An Integrated Supplier Selection and Order Allocation with Quantity Discounts under Uncertainty

Rouhollah Karimi[1]

Ph.D. Candidate, Department of Industrial Engineering, Imam Hossein University (IHU), Tehran Iran

(Email: Arashkarimi1989@gmail.com)

Masood Mosadegh-Khah *[2]

Associate Professor, Department of Industrial Engineering, Imam Hossein University (IHU), Tehran Iran

(Email: m.mosadeghkhah@ihu.ac.ir)

* Corresponding Author

Saeed Ramezani[3]

Assistant Professor, Department of Industrial Engineering, Imam Hossein University (IHU), Tehran Iran

(Email: ramezani.sr@gmail.com)

Abstract

Supplier selection and order allocation decisions are the main parties of a supply chain network which has a high impact on the economic performance of this network. This study using an Economic Order Quantity (EOQ) concept proposes an optimization model for the integrated supplier selection and order allocation problem where lot sizing, discounts, and disruptions are contributed. To address the uncertainty, scenario-based stochastic programming is employed to consider both operational and disruption uncertainties. For solving the proposed model, not only the exact solver is employed but also an innovative algorithm based on a hybrid algorithm using Particle Swarm Optimization (PSO) and the Imperialist Competitive Algorithm (ICA) is utilized. To enhance the performance of our metaheuristic algorithm, the Taguchi experimental design method is employed. Some sensitivity analyses on the key parameters of our optimization model

[1] present Email address: r.karimi@ihu.ac.ir

[2] present Email address: mmosdegh@ihu.ac.ir

[3] present Email address: sramezani@ihu.ac.ir

are done accordingly. The main findings are the performance of the proposed algorithm for solving large-scale tests and the practicality of the proposed model to address lot sizing, discounts, and disruptions.

Keywords*:* Supplier selection; Discounts; Disruptions; Lot sizing; Metaheuristics;

1. Introduction and literature review

Supply chain management is one solution to reduce the total cost of the operations and processes from supplying the raw materials and allocation of the final products to the customers [1-2]. In this regard, supplier selection and order allocation decisions play a key role to optimize the supply chain networks [3-4]. Although there are many studies for integrating supplier selection and order allocation [5-6], the majority of them ignore the uncertainty as well as real-life constraints such as lot sizing and discounts. These drawbacks of existing studies motivate us to develop a scenario-based stochastic programming solution for an integrated supplier selection and order allocation problem considering lot sizing, discounts, and disruptions through the supply chain network.

Recently, many studies are contributing to the uncertainty in supply chain management. Although most of them focus on operational uncertainties like travel time, prices, demand, order time, etc., a few studies are focusing on the disruptions and disasters like earthquakes, floods, forest fires, imminent attacks on facilities, etc. [7-8]. Evaluating the supply chain management with both operational and disruption uncertainties not only helps to revise critical operations like supplier selection and order allocation efficiently [9-12], but also defines a robust plan to control such disruptions in the supply chain management [13-16].

To focus on the economic performance of supplier selection and order allocation operations [17,15,18], the Economic Order Quantity (EOQ) model gives us this opportunity for planning in long term [19-21] while considering real-life constraints lot sizing of products and quantity discounts [22-26]. To show that our contributions to the supplier selection and order allocation using an EOQ model, lot sizing and quantity discounts under both operational and disruption uncertainties are rarely studied in the literature review, the following relevant works are studied.

From the literature on supplier selection and order allocation, using Multi-Criterion Decision-Making (MCDM) tools are widely used in the literature [12,27,28,21]. For example, the Analytic Hierarchy Process (AHP) model is very popular for evaluating and ranking the criteria for supplier selection. Akarte et al. studied supplier selection for car manufacturing using the AHP model [13]. Chan et al. considered the uncertainty using a robust optimization model while integrating supplier selection and order allocation [29]. Dweiri et al. proposed an optimization model to integrate supplier selection and order allocation decisions for the application of the car manufacturing industry [15].

The lot sizing was added to the supplier selection and order application problem by Mazdeh et al. [16]. To handle the complexity of their model in large-scale networks, they proposed a constructive heuristic algorithm. One of the earliest studies for adding the quantity discount to the supplier selection and order allocation problem was by Nourmohamadi Shalke et al. who also considered different sustainability criteria [3]. To rank them, they proposed the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). As an extension to their model, Cheraghalipour and Farsad optimized the total cost and environmental pollution for a supplier selection and order allocation problem considering quantity discount [18]. In another study for sustainable supplier selection with quantity discounts, Arabsheybani et al. proposed a fuzzy multi-objective model using the ratio analysis where the suppliers' risks were evaluated by the failure mode and effects analysis [30]. However, they did not consider lot sizing and disruptions using an EOQ model in comparison with the present study.

The EOQ was contributed to the supplier selection for the first time by Jaśkowski et al. where the application of the construction industry was contributed [31]. In a fuzzy environment, Safaeian et al. formulated a multi-objective supplier selection and order allocation problem to minimize the total cost while maximizing the reliability, service levels, and quality [4]. To solve it, a Non-dominated Sorting Genetic Algorithm (NSGA-II) was used to find Pareto solutions for the proposed problem. In another paper, a fuzzy grey TOPSIS was applied by Feng et al. to rank the suppliers for automobile manufacturing in China [32]. Liu et al. studied a combination of TOPSIS with cloud theory and fuzzy group entropy to rank the suppliers for the application of maritime ships [33]. Based on sustainability criteria, they suggested a green degree for maritime ships. Nezhadroshan et al. combined the Decision-Making Trial and Evaluation Laboratory Method (DEMATEL) and fuzzy AHP to address the supplier selection based on resiliency criteria where an earthquake from Mazandaran province in Iran was simulated [7]. Ali et al. developed a hybrid approach based on fuzzy AHP and Delphi method for analyzing the main factors and ranking the suppliers for an industrial case study in Bangladesh [12]. Beiki et al.

studied another sustainable supplier selection and order allocation problem using a case study of the car manufacturing industry in China [5].

Using a stochastic programming and Lagrangian relaxation theory, the resiliency criteria were evaluated by Fathollahi-Fard et al. for the water distribution network [34]. The green supplier selection based on vendor managed inventory contracts was formulated by Karampour et al. who suggested NSGA-II, multi-objective Keshtel algorithm and multi-objective red deer algorithm were proposed against the epsilon constraint method [35]. Fallahpour et al. developed a sustainable-resilient supplier selection model using a hyperheuristic fuzzy programming model integrated with a two-stage fuzzy inference system [27]. Mojtahedi et al. developed a sustainable coordinated solid waste management framework using routing optimization and adaptive memory search [36]. In another paper, Fallahpour et al. studied the supplier selection based on sustainability and Industry 4.0 criteria using a fuzzy best-worst method revising by the two-stage fuzzy inference system [37]. In another paper, Fathollahi-Fard et al. analyzed sustainability criteria for the water distribution network using an adaptive memory search algorithm and multi-objective optimization [38]. To study more papers in this research area, the interested readers may see review papers in this field [19,20,21,39].

Please insert Table 1 here

To study the literature review comprehensively, **Table 1** is provided to analyze the latest papers in this field. In this regard, we have classified the supply chain into three groups, i.e., general, green, and sustainable supply chains. The application of the papers is divided into a specific industry or a general case study. In addition, there are two types of discounts in the literature including exponential and quantity discounts. Lot sizing and disruptions are other criteria to evaluate the literature review. Finally, the solution methods are divided into MCDM, exact, and metaheuristic algorithms. Having a look at **Table 1**, we can conclude the following findings:

- Most of supply chain systems are general like our study. However, some of them are considered the green and sustainable conditions to develop a supply chain network;
- Discount was not popular in many studies. Among the papers, there is more interest in the quantity discount in comparison with the exponential ones.
- There is a great deal of interest in lot sizing to be considered in the supplier selection and order allocation decisions.

- Disruptions are also considered by many studies recently.
- Most of the studies focusing on the development of MCDM models instead of optimization algorithms.
- There is no study to consider disruptions, lot sizing and quantity discounts for the supplier selection and order allocation.

To fill these research gaps, this study for the first time proposes an EOQ model for formulating an integrated supplier selection and order allocation problem with both operational disruption uncertainties while offering an innovative optimization algorithm based on Particle Swarm Optimization (PSO) and the Imperialist Competitive Algorithm (ICA).

The rest of this paper is summarized as follows: Section 2 is the model development where the problem settings, assumptions, notations, and formulations are deployed. Section 3 is the solution algorithm development where the solution presentation and the search space are designed and a hybrid metaheuristic algorithm is applied. Section 4 does different tests, analyses, and discussions to study the performance of the proposed optimization model and the developed solution algorithm. Finally, Section 5 concludes this paper with findings, limitations, and recommendations.

2. Model development

The developed model in this study is inspired by the optimization model reported in Mazdeh et al. where we add the quantity discount, disruptions and EOQ model [16]. The proposed model includes four main sets including the set of suppliers $\left(i=\{1,2,\ldots,n\}\right)$, products $\left(j=\{1,2,\ldots,m\}\right)$, discounts $\left(k_i=\{1,2,\ldots,K_I\}\right)$ and scenarios $\left(s=\{1,2,\ldots,2^n\}\right)$. Among the suppliers, we show a disrupted supplier by I_S where a supplier with no disruption is shown by $I_s^{'}$. In addition, the scenarios are indexed by 2^n to show that they are two statuses for each supplier. As we considered different discounts and products, we want to show that different suppliers have different transportation modes for transferring the products and they also have different discount levels which are specialized for each supplier. A graphical presentation of the proposed problem is shown in Figure 1.

Please insert Figure 1 here

To explain the problem settings, we first should note that each supplier has a fixed capacity which cannot be changed during a disaster $\left(Cap_i\right)$. To order a product from a supplier, there are two types of costs including the fixed cost of ordering $\left(a_{ij}\right)$ and the operation cost of ordering $\left(s_{ij}\right)$. To manufacture a product, each supplier may have different costs as well $\left(C_{ij}\right)$. To repair a product, each supplier has two types of costs including the fixed cost of maintenance for the operator who wants to repair the product $\left(h_i^v\right)$ and the purchasing cost for the components of this product $\left(h_i^b\right)$.

