heating and gravity variance on the device stability using the perturbation technique. Regardless, nonlinear gravity field variety with profundity can happen in sedimentary bowls, orogenic and epeirogenic developments of the crustal designs and Earth's outside (Shi et al. [19], Nagarathnamma et al.[20], Kiran et al.[21], and Ananda et al. [22]). The ferrofluid saturated porous layer convection problem is investigated in the changeable gravitational field by Mahajan and Parashar [23]. Srinivasacharya and Dipak [24] investigated the effects of a Soret parameter, a changing gravity field, and viscous dissipation on the stability of a vertical throughflow. In a porous medium with variable gravity and throughflow, Tripathi and Mahajan [25] investigated the linear and nonlinear stability assessments for the double-diffusive convection problem. Yasiri et al. [26] used a 3-D approximation to examine the validity of the linear instability limits. Ragoju et al. [27] look at the impact of internal heat and a fluctuating gravity field on convection in a porous layer. The analysis of the exponential kind of heat source on parabolic flow on non-Newtonian fluid was studied by Samrat et al.[28]. For the composite layers, the double-component convection with profiles and a heat source was studied by Manjunatha et al. [29] and the triple-component convection with temperature gradients and a heat source was studied by Yellamma et al. [30]. They obtained the Marangoni effects for a two-layer configuration. All of the works mentioned above assume the internal heat source term to be uniform. However, the nature of the heat source patterns is often non-uniform in real issues and applications due to a variety of internal elements, including heat release from chemical reactions occurring in fluids, heat source created by radiation from an external medium, radioactive decay, and others.

The combined influence of variable heat and variable viscosity pattern on the beginning of convective moment in a porous bed with gravity variance is therefore examined in this work. We considered the three types of combinations of heat source and gravity field: case(i): $\chi(z)=z, \psi(z)=-z$, case(ii): $\chi(z)=z$, $\psi(z)=-\left(e^{z}-1\right)$, case(iii): $\chi(z)=z^{3}$, $\psi(z)=-z^{2}$. The variables viscosity parameter, gravity variance parameter, and heat variance parameter have all been examined in the computations.

2. Conceptual Model

Figure 1 demonstrates the physical structure of the current study. The horizontal porous matrix is bounded between planes at $z=0$ & $z=d$ with downward gravity $g(z)$. We presume that the viscosity depends exponentially on the temperature of the form $\mu=\mu_0 \exp\left[-A\left(T-T_0\right)\right]$, and the gravity vector $\vec{g}$ is, $\vec{g}(z)=-g_0\left(1+\lambda\psi(z)\right)\hat{k}$.

The appropriate basic equations of the asymmetric arrangement of the porous matrix are

$$\nabla\cdot\vec{V}=0, \tag{1}$$

$$0=-\nabla p-\frac{\mu(T)}{K}\vec{V}+\rho_0\left[1-\beta\left(T-T_0\right)\right]\vec{g}(z), \tag{2}$$

$$A\frac{\partial T}{\partial t}+\left(\vec{V}\cdot\nabla\right)T=k_m\nabla^2 T++Q(z), \tag{3}$$

where $\vec{V}$ is the velocity vector, μ, K and p are viscosity, permeability, and pressure, respectively.

It is supposed that the basic state of being time-independent and of the form:

$$\left(u,v,w,p,T\right)=\left(0,0,w_0,p_b(z),T_b(z)\right), \tag{4}$$

Then, for basic temperature T_b, Eq.(3) can be written as:

$$\frac{d^2T_b}{dz^2}-\frac{1}{\kappa}Q\left(z\right)=0, \tag{5}$$

$$T_b\left(z\right)=\frac{-1}{\kappa}\int_0^z\int_0^{\xi}Q\left(\tau\right)d\tau\,d\xi, \tag{6}$$

