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# Study of peristaltic transport of a dusty second-grade fluid in a curved configuration

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**Abstract.** This study attempts to analyze the effect of diverse parameters of the peristaltic flow of second grade dusty fluid through a curved configuration. Stream function conversions are used to model the separate system of equations for the dust particles and fluid. The Perturbation technique is used to get the analytical solutions and the results are verified through graphs. It is noticeable that the trapped bolus, compresses both the dust particles and fluid with an increase in 'the second grade parameter'. Moreover, a surge in 'the Reynolds number' $Re$ effects the bolus trapped in the lower portion of the channel for fluid. Decrement in the velocity is observed with a rise in 'the wave number' $\delta$ and 'the second grade parameter' $\alpha_1$.



## 1. Introduction

Peristalsis is referred as the sinusoidal waves drifting along the tube walls, passages and channels. Many physiological systems naturally follow this mechanism. The fertile egg motion through the uterine tubes, ureters carrying urine, blood vessels are the few examples of peristaltic movement within a living being. This mechanism can be observed in many industrial situations as well. Peristaltic pumps are mounted in machines to transport fluids like slurry fluids, unpurified crude oil, chemicals etc. Blood movement activity in the heart-lung machine is also carried out by such pumps. Fluids around us are mostly non-Newtonian in nature. Due to the wide application of non-Newtonian fluids and peristalsis, researchers gave much attention to study them extensively [1–15].

Over the past few years, fluid consisting of small dust granules have derived the attention of many researchers. Many paints available nowadays have powder, glitters and different materials suspended in them. Juices containing pulp, polluted water, infected urine, unpurified petroleumcan be few examples of such fluids [16]. Crooke [17] investigated the dusty fluid model for various situations and by assuming diverse boundary conditions. Dusty fluid model was then studied by Charya [18]. Srinivasa Charya and Rudha Krishnama Charya [19] presented an article that investigated the dusty fluid under wall properties influence. They studied the flow rate and the trapping of the dusty fluid. Muthuraj et al. [20] studied the impressions of Magnetohydrodynamics (MHD) on the dusty fluid. They also investigated the chemical reaction influence on viscous fluid. Dusty fluid with viscoelastic properties was studied by Bhatti et al. [21].

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The surface of the passage in this study was considered to be porous. Few recent studies that deal with dusty fluids are [22–30].

In physiological systems, many glandular ducts, tubes are curved in nature. The investigators gave huge importance to the studies that involved curved passages and also exhibiting peristaltic motion. Ali et al. [31] investigated the peristaltic aspect of the fluid flowing past a curved passage. The wavelength was taken long to give an initial understanding of the problem. In a curved passage the MHD impact of the peristaltic fluid flow was examined by Hayat et al. [32]. Shehzad et al. [33] examined the effect of slip on the boundaries of the fluid flowing through a curved passage. The Sisko fluid with variable viscosity, in a curved configuration was studied by Bibi et al. [34]. Two phase movement of nanofluid was investigated by Nadeem and Shahzadi [35]. Recently, Riaz et al. [36] studied the fluid with nanoparticles suspended in it, flowing past a curved channel. The channel walls were considered to be porous and partial slip condition was imposed on the boundary. The flow of Williamson fluid flowing through a curved channel is examined by Rashid et al. [37].

Literature survey indicates that no effort has been made to study the effects of peristaltic second grade dusty fluid in a curved configuration. The present study intends to provide the missing links in this context. This study is an effort to investigate the behavior of dusty second-grade fluid in a curved passage. The initiative of this study is the hope that such a results may be applicable in many clinical applications.

## 2. Mathematical modeling

We have considered a 2D curved channel with thickness $2a$. Furhter, it is considered that the second grade fluid with dust grains embedded is flowing past with channel having $(u, v)$ the radial $(r)$ and axial $(a)$ velocities of the fluid while $(u_s, v_s)$ be the velocities of the dust grains (see Figure 1). The mathematical description of the upper and lower walls, provided by Ali et al. [31], of the channel is stated as:

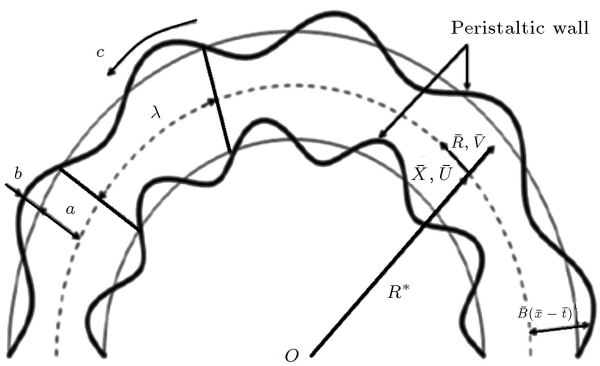


**Figure 1.** Curved configuration of the problem.

$$\bar{H}(\bar{X}, t) = a + b \sin \frac{2\pi(\bar{X} - c\bar{t})}{\lambda}, \tag{1}$$

$$-\bar{H}(\bar{X}, t) = -a - b \sin \frac{2\pi(\bar{X} - c\bar{t})}{\lambda}. \tag{2}$$

The continuity equations are:

$$\left(R\frac{\partial}{\partial \bar{x}} + \frac{\partial}{\partial \bar{r}}(R + \bar{r})\right)\bar{V} = 0, \tag{3}$$

$$\left(\frac{\partial}{\partial \bar{r}}(R + \bar{r}) + R\frac{\partial}{\partial \bar{x}}\right)\bar{V}_s = 0, \tag{4}$$

where the fluid's velocity is given by $\bar{V} = (\bar{u}, \bar{v})$ and the dust particle's velocity is $\bar{V}_s = (\bar{u_s}, \bar{v_s})$. The constitutive momentum equations which governs the flow are given as:

$$\rho\left(\frac{\partial \bar{V}}{\partial t} + (\bar{V} \cdot \nabla)\bar{V}\right) = -\nabla \bar{P} + \nabla \bar{T} + lN(\bar{V_s} - \bar{V}), \tag{5}$$

$$\left(\frac{\partial \bar{V_s}}{\partial t} + (\bar{V_s} \cdot \nabla)\bar{V_s}\right) = \frac{l}{m}(\bar{V} - \bar{V_s}). \tag{6}$$

The number density of dust granules suspended in the fluid are considered to be constant. $\bar{T}$ is the extra stress tensor of the second-grade fluid.

$$\bar{T} = \mu \bar{A_1} + \bar{\alpha_1}\bar{A_2} + \bar{\alpha_2}\bar{A_1^2}, \tag{7}$$

$$\bar{A_1} = (grad\ \bar{V}) + (grad\ \bar{V})^t, \tag{8}$$

$$\bar{A_2} = \frac{d\bar{A_1}}{dt} + \bar{A_1}(grad\ \bar{V}) + (grad\ \bar{V})^t \bar{A_1}. \tag{9}$$

The reference frame $(\bar{R}, \bar{X})$ is converted into laboratory frame $(\bar{r}, \bar{x})$ by using the following conversions:

$$\bar{u_s} = U_s - c, \quad \bar{v_s} = V_s, \quad \bar{x} = \bar{X} - c\bar{t},$$

$$\bar{r} = \bar{R}, \quad \bar{u} = \bar{U} - c, \quad \bar{v} = \bar{V}. \tag{10}$$