The suppliers try to satisfy the demand of markets for each type of products $\left(D_j\right)$. Having a competition among the suppliers, they offer different types of discounts. The domain discount for each product has a lower bound $\left(LO_{ijk_i}\right)$ and an upper bound $\left(UP_{ijk_i}\right)$ which are different for each supplier. Based on the quantity of purchased products from each supplier, there are three types prices for each product, i.e., the price with the general discount $w_{ijk_i}^{A}$, the price with the incremental discount $\left(w_{ijk_i}^{I}\right)$ and the price with no discount $\left(w_{ij}\right)$. It should be noted that all the supplier cannot offer all these prices. In this regard, we have defined binary parameters to show the availability of each supplier for the general discount $\left(d_i^A\right)$, incremental discount $\left(d_i^I\right)$ and without a discount $\left(d_i^N\right)$ and $d_i^N + d_i^A + d_i^I = 1$. If the suppliers cannot satisfy the demand of markets, there is a shortage cost for each product $\left(B_j\right)$.

Each supplier has a risk level of disruptions and this probability shows the risk of selecting a supplier $\left(\alpha_i\right)$. In this regard, we define α_i as the occurrence probability of a local disruption for a supplier. This means that for each supplier, based on the probability of $1-\alpha_i$, we can purchase the products from this supplier without a disruption. In addition, we define δ_s as the occurrence probability of a disruption under each scenario. The occurrence probability of disruption for each supplier is independent from other suppliers. In this regard, the probability of disruption is estimated as follows:

$$\delta_s = \prod_{i \in I_s} \left(1-\alpha_i\right). \prod_{i \notin I_s} \alpha_i \tag{1}$$

In addition to the local disruption for each supplier, there is a global disruption where no supplier is not available. The probability of this event is very low like the case of international sanctions to one country like Russia or Iran in 2022 (https://home.treasury.gov/news/press-releases/jy1104). Based on the probability of δ_s, the global disruption probability is computed as follows:

$$\delta_s^* = \begin{cases} \left(1-\alpha^*\right)\delta_s & I_s \neq \varnothing \\ \alpha^* + \left(1-\alpha^*\right)\prod_{i \in I}\alpha_i & I_s = \varnothing \end{cases} \tag{2}$$

Based on the above problem definition, the proposed model aims to make the decisions for the number of ordered products $\left(Q_j\right)$ and the selection of suppliers as a binary variable $\left(x_i\right)$. Other decision variables are including the portion of demand for each product which has been applied from a supplier in each scenario $\left(y_{ij}^{s}\right)$, the selection of discount domain for each supplier for the case of general discount $\left(p_{ijk_i}^{'A}\right)$ or incremental discount $\left(p_{ijk_i}^{'I}\right)$. Finally, the amount of shortage for each product under each scenario $\left(u_j^s\right)$.

To establish the proposed optimization model, we must follow the EOQ concept. The average of inventory during the order period for each supplier is uniformly changed from zero to Q_i. From the EOQ, the order period is $\frac{\mathrm{Q}}{\mathrm{D}}$. Hence, the average inventory $\left(\bar{I}\right)$ for each supplier is estimated as follows:

$$\bar{I} = \frac{\frac{1}{2} \times Q_i \times \frac{Q_i}{D}}{\frac{Q}{D}} = \frac{Q_i^2}{2Q} = \frac{Qy_i^2}{2} \tag{3}$$

Based on the EOQ model, the total cost for purchasing the products based on the average inventory is computed as follows:

$$\begin{aligned} cost^b = & \sum_{j=1}^{m}\sum_{i \in I}\frac{D_j}{Q_j}a_{ij}x_i + \sum_{s \in S}\sum_{j \in J}\sum_{i \in I_s}\delta_s^*\frac{Q_j y_{ij}^{s\,2} h_i^b}{2} \\ & + \sum_{s \in S}\sum_{j \in J}\sum_{i \in I_s}\delta_s^* D_j y_{ij}^{s} w_{ij} d_i^N + \sum_{s \in S}\sum_{j \in J}\sum_{i \in I_s}\sum_{k_i=1}^{K_I}\delta_s^* D_j y_{ij}^{s} w_{ijk_i}^{A} p_{ijk_i}^{'A} d_i^A \\ & + \sum_{s \in S}\sum_{j \in J}\sum_{i \in I_s}\sum_{k_i=1}^{K_I}\delta_s^*[\left(D_j y_{ij}^{s} - UP_{ijk_i-1}\right)\times w_{ijk_i}^{I} + \sum_{k_i^{'}=0}^{k_i-1}\left(UP_{ijk_i^{'}} - LO_{ijk_i^{'}}\right)\times w_{ijk_i^{'}}^{I}] \times p_{ijk_i}^{'I} \times d_i^I \end{aligned} \tag{4}$$

In this regard, the total cost of suppliers is computed as the following formula:

$$cost_i^v = \sum_{j\in J}\left(Cap_i C_{ij} + \frac{D_j}{Q_j} S_{ij} + \frac{Q_j Cap_i h_i^v y_{ij}}{2D_j} \right) \frac{D_j y_{ij}}{Cap_i} \tag{5}$$

Based on computations in Eq. (4) and (5), the total cost for the integrated supplier selection and order allocation system is as follows:

$$\begin{aligned} cost^{SC} &= cost^b + \sum_{s\in S}\delta_s^* \sum_{i\in I_s} cost_i^v \\ &= \sum_{j=1}^{m}\sum_{i\in I}\frac{D_j}{Q_j} a_{ij} x_i + \sum_{s\in S}\sum_{j\in J}\sum_{i\in I_s}\delta_s^* \frac{Q_j y_{ij}^{s\,2} h_i^b}{2} \\ &+ \sum_{s\in S}\sum_{j\in J}\sum_{i\in I_s}\sum_{k_i=1}^{K_I}\delta_s^* D_j y_{ij}^s w_{ijk_i}^A p_{ijk_i}^{'A} d_i^A + \sum_{s\in S}\sum_{j\in J}\sum_{i\in I_s}\sum_{k_i=1}^{K_I}\delta_s^* [\left(D_j y_{ij}^s - UP_{ijk_i-1}\right)\times w_{ijk_i}^I \\ &+ \sum_{k_i'=0}^{k_i-1}\left(UP_{ijk_i'} - LO_{ijk_i'}\right)\times w_{ijk_i'}^I]\times p_{ijk_i}^{'I} \times d_i^I + \sum_{s\in S}\sum_{i\in I_s}\sum_{j\in J}\delta_s^* C_{ij} D_j y_{ij}^s + \sum_{s\in S}\sum_{i\in I_s}\sum_{j\in J}\delta_s^* \frac{D_j^2 y_{ij}^s}{Q_j Cap_i} S_{ij} \\ &+ \sum_{s\in S}\sum_{i\in I_s}\sum_{j\in J}\delta_s^* \frac{Q_j y_{ij}^{s\,2} h_i^v}{2} \end{aligned} \tag{6}$$

Referring to the above formula, it is convex and non-linear based on Q_j. We transform Eq. (6) to a derivative one which is equaled to zero to find the optimal value for the Q_j.

$$\frac{\partial cost^{sc}}{\partial Q_j} = -\frac{D_j \sum_{i\in I} a_{ij} x_i}{Q_j^2} + \frac{\sum_{s\in S}\sum_{i\in I_s}\delta_s^* y_{ij}^{s\,2}\left(h_i^b + h_i^v\right)}{2} - \frac{\sum_{s\in S}\sum_{i\in I_s}\delta_s^* \frac{D_j^2 y_{ij}^s S_{ij}}{Cap_i}}{Q_j^2} =$$

$$\Rightarrow \frac{\sum_{s\in S}\sum_{i\in I_s}\delta_s^* y_{ij}^{s\,2}\left(h_i^b + h_i^v\right)}{2} = \frac{D_j \sum_{i\in I} a_{ij} x_i}{Q_j^2} + \frac{\sum_{s\in S}\sum_{i\in I_s}\delta_s^* \frac{D_j^2 S_{ij} y_{ij}^s}{Cap_i}}{Q_j^2}$$

$$\Rightarrow Q_j = \sqrt{\frac{2D_j\left(\sum_{i\in I} a_{ij} x_i + \sum_{s\in S}\sum_{i\in I_s}\delta_s^* \frac{D_j S_{ij} y_{ij}^s}{Cap_i}\right)}{\sum_{s\in S}\sum_{i\in I_s}\delta_s^* y_{ij}^{s\,2}\left(h_i^b + h_i^v\right)}}$$

(7)

After replacing the optimal value of Q_j into the objective function, we can revise the total cost as follows:

$$cost^{sc}=\sum_{j=1}^{m}\sqrt{2D_j\left(\sum_{i\in I}a_{ij}x_i+\sum_{s\in S}\sum_{i\in I_s}\delta_s^*\frac{D_jS_{ij}y_{ij}^s}{Cap_i}\right)\left(\sum_{s\in S}\sum_{i\in I_s}\delta_s^* y_{ij}^{s\,2}\left(h_i^b+h_i^v\right)\right)}$$
$$+\sum_{s\in S}\sum_{i\in I_s}\sum_{j=1}^{m}\delta_s^* C_{ij}D_jy_{ij}^s+\sum_{s\in S}\sum_{j\in J}\sum_{i\in I_s}\delta_s^* D_jy_{ij}^s w_{ij}d_i^N \qquad (8)$$
$$\sum_{s\in S}\sum_{j\in J}\sum_{i\in I_s}\sum_{k_i=1}^{K_I}\delta_s^* D_jy_{ij}^s w_{ijk_i}^A p_{ijk_i}^{'A}d_i^A+\sum_{s\in S}\sum_{j\in J}\sum_{i\in I_s}\sum_{k_i=1}^{K_I}\delta_s^*[\left(D_jy_{ij}^s-UP_{ijk_i-1}\right)\times w_{ijk_i}^I$$
$$+\sum_{k_i^{'}=0}^{k_i-1}\left(UP_{ijk_i^{'}}-LO_{ijk_i^{'}}\right)\times w_{ijk_i^{'}}^I]\times p_{ijk_i}^{'I}\times d_i^I$$

Adding the constraints related to the supplier selection, order allocation, lot sizing, discounts and disruptions, the final formulation of the proposed model is as follows:

$$cost^{sc}=\sum_{j=1}^{m}\sqrt{2D_j\left(\sum_{i\in I}a_{ij}x_i+\sum_{s\in S}\sum_{i\in I_s}\delta_s^*\frac{D_jS_{ij}y_{ij}^s}{Cap_i}\right)\left(\sum_{s\in S}\sum_{i\in I_s}\delta_s^* y_{ij}^{s\,2}\left(h_i^b+h_i^v\right)\right)}$$
$$+\sum_{s\in S}\sum_{i\in I_s}\sum_{j=1}^{m}\delta_s^* C_{ij}D_jy_{ij}^s+\sum_{s\in S}\sum_{j\in J}\sum_{i\in I_s}\delta_s^* D_jy_{ij}^s w_{ij}d_i^N \qquad (9)$$
$$+\sum_{s\in S}\sum_{j\in J}\sum_{i\in I_s}\sum_{k_i=1}^{K_I}\delta_s^* D_jy_{ij}^s w_{ijk_i}^A p_{ijk_i}^{'A}d_i^A+\sum_{s\in S}\sum_{j\in J}\sum_{i\in I_s}\sum_{k_i=1}^{K_I}\delta_s^*[\left(D_jy_{ij}^s-UP_{ijk_i-1}\right)\times w_{ijk_i}^I$$
$$+\sum_{k_i^{'}=0}^{k_i-1}\left(UP_{ijk_i^{'}}-LO_{ijk_i^{'}}\right)\times w_{ijk_i^{'}}^I]\times p_{ijk_i}^{'I}\times d_i^I+\sum_{j=1}^{m}\sum_{s\in S}\delta_s^* D_jB_ju_j^s$$

The objective is to minimize the total cost reported in Eq. (9) and the constraints (10) to (18) limit the feasible values for this objective. Constraint set (10) confirms that we can satisfy the demand or consider it as a shortage. Constraint set (11) shows the capacity limitation for the suppliers. Constraints (12) to (14) show the discount domain of general discount for the suppliers. As such, constraints (15) to (17) show the discount domain of incremental discount for the suppliers. Finally, the decision variables are supported in relation (18).