Using the boundary conditions, we obtain

$$T_b\left(z\right)=\frac{-1}{\kappa}\int_0^z\int_0^{\xi}Q\left(\tau\right)d\tau\,d\xi-C\,z+T_l\,, \tag{7}$$

Where $C=\frac{1}{d}\left(T_l-T_u\right)-\frac{1}{\kappa\,d}\int_0^d\int_0^{\xi}Q\left(\tau\right)d\tau\,d\xi.$

The perturbed quantities are

$$\vec{V}=\vec{V}',\; p=p_b(z)+p',\quad T=T_b\left(z\right)+\theta, \tag{8}$$

Applying Eq. (8) into Eqs.(1)-(3), the linear stability equations become:

$$f\left(z\right)\nabla^2w+f'\left(z_m\right)\frac{\partial w}{\partial z}=R\left(1+\lambda\psi(z)\right)\nabla_{hm}{}^2T', \tag{9}$$

$$\left(A\frac{\partial}{\partial t}-\nabla^2\right)T_m=w, \tag{10}$$

Where $f(z)=\exp\left[B\left(z-\frac{1}{2}\right)\right],\quad B=\left(\frac{\nu_{\max}}{\nu_{\min}}\right).$

The solution is assumed to be of the form

$$\left(w,T\right)=\left[W\left(z\right),\Theta\left(z\right)\right]\;e^{i\left(lx+my\right)}, \tag{11}$$

Substituting Eq. (11) into Eqs. $(9)-(10)$, we arrive

$$f\left(z\right)\left(D^2-a^2\right)w+f'\left(z\right)Dw=-Ra^2\left(1+\lambda\psi\left(z\right)\right)\Theta, \tag{12}$$

$$\left(D^2-a^2\right)\Theta=-\left(1+\delta\;\chi(z)\right)W, \tag{13}$$

Where Θ is the disturbed temperature amplitude and $R=\alpha g\left(T_l-T_u\right)d^3/\nu\kappa$ is the Rayleigh number.

The required boundary positions are as:

$$W=\Theta=0\quad\text{at}\quad z=0,1. \tag{14}$$

3. Method of Solution

To solve the system of Eqs. (12) and (13), we use the Galerkin weighted residuals approach. Consequently W & Θ are considered as

$$W=\sum_{i=1}^{n}A_i\,W_i\,,\quad\Theta=\sum_{i=1}^{n}B_i\,\Theta_i\,, \tag{15}$$

With trial functions

$$W_i=\Theta_i=\sin\left(i\pi z\right), \tag{16}$$

Using the governing parameters (δ, B, a), the eigenvalue R^c can be obtained.

4. Results and Discussion

The combined influence of a changeable heat source and temperature-dependent viscosity on the convective moment in a porous bed in the presence of gravity variance is investigated using the Galerkin process. Three different types of heat and gravity source combinations are considered, with the viscosity variation being of the exponential type. In the present analysis, the governing parameters considered are the viscosity parameter (B), gravity variance parameter (λ) , and heat variance parameter (δ). The stability of the configuration is attained in terms of R^c and a_c by referring to different values B, λ and δ. The results obtained for constant viscosity and without an internal heat source were compared to those indicated by Rionero and Straughan [3] in Table 1 to validate the current analysis.

The effects of the viscosity parameter on the stability of the configuration are shown in Figs. 2(a, b, c) for all three types of combinations of heat and gravity source variations. The effect of increasing B is to increase the marginal curves in all types of heat source variations, and hence the system is stabilized. In addition, the configuration is more unstable for the case (iii) combination and more stable for the case (ii) combination of heat source and gravity variance.