Below are the streamline functions and dimensionless quantities that have been used:

$$T = \frac{a\bar{T}}{\mu c}, \quad u = \frac{\bar{u}}{c}, \quad A = \frac{lNa^2}{\mu}, \quad v_s = \frac{\bar{v_s}}{\delta c},$$

$$\delta = \frac{a}{\lambda}, \quad r = \frac{\bar{r}}{a}, \quad v = \frac{\bar{v}}{\delta c}, \quad u_s = \frac{\bar{u_s}}{c},$$

$$Re = \frac{\rho c a}{\mu}, \quad x = \frac{\bar{x}}{\lambda}, \quad \alpha_1 = \frac{c\alpha_1}{\mu a}, \quad P = \frac{a^2}{\lambda \mu c}\bar{P},$$

$$v_s = \frac{\Omega}{r + \Omega}\frac{\partial \phi}{\partial x}, \quad \Omega = \frac{R}{a}, \quad v = \frac{\Omega}{r + \Omega}\frac{\partial \psi}{\partial x},$$

$$B = \frac{la}{mc}, \quad u_s = -\frac{\partial \phi}{\partial r}, \quad u = -\frac{\partial \psi}{\partial r}. \tag{11}$$

After removing the pressure terms, the compatibility

equations of the fluid and solid granules are as under:

$$
\begin{aligned}
&-\frac{\Omega}{r+\Omega}\frac{\partial^2\psi}{\partial r^2}\frac{\partial\psi}{\partial x}+\frac{1}{(r+\Omega)^2}\frac{\partial\psi}{\partial x}\left(1-\frac{\partial\psi}{\partial r}\right)\\
&\quad-\left(1-\frac{\partial\psi}{\partial r}\right)\frac{1}{r+\Omega}\frac{\partial^2\psi}{\partial x\partial r}\\
&\quad+\left(\frac{r}{\Omega}+1\right)\frac{\partial^3\psi}{\partial x\partial r^2}-\frac{\partial\psi}{\partial x}\frac{\partial^3\psi}{\partial r^3}\\
&\quad-\left(\frac{r+\Omega}{\Omega}\right)\left(1-\frac{\partial\psi}{\partial r}\right)\frac{\partial^2}{\partial r^2}\left(\frac{\partial\psi}{\partial x}\frac{\Omega}{\Omega+r}\right)\\
&\quad+\delta^2\frac{\Omega}{\Omega+r}\frac{\partial^3\psi}{\partial x^3}+\delta^2\frac{2\Omega^2}{(\Omega+r)^3}\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial x^2}\\
&\quad-\delta^2\frac{\Omega^2}{(\Omega+r)^2}\frac{\partial\psi}{\partial x}\frac{\partial^3\psi}{\partial x^2\partial r}+\frac{1}{\Omega}\frac{\partial^2\psi}{\partial r\partial x}\\
&\quad-\delta^2\frac{\partial^3\psi}{\partial r^3}\frac{\Omega^2}{(\Omega+r)^2}\left(1-\frac{\partial\psi}{\partial r}\right)\\
&\quad-\frac{\partial^2\psi}{\partial r\partial x}\left(1-\frac{\partial\psi}{\partial r}\right)\frac{2}{\Omega+r}\\
&\quad=\frac{1}{Re}\left(\frac{\partial^2}{\partial x\partial r}+\frac{\partial}{\partial x}\frac{1}{\Omega+r}\right)T_{xx}\\
&\quad+\frac{1}{\delta Re}\left[\frac{1}{\Omega}\frac{\partial}{\partial r}\left(\frac{1}{\Omega+r}\frac{\partial}{\partial r}(\Omega+r)^2\right)\right.\\
&\quad\left.-\delta^2\frac{\Omega}{\Omega+r}\frac{\partial^2}{\partial x^2}\right]T_{rx}-\frac{1}{Re}\frac{1}{\Omega+r}\frac{\partial^2}{\partial r\partial x}\left[(r+\Omega)T_{rr}\right]\\
&\quad+\frac{1}{\delta Re}A\left[\left(\frac{1}{\Omega}\frac{\partial}{\partial r}+\frac{\Omega}{\Omega+r}\frac{\partial^2}{\partial r^2}+\delta^2\frac{\partial^2}{\partial x^2}\right)(\psi-\phi)\right],\\
&\frac{1}{\Omega}\frac{\partial^2\phi}{\partial r\partial x}-\frac{1}{r+\Omega}\frac{\partial\phi}{\partial x}\frac{\partial^2\phi}{\partial r^2}+\frac{1}{(\Omega+r)^2}\left(1-\frac{\partial\phi}{\partial r}\right)\frac{\partial\phi}{\partial x}\\
&\quad-\frac{\partial\phi}{\partial x}\frac{\partial^3\phi}{\partial r^3}-\left(1-\frac{\partial\phi}{\partial r}\right)\frac{\partial^2\phi}{\partial r\partial x}\frac{1}{\Omega+r}\\
&\quad+\left(\frac{r+\Omega}{\Omega}\right)\frac{\partial^3\phi}{\partial x\partial r^2}-\frac{2}{\Omega}\frac{\partial^2\phi}{\partial x\partial r}\left(1-\frac{\partial\phi}{\partial r}\right)+\delta^2\frac{\partial^3\phi}{\partial x^3}\\
&\quad+\delta^2\frac{\partial^2\phi}{\partial x^2}-\delta^2\frac{\partial^3\phi}{\partial x^2\partial r}-\delta^2\frac{\partial^3\phi}{\partial x^3}\left(1-\frac{\partial\phi}{\partial r}\right)\\
&\quad\frac{\Omega}{\Omega+r}-\frac{\Omega}{\Omega+r}\left(1-\frac{\partial\phi}{\partial r}\right)\frac{\partial}{\partial r}\left(-\frac{\Omega}{(\Omega+r)^2}\frac{\partial\phi}{\partial x}\right.\\
&\quad\left.+\frac{\partial^2\phi}{\partial x\partial r}\frac{\Omega}{\Omega+r}\right)\\
&\quad=\frac{1}{\delta}B\left[\frac{\Omega}{\Omega+r}\frac{\partial^2}{\partial r^2}+\frac{1}{\Omega}\frac{\partial}{\partial r}+\delta^2\frac{\partial^2}{\partial x^2}\right](\psi-\phi).
\end{aligned}
$$