The proposed model is still non-linear. For solving it using an exact solver, we need to linearize it as much as possible. In this regard, the terms of $y_{ij}^s\times p_{ijk_i}^{'I}$ and $y_{ij}^s\times p_{ijk_i}^{'A}$ are replaced by new variables $fi_{ijk_i}^s$ and $fa_{ijk_i}^s$ respectively and new constraints are added to the model. In this regard, we should consider the following equations:

$$fa^{s}_{ijk_i} = y^{s}_{ij} \times p^{'A}_{ijk_i} \qquad fi^{s}_{ijk_i} = y^{s}_{ij} \times p^{'I}_{ijk_i}$$

$$fa^{s}_{ijk_i} \le M * p^{'A}_{ijk_i} \qquad fi^{s}_{ijk_i} \le M * p^{'I}_{ijk_i}$$

$$fa^{s}_{ijk_i} \le y^{s}_{ij} \qquad fi^{s}_{ijk_i} \le y^{s}_{ij} \tag{19}$$

$$fa^{s}_{ijk_i} \ge y^{s}_{ij} - M * \left(1 - p^{'A}_{ijk_i}\right) \qquad fi^{s}_{ijk_i} \ge y^{s}_{ij} - M * \left(1 - p^{'I}_{ijk_i}\right)$$

Regarding Eq. (19), the final linear model is presented as follows:

$$\begin{aligned}
&\sum_{j=1}^{m} \sqrt{2D_j \left(\sum_{i \in I} a_{ij} x_i + \sum_{s \in S}\sum_{i \in I_s} \delta^{*}_{s} \frac{D_j S_{ij} y^{s}_{ij}}{Cap_i}\right)\left(\sum_{s \in S}\sum_{i \in I_s} \delta^{*}_{s} y^{s\,2}_{ij} \left(h^{b}_{i} + h^{v}_{i}\right)\right)} \\
&+ \sum_{s \in S}\sum_{i \in I_s}\sum_{j=1}^{m} \delta^{*}_{s} C_{ij} D_j y^{s}_{ij} + \sum_{s \in S}\sum_{j \in J}\sum_{i \in I_s} \delta^{*}_{s} D_j y^{s}_{ij} w_{ij} d^{N}_{i} \\
&+ \sum_{s \in S}\sum_{j \in J}\sum_{i \in I_s}\sum_{k_i=1}^{K_I} \delta^{*}_{s} D_j fa^{s}_{ijk_i} w^{A}_{ijk_i} d^{A}_{i} + \sum_{s \in S}\sum_{j \in J}\sum_{i \in I_s}\sum_{k_i=1}^{K_I} \delta^{*}_{s} [\left(D_j fi^{s}_{ijk_i} - UP_{ijk_i-1} \times p^{'I}_{ijk_i}\right) \times w^{I}_{ijk_i} \\
&+ \sum_{k'_i=0}^{k_i-1} \left(UP_{ijk'_i} - LO_{ijk'_i}\right) \times w^{I}_{ijk'_i} \times p^{'I}_{ijk_i}] \times d^{I}_{i} + \sum_{j=1}^{m}\sum_{s \in S} \delta^{*}_{s} D_j B_j u^{s}_{j}
\end{aligned} \tag{20}$$

To show the complexity of proposed model, a numerical presentation of developed formulation is provided, here. In the coidered instance, there are two suppliers with one product along with one bound of discount. Accordingly, parameters are valued as follows.

$a_{ij} = [50{,}100]$; $s_{ij} = [300, 250]$; $C_{ij} = [4{,}3]$; $Cap_{ij} = [700{,}1200]$; $h^{b}_{i} = [3{,}4]$; $h^{v}_{i} = [100{,}80]$;

$D_j = [1000]$; $LO_{ijki} = [7{,}7]$; $UP_{ijki} = [10{,}10]$; $w^{A}_{ijki} = [9, M]$; $w^{I}_{ijki} = [M, 8]$; $w_{ij} = [M, M]$;

$d^{N}_{i} = [0, 0]$; $d^{A}_{i} = [1{,}0]$; $d^{I}_{i} = [0{,}1]$; $a_i = [0.1{,}0.1]$; $a^{*} = [0.01]$; $B_j = [1000]$; ${}_{j} = [0.4, 0.6]$;

As such, the following formulation shows the validation of proposed mathematical model.

$$cost^{sc} = \sqrt{\begin{gathered} 2*1000(50*x_1 + 100*x_2 + 0.4*\frac{1000*300*y_{11}^{1}}{700} + \\ 0.4*\frac{1000*250*y_{21}^{1}}{1200} + 0.6*\frac{1000*300*y_{11}^{2}}{700} + 0.6*\frac{1000*250*y_{21}^{2}}{1200})(0.4*y_{11}^{1\,2}\left(3+100\right) + \\ 0.6*y_{11}^{2\,2}\left(3+80\right) + 0.4*y_{21}^{1\,2}\left(4+100\right) + 0.6*y_{21}^{2\,2}\left(4+80\right)) \end{gathered}}$$

$$+0.4*4*1000*y_{11}^{1} + 0.4*3*1000*y_{21}^{1} + 0.6*4*1000*y_{11}^{2} + 0.6*3*1000*y_{21}^{2} +$$

$$0.4*1000*50*9*1*fi_{111}^{1} + 0.4*\left[\left(1000*fi_{211}^{1} - 10*p_{211}^{'I}\right)*8*\left(10-7\right)*p_{211}^{'I}\right]*1 +$$

$$0.6*\left[\left(1000*fi_{211}^{2} - 10*p_{211}^{'I}\right)*8*\left(10-7\right)*p_{211}^{'I}\right]*1 + 0.4*1000*1000*u_1^1 + 0.6*1000*1000*u_1^2$$

s.t

$$y_{11}^{1}+u_{1}^{1}+y_{21}^{1}=1;\ y_{11}^{2}+u_{1}^{2}+y_{21}^{2}=1;$$

$$y_{11}^{1}\le x_{1}*\frac{700}{1000};y_{11}^{2}\le x_{1}*\frac{700}{1000};y_{21}^{1}\le x_{2}*\frac{1200}{1000};y_{21}^{2}\le x_{2}*\frac{1200}{1000};$$

$$p'^{A}_{111}=x_{1}*1;$$

$$y_{11}^{1}*1000\le 10+M\left(1-p'^{A}_{111}*1\right);$$

$$y_{11}^{2}*1000\le 10+M\left(1-p'^{A}_{111}*1\right);$$

$$y_{11}^{1}*1000>7-M\left(1-p'^{A}_{111}*1\right)\quad ;y_{11}^{2}*1000>7-M\left(1-p'^{A}_{111}*1\right)$$

$$fa_{111}^{1}\le M*p'^{A}_{111};\qquad fa_{111}^{2}\le M*p'^{A}_{111};$$

$$fa_{111}^{1}\le y_{11}^{1}\ ;\qquad fa_{111}^{2}\le y_{11}^{2}$$

$$fa_{111}^{1}\ge y_{11}^{1}-M*\left(1-p'^{A}_{111}\right);fa_{111}^{2}\ge y_{11}^{1}-M*\left(1-p'^{A}_{111}\right)$$

$$p'^{I}_{211}=x_{2}*1$$

$$y_{21}^{1}*1000\le 10+M\left(1-p'^{I}_{211}*1\right);$$

$$y_{21}^{2}*1000\le 10+M\left(1-p'^{I}_{211}*1\right);$$

$$y_{21}^{1}*1000>7-M\left(1-p'^{I}_{211}*1\right)\quad ;y_{21}^{2}*1000>7-M\left(1-p'^{I}_{211}*1\right)$$

$$fa_{211}^{1}\le M*p'^{I}_{211};\qquad fa_{111}^{2}\le M*p'^{I}_{211};$$

$$fa_{211}^{1}\le y_{11}^{1}\ ;\qquad fa_{111}^{2}\le y_{11}^{2}$$

$$fa_{211}^{1}\ge y_{11}^{1}-M*\left(1-p'^{I}_{211}\right);fa_{111}^{2}\ge y_{11}^{1}-M*\left(1-p'^{I}_{211}\right)$$

$$x_{1},x_{2},p'^{I}_{211},p'^{A}_{111}\in\{0,1\}\ ,\quad y_{11}^{1},y_{11}^{2},y_{21}^{1},y_{21}^{2},u_{1}^{1}u_{1}^{2}\ge 0$$

3. Proposed solution algorithm

To solve the proposed model in large-scale instances, no exact solver was able to solve it in a reasonable time. The order allocation is an NP-hard problem and the proposed model which is more complex than any order allocation problem is NP-hard, too (Snyder, & Daskin, 2006)[48].

This fact highlights the need for the development of an efficient metaheuristic algorithm for solving the proposed model.

Based on the no-free lunch theory, existing algorithms may not be efficient for solving new NP-hard optimization models [49]. In this regard, we may need to revise, modify, and hybrid the existing metaheuristic algorithms to make them stronger [50,51,39,52,53]. This study proposes a combination of ICA and PSO for solving the proposed model.

Here, we first explain the solution definition and the search space for solving the proposed optimization model. Finally, the proposed hybrid optimization algorithm is studied.

3.1. Solution definition and search space

This study develops a metaheuristic algorithm using a continuous search space. The proposed metaheuristic algorithm at each iteration selects a set of solutions randomly from the search space [54-55]. To show how a random solution is created and how this random solution transforms into an integer solution meeting the constraints of our optimization model [56,57,25], we have defined the solution in our metaheuristic algorithm as follows:

The random-key technique is suitable for transforming a solution from a continuous search space into a feasible solution [48]. The random-key method was used for addressing different integer programming models using metaheuristic algorithms in diverse applications like scheduling [23], supply chains [34,38,58], and transportation and cross-docking centers [35,25].