Figures 3(a, b, c) and 4(a, b, c) illustrate the elucidates the deviation of the R^c and a_c with B for different values of λ and δ for three unique instances of difference of the gravity and heat effects: case(i) $\chi(z) = z$, $\psi(z) = -z$ case(ii) $\chi(z) = z$, $\psi(z) = -\left(e^z - 1\right)$, case(iii) $\chi(z) = z^3$, $\psi(z) = -z^2$, respectively. These results show that changes in the heat variance parameter value have a destabilizing influence on the configuration. With an increase in the heat source parameter, the critical Rayleigh number R^c drops. As a result, when the energy of the heat source drops, large values are required for the beginning of convection. Viscosity and gravity parameters, therefore, have a stabilizing effect on the device. Additionally, it can be shown from these figures that they have dual effects on the two parameters. In addition, from Figure 5, the fluidic system is noted to be more inconsistent for the case (iii) combination and more stable for the case (ii) combination (see Figure 5).

5. Conclusions

Convective unsteadiness in a porous matrix is mathematically studied, along with the joint effects of a changing heat source and a changing gravity source with varying viscosity. The investigation was directed at three unique patterns of heat and gravity source variations: case(i): $\chi(z) = z, \psi(z) = -z$, case(ii): $\chi(z) = z$, $\psi(z) = -\left(e^z - 1\right)$, and case(iii): $\chi(z) = z^3$ **,** $\psi(z) = -z^2$**.** The key outcomes of the study of linear stability are defined as follows:

- Stabilization of the porous convection is achieved by increasing the viscosity and gravity variance parameters and decreasing the internal heat variance parameter.
- It is noted that the convective moment is more consistent for case(ii) the combination of heat source and gravity variance functions, while the fluidic system is more inconsistent in case(iii) the combination of heat source and gravity variance functions.

- The variable heat source parameter δ and gravity fluctuation parameter λ are stabilizing impacts on stationary convection.

Nomenclature

a	wave number
B	viscosity parameter
D	differential operator d/dz
$\vec{g}$	gravity vector
K	permeability
p	pressure
R	Rayleigh number
T	temperature
$\vec{V}$	velocity vector
W	perturbed vertical velocity
∇^2	Laplacian operator
$f(z)$	variable viscosity function
$\chi(z)$	variable heat source function
$\psi(z)$	variable gravity function
μ	fluid viscosity
λ	gravity parameter
κ	thermal diffusivity
Θ	perturbed temperature
δ	heat variance parameter
β	slip parameter
ν	dynamic viscosity
∇_h^2	horizontal Laplacian operator

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Biographies

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M. Sankar, working in UTAS-Ibri in Sultanate of Oman, received Ph.D. degree in 2003 and published 60 research papers. He is actively working in the area of heat transfer of nanofluids and CFD. He guided 8 scholars for doctorate degree and has 25 years of teaching and research experience. He has completed Two Funded projects on Heat Transfer. He is also reviewer for more than 30 journals and Guest Edited Two conference proceedings from IoP. He has traveled across the globe to several countries related to research.

Figures and Table captions

Figures

Figure 1. Physical configuration of the problem

Figure 2(a, b, c) The plot of critical Rayleigh number R^c versus critical wavenumber a_c with $\delta = 0.1 = \lambda$ for different values of $B = 1, 2, 5, 10$ for all three cases.

Figure 3(a, b, c) The plot of critical Rayleigh number R^c versus viscosity parameter B with $\delta = 0$ for various values of $\lambda = 0.1, 0.3, 0.5$ for all three cases.

Figure 4(a, b, c) The plot of critical Rayleigh number R^c versus viscosity parameter B with $\delta = 0.5$ for various values of $\lambda = 0.1, 0.3, 0.5$ for all three cases.

Figure 5 The plot of critical Rayleigh number R^c versus λ with $\delta = 0.5$ & $B = 5$ for all three cases comparison.

Tables

Table 1. Critical Rayleigh number R^c and critical wavenumber a_c with gravity parameter λ in the case of constant viscosity case $(B = 0)$ and nonexistence of internal heating $(\delta = 0)$ for $\chi(z) = -z$ compared with Rionero and Straughan [3].