The elements of extra stress tensor are:

$$
\begin{aligned}
T_{rr}&=2\delta\frac{\partial}{\partial r}\left(\frac{\Omega}{\Omega+r}\frac{\partial\psi}{\partial x}\right)\\
&\quad+\alpha_1\left[-2\Omega\delta^2\frac{\partial^2}{\partial r\partial x}\left(\frac{1}{\Omega+r}\frac{\partial\psi}{\partial x}\right)\right.\\
&\quad+\left(1-\frac{\partial\psi}{\partial r}\right)\frac{2\delta^2\Omega^2}{\Omega+r}\frac{\partial^2}{\partial x\partial r}\left(\frac{1}{\Omega+r}\frac{\partial\psi}{\partial x}\right)\\
&\quad+\frac{2\Omega^4\delta^4}{(\Omega+r)^4}\left(\frac{\partial^2\psi}{\partial x^2}\right)^2\\
&\quad+2\Omega\delta^2\frac{\partial^2}{\partial r^2}\left(\frac{\Omega}{\Omega+r}\frac{\partial\psi}{\partial x}\right)\frac{\partial\psi}{\partial x}\frac{1}{\Omega+r}\\
&\quad+4\delta^2\left(\frac{\partial}{\partial r}\left(\frac{\Omega}{\Omega+r}\frac{\partial\psi}{\partial x}\right)\right)^2\\
&\quad-2\delta^2\frac{\Omega^2}{(\Omega+r)^2}\frac{\partial^2\psi}{\partial r^2}\frac{\partial^2\psi}{\partial x^2}\\
&\quad+\left(1-\frac{\partial\psi}{\partial r}\right)\frac{\partial^2\psi}{\partial r^2}\frac{2}{\Omega+r}\\
&\quad-\frac{4\Omega^2\delta^2}{(\Omega+r)^3}\frac{\partial}{\partial x}\frac{\partial\psi}{\partial x}\left(1-\frac{\partial\psi}{\partial r}\right)\\
&\quad\left.+\frac{2}{(\Omega+r)^2}\left(1-\frac{\partial\psi}{\partial r}\right)^2\right]\\
&\quad+\alpha_2\left[4\delta^2\left(\frac{\partial}{\partial r}\left(\frac{\partial\psi}{\partial x}\frac{\Omega}{\Omega+r}\right)\right)^2\right.\\
&\quad+\left(\frac{\partial}{\partial r}\left(-\frac{\partial\psi}{\partial r}\right)+\delta^2\frac{\partial^2\psi}{\partial x^2}\frac{\Omega^2}{(\Omega+r)^2}\right.\\
&\quad\left.\left.-\left(1-\frac{\partial\psi}{\partial r}\right)\frac{1}{\Omega+r}\right)^2\right],\\
T_{rx}&=-\frac{\partial^2\psi}{\partial r^2}+\frac{\Omega^2\delta^2}{(\Omega+r)^2}\frac{\partial^2\psi}{\partial x^2}-\left(1-\frac{\partial\psi}{\partial r}\right)\frac{1}{\Omega+r}\\
&\quad+\alpha_1\left[-\delta\frac{\partial^2\psi}{\partial x\partial r}\frac{1}{\Omega+r}+\delta\frac{\partial^3\psi}{\partial x\partial r^2}\right.\\
&\quad\left.-\delta^3\frac{\partial^3\psi}{\partial x^3}\frac{\Omega^2}{(\Omega+r)^2}-\delta\left(1-\frac{\partial\psi}{\partial r}\right)\frac{\partial^3\psi}{\partial x\partial r^2}\frac{\Omega}{\Omega+r}\right.
\end{aligned}
$$

$$\psi = -\frac{F}{2}, \quad \frac{\partial \psi}{\partial r} = -1, \quad \phi = -\frac{E}{2} \quad \text{at} \quad r = h = 1 + \Phi \sin(x)$$
$$\psi = +\frac{F}{2}, \quad \frac{\partial \psi}{\partial r} = -1, \quad \phi = \frac{E}{2} \quad \text{at} \quad r = -h = -(1 + \Phi \sin(x)) \tag{12}$$

Box I

$$\begin{aligned}
&+ \delta^3 \left(1 - \frac{\partial \psi}{\partial r}\right) \frac{\partial^2}{\partial x^2} \left(\frac{\Omega}{\Omega + r} \frac{\partial \psi}{\partial x}\right) \frac{\Omega^2}{(\Omega + r)^2} \\
&+ \delta \frac{\Omega}{(\Omega + r)^2} \left(1 - \frac{\partial \psi}{\partial r}\right) \frac{\partial^2 \psi}{\partial x \partial r} \\
&- \delta \frac{\partial \psi}{\partial x} \frac{\Omega}{\Omega + r} \frac{\partial^3 \psi}{\partial r^3} - \delta^3 \frac{\Omega^3}{(\Omega + r)^4} \frac{\partial \psi}{\partial x} \frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial x}\right) \\
&+ \delta^3 \frac{\Omega^2}{(\Omega + r)^2} \frac{\partial \psi}{\partial x} \frac{\partial^2}{\partial r \partial x} \left(\frac{\partial \psi}{\partial x} \frac{\Omega}{\Omega + r}\right) \\
&+ \delta \frac{\Omega}{(\Omega + r)^3} \frac{\partial \psi}{\partial x} \left(1 - \frac{\partial \psi}{\partial r}\right) + \delta \frac{\Omega}{(\Omega + r)^2} \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial r^2} \\
&+ 2\delta \frac{\partial}{\partial r} \left(-\frac{\partial \psi}{\partial r}\right) \frac{\partial}{\partial r} \left(\frac{\Omega}{\Omega + r} \frac{\partial \psi}{\partial x}\right) \\
&- 2\delta^3 \frac{\Omega^2}{(\Omega + r)^2} \frac{\partial}{\partial r} \left(\frac{\Omega}{\Omega + r} \frac{\partial \psi}{\partial x}\right) \frac{\partial^2 \psi}{\partial x^2} \\
&+ \frac{2\delta}{\Omega + r} \frac{\partial}{\partial r} \left(\frac{\Omega}{\Omega + r} \frac{\partial \psi}{\partial x}\right) \left(1 - \frac{\partial \psi}{\partial r}\right) \Bigg],
\end{aligned}$$

$$\begin{aligned}
T_{xx} = &- 2\delta \frac{\partial}{\partial r} \left(\frac{\Omega}{\Omega + r} \frac{\partial \psi}{\partial x}\right) \\
&+ \alpha_1 \Bigg[2\delta^2 \frac{\partial^2}{\partial r \partial x} \left(\frac{\Omega}{\Omega + r} \frac{\partial \psi}{\partial x}\right) - \delta^2 \frac{\partial^2}{\partial r \partial x} \\
&\left(\frac{1}{\Omega + r} \frac{\partial \psi}{\partial x}\right) \frac{2\Omega}{\Omega + r} \left(1 - \frac{\partial \psi}{\partial r}\right) - 2 \left(-\frac{\partial^2 \psi}{\partial r^2}\right)^2 \\
&- 2\delta^2 \frac{\partial^2}{\partial r^2} \left(\frac{\Omega}{\Omega + r} \frac{\partial \psi}{\partial x}\right) \frac{\partial \psi}{\partial x} \frac{\Omega}{\Omega + r} \\
&- 2\delta^2 \frac{\Omega^2}{(\Omega + r)^2} \frac{\partial^2 \psi}{\partial r^2} \frac{\partial^2 \psi}{\partial x^2} \\
&+ \frac{\partial^2 \psi}{\partial r^2} \left(1 - \frac{\partial \psi}{\partial r}\right) \frac{2}{\Omega + r} \\
&+ 4\delta^2 \left(\frac{\partial}{\partial r} \left(\frac{\partial \psi}{\partial x} \frac{\Omega}{\Omega + r}\right)\right)^2 \Bigg] \\
&+ \alpha_2 \Bigg[+ 4\delta^2 \left(\frac{\partial}{\partial r} \left(\frac{\partial \psi}{\partial x} \frac{\Omega}{\Omega + r}\right)\right)^2 \\
&+ \left(-\frac{\partial^2 \psi}{\partial r^2} - \left(1 - \frac{\partial \psi}{\partial r}\right) \frac{1}{\Omega + r}\right)^2 \\
&+ \delta^2 \frac{\partial^2 \psi}{\partial x^2} \frac{\Omega^2}{(\Omega + r)^2}\Bigg)^2 \Bigg],
\end{aligned}$$

and Eq. (12) obtained as shown in Box I.