As mentioned earlier, the proposed model has five variables including two integer ones $\left(y_i^s, u_j^s\right)$ and three binary variables $\left(x_i, p'^{I}_{ijk_i}, p'^{A}_{ijk_i}\right)$. Among them, we can consider x_i and y_i^s as main design variables and other variables can be computed using the constraints. For the selection of suppliers, the solution representation is shown in **Figure 2** where five suppliers (i.e., P_1 to P_5) are existed and we want to select a number of them to satisfy the demand as much as possible. The search space uses continuous variables between zero and one. We round these values and select some of them to get binary values. For example, in this example shown in **Figure 2**, after rounding the numbers, the second supplier, fourth supplier and the fifth supplier have been selected.

Please insert Figure 2 here

Based on the selected suppliers, we now want to define the portion of satisfied demand from these suppliers. Based on the feasible range of these values, we transform the continuous values between zero and one to feasible values using the following formula:

$$y_i^{"} = y_i \times (UP - LO) + LO \qquad (36)$$

Where UP and LO are defined respectively as the upper and lower bound of the number of shipped products for each supplier. **Figure 3** shows the example where UP and LO are respectively 80 and 20. For each scenario, these random values are generated randomly from the search space and then we transform them into feasible values for the selected suppliers.

Please insert Figure 3 here

3.2. Proposed ICA-PSO algorithm

The ICA proposed by Atashpaz-Gargari and Lucas [59] is inspired by the competition of a set of colonies and they are imperialists to get them iteratively using an evolutionary mechanism [1]. After generating a set of random solutions, they are divided into two groups, i.e., colonies and their imperialist [58,60]. We assign the colonies randomly using the roulette wheel selection to the imperialist where a portion of each imperialist is directed to its objective value [58]. The best imperialist will get more colonies in this classification [10]. In the main loop of ICA, the colonies first assimilate to their imperialists. This phase makes small changes in the colonies to do a local search. If a colony gets a better value of the objective function in comparison with its empire, we exchange their positions. Then, we randomly generate new solutions for a number of colonies and call this procedure as the revolution in these colonies. The weakest colonies from the weakest empire are picked up and delivered to the imperialists using the roulette wheel selection. If an empire has no colony, it is deleted and it would be considered as a colony to the best empire. We repeat these steps to satisfy the termination criterion of the algorithm.

Another algorithm that has been used in our hybrid optimization algorithm is the PSO. This algorithm based on swarm intelligence was proposed by Eberhart and Kennedy [61]. The main inspiration for PSO is taken from the social behavior of birds and fishes [62,63]. After generating a set of random solutions, the best solution is considered the global best. In this algorithm, each solution moves to its local best solution and the global best. At each iteration, we update the global best solution if a solution gets a better value in the objective function. These steps are repeated once the maximum number of iterations is terminated.

Based on the benefits of ICA and PSO, we propose a hybrid optimization algorithm for solving the proposed problem. In this regard, the base algorithm is the ICA and the PSO is considered a subloop. The main change in the ICA is to use the procedures of PSO instead of the assimilation phase in the ICA. In this regard, for each empire and its colonies, the global best is the empire and the local best is the colonies. We do these procedures instead of original assimilation in the ICA. To show the details of the implementation of this hybrid algorithm, **Figure 4** shows the pseudo-code of this algorithm.

Please insert Figure 4 here

4. Computational results

Here, we want to analyze the proposed model using the developed hybrid optimization algorithm. We first design the instances to evaluate the proposed model in different complexity levels. Then, the proposed algorithm is tuned to have an unbiased comparison. Then, the proposed model is validated against the exact solver to analyze the optimality gap for our algorithm. An extensive comparison is done consequently for analyzing large-scale instances. Finally, some sensitivity analyses were done on the proposed model. It should be noted that the coding of metaheuristics was written in MATLAB software and the coding of the exact solver was written in GAMS software. All the tests were run in a computer with INTEL Core 2 CPU using 2.4 GHz processor and 2 GB RAM.

4.1. Tests

To design the tests for the proposed model, we have used the benchmarks from Mazdeh et al. [16]. In this regard, as reported in **Table 2**, the tests are divided into three complexity levels from small, medium and large sizes. Totally, 12 tests are provided as reported in **Table 2**. Most importantly, we have defined the maximum time of search for the algorithm. In this regard, for solving a large-scale instance, the maximum time given to an algorithm for finding a solution is 120 seconds.

Please insert Table 2 here

4.2. Tuning

To improve the performance of the proposed algorithm, we need to tune its parameters. Like other metaheuristics, the proposed hybrid algorithm is also sensitive to its input parameters [62]. In this regard, one of the popular methods is the Taguchi method [64]. In this method, we first reduce the number of experiments for tuning the algorithm. Then, we use evaluation metrics for running the selected experiments to find the optimum value for the parameters of our algorithm. To see more information for the Taguchi method, interested readers can read: Pasha et al.; Fard & Hajiaghaei-Keshteli; Pasha et al.; Hajiaghaei-Keshteli & Aminnayeri [50,62,51,57].

In this study, we use two evaluation metrics to tune the proposed hybrid optimization algorithm. They are Signal to Noise $\left(S/N \right)$ and Relative Percentage Deviation (RPD) metrics. In the Taguchi method, we call the input parameters as the factors. Their values are the levels for these factors. To define the $\left(S/N \right)$ metric, we can consider the following formula for the proposed minimization problem:

$$S / N = -10 \log_{10} \left(cost^{sc} \right)^2 \tag{37}$$

where the value of the objective function from the algorithm is called as $cost^{sc}$. For this metric, a higher value brings the optimality of the selected level for the factors of the algorithm.

In a similar way, the RPD metric for a minimization problem is defined as follows:

$$RPD = \frac{Alg_{sol} - Min_{sol}}{Min_{sol}} \tag{38}$$

For the selected experiments, Min_{sol} is the minimum value found by the algorithm and Alg_{sol} is the solution from the metaheuristic algorithm in an experiment. A lower value of RPD is preferable.

To start with the tuning of the proposed hybrid optimization algorithm, the candidate values for the proposed algorithm are reported in **Table 3**.

Please insert Table 3 here

The proposed hybrid algorithm has seven factors with three levels. It means that the total of experiments is 3^7. Since it is too time-consuming to run these experiments, we have used the orthogonal array suggested by the Taguchi method. In this regard, 27 selected experiments using

L_{27}. is used. To show the average value of $\left({S}/{N}\right)$ and RPD metrics for each candidate value of parameters, **Figures 5** and **6** are provided respectively. Based on these optimum values, the tuned values of the parameters are reported in **ble 3**.

Please insert Figure 5 here

Please insert Figure 6 here

4.3. Validation

For solving small-scale instances, it is possible to use the exact solver (EX) by the GAMS software. In this regard, we not only compare the proposed algorithm with the exact solution, but also the solutions from original PSO and ICA for solving the proposed problem. In this regard, we run the proposed algorithm for thirty times. Then, the best (B) and worst (W) solutions are noted. We also report the average of solutions for these thirty run times (OUT). We also compute the standard deviation of these solutions (STD). Another criterion is the average hitting time (HT) representing the time to find the best solution and after that no improvement is done. The last criterion is the optimality gap from the best solution from the proposed metaheuristic algorithm and the optimal solution found by the exact solver. All these criteria are reported in **Table 4**.

Please insert Table 4 here

From **Table 5**, the best values in each criterion and test problem are shown in bold. We can see that in all these tests, the best value obtained by the ICA-PSO is better than PSO and ICA individually. To further analyze these results, we compare the CPU time of the exact solver with the hitting time of our metaheuristics as shown in **Figure 7**. At last but not least, the behavior of algorithms in terms of the optimality gap is depicted in **Figure 8**.

As shown in **Figure 7**, there is a clear difference between the computational time of the exact solver with metaheuristic algorithms. The metaheuristic algorithms are highly quicker than the exact solver. Among them, we can see that the proposed hybrid optimization algorithm is slower than PSO and ICA individually.

What can be seen in **Figure 8** reveals that the proposed hybrid optimization algorithm is stronger than ICA and PSO generally. We can see the proposed algorithm finds the optimal solution in two tests, i.e., P1 and P2. Except for P4, in other small tests, the proposed hybrid algorithm is the best.

Please insert Figure 7 here

Please insert Figure 8 here

4.4. Comparison

To show the high efficiency of the proposed algorithm in comparison with ICA and PSO, we solve medium and large data for the proposed problem. **Table 5** reports the results of solving medium and large instances. The metrics reported in this table for the evaluation of algorithms are the same as the metrics reported in **Table 4**. The best values in each metric are shown in bold. We can see that in most instances except P7 and P8, in other test instances, the proposed hybrid optimization algorithm outperforms the best performance for finding a better solution in comparison with other algorithms. To analyze the algorithms based on the hitting time, **Figure 9** shows the comparison of algorithms based on this metric. Finally, for analyzing the algorithms statistically, we have applied the RPD metric for the standard deviation of algorithms for analyzing the accuracy of the algorithms. Hence, the interval plot based on a 95% confidence level using the analysis of variance is shown in **Figure 10**. Generally, the best algorithm in this study shows the best performance in comparison with both PSO and ICA.

Please insert Table 5 here

Please insert Figure 9 here

Please insert Figure 10 here

4.5. Sensitivity analysis

To do the sensitivity analyses, we have focused on three factors including the fixed cost of ordering $\left(a_{ij}\right)$ and the operation cost of ordering $\left(s_{ij}\right)$ which have a high impact on the total cost. For each analysis, we have regenerated a test problem like P5 and solved it by the exact solver. The results of these sensitivity analyses are reported in **Table 6** and **Table 7**.

The first sensitivity analysis is performed on the fixed cost of ordering where some changes are done to the values of this parameter randomly. We have considered three cases numbered C1 to C3. Then, the values for the objective function and the CPU time are reported in **Table 6**. An increase to this parameter leads to an increase in the total cost. It should be noted that these changes do have not a high impact on the complexity of solving as the CPU time has a few variations.

Please insert Table 6 here

Another parameter is the rates of variable cost or operating cost of orders from suppliers. As reported in **Table 7**, we have done three sensitivity analyses numbered W1, W2, and W3. The changes in the total cost and CPU time are studied. Generally, as the variable cost has been increased, there is no significant variations for the total cost in comparison with the analyses reported for the fixed cost in **Table 7**. The last finding from **Table 7** is that an increase in the variable cost can reduce the complexity of solving as the CPU time is decreased generally.