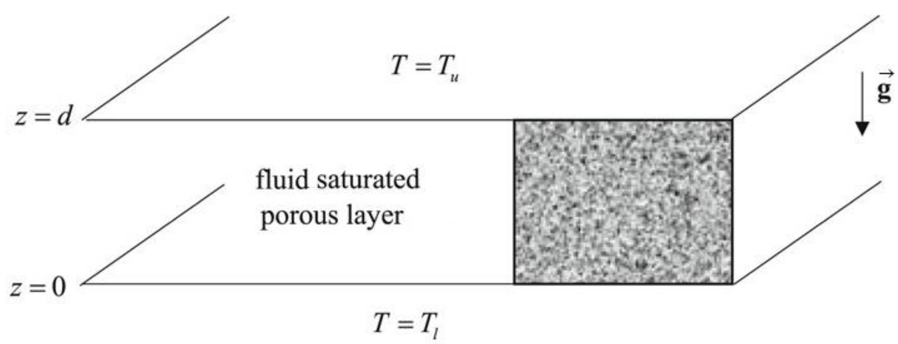
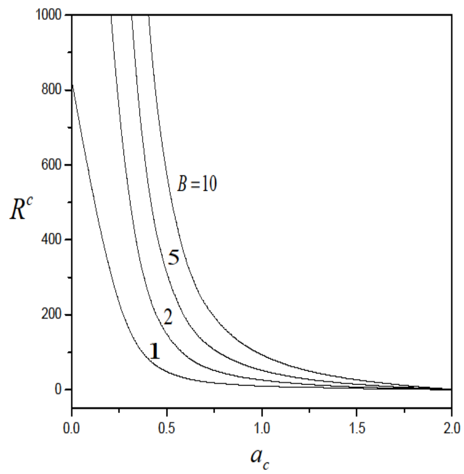


Figure 1



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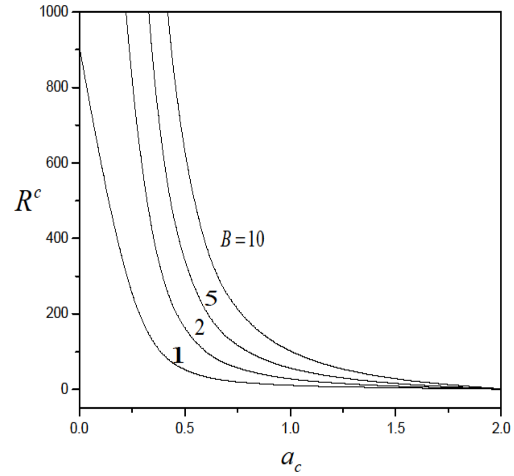

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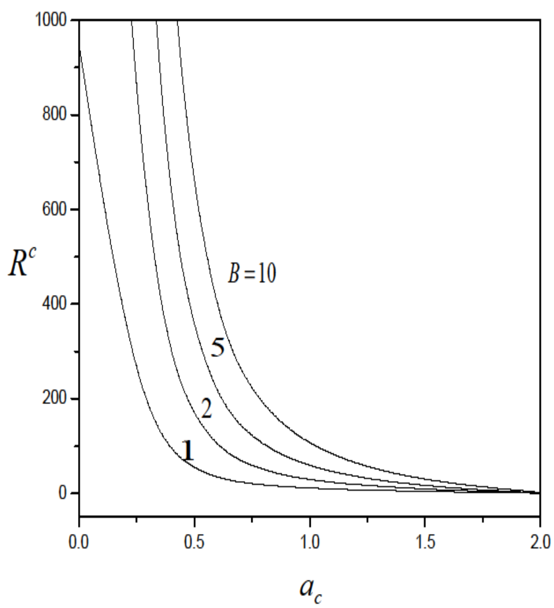