Dimensionless $E$ and $F$ are time mean flow rate for dust granules and fluid and are given as below:

$$Q = F + 2, \tag{13}$$

$$F = -\int_{-h}^{+h} \frac{\partial \psi}{\partial r} dr, \tag{14}$$

$$Q_s = E + 2, \tag{15}$$

$$F_s = -\int_{-h}^{+h} \frac{\partial \phi}{\partial r} dr, \tag{16}$$

here $Q$ and $Q_s$ are dimensionless time mean flow.

## 3. Solution methodology

By considering the series solution for $\phi$ and $\psi$ as:

$$\begin{aligned}
&\phi = \phi_0 + \delta \phi_1 + \mathcal{O}(\delta^2), \\
&\psi = \psi_0 + \delta \psi_1 + \mathcal{O}(\delta^2), \\
&F = F_0 + \delta F_1 + \mathcal{O}(\delta^2), \\
&E = E_0 + \delta E_1 + \mathcal{O}(\delta^2).
\end{aligned} \tag{17}$$

### *3.1. Zeroth order system*

The zeroth order system of equations are:

$$\begin{aligned}
&\frac{1}{\Omega} \frac{\partial}{\partial r} \left(\frac{1}{\Omega + r} \frac{\partial}{\partial r} (\Omega + r)^2\right) T_{0rx} \\
&\quad + A \left[\left(1 + \frac{r}{\Omega}\right) \frac{\partial^2}{\partial r^2} + \frac{1}{\Omega} \frac{\partial}{\partial r}\right] (\psi_0 - \phi_0) = 0,
\end{aligned} \tag{18}$$

$$B \left[\frac{1}{\Omega} \frac{\partial}{\partial r} + \left(1 + \frac{r}{\Omega}\right) \frac{\partial^2}{\partial r^2}\right] (\phi_0 - \psi_0) = 0, \tag{19}$$

where,

$$T_{0rx} = -\frac{\partial^2 \psi_0}{\partial r^2} + \frac{1}{\Omega + r}\left(\frac{\partial \psi_0}{\partial r} - 1\right), \tag{20}$$

with

$$\psi_0 = -\frac{F_0}{2}, \quad \frac{\partial \psi_0}{\partial r} = -1, \quad \phi_0 = -\frac{E_0}{2},$$

$$\text{at} \quad r = h = 1 + \Phi \sin(x),$$

$$\psi_0 = +\frac{F_0}{2}, \quad \frac{\partial \psi_0}{\partial r} = -1, \quad \phi_0 = +\frac{E_0}{2},$$

$$\text{at} \quad r = -h = -(1 + \Phi \sin(x)). \tag{21}$$

### *3.2. First order system*

The first order system of equations are as under:

$$\begin{aligned}
&\frac{1}{\Omega}\frac{\partial^2 \psi_0}{\partial r \partial x} - \frac{\partial^2 \psi_0}{\partial r^2}\frac{1}{\Omega + r}\frac{\partial \psi_0}{\partial x} + \frac{\partial \psi_0}{\partial x}\frac{1}{(\Omega + r)^2} \\
&\left(1 - \frac{\partial \psi_0}{\partial r}\right) - \left(1 - \frac{\partial \psi_0}{\partial r}\right)\frac{1}{\Omega + r}\frac{\partial^2 \psi_0}{\partial r \partial x} \\
&+ \frac{\partial^3 \psi_0}{\partial x \partial r^2}\left(1 + \frac{r}{\Omega}\right) - \frac{\partial^3 \psi_0}{\partial r^3}\frac{\partial \psi_0}{\partial x} \\
&- \left(1 + \frac{r}{\Omega}\right)\frac{\partial^2}{\partial r^2}\left(\frac{\Omega}{\Omega + r}\frac{\partial \psi_0}{\partial x}\right)\left(1 - \frac{\partial \psi_0}{\partial r}\right) \\
&- \frac{\partial^2 \psi_0}{\partial x \partial r}\frac{2}{\Omega + r}\left(1 - \frac{\partial \psi_0}{\partial r}\right) \\
&= \left(\frac{1}{\Omega + r}\frac{\partial}{\partial x} + \frac{\partial^2}{\partial r \partial x}\right) Re^{-1/2} T_{0xx} \\
&+ \frac{1}{\Omega}\left[\frac{\partial}{\partial r}\left(\frac{1}{\Omega + r}\frac{\partial}{\partial r}(\Omega + r)^2\right)\right] Re^{-1/2} T_{1rx} \\
&- \left[\frac{1}{\Omega + r}\frac{\partial^2}{\partial r \partial x}(\Omega + r)\right] Re^{-1/2} T_{0rr} \\
&+ A\ Re^{-1/2}\left[\frac{r}{\Omega + r}\frac{\partial^2}{\partial r^2} + \frac{1}{\Omega}\frac{\partial}{\partial r}\right](\psi_1 - \phi_1),
\end{aligned} \tag{22}$$

$$\begin{aligned}
&\frac{1}{\Omega}\frac{\partial^2 \phi_0}{\partial r \partial x} - \frac{\partial \phi_0}{\partial x}\frac{\partial^2 \phi_0}{\partial r^2}\frac{1}{\Omega + r} + \frac{1}{(\Omega + r)^2} \\
&\left(1 - \frac{\partial \phi_0}{\partial r}\right)\frac{\partial \phi_0}{\partial x} - \left(1 - \frac{\partial \phi_0}{\partial r}\right)\frac{1}{\Omega + r}\frac{\partial^2 \phi_0}{\partial x \partial r} \\
&+ \frac{\partial^3 \phi_0}{\partial x \partial r^2}\left(1 + \frac{r}{\Omega}\right) - \left(1 + \frac{r}{\Omega}\right)\frac{\partial}{\partial r} \\
&\left(-\frac{\partial \phi_0}{\partial x}\frac{\Omega}{(\Omega + r)^2} + \frac{\Omega}{\Omega + r}\frac{\partial^2 \phi_0}{\partial r \partial x}\right)\left(1 - \frac{\partial \phi_0}{\partial r}\right) \\
&- \frac{2}{\Omega}\frac{\partial^2 \phi_0}{\partial r \partial x}\left(1 - \frac{\partial \phi_0}{\partial r}\right) - \frac{\partial \phi_0}{\partial x}\frac{\partial^3 \phi_0}{\partial r^3} \\
&= B\left[\left(1 + \frac{r}{\Omega}\right)\frac{\partial^2}{\partial r^2} + \frac{1}{\Omega}\frac{\partial}{\partial r}\right](\phi_1 - \psi_1),
\end{aligned} \tag{23}$$

where:

$$\begin{aligned}
T_{0xx} &= \alpha_1\left[2\left(\frac{\partial^2 \psi_0}{\partial r^2}\right)^2 + \left(1 - \frac{\partial \psi_0}{\partial r}\right)\frac{\partial^2 \psi_0}{\partial r^2}\frac{2}{\Omega + r}\right] \\
&+ \alpha_2\left[\left(-\frac{\partial^2 \psi_0}{\partial r^2} - \left(1 - \frac{\partial \psi_0}{\partial r}\right)\frac{1}{\Omega + r}\right)^2\right],
\end{aligned} \tag{24}$$

$$\begin{aligned}
T_{0rr} &= \alpha_1\left[\frac{2}{\Omega + r}\left(1 - \frac{\partial \psi_0}{\partial r}\right)\right. \\
&\left. + \left(1 - \frac{\partial \psi_0}{\partial r}\right)^2 \frac{2}{(\Omega + r)^2}\right] + \alpha_2 \\
&\left[\left(-\frac{\partial^2 \psi_0}{\partial r^2} - \frac{1}{\Omega + r}\left(1 - \frac{\partial \psi_0}{\partial r}\right)\right)^2\right],
\end{aligned} \tag{25}$$

$$\begin{aligned}
T_{1rx} &= -\frac{\partial^2 \psi_1}{\partial r^2} + \frac{1}{\Omega + r}\frac{\partial \psi_1}{\partial r} \\
&+ \alpha_1\left[\frac{\partial^3 \psi_0}{\partial x \partial r^2} - \frac{\Omega}{\Omega + r}\left(1 - \frac{\partial \psi_0}{\partial r}\right)\right. \\
&+ \frac{\partial^2 \psi_0}{\partial r^2}\frac{\partial \psi_0}{\partial x}\frac{\Omega}{(\Omega + r)^2} + \frac{\partial^2 \psi_0}{\partial x \partial r}\left(1 - \frac{\partial \psi_0}{\partial r}\right) \\
&\frac{\Omega}{(\Omega + r)^2} + \frac{\Omega}{(\Omega + r)^3}\frac{\partial \psi_0}{\partial x}\left(1 - \frac{\partial \psi_0}{\partial r}\right) \\
&- 2\frac{\partial^2 \psi_0}{\partial r^2}\left(-\frac{\partial \psi_0}{\partial x}\frac{\Omega}{(\Omega + r)^2} + \frac{\Omega}{\Omega + r}\frac{\partial^2 \psi_0}{\partial x \partial r}\right) \\
&+ \frac{2}{\Omega + r}\left(1 - \frac{\partial \psi_0}{\partial r}\right)\left(-\frac{\Omega}{\Omega + r}\frac{\partial \psi_0}{\partial x}\right. \\
&\left. + \frac{\partial^2 \psi_0}{\partial r \partial x}\frac{\Omega}{\Omega + r}\right) + \frac{\partial^2 \psi_0}{\partial x \partial r}\frac{1}{\Omega + r} - \frac{\Omega}{\Omega + r} \\
&\left.\frac{\partial^3 \psi_0}{\partial r^3}\frac{\partial \psi_0}{\partial x}\right],
\end{aligned} \tag{26}$$

with:

$$\psi_1 = -\frac{F_1}{2}, \quad \frac{\partial \psi_1}{\partial r} = 0, \quad \phi_1 = -\frac{E_1}{2},$$

$$\text{at} \quad r = h = 1 + \Phi \sin(x),$$

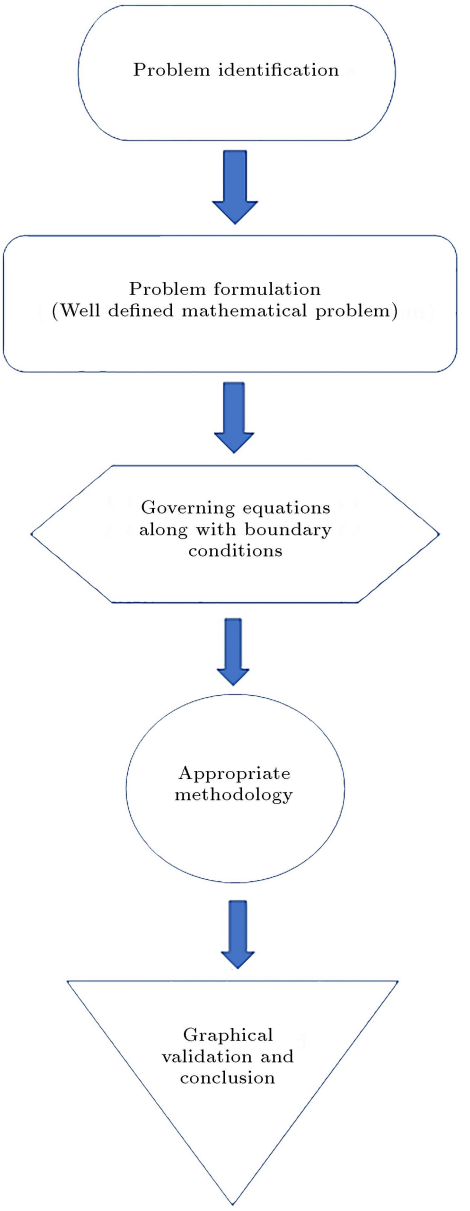


**Figure 2.** Flow chart of the problem

$$\psi_1 = +\frac{F_1}{2}, \quad \frac{\partial \psi_1}{\partial r} = 0, \quad \phi_1 = +\frac{E_1}{2},$$

$$\text{at} \quad r = -h = -(1 + \Phi \sin(x)). \tag{27}$$

DSolver command in Mathematica software have been utilized to get the solutions and graphs of the above mentioned system of equations.

Figure 2 display the flow chart that describes the pattern of complete manuscript.

## 4. Results interpretation

The results obtained analytically are plotted to check the validity and effects of physical parameters on the velocity and streamlines of the dusty second grade fluid. Figures 3–7 show the impact of wave number $\delta$, curvature parameter $\Omega$ and the second-grade parameter $\alpha_1$ on the fluid and dust grains velocity.

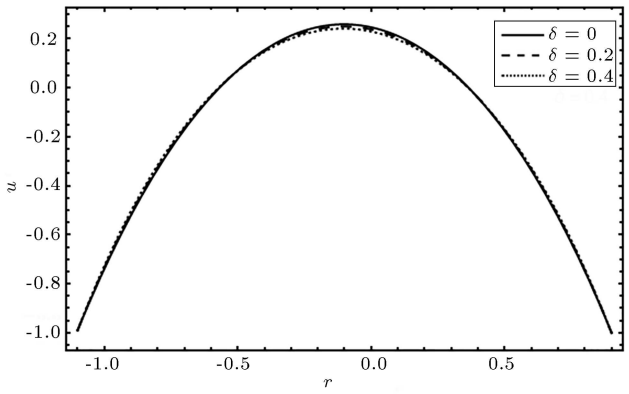


**Figure 3.** Fluid velocity for the parameter $\delta$ with $B = 0.7$, $\phi = 0.4$, $Q = 1.5$, $Re = 5$, $A = 0.8$, $\Omega = 3.5$, $Q_s = 1.8$, $\alpha_1 = 1$.