Please insert Table 7 here

5. Conclusions, managerial insights and future research

In this paper, the EOQ model is combined with the integration of supplier selection and order allocation where lot sizing, discounts, and disruptions are contributed among the first studies in this research area. The proposed model was formulated by scenario-based stochastic programming where local and global disruptions were contributed to the model. For solving the proposed model, not only the exact solver was employed but also an innovative algorithm based on a hybrid algorithm using the PSO and the ICA was utilized. To enhance the performance of our metaheuristic algorithm, the Taguchi experimental design method was employed. Some sensitivity analyses on the key parameters of our optimization model focusing on the fixed cost and operating costs were done. The proposed model was successful in addressing lot sizing, discounts, and disruptions to supplier selection and order allocation. Based on extensive comparison of the proposed hybrid algorithm against the exact solver, ICA and PSO individually, the proposed hybrid algorithm was reliable for solving small-scale instances and it is highly efficient for solving large-scale tests.

Based on all these results and analyses, the following managerial insights can be concluded. The first one is to shift the traditional supplier selection and order allocation problem to a modern one considering lot sizing, discounts, and disruptions using a scenario-based stochastic programming model. The second managerial insight is to recommend the developed algorithm, i.e., ICA-PSO for analyzing very large-scale instances efficiently. Other managerial insights can be referred to in our sensitivity analyses where the fixed cost of order plays a key role in the financial issues. Hence, the practitioners of supplier selection and order allocation decisions should pay more attention to the fixed cost of ordering instead of variable cost. It is also recommended to select suppliers with low disruption risk to improve the reliability of supply chain contracts.

Although this study examined a significant contribution to merging the supplier selection and order allocation considering lot sizing, discounts, and disruptions, there were some limitations to our model and solution algorithms which can be studied in our future works. First of all, we can use real-time optimization or online optimization for addressing the uncertainty and disruptions in supplier selection and order allocation decisions. Last but not least, the proposed model may need to be reformulated by Benders decomposition or Lagrangian relaxation theories. Finally, new heuristics and metaheuristics can be applied to the proposed model in comparison with the presented results in this paper.

References:

1. Golmohamadi, S., Tavakkoli-Moghaddam, R., & Hajiaghaei-Keshteli, M. “Solving a fuzzy fixed charge solid transportation problem using batch transferring by new approaches in meta-heuristic”, *Electronic Notes in Discrete Mathematics*, **58**, pp. 143-150 (2017).
2. Fallahpour, A., Olugu, E. U., Musa, S. N., Khezrimotlagh, D., et al. “An integrated model for green supplier selection under fuzzy environment: application of data envelopment analysis and genetic programming approach”, *Neural Computing and Applications*, **27**(3), pp. 707-725 (2016).
3. Nourmohamadi Shalke, P., Paydar, M. M., & Hajiaghaei-Keshteli, M. “Sustainable supplier selection and order allocation through quantity discounts”, *International Journal of Management Science and Engineering Management*, **13**(1), pp. 20-32 (2018).
4. Safaeian, M., Fathollahi-Fard, A. M., Tian, G., et al. “A multi-objective supplier selection and order allocation through incremental discount in a fuzzy environment”, *Journal of Intelligent & Fuzzy Systems*, **37**(1), pp. 1435-1455 (2019).
5. Beiki, H., Mohammad Seyedhosseini, S., V. Ponkratov, V., et al. “Addressing a sustainable supplier selection and order allocation problem by an integrated approach: a case of automobile manufacturing”, *Journal of Industrial and Production Engineering*, **38**(4), pp. 239-253 (2021).
6. Hajiaghaei-Keshteli, M., & Fathollahi Fard, A. M. “Sustainable closed-loop supply chain network design with discount supposition”, *Neural Computing and Applications*, **31**(9), pp. 5343-5377 (2019).
7. Nezhadroshan, A. M., Fathollahi-Fard, A. M., & Hajiaghaei-Keshteli, M. “A scenario-based possibilistic-stochastic programming approach to address resilient humanitarian logistics

considering travel time and resilience levels of facilities", *International Journal of Systems Science: Operations & Logistics*, **8**(4), pp. 321-347 (2021).

8. Tian, G., Fathollahi-Fard, A. M., Ren, Y., et al. "Multi-objective scheduling of priority-based rescue vehicles to extinguish forest fires using a multi-objective discrete gravitational search algorithm", *Information Sciences*, **608**, pp. 578-596 (2022).
9. Galindo, G. & Batta, R. "Review of recent developments in OR/MS research in disaster operations management", *European Journal of Operational Research*, **230**, pp. 201-211 (2013).
10. Fard, A. M. F., Gholian-Jouybari, F., Paydar, M. M., et al. "A Bi-Objective Stochastic Closed-loop Supply Chain Network Design Problem Considering Downside Risk", *Industrial Engineering and Management Systems*, **16**(3), pp. 342-362 (2017).
11. Fard, A. M. F., & Hajaghaei-Keshteli, M. "A tri-level location-allocation model for forward/reverse supply chain", *Applied Soft Computing*, **62**, pp. 328-346 (2018).
12. Ali, S. M., Paul, S. K., Chowdhury, P., et al. "Modelling of supply chain disruption analytics using an integrated approach: An emerging economy example", *Expert Systems with Applications*, **173**, pp. 114690 (2021).
13. Akarte, G.-H., Chaing, C.-H. & LI, C.-W. "Evaluating intertwined effects in e-learning programs: A novel hybrid MCDM model based on factor analysis and DEMATEL", *Expert systems with Applications,* **32,** pp. 1028-1044 (2007).
14. Chang, B., Chang, C.-W. & WU, C.-H. "Fuzzy DEMATEL method for developing supplier selection criteria", *Expert Systems with Applications,* **38,** pp. 1850-1858 (2012).
15. Dweiri A., Lewandski, L., & Apte, A. "Stochastic optimization for natural disaster asset prepositioning", *Production and Operations Management,* **19,** pp. 561-574 (2015).
16. Mazdeh, M. M., Emadikhiav, M., & Parsa, I. "A heuristic to solve the dynamic lot sizing problem with supplier selection and quantity discounts", *Computers & Industrial Engineering*, **85**, pp. 33-43 (2015).
17. Jahre, M., Persson, G., Kovacs, G. et al. "Humanitarian logistics in disaster relief operations", *International Journal of Physical Distribution & Logistics Management*, **37,** pp. 99-114 (2007).
18. Cheraghalipour, A., & Farsad, S. "A bi-objective sustainable supplier selection and order allocation considering quantity discounts under disruption risks: A case study in plastic industry", *Computers & Industrial Engineering*, **118**, pp. 237-250 (2018).
19. Caunhye, A. M., Nie, X. & Pokheral, S. "Optimization models in emergency logistics: A literature review", *Socio-Economic Planning Sciences,* **46**, pp. 4-13 (2012).

20. Bozorgi-Amiri, A., Jabalameli, M. & Al-e-hashem, S. M. "A multi-objective robust stochastic programming model for disaster relief logistics under uncertainty", *OR spectrum,* **35**, pp. 905-933 (2013).
21. Özdamar, L. & Ertem, M. A. "Models, solutions and enabling technologies in humanitarian logistics", *European Journal of Operational Research,* **244**, pp. 55-65 (2015).
22. Ha, D. E., Murray, A. T. & Li, T. C. "Decision support for network disruption mitigation", *Decision Support Systems,* **44**, pp. 954-969 (2008).
23. Golshahi-Roudbaneh, A., Hajiaghaei-Keshteli, M., & Paydar, M. M. "Developing a lower bound and strong heuristics for a truck scheduling problem in a cross-docking center", *Knowledge-Based Systems*, **129**, pp. 17-38 (2017).
24. Samadi, A., Mehranfar, N., Fathollahi Fard, A. M., et al. "Heuristic-based metaheuristics to address a sustainable supply chain network design problem", *Journal of Industrial and Production Engineering,* **35**(2), pp.102-117 (2018).
25. Sadeghi-Moghaddam, S., Hajiaghaei-Keshteli, M., & Mahmoodjanloo, M. "New approaches in metaheuristics to solve the fixed charge transportation problem in a fuzzy environment" *Neural computing and applications*, **31**(1), pp. 477-497 (2019).
26. Yu, H., Dai, H., Tian, G., et al. "Key technology and application analysis of quick coding for recovery of retired energy vehicle battery", *Renewable and Sustainable Energy Reviews*, **135**, pp. 110129 (2021).
27. Fallahpour, A., Nayeri, S., Sheikhalishahi, M., et al. "A hyper-hybrid fuzzy decision-making framework for the sustainable-resilient supplier selection problem: a case study of Malaysian Palm oil industry", *Environmental Science and Pollution Research*, pp. 1-21 (2021).
28. Zhang, C., Tian, G., Fathollahi-Fard, A. M., et al. "Interval-valued intuitionistic uncertain linguistic cloud petri net and its application to risk assessment for subway fire accident", *IEEE transactions on automation science and engineering*, (2020).
29. Chan, F. T., Kumar, N., Tiwari, M. K., et al. "Global supplier selection: a fuzzy-AHP approach." *International Journal of production research*, *46*(14), 3825-3857, (2008).
30. Arabsheybani, A., Paydar, M. M., & Safaei, A. S. "An integrated fuzzy MOORA method and FMEA technique for sustainable supplier selection considering quantity discounts and supplier's risk", *Journal of cleaner production*, **190**, pp. 577-591 (2018).
31. Jaśkowski, P., Sobotka, A., & Czarnigowska, A. "Decision model for planning material supply channels in construction", *Automation in Construction*, **90**, pp. 235-242 (2018).
32. Feng, Y., Zhang, Z., Tian, G., et al. "A novel hybrid fuzzy grey TOPSIS method: supplier evaluation of a collaborative manufacturing enterprise", *Applied Sciences*, **9**(18), pp. 3770 (2019).