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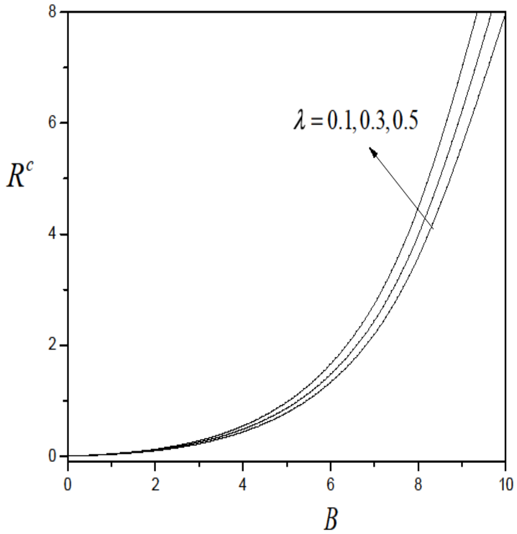

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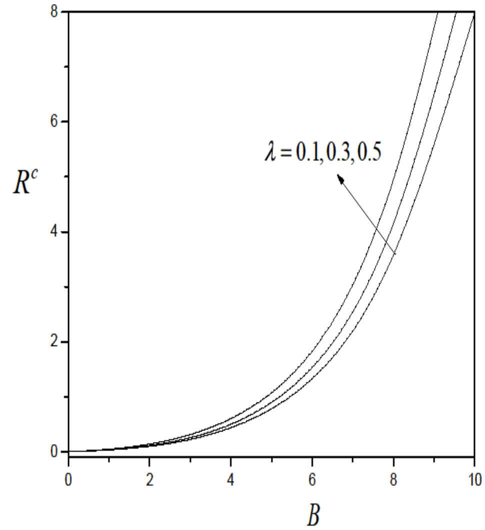

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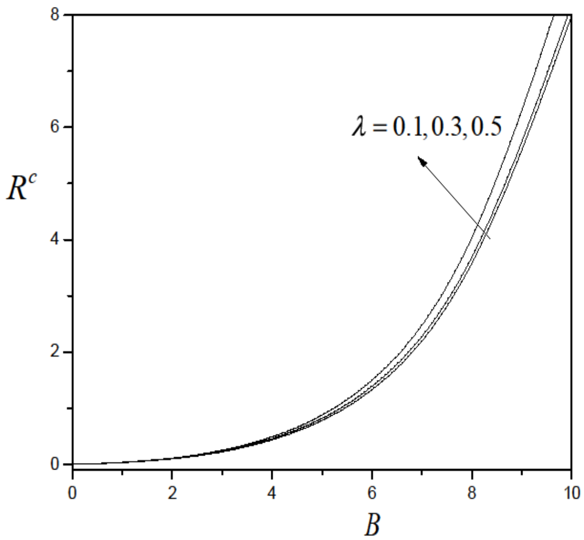

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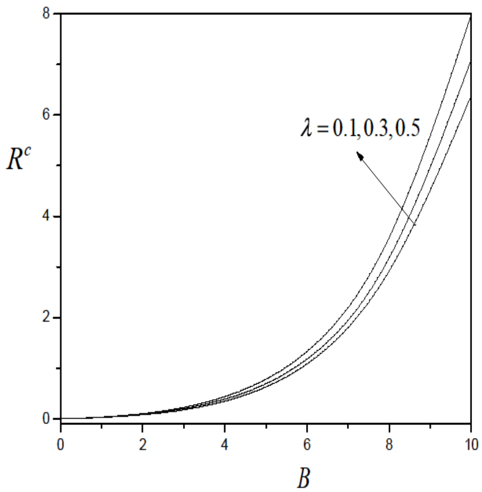