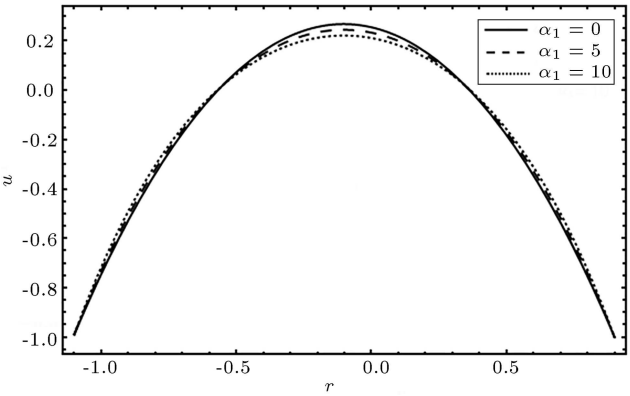


**Figure 4.** Fluid velocity for the parameter $\alpha_1$ with $B = 0.7$, $\phi = 0.4$, $Q = 1.5$, $Re = 5$, $A = 0.8$, $\Omega = 3.5$, $Q_s = 1.8$, $\delta = 1$.

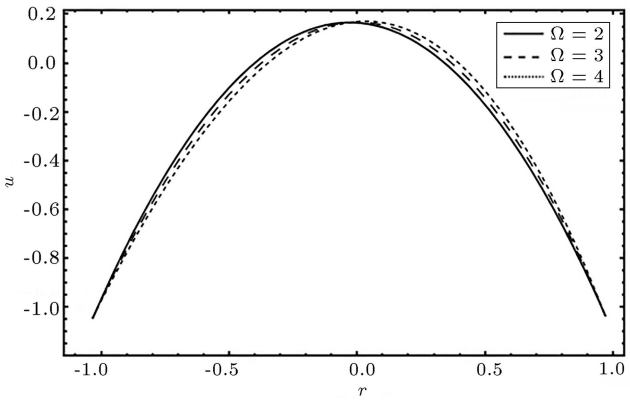


**Figure 5.** Fluid velocity for the parameter $\Omega$ with $B = 0.8$, $\phi = 0.45$, $Q = 1.5$, $Re = 8$, $A = 0.8$, $\delta = 1$, $Q_s = 1.8$, $\alpha_1 = 0.8$.

### *4.1. Velocity*

In Figure 3, the effect of wave number can be observed. At the center of the passage, fluid's velocity decreases

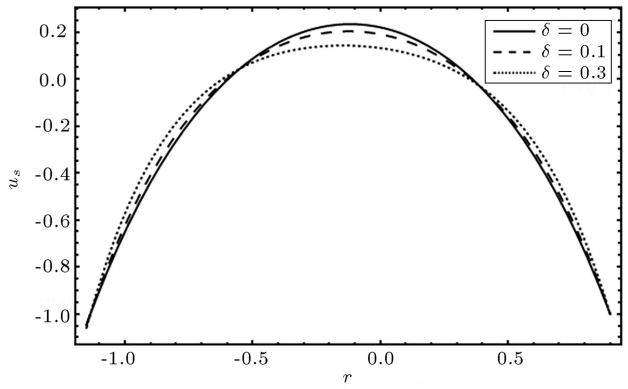


**Figure 6.** Dust granules velocity for the parameter $\delta$ with $B = 0.7$, $\phi = 0.4$, $Q = 1.5$, $Re = 5$, $A = 0.8$, $\Omega = 3.5$, $Q_s = 1.8$, $\alpha_1 = 1$.

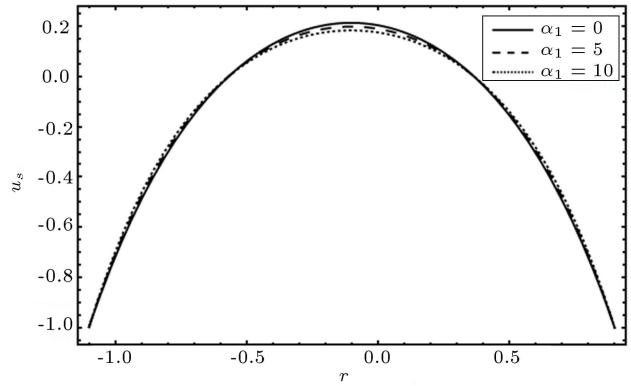


**Figure 7.** Dust granules velocity for the parameter $\alpha_1$ with $B = 0.7$, $\phi = 0.4$, $Q = 1.5$, $Re = 5$, $A = 0.8$, $\Omega = 3.5$, $Q_s = 1.8$, $\delta = 0.01$.

as wave number has an inverse relation to wavelength thus with the increase in wave number, decrease in wavelength happens thus causing lesser turbulence and resulting reduction in fluid velocity. For the higher value of second-grade parameter, the fluid velocity drops as shown in Figure 4. Decline in the center of the passage is spotted with the increase in second-grade parameter, this may be because second grade parameter affects the viscous nature of the fluid thus causing difficulties for it to flow. Figure 5 shows the impact of the curvature parameter on the velocity profile. A decline in the velocity is observed along the lower portion of the channel as the values of the curvature are increased. An opposite behavior emerges near the centerline of the channel that is the velocity inclines along the upper portion of the channel. Figure 6 demonstrates the impression of the velocity of solid dust particles under the effect of the wave number. The velocity at the boundaries remains unchanged while at the center of the channel, velocity declines with an enhance in the wave number. Surge in the second-grade parameter lowers down dust granules velocity as given in Figure 7.

### *4.2. Streamline patterns*

#### *4.2.1. For fluid*

Streamline patterns are a useful tool to understand the flow of the fluid. Figure 8 portrays to see the effect of 'the Reynolds number', '*Re*' on the trapped bolus of the fluid. The bolus trapped shrinks with the higher parametric values of *Re* in upper part of the channel while in the lower part it contracts and gradually vanishes. With the increase in Reynolds number, the effect of viscosity declines thus the trapped bolus flows past the curved configuration more steadily. In Figure 9 the impact of 'the curvature parameter' '$\Omega$' on streamlines can be observed. In upper part of the channel, the bolus compresses while formation of bolus can be seen in the bottom of the channel. One can deduce that rise in the curvature parameter may cause unsteadiness in the fluid flow. Figure 10 is disclosed to observe the impact of 'the second-grade parameter' '$\alpha_1$' on the fluid's trapped bolus. From Figure 10, it is witnessed that the viscous nature of fluid is enhanced

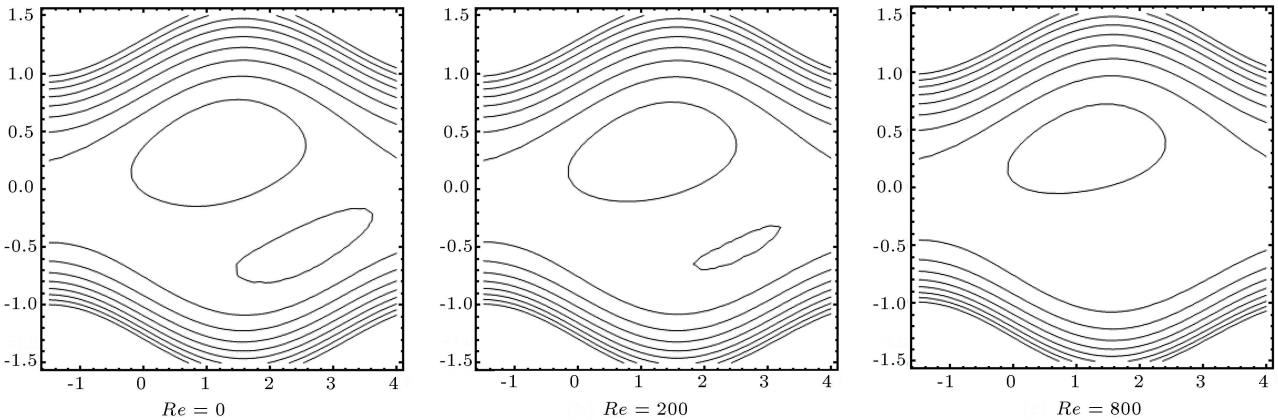


**Figure 8.** Fluid streamlines for the parameter $Re$ with $B = 0.7$, $\phi = 0.15$, $Q = 5$, $A = 0.8$, $\Omega = 7$, $Q_s = 1.8$, $\delta = 0.01$, $\alpha_1 = 0.5$.