33. Liu, X., Tian, G., Fathollahi-Fard, A. M., et al. “Evaluation of ship’s green degree using a novel hybrid approach combining group fuzzy entropy and cloud technique for the order of preference by similarity to the ideal solution theory”, *Clean Technologies and Environmental Policy*, **22**(2), pp. 493-512 (2020).
34. Fathollahi-Fard, A. M., Hajiaghaei-Keshteli, M., Tian, G., et al. “An adaptive Lagrangian relaxation-based algorithm for a coordinated water supply and wastewater collection network design problem”, *Information Sciences*, **512**, pp. 1335-1359 (2020).
35. Karampour, M. M., Hajiaghaei-Keshteli, M., Fathollahi-Fard, A. M., et al. “Metaheuristics for a bi-objective green vendor managed inventory problem in a two-echelon supply chain network”, *Scientia Iranica*, **29**(2), pp. 816-837 (2022).
36. Mojtahedi, M., Fathollahi-Fard, A. M., Tavakkoli-Moghaddam, R., et al. “Sustainable vehicle routing problem for coordinated solid waste management”, *Journal of Industrial Information Integration*, **23**, pp. 100220 (2021).
37. Fallahpour, A., Wong, K. Y., Rajoo, S., et al. “An integrated approach for a sustainable supplier selection based on Industry 4.0 concept”, *Environmental science and pollution research*, pp. 1-19 (2021b).
38. Fathollahi-Fard, A. M., Ahmadi, A., & Mirzapour Al-e-Hashem, S. M. J. “Sustainable Closed-loop Supply Chain Network for an Integrated Water Supply and Wastewater Collection System under Uncertainty”, *Journal of Environmental Management*, **275**, pp. 111277 (2020b).
39. Moosavi, J., Naeni, L. M., Fathollahi-Fard, A. M., et al. “Blockchain in supply chain management: a review, bibliometric, and network analysis”, *Environmental Science and Pollution Research*, pp. 1-15 (2021).
40. Önüt, S., Kara, S. S., & Işik, E. “Long term supplier selection using a combined fuzzy MCDM approach: A case study for a telecommunication company”, *Expert systems with applications*, **36**(2), pp. 3887-3895 (2009).
41. Şenyiğit, E., Düğenci, M., Aydin, M. E., et al. “Heuristic-based neural networks for stochastic dynamic lot sizing problem”, *Applied Soft Computing*, **13**(3), pp. 1332-1339 (2013).
42. Liao, W., Lin, J. X., & Leonard, W. J. “Interleukin-2 at the crossroads of effector responses, tolerance, and immunotherapy”, *Immunity*, **38**(1), pp. 13-25 (2013).
43. Eslamipoor, R., & Sepehriar, A. “Firm relocation as a potential solution for environment improvement using a SWOT-AHP hybrid method”, *Process safety and environmental protection*, **92**(3), pp. 269-276 (2014).

44. Gitinavard, H., Makui, A., & Jabbarzadeh, A. “Interval-valued hesitant fuzzy method based on group decision analysis for estimating weights of decision makers”, *Journal of Industrial and Systems Engineering*, **9**(3), pp. 96-110 (2016).

45. Bohner, C., & Minner, S. “Supplier selection under failure risk, quantity and business volume discounts”, *Computers & Industrial Engineering*, **104**, pp. 145-155 (2017).

46. Hamdan, S., & Cheaitou, A. “Supplier selection and order allocation with green criteria: An MCDM and multi-objective optimization approach”, *Computers & Operations Research*, **81**, pp. 282-304 (2017).

47. Venegas, B. B., & Ventura, J. A. “A two-stage supply chain coordination mechanism considering price sensitive demand and quantity discounts”, *European Journal of Operational Research*, **264**(2), pp. 524-533 (2018).

48. Snyder, L. V. & Daskin, M. S. “A random-key genetic algorithm for the generalized traveling salesman problem”, *European Journal of Operational Research*, **174**, pp. 38-53 (2006).

49. Wolpert, D. H., & Macready, W. G. “No free lunch theorems for optimization”, *IEEE transactions on evolutionary computation*, **1**(1), pp. 67-82 (1997).

50. Pasha, J., Dulebenets, M. A., Fathollahi-Fard, A. M., et al. “An integrated optimization method for tactical-level planning in liner shipping with heterogeneous ship fleet and environmental considerations”, *Advanced Engineering Informatics*, **48**, pp. 101299 (2021).

51. Pasha, J., Nwodu, A. L., Fathollahi-Fard, A. M., et al. “Exact and metaheuristic algorithms for the vehicle routing problem with a factory-in-a-box in multi-objective settings”, *Advanced Engineering Informatics*, **52**, pp. 101623 (2022).

52. Fathollahi-Fard, A. M., Hajiaghaei-Keshteli, M., & Tavakkoli-Moghaddam, R. “Red deer algorithm (RDA): a new nature-inspired meta-heuristic”, *Soft Computing*, **24**(19), pp. 14637-14665 (2020).

53. Fathollahi-Fard, A. M., Hajiaghaei-Keshteli, M., & Mirjalili, S. “A set of efficient heuristics for a home healthcare problem”, *Neural Computing and Applications*, **32**(10), pp. 6185-6205 (2020).

54. Wang, W., Zhou, X., Tian, G., et al. “Multi-objective low-carbon hybrid flow shop scheduling via an improved teaching-learning-based optimization algorithm”, *Scientia Iranica*, (2022).

55. Fathollahi-Fard, A. M., Niaz Azari, M., & Hajiaghaei-Keshteli, M. “An improved red deer algorithm for addressing a direct current brushless motor design problem”, *Scientia Iranica*, **28**(3), pp. 1750-1764 (2021).

56. Hajiaghaei-Keshteli, M., & Aminnayeri, M. “Keshtel Algorithm (KA); a new optimization algorithm inspired by Keshtels’ feeding”, In *Proceeding in IEEE Conference on Industrial Engineering and Management Systems*, pp. 2249-2253 (2013).

57. Hajiaghaei-Keshteli, M., Aminnayeri, M., & Ghomi, S. F. "Integrated scheduling of production and rail transportation", *Computers & Industrial Engineering*, **74**, 240-256 (2014).
58. Devika, K., Jafarian, A., & Kaviani, A. "Sustainable closed-loop supply chain network design: Hybrid metaheuristic algorithms based on triple line theories", *European Journal of Operational Research*, **25**, 243-257 (2014).
59. Atashpaz-Gargari, E., & Lucas, C. "Imperialist competitive algorithm: an algorithm for optimization inspired by imperialistic competition", In *Evolutionary computation, 2007. CEC 2007. IEEE Congress on*, pp. 4661-4667 (2007).
60. Molla-Alizadeh-Zavardehi, S., Tavakkoli-Moghaddam, R., & Lotfi, F. H. "A modified imperialist competitive algorithm for scheduling single batch-processing machine with fuzzy due date", *The International Journal of Advanced Manufacturing Technology*, **85**(9-12), pp. 2439-2458 (2016).
61. Kennedy, J., & Eberhart, R. "Particle swarm optimization", In *Proceedings of ICNN'95-international conference on neural networks*, pp. 1942-1948 (1995).
62. Fathollahi-Fard, A. M., Hajiaghaei-Keshteli, M., & Tavakkoli-Moghaddam, R. "The social engineering optimizer (SEO)", *Engineering Applications of Artificial Intelligence*, **72**, 267-293 (2018).
63. Tavakkoli-Moghaddam, R., Hajiaghaei-Keshteli, M., Mousavi, M., et al. "Two meta-heuristics to solve a coordinated air transportation and production scheduling problem with time windows for the due date", In *Systems, Man, and Cybernetics (SMC), 2016 IEEE International Conference on*, pp. 004663-004668 (2016).
64. Taguchi, G., & Jugulum, R. "The Mahalanobis-Taguchi strategy: A pattern technology system", *John Wiley & Sons*, (2002).

List of Figures

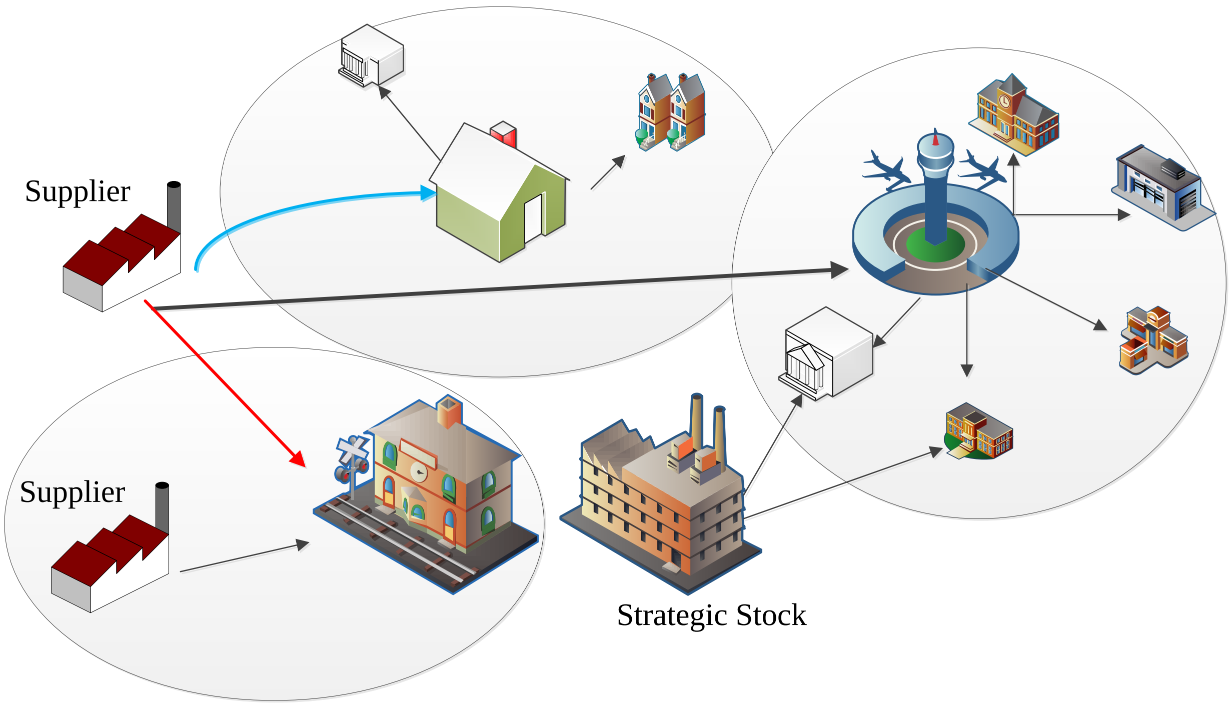

Figure 1. Overall structure of supplier selection and order allocation problem [7]