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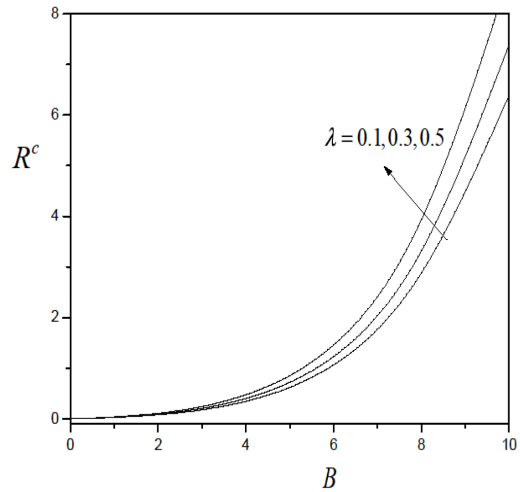

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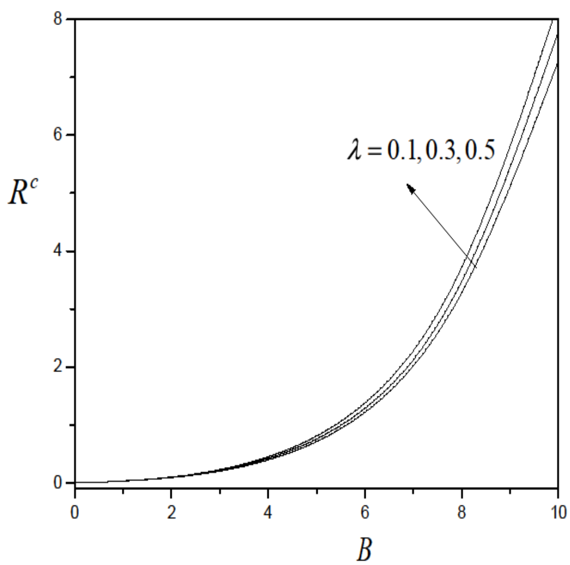

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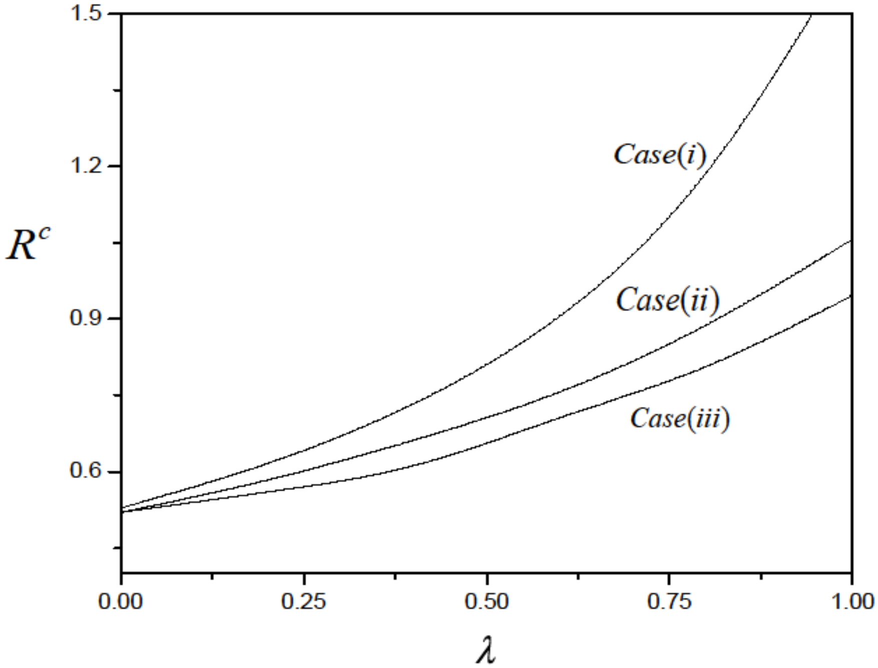

Figure 5

Table 1

	Present study $\chi(z)=-z$		Rionero and Straughan [3]	
λ	R^c	a_c^2	R^c	a_c^2
0	39.47	9.87	39.47	9.87
1	77.076	10.20	77.02	10.20
1.5	132.01	12.30	132.00	12.31
1.8	189.96	17.17	189.98	17.19
1.9	212.25	19.45	212.28	19.47