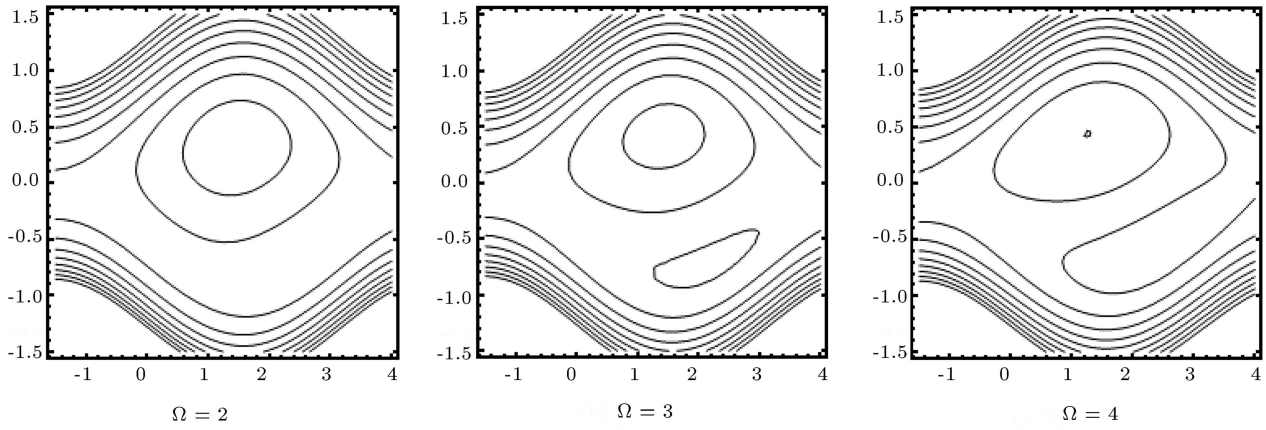


**Figure 9.** Fluid streamlines for the parameter $\Omega$ with $B = 0.7$, $\phi = 0.4$, $Q = 1.5$, $A = 0.8$, $Re = 5$, $Q_s = 1.8$, $\delta = 0.01$, $\alpha_1 = 2$.

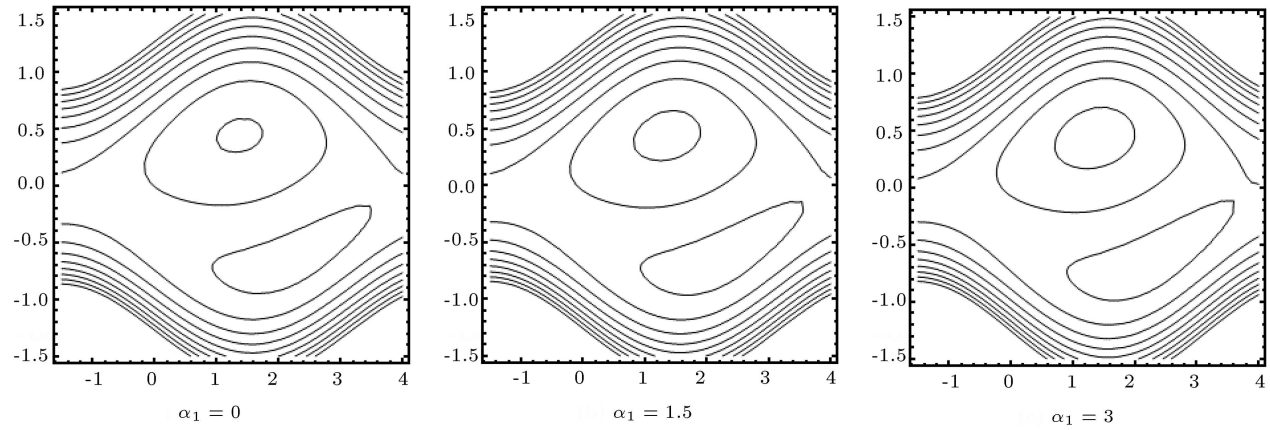


**Figure 10.** Fluid streamlines for the parameter $\alpha_1$ with $B = 0.7$, $\phi = 0.4$, $Q = 1.5$, $A = 0.8$, $Re = 5$, $Q_s = 1.8$, $\delta = 0.01$, $\Omega = 3.5$.

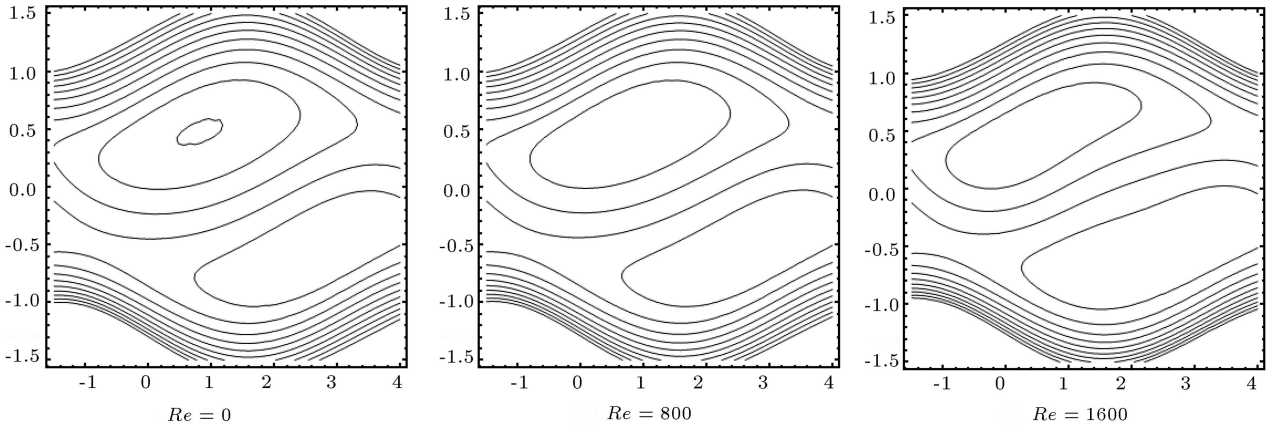


**Figure 11.** Dust granules streamlines for the parameter $Re$ with $B = 0.7$, $\phi = 0.3$, $Q = 1.5$, $A = 0.8$, $\alpha_1 = 3$, $Q_s = 1.8$, $\delta = 0.02$, $\Omega = 4$.

for an increase in $\alpha_1$ and hence the trapped bolus in the upward part of the channel grows in its size.