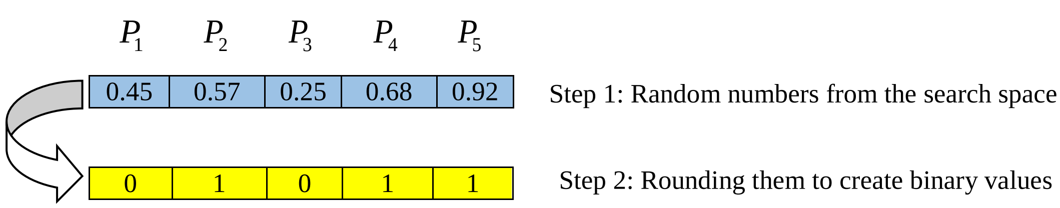

Figure 2. The random-key method for selecting the suppliers

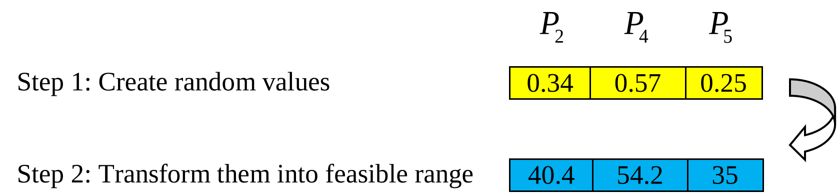

Figure 3. Proposed random key for the portion of shipped products for each selected supplier

```
Create a set of random solutions using the random-key method.
Divide these solutions into two groups, i.e., colonies and empires.
X* is the best solution which is one of empires.
it=1;  %Counter of iterations
Maxit;  %Maximum number of iterations
while (it<=Maxit)
      for each empire
        for each colony
          v= w*v+c1*rand*(X*-p)+c2*rand*(the empire-p);
          Update the objective function.
        endfor
        Do a revolution.
        if we can update the empire with a colony
          Exchange this colony with its imperialist.
        endif
        Pick the weakest colonies from the weakest empire.
        Assign them to the best empires.
      endfor
      if there is an imperialist which has no colony
        Remove imperialist.
      endif
      Update the X*
      w=w*α;
      it=it+1;
end while
return X*
```
Figure 4. Pseudo-code of the proposed hybrid optimization algorithm

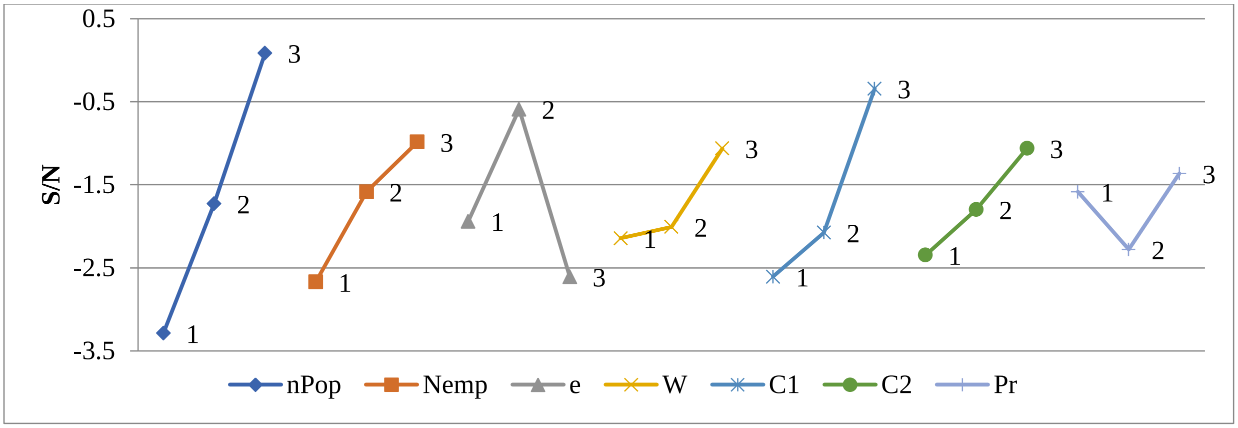

Figure 5. Average value of S/N metric for the proposed hybrid optimization algorithm

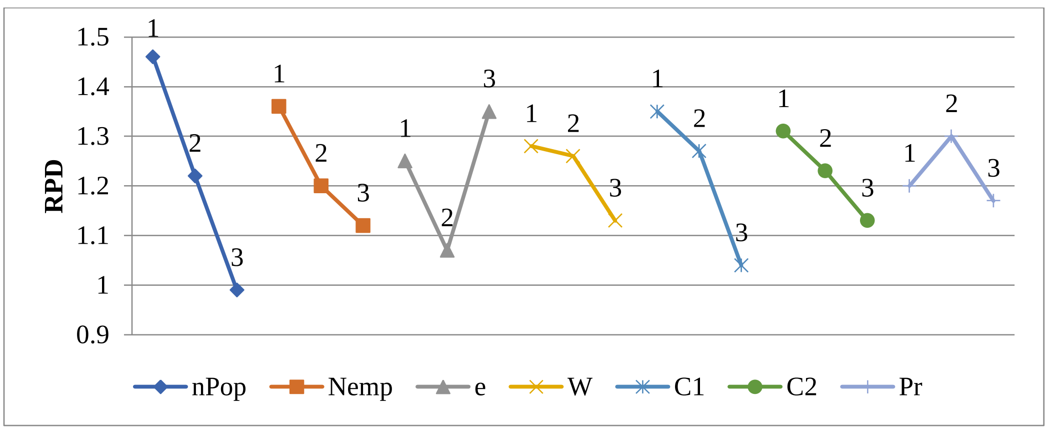

Figure 6. Average value of RPD metric for the proposed hybrid optimization algorithm

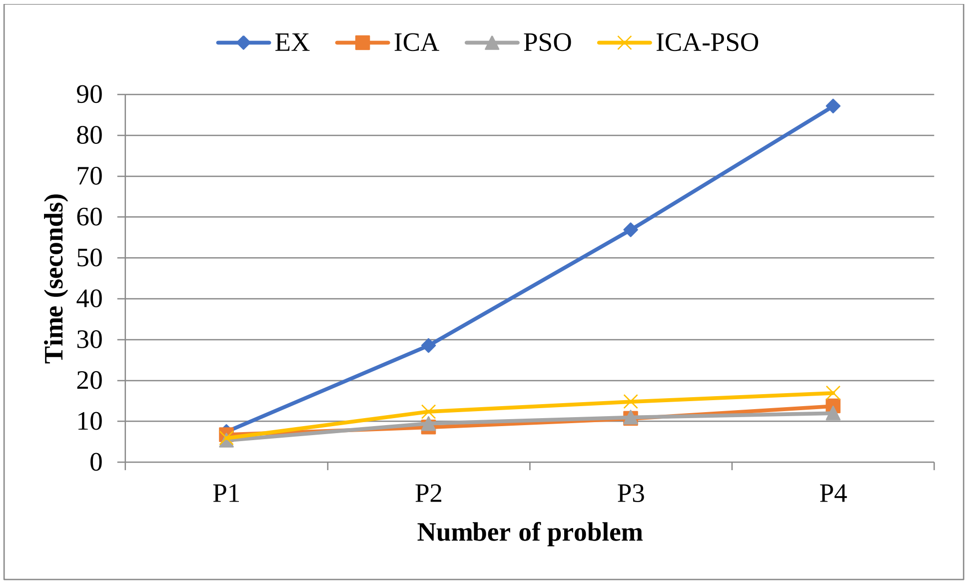

Figure 7. Comparison of computational time of the exact solver with the hitting time of algorithms

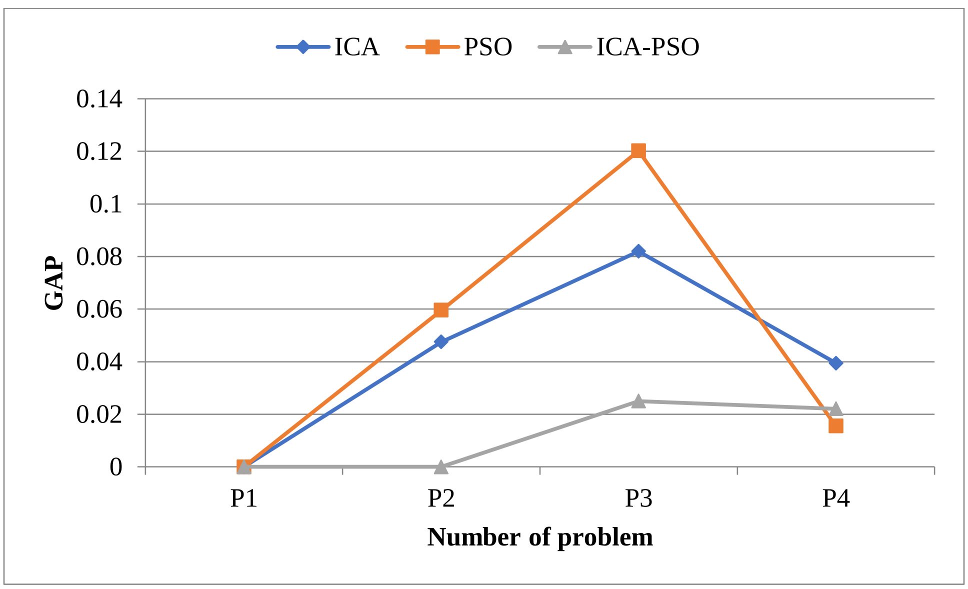

Figure 8. Optimality gap for the metaheuristic algorithms

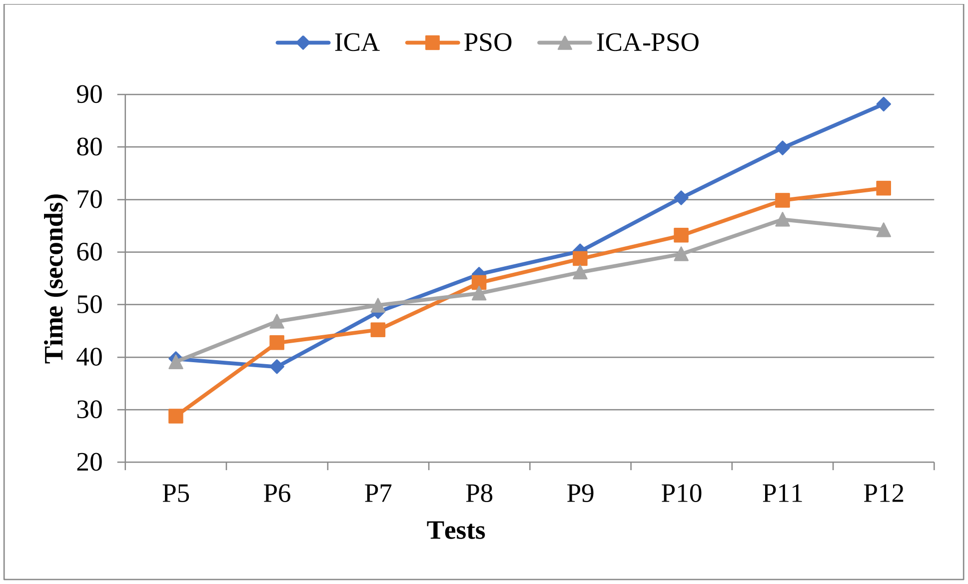

Figure 9. Comparison of algorithms in terms of hitting time

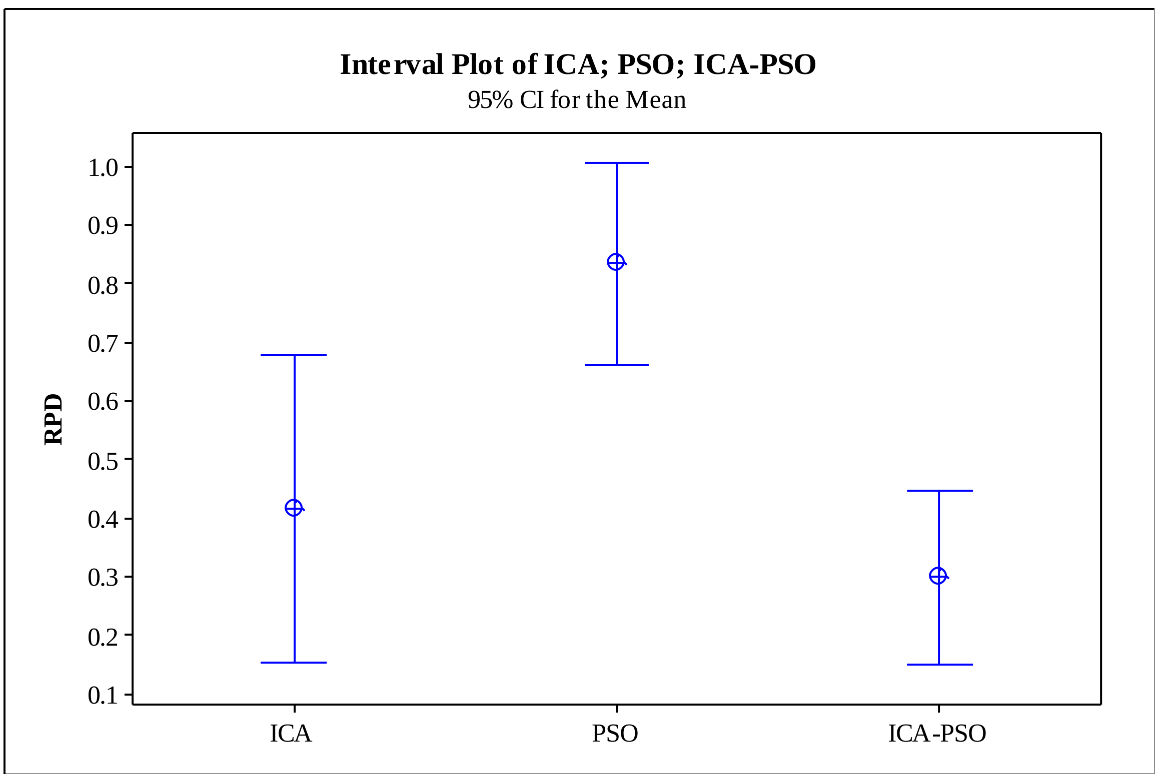

Figure 10. Interval plot with 95% confidence level for the algorithms

List of Tables

Table 1. Relevant studies for the supplier selection and order allocation studies

References	Supply chain types			Industry		Discounts		Lot sizing	Disruptions	Solution approach		
	General	Green	Sustainable	General	Special	Quantity	Exponential			MCDM	Exact	Metaheuristic (or Heuristics)
[22]	√	-	-	-	Automatic transmission park	-	-	-	-	AHP	-	-
[40]	√	-	-	-	Telecommunication Industry	-	-	√	-	Fuzzy ANP, Fuzzy TOPSIS		
[41]	√	-	-	-	Furniture Industry	-	-	-	-	-	√	√
[42]	√	-	-	-	Thin-film transistor-liquid-crystal display	-	√	-	-	Fuzzy inference based on quality index	-	-
[43]	√	-	-	√	-	√	-	-	-	Fuzzy ELECTRE	√	-
[44]	√	-	-	-	Building factory side	-	-	√	-	interval-valued hesitant fuzzy sets (IVHF-MCWR); TOPSIS	√	-
[16]	√	-	-	√	-	√	-	√	-	-	√	√
[2]	-	√	-	-	Garment industry	√	-	-	-	DEA approach	√	√
[45]	√	-	-	√	-	√	-	√	-	-	√	-
[46]	√	-	-	√	-	√	-	-	√	-	√	-
[3]	-	-	√	-	Protein materials packing industry	√	-	-	-	TOPSIS	-	-
[18]	-	-	√	-	Plastic industry	√	√	√	√	A hybrid MCDC-MILP approach	-	-
[47]	√	-	-	√	-	√	√	√	√	-	√	-
[30]	-	-	√	√	-	√	-	-	-	Fuzzy MOORA	-	-
[4]	√	-	-	√	-	-	√	-	-	-	-	√
[32]	√	-	-	-	Production system in China	-	-	-	-	Fuzzy grey TOPSIS	-	-

[33]	√	-	-	-	Maritime ships	-	-	-	-	Fuzzy entropy and cloud TOPSIS	-	-
[7]	√	-	-	√	-	-	-	√	√	AHP and DEMATEL	-	-
[34]	√	-	-	-	Water network	-	-	-	√	-	-	√
[35]	-	√	-	√	-	-	-	√	-	-	-	√
[38]	-	-	√	-	Water network	-	-	-	√	-	-	√
[27]	-	-	√	-	Palm oil industry	-	-	-	√	Hyper heuristic Fuzzy inference system	-	-
[37]	-	-	√	-	Textile industry	-	-	-	-	Fuzzy best-worst method and two-stage fuzzy inference system	-	-
[12]	√	-	-	√	-	-	-	-	√	Fuzzy AHP and Delphi method	-	-
[36]	-	-	√	-	Municipal waste	-	-	-	-	-	√	√
[5]	-	-	√	√	Car manufacturing	-	-	-	-	-	√	-
This study	√	-	-	√	-	√	-	√	√	-	√	√

Table 2. Size of tests

Levels of complexity	Tests	Size (i, j, s, k_i)	Time (Seconds)
Small	P1	(3, 5, 5, 3)	10
	P2	(5, 8, 6, 4)	15
	P3	(6, 10, 8, 4)	20
	P4	(8, 10, 10, 6)	30
Medium	P5	(12, 12, 10, 6)	50
	P6	(14, 12, 10, 6)	60
	P7	(18, 14, 14, 8)	70
	P8	(20, 16, 14, 8)	75
Large	P9	(22, 16, 18, 8)	90
	P10	(24, 16, 18, 10)	100
	P11	(24, 18, 20, 10)	110
	P12	(26, 20, 20, 12)	120

Table 3. Candidate values for the proposed hybrid optimization algorithm and its tuned values.

Factor	Level			Best level
	1	2	3	
A: nPop=number of countries	100	150	200	200
B: Nemp=number of empires	8	10	14	14
C: e=colonies mean cost coefficient	0.05	0.07	0.1	0.07
D: W=inertia weight of particle	0.75	0.85	0.95	0.95
E: C1=acceleration coefficient of local optimum	1.5	2	2.15	2.15
F: C2=acceleration coefficient of global optimum	1.5	2	2.15	2.15
G: Pr=the rate of revolution	0.05	0.1	0.15	0.15

Table 4. Results of metaheuristics in small-scale instances

Algorithm		P1	P2	P3	P4
EX	OUT	1284	1326	1439	1673
	CPU	7.46	28.54	56.88	87.14
ICA	B	1284	1389	1557	1739
	W	1318	1449	1720	1855
	OUT	1300	1411	1628.5	1792
	STD	145	157	178	204
	HT	6.74	8.58	10.68	13.74
	GAP	**0**	0.047511	0.082001	0.03945
PSO	B	1284	1405	1612	1699
	W	1309	1512	1728	1753
	OUT	1295	1455	1660	1716
	STD	**133**	**152**	173	182
	HT	5.37	9.48	10.99	12.01
	GAP	**0**	0.059578	0.120222	0.015541
ICA-PSO	B	1284	1326	1475	1710
	W	1305	1428	1558	1833
	OUT	1294	1370	1515	1771
	STD	**133**	158	**166**	**179**
	HT	5.89	12.37	14.83	16.95
	GAP	**0**	0	0.025017	0.022116

Table 5. Results of algorithms for medium and large instances

Algorithms		P5	P6	P7	P8	P9	P10	P11	P12
ICA	B	1802	1987	**2146**	**2466**	2788	3187	3291	3455
	W	2087	**2057**	**2444**	**2609**	2906	3435	**3344**	3732
	OUT	1944	2022	2295	2537	2847	3311	3317	3593
	STD	193	**200**	211	245	261	290	322	**251**
	HT	39.68	38.16	48.63	55.78	60.19	70.33	79.82	88.15
PSO	B	1833	2042	2317	2591	2688	2903	3215	3466
	W	2026	2272	2518	2878	2974	3028	3400	3711
	OUT	1925	2157	2415	2734	2831	2965	3307	3588
	STD	199	224	257	288	299	299	337	328
	HT	28.76	42.73	45.19	54.18	58.72	63.19	69.85	72.14
ICA-PSO	B	**1785**	**1958**	2284	2476	**2571**	**2816**	**3105**	**3366**
	W	**1875**	2232	2555	2613	**2692**	**2893**	3405	**3662**
	OUT	1830	2095	2419	2544	2631	2854	3255	3514
	STD	**189**	204	**207**	**235**	**257**	**288**	**318**	**289**
	HT	39.12	46.82	49.88	52.17	56.18	59.64	66.23	64.23

Table 6. Sensitivity analysis of the fixed cost of ordering

Number of cases	a_{ij}	$cost^{sc}$	CPU time
C1	[57, 74]	3885	208.65
C2	[68, 85]	5002	178.2219
C3	[72, 95]	5645	191.2625

Table 7. Sensitivity analysis of the operating cost of orders

Number of cases	s_{ij}	$cost^{sc}$	CPU time
W1	$[1, 10]$	3885	208.65
W2	$[5, 15]$	3767	199.9563
W3	$[10, 15]$	40751	183.1378

Biographies:

Roohollah Karimi was born and raised in Mazandaran Province. He is a Ph.D. Candidate in Industrial Engineering at Imam Hossein University, Tehran, Iran. His main fields of research are supply chain management, supplier selection, order allocation, uncertainty modeling, and disruptions management.

Masood Mosadegh-Khah was born and raised in Tehran, Iran. He is now an Associate Professor at the faculty of technical and Engineering at Imam Hossein University, Tehran, Iran. His main fields of research are strategic management, supply chain management, operations research, optimization, and disruptions management.

Saeed Ramezani was born and raised in Mashhad, Iran. He is now an Assistant Professor at the faculty of technical and Engineering at Imam Hossein University, Tehran, Iran. His main fields of research are strategic management, supply chain management, and disruptions management.