*4.2.2. For solid particles*

The bolus in the bottom part of the channel gets bigger with higher values of 'the Reynolds number' '$Re$' as depicted in Figures 11 and 12 is displaying the impact of 'the curvature parameter' '$\Omega$' on the streamlines of the dust particles. It is observed that in the lower section of the passage, a bolus forms with an upturn in $\Omega$ which continues to be growing with the rise in $\Omega$. Due the increase in curvature parameter, the trapped

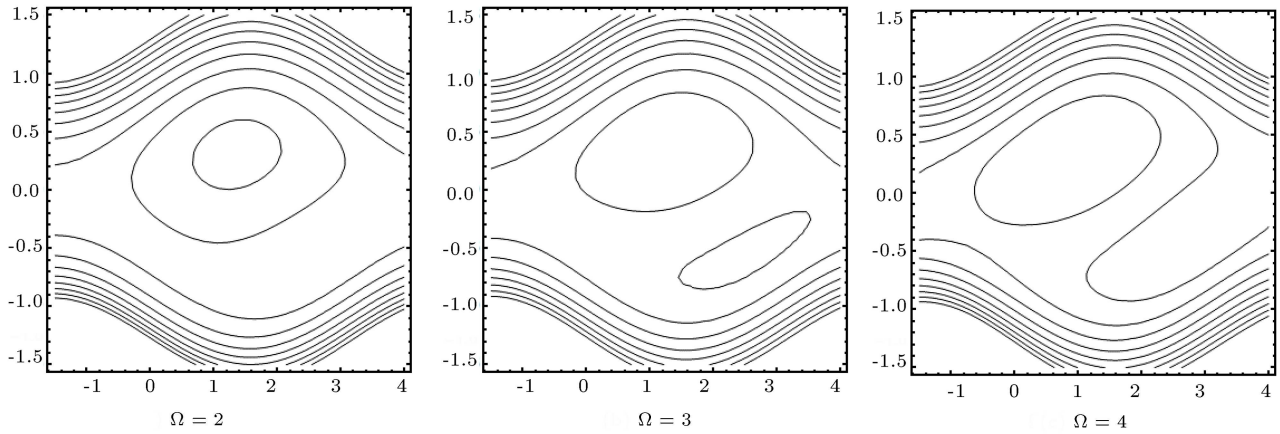


**Figure 12.** Dust granules streamlines for the parameter $\Omega$ with $B = 0.7$, $\phi = 0.3$, $Q = 1.5$, $A = 0.8$, $\alpha_1 = 2$, $Q_s = 1.8$, $\delta = 0.01$, $Re = 5$.

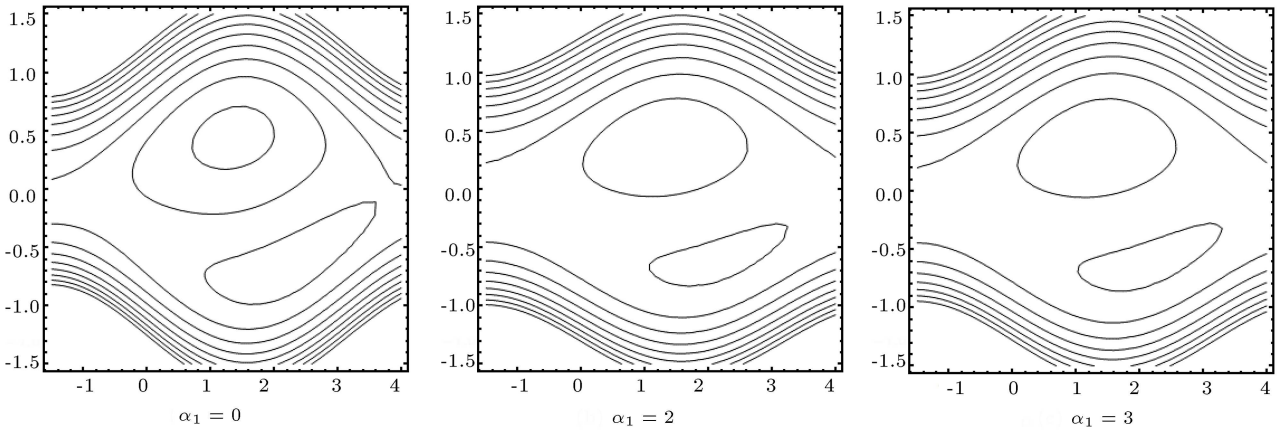


**Figure 13.** Dust granules streamlines for the parameter $\alpha_1$ with $B = 0.7$, $\phi = 0.3$, $Q = 1.5$, $A = 0.8$, $\Omega = 4$, $Q_s = 1.8$, $\delta = 0.01$, $Re = 5$.

bolus is not symmetrical in upper and lower part of channel. A rise in 'the second-grade parameter' '$\alpha_1$' causes contraction in the volume of the trapped bolus as demostrated in Figure 13.

## 5. Concluding remarks

In a curved configuration, the peristaltic activity of second-grade dusty fluid is discussed. The results are obtained by using perturbation technique. Few notable points are given below:

- For fluid and dust particles formation of bolus can be spotted in the bottom part of the channel with increase in $\Omega$;
- The curvature parameter $\Omega$ enhances the velocity of the fluid in the right side of the channel;
- The trapped bolus contracts with the surge in the second-grade parameter $\alpha_1$ for both solid and fluid particles;
- The trapped bolus in the lower portion of channel eventually vanishes with an increase in $Re$ for fluid;
- A decay in the velocity of the dust grains and fluid is witnessed for an augmentation in the wave number $\delta$ and second-grade parameter $\alpha_1$.

This work can be extended by adding various body forces to this model.

## Nomenclature

| | |
|---|---|
| $A, B$ | Non-dimensional parameters $A = \frac{lNa^2}{\nu}$, $B = \frac{la}{mc}$ |
| $a$ | Channel thickness |
| $b$ | Wave amplitude |
| $c$ | Wave speed |
| $l$ | Stokes's co-efficient of resistance |
| $m$ | Mass of the solid particles |
| $N$ | Number density of solid particles |

| | |
|---|---|
| $Re$ | Reynolds number |
| $\bar{T}$ | Extra stress tensor in reference frame |
| $T$ | Extra stress tensor in laboratory frame |
| $\bar{t}$ | Time in reference frame |
| $(\bar{U}, \bar{V})$ | Fluid velocities in reference frame |
| $(\bar{U}_s, \bar{V}_s)$ | Dust particles velocities in reference frame |
| $(U, V)$ | Fluid velocities in laboratory frame |
| $(U_s, V_s)$ | Dust particles velocities in laboratory frame |
| $(u, v)$ | Non-dimensional velocities of fluid |
| $(u_s, v_s)$ | Non-dimensional velocities of dust particles |
| $\alpha_1$ | Second-grade parameter in dimensionless form |
| $\Omega$ | Curvature of the channel |
| $\delta$ | Wave number |
| $\lambda$ | Wavelength |
| $\rho$ | Fluid density |

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